THE SHARP RAZOR:
Deflating the Sharpe Ratio by asking for a Minimum Track Record Length

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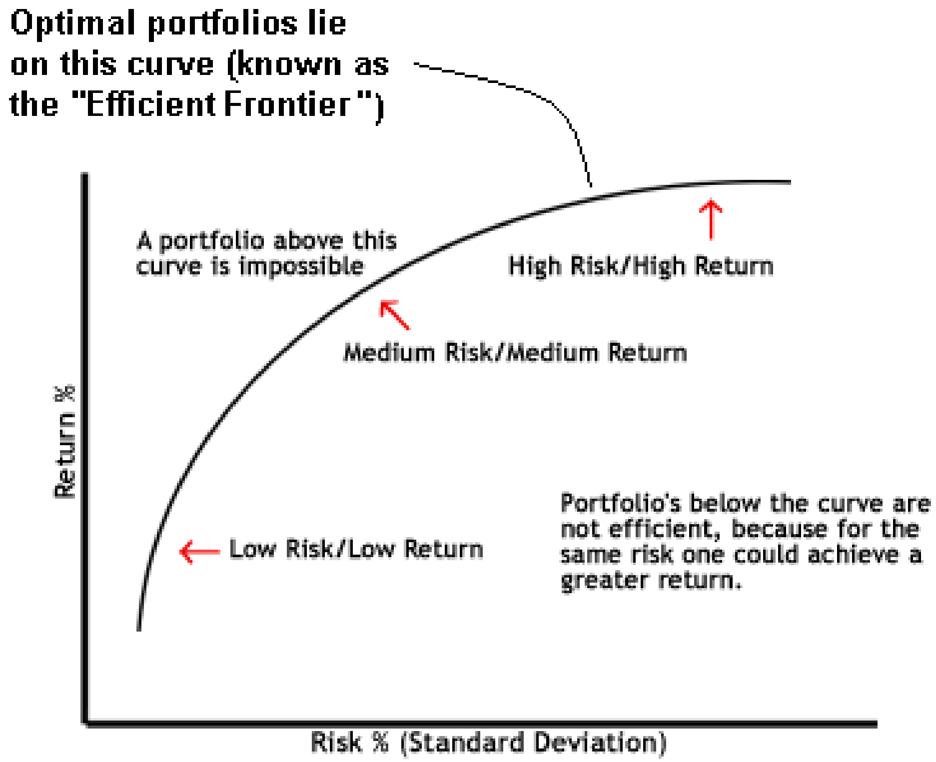
Key Points

- Because the Sharpe ratio only takes into account the first two moments, it wrongly "translates" skewness and excess kurtosis into standard deviation.
- As a result,
  - It *deflates* the skill measured on "well-behaved" investments (positive skewness, negative excess kurtosis).
  - It *inflates* the skill measure on "badly-behaved" investments (negative skewness, positive excess kurtosis).
- **Sharpe ratio estimates need to account for Higher Moments, even if you assume that investors only care about two moments (Markowitz framework)!**

SECTION I
The Mean-Variance framework

Modern Portfolio Theory

- Markowitz introduced “Modern Portfolio Theory” in his 1952 paper “Portfolio Selection” [Journal of Finance].



Among other assumptions:

- Investors are rational and risk-averse.
- Investors are only sensitive to the first two moments, thus the name “Mean-Variance Optimization” (MVO).
- Future Mean and Variance can be exactly predicted.

The Sharpe ratio (1/2)

- Sharpe (1975) applied Markowitz's mean-variance framework to the evaluation of investment performance [Journal of Portfolio Management].
- Suppose that a strategy's excess returns (or risk premiums), r_t, are *IID*

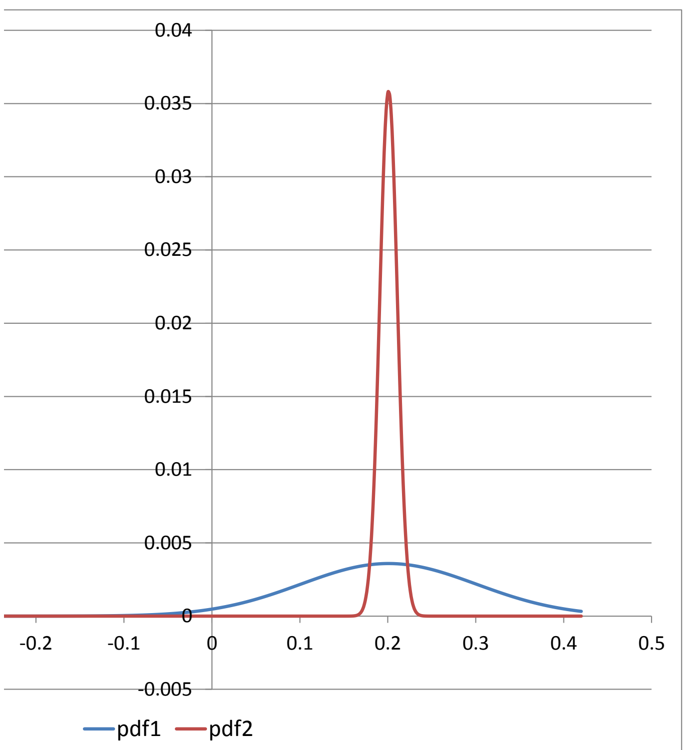
$$r \sim N(\mu, \sigma^2)$$

where *N* represents a Normal distribution with mean μ and variance σ^2. The purpose of the Sharpe ratio (SR) is to evaluate the skills of a particular strategy or investor.

$$SR = \frac{\mu}{\sigma}$$

The Sharpe ratio (2/2)

- What is the idea behind computing the ratio of excess returns and standard deviation?
- There are two interpretations:



- **Mathematical**: The standard deviation measures "dispersion" around the mean. The more dispersion, the more uncertainty regarding the outcomes.
- **Financial**: It is a "return on risk", rather than a "return on capital". The Sharpe ratio is invariant to scale changes, hence to leverage (as long as it is kept constant for each investment).

