

# Labor markets over the business cycle

## Indivisible labor. Search and matching

### Applied Macroeconomics

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# RBC model vs data comparison

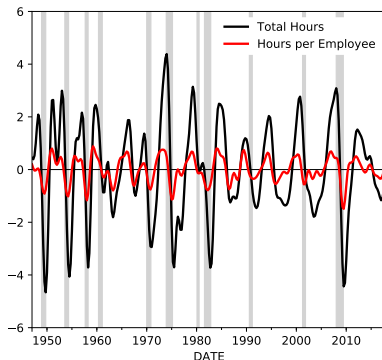
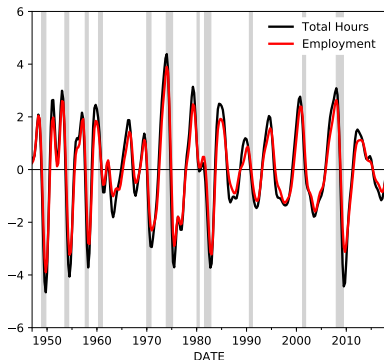
		Std. Dev.		Corr. w. $y$		Autocorr.	
		Data	Model	Data	Model	Data	Model
Output	$y$	1.60	1.60	1.00	1.00	0.85	0.72
Consumption	$c$	0.86	0.57	0.76	0.92	0.83	0.80
Investment	$i$	4.54	5.14	0.79	0.99	0.87	0.71
Capital	$k$	0.57	0.46	0.36	0.08	0.97	0.96
Hours	$h$	<b>1.60</b>	<b>0.73</b>	0.81	0.98	0.90	0.71
Wage	$w$	0.84	0.73	<b>0.10</b>	<b>0.99</b>	0.65	0.75
Interest rate	$r$	0.39	0.06	<b>-0.01</b>	<b>0.96</b>	0.40	0.71
TFP	$z$	1.00	1.15	<b>0.67</b>	<b>1.00</b>	0.71	0.72
Productivity	$y/h$	1.30	0.95	<b>0.51</b>	<b>0.99</b>	0.65	0.75

# RBC model vs data comparison

- ▶ Model performance is quite good – it was a big surprise in the 1980s!
- ▶ There are some problems with it though
  - ▶ In the data, hours are just as volatile as output
  - ▶ In the model, hours are less than half as volatile as output
  - ▶ In the data, real wage can be either pro- or countercyclical
  - ▶ In the model, real wage is strongly procyclical
  - ▶ In the data TFP and productivity are mildly correlated with output
  - ▶ In the model both are 1:1 correlated with output
- ▶ These results suggest that
  - ▶ Need some room for nominal variables
  - ▶ More shocks than just TFP are needed
  - ▶ **We need to focus more on labor market**
    - **should improve behavior of hours and real wage**

# Indivisible labor: introduction

Most of the variation in hours worked is on the *extensive* margin (employment-unemployment) rather than on the *intensive* margin (hours worked by individual employees)



# Indivisible labor: introduction

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$$H_t = L_t h_t \quad \longrightarrow \quad \log H_t = \log L_t + \log h_t$$

$$\text{Var}(\log H) = \text{Var}(\log L) + \text{Var}(\log h) + 2 \cdot \text{Cov}(\log N, \log h)$$

Variance-covariance matrix of Hodrick-Prescott deviations

	Total Hours	Employment	Hours per Employee
Total Hours	<b>3.52</b>		
Employment		<b>2.47</b>	<b>0.40</b>
Hours per Employee		<b>0.40</b>	<b>0.24</b>

About 70% of variance of total hours worked is accounted for by variance of employment level and only 7% is accounted for by variance of hours worked by individual employees (the rest is accounted for by covariance)

# Indivisible labor: setup

- ▶ “Realistic” hours worked variation results from a two-step process
  - ▶ Decision between working and not working
  - ▶ Conditional on working, how much to work
- ▶ This is difficult to model – we’ll focus on the first step only
- ▶ Gary Hansen (1985) and Richard Rogerson (1988) invented a clever technical solution
- ▶ In the RBC model households choose how much to work
- ▶ Here they will choose the probability  $p$  of working  $\bar{h}$  hours
  - ▶ All workers are identical
  - ▶ Each worker can work either 0 hours or a fixed number of hours  $\bar{h}$
  - ▶ Each worker is a part of big family and consumes the same amount regardless of working or not
  - ▶ As a consequence all workers choose the same probability of working

# Households' problem

Consider first a single-period problem

$$\max \quad U = \log c + E [\phi \log (1 - h) | p]$$

Expand the expected term

$$E [\phi \log (1 - h) | p] = p\phi \log (1 - \bar{h}) + (1 - p)\phi \log (1 - 0) = p\phi \log (1 - \bar{h})$$

Since all workers choose the same  $p$ , the average number of hours per worker household  $h$  is equal to probability  $p$  times working hours per employed  $\bar{h}$

$$h = p\bar{h} \quad \longrightarrow \quad p = h/\bar{h}$$

Going back to the expected term

$$E [\phi \log (1 - h) | p] = p\phi \log (1 - \bar{h}) = h \frac{\phi \log (1 - \bar{h})}{\bar{h}} = -Bh$$

where  $B = (-\phi \log (1 - \bar{h}) / \bar{h}) > 0$ . Utility becomes linear in  $h$ !

# Households' solution I

A representative household solves expected utility maximization problem

$$\begin{aligned} \max \quad & U_0 = E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\log c_t - B h_t) \right] \\ \text{subject to} \quad & a_{t+1} + c_t = (1 + r_t) a_t + w_t h_t + \text{div}_t \end{aligned}$$

Lagrangian

$$\begin{aligned} \mathcal{L} = & \sum_{t=0}^{\infty} \beta^t E_0 [\log c_t - B h_t] \\ & + \sum_{t=0}^{\infty} \beta^t E_0 [\lambda_t [(1 + r_t) a_t + w_t h_t + \text{div}_t - a_{t+1} - c_t]] \end{aligned}$$



# Households' solution II

Lagrangian

$$\begin{aligned}\mathcal{L} = & \sum_{t=0}^{\infty} \beta^t E_0 [\log c_t - B h_t] \\ & + \sum_{t=0}^{\infty} \beta^t E_0 [\lambda_t [(1 + r_t) a_t + w_t h_t + div_t - a_{t+1} - c_t]]\end{aligned}$$

First Order Conditions

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t E_0 \left[ \frac{1}{c_t} \right] - \beta^t E_0 [\lambda_t] = 0 \quad \longrightarrow \quad \lambda_t = \frac{1}{c_t}$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = \beta^t \cdot E_0 [-B] + \beta^t E_0 [\lambda_t w_t] = 0 \quad \longrightarrow \quad \lambda_t = \frac{B}{w_t}$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}} = -E_0 [\lambda_t] + \beta E_0 [\lambda_{t+1} (1 + r_{t+1})] = 0$$

$$\longrightarrow \quad \lambda_t = \beta E_t [\lambda_{t+1} (1 + r_{t+1})]$$

# Households' solution III

## First Order Conditions

$$c_t : \lambda_t = \frac{1}{c_t}$$

$$h_t : \lambda_t = \frac{B}{w_t}$$

$$a_{t+1} : \lambda_t = \beta E_t [\lambda_{t+1} (1 + r_{t+1})]$$

## Resulting

$$\text{Intertemporal condition } (c + a) : 1 = \beta E_t \left[ \frac{c_t}{c_{t+1}} (1 + r_{t+1}) \right]$$

