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Brief Paper

Adaptive control schemes for mobile robot formations with triangularised structures

J. Guo[1] Z. Lin[1] M. Cao[2] G. Yan[1]

[1]Department of Systems Science and Engineering, Zhejiang University, 38 Zheda Road, Hangzhou 310027, People's Republic of China
[2]Faculty of Mathematics and Natural Sciences, University of Groningen, Nijenborgh 4,9747 AG, Groningen, The Netherlands
E-mail: linz@zju.edu.cn

Abstract: The study investigates the leader–follower formation control problem, for which the objective is to control a group of robots such that they move as a rigid formation with a prescribed constant velocity. It is assumed in the study that there are two leader robots, who are the only robots in the group that are informed about the prescribed velocity. All the other robots are followers and do not have the reference velocity information. The authors take the robotic formation as coupled triangular sub-formations and develop adaptive control strategies to enable each follower robot to attain and maintain a stable triangular formation with respect to its two leading neighbours. As a result, the whole group forms a rigid formation. Analyses on convergence and stability properties of equilibrium formations are provided, which show that the desired formation is asymptotically stable. Finally, simulations are given to illustrate our results.

1 Introduction

In recent years, the control of networks of autonomous vehicles and mobile robots has attracted considerable research attention. Under carefully deigned coordinated and cooperative control strategies, networks of robots can create collective intelligence and perform tasks as a whole that are far beyond the capabilities of individual members. Among the large numbers of problems for multi-vehicle systems, formation control has been one of the most important problems because of its broad applications. There are different methods dealing with group formation problems, such as behaviour-based approach [1, 2], potential function approach [3, 4], virtual structure approach [5, 6] and leader-following approach [7–9]. Cao *et al.* [10] provide a global convergence analysis for a team of three robots to achieve a stationary formation with an acyclic sensing graph, whereas Anderson *et al.* [11] study a formation of three robots with a cyclic sensing graph. Chen and Tian [12] describe each agent as a double integrator and further investigate the three-coleader formation control problem through a backstepping method. In addition, Bai *et al.* [13] design an adaptive control law based on the passivity theory to steer a group of agents to a formation that moves with a prescribed reference velocity. Related distance-constrained formation maintenance problems with directional sensing information flow are also discussed in [14–17].

In some literature on formation control using the leader-following approach, it is assumed that there is only one group leader in the team and the leader's information such as moving velocity is known by all the follower robots. However, in real applications, the leader's information may not be available to all the follower robots, because of practical constraints. Instead, each follower robot can only obtain the information about its neighbours. So it takes its neighbouring robots as its leaders. Thus, the information about the group leader does not have to be broadcast directly to all the follower robots [9]. In this paper, we label a group of robots from 1 to n such that robots 1 and 2 are leader robots and robots i ($i = 3, \ldots, n$) are follower robots. The neighbour relationships are defined as follows. The leader robots 1 and 2 can sense only the relative positions of each other. For the follower robots, each robot has two neighbour robots whose relative positions are measured in their own coordinate systems. For instance, robot 3 has robots 1 and 2 as its neighbours, robot 4 has robots 2 and 3 as its neighbours and so on. Moreover, in our paper, it is assumed that the reference velocity is only available to the two leader robots and cannot be known by the follower robots. The control objective is to devise a controller that enables the robots to form a rigid formation and to maintain the formation while moving as a whole at the desired constant reference velocity. We first introduce a control law for the leader robots to make them converge to a configuration with a prescribed distance moving with the desired reference velocity. Then an adaptive scheme is proposed for each follower robot so that it forms a desired triangular formation with respect to its two leading neighbours. However, the proposed controller also introduces another undesirable equilibrium for the triangular sub-formations, which is the collinear configuration. By investigating the stability properties of these equilibrium formations using the Lyapunov indirect method, it is shown that the triangular formation is stable whereas the collinear one is not.

Our work is motivated by Cao *et al.* [10], where they investigate how to form a stationary formation globally. Compared to [10], this paper extends the approach to the formation control problem in motion and a new adaptive scheme is proposed such that all the robots eventually move with the same prescribed reference velocity that is not known by the follower robots. Also, in contrast to [13] where the information flow graph has to be bidirectional in order to apply passivity analysis tools to the formation control problem in motion, this paper does not require bidirectional information exchanges except for the two leaders. Moreover, the adaptive scheme we proposed assures that the robots can be adaptively recovered to the desired formation even in the event of an abrupt change in the reference velocity. Finally, the approach relies only on local information and each follower robot only needs to know the relative position information of its two neighbours. So the approach is scalable in the sense that one does not need to redesign the control strategy and it does not bring any additional difficulty in analysing their group behaviours when the number of robots in the group increases.

2 Problem statement

Consider a group of n identical mobile robots with dynamics

$$\dot{x}_i = u_i, \quad i = 1, \dots, n \tag{1}$$

where $x_i \in \mathbb{R}^2$ is the position of robot i in the plane and $u_i \in \mathbb{R}^2$ is its velocity control.

We assume each robot carries an onboard sensor and is able to sense the relative position of robot j (namely $x_j - x_i$) when robot j is its neighbour. In our setup, the group has two leader robots with the knowledge of the reference velocity v_0, which is constant or piecewise constant. Two leader robots labelled 1 and 2 are required to achieve and keep a desired distance between each other. The rest of the robots are followers who do not know the reference velocity. Each follower follows its two leading robots and maintains a triangulation formation with them. For instance, robot 3 follows robots 1 and 2, robot 4 follows robots 2 and 3 and so on. The interaction directed graph for our setup is illustrated in Fig. 1 where a directed arc from node i to node j means that robot i uses the relative position information of robot j for the purpose of formation control.

The problem of formation control for a group of robots with two leaders is described as follows.

Problem: Find a distributed control law for each robot using only the available local information such that

(1) two leader robots 1 and 2 move with velocity v_0 and keep a desired distance from each other,

(2) each follower robot i ($i \geq 3$) achieves and maintains a desired triangular formation with respect to robots $i-2$ and $i-1$.

