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System take Home 1A

$m\ddot{z} = mg - k\,(i/z)^2$

a) $x_1 = z$ (position) / $x_2 = \dot{z}$

$\dot{x}_1 = x_2$ / $\dot{x}_2 = 0$ / $i = x_3$

$\ddot{x} = g - k/m\,(i/x_1)^2$

$\frac{dx_2}{dt} = g - k/m\,(x_3/x_1)^2$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ g - k/m\left(\frac{x_3}{x_1}\right)^2 \end{bmatrix}$$

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$x_3 = i$ (current)

b) $\ddot{z} = 0$ $\quad di/dt = 0$ $\quad \rightarrow 0 = g - \frac{i^2 k}{mz^2}$

$$\boxed{i_e = z_e\sqrt{\frac{mg}{k}}}$$

c) linearization.

$\dot{\hat{x}} = A\hat{x} + B\hat{u}$, $\quad \hat{x} = x - x_e$, $\quad \hat{u} = u - u_e$

$y = Cx + Du$ $\qquad u_e = i_e = z_e\sqrt{mg/k}$

$$A = \partial f/\partial x|_e = \begin{pmatrix} \frac{\partial \dot{z}}{\partial z} & \frac{\partial \dot{z}}{\partial \dot{z}} \\ \partial f/\partial z & \partial f/\partial \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{2kz_e^2 mg}{kmz_e^3} & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \frac{2g}{z_e} & 0 \end{pmatrix}.$$

$$B = \partial f/\partial u|_e = \begin{pmatrix} 0 \\ \frac{-2k}{mz^2}i \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{-2k}{mz_e}\left(\sqrt{\frac{mg}{k}}\right) \end{pmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

d) $TF = C(SI - A)^{-1}B + D$

$$(SI - A)^{-1} = \begin{bmatrix} s & -1 \\ -2g/z_e & s \end{bmatrix}^{-1} = \frac{1}{s^2 - \frac{2g}{z_e}}\begin{pmatrix} s & 1 \\ \frac{2g}{z_e} & s \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ \frac{-2k}{mz_e}\sqrt{\frac{mg}{k}} \end{pmatrix}$$

$$TF = \begin{pmatrix} \left(\frac{-2k}{mz_e}\right)\left(\sqrt{\frac{mg}{k}}\right) / \left(s^2 - \frac{2g}{z_e}\right) \\ \left(\frac{-2k}{mz_e}\right)(s)\left(\sqrt{\frac{mg}{k}}\right) / \left(s^2 - \frac{2g}{z_e}\right) \end{pmatrix}$$

FB State Space

e) The system unstable, one of the poles are imaginary, meaning there will be oscillatory response.

syst final (2)

a) $T_1 = T_1 - T_i \qquad T_2 = T_2 - T_i$ ... so on

$$\frac{dT_1}{dt} = \frac{T_i - T_1}{R_a C_1} \qquad \frac{dT_i}{dt} = \frac{T_1 - T_2}{R_b C_1}$$

$$\frac{dT_1}{dt} - \frac{dT_i}{dt} = \frac{(T_i - T_1)}{R_a C_1} - \frac{(T_1 - T_2)}{R_b C_1}$$

$$\frac{dT_2 - dT_i}{dt} = \frac{T_1 - T_2}{R_b C_2} - \frac{(T_2 - T_3)}{R_c C_2}$$

$$\frac{d(T_3 - T_i)}{dt} = \frac{T_2 - T_3}{R_c C_3} - \frac{(T_3 - T_4)}{R_d C_3}$$

$$\frac{d(T_4 - T_i)}{dt} = \frac{(T_3 - T_4)}{R_d C_4} - \frac{(T_4 - T_o)}{R_e C_4}$$

b) Total heat is conserved.

for: $T_1 = T_1 - T_i$, then $T_i = T_1 - T_1 = 0$

$T_2 = T_2 - T_i$, then $T_i = T_2 - T_2 = 0$

$\therefore T_i = 0$ then

$T_1 - T_2 = T_2 - T_3 = T_3 - T_4 = T_1 - T_o$

$(T_i\, X)$ meaning T_i doesn't matter and inside Temp is conserved regardless of T_i

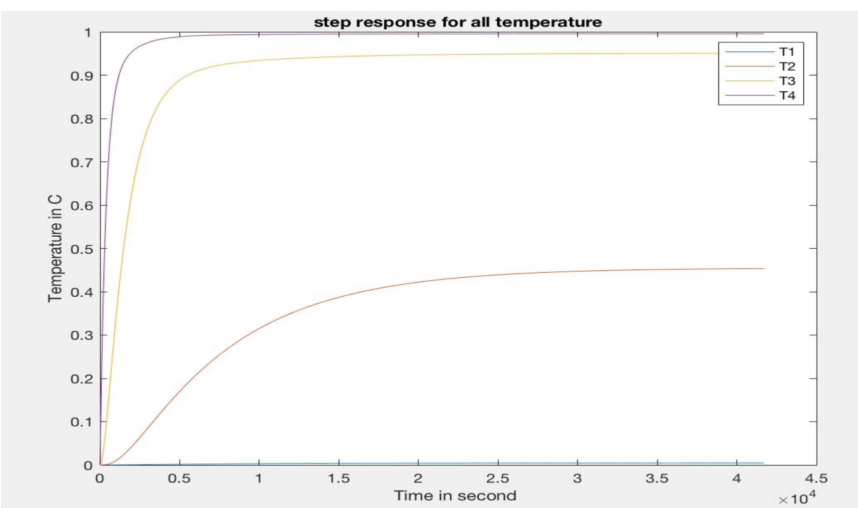
c)
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \left(-\frac{1}{R_a C_1} - \frac{1}{R_b C_1}\right) & \left(\frac{1}{R_b C_1}\right) & 0 & 0 \\ \frac{1}{R_b C_2} & \left(-\frac{1}{R_b C_2} - \frac{1}{R_e C_2}\right) & \left(\frac{1}{R_e C_2}\right) & 0 \\ \left(\frac{1}{R_e C_3}\right) & \left(-\frac{1}{R_e C_3} - \frac{1}{R_d C_3}\right) & \left(\frac{1}{R_d C_3}\right) & 0 \\ 0 & 0 & \frac{1}{R_d C_4} & \left(-\frac{1}{R_d C_4} - \frac{1}{R_e C_4}\right) \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} +$$

