

# Calculus - I.

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102)

## Types of data.

Data.

Information, facts, lib. bld. about spes.

Categorical

(Define categories  
or groups)

Numerical

Discrete

b counted items

Continuous.

b time, pressure

→ Benefit of visual representation of data:

→ Create picture of data: makes abstract data more concrete and understandable.

→ make cognitive processing easier

→ content view at glance: giving users comprehensive understanding of data.

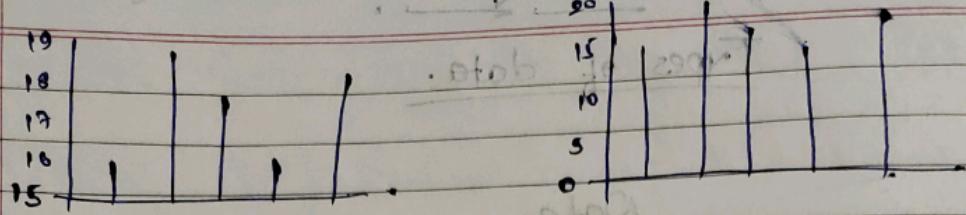
→ direct attention toward content.

→ describe, explore, and summarize.

# FOUR UMBRELLA PRINCIPLE of effective visualization.

→ KNOW PURPOSE :- having purpose statement for every table or graph. You create message. Purpose is not necessary a message

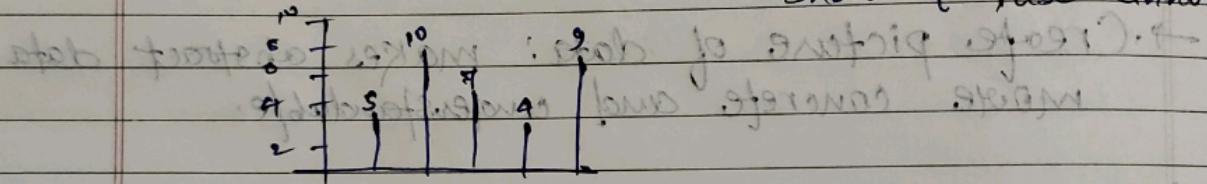
→ ENSURE INTEGRITY :- accuracy & consistency



some data but different represented  
says different information.

3) MAXIMIZE DATA INK ; use less amount of  
MINIMIZE NON-DATA INK ink to convey most  
info. between amount of info.

4) SHOW YOUR DATA : ANNOTED or don't hide data,  
Show it, use annotations



raise writing writing style

# EXECUTING YOUR INFO display that 3-step  
PROCESS :

1) DEFINING MESSAGE

↳ what I'm trying to communicate

↳ what's message

↳ how do i make msg clear at glance

2) CHOOSING FORMS

↳ what should I use : text, table, graph  
chart, diagram

3) CREATING DESIGN ; for good design people pay  
for it, "presented" matters.

## Q. 1) DEFINING MESSAGE:

→ In what you want your audience to understand and remember. It could be data trends, relationship b/w various variables or summary.

→ tab patterns and layout → worthiness.

## Q. 2) CHOOSING FORM - TABLE or CHART?

### 1) Choose a Table :-

- display complete dataset
- present wider range of dataset that is difficult to scale using graph.
- focus on specific item

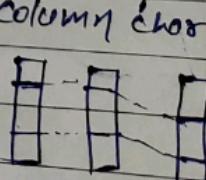
→ explain how numbers are derived - how results were calculated.

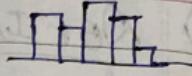
### 2) Choose a chart: In brief for : broad area

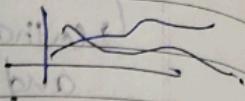
- compare a slice of information
- show changes over time
- show "causes of changes" and last
- show patterns of data distribution

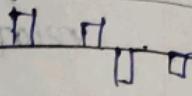
### CHOOSE APPROPRIATE GRAPH:

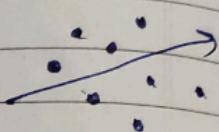
- components of one item → pie chart
- Components of multiple items → 100% column / stacked column chart



• Item comparison → BAR CHART 

• Change over time → column / line chart 

• Frequency distribution → Histogram 

• correlation → Paired bar, scatter dot 

### 3) CREATING DESIGNS:

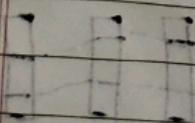
- avoid 3rd effect
- avoid legends; consider using labels
- avoid contrasting borders around objects
- use annotations to highlight keys

### 4) VISUALIZATION ON DASHBOARD:

Dashboard: A visual display of most imp. info  
Should on Dashboard | Should not.

