ITP 20002-{01, 02} Discrete Mathematics, Fall 2020

Final Exam

10 problems

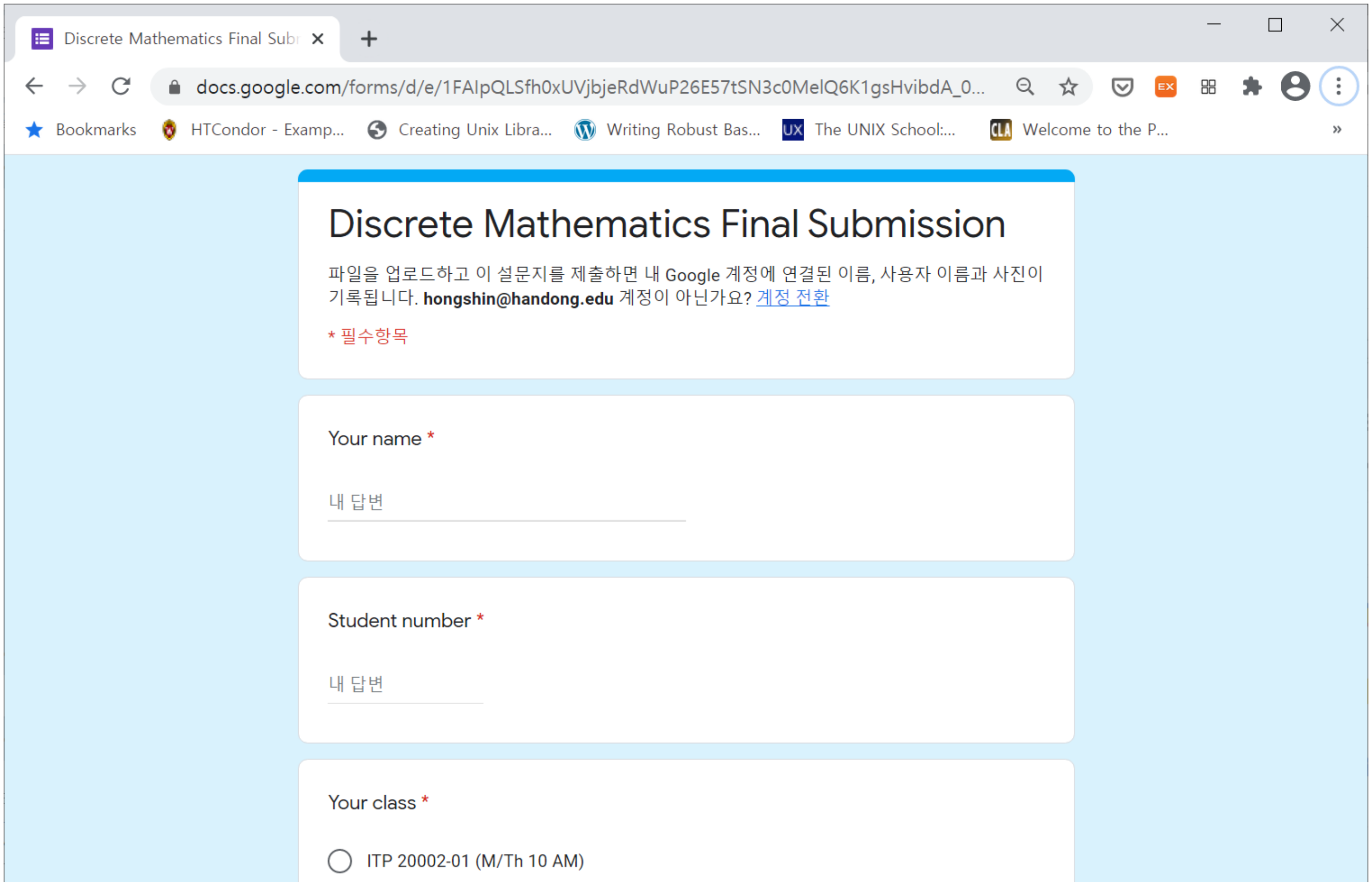
7 PM—9 PM (2 hours), 17 Dec 2020

Overview

Problem 1	16 points	**Problem 6**	7 points
Problem 2	10 points	**Problem 7**	9 points
Problem 3	8 points	**Problem 8**	8 points
Problem 4	6 points	**Problem 9**	8 points
Problem 5	13 points	**Problem 10**	15 points