In what follows, the notation $\|\cdot\|$ is used to denote the Euclidean norm. Let d_{ij} be the desired distance between robots i and j. The control objective can be written more clearly as follows: As $t \to \infty$

(1)

$$\dot{x}_1(t) \to v_0$$
$$\dot{x}_2(t) \to v_0$$
$$\|x_1(t) - x_2(t)\| \to d_{12}$$

(2) for $i \geq 3$

$$\|x_i(t) - x_{i-2}(t)\| \to d_{(i-2)i}$$
$$\|x_i(t) - x_{i-1}(t)\| \to d_{(i-1)i}$$

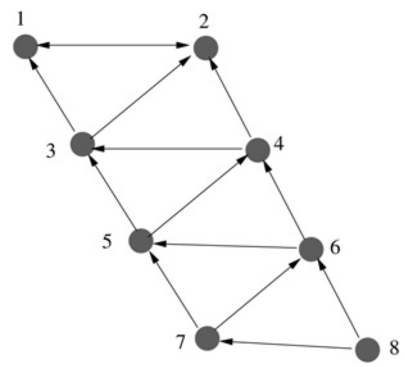

Figure 1 *Interaction directed graph*

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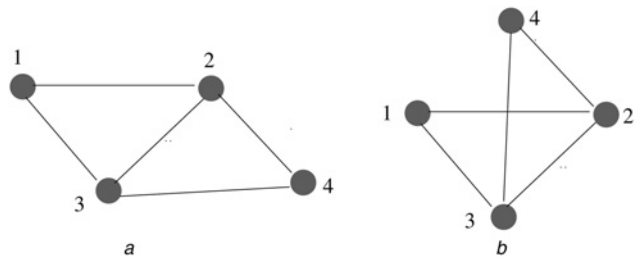

Figure 2 *Two rigid formations with the same distance constraints*

a One formation $\mathbb{F}_d$
b Another formation $\mathbb{F}_{d'}$

In fact, when

$$\begin{aligned} \|x_{i-1}(t) - x_{i-2}(t)\| &= d_{(i-1)(i-2)} \\ \|x_i(t) - x_{i-2}(t)\| &= d_{(i-2)i} \\ \|x_i(t) - x_{i-1}(t)\| &= d_{(i-1)i} \quad \text{for all } i \geq 3 \end{aligned} \tag{2}$$

we say the group of n robots are in a formation. Let $\mathbb{F}_d$ be a formation satisfying (2). Note that the formation $\mathbb{F}_d$ is rigid, but not globally rigid [14]. For example, a four-agent rigid formation $\mathbb{F}_d$ satisfying (2) shown in Fig. 2*a* has a discontinuous deformation in Fig. 2*b*.

To make the formation realisable, the desired distances must satisfy the following triangle inequality constraints

$$\begin{aligned} d_{(i-2)(i-1)} &< d_{(i-2)i} + d_{(i-1)i} \\ d_{(i-2)i} &< d_{(i-2)(i-1)} + d_{(i-1)i} \\ d_{(i-1)i} &< d_{(i-2)(i-1)} + d_{(i-2)i} \end{aligned}$$

3 Control synthesis

In this section, we synthesise control laws. First, we present the control law for the two leader robots. Second, we propose an adaptive control law for the follower robots and investigate convergence and stability properties.

3.1 Control law for leader robots

In this subsection, we study control laws for the two leader robots. Since the motion of these two leader robots is not affected by their followers, we can analyse their dynamical behaviour independently. Recall that the reference velocity v_0 is known only to these two leader robots. We then design the following control law using only locally sensed relative position information

$$\begin{aligned} u_1 &= v_0 + (x_2 - x_1)(\|x_2 - x_1\|^2 - d_{12}^2) \\ u_2 &= v_0 + (x_1 - x_2)(\|x_1 - x_2\|^2 - d_{12}^2) \end{aligned} \tag{3}$$

With the above control, we then have the following dynamics for these two leader robots

$$\begin{aligned} \dot{x}_1 &= v_0 + (x_2 - x_1)(\|x_2 - x_1\|^2 - d_{12}^2) \\ \dot{x}_2 &= v_0 + (x_1 - x_2)(\|x_1 - x_2\|^2 - d_{12}^2) \end{aligned} \tag{4}$$

Next we show that with the control law above, these two leader robots will eventually converge to a formation with the desired distance d_{12} and move with the reference velocity v_0 if they are not initially coincident.

Theorem 1: For the two leader robots with the control law (3), if $x_1(0) \neq x_2(0)$, then

$$\left.\begin{aligned} \|x_1(t) - x_2(t)\| &\to d_{12} \\ \dot{x}_1(t) &\to v_0 \\ \dot{x}_2(t) &\to v_0 \end{aligned}\right\} \quad \text{exponentially as } t \to \infty$$

Proof: Let $z_{12} = x_1 - x_2$ and $e_{12} = \|z_{12}\|^2 - d_{12}^2$. It holds that

$$\begin{aligned} \dot{z}_{12} &= \dot{x}_1 - \dot{x}_2 \\ &= -2(x_1 - x_2)(\|x_1 - x_2\|^2 - d_{12}^2) \\ &= -2z_{12}e_{12} \end{aligned} \tag{5}$$

For the remaining, we refer to the proof of Lemma 4 in [10] from which one obtains $e_{12} \to 0$ exponentially fast. Thus, $\|z_{12}(t)\|$ tends to d_{12} exponentially and $\dot{x}_1(t) \to v_0$, $\dot{x}_2(t) \to v_0$ exponentially. □

Note that the bidirectional information flow between the two leader robots removes the rotational degree of freedom from the formation.

3.2 Control law for follower robots

In the last subsection, we have discussed the control law for the two leader robots. In this subsection, we investigate control laws for the follower robots. First, we consider the control for robot 3 to see how it achieves a triangular formation with the two leader robots 1 and 2. This is critical, since it will serve as the base to obtain control laws for all the other follower robots.

As the reference velocity is not known by the follower robots, we consider an adaptive control. To be more specific, for robot 3, we use the following control utilising only the relative position information about robots 1 and 2

$$\begin{aligned} \dot{\theta}_3 &= (x_1 - x_3)(\|x_1 - x_3\|^2 - d_{13}^2) \\ &\quad + (x_2 - x_3)(\|x_2 - x_3\|^2 - d_{23}^2) \\ u_3 &= \theta_3 + (x_1 - x_3)(\|x_1 - x_3\|^2 - d_{13}^2) \\ &\quad + (x_2 - x_3)(\|x_2 - x_3\|^2 - d_{23}^2) \end{aligned} \tag{6}$$

With the above control law for the third robot, we obtain the dynamics for these three robots (two leaders and one follower) as

$$
\begin{aligned}
\dot{x}_1 &= v_0 + (x_2 - x_1)(\|x_2 - x_1\|^2 - d_{12}^2) \\
\dot{x}_2 &= v_0 + (x_1 - x_2)(\|x_1 - x_2\|^2 - d_{12}^2) \\
\dot{x}_3 &= \theta_3 + (x_1 - x_3)(\|x_1 - x_3\|^2 - d_{13}^2) \\
&\quad + (x_2 - x_3)(\|x_2 - x_3\|^2 - d_{23}^2) \\
\dot{\theta}_3 &= (x_1 - x_3)(\|x_1 - x_3\|^2 - d_{13}^2) \\
&\quad + (x_2 - x_3)(\|x_2 - x_3\|^2 - d_{23}^2)
\end{aligned} \tag{7}
$$

Introduce the coordinate transformation

$$z_{13} = x_1 - x_3, \quad z_{23} = x_2 - x_3$$

and let

$$e_{13} = \|z_{13}\|^2 - d_{13}^2, \quad e_{23} = \|z_{23}\|^2 - d_{23}^2$$

Then we obtain

$$
\begin{aligned}
\dot{z}_{13} &= \dot{x}_1 - (\theta_3 + z_{13}e_{13} + z_{23}e_{23}) \\
\dot{z}_{23} &= \dot{x}_2 - (\theta_3 + z_{13}e_{13} + z_{23}e_{23}) \\
\dot{\theta}_3 &= z_{13}e_{13} + z_{23}e_{23}
\end{aligned} \tag{8}
$$

where $\dot{x}_1 = v_0 - z_{12}e_{12}$ and $\dot{x}_2 = v_0 + z_{12}e_{12}$ from (4).