Introducing Confidence

- One of the problems with this approach is that Mean and Variance are usually unknown. Thus, the true value of SR cannot be know for certain.
- Applying the *Central Limit Theorem*, Lo (2002) derived the Sharpe ratio's confidence band assuming that the returns are IID Normal [Financial Analysts Journal].
- Asymptotically, the estimated $\widehat{SR}$ converges to

$$\left(\widehat{SR} - SR\right) \xrightarrow{a} N\left(0, \frac{1 + \frac{1}{2}SR^2}{n}\right)$$

where *n* is the number of observations.

SECTION II
Relaxing the Model's Assumptions

Dropping the Normality Assumption

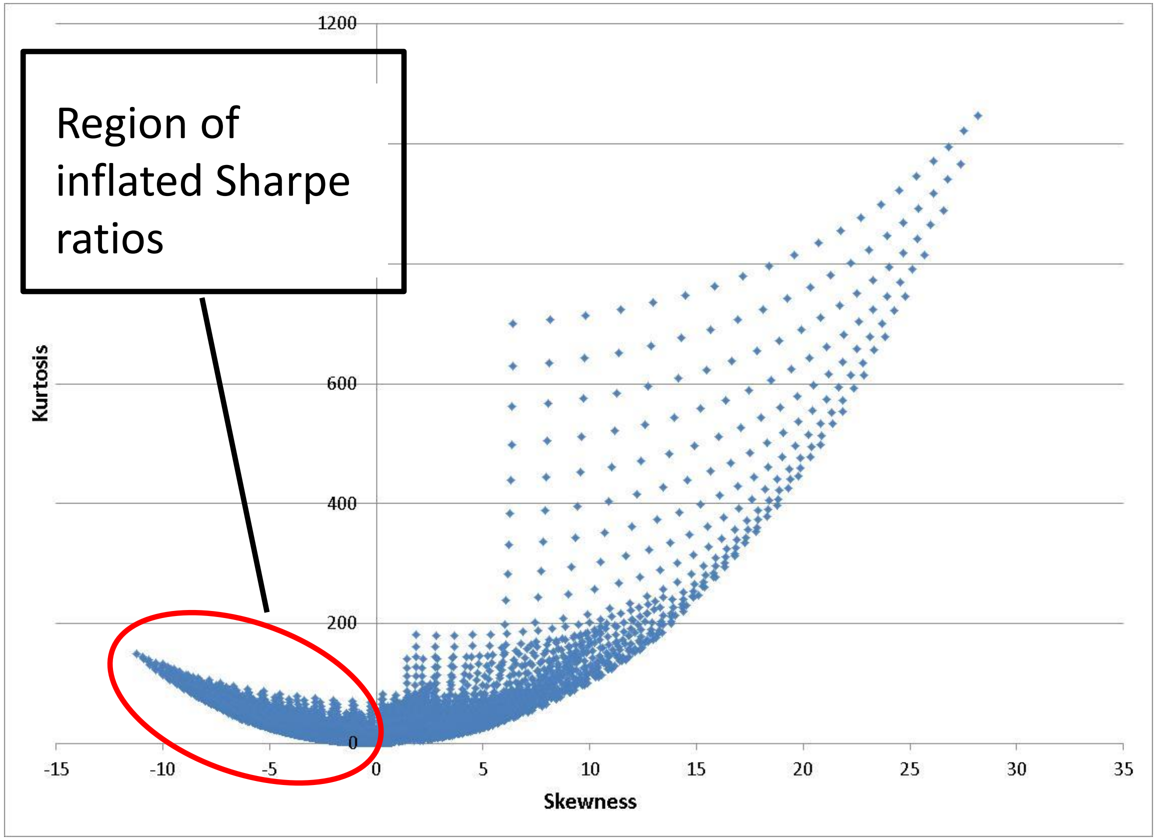
- Mertens (2002) proves that *the Normality assumption on returns could be dropped*, and still the estimated Sharpe ratio converges to a Normal distribution with parameters

$$\left(\widehat{SR} - SR\right) \xrightarrow{a} N\left(0, \frac{1 + \frac{1}{2}SR^2 - \boldsymbol{\gamma_3} SR + \frac{\boldsymbol{\gamma_4 - 3}}{\boldsymbol{4}} SR^2}{n}\right)$$

- Note how the signs associated with the moments make sense!: The variance of Sharpe ratios increases with *negative skewness* and *positive excess kurtosis*.
- <u>Conclusion 1</u>: SR follows a Normal distribution, *even if the returns do not*.