$$\text{Intratemporal condition } (c + h) : B = \frac{w_t}{c_t}$$

# Full set of equilibrium conditions

System of 8 equations and 8 unknowns:  $\{c, h, y, r, w, k, i, z\}$

$$\text{Euler equation} : 1 = \beta E_t \left[ \frac{c_t}{c_{t+1}} (1 + r_{t+1}) \right]$$

$$\text{Consumption-hours choice} : B = \frac{w_t}{c_t}$$

$$\text{Production function} : y_t = z_t k_t^\alpha h_t^{1-\alpha}$$

$$\text{Real interest rate} : r_t = \alpha \frac{y_t}{k_t} - \delta$$

$$\text{Real hourly wage} : w_t = (1 - \alpha) \frac{y_t}{h_t}$$

$$\text{Investment} : i_t = k_{t+1} - (1 - \delta) k_t$$

$$\text{Output accounting} : y_t = c_t + i_t$$

$$\text{TFP AR(1) process} : \log z_t = \rho_z \log z_{t-1} + \varepsilon_t$$

## Steady state – closed form solution

Start with the Euler equation

$$1 = \beta(1 + r) \quad \longrightarrow \quad r = \frac{1}{\beta} - 1$$

From the interest rate equation obtain the  $k/h$  ratio

$$r = \alpha k^{\alpha-1} h^{1-\alpha} - \delta \quad \longrightarrow \quad \left(\frac{k}{h}\right)^{\alpha-1} = \frac{r + \delta}{\alpha} \quad \longrightarrow \quad \frac{k}{h} = \left(\frac{\alpha}{r + \delta}\right)^{\frac{1}{1-\alpha}}$$

From the production function obtain the  $y/h$  ratio and use it to get wage

$$y = k^{\alpha} h^{1-\alpha} \quad \longrightarrow \quad \frac{y}{h} = \left(\frac{k}{h}\right)^{\alpha} \quad \text{and} \quad w = (1 - \alpha) \frac{y}{h}$$

From investment and output accounting equations obtain the  $c/h$  ratio

$$i = \delta k \quad \longrightarrow \quad y = c + \delta k \quad \longrightarrow \quad \frac{c}{h} = \frac{y}{h} - \delta \frac{k}{h}$$

Get  $c$  from the consumption-hours choice. Then obtain  $h$ .

The rest follows from  $h$ .

$$c = \frac{w}{B} \quad \text{and} \quad h = \frac{c}{c/h}$$

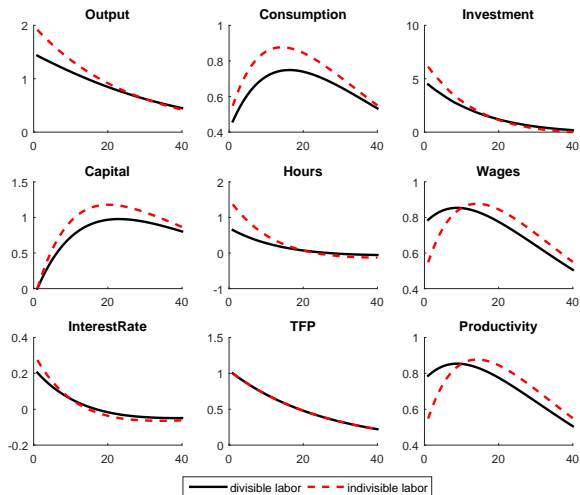
# Parameters

- ▶ To best compare our two models,  
we need them to generate identical steady states
- ▶ We replace parameter  $\phi$  with parameter  $B$
- ▶ We choose the value for  $B$  so that it matches  $h = 1/3$
- ▶ For this model  $B = 2.63$

# Model comparison: impulse response functions

RBC model IRF: black solid lines

Indivisible labor IRF: red dashed lines

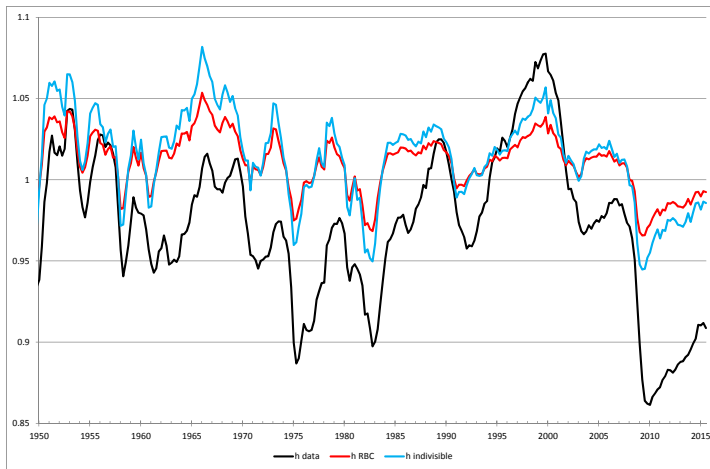


Percentage deviations from steady state (percentage points for  $r$ )

## Model comparison: moments

	Std. Dev.			Corr. w. $y$			Autocorr.		
	Data	RBC	Ind	Data	RBC	Ind	Data	RBC	Ind
$y$	1.60	1.60	1.60	1.00	1.00	1.00	0.85	0.72	0.72
$c$	0.86	0.57	0.53	0.76	0.92	0.90	0.83	0.80	0.81
$i$	4.54	5.14	5.33	0.79	0.99	0.99	0.87	0.71	0.71
$k$	0.57	0.46	0.47	0.36	0.08	0.08	0.97	0.96	0.96
$h$	<b>1.60</b>	<b>0.73</b>	<b>1.15</b>	0.81	0.98	0.98	0.90	0.71	0.70
$w$	0.84	0.73	0.53	0.10	0.99	0.90	0.65	0.75	0.81
$z$	1.00	1.15	0.83	0.67	1.00	1.00	0.71	0.72	0.72
$y/h$	1.30	0.95	0.53	0.51	0.99	0.90	0.65	0.75	0.81

# Model comparison: model-generated hours worked





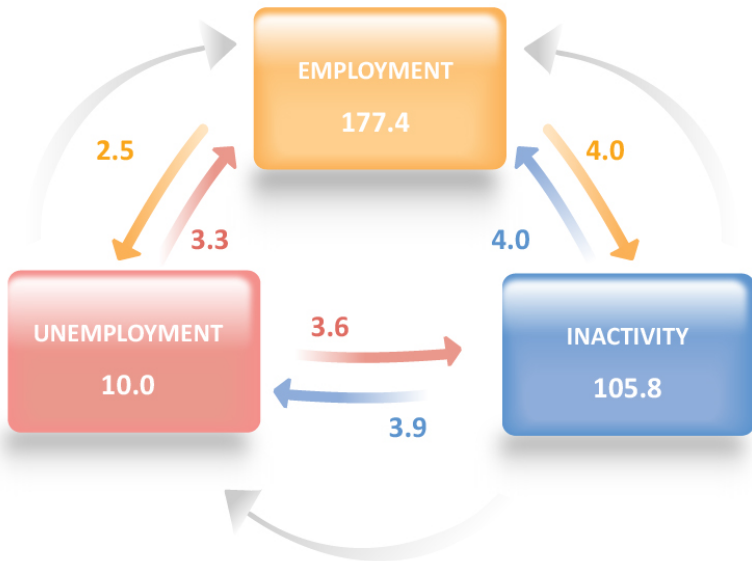
## Indivisible labor: summary

- ▶ Model enhances hours volatility – but it's still too low
- ▶ Improves a bit correlation of wages and productivity with output
- ▶ Slightly decreases empirical match in other dimensions
- ▶ Technical advantage – requires smaller TFP shocks
- ▶ Philosophical advantage – more “realistic” labor market