We first show that $z_{13}e_{13} + z_{23}e_{23}$ converges to 0 and θ_3 converges to v_0 as $t \to \infty$. That is, the solution to the above system approaches the equilibrium set.

We recall the Barbalat's lemma, which is helpful in the proof of the main result that we want to prove.

Lemma 1 (Barbalat's lemma) [18]: Let φ: $\mathbb{R} \to \mathbb{R}$ be a uniformly continuous function on $[0, \infty)$. Suppose that $\lim_{t\to\infty} \int_0^t \varphi(\tau)\,\mathrm{d}\tau$ exists and is finite. Then, $\varphi(t) \to 0$ as $t \to \infty$.

Theorem 2: For robot 3 with the control law (6)

$$
\left.
\begin{aligned}
z_{13}(t)e_{13}(t) + z_{23}(t)e_{23}(t) &\to 0 \\
\theta_3(t) &\to v_0 \\
\dot{x}_3(t) &\to v_0
\end{aligned}
\right. \quad \text{as } t \to \infty
$$

Proof: Define a continuously differentiable function

$$V = \frac{1}{4}e_{13}^2 + \frac{1}{4}e_{23}^2 + \frac{1}{2}\|\theta_3\|^2$$

Taking the time derivative of V along the solution of system (8), one obtains

$$
\begin{aligned}
\dot{V} &= \frac{1}{2}e_{13}\dot{e}_{13} + \frac{1}{2}e_{23}\dot{e}_{23} + \theta_3^{\mathrm{T}}\dot{\theta}_3 \\
&= e_{13}z_{13}^{\mathrm{T}}[\dot{x}_1 - (\theta_3 + z_{13}e_{13} + z_{23}e_{23})] \\
&\quad + e_{23}z_{23}^{\mathrm{T}}[\dot{x}_2 - (\theta_3 + z_{13}e_{13} + z_{23}e_{23})] \\
&\quad + \theta_3^{\mathrm{T}}(z_{13}e_{13} + z_{23}e_{23}) \\
&= e_{13}z_{13}^{\mathrm{T}}\dot{x}_1 + e_{23}z_{23}^{\mathrm{T}}\dot{x}_2 - \|e_{13}z_{13} + e_{23}z_{23}\|^2 \\
&= [e_{13}z_{13}^{\mathrm{T}}\ e_{23}z_{23}^{\mathrm{T}}]\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} - \|e_{13}z_{13} + e_{23}z_{23}\|^2 \\
&\le \|e_{13}z_{13}^{\mathrm{T}}\ e_{23}z_{23}^{\mathrm{T}}\|\left\|\begin{matrix}\dot{x}_1 \\ \dot{x}_2\end{matrix}\right\| - \|e_{13}z_{13} + e_{23}z_{23}\|^2
\end{aligned}
$$

Now we show that $V(t)$ is bounded. Suppose by contradiction that it is unbounded. Then there exists $T > 0$ large enough such that $\|V(t)\| = c$ and $\|V(t)\| > c$ for t in the interval $(T, T + \delta)$ where c is a large number and $\delta > 0$ is a small constant. Since $V(t)$ is unbounded, then it must be true that at least one of $e_{13}(t)$, $e_{23}(t)$ and $\theta_3(t)$ is unbounded. Without loss of generality, we assume $e_{13}(t)$ is unbounded as the argument will be similar when assuming either of the other two is unbounded. Hence, $\|e_{13}(T)\|$ is also very large (we take it as far larger than d_{12}) and so is $\|z_{13}(T)\|$. Notice that from Theorem 1 $\|z_{12}(t)\| \to d_{12}$. Without loss of generality, we assume that for this large T, $\|z_{12}(T)\|$ is close to d_{12}. Thus $\|z_{23}(T)\|$ is also very large because of the fact that $\|z_{23}\| \ge \|z_{13}\| - \|z_{12}\|$. Moreover, we can obtain that $e_{13}(T) > 0$, $e_{23}(T) > 0$ and the angle between $z_{13}(T)$ and $z_{23}(T)$ is no more than $\pi/2$ (namely $z_{13}(T)^{\mathrm{T}}z_{23}(T) > 0$). (For notation simplicity, we omit the time T in the following inequalities, which hold only at the time instant T.) Hence

$$\|e_{13}z_{13}^{\mathrm{T}}\ e_{23}z_{23}^{\mathrm{T}}\|^2 \le \|e_{13}z_{13} + e_{23}z_{23}\|^2$$

Owing to the above inequality, it follows that

$$\dot{V}(T) \le \|e_{13}z_{13} + e_{23}z_{23}\|\left\|\begin{matrix}\dot{x}_1 \\ \dot{x}_2\end{matrix}\right\| - \|e_{13}z_{13} + e_{23}z_{23}\|^2$$

Recall from Theorem 1 that $\dot{x}_1(t) \to v_0$ and $\dot{x}_2(t) \to v_0$. Hence, it must hold that at T

$$\left\|\begin{matrix}\dot{x}_1 \\ \dot{x}_2\end{matrix}\right\| < \|e_{13}z_{13} + e_{23}z_{23}\|$$

Thus, we have $\dot{V}(T) < 0$, a contradiction to the assumption that $V(T) = c$ and $V(t) > c$ for $t \in (T, T + \delta)$.