$$\underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{R_e C_4} \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} T_o - T_i \end{bmatrix}}_{u}$$

$$\begin{bmatrix} y \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{0}_{D}$$

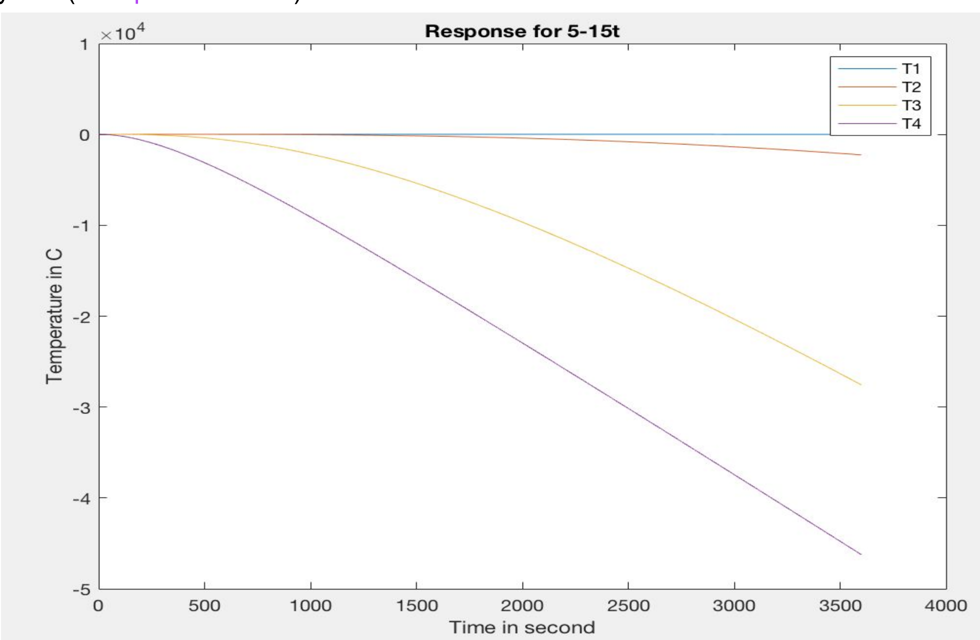
close all; clear all; clc;

```
%% D) step response of state space system
% parameters
% resistors
Ra = 0.02;
Rb = 2.00;
Rc = 2.2;
Rd = 0.2;
Re = 0.02;
% capacitors
C1 = 8700;
C2 = 6200;
C3 = 6600;
C4 = 20000;
% state space matrix
A1 = [ (-1/C1)*(1/Ra + 1/Rb) (1/C1)*(1/Rb) 0 0 ;
    (1/C2)*(1/Rb) (-1/C2)*(1/Rb + 1/Rc) (1/C2)*(1/Rc) 0];
A2 = [0 (1/C3)*(1/Rc) (-1/C3)*(1/Rc + 1/Rd) (1/C3)*(1/Rd);
    0 0 (1/C4)*(1/Rd) (-1/C4)*(1/Rd + 1/Re) ];
A = [A1; A2]
B = [ 0 ;
     0 ;
     0 ;
    (1/C4)*(1/Re)];
C = eye(4);
D = [ 0;
     0;
     0;
     0 ];
sys = ss(A,B,C,D);
[Y,T] = step(sys);
```




```
% state space graph
figure(1)
plot(T,Y)
legend('T1', 'T2', 'T3', 'T4')
title('step response for all temperature')
xlabel('Time in second ')
ylabel('Temperature in C')
```

```
%% E)
% time condition
t = 0:0.1:3600;
% state space graph
sys = ss(A,B,C,D);
x0 = [ -5;
       -5;
       -5;
       -5 ];
u = -15 - 15*t;
[Y, T] = lsim(sys,u,t,x0);
T1 = Y(:,1)+20;
T2 = Y(:,2)+20;
T3 = Y(:,3)+20;
T4 = Y(:,4)+20;
figure(2)
plot(T,T1,T,T2,T,T3,T,T4)
legend('T1','T2','T3','T4')
title('state space graph with time condition ')
xlabel('Time in second')
ylabel('Temperature in C')
```




```
%% F)
% calculate the poles of the system
SP=ss(A,B,C,D)
TF=tf(SP)
%poles natural frequnecy and damping
[wn,zeta,p]=damp(TF)
Ra=0.02;C1=8700

% the poles graph shows that 0.0058 is the fastest
% and 0.0001 is the slowest
```

```
wn =

    0.0001
    0.0007
    0.0028
    0.0058


zeta =

      1
      1
      1
      1


p =

   -0.0001
   -0.0007
   -0.0028
   -0.0058
```

```
%% G)
%time constant approximation
figure(4)
sys = ss(A,B,C,D);
[Y,T] = step(sys);
plot(T,Y(:,1))
title('Step Response for T1')
xlabel('Time in second')
ylabel('Temperature in C')
```