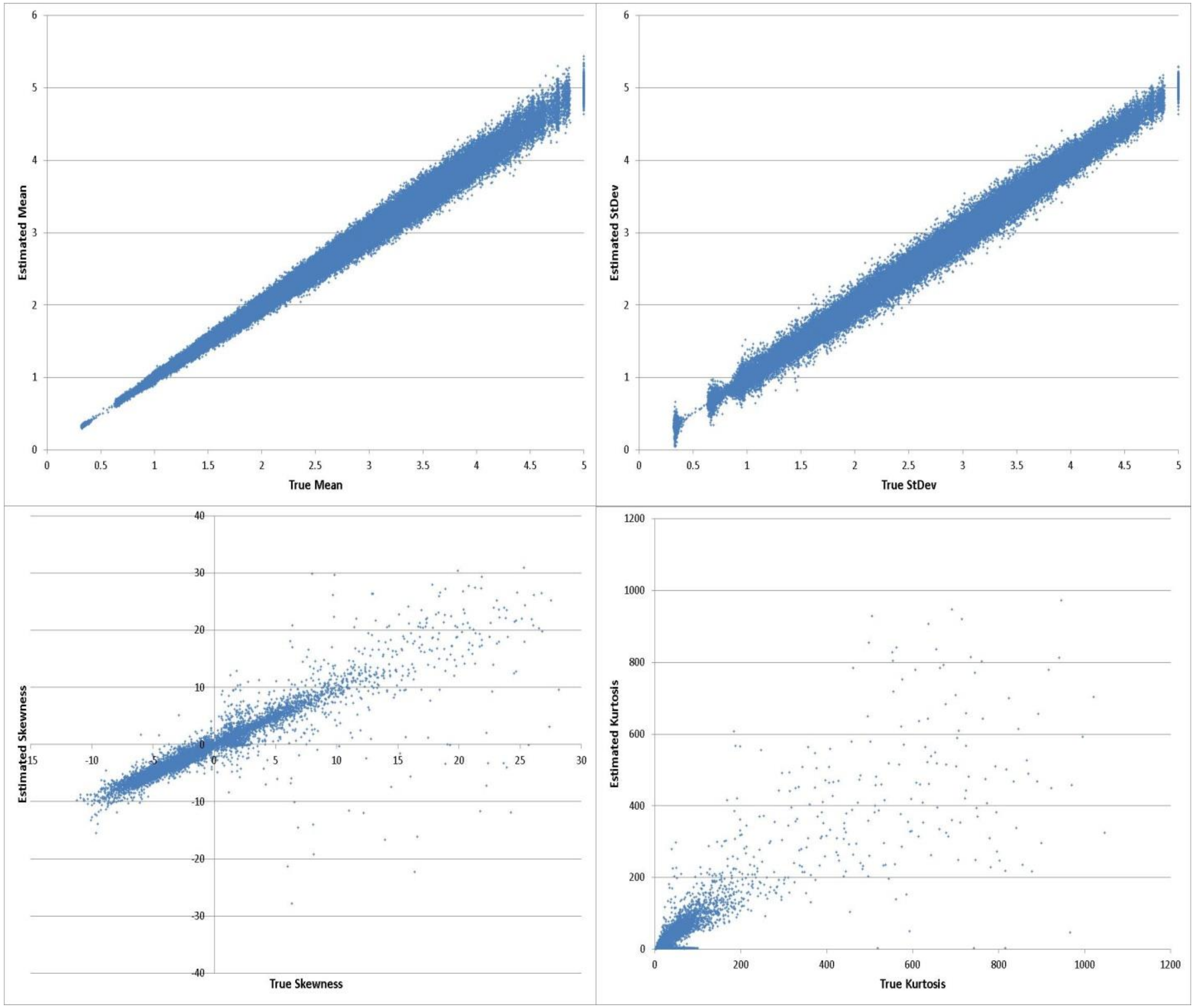
Introducing Higher Moments

- Higher Moments (e.g., Skewness and Kurtosis) do not affect the point estimate of SR.



- However, Skewness and Kurtosis greatly impact the confidence bands of SR.
- *Mixtures of Two Gaussians* produce an infinite number of Non-Normal distributions, all with the same Sharpe ratio (e.g., $\widehat{SR} = 1$).
- High readings of SR may come from extremely risky distributions, like negative skewness and positive kurtosis.

Why Four Moments?



Beyond the 4th moment (kurtosis):

- Estimates become very inaccurate.
- There isn't a good theoretical explanation as to their meaning.

Here we plot the True value vs. the Estimated value (using 1,000 observations per estimate) for 4 moments on 96,551 Mixtures of two Gaussians ($SR^* = 1$).

Dropping the IID Assumption

- Mertens (2002) originally assumed IID returns.
- Christie (2005) uses a GMM approach to derive a limiting distribution that
  - Only assumes Stationary and Ergodic returns.
  - Allows for time-varying conditional volatilities, serial correlation (non-IID returns).
- Surprisingly, Opdyke (2007) proved that the expressions in Mertens (2002) and Christie (2005) are equivalent!
- Conclusion 2: Mertens' result is valid under the more general assumption of *stationary and ergodic returns*, and not only IID.

Portfolio choice with Higher Moments

- Markowitz assumed that the investor's utility function only cares about Mean and Variance.
- Unfortunately, the *confidence* around Mean and Variance estimates are affected by returns' Non-Normality.
- Thus, the confidence around Sharpe ratio estimates is also affected by Higher Moments.
- Conclusion 3: **Sharpe ratio estimates need to account for Higher Moments, even in a Markowitz setting!**
- Hedge Funds' Sharpe ratios are typically inflated by *negative skewness* and *positive excess kurtosis* (Brooks and Kat, (2002), Ingersoll et al. (2007)).

SECTION III
Probabilistic Sharpe Ratio

Probabilistic Sharpe Ratio (PSR)

- We can use Merten's great result to redefine Sharpe ratio, in a probabilistic way.
- Bailey and López de Prado (2012) derive the expression [Journal of Risk]

$$\widehat{PSR}(SR^*) = Z\left[\frac{\left(\widehat{SR} - SR^*\right)\sqrt{n-1}}{\sqrt{1 - \hat{\gamma}_3\widehat{SR} + \frac{\hat{\gamma}_4 - 1}{4}\widehat{SR}^2}}\right]$$

where *Z* is the *cdf* of the Standard Normal distribution and SR^* is a user-defined benchmark SR value.

- Conclusion 4: PSR computes with what probability the estimated $\widehat{SR}$ beats a benchmark SR^*, after correcting for skewness and kurtosis.