We just showed that $V(T)$ is bounded. From the expression of V, then we know $e_{13}(t)$, $e_{23}(t)$ and $\theta_3(t)$ are bounded and so are $z_{13}(t)$ and $z_{23}(t)$. Since

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$\dot{\theta}_3 = z_{13}e_{13} + z_{23}e_{23}$, one obtains

$$\theta_3(t) = \int_0^t [z_{13}(\tau)e_{13}(\tau) + z_{23}(\tau)e_{23}(\tau)]\, d\tau + \theta_3(0)$$

Note that $\theta_3(t)$ is bounded, so $\lim_{t\to\infty} \int_0^t (z_{13}(\tau)e_{13}(\tau) + z_{23}(\tau)e_{23}(\tau))\, d\tau$ exists and is finite. Moreover, $z_{13}e_{13} + z_{23}e_{23}$ is uniformly continuous. Hence, from Lemma 1

$$z_{13}(t)e_{13}(t) + z_{23}(t)e_{23}(t) \to 0 \quad \text{as } t \to \infty$$

Next we prove that $\theta_3(t) \to v_0$. Since $\theta_3(t)$ is bounded and $\dot{\theta}_3(t) \to 0$, we obtain $\theta_3(t) \to a$ where a is a constant. From Theorem 1, we know $e_{12}(t) \to 0$ as $t \to \infty$. Together with $z_{13}(t)e_{13}(t) + z_{23}(t)e_{23}(t) \to 0$, it follows from (8) that $\dot{z}_{13}(t) \to (v_0 - a)$ and $\dot{z}_{23}(t) \to (v_0 - a)$. If $a \neq v_0$, $z_{13}(t)$ and $z_{23}(t)$ would tend to ∞, which contradicts with the conclusion that $z_{13}(t)$ and $z_{23}(t)$ are bounded. Therefore the constant a must be v_0. That is $\theta_3(t) \to v_0$.

Finally, from (6) we obtain $\dot{x}_3(t) \to v_0$. □

Note that $z_{13}e_{13} + z_{23}e_{23} = 0$ implies that either

1. $e_{13} = 0$, $e_{23} = 0$, or

2. $z_{13}e_{13} + z_{23}e_{23} = 0$ but e_{13} or $e_{23} \neq 0$.

For the first case, if $e_{12} = 0$, it corresponds to a desired triangular formation. That is, the three robots 1, 2 and 3 form a triangle with desired edge lengths d_{12}, d_{13} and d_{23} (see Fig. 3). The second case corresponds to a collinear formation. That is, the three robots are positioned on a line (see Fig. 4).

Recall from Theorem 1 that for two leader robots with the control law (3), if $x_1(0) \neq x_2(0)$, then $e_{12}(t) \to 0$. And Theorem 2 tells us that robot 3 converges to either a triangular formation or a collinear formation with robots 1 and 2. Next, we show that the triangular formation is asymptotically stable while the collinear formation is unstable. When we say the triangular formation is asymptotically stable, it means that the equilibrium manifold corresponding to the triangular formation is asymptotically stable.

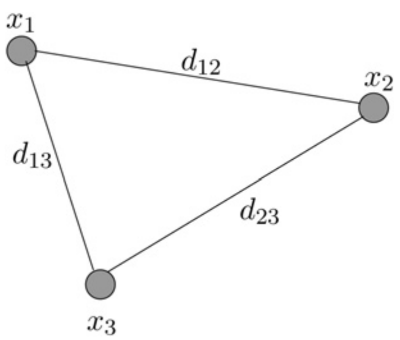

Figure 3 *Triangular formation*

Figure 4 *Collinear formation*

The proof of Theorem 3 requires the following two lemmas.

Lemma 2 [17]: Let $\mathcal{E}$ be a k-dimensional equilibrium manifold of $\dot{x} = f(x)$. Let $J_f(x_0)$ be the Jacobian matrix at x_0. If, for all $x_0 \in \mathcal{E}$, $J_f(x_0)$ has all stable eigenvalues, except for k eigenvalues at zero, then $\mathcal{E}$ is locally asymptotically stable.

Lemma 3: For any two vectors $z_1, z_2 \in \mathbb{R}^2$, the following holds

$$\|z_1\|^2 + \|z_2\|^2 \geq 2 \det[z_1\ z_2]$$

Proof: Let $z_1 = (x_1\ y_1)^T$ and $z_2 = (x_2\ y_2)^T$. Then we can write

$$\|z_1\|^2 + \|z_2\|^2 = x_1^2 + y_1^2 + x_2^2 + y_2^2$$
$$2 \det[z_1\ z_2] = 2x_1y_2 - 2x_2y_1$$

Thus

$$\begin{aligned} &\|z_1\|^2 + \|z_2\|^2 - 2\det[z_1\ z_2] \\ &= x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2x_1y_2 + 2x_2y_1 \\ &= (x_1 - y_2)^2 + (x_2 + y_1)^2 \geq 0 \end{aligned}$$

□

Theorem 3: For robots 1, 2 and 3 with the control laws (3) and (6), the triangular formation is asymptotically stable.

Proof: We prove it using Lyapunov's indirect method. For these three robots, the overall system is given in (5) and (8). Then the Jacobian matrix at any state satisfying $e_{12} = 0$ is calculated and has the following block lower triangular matrix form

$$J = \begin{pmatrix} -2A & 0 \\ E & D \end{pmatrix} \tag{9}$$

where

$$A = 2z_{12}z_{12}^T,\ E = (-A \quad A \quad 0)^T$$

$$D = \begin{pmatrix} -e_{13}I_2 - B & -e_{23}I_2 - C & -I_2 \\ -e_{13}I_2 - B & -e_{23}I_2 - C & -I_2 \\ e_{13}I_2 + B & e_{13}I_2 + C & 0 \end{pmatrix}$$

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and

$$B = 2z_{13}z_{13}^{\mathrm{T}}, \quad C = 2z_{23}z_{23}^{\mathrm{T}}$$

For the triangular formation (namely, $e_{12} = e_{13} = e_{23} = 0$), the matrix D then becomes

$$D = \begin{pmatrix} -B & -C & -I_2 \\ -B & -C & -I_2 \\ B & C & 0 \end{pmatrix}$$

One can check that it is obtained that the characteristic equation for the matrix D is

$$\lambda^2(a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4) = 0 \tag{10}$$

where

$$\begin{aligned} a_0 &= 1 \\ a_1 &= 2(\|z_{13}\|^2 + \|z_{23}\|^2) \\ a_2 &= a_1 + a_4 \\ a_3 &= 2a_4 \\ a_4 &= 4(\det[z_{13}\ z_{23}])^2 \end{aligned}$$

From (10), we immediately know that D has two zero eigenvalues. Next, we use Routh's criterion to check the roots of the fourth order polynomial in (10). For the triangular formation, it is clear that the three robots are not collinear, which means, $\det[z_{13}\ z_{23}] \neq 0$. Thus, we know that a_0, a_1, a_2, a_3 and a_4 are all positive numbers. Moreover, it can be checked that

$$a_1a_2 - a_0a_3 = a_1^2 - 2a_4 + a_1a_4$$

which is greater than 0 by Lemma 3. And in addition, using Lemma 3 again, we obtain

$$a_3(a_1a_2 - a_0a_4) - a_1^2a_4 = a_4(a_1^2 - 4a_4 + 2a_1a_4)$$

is also greater than 0. Thus, from Routh's criterion, these four eigenvalues all have negative real parts.

