BARRA

BARRA's Risk Models

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Investment decisions boil down to picking a risk-return combination with which one is comfortable. At one end of the spectrum lie nominally riskless savings accounts, whereas at the other end lie exotic derivative securities whose structures, let alone their risks, are difficult to understand. It is natural to think that greater risks are rewarded with greater average returns. This, however, is an oversimplification. Greater risk is rewarded only to the extent that the economy as a whole is concerned about the source of greater risk. Proper investment decisions, therefore, should begin by considering the set of investment opportunities that provide a given level of return for the *smallest* level of risk. This set is referred to as the *efficient* set. Within the efficient set, greater return may be obtained only by bearing greater risk.

Key to defining the efficient set is a definition and measurement of risk. A commonly used and eminently justifiable definition of risk is the dispersion of actual returns around the expected or average return. This dispersion is measured via the *standard deviation* of returns. Although this dispersion is readily quantified for individual securities, the dispersion of portfolio returns is crucially dependent on the degree of comovement in security returns. For example, consider two securities whose returns move in lock step. When one security returns 15 percent, the other returns 15 percent, and so on. Both securities are risky, but by selling one and buying the other, we can obtain a guaranteed return! Thus, in defining the efficient set, we need measures of security dispersion and comovement. These are contained in the *covariance matrix* of security returns.

There are a number of ways of estimating the covariance matrix of security returns. Substantial gains are made by recognizing that covariances are driven by common sources of returns across securities. These common sources of returns are called *common factors*. Estimating the covariance matrix of security returns thus depends on estimating a factor model for security returns.

In this article, we will discuss the benefits and costs of the different approaches to estimating factor models of security returns. We will begin by discussing how portfolio standard deviations are computed from the covariance matrix of security returns. Next, we will discuss how we might estimate the covariance matrix and the role that factor models play in this estimation. This leads us into the different types of factor models, and the strengths and weaknesses of each. We will focus on BARRA's approach to estimating factor models, and contrast it with other approaches. Empirical evidence regarding the accuracy of BARRA's risk forecasts will be presented, and the performance of BARRA's model relative to other approaches will be discussed. As we shall see, BARRA's risk model provides accurate, robust, and intuitively appealing risk forecasts.

Risk Measurement and the Covariance Matrix of Security Returns

The standard deviation is a natural measure of security risk because it measures the dispersion of possible returns around the mean or expected return. It also allows us to make statements about the likelihood of possible returns: about two thirds of the time the realized return will be within one standard deviation from the mean, and about 95 percent of the time the realized return will be within two standard deviations of the mean.[1]

Our interest lies in portfolio standard deviations. For a portfolio of two securities (e.g., IBM and EXXON) with 75 percent of the portfolio holdings in IBM and 25 percent of the holdings in EXXON, the portfolio standard deviation is given by

$$\sqrt{0.75^2\,\sigma_{IBM}^2 + 0.25^2\,\sigma_{EXXON}^2 + 2(0.75)(0.25)\sigma_{IBM,EXXON}} \tag{1}$$

where σ_{IBM} and σ_{EXXON} denote the standard deviations of IBM and EXXON's returns respectively, and $\sigma_{IBM,EXXON}$ denotes the covariance between IBM and EXXON's returns. The covariance is the expected or average value of the cross-product of deviations from the mean returns of the two stocks:

$$\sigma_{IBM,EXXON} = E\Big[\big(R_{IBM} - E(R_{IBM})\big)\big(R_{EXXON} - E(R_{EXXON})\big)\Big]. \tag{2}$$

The *correlation* between the returns of the two stocks, $\rho_{IBM,EXXON}$ is given by

$$\rho_{IBM,EXXON} = \frac{\sigma_{IBM,EXXON}}{\sigma_{IBM}\sigma_{EXXON}}. \tag{3}$$

The correlation measures the degree to which the two stocks move together. If two stocks are strongly positively correlated, then their returns tend to move

[1] Strictly, these probabilities apply to returns that are normally distributed.

together. Hence their risks do not offset in a portfolio. On the other hand, stocks that have low correlations will tend to cancel each other's risks, resulting in substantial risk reduction by combining the stocks in portfolios.

