Strategic Asset Allocation and Commodities

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ibbotson

225 North Michigan Avenue Suite 700 Chicago, IL 60601-7676 • (312) 616-1620

Study Prepared by:
Thomas M. Idzorek, CFA
Director of Research

Correspondence should be sent to:

Ibbotson Associates
225 North Michigan Avenue
Suite 700
Chicago, IL 60601-7676
Phone: 312 616 1620
Fax: 312 616 0404

About Ibbotson Associates

Ibbotson Associates opened its doors in 1977 to bridge the gap between modern financial theory and real-world investment practice. Professor Roger G. Ibbotson, the company founder and Chairman, pioneered collecting the requisite data used in asset allocation and in quantifying the benefits of diversification. The firm continues to provide solutions to investment and finance problems for a diverse set of markets. Ibbotson Associates fills a growing need in the finance industry as a single-source provider of investment knowledge, expertise, and technology.

Ibbotson Associates' investment management services include asset allocation design, back-testing of portfolio strategies, mutual fund analysis, assessment of investor risk tolerance, employee training, structuring and education for retirement plans, and client-specific research. Ibbotson Associates' products include client presentation materials, computer-based training, and asset allocation software tools.

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Executive Summary

This paper studies the role of commodities in a strategic asset allocation. Commodities are real return, real assets that are part of the consumable/transformable super asset class and the store-of-value super asset class. There are several methods of obtaining exposure to commodities. This paper focuses on the type of exposure to commodities produced by a fully collateralized total return commodity index. To study the historical return characteristics of commodities, we formed an equally weighted, monthly rebalanced composite of four commodity indices. Our historical analysis supports the claims that commodities have low correlations to traditional stocks and bonds, produce high returns, hedge against inflation, and provide diversification through superior returns when they are needed most.

Using historical capital market assumptions based on annual data from 1970 to 2004, we found that including commodities in the opportunity set resulted in a superior historical efficient frontier, which included large allocations to commodities. Over the common standard deviation range, the average improvement in historical return at each of the risk levels was approximately 133 basis points.

The various elements of Greer's commodity pricing model are the likely return drivers of commodities. Commodity returns come from collateral and a commodity strategy premium. An insurance premium, a convenience yield, and a diversification/rebalance return all contribute to the commodity strategy premium. We believe commodities offer an inherent or natural return that is not conditional on skill.

We developed an expanded working version of the hypothetical market portfolio that includes commodities. Doing so enabled us to create a set of expected returns based on the CAPM, a model that most practitioners believe does not apply to commodities. Under what can only be characterized as a very conservative commodity expected return estimate relative to historical commodity returns, we created forward-looking efficient frontiers *with* and *without* commodities included in the opportunity set. Using CAPM expected returns, the inclusion of commodities in the opportunity set improved the efficient frontier. Over the common standard deviation range of approximately 2.4% to 22.4%, the average improvement in return at each of the risk levels was approximately 35 basis points. Allocations to commodities ranged from 0% to nearly 25%.

Using expected returns based on the building blocks methodology led to similar results. These results were independent of market capitalization estimates. Over the common standard deviation range of approximately 2.4% to 17.2%, the average improvement in return at each of the risk levels was approximately 77 basis points. Allocations to commodities ranged from 0% to 31%.

In our third analysis, using the Black-Litterman model, CAPM expected returns were combined with expected returns based on the building blocks methodology. Once again, including commodities in the opportunity set led to an efficient frontier above an efficient frontier in which commodities were excluded from the opportunity set. Over the common standard deviation range of approximately 2.4% to 22.4%, the average improvement in return at each of the risk levels was approximately 36 basis points. Allocations to commodities ranged from 0% to nearly 28%.

In a sensitivity analysis, the three forward-looking optimizations were repeated under the assumption that the expected return on commodities will be 2% above the expected return of collateral. Despite the dramatic reductions in the expected returns of commodities, commodities continued to play a significant role in the forward-looking strategic asset allocations.

No matter which set of returns was used, including commodities in the opportunity set improved the risk-return characteristics of the efficient frontier. Furthermore, commodities played an important and significant role in the strategic asset allocations. Given the inherent return of commodities, there seems to be little risk that commodities will dramatically underperform the other asset classes on a risk-adjusted basis over any reasonably long time period. If anything, the risk is that commodities will continue to produce equity-like returns, in which case, the forward-looking strategic allocations to commodities are too low.