- |                              |                     |
|------------------------------|---------------------|
| → Real time data.            | → complex visual    |
| → visual summary             | → excessive details |
| → filter / drill down option | ↳ use hyperlinks.   |
| → comparative data           | → Redundant metrics |

↳ best practice: most common → most effective for decision making.



- 1) To show complete data → use tabular form
- 2) To show pattern → use graph rather table
- 3) One item proportional to total → pie chart

Multiple item components → multiple item components

Comparison b/w items → Bar chart.

Trend over time → Line chart ↗↘.

Histogram frequency of items → Bar chart compare b/w items

Outliers identification → Box plot

Correlat<sup>n</sup> representat<sup>n</sup> → Scatter plot

Bar chart → compare b/w items.

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## Ch 2.

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→ The data values themselves are used directly in simulation. This is called "trace-driven simulation".

→ "FIT" a theoretical distribution to the data (and check whether that "fit" is good?)

→ Using data, we build our own distributions.  
↳ this is Empirical distribution.

# → for SYMMETRIC DISTRIBUTION: mean, median, mode  
Should be equal.

→ if these values are sufficiently closer to each other we can think of symmetric distribution.

→ Coefficient of variation (CV): ~~ratio~~  $\frac{\text{std. dev}}{\text{mean}} = \frac{s}{\bar{x}}$

→ the CV is 1 for exponential distribution.

Low CV: less variability related to mean.

→ means data points are close to av. value.

High CV: greater variability related to mean.

→ means data points are more spread out.

→ Lexis ratio: same as CV for discrete distribution.

$$L = \frac{\text{observed variance}}{\text{expected variance}}$$

$$\frac{n}{(n-1)(n-2)} \sum \frac{(x_i - \bar{x})^2}{S^2}$$

$L < 1$ : data is more uniform than expected

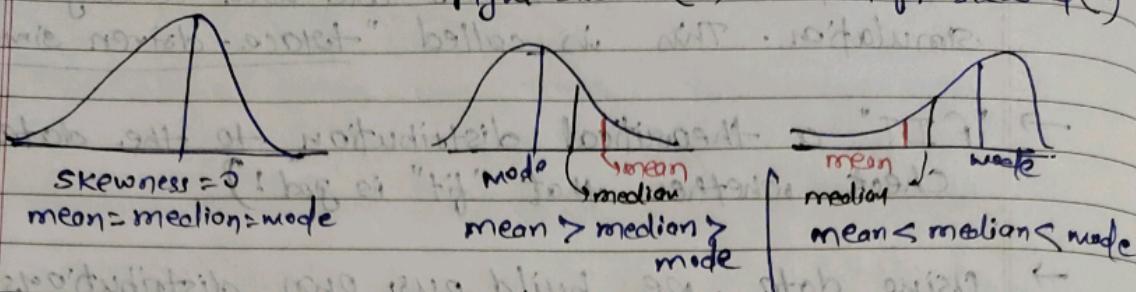
→ Skewness ( $\nu$ ): measure of symmetry of distribution.

for normal dist.  $\nu = 0$

for  $\nu > 0$ : right skewed (exponent dist.,  $\nu = 2$ )

for  $\nu < 0$ : left skewed

(i) right skewed (+) or left skewed (-)



### GOODNESS - OF - FIT:

↳ can be checked by several methods:

- i) frequency comparison (a bit technical)
- ii) probability plots (visual tool)
- iii) goodness-of-fit tests (statistical test of goodness)

### PROBABILITY PLOTS:

↳ Q-Q plot: Quantile-quantile plot

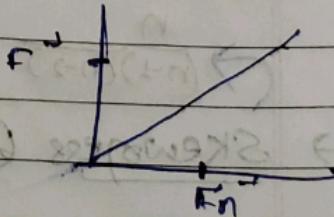
→ Graph of  $q_i$  - quantile of fitted (model) distribution versus  $q_i$  - quantile of sample distribution.

$$x_{q_i}^m = F^{-1}(q_i) \quad \text{and} \quad x_{q_i}^s = \tilde{F}_n^{-1}(q_i) = x_i$$

If this and this is equal then

Q-Q plot will be approximately linear with intercept 0 & slope 1

→ valid for discrete dataset only.



skewed (-)

2) P-P plot : prob-prob. plot.