In general, with holdings h_{IBM} and h_{EXXON}, the standard deviation of the portfolio's return is

$$\sqrt{h_{IBM}^2\sigma_{IBM}^2 + h_{EXXON}^2\sigma_{EXXON}^2 + 2h_{IBM}h_{EXXON}\sigma_{IBM,EXXON}}\,. \tag{4}$$

Characterization of the risk of any portfolio of these two securities thus requires estimates of the standard deviations (or variances) of their returns, as well as the covariance between their returns. This information is succinctly captured in the covariance matrix of returns, *V*, which contains all the asset variances and covariances. For a portfolio of many assets, with holdings represented by a vector *h*, the standard deviation of the portfolio's return is given by

$$\sigma_P = \sqrt{h^T V h}\,. \tag{5}$$

Accurate characterization of portfolio risk thus requires an accurate estimate of the covariance matrix of security returns.

Estimating the Covariance Matrix and Factor Models of Security Returns

A relatively simple way to estimate the covariance matrix is to use the history of security returns to compute each variance and covariance. This approach, however, suffers from two drawbacks. First, estimating a covariance matrix for 3000 stocks requires data for at least 3,000 periods. With monthly or weekly horizons, such a long history may simply not exist. Second, it is subject to estimation error: in any period, two stocks such as Weyerhaeuser and Ford may show very high correlation, higher than, say, GM and Ford. Our intuition suggests that the correlation between GM and Ford should be higher because they are in the same line of business. The simple method of estimating the covariance matrix does not capture our intuition.

This intuition, however, points to an alternative method for estimating the covariance matrix. Our feeling that GM and Ford should be more highly correlated than Weyerhaeuser and Ford comes from Ford and GM being in the same industry. Taking this further, we can argue that firms with similar characteristics, such as their line of business, should have returns that behave similarly. For example, Weyerhaeuser, Ford, and GM will all have a common component in their returns because they would all be affected by news that affects the stock market as a whole. The effects of such news may be captured by a stock market component in each stock's return. This common component may be the (weighted) average return to all U.S. stocks. The degree to which each of the three stocks responds to this stock market component depends on the sensitivity of each stock to the stock market component. Additionally, we would expect GM and Ford to respond to news affecting the automobile industry, whereas we would expect Weyerhaeuser to respond to news affecting the forest and paper products industry. The effects of such news may be captured by the average returns of stocks in the auto industry and the forest and paper products industry. There are, however, events that affect one stock without affecting the others. For example, a defect in the brake system of GM cars, that forces a recall and replacement of the system, will likely have a negative impact on GM's stock price. This event, however will most likely leave Weyerhaeuser and Ford stock prices unaltered.

These arguments lead us to the following representation for returns:

$$\begin{aligned} R_{GM} &= E\left[R_{GM}\right] + B_{1,GM} \cdot \left[R_M - E\left[R_M\right]\right] \\ &\quad + 1 \cdot \left[R_{AUTO} - E\left[R_{AUTO}\right]\right] + 0 \cdot \left[R_{FP} - E\left[R_{FP}\right]\right] + \in_{GM} \end{aligned} \tag{6}$$

where

R_{GM} denotes GM's realized return,

R_M denotes the realized average stock market return,

R_{AUTO} denotes the realized average return to automobile stocks,

R_{FP} denotes the realized average return to forest and paper products stocks,

$E[.]$ denotes expectations,

$B_{1,GM}$ denotes GM's sensitivity to stock market returns, and

$\in_{GM}$ captures the effect of GM specific news on GM returns.

This equation simply states that GM's realized return consists of an expected component and an unexpected component. The unexpected component depends on any unexpected events that affect stock returns in general $\left(R_M - E\left[R_M\right]\right)$, any unexpected events that affect the auto industry $\left(R_{AUTO} - E\left[R_{AUTO}\right]\right)$, and any unexpected events that affect GM alone $\left(\in_{GM}\right)$. Similar equations may be written for Ford and Weyerhaeuser.

