Jacobi Method Complexity:
The Jacobi Method is computed by

$$x_i^k = \frac{1}{a_{ii}}\left[b_i - \sum_{\substack{j=1 \\ j\neq i}}^{N}\left(a_{ij}x_j^{(k-1)}\right)\right] \tag{1}$$

where k is the iteration, a_{ij} is an element of $\mathbf{A}$, b_i is an element of $\mathbf{b}$, and N is the side length of the matrix $\mathbf{A}$.

Each iteration the sum has a multiplication, of which there are $N-1$. These are then summed for $N-2$ additional calculations. Then b_i is added for another 1 calculation. This results in $2N-1$ calculations for each x_i^k. These are given in the order that they appear as one is naturally doing the calculation.

There are then N times the calculations for a single x_i^k to account for all x_i^k. This results in $N(2N-1)$ calculations per iteration.

Final answer:

$$2N^2 - N \tag{2}$$

or

$$\mathcal{O}\left(N^2\right) \tag{3}$$

Gauss–Seidel Method Complexity:
The Gauss–Seidel method is computed by

$$x_i^k = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1}\left(a_{ij}x_j^{(k)}\right) - \sum_{j=i+1}^{N}\left(a_{ij}x_j^{(k-1)}\right)\right]. \tag{4}$$

One can see that equations (1) and (4) have the same amount of calculations for a given N. This is because the sum in equation (1) goes from 1 to $i-1$ and from $i+1$ to N, skipping i, and the sums in equation (4) do the same. This means the complexity of these two methods are the same per iteration as:

$$2N^2 - N \tag{5}$$

or

$$\mathcal{O}\left(N^2\right) \tag{6}$$

Cholesky Decomposition Complexity:
The Cholesky decomposition is computed by

$$l_{ii} = \sqrt{a_{ii} - \sum_{j=1}^{j=i-1} l_{ij}^2} \tag{7}$$

and

$$l_{ij} = \frac{1}{l_{jj}} \left(a_{ij} - \sum_{k=1}^{k=j-1} l_{jk} l_{ik} \right) \quad \text{for} \quad i > j \tag{8}$$

Each component under the diagonal l_{ij} $i > j$ requires 1 addition, $j-1$ multiplications, $j-2$ additions, and 1 division. These are given in the order that they appear as one is naturally doing the calculation.

This becomes $2j-1$ calculations for each l_{ij} $i > j$ or

$$\sum_{j=1}^{N} (N-j)(2j-1) \tag{9}$$

which decomposes to

$$(1+2N) \sum_{j=1}^{N} (j) + \sum_{j=1}^{N} (N) - 2 \sum_{j=1}^{N} \left(j^2\right) \tag{10}$$

which becomes

$$N^2 + .5(2N+1)\left(N^2+N\right) - 2\left(\frac{N(N+1)(2N+1)}{6}\right) \tag{11}$$

which becomes

$$\frac{1}{3}N^3 + \frac{3}{2}N^2 + \frac{1}{6}N. \tag{12}$$

The diagonal has 1 square root, 1 addition, $i-1$ multiplications, and $i-2$ additions. These are given in the order that they appear as one is naturally doing the calculation. This becomes $2i-1$ calculations for each l_{ii}. This can be written as

$$\sum_{i=1}^{N} (2i-1) \tag{13}$$

which becomes

$$N^2 + N - N \tag{14}$$

which simplifies to N.

Combining with the diagonal calculations yields

$$\frac{1}{3}N^3 + \frac{5}{2}N^2 + \frac{1}{6}N. \tag{15}$$

Final answer:

$$\frac{1}{3}N^3 + \frac{5}{2}N^2 + \frac{1}{6}N. \tag{16}$$

or

$$\mathcal{O}\left(N^3\right) \tag{17}$$