Answer Submission Site

- https://forms.gle/XdKStHNMjGPZLJTu8



Problem 1 (16 point)

Write in your own word a statement that you will uphold the honor code in taking this exam. As an exceptional case, you are allowed to use Korean in writing this statement.

c.f. the Handong CSEE Standard on Examination

1. Examination is an educational act necessary for evaluation of the students' achievement and for encouraging the students to absorb the material in the process of preparation.
2. Student should do their best to prepare for exams in order to improve her/his own knowledge and skill and should fully engage in the test during examination hour.
3. Accessing or providing unauthorized information, including other students' answer sheets, is regarded as cheating. The use of electronic devices, including cell phones and computers, without permission is strictly prohibited.
4. Entering or leaving the classroom during the examination before the finish time without permission is regarded as cheating.

Problem 2 (10 points)

Prove that a connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two verticies of odd degree.

Use the theorem on the existence of an Euler circuit in your proof after stating it clearly.

Problem 3 (8 points)

Let A be the set of all relations on a finite set S, each of which is transitive and anti-symmetric at the same time.

Prove or disprove that $(A, \subset)$ is a lattice.

Problem 4 (6 points)

Prove that $P(A \cap B \cap C) = P(A)\, P(B \mid A)\, P(C \mid A \cap B)$ for three events A, B, and C.

Problem 5 (13 points)

Answer each of the following questions:

(a) What is a undecideable problem?

(b) Is an algorithm countable? Give reasons to support your answer.

(c) Express the Turning's undecidability theorem (i.e., Halthing problem theorem) as a predicate formula with the following terms:

- D : a set of all input strings
- P : a set of all programs (algorithms) each of which receives a string as an input and returns a boolen value
- $m(s_1, s_2)$: a function that merges two strings s_1 and s_2 into one
- $T(p, s)$: a predicate that determines whether program p halts when it is executed with input s

Problem 6 (7 points)

Let f be a function from a finite set A to another finite set B. Show that there exist a set $\{a_1, a_2, ..., a_m\} \subseteq A$ such that $f(a_1) = f(a_2) = ... = f(a_m)$ for $m = \lceil |A|/|B| \rceil$

Problem 7 (9 points)

Answer to each of the following questions:

(a) How many distinct complete bipartiate graphs with total N verticies exist? $(2 \leq N)$

(b) How many distinct wheel graphs with N verticies (i.e. W_{N-1}) exist? $(4 \leq N)$

(c) How many distinct simple direct graphs with N verticies exist? $(1 \leq N)$

Problem 8 (8 points)

Prof. Hong has invented a test method that determines if a person is affected by COVID-19 with a voice recognition technique. Indeed!

To demonstrate its effectiveness, three studies are conducted for the people in the same city, and found the following results:

- the test shows posive in 90% for those who have COVID-19
- the test shows posive in 12% for the randomly selected people
- among those who test positive, 40% does not have COVID-19

Assume that each study uses a sufficiently large set of random samples and the COVID-19 trend did not change during the studies.

What is the probability of a person in this city to have COVID-19? Give reasons to support your answers using the Bayes theorem.

Problem 9 (8 points)

Show that for two functions from positive integers to positive real numbers, f_1 and f_2, if $f_1(\mathrm{x})$ is $O(g_1(\mathrm{x}))$ and $f_2(\mathrm{x})$ is $O(g_2(\mathrm{x}))$, then $f_1(\mathrm{x}) + f_2(\mathrm{x})$ is $O(\max(g_1(\mathrm{x}), g_2(\mathrm{x})))$

Problem 10 (15 points)

The following is an algorithm for finding the shortest path between two vertices in an undirected graph without weight:

```
Input G=(V, E),  a undirected graph
        a ∈ V, the starting vertext
        z ∈ V, the destination vertex
Procedure
        foreach v ∈ V:
                L(v) ← ∞
                L(a) ← 0
        S ← Ø
        while z ∉ S :
                u ← a vertex ∈ V \ S with a minimal L(u)
                S ← S ∪ {u}
                foreach v ∈ V \ S
                        if (u, v) ∈ E and L(u) + 1 < L(v) :
                                L(v) ← L(u) + 1
        return L(z)
```

Prove that this algorithm is correct
(Hint: remind the Dijstra's shortest path algorithm)