The sources of variation in GM's stock returns, thus, are variations in stock returns in general, variations in auto industry returns, and any variations that are specific to GM. Moreover, GM and Ford returns are likely to move together because both are exposed to stock market risk and auto industry risk. Weyerhaeuser and GM, and Weyerhaeuser and Ford, on the other hand, are likely to move together to a lesser degree because the only common component in their returns is the market return. Some additional correlation would arise, however, because auto and forest and paper products industry returns may exhibit some correlation.

By beginning with our intuition about the sources of comovement in security returns, we have made substantial progress in estimating the covariance matrix of security returns. What we need now is the covariance matrix of common sources in security returns, the variances of security specific returns, and estimates of the sensitivity of security returns to the common sources of variation in their returns. Because the common sources of risk are likely to be much fewer than the number of securities, we need to estimate a much smaller covariance matrix and hence a smaller history of returns is required. Moreover, because similar stocks are going to have larger sensitivities to similar common sources of risk, similar stocks will be more highly correlated than dissimilar stocks: our estimated correlation for GM and Ford will be larger than that for Ford and Weyerhaeuser.

The decomposition of security returns into common and specific sources of return is, in fact, a factor model of security returns. Denoting by R_i the return of security *i*, by *f* the vector of returns to portfolios representing common sources of return or *common factors*, and by $\in_i$ the specific return of the security, we have simply stated that

$$R_i = E[R_i] + X_i[f - E[f]] + \in_i \quad (7)$$

where X_i is the vector of the sensitivities of the security returns to the common factors. For the entire vector of security returns, *R*, we have

$$R - E[R] = X[f - E[f]] + \in \quad (8)$$

where *X* is the matrix of factor sensitivities and $\in$ is the vector of security specific returns. The covariance matrix of security returns is then given by

$$V = XFX^T + \Delta \quad (9)$$

where *F* is the covariance matrix of factor returns and Δ is the matrix with the variances of specific returns along the diagonal. Note that the off-diagonal elements of Δ, the covariances of the specific returns, should be zero because specific returns are driven by events that affect only the returns of a specific firm.

Estimation of *V* thus requires estimation of the sensitivities of the securities to the common factors (X), the covariance matrix of the factors (F), and the variances of security specific returns (Δ).

Estimation of Factor Models

There are three general approaches to estimating a factor model of returns. One approach is to estimate the security sensitivities (X) from fundamental information about the securities. For example, we would begin with a list of industries and assign each stock a weight in each industry depending on the proportion of value obtained from each industry, and treat these weights as the sensitivities of the firm to the industry factor returns. Similarly, we would construct indices of other firm characteristics (such as leverage), and treat these risk indices as sensitivities to the factors associated with those characteristics (such as a leverage factor). Thus, there is a factor corresponding to every industry, and a factor corresponding to every risk index. Given the firm sensitivities to the factors, the factor returns and specific returns are treated as unobservable and are estimated via monthly regressions of stock returns on their sensitivities. The covariance matrix of factors and specific returns is then computed from the time series of factor returns and specific returns. This approach is referred to as the Fundamental approach because it begins with data on firm fundamentals.

A second approach treats observable macroeconomic variables, such as GNP growth and unexpected inflation, as factors and estimates the sensitivities of the securities to these factors. The factor covariance matrix in this case may be estimated directly from the data on the factors. This approach is called the Macroeconomic or Observable factor approach.

The third approach treats both the factors as well as the sensitivities to those factors as unobservable. It takes the covariance matrix of realized returns and decomposes it into a factor component and a specific component. In the process, it estimates the security sensitivities to the factors. This is known as the statistical approach to estimating factor models.[2]

Comparing the Three Types of Factor Models

Risk Forecasts

The three types of factor models differ in their specification of factors, in their estimation method, and, consequently, in their inputs and outputs and their ability to model and capture changing risk. In judging the merits of each approach, we should keep sight of our objective: accurate measurement of security risk. To this end, we would prefer a procedure that is robust (less liable to pick up spurious correlations), capable of explaining the variability in returns (the common sources of risk are captured), and is dynamic (able to change risk predictions as the determinants of risk change). Let us examine each factor model along these dimensions.

