

Title : Compare and Contrast on Different Types of Differentiation

1. Compare and contrast symbolic differentiation, numerical differentiation, and automatic differentiation.

Numerical Differentiation	Finite difference approximation of derivatives using values of the original function evaluated at some sample points
Advantage	Simple
Disadvantage	Highly inaccurate with multiple n dimensions Truncation error/ round off error, two are in inverse trade off relationship depending on magnitude of step h.
Attempts to improve	Higher order finite differences, Richardson extrapolation to the limit, differential quadrature methods using weighted sums

Symbolic Differentiation	Automatic manipulation of expressions for obtaining derivative expression, carried out by applying transformations representing rules of differentiation.
Advantage	Can give valuable insight into the structure of the problem domain, and produce analytical solutions of extrema that can eliminate the needs for derivative calc wholesomely.
Disadvantage	Expression swell - producing exponentially large symbolic expression which takes correspondingly long to evaluate. (inefficient runtime calculator of derivative values)
Attempts to improve	Combining symbolic + numerical can yield ideal differentiation = AD

Automatic Differentiation	Non standard interpretation of a computer program that involves augmenting the standard computation with the calculation of various derivatives.
Advantage	Effective in classical closed -form, Also applicable in kaiing use of control flow such as branching, loops,. Recursion, producer call, (wider expressivity) “AD is blind with respect to aunty operation, which do not directly alter numeric values”

2. Compare and contrast forward-mode automatic differentiation with reverse-mode automatic differentiation.

Forward Mode AD	<p>Forward Mode applies general chain rule to each basic operation in the forward pass, obtaining a derivative trace. Compute the Jacobian of a function with n independent variables x and m dependent variable y_i</p> <p>(sets only one $x = 1$ and rest to zero (the i th unit vectors)) i</p> <p>Use of dual number</p> <p>$v + v\epsilon$: extract the derivative of a function by interpreting any non dual number v as $v + 0\epsilon$, and evaluate the function with initial input of coefficient 1 for ϵ .</p>
Advantage	When $m(\text{Dependent variable}) \gg n(\text{independent variable})$, reverse mode performs better than forward mode
Reverse Mode AD	<p>Reverse Mode corresponds to a generalized backpropagation algorithm, that propagates derivatives backward from the given output.</p> <p>Processed through two phases</p> <p>Forward phase - all intermediate variables are evaluated, stored in memory.</p> <p>Backward phase - propagates back the derivatives/ adjoints with help of chain rule</p> <p>Since Machine/ Deep learning is based on gradient of scalar valued objective with respect to a multiple parameters, the reverse mode is the mainstay technique in the form of the backpropagation algorithm</p>
Advantage	When $m(\text{Dependent variable}) \ll n(\text{independent variable})$, reverse mode performs better than forward mode
Disadvantage	Increased storage requirements in proportion to the number of the operation in the evaluated function.