→ A graph of model probability  $\hat{F}(x_i)$  against the sample probability  $\tilde{F}_n(x_i) = q$ .

→ valid for both continuous & discrete dataset.

if  $F(x)$  is correct distribution that is fitted, for a large sample size, then  $\hat{F}(x_i)$  &  $F(x_i)$  will be close together and P-P plot will be approx. linear with intercept 0 & slope 1.

# Q-Q plot amplify differences b/w tails of model distib'.

P-P plot amplify difference at middle point

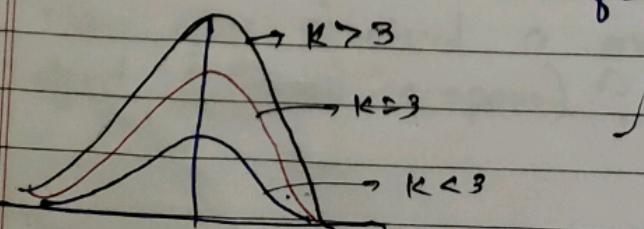


# KURTOSIS: describes the shape of distribution's tail ( $k$ ) → overall focusing on presence of outliers and "tailedness" of distribution.

•  $k=3$  (Meso kurtic) : Normal distrib' like behaviour with moderate tails.

*fatter tail peak sharper peak*  
•  $k > 3$  (Lepto kurtic) : more data in the tails, indicating higher likelihood of extreme outliers.

*thin tail flat peak*  
•  $k < 3$  (Platykurtic) : less data in the tails, indicating fewer outliers & flatter peak.



## Week-2 PA.

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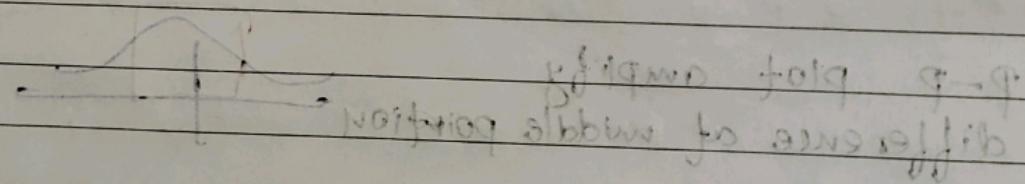
$$4) CV = \frac{\sigma}{\mu} = \frac{3.2}{10.7} \text{ folg } 0.299 \rightarrow \text{durch } 0.299$$

- 5) Binomial  $\rightarrow$  mean =  $n p$ , var =  $n p q$   
 Geometric  $\rightarrow$  Mean =  $\frac{1}{p}$ , var =  $\frac{q}{p^2}$

Poisson  $\rightarrow$  mean = var =  $\lambda$

6). Calculated value (19.70) is greater than tabulated value (8.25). This means calculated chi-square statistic exceeds critical value. In hypothesis test, if calculated value exceeds the tabulated (critical value) we reject null hypothesis ( $H_0$ )

→ since calculated value > critical  
 $p\text{-value} < 0.05$



folg 0.299  
 Werte liegen links von Mittelpunkt

• Chi-Square für "zufällig" mit zufälligen Ergebnissen für "zufällig" Nullhypothese  $\leftarrow H_0$   
 • Chi-Square für "zufällig" nur