Both fundamental and macroeconomic factor models are robust because they do not use the history of correlations to predict correlations going forward. Statistical factor models, however, are subject to picking up spurious correla-

[2] Details of the estimation method for each factor model are given in the appendix.

tions because they use the history of security correlations to estimate the factor covariance matrix, matrix of specific variances, and the sensitivities of security returns to the factors. Thus, if the observations come from a period in which Weyerhaeuser and Ford returns are highly correlated, then Weyerhaeuser and Ford will both have relatively large or similar estimated sensitivities to one (or more) of the factors. Using these estimated sensitivities to predict correlations will result in a high predicted correlation between Ford and Weyerhaeuser. It is useful to think of fundamental and macroeconomic factor models as filtering mechanisms, where the correlation between security returns is broken into a *persistent* component, which is factor related, and a *transitory* component, which is driven by one time events. By using prior information about the sources of true correlation among security returns, fundamental and macroeconomic models can recognize correlations that are transitory. Statistical factor models, on the other hand, are unable to do this because they consider only the correlation matrix of security returns and treat the entire correlation between securities as permanent. Statistical models work hard at constructing factors that explain the in-sample correlations in security returns. Consequently, their out-of-sample predicted correlations are similar to the in-sample estimated correlations.

Evidence in this regard is provided in Table 1. For a sample of 20 U.S. companies, the Table lists the most highly correlated firms according to BARRA's fundamental model and a statistical factor model. Note that BARRA's model always provides an intuitive match, whereas the statistical factor model often provides puzzling matches. Both models provide similar results for oil companies and financial companies, but the statistical model matches Chrysler with International Paper, American Products (tobacco) with Bausch and Lomb, CBS with First Bank, Kodak with Travelers Inc. (Life Insurance), Ford with Weyerhaeuser, and GM with Georgia Pac (Paper). Although it is easy to believe that Chrysler and International Paper were highly correlated during a particular historical period, there is little reason to expect such a high correlation in the future.

In the same vein, let us compare the in- and out-of-sample variability of the specific returns from BARRA's fundamental factor model and a statistical factor model. The statistical factor model will win such competitions in-sample because it is designed to minimize the in-sample variance of specific returns.

The relevant tests, therefore, are the out-of-sample performance of the models. The evidence here clearly and strongly favors fundamental factor models. For example, Table 2 contains the square root of the mean-squared-error from fitting a statistical factor model versus BARRA's fundamental factor model to Swiss stocks. In-sample, the statistical factor model outperforms BARRA's model, but out-of-sample BARRA's model dominates. Moreover, BARRA's fundamental model performs equally well both in and out-of-sample, indicating that the model is robust. The sharp drop in the performance of the statistical factor model in moving to the out-of-sample tests, on the other hand, shows that such models are distinctly overfitted and are liable to pick up spurious correlations in the in-sample period.

How much each type of factor model captures of security returns is an empirical issue. Note that the actual amount of common variation in security returns limits all models in their ability to capture the common variation in security returns . For example, suppose that, on average across many periods, common factors account for 30 percent of the variation in stock returns. Then no model can capture more than 30 percent of the variation in stock returns. The relative performance of each type of model in its ability to capture the variation in U.S. stock returns is documented in Table 3. As the table shows, both fundamental and statistical factor models are able to capture about 40 percent of the variation in stock returns, with BARRA's fundamental factor model capturing more of the variation in stock returns than statistical or macroeconomic factor models. Note further that the performance of macroeconomic factor models is, by comparison, dismal.