Introduction

Most strategic asset allocations consist primarily of allocations to the three "traditional" asset classes—stocks, bonds, and cash. Expanding the investable universe beyond these three traditional asset classes improves the risk-return characteristics of a strategic asset allocation. Asset classes with low correlations to the current opportunity set of asset classes provide the largest benefit. A quest to find asset classes with attractive correlations, a lower perceived forward-looking equity risk premium, the desire for "real returns," and lower nominal interest rates creates interest in commodities. Although the idea of commodities as an investable asset class has existed for a long time (see Greer [1978]), commodities' transition into a mainstream asset class that should be part of the strategic asset allocation decision is a more recent phenomenon. This paper studies the role of commodities in a strategic asset allocation.

Strategic asset allocation is both a process and a result. It is the process of determining the target long-term allocations to the available asset classes. It is widely agreed upon that the strategic asset allocation is the most important decision in the investment process, and for diversified portfolios, the dominant determinant of performance.[1]

Advocates of the commodity asset class claim that the asset class has low correlations to traditional stocks and bonds, produces high returns, hedges against inflation, and provides diversification through superior returns when they are needed most. Nevertheless, commodities are often excluded from the opportunity set of investable asset classes. Despite considerable academic research, no definitive conclusions regarding the role of commodities in a strategic asset allocation exist. Possible reasons that commodities are excluded from the opportunity set include a limited number of implementation vehicles; the major commodity indices have short histories that have been backfilled; ambiguity over what constitutes an asset class and an investment strategy; the role of commodities in the market portfolio is undefined; the lack of an accepted commodity pricing model; and the lack of an understanding of the inherent returns of commodities.

Two recent influential academic works on the role of commodities seem to reach differing conclusions regarding the role of commodities in a strategic asset allocation setting and the source of commodity returns. Erb and Harvey [2005] raises questions about a long-term strategic asset allocation to commodities and attributes commodity returns primarily to diversification and rebalancing. Gorton and Rouwenhorst [2005] finds that commodities are an attractive asset class for diversifying a traditional stock and bond asset allocation and attributes commodity returns primarily to an insurance premium.

Commodities are a unique asset class and it is this uniqueness that creates questions regarding the role of commodities in a strategic asset allocation. In addition to Erb and Harvey [2005] and Gorton and Rouwenhorst [2005], other works that investigate the role of commodities in an asset allocation include Anson [1999], Jensen, Johnson, and Mercer [2000], Lummer and Siegel

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[1] See Brinson, Hood, and Beebower [1986] and Ibbotson and Kaplan [2000].

[1993], and Kaplan and Lummer [1997].[2] Most of these asset allocation studies rely only on historical data. With this in mind, we investigate both the historical and forward-looking role of commodities in a strategic asset allocation using some of the most modern asset allocation tools.

The primary tool for investigating the role of asset classes in a strategic asset allocation is Harry Markowitz's mean-variance optimization (see Markowitz [1952, 1959]). Mean-variance optimization is at the heart of modern portfolio theory and over the last 50 years has become the dominant asset allocation model. Mean-variance optimization requires three sets of inputs for the asset classes that make up a given opportunity set—returns, standard deviations, and correlations. Mean-variance optimization results in an efficient frontier, where each point on the frontier represents the risk and return of an *efficient* asset allocation. Efficient asset allocations maximize expected return for a given level of risk, or equivalently, minimize risk for a given level of return.

The asset allocations that make up an efficient frontier are a function of the three inputs or capital market assumptions, as they are often called. Historical capital market assumptions are a fact and are known with certainty. Historical efficient frontiers tell us the asset allocations that were optimal in the *past*. We use historical capital market assumptions and the traditional mean-variance optimization framework to create historical efficient frontiers. We then compare the asset allocations from historical efficient frontiers *with* and *without* commodities included in the opportunity set.

Forward-looking capital market assumptions are estimates, and as such, are not known with certainty. The uncertainty in forward-looking capital market assumptions motivates the use of two relatively new techniques designed to improve the asset allocation process that help overcome some of the weaknesses of the traditional mean-variance asset allocation framework.

The first approach is resampled mean-variance optimization. Resampled mean-variance optimization combines traditional mean-variance optimization with Monte Carlo simulation to account for the uncertainty in forward-looking capital market assumptions.[3] The second approach is the reverse optimization method proposed in Sharpe [1974], which flows directly from Sharpe's work on the CAPM (see Sharpe [1964]). Reverse optimization helps spread estimation error of return estimates evenly across all of the return estimates, is a specialized version of the CAPM, and is the starting point for the Black-Litterman asset allocation model (a popular Bayesian asset allocation technique (see Black and Litterman [1992])). After developing forward-looking capital market assumptions based on Ibbotson Associates' building blocks methodology, we combine the reverse optimized CAPM returns with the building block returns using the Black-Litterman model.[4]

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[2] Both Lummer and Siegel [1993] and Kaplan and Lummer [1997] are former Ibbotson studies that analyzed the role of commodities using the Goldman Sachs Commodity Index. Our current study is based on more data, uses a more generalized commodities index, and explores several methods of estimating commodity returns.

