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Ques 1 what is the result of the matrix operation A+C?

Soln 1 :-

$$A = \begin{bmatrix} -4 & 0 & 2 \\ 3 & 1 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A+C = \begin{bmatrix} -4 & 0 & 2 \\ 3 & 1 & 5 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4+1 & 0+0 & 2+0 \\ 3+0 & 1+1 & 5+0 \\ 1+0 & 2+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 2 \\ 3 & 2 & 5 \\ 1 & 2 & 2 \end{bmatrix}$$

Ans:-

$$A+C = \begin{bmatrix} -3 & 0 & 2 \\ 3 & 2 & 5 \\ 1 & 2 & 2 \end{bmatrix}$$

Ques2 What is result of the matrix operation B+D?

Soln 2:

$$B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 4 & 2 \\ 3 & 5 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 3 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$$

Adding a 3×3 matrix to a 3×2 matrix directly, it is not possible because matrix addition requires matrices to have the same dimensions.

Ques3 What is the result of the matrix operation B^T?

Soln 3:

$$B = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 4 & 2 \\ 3 & 5 & 1 \end{bmatrix}$$

B^T → The transpose of the matrix, it switches the row and column indices of the matrix B, by producing other matrix, denoted by B^T

$$B^T = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 4 & 5 \\ -1 & 2 & 1 \end{bmatrix}$$

Ans:

$$B^T = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 4 & 5 \\ -1 & 2 & 1 \end{bmatrix}$$

Q-4 What is the Result of the matrix operation $2E-3D$?

Soln $E = \begin{bmatrix} 6 & -2 \\ 0 & 4 \\ 1 & 5 \end{bmatrix}$

$D = \begin{bmatrix} 3 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$

$2E = 2\begin{bmatrix} 6 & -2 \\ 0 & 4 \\ 1 & 5 \end{bmatrix}$ | $3D = 3\begin{bmatrix} 3 & 0 \\ 2 & -1 \\ 1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 6*2 & -2*2 \\ 0*2 & 2*4 \\ 1*2 & 2*5 \end{bmatrix}$ | $= \begin{bmatrix} 3*3 & 0*3 \\ 2*3 & -1*3 \\ 1*3 & -2*3 \end{bmatrix}$

$2E = \begin{bmatrix} 12 & -4 \\ 0 & 8 \\ 2 & 10 \end{bmatrix}$ | $3D = \begin{bmatrix} 9 & 0 \\ 6 & -3 \\ 3 & -6 \end{bmatrix}$

$2E-3D = \begin{bmatrix} 12 & -4 \\ 0 & 8 \\ 2 & 10 \end{bmatrix} - \begin{bmatrix} 9 & 0 \\ 6 & -3 \\ 3 & -6 \end{bmatrix}$

$= \begin{bmatrix} 12-9 & -4-0 \\ 0-6 & 8-(-3) \\ 2-3 & 10-(-6) \end{bmatrix}$

$$2E-3D = \begin{bmatrix} 3 & -4 \\ -6 & 11 \\ -1 & 16 \end{bmatrix}$$

Q5 What is the result of the matrix operator?

FH

Soln

$$F = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \end{bmatrix} \qquad H = \begin{bmatrix} 7 & -2 \\ 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$FH = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \end{bmatrix} \begin{bmatrix} 7 & -2 \\ 4 & 1 \\ 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3*7+(1)*(4)+4*2 & (-2)*3+1*1+4*3 \\ 2*7+0*4+(-1)*2 & 2*(-2)+0*1+(-1)*3 \end{bmatrix}$$

$$= \begin{bmatrix} 21+4+8 & -6+1+12 \\ 14+0-2 & -4+0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 33 & 7 \\ 12 & -7 \end{bmatrix}$$

Ans:- $$FH = \begin{bmatrix} 33 & 7 \\ 12 & -7 \end{bmatrix}$$

Q-6 What is the Result of the matrix operation HG?

Soln- $H = \begin{bmatrix} 7 & -2 \\ 4 & 1 \\ 2 & 3 \end{bmatrix}$ $G = \begin{bmatrix} 3 & 2 & 3 \\ 0 & 1 & -1 \\ 5 & 1 & 0 \end{bmatrix}$

It is not possible to multiply (3*2) matrix and 3×3 matrix

The Matrix of H column and the row of matrix G size need to be same.