• Chi-Square für "zufällig" :  $(\text{oben } 0.025\%) \delta = 2$

• Chi-Square für "zufällig"  $\delta = 2$

• Chi-Square für "zufällig" :  $(\text{oben } 0.05\%) \delta < 2$

• Chi-Square für "zufällig"  $\delta < 2$

### Canteek - 3

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Total 1200 candidate (960 male, 240 female)  
out of 1200, 324 were given offer letters

offer made	Male	Female	Total
Offered	288	36	324
Not offered	672	204	876
Total	960	240	1200

$$P(M \cap A) = 288/1200 = 0.24$$

M: male

F: female

A: admission offered

$$P(M \cap A^c) = 672/1200 = 0.56$$

$$P(F \cap A) = 36/1200 = 0.03$$

$$P(A) + P(A^c) = 1$$

$$P(F \cap A^c) = 204/1200 = 0.17$$

		Male	Female	Total	Marginal
A	Offer made	0.24	0.03	0.27	
	Not offered	0.56	0.17	0.73	
Total		0.8	0.2	1.0	

Joint Probability.

conditional  
prob:

$$P(A|M) = ? = \frac{P(A \cap M)}{P(M)} = \frac{0.24}{0.8}$$

$C = \frac{J}{M}$	$C = \text{conditional prob}$
$J = \text{joint prob}$	$M = \text{marginal prob}$



40% of emails are spam

the word "lottery" appears 30% of spam email

the word ... 5% of non-spam email

Now you received a mail contain word 'lottery'  
find  $P(\text{mail is spam})$ ?

$A = \text{spam mail}$        $B = \text{mail contains "lottery"}$   
 $A' = \text{not spam}$        $B' = \text{mail does not contain "lottery"}$

$$P(A|B) = ?$$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$$P(A) = 0.4$$

$$P(A') = 0.6$$

$$P(B) = 0.3$$

$$P(B') = 0.7$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A) \cdot P(B)}{P(A)} = P(B)$$

$$P(A) = 0.4$$

$$P(B|A) = 0.3$$

$$P(B|A') = 0.05$$

$$P(A|B) = 0.6$$

$$\text{Posterior probability: } P(B) = 0.3 \times 0.4 + 0.05 \times 0.6 = 0.15$$

$$P(A|B) = \frac{0.3 \times 0.4}{0.15} = 0.8$$

$$P(A|B) = \frac{0.3 \times 0.4}{0.15} = 0.8$$

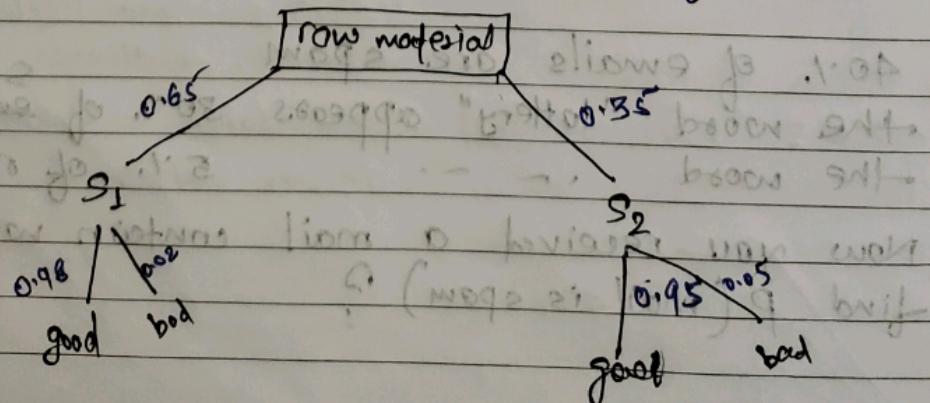
		Mail			
		Spam	Not spam		
Spam	0.4	0.2	0.8	0.6	0.4
	0.3	0.1	0.9		0.5
Not spam	0.1	0.9	0.1	0.05	0.95
	0.2	0.8	0.2		0.05

have word "lottery"

have word "lottery"

Bayes' rule

→ Bayes' rule is used to calculate posterior probability if we have initial belief (probability) and the additional sample information.



find  $P(\text{good quality comes from } S_1) = \frac{0.65 \times 0.98}{0.65 \times 0.98 + 0.35 \times 0.95} = \frac{0.637}{0.9895} = 0.65$

$$\frac{0.65 \times 0.98}{0.65 \times 0.98 + 0.35 \times 0.95} = \frac{0.637}{0.9895} = 0.65$$

find  $P(\text{bad quality comes from } S_1)$

$$\frac{0.65 \times 0.02}{0.65 \times 0.02 + 0.35 \times 0.05} = \frac{0.013}{0.013 + 0.0175} = \frac{0.013}{0.0305} = 0.42$$

### Chi-square distribution:

- Null hypothesis  $\rightarrow H_0$ : variables are independent.

