Week - 5
Tutorial
Solution of Quadratic Equations
Mathematics for Data Science - 1

1. Two curves representing the functions $y_1 = a_1x^2+b_1x+c$ and $y_2 = a_2x^2+b_2x+c$ intersect each other at two points, then what will be their $X-$ coordinates, where $(a_1 \neq a_2)$?

Use following information to solve question 2 and 3.
The approximate temperature (T) (in ^oC) variation at a particular place with time (t) is give in Table T-5.0.

t	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00	18:00	19:00	20:00
T	30	32	34	36	40	43	46	48	46	43	40	35	32

Table T-5.0

2. Anshu fit a quadratic equation for temperature during day time as $T(x) = -0.4x^2 + 5x + 25$ where x is the number of hours after 08:00 AM. If she will not go out of her home if temperature is greater than 40^oC (strictly greater than 40), then what is the minimum time gap when she will not come out?

3. Rather than fitting a quadratic in above case we can fit two linear equations ℓ_1 and ℓ_2 respectively as shown in Figure.

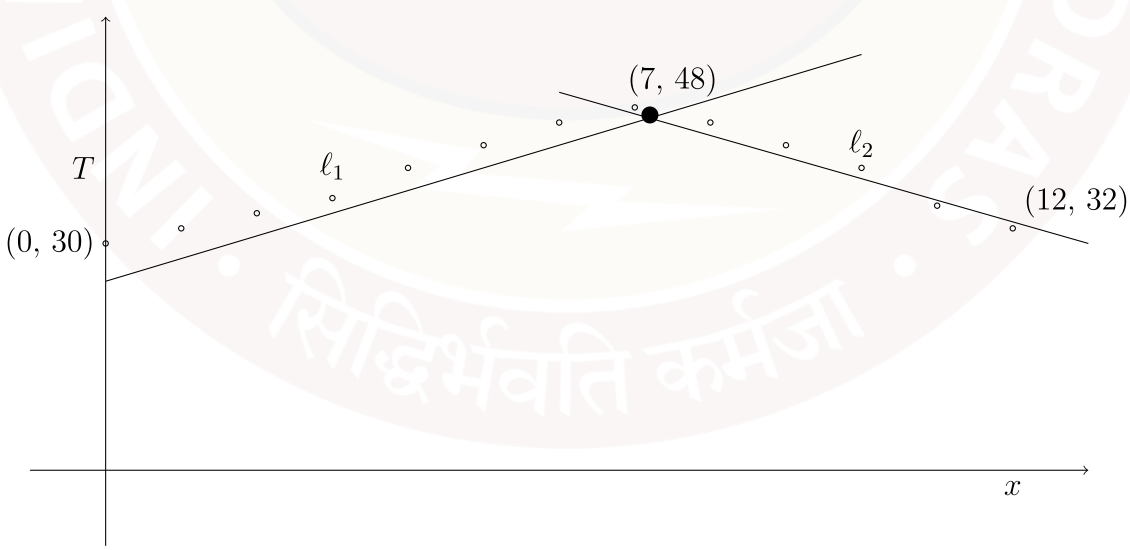

Figure T-5.1

Given that:
$\ell_1 \equiv T = 3x + 25, \quad x \in [0, 7]$
$\ell_2 \equiv T = -3x + 67, \quad x \in [7, 12]$
Draw a rough sketch of quadratic equation ($T(x) = -0.4x^2+5x+25$, vertex $\equiv (6.25, 40.625)$) mentioned in question 2 with respect to these two lines.

4. If $5x^2 + 8x + 1 = 0$, then answer the following.
   (a) Find the roots of above equation.
   (b) Calculate sum and product of roots.
   (c) With the help of above answers prove that sum and product of roots for any quadratic equation $ax^2 + bx + c = 0$ will be $-\frac{b}{a}$ and $\frac{c}{a}$ respectively.

5. Let M and N be the sets of all values of m and n respectively such that both equations $x^2 + mx + 4 = 0$ and $x^2 - nx + 1 = 0$ have always two real distinct roots each, then find the sets of M and N.
   Let C be a set of positive integers and values of m and n to be chosen randomly from C, then define the set C such that both the equations have two real distinct roots each.