# Search and matching: introduction

- ▶ Labor markets are in a state of constant flux
- ▶ At the same time there are job-seeking workers and worker-seeking firms
- ▶ Labor markets are decentralized and thus active search is needed
- ▶ Search friction leads to unemployment even in the steady state

## Labor market status and flows: EU 2017Q2-2017Q3



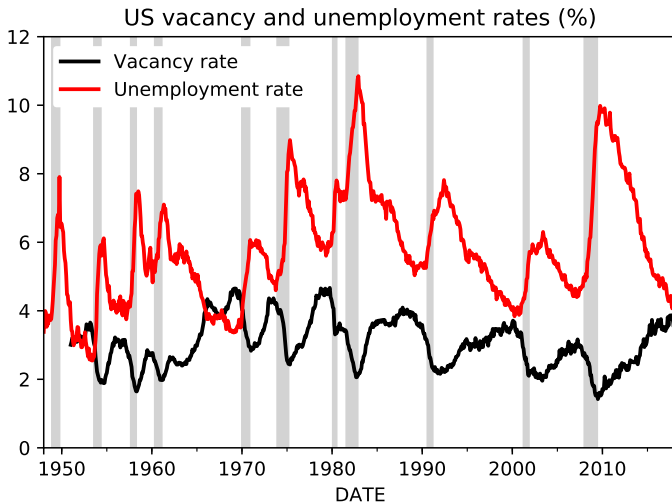
# Labor market status change probabilities in EU

	EMPLOYMENT 2017Q3	UNEMPLOYMENT 2017Q3	INACTIVITY 2017Q3
EMPLOYMENT 2017Q2	96.5 %	1.3 %	2.2 %
UNEMPLOYMENT 2017Q2	19.8 %	59.0 %	21.2 %
INACTIVITY 2017Q2	3.5 %	3.5 %	93.0 %

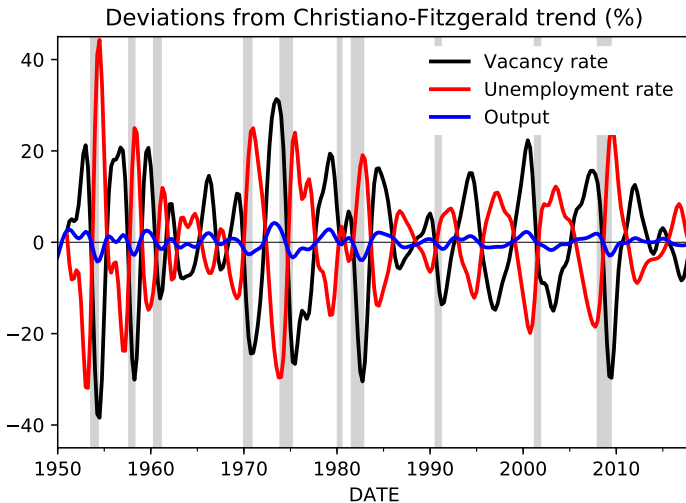
Source:

[http://ec.europa.eu/eurostat/statistics-explained/index.php/Labour\\_market\\_flow\\_statistics\\_in\\_the\\_EU](http://ec.europa.eu/eurostat/statistics-explained/index.php/Labour_market_flow_statistics_in_the_EU)

# Unemployment and vacancy rates: USA 1948Q1-2018Q1



# Labor market fluctuations: USA 1950Q1-2018Q1



# Matching function

- ▶ Firms create open job positions (openings, vacancies)
- ▶ Workers search for jobs
- ▶ Both jobs and workers are heterogeneous
  - not every possible match is attractive
- ▶ Matching function captures this feature
- ▶ New matches  $M$  are a function of the pool of unemployed  $U$  and vacancies  $V$

$$M_t = \chi V_t^\eta U_t^{1-\eta}$$

- ▶ After normalizing labor force to unity, match probability  $m$  is a function of unemployment rate  $u$  and vacancy rate  $v$

$$m_t = \chi v_t^\eta u_t^{1-\eta}$$

where  $\chi > 0$  and  $\eta \in (0, 1)$

# Job finding and job filling probabilities

- ▶ Unemployed workers are interested in job finding probability  $p$

$$p_t = \frac{m_t}{u_t} = \chi \left( \frac{v_t}{u_t} \right)^\eta = \chi \theta_t^\eta = q_t \theta_t$$

where  $\theta = v/u$  is called labor market tightness

- ▶ Firms with vacancies care about job filling probability  $q$

$$q_t = \frac{m_t}{v_t} = \chi \left( \frac{v_t}{u_t} \right)^{\eta-1} = \chi \theta_t^{\eta-1} = \frac{p_t}{\theta_t}$$

- ▶ Dual externality from congestion
  - ▶ High unemployment rate decreases  $p$  and increases  $q$
  - ▶ High vacancy rate increases  $p$  and decreases  $q$



# Employment dynamics

- ▶ Ignoring labor market inactivity, employment rate  $n$  and unemployment rate  $u$  sum to unity:

$$n_t + u_t = 1 \quad \longrightarrow \quad n_t = 1 - u_t$$

- ▶ Existing matches are destroyed with exogenous probability  $s$
- ▶ New matches increase next period employment

$$n_t = n_{t-1} - sn_{t-1} + m_{t-1}$$

$$u_t = u_{t-1} + sn_{t-1} - m_{t-1}$$

- ▶ We can find the steady state unemployment rate

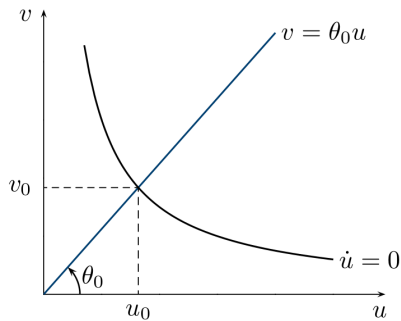
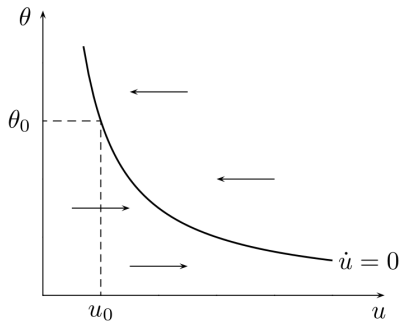
$$u = u + s(1 - u) - p(\theta)u$$

$$u = \frac{s}{s + p(\theta)}$$

as a function of separation and job finding probabilities

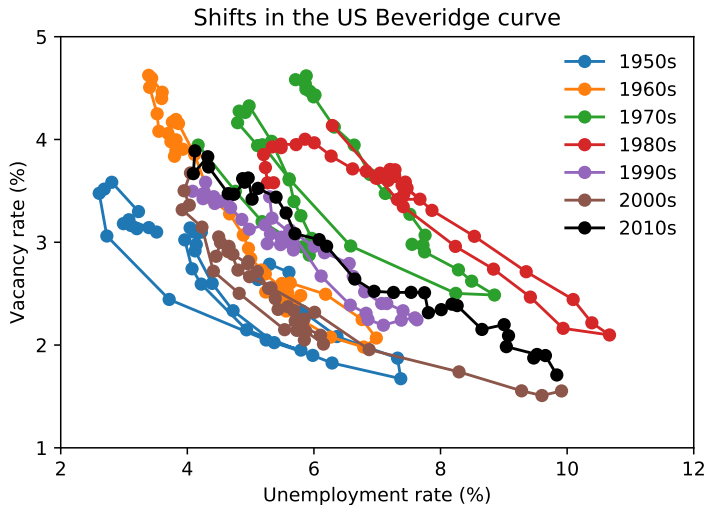
- ▶ If separation probability and matching function parameters do not change, then there exists a stable negative relationship between unemployment and vacancy rates known as the Beveridge curve

