Handling input: Why the "$" ?

In [254]:

```
import string                                                  O(1)
import math

def userInput():
    userMessage = input("Please enter what you would like to encode/pattern se
arch: ")
    userMessage += '$'   O(1)
    return userMessage
```

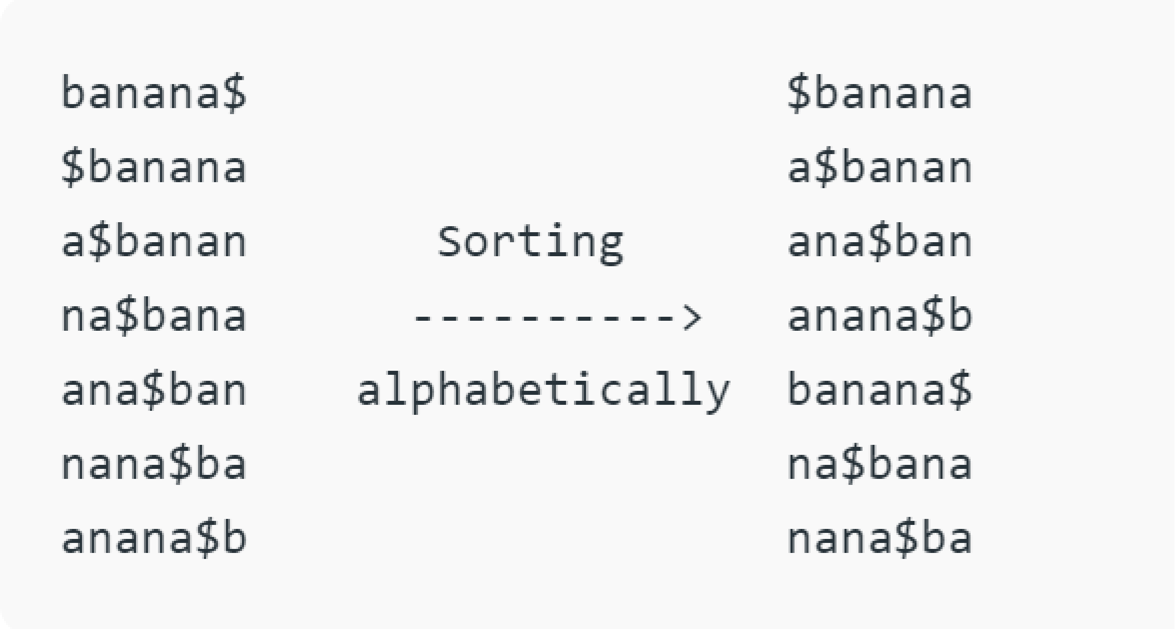
Constructing the suffix array

O(nlogn)

In [255]:

```
def constructArray(userMessage):                O(n)
    suffixArray = []
    for i in range(0, len(userMessage)):                        o(1)
        suffixArray.append((userMessage[i:len(userMessage)], i))
    suffixArray.sort(key=lambda tup: tup[0])   O(nlogn)
    return suffixArray
```

Construting the BWT



```
banana$                      $banana
$banana                      a$banan
a$banan        Sorting       ana$ban
na$bana     ----------->     anana$b
ana$ban   alphabetically     banana$
nana$ba                      na$bana
anana$b                      nana$ba
```

In [256]:

```
def constructBWT(userMessage, suffixArray):
    BWT = ''                                                O(n^2)
    for i in range(len(suffixArray)):  O(n)
        print(suffixArray[i])
        BWT += userMessage[suffixArray[i][1]-1]  O(n)
    return BWT
```

Move to front encoding: But Why

In [257]:
```
def moveToFront(BWT):

    asciis = [chr(i) for i in range(256)]                 O(n^2)
    encodedMTF = []

    for b in range(len(BWT)):  O(n)
        rank = asciis.index(BWT[b])  O(n)
        encodedMTF.append(str(rank))  O(1)
        asciis.pop(rank)  O(1)
        asciis.insert(0, BWT[b])  O(n)

    return encodedMTF
```

Undo It All!!!