Q-7 What is the result of the matrix operation (6F)(4I)?

$$(6F)(4I) = \left(6\begin{bmatrix} 3 & 1 & 4 \\ 2 & 0 & -1 \end{bmatrix}\right)\left(4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 6*3 & 1*6 & 4*6 \\ 6*2 & 0*6 & -1*6 \end{bmatrix} \begin{bmatrix} 1*4 & 0*4 & 0*4 \\ 0*4 & 1*4 & 0*4 \\ 0*4 & 0*4 & 1*4 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 6 & 24 \\ 12 & 0 & -6 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} (18*4)+(0\times6)+(0\times6) & (18\times0)+(6*4)+(24*0) & (0*18)+(6\times0)+(24*4) \\ (12*4)+(0\times0)+(-6\times0) & (12\times0)+(0\times4)+(-6\times0) & (12\times0)+(0\times0)+(-6\times4) \end{bmatrix}$$

$$= \begin{bmatrix} 72+0+0 & 0+24+0 & 0+0+96 \\ 48+0+0 & 0+0+0 & 0+0-24 \end{bmatrix}$$

$$\begin{bmatrix} 72 & 24 & 96 \\ 48 & 0 & -24 \end{bmatrix}$$

Ques Are the two matrixes below inverse?

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 2 & 0 \end{bmatrix}$$

Soln:- To find the matrixes are inverse of each other we used the formula

$$(\text{Matrix})(\text{Matrix})^{-1} = I \quad (\text{Identity matrix})$$

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 2 & 0 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 4 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1\times1)+(0\times0)+(4\times4) & (1\times0)+(0\times1)+(4\times2) & (1\times1)+(0\times0)+(4\times0) \\ (0\times1)+(1\times0)+(2\times4) & (0\times0)+(1\times1)+(2\times2) & (0\times1)+(1\times0)+(2\times0) \\ (1\times1)+(0\times0)+(0\times4) & (1\times0)+(0\times1)+(0\times2) & (1\times1)+(0\times0)+(0\times0) \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+16 & 0+0+8 & 1+0+0 \\ 0+0+8 & 0+1+4 & 0+0+0 \\ 1+0+0 & 0+0+0 & 1+0+0 \end{bmatrix}$$

$$\begin{bmatrix} 17 & 8 & 1 \\ 8 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Not inverse, The two matrix are not inverse.

Q-9 If the matrix below is A, find A^{-1} if possible

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & -1 \\ -2 & -1 & -2 \end{bmatrix}$$

Soln:-

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ -2 & -1 & -2 & 0 & 0 & 1 \end{array}\right]$$

(i) operation :-

Add 2 times row1 to row3

$R_3 \longrightarrow R_3 + 2R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ -2+(2\times1) & -1+(2\times2) & -2+(2\times-1) & 0+(1\times2) & 0+(0\times2) & 1+(0\times2) \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ -2+2 & -1+4 & -2+(-2) & 0+2 & 0+0 & 1+0 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & -1 & 1 & 0 & 0 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(ii) Operate :- $R_1 \longrightarrow -4R_1 + R_3$

(Add 4 times R_1 to R_3)

Add Row 3 to (-4) times Row 1

$$\left[\begin{array}{ccc|ccc} (1\times-4)+0 & 2\times(-4)+3 & (-4\times-1)+(-4) & (-4\times1)+2 & (-4\times0)+0 & (-4\times0)+1 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4+0 & -8+3 & 4+0-4 & -4+2 & 0+0 & 0+1 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 4-4 & -2 & 0 & 1 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 3 & 5 & -1 & 0 & 1 & 0 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(iii) Operate

$R_2 \longrightarrow R_2 + R_1$

Add row 1 to Row 2

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 3+(-4) & 5+(-5) & -1+0 & 0+(-2) & 1+0 & 0+1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 3-4 & 5-5 & -1+0 & 0-2 & 1+0 & 0+1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ -1 & 0 & -1 & -2 & 1 & 1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(iv) operate $R_2 \longrightarrow -4R_2 + R_1$

Add Row1 to -4 times Row2

$$\left[\begin{array}{ccc} -4 & -5 & 0 \\ (-4\times-1)+(-4) & (-4\times0)+(-5) & (-4\times-1)+0 \\ 0 & 3 & -4 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ (-4\times-1)+(-4) & (-4\times0)+(-5) & (-4\times-1)+0 & (-4\times-2)-2 & (-4\times1)+0 & (-4\times1)+1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 4-4 & -5-5 & 4+0 & 8-2 & -4+0 & -4+1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 0 & -5 & 4 & 6 & -4 & -3 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(v) $R_2 \longrightarrow R_2' + R_3$