More evidence in favor of fundamental factor models may be provided by examining how well BARRA's fundamental factor model captures the returns of well diversified portfolios. Such portfolios carry negligible specific risk because these risks cancel across a large number of securities. A good factor model should thus explain a large proportion of the variance of returns for such portfolios. Table 4 contains the proportion of variance that is captured by BARRA's model for various index portfolios. The table clearly shows that the factors are able to capture almost all of the variation in the returns of well diversified portfolios, i.e., there is negligible realized specific risk for well diversified portfolios. A related test of the accuracy of BARRA's model is to test whether the model's portfolio risk predictions are accurate. Table 4 contains tests of the biases in BARRA's risk forecasts for different portfolios in different countries. As the table shows, the risk forecasts are accurate, on average, across a number of countries and portfolios.

Fundamental factor models are also better at capturing changing risks than statistical or macroeconomic factor models. This is because fundamental factor models allow the sensitivities of the securities to the common factors to change over time. In contrast, both statistical and macroeconomic models allow sensitivities to change only slowly, as more data become available to estimate the sensitivities. For example, as a firm's leverage increases, its risk increases. Similarly, as a firm increases its operations in a industry different from its initial industry, the firm's risk changes. Statistical and macroeconomic factor models, however, are unable to capture these risk changes because the estimated sensitivities of the firms do not use this information. In contrast, fundamental factor models would capture these changes in a timely manner because the firm's sensitivity to the leverage factor, and its exposure to new industries would change as information regarding these changes becomes available.

Of particular importance here are the *leverage* and *momentum* factors (such as Success). The data show that factors related to these firm characteristics explain a significant portion of the variation in stock returns. Sensitivities to these factors, however, may change rapidly. Fundamental factor models capture such changes, whereas macroeconomic and statistical models do not. The in-ability of macroeconomic and statistical models to capture momentum is especially significant in a high turnover environment, such as a broker/dealer or hedge fund, where momentum may be the single most important risk factor.

As an example of the ability of fundamental models to capture changing risk, let us examine the performance of fundamental models in terms of predicting asset betas. Table 6 contains the root-mean-squared error in predicted betas from BARRA's U.S. fundamental model, and compares BARRA's betas with historical betas. As the Table shows, BARRA's predicted betas are clearly better predictors of future betas. Further analysis reveals that BARRA's predicted betas are also more strongly correlated with realized future betas.

Moreover, macroeconomic and statistical factor models do not work for new issues because enough return data are not available to estimate the factor sensitivities of such securities. Fundamental factor models, on the other hand, may rapidly incorporate such securities because a relatively short history of fundamental data is necessary for computing the risk indices and industry sensitivities.

Additional Considerations

Our discussion thus far has clearly favored fundamental models: they have greater explanatory power and are more flexible in modeling changing risk. There are, however, potential drawbacks to fundamental factor models. By their nature, fundamental models are data and labor intensive. Computation of sensitivities requires a large amount of fundamental data, and accurate computation of the sensitivities requires intensive data checking and analysis. In contrast, statistical and macroeconomic factor models require only security return and macroeconomic data. Fundamental factor models, therefore, are relatively costly to produce. This, however, is best viewed as a cost that is well worth the additional benefits of fundamental models.

Nevertheless, a large number of factors are needed in fundamental factor models. This implies that there are large number of sensitivities that are used to estimate factor returns. This may be a problem because some of the sensitivities may be linearly related to the others, or there is *multicolinearity* in the sensitivities. This is unlikely for the industry sensitivities because most firms tend to belong to only a few industries. It may be more of a problem for the risk indices, where one risk index may be linearly related to some of the others. The effect of multicolinearity is to make the estimated factors, and hence the factor covariance matrix, imprecise. Most of BARRA's risk indices show little evidence of multicolinearity. In the USE2 model, for example, there are two risk indices that exhibit multicolinearity. These are Growth and Dividend Yield, and the multicolinearity arises because high growth firms are low yield firms and vice versa. Such instances of simple colinearity between two fundamentals, however, are easily detected and easily remedied by using appropriate statistical techniques. Thus, for example, the evidence from BARRA's USE2 model indicates that Growth and Dividend Yield capture the effects of the same fundamental factor, and BARRA's USE3 model will capture that factor in only one risk index.