Example of SR vs. PSR (1/2)

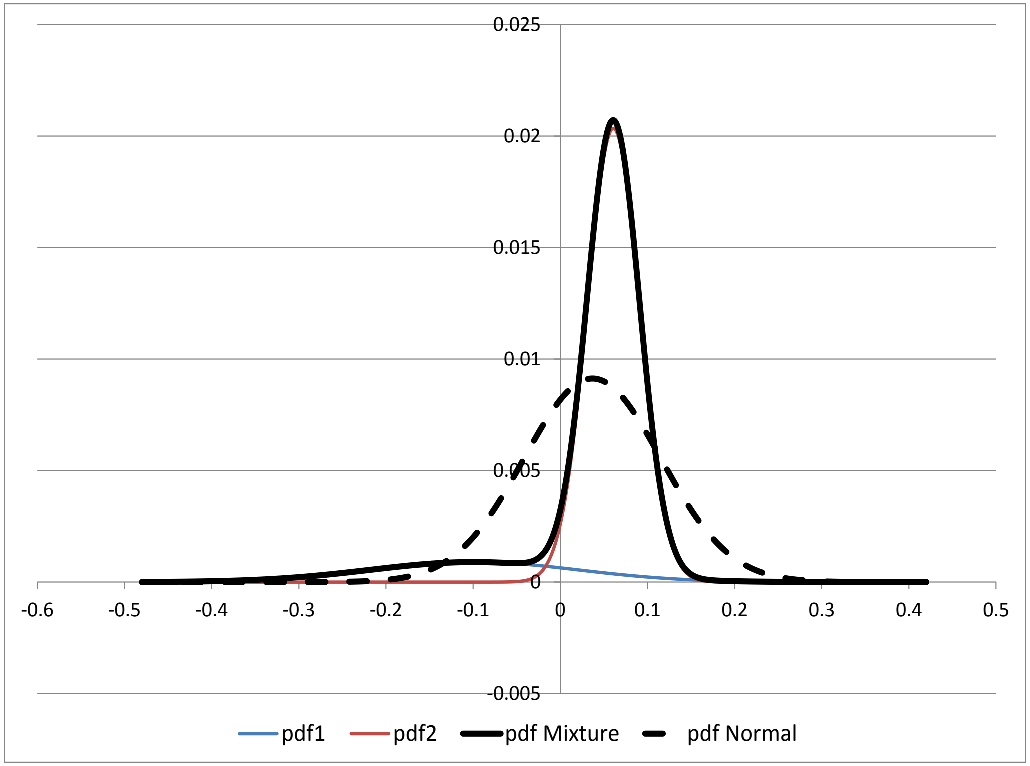
- Suppose a fund with the following statistics over a two years sample of monthly returns:

Stats	Values
Mean	0.036
StDev	0.079
Skew	-2.448
Kurt	10.164
SR	0.458
Ann. SR	1.585

At first sight, an annualized Sharpe ratio of 1.59 over the last two years seems high enough to reject the hypothesis that it has been achieved by sheer luck.

- The question is, *"how inflated is this annualized Sharpe ratio due to the track record's non-normality, length and sampling frequency?"*

Example of SR vs. PSR (2/2)



The Non-Normal dist. is consistent with the fund's stats. The Normal dist. has the same Sharpe ratio estimate (1.59). However, the confidence around these two $\widehat{SR}$ estimates is very different :

- Normal: $\sigma_{\widehat{SR}} = 0.22$
- Non-Normal: $\sigma_{\widehat{SR}} = 0.34$

A rational investor would prefer the fund with Normal returns, because it delivers *the same Sharpe ratio with greater confidence*.

PSR incorporates this confidence information by estimating the probability that the estimated $\widehat{SR}$ is in reality greater than a given benchmark value, SR^*. For example, for $SR^* = 0$ (skill-less benchmark), $\widehat{PSR}(0) = 0.982$ for the Normal dist. fund, compared to the $\widehat{PSR}(0) = 0.913$ for the Non-Normal dist. fund.

SECTION IV
Minimum Track Record Length

Track Record Length and Investment Skill

- The previous example was not meant to imply that a track record of 1.59 Sharpe ratio is “insignificant”.
- As a matter of fact, should we have 3 years instead of 2, $\widehat{PSR}(0) = 0.953$, typically enough to reject the hypothesis of skill-less performance… even after accounting for *skewness* and *kurtosis*!
- In other words, a longer track record may be able to compensate for the uncertainty introduced by non-Normal returns.
- How can we formulate that “compensation effect” between non-Normality and the track record’s length?

Minimum Track Record Length (MinTRL)

- Question: *"How long should a track record be in order to have statistical confidence that its Sharpe ratio is above a given threshold?"*
- Bailey and López de Prado (2012) computed the answer:

$$MinTRL = 1 + \left[1 - \hat{\gamma}_3 \widehat{SR} + \frac{\hat{\gamma}_4 - 1}{4} \widehat{SR}^2\right] \left(\frac{Z_\alpha}{\widehat{SR} - SR^*}\right)^2$$

- Conclusion 5: A longer track record will be required the
  - smaller $\widehat{SR}$ is, or
  - the more negatively skewed returns are, or
  - the greater the fat tails, or the greater our required level of confidence.

SECTION V
Numerical Examples

MinTRL for a Daily IID Normal returns

- Minimum track record lengths (*MinTRL*) in years required for various combinations of measured $\widehat{SR}$ (rows) and benchmarked SR^* (columns) at a 95% confidence level, based upon *daily* IID Normal returns.

		True Sharpe Ratio									
		0	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**
Observed Sharpe	**0**										
	0.5	10.83									
	1	2.71	10.85								
	1.5	1.21	2.72	10.87							
	2	0.69	1.22	**2.73**	10.91						
	2.5	0.44	0.69	1.22	2.74	10.96					
	3	0.31	0.44	0.69	1.23	2.76	11.02				
	3.5	0.23	0.31	0.45	0.70	1.24	2.78	11.09			
	4	0.18	0.23	0.31	0.45	0.70	1.24	2.80	11.17		
	4.5	0.14	0.18	0.23	0.32	0.45	0.71	1.25	2.82	11.26	
	5	0.12	0.14	0.18	0.24	0.32	0.46	0.71	1.27	2.84	11.36

For example, a ***2.73*** years track record is required for an annualized Sharpe of 2 to be considered greater than 1 at a 95% confidence level.