$H_0: \text{the categorical variables are independent.}$

- Alternate hypothesis  $\rightarrow H_1$ : variables are not independent.

Let  $b_{ij}$  be the observed frequency (from sample)

Let  $e_{ij}$  be expected frequency

$\rightarrow$  The expected frequency ( $e_{ij}$ ) for a cell equals the product of row & column total for that cell, divided by total sample size.

City	Brand A	Brand B	Brand C	Total
Mumbai	279 (261.4)	73 (40.7)	225 (244.9)	577
Chennai	165 (182.6)	47 (49.3)	191 (171.1)	403
Total	444	120	416	980

$\rightarrow$  observed frequency

$$261.4 = \frac{577 \times 444}{980}$$

$\rightarrow$  expected frequency

$$182.6 = \frac{403 \times 444}{980}$$

- Chi-squared test statistic:

$$\chi^2 = \frac{\sum (b_{ij} - e_{ij})^2}{e_{ij}}$$

$$49.3 = \frac{403 \times 120}{980}$$

$$\chi^2 = \frac{(279 - 291.4)^2}{291.4} + \frac{(73 - 70.7)^2}{70.7} + \dots \text{for total 6 terms}$$

$$\chi^2_{\text{cal}} = 7.0 \quad (\text{this is Calculated test statistics value})$$

→ degree of freedom (df) = (row-1)(column-1)

$$(K-P-1) \quad (2-1)(3-1) = 2$$

(bins) → (parameters needs to calculate)

- df = 2, so at  $\alpha = 0.05$  (95% confidence), tabular value of test statistic. = 5.99

$$\therefore \chi^2_{\text{cal}} = 7.0 > \chi^2_{\text{test}} = 5.99$$

reject null hypothesis ( $H_0$ )

• ~~just based on~~ SSP additor for categories  $\rightarrow$   $H_1$

• df = 2, so at  $\alpha = 0.01$  (99% confidence), tabular value of

• ~~just based on~~ test statistic = 10.9.2  $\rightarrow$   $H_1$

(~~just based on~~)  $\chi^2_{\text{cal}} = 7.0 < \chi^2_{\text{test}} (9.21)$

~~just based on~~  $\chi^2_{\text{cal}} = 7.0 < \chi^2_{\text{test}} (9.21)$

~~just based on~~ accept null hypothesis ( $H_0$ )

~~just based on~~ accept null hypothesis ( $H_0$ )

lotot	2 hours	1 hour	1 hour	8 hours
FF2	222	222	222	822
FF4	201	201	201	801
FFD	814	814	814	814

~~just based on~~ frequency

~~just based on~~ frequency

for total 6 time  
p-value)

value

brief

p-value

value of

)

- at  $\alpha = 0.01$ .
- ① → if p-value is greater than  $\alpha = 0.01$  : then accept  $H_0$   
→ if p-value is less than  $\alpha = 0.01$  : then reject  $H_0$
- ② → if  $\chi^2_{cal} > \chi^2_{test}$  [reject  $H_0$ ]  
→ if  $\chi^2_{cal} < \chi^2_{test}$  [accept  $H_0$ ]

? = (which row fit in a major area of study) ?

	Sec-A	Sec-B	Total	— observed freq.
Sci	288 (269.7)	60 (78.25)	348	— expected freq.
Math	622 (640.25)	204 (185.7)	826	$269.7 = 348 \times 910 / 1174$
Total	910	264	1174	

$$\frac{\text{total Sci student}}{\text{total stud.}} \rightarrow \frac{348}{1174} \approx 0.296$$

$$③ P(\text{Math} | \text{Sec-B}) = \frac{P(\text{Math} \cap \text{Sec-B})}{P(\text{Sec-B})} = \frac{204}{264} = 0.77$$

$$④ \chi^2 = \frac{(288 - 269.7)^2}{269.7} + \frac{(60 - 78.25)^2}{78.25} + \frac{(622 - 640.25)^2}{640.25} + \frac{(204 - 185.7)^2}{185.7}$$

Using excel p-value = 0.005

= (CHISQ.Test).