6. A sniper shoots a bullet at some inclination from the ground towards a bird flying in $-ve\,X-$ direction at a constant height of 1600 ft. Because of gravity, the path of the bullet is a projectile as shown in Figure T-5.2. The height y (in ft) of the bullet after t seconds varies as $y(t) = u_y t - \frac{1}{2}gt^2$, where u_y is the initial vertical speed of bullet in m/s. Further, distance travelled by the bullet in $X-$ direction can be measured as $x = u_x t$ where u_x is the speed of bullet in $X-$ direction. Given that $u_x = u_y = 400\ ft/s,\ g = 32\ ft/s^2,$ one unit = one ft, and neglect the effect of wind, then find the position of hitting.

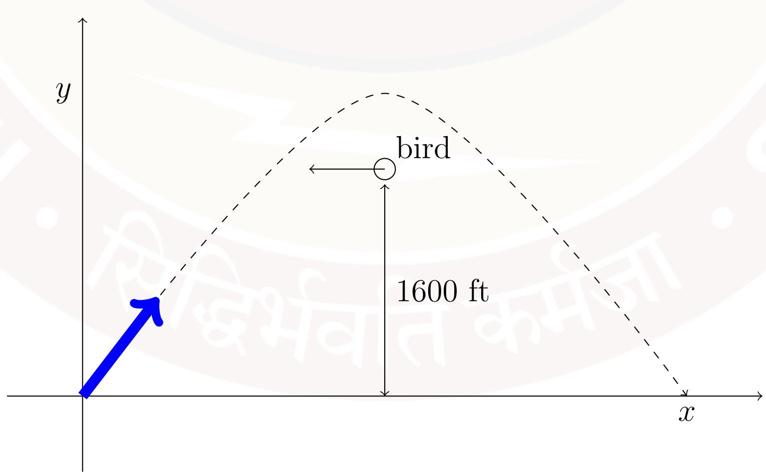

Figure T-5.2

7. Figure T-5.3 shows the curves C_1 and C_2, and line ℓ with their representing functions F_1 and F_2 respectively. If C_1' and C_2' are the functions F_1' and F_2' which are reflections of C_1 and C_2 respectively around ℓ.

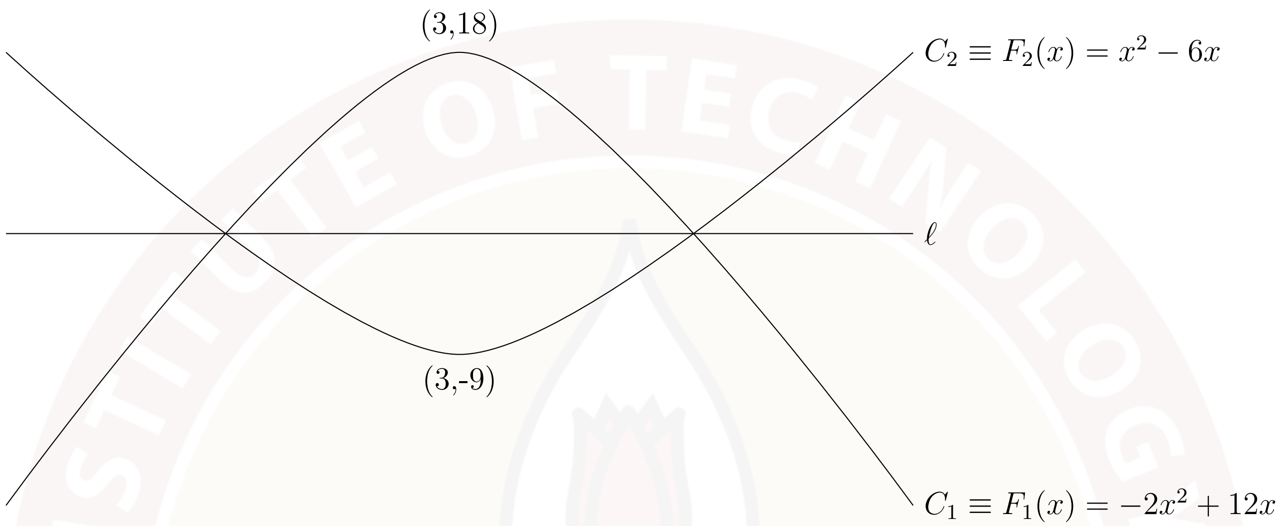

Figure T-5.3