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Problem 1

Part 1: Solve Diff Eqs

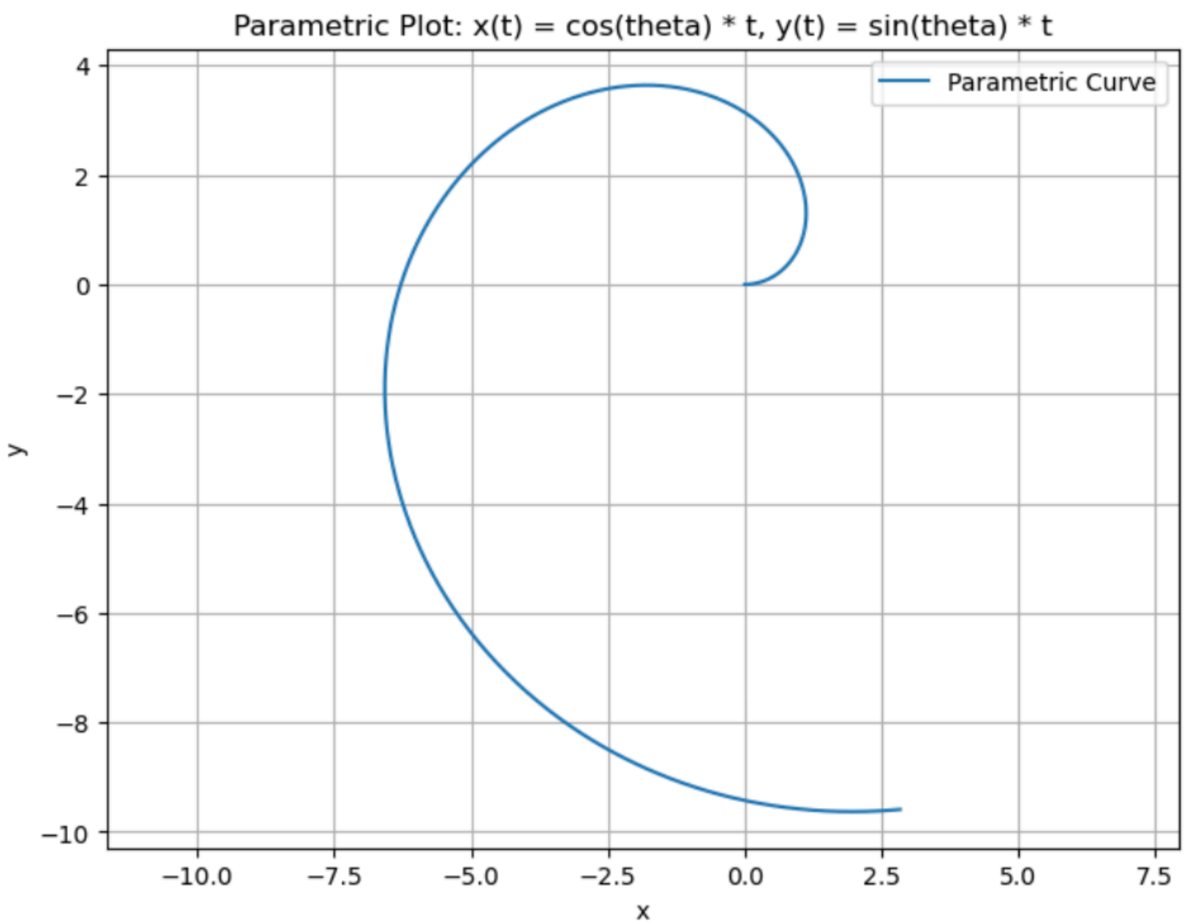
Given: $u = 0.5$ and $\frac{d\theta}{dx} = u$, we know that $\theta = 0.5t$

So, we can directly plug in theta into $\frac{dx}{dt}$ and $\frac{dy}{dt}$ and find the equations for the car's trajectory since v = 1:

$$x(t) = cos(\theta) * t$$

$$y(t) = sin(\theta) * t$$

Part 2: Plot



Part 3: Describe path

The path taken by the car follows a spiral trajectory due to the sinusoidal functions for x and y. Increasing the turning rate would lead to sharper turns and a tighter spiral

Problem 2

Is the system asymptotically stable for b = +/−4? Explain why, considering the eigenvalues of A.

$$A = \begin{bmatrix} 1 & 4 \\ -4 & b \end{bmatrix}$$

Find Eigenvectors

$$|\begin{bmatrix} 1-\lambda & 4 \\ -4 & b-\lambda \end{bmatrix}|$$

$$16 + (1-\lambda)(b-\lambda) = 0$$

$$b - (b+1)\lambda + \lambda^2 + 16 = 0$$

$$\lambda = \frac{b+1 \pm \sqrt{(b+1)^2 - 4(b+16)}}{2}$$

When **b = -4**, the eigenvalue has a negative real part so it's **asymptotically stable**

$$\lambda = \frac{-3 \pm \sqrt{9 - 4(12)}}{2}$$

When **b = 4**, the eigenvalue has a negative real part so it's **unstable**

$$\lambda = \frac{5 \pm \sqrt{25 - 4(20)}}{2}$$

Problem 3

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = Ax + Bu$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -\begin{bmatrix} 2 & k_{12} \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$u = -Kx$$

$$\dot{x} = [A - Bk]x$$

$$A - Bk = \begin{bmatrix} -1 & 2 - k_{12} \\ 1 & 0 \end{bmatrix}$$

Characteristic Equation

$$\det[sI - [A - Bk]] = 0$$

$$\det \begin{bmatrix} s+1 & -2 + k_{12} \\ -1 & s \end{bmatrix}$$

$$s^2 + s - 2 + k_{12} = 0$$

$$s = \frac{-1 \pm \sqrt{1 - 4(k_{12} - 2)}}{2}$$

System is stable if s has negative real parts, so $k_{12} > 2$ **returns stable system**