MinTRL for a Weekly IID Normal returns

- Minimum track record lengths ($MinTRL$) in years required for various combinations of measured $\widehat{SR}$ (rows) and benchmarked SR^* (columns) at a 95% confidence level, based upon *weekly* IID Normal returns.

	True Sharpe Ratio									
Observed Sharpe	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**
0										
0.5	10.87									
1	2.75	10.95								
1.5	1.25	2.78	11.08							
2	0.72	1.27	**2.83**	11.26						
2.5	0.48	0.74	1.29	2.89	11.49					
3	0.35	0.49	0.75	1.33	2.96	11.78				
3.5	0.27	0.36	0.50	0.78	1.36	3.04	12.12			
4	0.21	0.27	0.37	0.52	0.80	1.41	3.14	12.51		
4.5	0.18	0.22	0.28	0.38	0.54	0.83	1.46	3.25	12.95	
5	0.15	0.18	0.23	0.29	0.39	0.56	0.86	1.51	3.38	13.44

For example, a ***2.83*** years track record is required for an annualized Sharpe of 2 to be considered greater than 1 at a 95% confidence level.

MinTRL for a Monthly IID Normal returns

- Minimum track record lengths (*MinTRL*) in years required for various combinations of measured $\widehat{SR}$ (rows) and benchmarked SR^* (columns) at a 95% confidence level, based upon *monthly* IID Normal returns.

		True Sharpe Ratio									
		0	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**
Observed Sharpe	**0**										
	0.5	11.02									
	1	2.90	11.36								
	1.5	1.40	3.04	11.92							
	2	0.87	1.49	**3.24**	12.71						
	2.5	0.63	0.94	1.60	3.49	13.72					
	3	0.50	0.68	1.01	1.74	3.80	14.96				
	3.5	0.42	0.54	0.74	1.10	1.90	4.17	16.43			
	4	0.37	0.45	0.58	0.80	1.21	2.09	4.59	18.12		
	4.5	0.33	0.40	0.49	0.64	0.88	1.33	2.30	5.07	20.04	
	5	0.30	0.36	0.43	0.53	0.70	0.97	1.46	2.54	5.61	22.18

For example, a ***3.24*** years track record is required for an annualized Sharpe of 2 to be considered greater than 1 at a 95% confidence level.

MinTRL for a Monthly IID Non-Normal returns

- Minimum track record lengths (*MinTRL*) in years required for various combinations of measured $\widehat{SR}$ (rows) and benchmarked SR^* (columns) at a 95% confidence level, based upon *monthly* IID with $\hat{\gamma}_3 = -0.72, \hat{\gamma}_4 = 5.78$.

	True Sharpe Ratio									
Observed Sharpe	**0**	**0.5**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**
0										
0.5	12.30									
1	3.62	14.23								
1.5	1.93	4.24	16.70							
2	1.31	2.26	**4.99**	19.72						
2.5	1.01	1.53	2.66	5.88	23.26					
3	0.84	1.17	1.79	3.11	6.90	27.35				
3.5	0.73	0.97	1.36	2.08	3.63	8.06	31.98			
4	0.66	0.84	1.11	1.57	2.40	4.20	9.35	37.15		
4.5	0.61	0.75	0.96	1.27	1.79	2.76	4.84	10.78	42.85	
5	0.57	0.69	0.85	1.08	1.44	2.04	3.15	5.53	12.34	49.09

For example, a ***4.99*** years track record is required for an annualized Sharpe of 2 to be considered greater than 1 at a 95% confidence level.

SECTION VI
Skillful Hedge Fund Styles

Which Hedge Fund Styles are skillful?

- Conclusion 6: After adjusting for skewness and kurtosis, only a few hedge fund styles deliver performance beyond what would be expected by sheer luck.

HFR Index	Code	SR	StDev(SR)	An. SR	Low An. SR	PSR(0)	PSR(0.5)	MinTRL (0)	MinTRL (0.5)
Conserv	HFRIFOFC Index	0.251	0.116	0.871	0.210	0.985	0.822	6.456	35.243
Conv Arbit	HFRICAI Index	0.253	0.124	0.875	0.170	0.979	0.809	7.282	39.246
Dist Secur	HFRIDSI Index	0.414	0.116	1.433	0.771	1.000	0.990	2.448	5.661
Divers	HFRIFOFD Index	0.208	0.099	0.719	0.158	0.982	0.740	6.841	72.870
EM Asia	HFRIEMA Index	0.200	0.092	0.691	0.168	0.985	0.726	6.423	82.857
EM Global	HFRIEMG Index	0.258	0.100	0.892	0.325	0.995	0.872	4.559	23.242
EM Latin Amer	HFRIEMLA Index	0.173	0.093	0.598	0.068	0.968	0.620	8.782	323.473
Emerg Mkt	HFRIEM Index	0.259	0.100	0.896	0.324	0.995	0.873	4.602	23.214
Equity Hedge	HFRIEHI Index	0.196	0.092	0.681	0.158	0.984	0.715	6.608	92.752
Equity Neutral	HFRIEMNI Index	0.413	0.099	1.432	0.866	1.000	0.997	1.817	4.176
Event Driven	HFRIEDI Index	0.348	0.108	1.205	0.589	0.999	0.970	2.982	8.548
Fixed Asset-Back	HFRIFIMB Index	0.657	0.153	2.276	1.405	1.000	1.000	1.706	2.749
Fixed Hig	HFRIFIHY Index	0.283	0.120	0.980	0.294	0.991	0.875	5.513	22.716
Fund of Funds	HFRIFOF Index	0.213	0.099	0.739	0.174	0.984	0.757	6.560	61.984
Macro	HFRIMI Index	0.381	0.087	1.320	0.824	1.000	0.997	1.649	4.138
Mkt Defens	HFRIFOFM Index	0.388	0.087	1.343	0.847	1.000	0.997	1.596	3.922
Mrg Arbit	HFRIMAI Index	0.496	0.112	1.717	1.080	1.000	0.999	1.611	3.124
Multi-Strategy	HFRIFI Index	0.361	0.138	1.252	0.468	0.996	0.943	4.426	12.118
Priv/Regulation	HFRIREGD Index	0.225	0.082	0.780	0.312	0.997	0.837	4.083	31.061
Quant Direct	HFRIENHI Index	0.146	0.090	0.506	-0.005	0.948	0.508	11.400	77398.739
Relative Value	HFRIRVA Index	0.470	0.163	1.630	0.702	0.998	0.977	3.676	7.561
Russia-East Euro	HFRICIS Index	0.278	0.104	0.964	0.369	0.996	0.900	4.303	18.285
Sec Energy	HFRISEN Index	0.278	0.094	0.963	0.427	0.998	0.922	3.522	14.951
Sec Techno	HFRISTI Index	0.067	0.086	0.231	-0.261	0.780	0.184	50.420	n/a
Short Bias	HFRISHSE Index	0.043	0.086	0.148	-0.344	0.690	0.120	122.495	n/a
Strategic	HFRIFOFS Index	0.149	0.091	0.517	-0.004	0.949	0.521	11.348	10935.740
Sys Diversified	HFRIMTI Index	0.316	0.085	1.094	0.610	1.000	0.978	2.252	7.434
Wgt Comp	HFRIFWI Index	0.287	0.097	0.994	0.441	0.998	0.929	3.515	13.974
Wgt Comp CHF	HFRIFWIC Index	0.229	0.088	0.792	0.291	0.995	0.831	4.513	32.660
Wgt Comp GBP	HFRIFWIG Index	0.181	0.093	0.626	0.097	0.974	0.653	7.986	194.050
Wgt Comp GBP	HFRIFWIG Index	0.181	0.093	0.626	0.097	0.974	0.653	7.986	194.050
Wgt Comp JPY	HFRIFWIJ Index	0.167	0.090	0.580	0.065	0.968	0.601	8.805	459.523
Yld Alternative	HFRISRE Index	0.310	0.108	1.073	0.456	0.998	0.937	3.748	12.926