# Beveridge curve: theory



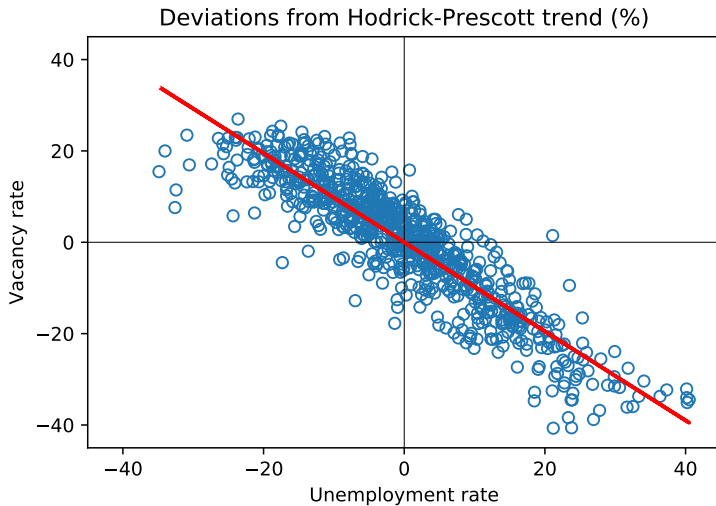
Graph by Leszek Wincenciak

# Beveridge curve: data

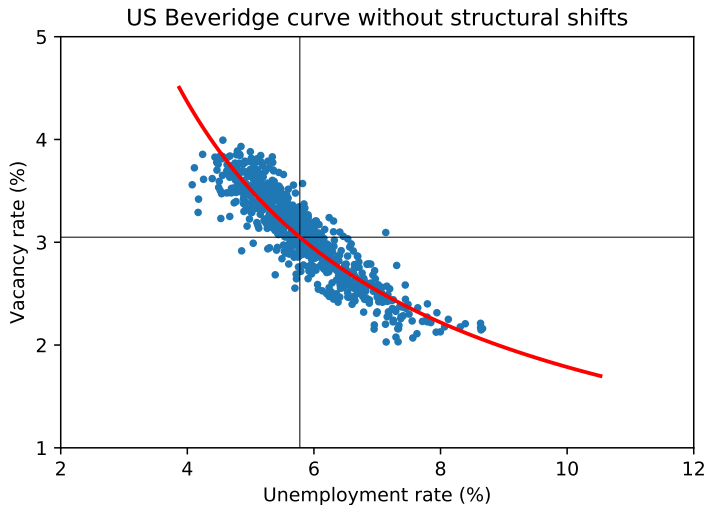


## Beveridge curve: data

Detrending with Hodrick-Prescott filter takes out structural shifts



# Beveridge curve: “estimation”



## Firm side

- ▶ Assume firms and workers discount future with  $\beta$
- ▶ Period net gain from a filled job equals marginal product of employee less wage
- ▶ With probability  $(1 - s)$  the match will survive into the next period

$$\mathcal{J}_t = (mpn_t - w_t) + \beta E_t [(1 - s) \mathcal{J}_{t+1} + s \mathcal{V}_{t+1}]$$

- ▶ Period net loss from open vacancy is its cost  $\kappa$  (advertising, interviewing)
- ▶ With probability  $q$  the vacancy will be filled

$$\mathcal{V}_t = -\kappa + \beta E_t [q_t \mathcal{J}_{t+1} + (1 - q_t) \mathcal{V}_{t+1}]$$

- ▶ Free entry in vacancies ensures that always  $\mathcal{V} = 0$

$$\frac{\kappa}{q_t} = \beta E_t [\mathcal{J}_{t+1}]$$

$$\mathcal{J}_t = (mpn_t - w_t) + \beta E_t [(1 - s) \mathcal{J}_{t+1}]$$

- ▶ In the steady state ( $r = 1/\beta - 1$ )

$$w = mpn - (r + s) \frac{\kappa}{q(\theta)}$$

## Worker side

- ▶ Period net gain from employment equals wage
- ▶ With probability  $(1 - s)$  the match will survive into the next period

$$\mathcal{E}_t = w_t + \beta E_t [(1 - s) \mathcal{E}_{t+1} + s \mathcal{U}_{t+1}]$$

- ▶ Period net gain from unemployment equals benefits (and possibly utility from leisure)
- ▶ With probability  $p$  unemployed finds a job

$$\mathcal{U}_t = b + \beta E_t [p_t \mathcal{E}_{t+1} + (1 - p_t) \mathcal{U}_{t+1}]$$

# Wage setting I

- ▶ In principle, wage can be as low as gain from unemployment  $b$  or as high as marginal product of employee  $mpn$  plus match gain
- ▶ Negotiated wage will be somewhere between those two values
- ▶ An easy way to pin down wage is Nash bargaining
- ▶ Let  $\gamma \in [0, 1]$  denote the relative bargaining power of firms
- ▶ Intuitively  $w \rightarrow b$  if  $\gamma \rightarrow 1$  and  $w \rightarrow mpn + \kappa\theta$  if  $\gamma \rightarrow 0$
- ▶ The negotiated wage is the solution of the problem

$$\max_{w_t} (\mathcal{J}_t(w_t))^\gamma (\mathcal{E}_t(w_t) - \mathcal{U}_t)^{1-\gamma}$$

- ▶ Solving the problem results in

$$\gamma(\mathcal{E}_t - \mathcal{U}_t) = (1 - \gamma) \mathcal{J}_t$$

- ▶ Alternatively: total match surplus  $\mathcal{S}_t = (\mathcal{E}_t - \mathcal{U}_t) + \mathcal{J}_t$

$$\mathcal{E}_t - \mathcal{U}_t = (1 - \gamma) \mathcal{S}_t \quad \text{and} \quad \mathcal{J}_t = \gamma \mathcal{S}_t$$



## Wage setting II

$$\gamma (\mathcal{E}_t - \mathcal{U}_t) = (1 - \gamma) \mathcal{J}_t$$

Plug in expressions for  $\mathcal{E}_t$ ,  $\mathcal{U}_t$  and  $\mathcal{J}_t$

$$\begin{aligned} \gamma \{ (w_t - b) + \beta (1 - s - p_t) E_t [\mathcal{E}_{t+1} - \mathcal{U}_{t+1}] \} \\ = (1 - \gamma) \{ (mpn_t - w_t) + \beta E_t [(1 - s) \mathcal{J}_{t+1}] \} \end{aligned}$$

$$\begin{aligned} w_t - \gamma b + (1 - s - p_t) \beta E_t [\gamma (\mathcal{E}_{t+1} - \mathcal{U}_{t+1})] \\ = (1 - \gamma) mpn_t + (1 - s) \beta E_t [(1 - \gamma) \mathcal{J}_{t+1}] \end{aligned}$$

$$\begin{aligned} w_t - \gamma b + (1 - s - p_t) \beta E_t [(1 - \gamma) \mathcal{J}_{t+1}] \\ = (1 - \gamma) mpn_t + (1 - s) \beta E_t [(1 - \gamma) \mathcal{J}_{t+1}] \end{aligned}$$