On the other hand, note that the 2×2 matrix A is of rank one and is positive semi-definite, and so $-2A$ has one zero eigenvalue and the other eigenvalue has a negative real part. Hence, the Jacobian matrix J at any state satisfying $e_{12} = e_{13} = e_{23} = 0$ has three zero eigenvalues and five eigenvalues with negative real part. Moreover, it can be checked that the equilibrium manifold corresponding to the triangular formation [namely $\{(z_{12}, z_{13}, z_{23}, \theta_3) | e_{12} = e_{13} = e_{23} = 0, \theta_3 = v_0\}$] is of dimension three. Then, by Lemma 2, it follows that the triangular formation is asymptotically stable. □

Next, we show that the collinear formation is unstable. Before presenting the result, we introduce a lemma first, which will be used in the proof.

Lemma 4 [10]: If three robots are in the collinear formation, then $e_{13} + e_{23} < 0$.

Theorem 4: For robots 1, 2 and 3 with the control laws (3) and (6), the collinear formation is unstable.

Proof: For collinear formation, $z_{13}e_{13} + z_{23}e_{23} = 0$ but e_{13} or $e_{23} \neq 0$. Without loss of generality, assume $e_{13} \neq 0$. Then we write

$$z_{13} = -\frac{e_{23}}{e_{13}}z_{23}$$

and substitute it into (9). Thus, the Jacobian matrix at any state corresponding to collinear formation is still of the form (9) but the sub-block D in the Jacobian matrix J becomes

$$D = \begin{pmatrix} -e_{13}I_2 - \left(\frac{e_{23}}{e_{13}}\right)^2 C & -e_{23}I_2 - C & -I_2 \\ -e_{13}I_2 - \left(\frac{e_{23}}{e_{13}}\right)^2 C & -e_{23}I_2 - C & -I_2 \\ e_{13}I_2 + \left(\frac{e_{23}}{e_{13}}\right)^2 C & e_{23}I_2 + C & 0 \end{pmatrix}$$

where C is still the same. Let λ be a complex number. It can be checked that $\lambda = 1$ is not an eigenvalue of D. So after several elementary row and column operations for the matrix $D - \lambda I$, we obtain

$$\begin{pmatrix} 0 & 0 & -\lambda I \\ 0 & -\lambda I & -\lambda I \\ M & e_{23}I + C & \dfrac{\lambda^2}{\lambda - 1}I \end{pmatrix}$$

where

$$M = \left(e_{13} + e_{23} + \frac{\lambda^2}{\lambda - 1}\right)I + \left[\left(\frac{e_{23}}{e_{13}}\right)^2 + 1\right]C$$

Since the rank of C is 1, if λ satisfies

$$e_{13} + e_{23} + \frac{\lambda^2}{\lambda - 1} = 0 \tag{11}$$

then M is not full rank, which means this λ is an eigenvalue of D. From (11), we solve for

$$\lambda_{1,2} = \frac{1}{2}\left[-(e_{13} + e_{23}) \pm \sqrt{(e_{13} + e_{23})^2 + 4(e_{13} + e_{23})}\,\right]$$

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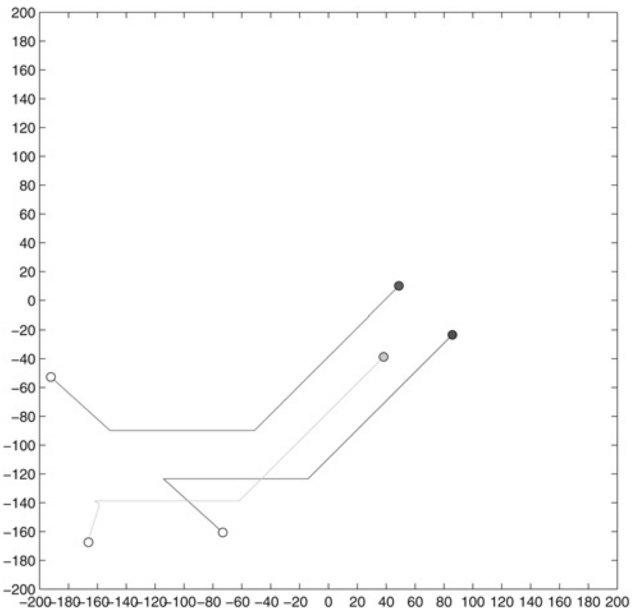

Figure 5 *Three robots form a triangular formation and move in the plane*

By Lemma 3, we know that $(e_{13} + e_{23}) < 0$ at the collinear formation. So one of these two eigenvalues must have a positive real part. Hence, the collinear formation is unstable. □

If the reference velocity v_0 is piecewise constant, the control law proposed in the paper is still able to maintain the formation in motion after a transient period as long as it does not change its value too often. We have now investigated the convergence and stability properties of two possible formations (namely triangular formation and collinear formation). It extends the previous work [10] where the speed of the formation is zero to the case where the formation travels at a non-zero velocity.

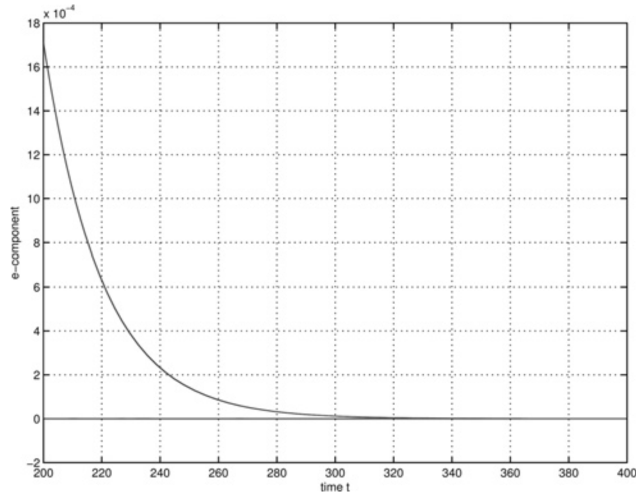

Figure 7 *Formation errors $e_{ij}(t)$ for $t \in [200, 400]$*

In what follows, we generalise the adaptive control law to any follower robot. That is, for any robot i $(i \geq 3)$, let the control law be

$$\begin{aligned} \dot{\theta}_i &= (x_{i-2} - x_i)(\|x_{i-2} - x_i\|^2 - d^2_{(i-2)i}) \\ &\quad + (x_{i-1} - x_i)(\|x_{i-1} - x_i\|^2 - d^2_{(i-1)i}) \\ u_i &= \theta_i + (x_{i-2} - x_i)(\|x_{i-2} - x_i\|^2 - d^2_{(i-2)i}) \\ &\quad + (x_{i-1} - x_i)(\|x_{i-1} - x_i\|^2 - d^2_{(i-1)i}) \end{aligned} \tag{12}$$

which uses only the relative position information of its precedent two neighbour robots according to their labels.