Fundamental and macroeconomic factor models have the additional benefits that the factors are meaningful and intuitive. For example, unexpected inflation or Oil industry returns are meaningful concepts, and their effects on Oil stock returns are also directly measurable. In contrast, statistical factors do not allow such an interpretation. This intuitive appeal and observability of fundamental and macroeconomic factor models also makes them extremely useful for risk characterization and performance analysis. In particular, one is able to identify the bets that a portfolio manager is taking. Moreover, one is able to examine, ex-post, which bets paid off and which did not.

Most fund managers are identified by an investment style, such as Index, or Growth, or Value. As such, fund manager performance is judged relative to a benchmark that is appropriate for their style. By examining how the factor sensitivity of their portfolios differ from those of the benchmark, managers are able to identify the bets they are placing. Some of these bets may be intentional: for example, a growth manager may believe that computer stocks are going to outperform other growth stocks. As a result, the manager may tilt her portfolio toward computer stocks. In comparing the sensitivities of her portfolio to the benchmark, this manager will find that her portfolio has a greater exposure to the computer industry. This is an intentional bet on the computer industry. Suppose, however, that computer firms also are more leveraged than other firms. The fund manager will find that her portfolio is also more exposed to the leverage factor. This is an unintentional bet. Unless she believes that more leveraged firms are going to outperform less leveraged firms, the fund manager may want to revise her holdings to reduce the leverage exposure while maintaining her computer bet.

This leads us into a discussion of performance analysis. Ex-post, the manager can observe the factor returns and examine whether the computer factor returns were larger than the consensus forecast, i.e., whether the computer bet paid off. Moreover, portfolio returns can be decomposed along each factor, and the total portfolio return may be attributed to each of the factors. Such an analysis is straightforward with BARRA's fundamental factor model, but is difficult with unintelligible statistical factors.

Testing and Improving BARRA's Risk Models

We have already discussed evidence regarding the accuracy of BARRA's risk forecasts (Table 4). This evidence comes from continuous procedures that test the accuracy of BARRA's models over the previous year, as well as over longer time periods. The results of these tests point out directions for further improvement in BARRA's risk forecasts. Moreover, BARRA's research group keeps abreast of developments in the professional and academic literature, and investigates the efficacy of new risk modeling techniques. Techniques that are found to be useful are incorporated in the risk models.

Two recent enhancements to BARRA's risk models focus on the covariances among the factors, as well as forecasts of specific risks of securities. The research process examined whether recent correlations among factors are more informative about correlations in the near future, and whether alternative risk forecasts for a market index may be used to improve the forecasts obtained from BARRA's models. These investigations led to new BARRA factor covariance matrices. The new covariance matrices gives greater weight to more recent factor returns, and scale the resulting covariance matrices to match a forecast for the volatility of a market index (such as the HICAP for USE2). The market volatility forecast comes from a model that incorporates the observation that equity volatilities increase after periods with large absolute returns. The results of these research efforts have been implemented in BARRA's Equity risk models. Continuous tests of the new covariance matrices show that they do indeed provide improved risk forecasts.

Current research efforts are directed at developing a new U.S. Equity risk model (USE3), constructing a model of the impact of trades on security prices so that trades may be placed in a manner that minimizes costs, improvements in equity valuation models, and incorporating derivative securities in the risk models. The impetus for these research projects came out of the results of diagnostic tests of our models, new discoveries by professional and academic researchers, and client feedback about BARRA's models. The goal of these projects is to provide risk management tools that incorporate current knowledge and are responsive to client needs.

Summary and Conclusion

Characterization of portfolio risk requires the covariance matrix of security returns. Factor models of security returns build the covariance matrix by decomposing a security's returns, and hence risks, into those that are driven by a set of variables that are common to all securities, and a component that is specific to the security. In implementing factor models, we have a choice between three types of models. Macroeconomic factor models assume that security returns are driven largely by a set of observable macroeconomic variables. Fundamental factor models assume that the factors are related to the fundamentals of the firms. Statistical factor models treat the factors as unobservable.

