

### Homework 3. Cardinality and Countability

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Write by your hand a proof that answers each of the following four questions:

1. Show that the power set of a countably infinite set is uncountable  
(from midterm exam of Discrete Mathematics, Fall 2017)
2. Prove or disprove that the set of all functions mapping  $\mathbb{N}$  to  $\mathbb{N}$  is countable  
(from midterm exam of Discrete Mathematics, Fall 2018)
3. Prove or disprove that  $|\mathcal{P}(S)| < |\mathcal{P}(\mathcal{P}(S))|$  for a countably infinite set  $S$   
(from midterm exam of Discrete Mathematics, Fall 2019)
4. Compare the cardinalities of a range of real numbers  $[0, 1)$  and  $\mathcal{P}(\mathbb{N})$   
(from midterm exam of Discrete Mathematics, Fall 2019)

You must turn in your papers by **11 PM, Oct 26 (Mon)**. You can drop a hardcopy into a homework box at OH 313. Or, you can submit an electric copy by scanning your papers. You must create an PDF file with the scanned images and submit it via Hisnet.

All answers must be handwritten. There is no restriction on the number of pages that you submit.