∴ p-value <  $\alpha$

so reject  $H_0$

⑤ p-value is  $\alpha = 0.01$

still p-value (0.005) <  $\alpha$

reject  $H_0$

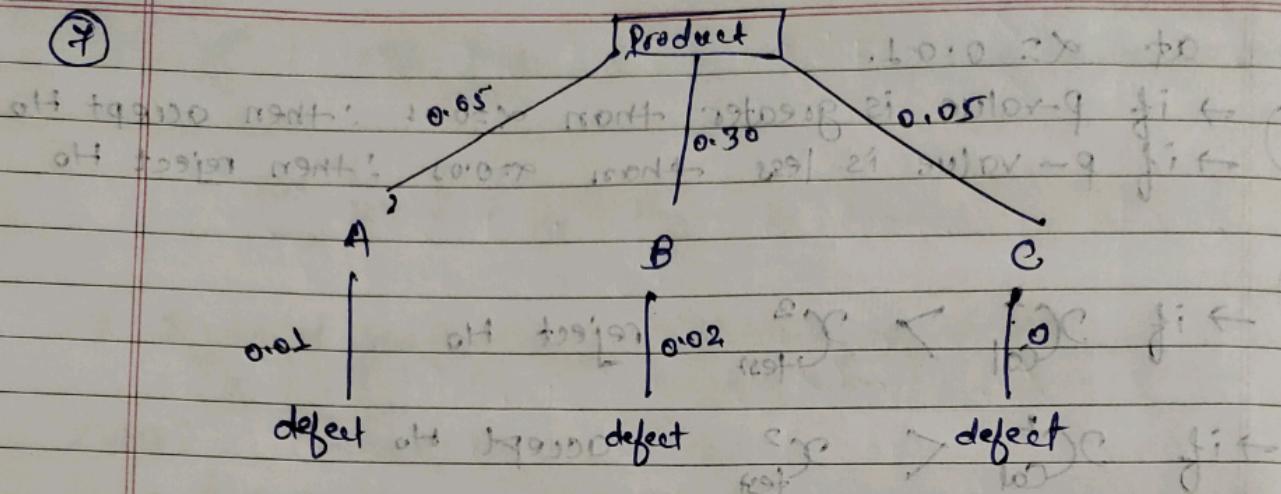
⑥  $\alpha = 0.005$

p-value (0.005) >  $\alpha$

accept  $H_0$

E - AP

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$$P(\text{Product come from } B \mid \text{it was defective}) = ?$$

$$\begin{aligned}
 &\text{prob based on lot size} = \frac{\text{lot size}}{\text{total}} = \frac{8-282}{8+82} = \frac{60}{125} = 0.48 \\
 &\text{prob based on defect rate} = \frac{0.30}{0.30 + 0.02 + 0.05} = \frac{0.30}{0.37} = \frac{30}{37} = 0.81 \\
 &= \frac{60}{65 + 60} = \frac{60}{125} = 0.48
 \end{aligned}$$

$$\frac{60}{125} = \frac{(8-282) \cap (\text{defective})}{(8-282) \cup (\text{defective})} = \frac{(8-282) \cap (\text{defective})}{(8-282) \cup (\text{good})} = \frac{(8-282) \cap (\text{defective})}{8+82} = \frac{60}{125} = 0.48$$

$$\frac{60}{125} = \frac{(8-282) \cap (\text{defective})}{20-82} + \frac{(20-82) \cap (\text{defective})}{20-82} + \frac{(20-82) \cap (\text{good})}{20-82} = \frac{60}{125} = 0.48$$

$$\frac{60}{125} = \frac{20-82 \cap (\text{defective})}{20-82} + \frac{20-82 \cap (\text{good})}{20-82} = \frac{60}{125} = 0.48$$