In evaluating the three types of factor models, we examined whether they capture the common sources of security returns, are able to model changing risk, and whether the factors are intuitive and sensible. Macroeconomic factor models are intuitively appealing, but they capture only a small part of the variation in stock returns. BARRA's fundamental factor models inherit the intuitive appeal of macroeconomic factor models, yet outperform even statistical factor models in capturing the common sources of risk. Moreover, fundamental factor models readily capture the changing risk characteristics of firms. On the other hand, macroeconomic and statistical factor models are unable to accurately model changing risk because they do not allow timely changes in the sensitivities of the securities to the factors. These considerations point to fundamental factor models, properly implemented, as the preferred approach to estimating the covariance matrix of security returns. Our discussion of BARRA's risk models reveals that these models provide robust and accurate risk forecasts. Moreover, BARRA's models are constantly monitored, and there is a continuous research effort to improve the accuracy of BARRA's models.

Appendix: Estimation of Factor Models

Fundamental factor models begin with observations on firm characteristics at the beginning of every period. These firm characteristics are used to compute the sensitivities of the firms to the factors. These sensitivities constitute X_t, measured at the beginning of period *t*. The factor returns, f_t, and the specific returns, $\in_t$, are estimated by regressing the excess returns, r_t, on the sensitivities:

$$f_t = \left[X_t^T W_T^{-1} X_t\right]^{-1} X_t^T W_T^{-1} r_t \tag{10}$$

$$\in_t = r_t - X_t f_t \tag{11}$$

where W_t is a weighting matrix for the Generalized Least Squares (GLS) regression. The history of estimated factor and specific returns is then used to estimate the factor covariance matrix. This last estimation may include weighting the observations and scaling the variance estimates to better match the volatility of a market index.

Macroeconomic factor models assume that security returns are related to a set of observable macroeconomic factors, such as unexpected changes in inflation, unemployment, and net business formation. Given the observed history of factors, *f*, the sensitivities of a firm to the factors, X_i, and the firm's specific returns are estimated by regressing the firm's excess returns on the factors:

$$X_i = \left[f^T f\right]^{-1} f^T r_i \tag{12}$$

$$\in_t = r_t - X_i f_t. \tag{13}$$

The factor covariance matrix is obtained directly from the observed factor series. The specific variances are obtained from the estimated specific returns.

Statistical factor models treat both the factors and the sensitivities as unobservable. The history of security returns is used to estimate the security return covariance matrix, *V.* Using statistical techniques such as Maximum Likelihood Factor Analysis, the security sensitivities, *X,* the factor covariance matrix, *F,* and the matrix of specific variances, Δ, are estimated from the estimated covariance matrix of returns, *V.*

Table 1

Companies with Highest Predicted Correlations BARRA vs. Statistical Factor Models

Company		Fundamental Factor Model		Statistical Factor Model	
ARC	Atlantic Richfield Domestic petroleum res	**AN**	Amoco Domestic petroleum res	**UCL**	Unocal Domestic petroleum res
AHP	American Home Prod Co Drugs, medicine	**BMY**	Bristol Myers Squibb Co Drugs, medicine	**PFE**	Pfizer Inc Drugs, medicine
BUD	Anheuser Busch Ind Liquor	**VO**	Seagram Ltd Liquor	**AHP**	American Home Prods Co Drugs, medicine
AIT	Ameritech Corp New Telephone, telegraph	**BLS**	Bellsouth Corp Telephone, telegraph	**FPC**	Florida Progress Corp Electric Utilities
AL	Alcan Aluminum Ltd NE Aluminum	**AA**	Aluminum Co Amer Aluminum	**N**	Inco Ltd Misc mining and metals
AMR	AMR Corp Air transport	**DAL**	Delta Air Line Del Air transport	**GE**	General Electric Co Producers goods
AMB	American Brands Inc De Tobacco	**MO**	Philip Morris Cos Inc Tobacco	**BOL**	Bausch & Lomb, Inc Health care (nondrug)
AXP	American Express Co Misc Finance	**FNM**	Federal Nat'l Mtg Assn Misc Finance	**PNC**	PNC Bk Corp Banks

Table 1 (continued)