In [258]:
```
def unMoveToFront(encodedMTF):

    asciis = [chr(i) for i in range(256)]                 O(n^2)
    decodedMTF = ''

    for b in range(len(encodedMTF)):  O(n)
        character = asciis[int(encodedMTF[b])]  O(1)
        decodedMTF += character  O(n)
        asciis.pop(asciis.index(character))  O(1)
        asciis.insert(0, character)  O(n)

    return decodedMTF
```

0	1	2	3	4	5	6	7	8
l	l$	l$t	l$ta	l$tar	l$tarh	l$tarhe	l$tarhee	$tarheel
t	ta	tar	tarh	tarhe	tarhee	tarheel	tarheel$	arheel$t
h	he	hee	heel	heel$	heel$t	heel$ta	heel$tar	eel$tarh
e	ee	eel	eel$	eel$t	eel$ta	eel$tar	eel$tarh	el$tarhe
r	rh	rhe	rhee	rheel	rheel$	rheel$t	rheel$ta	heel$tar
e	el	el$	el$t	el$ta	el$tar	el$tarh	el$tarhe	l$tarhee
a	ar	arh	arhe	arhee	arheel	arheel$	arheel$t	rheel$ta
$	$t	$ta	$tar	$tarh	$tarhe	$tarhee	$tarheel	tarheel$

In [259]:
```
def inverseBWT(BWT):                                      O(n^2 log n)
    table = ['' for c in BWT]
    for j in range(len(BWT)):  O(n)
        table = sorted([c+table[i] for i, c in enumerate(BWT)])  O(nlogn)
    return table[BWT.index("$")]
```

In [260]:

```
def printArray(array):
    print(*array, sep = ", ")
```

O(n)

Pattern Searching

[redacted]

[redacted]

In [261]:

```
#INCLUDED FOR COMPARISON ONLY

def findLast(pattern, userMessage, suffixarray):
    lo, hi = 0, len(userMessage)
    while (lo < hi):
        middle = (lo+hi)//2
        #print(middle)
        if userMessage[suffixarray[middle][1]:suffixarray[middle][1]+len(patte
rn)] <= pattern:
            lo = middle + 1
        else:
            hi = middle
    return lo
def findFirst(pattern, userMessage, suffixarray):
    lo, hi = 0, len(userMessage)
    while (lo < hi):
        middle = int((lo+hi)/2)
        #print(middle)
        if userMessage[suffixarray[middle][1]:] < pattern:
            lo = middle + 1
        else:
            hi = middle
    return lo

def findAllPatterns(pattern, userMessage, suffixArray):
    first = findFirst(pattern, userMessage, suffixArray)
    last = findLast(pattern, userMessage, suffixArray)
    print ("Total matches: " + str(last - first))
    return (last-first)
```

$O(n \log M)$

n = length of pattern

M = length of Messege

||

Same complexity

||

In [262]:

```
def FMIndex(bwt):
    fm = [{c: 0 for c in bwt}]
    for c in bwt:
        row = {symbol: count + 1 if (symbol == c) else count for symbol, count
in fm[-1].items()}
        fm.append(row)
    offset = {}
    N = 0
    for symbol in sorted(row.keys()):
        offset[symbol] = N
        N += row[symbol]
    return fm, offset


def findBWT(pattern, FMIndex, Offset):
    lo = 0
    hi = len(FMIndex) - 1
    for symbol in reversed(pattern):
        lo = Offset[symbol] + FMIndex[lo][symbol]
        hi = Offset[symbol] + FMIndex[hi][symbol]
    return lo, hi
```

$O(n)$

$O(n)$

$O(n)$

$O(n)$

Please reference my other project :)