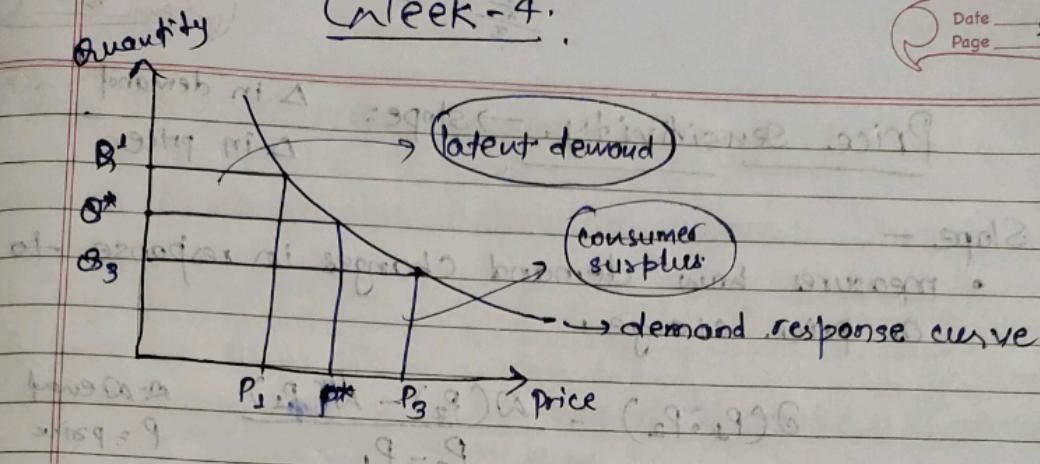
$$\frac{60}{125} = \frac{20-82 \cap (\text{defective})}{20-82} + \frac{20-82 \cap (\text{good})}{20-82} = \frac{60}{125} = 0.48$$

## Week - 4.

classmate

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### Demand Response Curve.

→ A function that describes how demand for a product  $d(P)$  varies as a function of its price.

→ Similar to the demand curve in economics, but for a single seller, in a single market.

### # Four properties:

1. Non-negative (both quantity & price)
2. continuous
3. Differentiable
4. Downward sloping

Latent Demand: a situation where there is a potential desire for a product, but market does not currently fulfill that demand. It might be because of

Product isn't available, price is very high or consumer aren't yet aware of product.

Consumer Surplus: difference b/w what consumers are willing to pay for good and what they actually pay.

Price sensitivity,  $\rightarrow$  slope =  $\frac{\Delta \text{ in demand}}{\Delta \text{ in price}}$

Slope -

- measures how demand changes in response to a price change.

$$\frac{\partial Q(P_1, P_2)}{\partial P_1} = \frac{Q(P_2) - Q(P_1)}{P_2 - P_1}$$

Q = demand  
P = price

① ELASTICITY: ratio of % change in demand to % change in price.

Elasticity =  $\frac{\% \text{ change in quantity}}{\% \text{ change in price}}$

$\rightarrow$  if  $E > 1$ , demand is elastic, means consumers are highly responsive to price change.  
(e.g. if price increases, demand decreases)

$\rightarrow E < 1$ , demand is Inelastic, mean even large price increase don't significantly reduce demand

$\rightarrow E = 0$ , demand is perfect inelastic, demand doesn't change at all with response to price changes.

$\rightarrow E = \infty$ , demand is perfect elastic, mean consumer will only buy at one price.

$\rightarrow$  Elasticity of 2 means to 1% reduction in price will yield a 2% increase in sale.

$\rightarrow E = 2 \rightarrow$  1% increase in price leads to 2% decrease in demand.

$\rightarrow E = 0.5 \rightarrow$  1% increase in price leads to 0.5% decrease in demand.

