ITP 20002-{01, 02} Discrete Mathematics, Fall 2020

Midterm

10 problems

8 PM—10 PM, 22 Oct 2020

Overview

Problem 1	15 points	**Problem 6**	6 points
Problem 2	12 points	**Problem 7**	4 points
Problem 3	8 points	**Problem 8**	3 points
Problem 4	9 points	**Problem 9**	3 points
Problem 5	15 points	**Problem 10**	21 points

Problem 1 (15 point)

Read the following statements quoted from Handong CSEE Standard and then write "*I agree to uphold Handong Honor Code and Handong CSEE Standard in taking this exam.*" by yourself on the answer box.

Examination

1. Examination is an educational act necessary for evaluation of the students' achievement and for encouraging the students to absorb the material in the process of preparation.
2. Student should do their best to prepare for exams in order to improve her/his own knowledge and skill and should fully engage in the test during examination hour.
3. Accessing or providing unauthorized information, including other students' answer sheets, is regarded as cheating. The use of electronic devices, including cell phones and computers, without permission is strictly prohibited.
4. Entering or leaving the classroom during the examination before the finish time without permission is regarded as cheating.

Problem 2 (12 points)

Write a proof using the rules of inferences to show that $\forall x \in D\big(P(x) \rightarrow \neg Q(x)\big)$ follows from $\nexists x \in D\big(\neg S(x) \wedge R(x)\big)$, $\forall x \in D\ \big(P(x) \rightarrow R(x)\big)$, and $\nexists x \in D\big(S(x) \wedge Q(x)\big)$.

* You can find the list of inference rules in the next slide

p $p \to q$ $\therefore q$	Modus ponens
$\neg q$ $p \to q$ $\therefore \neg p$	Modus tollens
$p \to q$ $q \to r$ $\therefore p \to r$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	Disjunctive syllogism
p $\therefore p \vee q$	Addition
$p \wedge q$ $\therefore p$	Simplification
p q $\therefore p \wedge q$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore q \vee r$	Resolution

De Morgan's Law

$$\frac{\neg(p \wedge q)}{\therefore \neg p \vee \neg q} \qquad \frac{\neg(p \vee q)}{\therefore \neg p \wedge \neg q}$$

$$\frac{\forall x(P(x) \wedge Q(x))}{\therefore \forall x\, P(x) \wedge \forall x Q(x)} \qquad \frac{\exists x(P(x) \vee Q(x))}{\therefore \exists x\, P(x) \vee \exists x Q(x)}$$

$$\frac{\neg \forall x P(x)}{\therefore \exists x\, \neg P(x)} \qquad \frac{\neg \exists x P(x)}{\therefore \forall x\, \neg P(x)}$$

Universal instantiation

$$\frac{\forall x P(x)}{\therefore P(c)}$$

Universal generalization

$$\frac{P(c) \text{ for an arbitrary } c}{\therefore \forall x P(x)}$$

Existential instantiation

$$\frac{\exists x P(x)}{\therefore P(c) \text{ for some element } c}$$

Existential generalization

$$\frac{P(c) \text{ for some element } c}{\therefore \exists x P(x)}$$

Problem 3 (8 points)

Suppose that $f : A \to B$ and $g : B \to C$ are given.

Use the set builder notation to represent $g \cdot f$.

Problem 4 (9 points)

Give a recursive definition of the set of bit strings (i.e., strings with 0 and/or 1) that have more zeros than ones.

Problem 5 (15 points)

Prove that, for a given infinite sequence of real numbers, the set of all its subsequences is uncountable.

Your proof must use a structure of the Cantor's diagonal argument.

Problem 6 (8 points)

Use mathematical induction to show that

$\frac{1}{1\times3}+\frac{1}{3\times5}+\ \dots+\frac{1}{(2n-1)\times(2n+1)}=\frac{n}{2n+1}$ holds for $n\in\mathbb{N}$.

Problem 7 (6 points)

Explain what is *proof-by-contradiction*, and why it works.

Problem 8 (3 points)

A Sudoku puzzle instance may have multiple solutions.

How can we find two different solutions of a given Sudoku puzzle instance, if there exist, using a SAT solver? Explain your idea briefly.

Problem 9 (3 points)

What is the cardinality of $\mathcal{P}(\{\emptyset, \mathcal{P}(\emptyset)\})$?

Problem 10 (21 points)

Suppose that you are given a propositional formula ϕ defined over a set of propositional variables V. ϕ is an element of the well-formed formula set F recursively defined as follows:

- Basis step: **True** $\in F$, **False** $\in F$, and $v \in F$ for $v \in V$
- Recursive step: $(\neg e_1) \in F$, $(e_1 \wedge e_2) \in F$, and $(e_1 \vee e_2) \in F$ for $e_1 \in F$ and $e_2 \in F$

Answer to each of the following questions:

(a) Define M, the set of all possible assignments (i.e., models) for ϕ

(b) Give a recursive definition of a function $eval : F \times M \rightarrow \{\mathbf{True}, \mathbf{False}\}$ which finds the evaluation of a given propositional formula with respect to a given assignment.

(c) Write a predicate formula that specifies the condition that ϕ is valid.