Add Row2 to Row3

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 0+0 & -5+3 & -4+4 & 6+2 & -4+0 & -3+1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 0 & -2 & 0 & 8 & -4 & -2 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(vi) operate $R_2 \longrightarrow R_2 \times \frac{-1}{2}$

Multiply Row 2 by $(-1/2)$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 0\times(-1/2) & -2\times(-1/2) & 0\times(-1/2) & 8\times(-1/2) & -4\times(-1/2) & -2\times(-1/2) \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5 & 0 & -2 & 0 & 1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(vii) operate :- Add five time Row 2 to Row 1

$R_1 \longrightarrow R_1 + 5R_2$

$$\left[\begin{array}{ccc|ccc} -4+10\times5) & -5+(1\times5) & 0+(0\times5) & -2+(5\times-4) & 0+(2\times5) & 1+(5\times1) \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & -5+5 & 0 & -2+(-20) & 0+10 & 1+5 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} -4 & 0 & 0 & -22 & 10 & 6 \\ 0 & 1 & 0 & -4 & +2 & 1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(viii) Operate

Multiply Row 1 by $-1/4$

$R_1 \rightarrow R_1 \times -1/4$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 3 & -4 & 2 & 0 & 1 \end{array}\right]$$

(ix) operate :- Multiply Add (three) times Row 2 to Row 3

(ix) operate :- Add (-3) times Row 2 to Row 3

$R_3 \rightarrow R_3 + (-3) R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0+0 & 3+(1\times-3) & -4+(-3\times0) & 2+(-4\times-3) & 0+(-3\times2) & 1+(-3\times1) \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 3-3 & -4+0 & 2+12 & 0-6 & 1-3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & -4 & 14 & -6 & -2 \end{array}\right]$$

(x) operate :- Multiply Row 3 by $-1/4$

$$R_3 \to R_3 \times -1/4$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -7/2 & 3/2 & 1/2 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & -4\times -1/4 & 14\times -1/4 & -6\times -1/4 & -2\times -1/4 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 11/2 & -5/2 & -3/2 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -7/2 & 3/2 & 1/2 \end{array}\right]$$

Ans $A^{-1} = \begin{bmatrix} 11/2 & -5/2 & -3/2 \\ -4 & 2 & 1 \\ -7/2 & 3/2 & 1/2 \end{bmatrix}$

Check find:- $AA^{-1} = I_3$

$$\begin{bmatrix} 1 & 2 & -1 \\ 3 & 5 & -1 \\ -2 & -1 & -2 \end{bmatrix} \begin{bmatrix} 11/2 & -5/2 & -3/2 \\ -4 & 2 & 1 \\ -7/2 & 3/2 & 1/2 \end{bmatrix}$$

$$\begin{bmatrix} 1\times\frac{11}{2} + 2\times -4 + -1\times -7/2 & 1\times -5/2 + 2\times 2 + -1\times 3/2 & (1\times -3/2)+2\times 1+(-1\times 1/2) \\ 3\times 11/2 + 5\times(-4) + (-1)\times(-7/2) & (3\times -5/2) + 5\times 2 + (-1\times 3/2) & (3\times -3/2)+(5\times 1)+(-1\times 1/2) \\ -2\times 11/2 + (-1)(-4) + (-2)(-7/2) & (-2\times -5/2)+(-1\times 2)+(-2\times 3/2) & \left(-2\times\frac{-3}{2}\right)+(-1\times 1)+(-2\times 1/2) \end{bmatrix}$$

$$\begin{bmatrix} \frac{11}{2} - 8 + \frac{7}{2} & \frac{-5}{2} + 4 - \frac{3}{2} & \frac{-3}{2} + 2 - \frac{1}{2} \\ \frac{33}{2} - 20 + \frac{7}{2} & \frac{-15}{2} + 10 - \frac{3}{2} & \frac{-9}{2} + 5 - \frac{1}{2} \\ \frac{-22}{2} + 4 + \frac{14}{2} & \frac{10}{2} - 2 - \frac{6}{2} & \frac{6}{2} - 1 - \frac{2}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{18}{2} - 8 & \frac{-8}{2} + 4 & \frac{-4}{2} + 2 \\ \frac{40}{2} - 20 & \frac{-18}{2} + 10 & \frac{-10}{2} + 5 \\ \frac{-8}{2} + 4 & \frac{4}{2} - 2 & \frac{4}{2} - 1 \end{bmatrix}$$

$$\begin{bmatrix} 9-8 & -4+4 & -2+2 \\ 20-20 & -9+10 & -5+5 \\ -4+4 & 2-2 & 2-1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$$AA^{-1} = I_3$$

Hence proved.