$$w_t = \gamma b + (1 - \gamma) \{ mpn_t + p_t \beta E_t [\mathcal{J}_{t+1}] \}$$

$$\kappa/q_t = \beta E_t [\mathcal{J}_{t+1}]$$

$$w_t = \gamma b + (1 - \gamma) (mpn_t + p_t \kappa/q_t)$$

$$w_t = \gamma b + (1 - \gamma) (mpn_t + \kappa \theta_t)$$

# Full set of equilibrium conditions

System of 9 equations and 9 unknowns:  $\{w, mpn, \theta, \mathcal{J}, q, u, n, m, v\}$

$$w_t = \gamma b + (1 - \gamma) (mpn_t + \kappa \theta_t)$$

$$\mathcal{J}_t = (mpn_t - w_t) + \beta E_t [(1 - s) \mathcal{J}_{t+1}]$$

$$\frac{\kappa}{q_t} = \beta E_t [\mathcal{J}_{t+1}]$$

$$u_t = 1 - n_t$$

$$n_t = (1 - s) n_{t-1} + m_{t-1}$$

$$q_t = \chi \theta_t^{\eta-1}$$

$$\theta_t = \frac{v_t}{u_t}$$

$$m_t = \chi v_t^\eta u_t^{1-\eta}$$

$$\ln mpn_t = \rho_{mpn} \ln mpn_{t-1} + \varepsilon_t$$

## Steady state: key equations

In the steady state the model is fully summarized by the following three key equations:

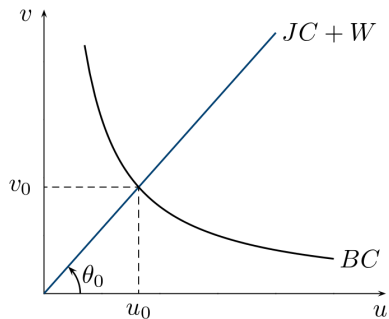
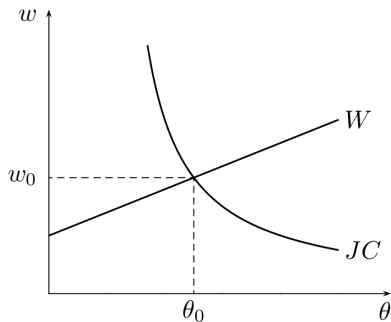
$$\text{Beveridge curve (BC)} : u = \frac{s}{s + p(\theta)}$$

$$\text{Job (vacancy) creation (JC)} : w = mpn - (r + s) \frac{\kappa}{q(\theta)}$$

$$\text{Wage setting (W)} : w = \gamma b + (1 - \gamma)(mpn + \kappa\theta)$$

Can be even reduced further to equations in  $u$  and  $\theta$

## Steady state: graphical solution



Graph by Leszek Wincenciak

## Steady state: algebraic solution

- ▶ In this model the crucial variable is labor market tightness  $\theta$
- ▶ We can find it by solving the following system

$$w = \gamma b + (1 - \gamma) (mpn + \kappa \theta)$$

$$w = mpn - (r + s) \frac{\kappa}{q(\theta)}$$

- ▶ After some rearrangement

$$(r + s) \frac{\kappa}{\chi} \theta^{1-\eta} = \gamma (mpn - b) - (1 - \gamma) \kappa \theta$$

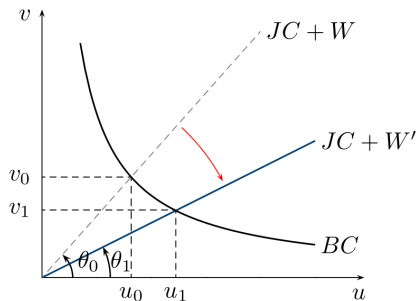
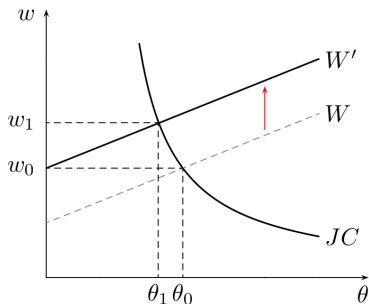
- ▶ The above equation does not have a closed form solution for  $\theta$
- ▶ We can solve it easily via numerical methods
- ▶ We can also use a trick – set  $\theta = 1$  and solve for  $\chi$   
(but loose a degree of freedom for calibration)

$$\chi = [(r + s) \kappa] / [\gamma (mpn - b) - (1 - \gamma) \kappa]$$

# Comparative statics I

Effects of an increase in unemployment benefits ( $b \uparrow$ )  
or in workers' bargaining power ( $\gamma \downarrow$ ):

- ▶ Increase in real wage  $w$
- ▶ Decrease in labor market tightness  $\theta$
- ▶ Decrease in vacancy rate  $v$
- ▶ Increase in unemployment rate  $u$

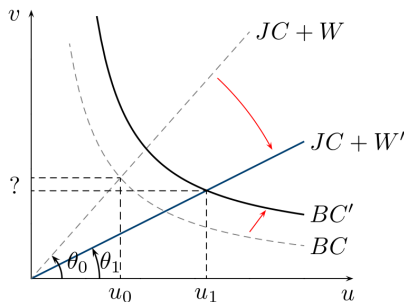
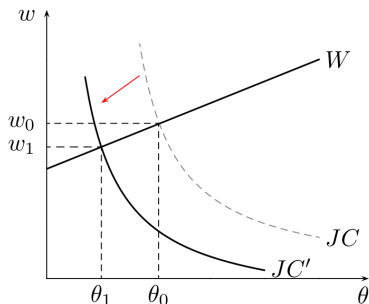


Graph by Leszek Wincenciak

# Comparative statics II

Effects of an increase in separation rate ( $s \uparrow$ )  
or a decrease in matching efficiency ( $\chi \downarrow$ ):

- ▶ Decrease in real wage  $w$
- ▶ Decrease in labor market tightness  $\theta$
- ▶ Ambiguous effect on vacancy rate  $v$  (depends on parameter values)
- ▶ Increase in unemployment rate  $u$

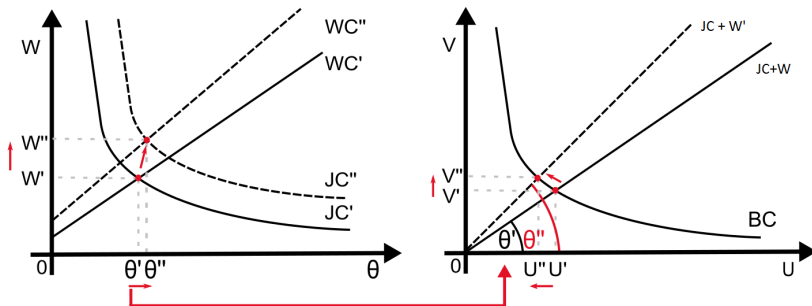


Graph by Leszek Wincenciak

# Comparative statics III

Effects of an increase in labor productivity ( $mpn \uparrow$ ):

- ▶ Increase in real wage  $w$
- ▶ Increase in labor market tightness  $\theta$
- ▶ Increase in vacancy rate  $v$
- ▶ Decrease in unemployment rate  $u$