Company		Fundamental Factor Model		Statistical Factor Model	
BEL	Bell Atlantic Corp Telephone, telegraph	**BLS**	Bellsouth Corp Telephone, telegraph	**DUK**	Duke Power Co Electric utilities
BAC	BankAmerica Corp Banks	**NB**	NCNB Corp Banks	**BBI**	Barnett Banks Inc Banks
C	Chrysler Corp Motor vehicles	**F**	Ford Motor Co Del Motor vehicles	**IP**	International Paper Co Paper
DD	Dupont EI De Nemours Chemicals	**AN**	AMOCO Corp Domestic petroleum res	**N**	Inco Ltd Misc mining and metals
CBS	CBS Inc Media	**CCB**	Capital Cities ABC Inc Media	**FBS**	First Bank Sys Inc Banks
DOW	Dow Chemical Co Chemicals	**GE**	General Electric Co Producers goods	**IP**	International Paper Co Paper
EK	Eastman Kodak Co Photographic, optical	**MMM**	Minnesota Mng & Mgf Co Chemicals	**TRV**	Travelers, Inc Life insurance
F	Ford Motor Co Del Motor vehicles	**GM**	General Mtrs Corp Motor vehicles	**WY**	Weyerhaeuser Co Paper
CCI	Citicorp Banks	**BAC**	BankAmerica Corp Banks	**CMB**	Chase Manhattan Corp Banks

Table 1 (continued)

Company		Fundamental Factor Model		Statistical Factor Model	
FNM	Federal Nat'l Mtg Assn Misc finance	**AIG**	American Int'l Group Inc Other Insurance	**JPM**	Morgan JP & Co Inc Banks
GE	General Electric Co Producers goods	**EMR**	Emerson Electric Co Producers goods	**STI**	Suntrust Bks Inc Banks
GM	General Mtrs Corp Motor vehicles	**F**	Ford Motor Co Del Motor vehicles	**GP**	Georgia Pac Corp Paper
XON	Exxon Corp International oil	**MOB**	Mobil Corp International oil	**TX**	Texaco Inc International oil

Table 2

Root-Mean-Squared-Error of Factor Models for Switzerland

Model	In-Sample (%)	Out-of-Sample (%)
Fundamental (BARRA)	5.13	5.42
Statistical	3.78	7.21

Source: Andrew Rudd, "On Factor Models," *BARRA Newsletter,* September/October 1992.

Table 3

The Explanatory Power of the Three Types of Factor Models

Model	Average Variation Explained (%)
Macroeconomic	10.9
Statistical	39.0
Fundamental (BARRA)	42.6

Source: Gregory Connor, "The Three Types of Factor Models: A Comparison of Their Explanatory Power," forthcoming in the *Financial Analysts Journal.*

Table 4

Explanatory Power of BARRA's Risk Model for Index Portfolios

Country	Portfolio	R^2
U.S.	S&P 500	0.9967
	MIDCAP	0.9963
	HICAP	0.9978
	NYSE	0.9976
U.K.	FT100	0.9878
Japan	TSE1	0.9971
	NK225	0.9949
Germany	FAZ	0.9867

Source: Andrew Rudd, "On Factor Models," *BARRA Newsletter,* September/October 1992.

Table 5

Biases in BARRA's Risk Forecasts

Country/Index	Bias Test Result
Australia	
ALLORD	No Bias
ALLRES	Risk Overestimated
ALLIND	No Bias
LEAD 20	No Bias
LEAD 50	No Bias
Canada	
TSE300	No Bias
Japan	
TSE1	No Bias
NK225	No Bias
TSE2	No Bias
U.S.	
HICAP	No Bias
SMALLCAP	No Bias

Source: Aamir Sheikh, "BARRA's New Risk Forecasts," *BARRA Newsletter,* Winter 1994, and Section D, "BARRA's New Covariance Matrices," of the 18th Annual BARRA Equity Research Seminar, June 1994.

Table 6

Historical Betas vs. BARRA Betas

Beta Prediction	Root Mean Square Error
Historical Beta	0.47
BARRA Beta	0.42