Q10 What is the determinant of the matrix below?

$$\begin{bmatrix} 2 & -8 \\ 3 & -6 \end{bmatrix}$$

Soln

Determinant, is denoted $|A|$ of a 2×2 matrix is the product of two elements on the main diagonal minus the product of the other two elements

$$A = \begin{bmatrix} 2 & -8 \\ 3 & -6 \end{bmatrix}$$

$$|A| = (-6 \times 2) - (-8 \times 3)$$
$$|A| = -12 - (-24)$$
$$|A| = -12 + 24$$
$$\boxed{|A| = 12}$$

Ans

$$\boxed{\text{Determinent} = 12}$$

Determinent of A = 12

Q11 Gaussian Equations

$$x + y + (-z) = -2$$
$$2x - y + z = 5$$
$$-x + 2y + 2z = 1$$

Soln:-

$$x + y - z = -2$$
$$2x - y + z = 5$$
$$-x + 2y + 2z = 1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

(i) operate :- Add row 2 to row 1

$R_1 \rightarrow R_1 + R_2$

$$\left[\begin{array}{ccc|c} 1+2 & 1-1 & -1+1 & -2+5 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 3 & 0 & 0 & 3 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

(ii) operate :- Multiply row 1 by $\frac{1}{3}$

$R_1 \rightarrow R_1 \times \frac{1}{3}$

$$\left[\begin{array}{ccc|c} 3\times 1/3 & 0\times\frac{1}{3} & 0\times 1/3 & 3\times\frac{1}{3} \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 5 \\ -1 & 2 & 2 & 1 \end{array}\right]$$

(iii) operate :- Add row 2 to twotimes row 3

$R_3 \rightarrow 2R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 5 \\ (-1\times2)+2 & (2\times2)-1 & (2\times2)+1 & (2\times1)+5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 5 \\ -2+2 & 4-1 & 4+1 & 2+5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2 & -1 & 1 & 5 \\ 0 & 3 & 5 & 7 \end{array}\right]$$

(iv) Operate :- add (-2) times row 1 to Row 2

$R_2 \longrightarrow R_2 + (-2)R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2+(-2\times1) & -1+(-2\times0) & 1+(-2\times0) & 5+(-2\times1) \\ 0 & 3 & 5 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 2-2 & -1+0 & 1+0 & 5-2 \\ 0 & 3 & 5 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 3 & 5 & 7 \end{array}\right]$$

(v) operate :- add -3 times Row 2 to Row 3

$R_3 \longrightarrow R_3 + 3R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 3 \\ 0+(0\times3) & 3+(-1\times3) & 5+(1\times3) & 7+(3\times3) \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 8 & 9+7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 8 & 16 \end{array}\right]$$

(vi) operate :- multiply $\frac{1}{8}$ by R_3

$$R_3 \longrightarrow R_3 \times \frac{1}{8}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 3 \\ 0\times\frac{1}{8} & 0\times\frac{1}{8} & 8\times\frac{1}{8} & 16\times\frac{1}{8} \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

(vii) operate: add (-1) times row 3 to Row 2

$$R_2 \longrightarrow R_2 + (-1)R_3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0+(-1\times0) & -1+(-1\times0) & 1+(-1\times1) & 3+(-1\times2) \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 1-1 & 3-2 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

(viii) operate :- multiply R_2 by (-1) to row

$$R_2 \longrightarrow R_2 \times (-1)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ (-1\times0) & (-1\times1) & (0\times-1) & 1\times(-1) \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1\times-1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Solution :-

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$x=1, y=-1, z=2$

$(1, -1, 2)$

Check:-

$(x+y-z=-2) \times 2$

$2x-y+z=5$

$2x+2y-2z=-4$

$2x-y+z=5$

$-\quad +\quad -\quad -$

$3y-3z=-9$

$3(y-z)=-9$

$\boxed{y-z=-3}$

$x+y-z=-2$

$-x+2y+2z=1$

$x+y-z=-2$

$-x+2y+2z=1$

$3y+z=-1$

$\boxed{3y+z=-1}$

$y-z=-3$

$3y+z=-1$

$4y=-4$

$\boxed{y=-1}$

$3y+z=-1$

$3(-1)+z=-1$

$-3+z=-1$

$z=-1+3$

$\boxed{z=2}$

$x+y-z=-2$

$x+(-1)-(2)=-2$

$x-1-2=-2$

$x-3=-2$

$x=-2+3$

$\boxed{x=1}$

$x = 1$

$y = -1$

$z = 2$

hence proved.

The solution of the equation is $(1, -1, 2)$