Data Structure

Sorting

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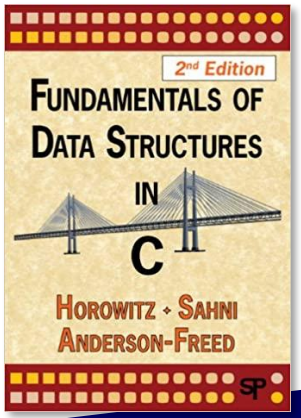
Ch. 7 Sorting



Motivation – Search

- checking whether or not an item exists in a list of *n* items
  - sequential search:
    check each item by iterating over the list
  - binary search:
    assume that the list is sorted ;
    compare the middle point of the list with the target item, and continue the binary search on a half of the list where the item may be found

Sorting Problem

- Given a list of n items $(a_1, a_2, \cdots, a_n)$, find a permutation σ $(a_{\sigma 1}, a_{\sigma 2}, ..., a_{\sigma n})$ s.t. $key(a_{\sigma i}) \leq key(a_{\sigma\, i+1})$ for $1 \leq i < n$
  - assume that the equivalance and the ordering in items are well defined
- A sorting is stable if the permutation satisfies the following condition:

  if $key(a_{\sigma i}) = key(a_{\sigma j})$ and $i < j$, $\sigma_i < \sigma_j$

Insetion Sort - Idea

- Given an unsorted list *U* of items, repeat the following two steps until the list becomes empty:
  (1) remove a minimum/maximum item in the *U*
  (2) insert the removed item to the sorted list

- We can use a prefix *U*[0 .. *i*] as a sorted list for *i*-th turn to make sorting happen within the given list

Insetion Sort - Algorithm

- Algorithm

  Input: a given a list of N items, $L[0 .. N\text{-}1]$

  Output: a sorted list of item L

  Procedure

  for i in $[0 .. N\text{-}2]$

  find j such that L[j] is minimum among $L[i .. N\text{-}1]$

  swap $L[i]$ and $L[j]$

  end for

- Example

(7, b)	(3, d)	(2, e)	(9, a)	(5, g)	(3, a)	(2, h)

Insertion Sort - Analysis

- Algorithm

  Input: a given a list of N items, $L[0 .. N\text{-}1]$

  Output: a sorted list of item L

  Procedure

  for i in $[0 .. N\text{-}2]$

  find j such that L[j] is minimum among $L[i .. N\text{-}1]$

  swap $L[i]$ and $L[j]$

  end for

- Time complexity

$$O\left(\sum_{i=0}^{N-2}(i+1)\right) = O(n^2)$$

Quick Sort

- Given a list of items $L[0 .. N-1]$,

  reorder L such that all elements in $L[0 .. p]$ are less than equal to all elements in $L[p+1 .. N-1]$, and
  sort two sublists $L[0.. p]$ and $L[p+1 .. N-1]$ independently

- Algorithm

```
Sort (L[0 .. N-1], left, right)
    if left == right then return
    p = left  ; l = left +1 ; r = right
    while l < r begin
        while L[l] < L[p] /\ l ≤ right  begin  l++ end while
        while L[p] < L[r] /\ left ≤ r   begin  r-- end while
        if  l < r  then swap L[l] and L[r]  ; l++ ; r--
    end while
    swap L[p] and L[r]
    Sort (L, left, r – 1)
    Sort (L, r, right)
```

Quick Sort - Example

5	3	1	9	7	3	2	2

Quick Sort - Analysis


```
Sort (L[0 .. N-1], left, right)
  if left == right then return
  p = left  ; l = left +1 ; r = right
  while l < r begin
     while L[l] < L[p] /\ l ≤ right begin
        l++ end while
     while L[p] < L[r] /\ left ≤ r begin
        r--  end while
     if   l < r  then
        swap L[l] and L[r]
        l++ ; r--
     end if
   end while
   swap L[p] and L[r]
   Sort (L, left, r – 1)
   Sort (L, r, right)
```

$T(N) \leq cN + 2T(N/2)$