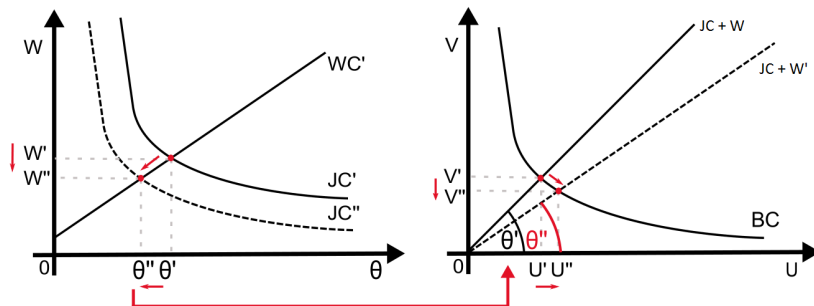
Graph by Matthias Hertweck



# Comparative statics IV

Effects of an increase in interest rate ( $r \uparrow$ )  
or an increase in impatience ( $\rho \uparrow \rightarrow \beta \downarrow$ ):

- ▶ Decrease in real wage  $w$
- ▶ Decrease in labor market tightness  $\theta$
- ▶ Decrease in vacancy rate  $v$
- ▶ Increase in unemployment rate  $u$



Graph by Matthias Hertweck

# Transitional dynamics

Reduced form of the model:

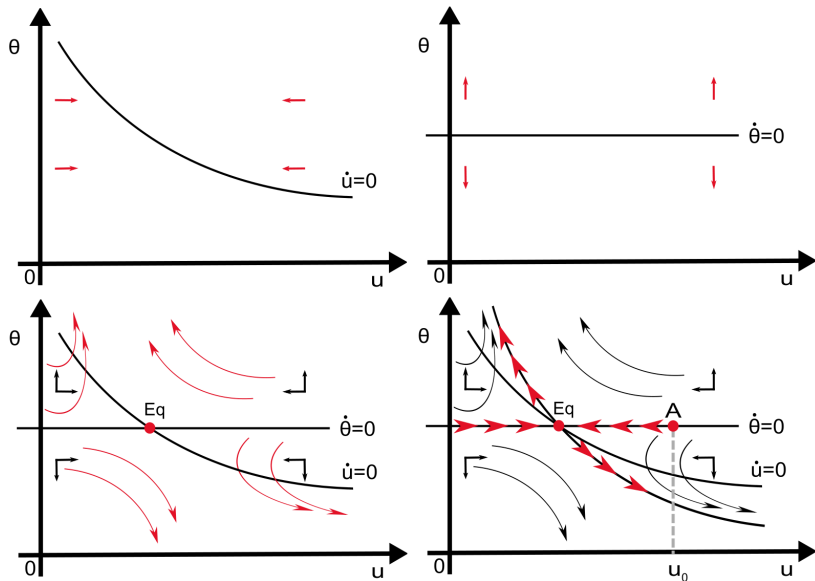
$$\Delta u = 0 \quad \longrightarrow \quad u = \frac{s}{s + \chi \theta^\eta}$$

$$\Delta \theta = \frac{\theta}{1 - \eta} \left[ (r + s) - \gamma (mpn - b) \frac{\chi \theta^{\eta-1}}{\kappa} + (1 - \gamma) \chi \theta^\eta \right]$$

The dynamic equation for  $\theta$  is independent of  $u$

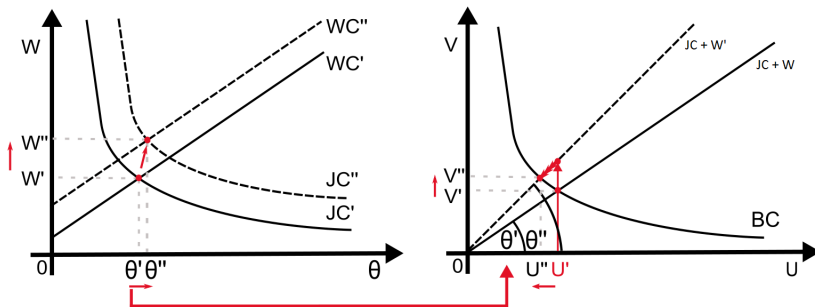
–  $\Delta \theta = 0$  is a flat line in  $(u, \theta)$  space

# Transitional dynamics: phase diagram



Graphs by Matthias Hertweck

# Transitional dynamics: positive productivity shock



Graph by Matthias Hertweck

# Parameters

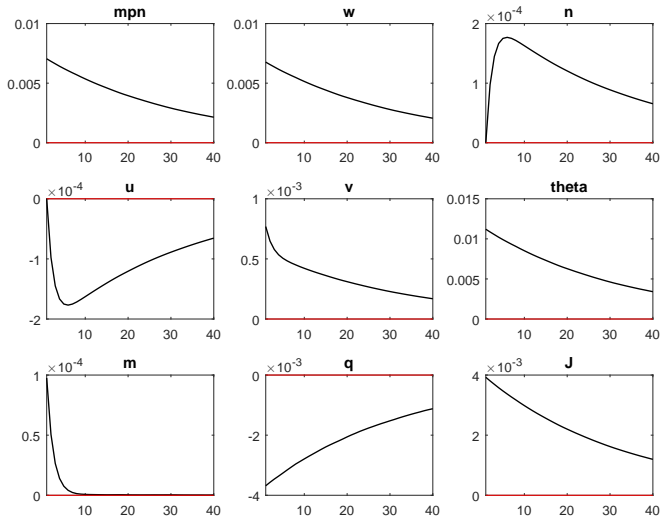
Values come from Shimer (2005, AER)

	Description	Value
$\chi$	matching efficiency	0.45
$\eta$	matching elasticity of $v$	0.28
$s$	separation probability	0.033
$\beta$	discount factor	0.99
$mpn$	steady state marginal product	1
$\kappa$	vacancy cost	0.21
$b$	unemployment benefit	0.4
$\gamma$	firm bargaining power	0.28

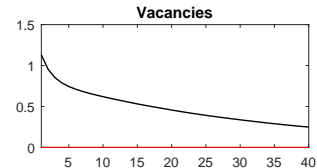
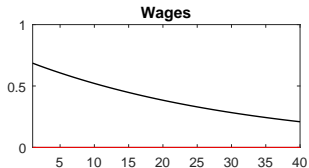
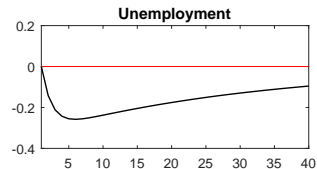
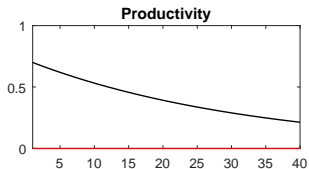
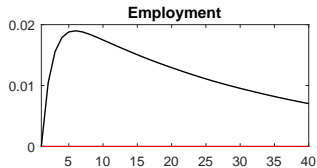
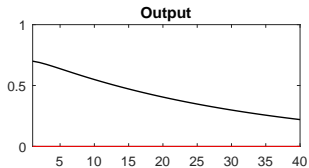
## Implied steady state values

	Description	Value
$u$	unemployment rate	0.0687
$v$	vacancy rate	0.0674
$m$	new matches	0.031
$\theta$	tightness	0.98
$p$	job finding probability	0.448
$q$	job filling probability	0.456
$w$	wage	0.98