[3] Ibbotson Associates' implementation of resampling grew out of the pioneering work of Jobson and Korkie [1980, 1981], Jorion [1992], DiBartolomeo [1993], and Michaud [1998].

[4] Ibbotson Associates' methodology for forecasting asset class optimization inputs is based on Ibbotson and Sinquefield [1976a, 1976b]. Refinements to the methodology are presented in Lummer, Riepe, and Siegel [1994] and Ibbotson and Chen [2003].

In order to apply the building blocks methodology to commodities, we decompose commodity returns into various building blocks and then use forecasts of the building block components to create a complete set of forward-looking optimization inputs.

This paper is organized into three sections. In section 1, we select an opportunity set of asset classes that constitutes a relatively typical opportunity set. This includes a framework for defining and conceptualizing what is an asset class. We explore several methods of obtaining exposure to commodities. Market proxies for the asset classes are established, which enables us to study the historical returns of the asset classes.

In section 2, we compare efficient frontiers *with* and *without* commodities based on historical inputs. The construction of efficient frontiers *with* and *without* commodities based on historical inputs identifies the asset allocations that would have been optimal in the past.

In section 3, based on a desire to investigate the forward-looking role of commodities in a strategic asset allocation, we look at the fundamental return drivers for commodities as we develop three reasonable sets of forward-looking capital market assumptions. We create and then compare efficient frontiers *with* and *without* commodities based on the three different sets of forward-looking inputs. Finally, we perform a sensitivity analysis in which we repeat the three forward-looking optimizations with the expected return of commodities set to a relatively conservative level, 2% above the expected return on collateral.

Section 1: Strategic asset allocation and the opportunity set

Strategic asset allocation is the core of an investment plan. More formally, the strategic asset allocation is a set of long-term target allocations to applicable investable asset classes (proxied by market indices) with the highest likelihood of meeting long-term investment goals. The goal of the strategic asset allocation *process* is to determine the long-term exposure to the available asset classes. Most traditional definitions of asset allocation read something like this: *Asset allocation is a high level decision on how to allocate wealth to the broad asset classes, such as stocks, bonds, and cash.* This immediately leads to two questions: what constitutes an asset class and what are the asset classes?

There are no definitive definitions of what constitutes an asset class; nor are there definitive lists of asset classes. Asset allocation depends on how one chooses to identify the relevant asset classes that collectively form the opportunity set. It is very difficult to allocate wealth to asset classes without a framework for conceptualizing asset classes and then identifying which asset classes are relevant.

Asset classes can be defined in very broad terms, or in a more granular fashion. Even the seemingly black and white broad asset classes of stocks, bonds, and cash become gray when examined closely.

The identification of the investable opportunity set significantly changes the potential risk and return possibilities. Greer [1997] presents a classification framework based on three super asset classes: capital assets, consumable/transformable assets, and store-of-value assets. Capital assets, such as stocks, bonds, and real estate, provide an ongoing source of value that can be measured using the present value of future cash flows technique. Consumable/transformable assets, such as commodities, do not provide a source of ongoing cash flows, but a single cash flow. Store-of-value assets, such as currency and fine art, are not consumed and do not generate income; yet, they have monetary value. In practice, the lines between these super asset classes can be unclear. For example, some commodities, such as precious metals, are both a consumable/transformable asset and a store-of-value asset.

Asset classes can also be thought of as risk factors or beta exposures. Beta exposures produce a natural return, are not conditional on skill, and should be available for low fees. While investors have traditionally thought of beta exposures relative to equity markets, a much broader definition of beta is gaining popularity. Beta exposures are market exposures and include sensitivities to financial markets, interest rates, credit spreads, volatility, and other market-related forces. Well-known financial author, Clifford Asness, has defined beta as any trading strategy that can be written down. By this definition, commodities are certainly a beta exposure. **More importantly, we believe commodities offer an inherent or natural return that is not conditional on skill. Coupling this with the fact that commodities are the basic ingredients that build society, we believe commodities are a unique asset class and should be treated as such.**[5]

Commodities are part of a growing category or grouping of asset classes referred to as real return asset classes. In addition to commodities, real return asset classes include Treasury Inflation Protection Securities (TIPS) and real estate. The common link between these three asset classes is a relatively high correlation to inflation. High correlation with inflation provides purchasing power protection, i.e. a hedge against inflation.