- Distressed Securities
- Equity Market Neutral
- Event Driven
- Fixed Asset-Backed
- Macro
- Market Defensive
- Mortgage Arbitrage
- Relative Value
- Systematic Diversified

SECTION VII
The Sharpe Ratio Efficient Frontier

A new investment paradigm (1/4)

- Following Markowitz (1952), a portfolio *w* belongs to the Efficient Frontier if it delivers maximum expected excess *return on capital* ($E[rw]$) subject to the level of uncertainty surrounding those portfolios' excess returns ($\hat{\sigma}_{(rw)}$).

$$\max_{w} \quad E[rw] \Big| \hat{\sigma}_{(rw)} = \sigma^*$$

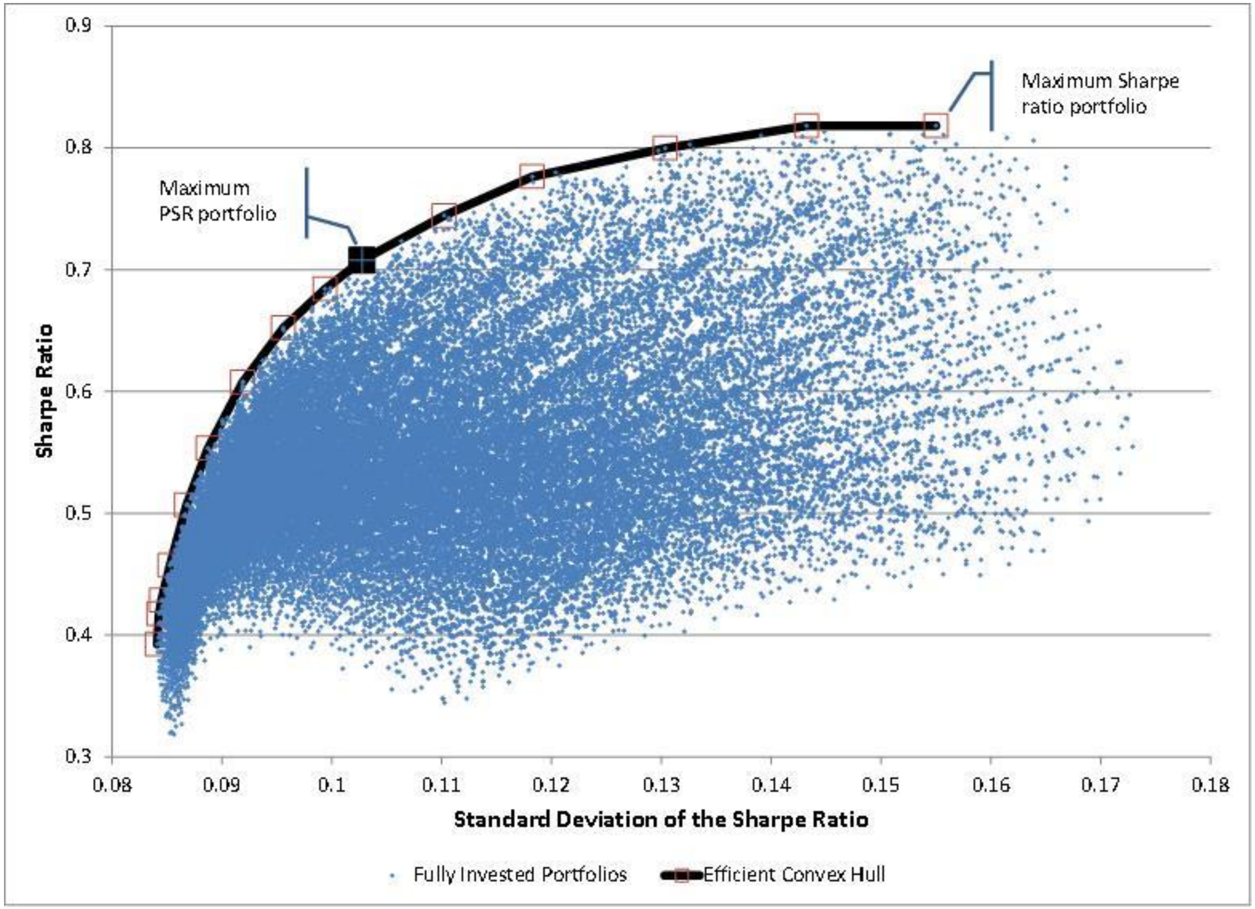
A new investment paradigm (2/4)