## LINEAR Response CURVE

Lee 3

Simplest price response curve:

$$D(P) = D_0 - m \cdot P$$

$D_0$  = demand at price = 0 (market size)  
 $m$  = slope

→ the price at which demand = 0 is called  
 (MP) ~~satisfying~~ satisfying price,  $P_s = \frac{D_0}{m}$

→ elasticity of this curve is

$$\epsilon = \frac{mP}{D_0 - mP}$$

$\epsilon = 0$  when  $P=0$

as  $P \rightarrow \frac{D_0}{m}$ ,  $\epsilon \rightarrow \infty$

$$x_1 + x_2 = (x = x_1 + x_2)$$

## Constant elasticity curve

(constant elasticity)

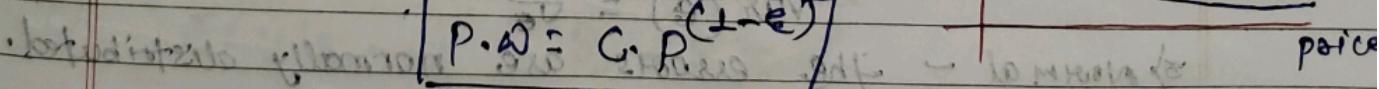
→ after algebraic transition, the constant elasticity curve is given by:

$$D = C \cdot P^{-\epsilon}$$

$C$  = constant (if it is demand when price = 1)

→ Revenue is.  $R = P \cdot D$

$$P \cdot D = C \cdot P^{(1-\epsilon)}$$



lec-5

## LINEAR DEMAND RESPONSE CURVE

- A SLR model can help to identify  $\beta_0$  ( $y$ -intercept) and  $\beta_1$  (slope)
- SLR tell us if the linear relationship is good fit for the data available from the market experiment

behavior at  $x = \text{firmsize}$  down to  $x = \text{size}$  suitable

### SIMPLE REGRESSION MODEL (SRM)

→ Equat<sup>bivariate</sup> of SRM describes how the conditional mean of  $y$  depends on  $x$ .

→ SRM shows that these means lie on a line with intercept  $\beta_0$  + slope  $\beta_1$ .

$$\mu_{y|x} = E(y|x=x) = \beta_0 + \beta_1 x$$

→ deviation of responses around  $\mu_{y|x}$  are called errors.

(predicted value)

→ Error, is denoted by  $\epsilon$ , and  $E(\epsilon) = 0$

# The SRM makes 3 assumptions about error term:

1) Independent - Errors are independent of each other.

2) Equal variance - all errors have some variance

$$\text{Var}(\epsilon) = \sigma^2$$

3) Normal - the errors are normally distributed.

1) Consumer surplus = (Willing to Pay) - (Actual price paid)

Product surplus = (Price received) - (min. price producer would accept)

Eg = if a company willing to sell at 500/- but sells it for 800/- then producer surplus is 300.

2) by reducing price

$$3) E = \frac{\% \text{ change in demand}}{\% \text{ change in price}} = \frac{(8-10)}{10} \times 100 = -20\%$$

$$\rightarrow \frac{(8.75 - 3.00)}{3.00} \times 100 = 187.5\% \quad \text{Ans: } 0.8$$

$$= \frac{5.75}{3.00} \times 100 = 187.5\% \quad \text{Ans: } 0.8$$

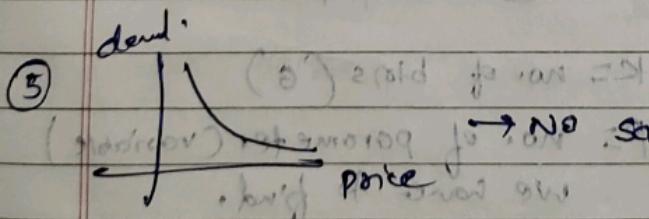
$$4) E = 0.9 = \frac{\% \text{ change in demand}}{\% \text{ change in price}} = \frac{(x - 1500)}{1500} \times 100$$

$$= \frac{x - 1800}{15} \quad \text{Ans: } 0.9$$

$$\frac{8000 - 10000}{18000} \times 100 = 0.9 = \frac{x - 1500}{15}$$

$$-20\% \quad \text{Ans: } 0.9$$

$$x = 240 + 1500 = 1740$$



$$5) E(Y/x) = \beta_0 + \beta_1 x$$

$$6) E(Y/x) = 1 - \beta_1 x \quad (\text{Ans: } E(Y/x) = 1 - \beta_1 x \text{ (Ans: } E(Y/x) = 1 - \beta_1 x))$$

7) linear correlation b/w X & Y is symmetric

$$E(Y/x) = 1 - \beta_1 x$$

Ans: linear reln b/w X & Y is symmetric

Ans: linear reln b/w X & Y is symmetric