Similarly, define the relative positions

$$z_{(i-2)i} = x_{i-2} - x_i$$

$$z_{(i-1)i} = x_{i-1} - x_i$$

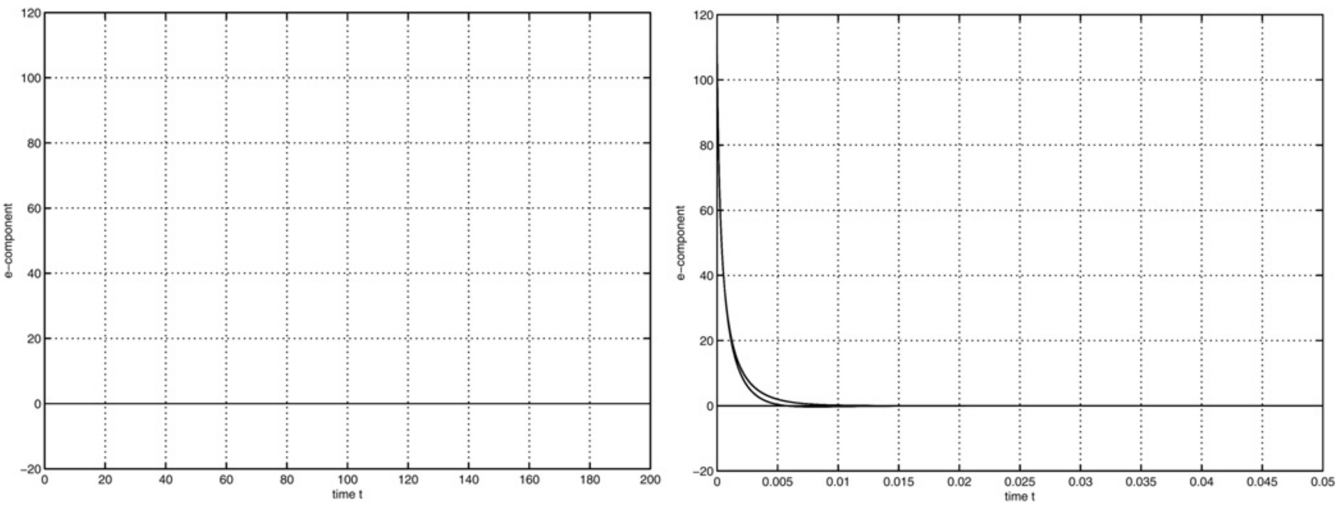

Figure 6 *Left: the formation errors $e_{ij}(t)$ for $t \in [0, 200)$, right: the zoomed-in plot of the subinterval $t \in [0, 0.05]$*

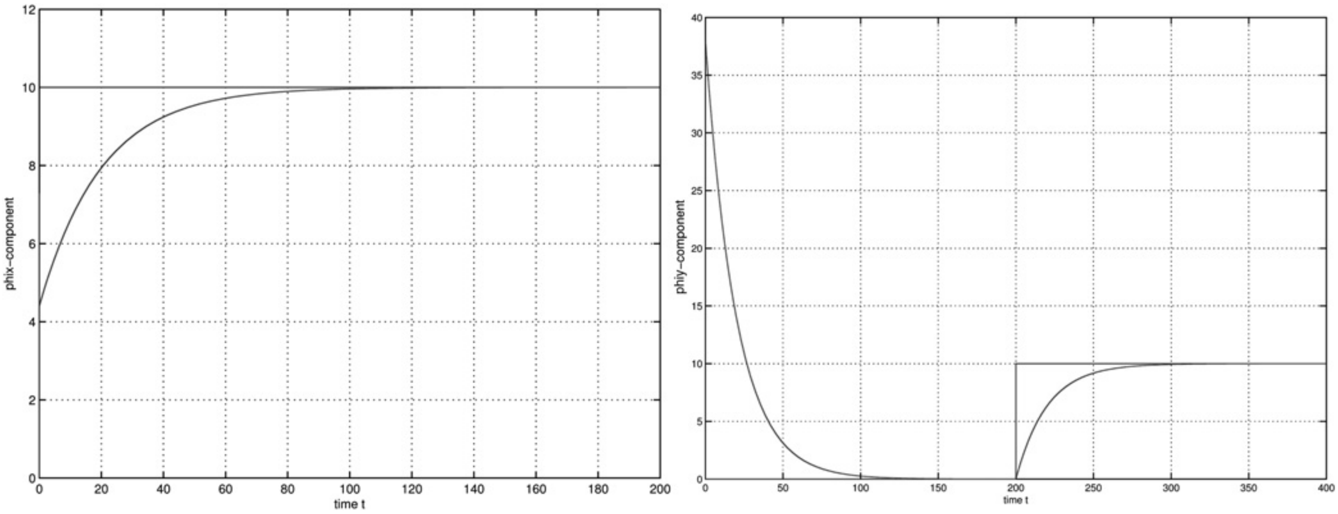

Figure 8 *Evolution of $\theta_3(t)$*
Left: the *x*-component of θ_3. Right: the *y*-component of θ_3

and let

$$e_{(i-2)i} = \|z_{(i-2)i}\|^2 - d^2_{(i-2)i}$$

$$e_{(i-1)i} = \|z_{(i-1)i}\|^2 - d^2_{(i-1)i}$$

We then obtain the dynamics

$$\begin{aligned}\dot{z}_{(i-2)i} &= \dot{x}_{i-2} - (\theta_i + z_{(i-2)i}e_{(i-2)i} + z_{(i-1)i}e_{(i-1)i})\\ \dot{z}_{(i-1)i} &= \dot{x}_{i-1} - (\theta_i + z_{(i-2)i}e_{(i-2)i} + z_{(i-1)i}e_{(i-1)i})\\ \dot{\theta}_i &= z_{(i-2)i}e_{(i-2)i} + z_{(i-1)i}e_{(i-1)i}\end{aligned} \tag{13}$$

where $\dot{x}_{i-2}$ and $\dot{x}_{i-1}$ are the position dynamics of its precedent two neighbour robots.

Note that the dynamics above have exactly the same form as (8). So by a similar argument, one is able to obtain the following result for any follower robot i ($i \geq 3$). That is, if the precedent two robots are not coincident, then robot i converges to form a triangular formation or a collinear formation with the precedent two neighbour robots.