- Similarly, we define what we denote the *Sharpe ratio Efficient Frontier* (SEF) as the set of portfolios $\{w\}$ that deliver the highest expected excess *return on risk* (as expressed by their Sharpe ratios) subject to the level of uncertainty surrounding those portfolios' excess returns on risk (the standard deviation of the Sharpe ratio).

$$\max_{w} \quad \widehat{SR}(rw) \middle| \hat{\sigma}_{\widehat{SR}(rw)} = \sigma^*$$

A new investment paradigm (3/4)

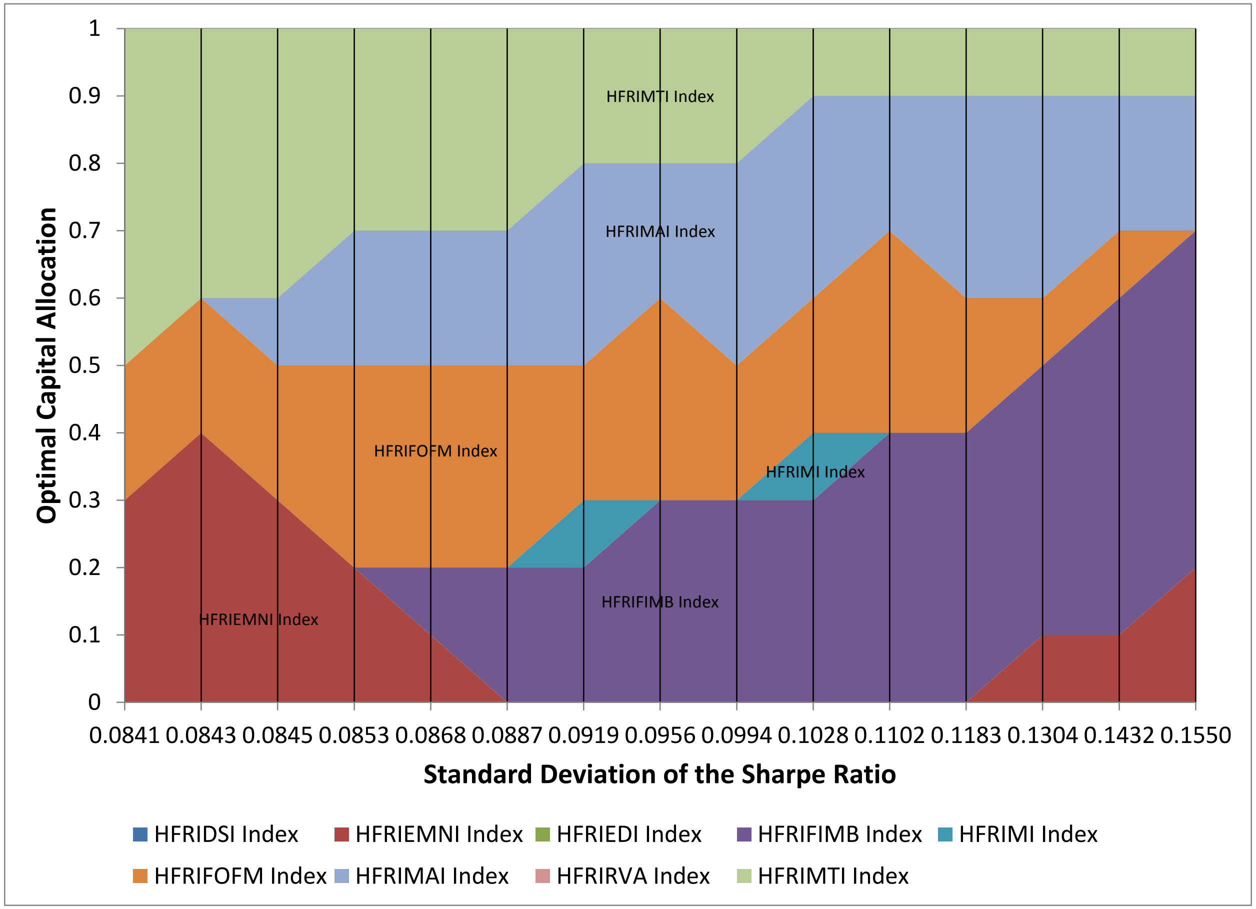
- The Sharpe ratio Efficient Frontier (SEF) is derived in terms of optimal mean-variance combinations of **risk-adjusted returns**.



The portfolio at the right end of the SEF is the traditional *Maximum Sharpe ratio portfolio*. It delivers a greater mean SR, however at a much lower confidence.

A new investment paradigm (4/4)

- We can compute the capital allocations that deliver maximum Sharpe ratios **for each confidence level**.



The key difference with Markowitz's Efficient Frontier is that *SEF* is computed on **risk-adjusted returns**, rather than **returns on capital**.

Computing the PSR Optimal Portfolio (1/2)

- For example, this is the optimal PSR capital allocation that results from using the HFR database (01/01/00-05/01/11).