In [263]:

```
from queue import PriorityQueue

class BinaryTree:
    def __init__(self, char, freq, left=None, right=None, parent=None):
        self.character = char
        self.frequency = freq
        self.leftChild = left
        self.rightChild = right                      O(1)
        self.parent = parent

    def __str__(self):
        return (str(self.character) + ", " + str(self.frequency))

    def getRightChild(self):
        return self.rightChild

    def getLeftChild(self):
        return self.leftChild

    def getCharVal(self):
        return self.character

    def getFreqVal(self):
        return self.frequency

    def isLeaf(self):
        return not (self.rightChild or self.leftChild)
    def isTop(self):
        return (self.getFreqVal == 1)
```

In [264]:

```
#https://stackoverflow.com/questions/991350/counting-repeated-characters-in-a-string-in-python

def makeFrequencies(msg):
    freq = {}
    setup = [(i,msg.count(i)) for i in set(msg)]       O(n)
    for i in range(len(setup)):
        freq[setup[i][0]] = (setup[i][1]/len(msg))
    return freq

def getKeys(dict):                  O(1)
    return dict.keys()
def getValues(dict):
    return dict.values()            O(1)
```

In [265]:

```
def makeQueue(freq):
    q = PriorityQueue()
    klist = list(getKeys(freq))
    flist = list(getValues(freq))
    for i in range(len(flist)):
        q.put((flist[i], id(BinaryTree(str(klist[i]), flist[i])), BinaryTree(s
tr(klist[i]), flist[i])))
    return q
```

$O(n\log n)$

In [266]:

```
def printTree(tree):
    print("Main val: " + str(tree.getFreqVal()))
    print("main tree left: " +str(tree.getLeftChild()))
    print("main tree right:" + str(tree.getRightChild()))
    print("left child, left child: " + str(tree.getLeftChild().getLeftChild
()))
    print("left child, right child: " + str(tree.getLeftChild().getRightChild
()))
    print("right child, left child: " + str(tree.getRightChild().getLeftChild
()))
    print("right child, right child: " + str(tree.getRightChild().getRightChil
d()))

def buildTree(q):
    size = q.qsize()
    if(size//2 != 0):
        node1 = q.get()
        node2 = q.get()
        nodeSums = node1[0] + node2[0]
        tree = BinaryTree("EMPTY", nodeSums, node1[2], node2[2])
        q.put((nodeSums, id(tree), tree))
        buildTree(q)
    return q
```

$O(1)$

$O(n\log n)$

In [267]:

```
def makeCode(root, ans, table):
    if (root != None):
        if (root.getLeftChild() != None):
            makeCode(root.getLeftChild(), ans+"0", table)
        if (root.getRightChild() != None):
            makeCode(root.getRightChild(), ans+"1", table)
        if (root.isLeaf()):
            table[root.getCharVal()] =  ans
    return table
```

$O(n)$

In [268]:

```
def encodeMessage(msg, table):
    ans = ""
    for i in range(len(msg)):
        ans += table[msg[i]] + " "
    return ans
```

$O(n)$

In [269]:

```
def bitCounter(msg):
    msg = msg.replace(" ", "")
    return len(msg)
```

$O(n)$

In [270]:

```
def decodeMessage(msg, tree):
    og = tree
    ans = ""
    for i in range(len(msg)):
        if(msg[i] == "0"):
            tree = tree.getLeftChild()
        elif(msg[i] == "1"):
            tree = tree.getRightChild()
        else:
            ans+= (tree.getCharVal() + ", ")
            tree = og
            i += 1
    size = len(ans)
    finalAns = ans[:size - 2]
    return finalAns
```

$O(n)$

Outcomes:

$\boxed{O(n^2 \log n)}$

```
In [271]: def main():

              #Ask user for original message

              userMsg = userInput()

              #Construct suffix array based on original message

              sA = constructArray(userMsg)

              #Perform Burrows Wheeler Transfrom using the suffix array

              BWT = constructBWT(userMsg, sA)

              #Locate patterns by creating an FM Index from the suffix array

              FM, Offset = FMIndex(BWT)

              #Hardcoded visualization of FM Index
              if(userMsg == "banana$"):
                  print ("%2s, %2s,%2s,%2s" % tuple([symbol for symbol in sorted(Offset.
          keys())]))
                  for row in FM:
                      print ("%2d, %2d,%2d,%2d" % tuple([row[symbol] for symbol in sorte
          d(row.keys())]))

                  pattern = "ana"
                  matches = findAllPatterns(pattern, userMsg, sA)
                  if(matches != 0):
                      print("Index of substring including pattern, exclusive of last ind
          ex: " + str(findBWT(pattern, FM, Offset)))

              #Move to front encoding of BWT and mapping from list to string
              MTF = moveToFront(BWT)
              readableMTF = ' '.join(map(str, MTF))

              #Creates frequency table for Huffman Encoding

              freq = makeFrequencies(MTF)
              freq2 = makeFrequencies(userMsg)

              #Creates original Priority Queue using frequency table

              queue = makeQueue(freq)
              queue2 = makeQueue(freq2)

              #Manipulates priority queue so that only one node remains

              tree = buildTree(queue)
              tree2 = buildTree(queue2)
              masterNode = tree.get()
              masterNode2 = tree2.get()

              #Creates individual binary codes for each charatcer based off of the maste
          r node
```

```python
    bitTable = {}
    bitTable = makeCode(masterNode[2], "", bitTable)
    bitTable2 = {}
    bitTable2 = makeCode(masterNode2[2], "", bitTable2)

    #Generates final encoded Huffman Message

    encodedMsg = encodeMessage(MTF, bitTable)
    totalBits = bitCounter(encodedMsg)

    encodedMsg2 = encodeMessage(userMsg, bitTable2)
    totalBits2 = bitCounter(encodedMsg2)

    #Performs both transforms and encoding in reverse order
    #Reversal of Huffman Encoding

    decodedMsg = decodeMessage(encodedMsg, masterNode[2])
    decodedList = list(decodedMsg.split(", "))

    #Reversal of Move to Front Encoding

    uMTF = unMoveToFront(decodedList)

    #Reversal of Burrows Wheeler Transform

    iBWT = inverseBWT(uMTF)

    #Print output
    print("Burrows Wheeler Transform: " + BWT)
    print("Move to front encoding: " + readableMTF)
    print("Encoded Huffman message: " + encodedMsg)

    #If string is only one charatcer (just "l" no spaces, no commas, etc.), it only uses whitespace to decode

    if(encodedMsg.isspace()):
        print("Total Huffman bits with Burrows Wheeler Transform: " + str(len(encodedMsg)))
    else:
        print("Total Huffman bits with Burrows Wheeler Transform: " + str(totalBits) + " vs. Total Huffman bits without Burrows Wheeler Transform: " + str(totalBits2))
        print("Percentage improvement: " + str((((totalBits2-totalBits))/totalBits2)*100) + "%")
    print("Total ASCII bits: " + str(len(userMsg) * 8))
    print("Decoded Huffman message: " + decodedMsg)
    print("Move to front decoding: " + uMTF)
    print("Burrows Wheeler Inverse: " + iBWT)

if __name__ == '__main__':
    main()
```

```
Please enter what you would like to encode/pattern search: banana
('$', 6)
('a$', 5)
('ana$', 3)
('anana$', 1)
('banana$', 0)
('na$', 4)
('nana$', 2)
 $,  a, b, n
 0,  0, 0, 0
 0,  1, 0, 0
 0,  1, 0, 1
 0,  1, 0, 2
 0,  1, 1, 2
 1,  1, 1, 2
 1,  2, 1, 2
 1,  3, 1, 2
Total matches: 2
Index of substring including pattern, exclusive of last index: (2, 4)
Burrows Wheeler Transform: annb$aa
Move to front encoding: 97 110 0 99 39 3 0
Encoded Huffman message: 110 101 01 00 100 111 01
Total Huffman bits with Burrows Wheeler Transform: 18 vs. Total Huffman bits
without Burrows Wheeler Transform: 13
Percentage improvement: -38.46153846153847%
Total ASCII bits: 56
Decoded Huffman message: 97, 110, 0, 99, 39, 3, 0
Move to front decoding: annb$aa
Burrows Wheeler Inverse: banana$
```