Theorem 5: Under the control law (12)

$$\left.\begin{aligned} z_{(i-2)i}(t)e_{(i-2)i}(t) + z_{(i-1)i}(t)e_{(i-1)i}(t) &\to 0\\ \theta_i(t) &\to v_0\\ \dot{x}_i(t) &\to v_0\end{aligned}\right. \quad \text{as } t \to \infty$$

The Jacobian matrix of the overall system evaluated at any

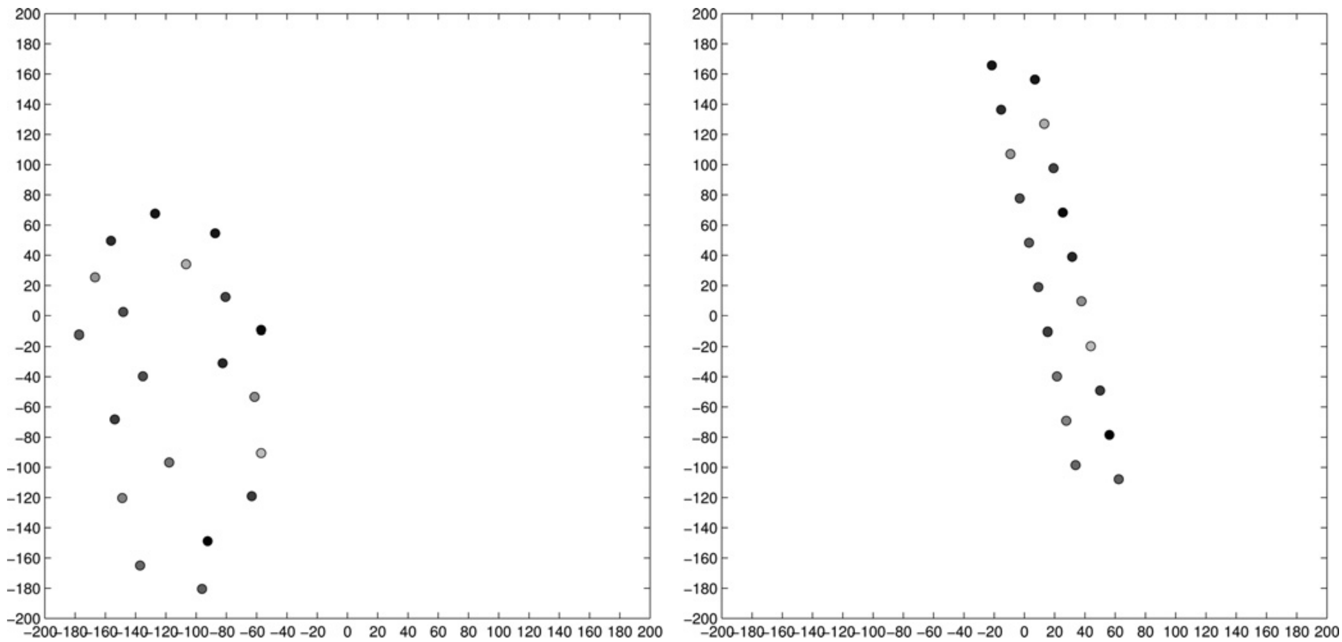

Figure 9 *Initial distribution (left) and final formation (right) of 20 robots*

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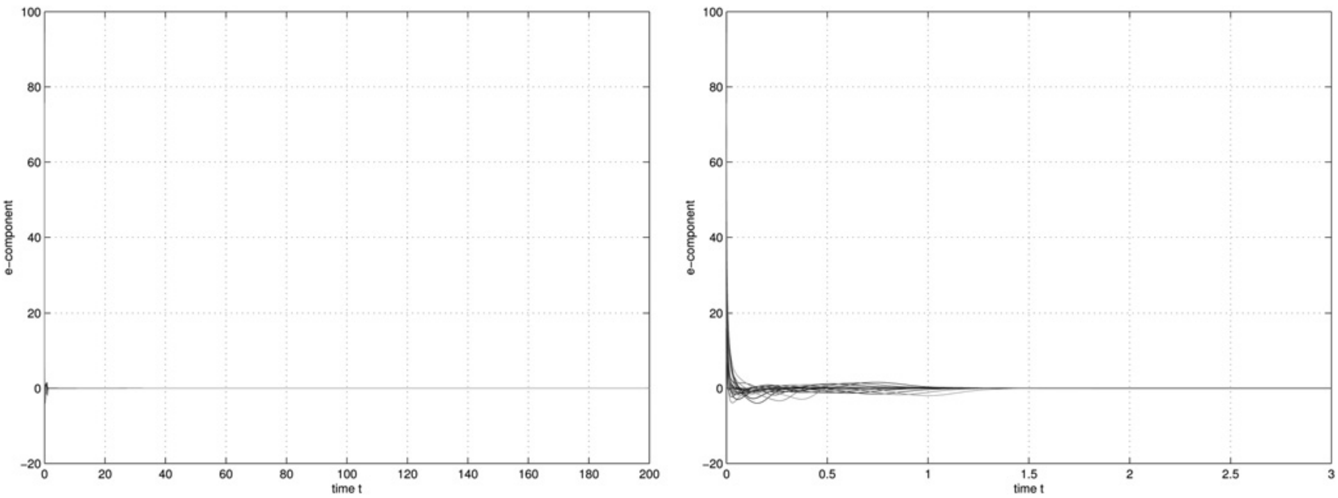

Figure 10 *Left: the formation errors for t ∈ [0, 200), right: the zoomed-in plot of the subinterval t ∈ [0, 3]*

state satisfying $e_{12} = 0$ has the following form

$$J = \left(\begin{array}{c|cccc} -2A & & 0 & & \\ \hline & D_3 & 0 & \cdots & 0 \\ & * & D_4 & \ddots & \vdots \\ * & \vdots & \ddots & \ddots & 0 \\ & 0 & \cdots & * & D_n \end{array}\right) \tag{14}$$

where $A = 2z_{12}z_{12}^{\mathrm{T}}$ and for $i \geq 3$

$$D_i = \begin{pmatrix} -e_{(i-2)i}I_2 - B_i & -e_{(i-1)i}I_2 - C_i & -I_2 \\ -e_{(i-2)i}I_2 - B_i & -e_{(i-1)i}I_2 - C_i & -I_2 \\ e_{(i-2)i}I_2 + B_i & e_{(i-1)i}I_2 + C_i & 0 \end{pmatrix}$$

$$B_i = z_{(i-2)i}z_{(i-2)i}^{\mathrm{T}}, \quad C_i = z_{(i-1)i}z_{(i-1)i}^{\mathrm{T}}$$

Note that the Jacobian matrix J is of the block lower triangular form. So we are able to check the stability from the location of D_i's eigenvalues for $i = 3, \ldots, n$. Hence, using the same technique as for Theorem 3 and 4, the following result is obtained.

Theorem 6: For a group of n robots with the control laws (3) and (12), any formation $\mathbb{F}_d$ satisfying (2) is asymptotically stable, and any formation not satisfying (2) is unstable.