HFR Index	Code	Max PSR	Max SR
Dist Secur	HFRIDSI Index	0	0
Equity Neutral	HFRIEMNI Index	0	0.2
Event Driven	HFRIEDI Index	0	0
Fixed Asset-Back	HFRIFIMB Index	0.3	0.5
Macro	HFRIMI Index	0.1	0
Mkt Defens	HFRIFOFM Index	0.2	0
Mrg Arbit	HFRIMAI Index	0.3	0.2
Relative Value	HFRIRVA Index	0	0
Sys Diversified	HFRIMTI Index	0.1	0.1

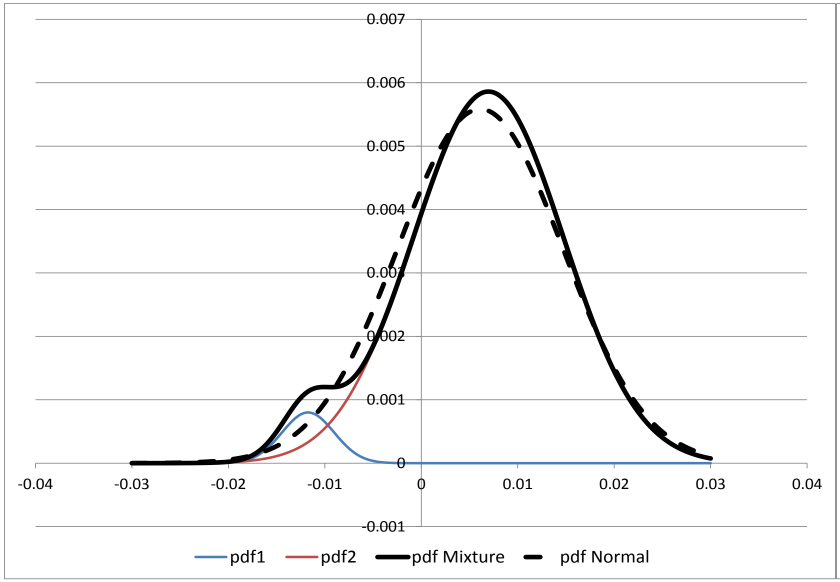
Stat	Max PSR	Max SR
Average	0.0061	0.0060
StDev	0.0086	0.0073
Skew	-0.2250	-1.4455
Kurt	2.9570	7.0497
Num	134	134
SR	0.7079	0.8183
StDev(SR)	0.1028	0.1550
An. SR	2.4523	2.8347
Low An. SR	1.8667	1.9515
PSR(0)	1.00000	1.00000
PSR(0.5)	1.00000	0.99999
MinTRL (0)	0.7152	1.1593
MinTRL (0.5)	1.0804	1.6695

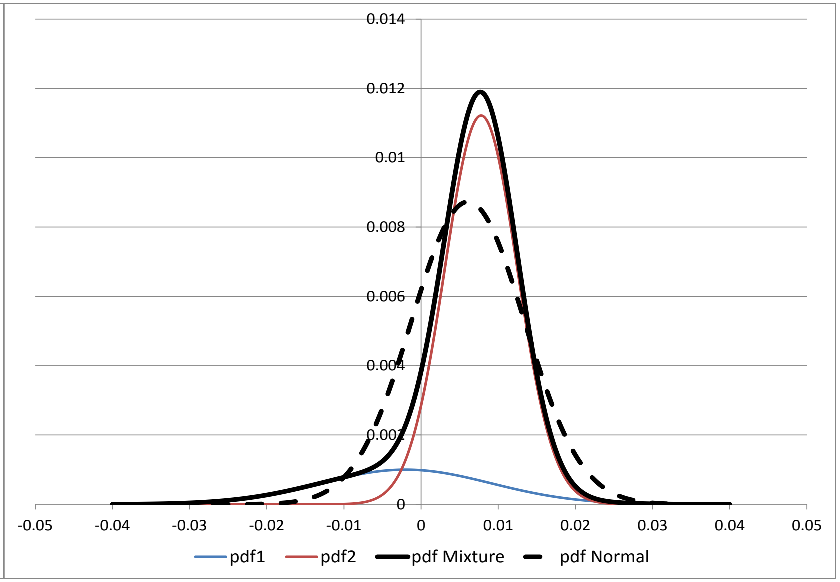
Max PSR solution is preferable:

- Although it delivers a lower Sharpe ratio than the *Max SR* portfolio (0.708 vs. 0.818 in monthly terms), its better diversified allocations allow for a much greater confidence (0.103 vs. 0.155 standard deviations).
- *Max PSR* invests in 5 styles, and the largest holding is 30%, compared to the 4 styles and 50% maximum holding of the *Max SR* portfolio.

Computing the PSR Optimal Portfolio (2/2)

- *Max PSR* is very close to Normal (left figure), while the *Max SR* portfolio features a risky left fat-tail (right figure).





- Conclusion 7: Taking into account higher moments has allowed us to *naturally* find a better balanced portfolio that is optimal in terms of uncertainty-adjusted SR.

SECTION VIII
Conclusions

Conclusions (1/2)

1. SR follows a Normal distribution, even if the returns do not.
2. Mertens' result is valid under the more general assumption of stationary and ergodic returns, and not only IID.
3. Sharpe ratio estimates need to account for Higher Moments, even in a Markowitz setting!
4. PSR computes with what probability the estimated $\widehat{SR}$ beats a benchmark SR^*, after correcting for skewness and kurtosis.

Conclusions (2/2)

5. A longer track record will be required the
   - smaller $\widehat{SR}$ is, or
   - the more negatively skewed returns are, or
   - the greater the fat tails, or the greater our required level of confidence.
6. After adjusting for skewness and kurtosis, only a few hedge fund styles deliver performance beyond what would be expected by sheer luck.
7. Taking into account higher moments has allowed us to *naturally* find a better balanced portfolio that is optimal in terms of uncertainty-adjusted SR.

THANKS FOR YOUR ATTENTION!

SECTION IX
The stuff nobody reads

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Bio

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The research contained in this presentation is the result of a continuing collaboration with

Dr. David H. Bailey, LBNL

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