# Impulse response functions I

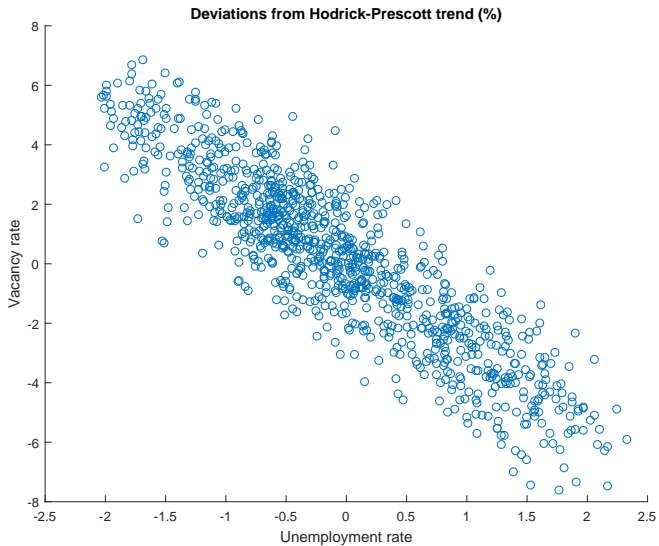


# Impulse response functions II





# Model generated Beveridge curve



# Summary

- ▶ We have a “realistic” model of the labor market
- ▶ Able to match both steady state (average) and some cyclical properties of the labor market
- ▶ Replicates the negative slope of the Beveridge curve
- ▶ Not enough variation in employment
- ▶ Beveridge curve too steep
- ▶ Too much variation in wages

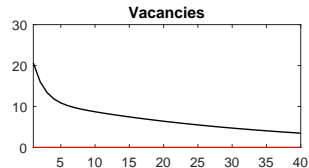
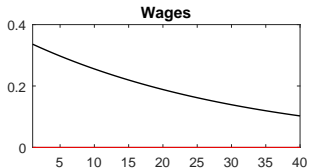
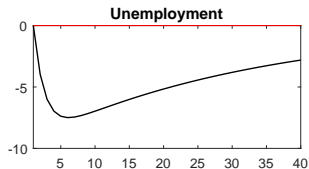
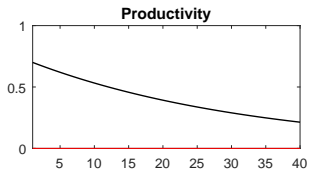
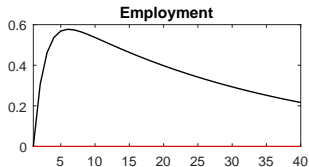
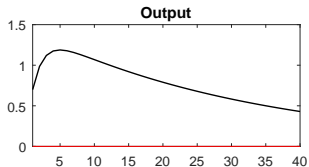
## Alternative parametrization

Values come from Hagedorn & Manovskii (2008, AER)

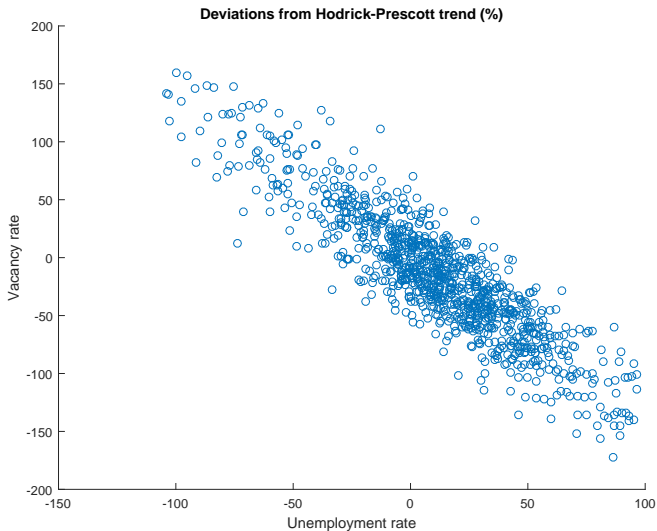
	Description	Value
$\eta$	matching elasticity of $v$	0.45
$b$	unemployment benefit	0.965
$\gamma$	firm bargaining power	0.928

- ▶ Firms have very strong bargaining position
- ▶ But unemployment gain includes leisure utility
- ▶ Steady state unchanged

# Hagedorn & Manovskii: Impulse response functions

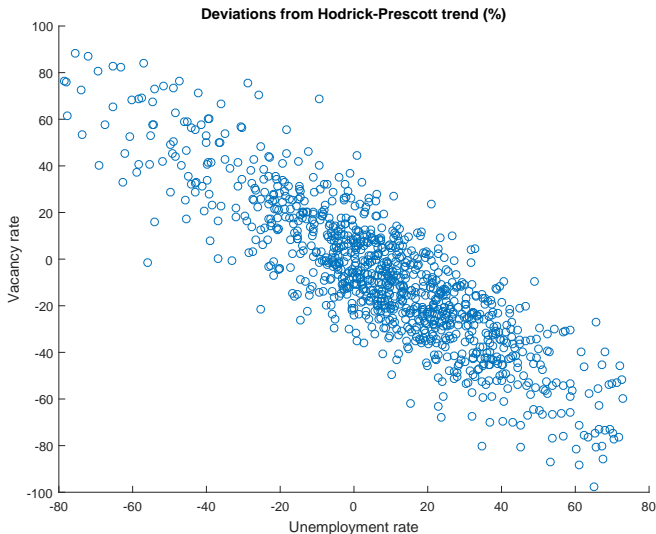


# Hagedorn & Manovskii: Beveridge curve



# Mortensen & Nagypal (2007): Beveridge curve

Set  $\eta = 0.54$ . Model BC replicates slope of the data BC



# Summary

- ▶ Alternative parametrizations yield better results
- ▶ Both unemployment and employment become more volatile
- ▶ Volatility of wages is diminished
- ▶ Key problem for the search and matching model identified
  - period-by-period Nash bargaining
- ▶ Further extensions make alternative assumptions about the wage setting process

# Integration with RBC framework

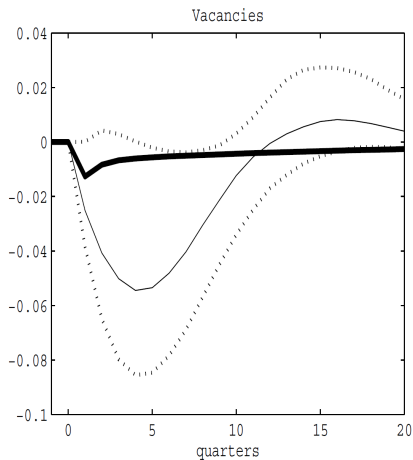
- ▶ Very easy
- ▶ Get  $mpn$  from the usual firm problem
- ▶ Adjust  $\beta$  for  $\beta \frac{\lambda_{t+1}}{\lambda_t}$  in the firm's valuation since the latter is the correct stochastic discounting factor
- ▶ Solve for labor market variables
- ▶ Get back to the RBC part
- ▶ Remember to include vacancy costs in the national accounting equation

$$y_t = c_t + i_t + \kappa v_t$$



# Observation of Fujita (2004)

Model IRF for vacancies is counterfactual



## Alternative hiring cost function

- ▶ We assumed linear vacancy posting costs

$$\begin{aligned}\psi(v_t) &= \kappa v_t \\ w_t &= \gamma b + (1 - \gamma)(mpn_t + \kappa \theta_t) \\ \frac{\kappa}{q_t} &= \beta E_t \left[ mpn_{t+1} - w_{t+1} + (1 - s) \frac{\kappa}{q_{t+1}} \right]\end{aligned}$$