As the interaction directed graph does not introduce any cycle when adding more and more follower robots, the convergence and stability properties of the overall system are not affected when the size of the group increases. However, the transient response of each follower robot may differ depending on how far away it is from the leaders in terms of path length in the interaction graph. It remains an open interesting problem about how the transient response is amplified from the first follower to the nth follower as n tends to ∞ and this is related to the notion of mesh stability [19].

4 Simulation

In this section, we present several simulations to illustrate our results.

First, we simulate three robots using the control laws (3) and (6) for two leader robots and one follower robot, respectively. The reference velocity v_0 is piecewise constant and it changes its value at $t = 200$ s. The initial distribution of three robots are arbitrary generated. The simulated trajectories of three robots under the control laws are given in Fig. 5. They form a triangular formation and move in the plane. After the abrupt change of the reference velocity at $t = 200$ s, three robots can be recovered to the triangular formation and move as a whole again with the new velocity. This can also be seen in Figs. 6 and 7, where the evolutions of $e_{12}(t)$ and $e_{13}(t) + e_{23}(t)$ are shown in Fig. 6 for the time interval [0, 200) and in Fig. 7 for the time interval [200, 400]. Notice that in Fig. 7 the

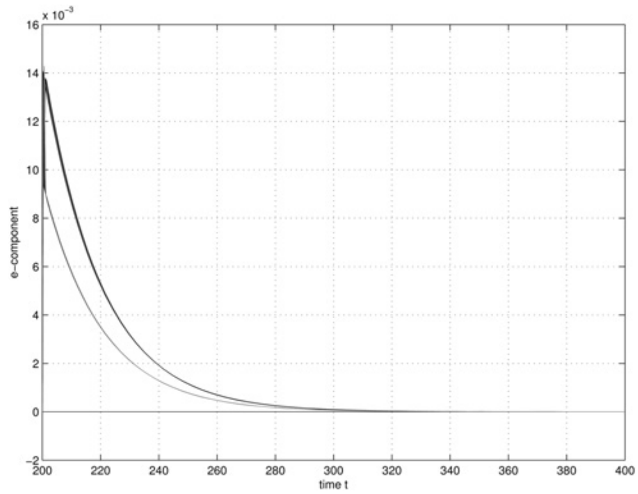

Figure 11 *Formation errors for t ∈ [200, 400]*

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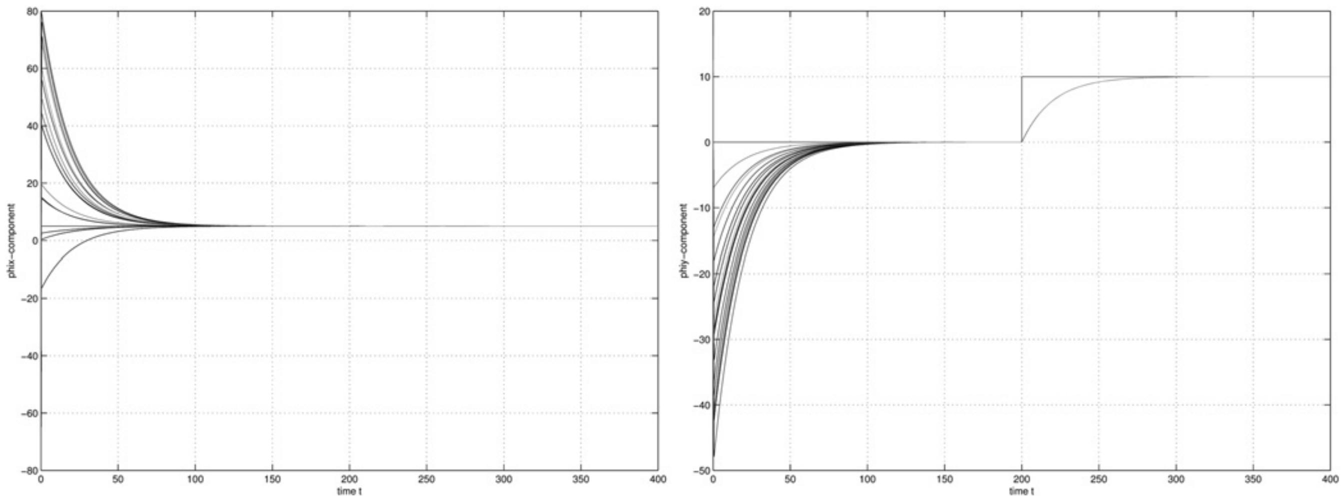

Figure 12 *Evolutions of $\theta_i(t)$ ($i = 3, \ldots, 20$)*

formation errors are a little perturbed away from zero at $t = 200$ s because of the change of the reference velocity and then converge to zero again. The evolution of $\theta_3(t)$ together with the reference velocity are given in Fig. 8. It can be seen that $\theta_3(t)$ adaptively tracks the reference velocity.

Second, we present a simulation in Fig. 9 for 20 robots that achieve a formation in moving. The evolutions of formation errors $e_{12}(t)$ and $e_{(i-2)i}(t) + e_{(i-1)i}(t)$, ($i = 3, \ldots, 20$) are shown in Fig. 10 for the time interval [0, 200) and in Fig. 11 for the time interval [200, 400]. The evolutions of $\theta_i(t)$ ($i = 3, \ldots, 20$) together with the reference velocity are given in Fig. 12. Again, the reference velocity changes to another value at $t = 200$ s in this simulation. From the simulation, we can see that the robots with our control strategies can be adaptively recovered to a desired formation even though the reference velocity suddenly changes.

5 Conclusion and future work

In this paper, we have discussed a leader–follower formation control problem. The mobile robots in a team are required to not only form a rigid formation but also move as a whole with a prescribed piecewise constant velocity. There are two leaders in the robot team, and the reference piecewise constant velocity is only available to the leader robots. Each leader robot has the other leader robot as its neighbour and controls the distance separation between them. Every follower robot has exactly two neighbours according to their labels and it determines its movement strategy using only local knowledge of the relative positions of its neighbours. A control law is designed for the leader robots first. Then an adaptive control law is investigated for every follower robot to achieve a triangular formation with its precedent two neighbour robots. Using Barbalat's lemma and Lyapunov indirect method, we study the convergence and stability properties in detail.

This control strategy can be applied to a large number of robots moving in a formation with a simple chain-like neighbour relationship. In this setup, one open issue is that the tracking (spacing) errors are expected not to be amplified downstream from robot to robot in the presence of disturbance at the leader robots. This property is very important for formations in motion as it ensures that small perturbations on the leader robots do not cause collisions among follower robots. Pant *et al.* [19] introduced the concept of mesh stability for interconnected systems. This idea may be explored in future work.

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