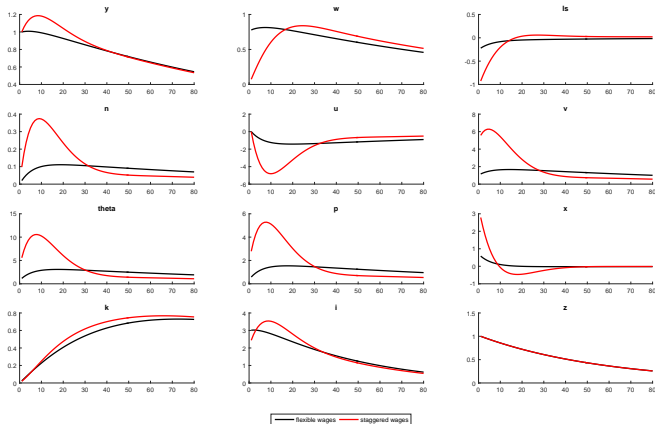
- ▶ Gertler & Trigari (2009, JPE) assume convex labor posting costs
- ▶ Define hiring rate  $x$  as the ratio of new hires to employed workers

$$\begin{aligned}x_t &= \frac{m_t}{n_t} \\ \psi(x_t) &= \frac{\kappa}{2} x_t^2 n_t \\ w_t &= \gamma b + (1 - \gamma) \left( mpn_t + \frac{\kappa}{2} x_t^2 + p_t \kappa x_t \right) \\ \kappa x_t &= \beta E_t \left[ mpn_{t+1} - w_{t+1} + (1 - s) \kappa x_{t+1} + \frac{\kappa}{2} x_t^2 \right]\end{aligned}$$

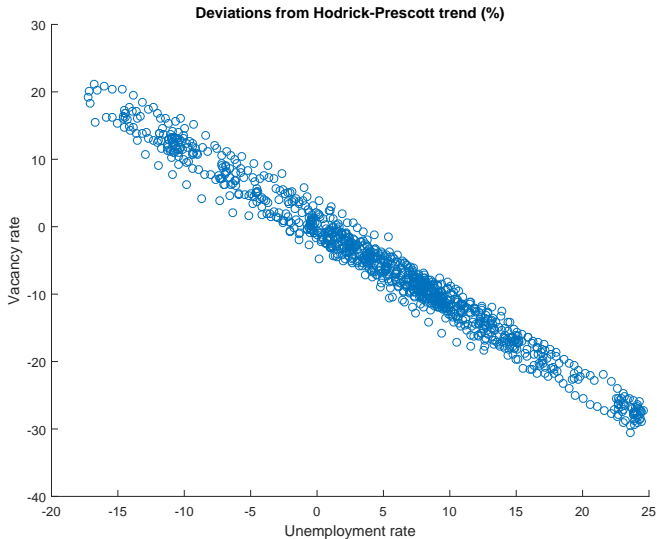
- ▶ They also consider staggered (multi-period) wage contracts where only a fraction of previous wage contracts are renegotiated

# Gertler & Trigari: Impulse response functions

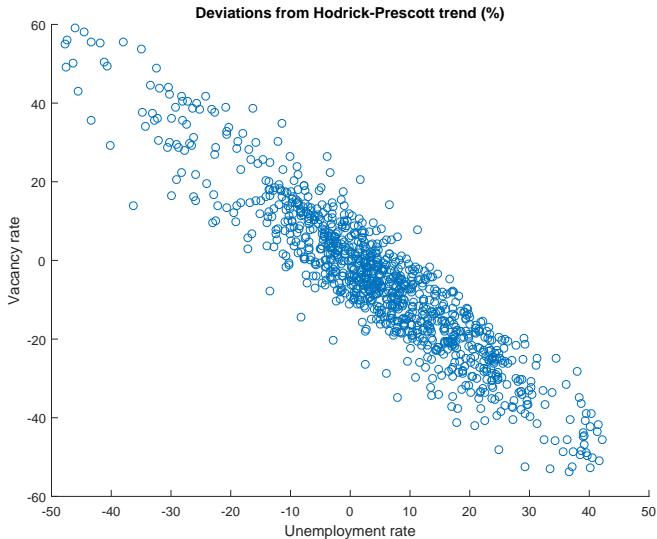
## Monthly period frequency



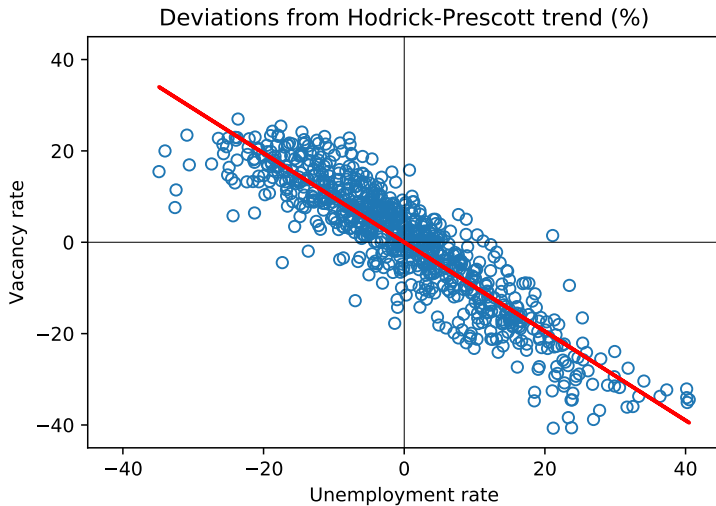
# Gertler & Trigari: Beveridge curve (flexible wages)



# Gertler & Trigari: Beveridge curve (staggered wages)



## Beveridge curve: data



# Gertler & Trigari: business cycle statistics

	$y$	$w$	$ls$	$n$	$u$	$v$	$\theta$	$a$	$i$	$c$
A. U.S. Economy, 1964:1–2005:1										
Relative standard deviation	1.00	.52	.51	.60	5.15	6.30	11.28	.61	2.71	.41
Autocorrelation	.87	.91	.73	.94	.91	.91	.91	.79	.85	.87
Correlation with $y$	1.00	.56	−.20	.78	−.86	.91	.90	.71	.94	.81
B. Model Economy, $\lambda = 0$ (Flexible Wages)										
Relative standard deviation	1.00	.87	.09	.10	1.24	1.58	2.72	.93	3.11	.37
Autocorrelation	.81	.81	.58	.92	.92	.86	.90	.78	.80	.85
Correlation with $y$	1.00	1.00	−.54	.59	−.59	.98	.92	1.00	.99	.93
C. Model Economy, $\lambda = 8/9$ (3 Quarters)										
Relative standard deviation	1.00	.56	.57	.35	4.44	5.81	9.84	.71	3.18	.35
Autocorrelation	.84	.95	.65	.90	.90	.82	.88	.76	.86	.86
Correlation with $y$	1.00	.66	−.56	.77	−.77	.91	.94	.97	.99	.90
D. Model Economy, $\lambda = 11/12$ (4 Quarters)										
Relative standard deviation	1.00	.48	.58	.44	5.68	7.28	12.52	.64	3.18	.34
Autocorrelation	.85	.96	.68	.91	.91	.86	.90	.74	.88	.86
Correlation with $y$	1.00	.55	−.59	.78	−.78	.93	.95	.95	.99	.90

# Summary

- ▶ After adding multi-period contracts, Gertler & Trigari obtain a very good empirical match of the RBC model with search & matching features
- ▶ This is one of the best matches for single-shock models
- ▶ Key to the success was
  - ▶ Convex vacancy posting
  - ▶ Staggered (multi-period) wage contracts



## Possible further extensions

- ▶ Endogenous (non-constant) separation rate
- ▶ On-the-job search
- ▶ Hours per worker adjustments