Commodities are *real return* assets that happen to be examples of *real* assets. Real assets are things you can touch, which distinguishes them from capital or financial assets, such as stocks and bonds. Commodities can be segmented into “hard” and “soft” commodities. Hard commodities are non-perishable real assets, such as energy (e.g. oil and gas), precious metals (e.g. gold, silver, and platinum), industrial metals (e.g. aluminum and copper), and timber. “Soft” commodities are perishable and consumable real assets, such as agricultural products and livestock. This barrage of classification jargon results in a lengthy definition of commodities. **Commodities are real return, real assets that are part of both the consumable/transformable super asset class and the store-of-value super asset class.**

Most asset allocation studies focus exclusively on capital assets, the first of the super asset classes. In this study, we look at a relatively broad set of asset classes that also includes commodities. The opportunity set includes U.S. Treasury bills (cash), Treasury Inflation Protection Securities (TIPS), U.S. bonds, international bonds, U.S. stocks, international stocks, and commodities. This opportunity set is more inclusive than that of the typical U.S. investor, which is often thought to be a 60% U.S. stock and 40% U.S. bond asset allocation. In a more narrowly defined opportunity set, we would expect commodities to play a *larger* role in the asset allocation, while in a more broadly defined opportunity set, we would expect commodities to

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[5] Consistent with Anson [2002], we believe that commodities are an asset class, while skill-based strategies, such as hedge funds, are not asset classes. For additional information on distinguishing skill-based strategies from asset classes, see Waring and Siegel [2003, 2005] and Dopfel [2005].

play a *smaller* role in the asset allocation. In the future, investors should continue to expand their investment opportunity set to include the asset classes that make up the all-inclusive, unobservable, market portfolio of the CAPM.

After determining the asset classes that form the opportunity set, representative asset class proxies must be selected. Asset class proxies enable us to measure the performance of the different asset classes. For some asset classes, multiple asset class proxies are available, many of which are reasonable substitutes for each other. For example, the Russell 3000, the S&P 1500 SuperComposite, and the Wilshire 5000 have similar performance characteristics. Yet for other asset classes, such as commodities, fewer choices and more disparity exist in the performance characteristics among the index choices. Prior to selecting an appropriate asset class proxy for commodities, we must first understand the three methods by which investors have traditionally sought exposure to commodities. When we refer to commodities, or the commodity asset class, we focus on a particular type of exposure to commodities.

Exposure to Commodities

Unlike many financial assets where the methods of obtaining exposure to the underlying asset class are relatively clear, this is not the case with commodities. The primary methods of obtaining exposure to commodities include:

- Direct physical investment
- A portfolio of commodity-related stocks
- Commodity futures

These three methods result in different exposures that can result in significantly different risk and return characteristics.

Generally, a direct physical investment in commodities is simply not practicable. Most commodities are perishable and thus cannot be stored for long periods of time.[6] The one exception in which a direct physical investment may be tenable is precious metals.

A portfolio of commodity-related stocks does not provide a direct or pure asset class exposure to commodities. It provides exposure to the management skills, practices, and the additional business lines of the companies represented in the portfolio. Commodity-related stocks are part of the broader equity asset class; therefore, a portfolio of commodity-related stocks is usually relatively highly correlated with the broader equity asset class and results in a form of double counting, in which the same stocks are included in two different asset classes.[7] In some cases, companies may hedge the prices of the commodities associated with their businesses, essentially negating exposure to the underlying commodities. Gorton and Rouwenhorst [2005] attempts to match individual commodities to "pure play" companies using Standard Industrial Classification

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[6] Till and Eagleeye [2003] finds that commodities that are more difficult to store seem to perform better than commodities that are not difficult to store.

[7] A common example of double counting relates to real estate investment trusts or REITs. Most publicly traded REITs are constituents of the broad equity indices. REITs are not missing from our opportunity set; they are part of the U.S. equity proxy.

(SIC) codes. It finds that over a 41 year period, the correlation of commodity-related companies to commodity futures is .4 and the correlation of commodity-related companies to the S&P 500 is a much higher .57, concluding that commodity-related companies act more like the S&P 500 than they act like commodity futures. A portfolio of commodity-related companies does not provide a pure exposure to commodities, may not provide a hedge against commodity price increases, and may not provide a hedge against inflation.[8]

The two primary methods for obtaining exposure to commodities with commodity futures are the hiring of a commodity trading advisor (CTA) and an investment in a passive or active product designed to track the performance of a commodity index or sub-index.

Most CTAs, or managed futures managers, are very active managers with exposures to commodities that vary over time. CTAs may have net short positions in commodities as well as large exposures outside of commodities, such as currency or equity exposures.

The two major types of commodity futures indices are spot price indices and total return fully collateralized indices. Spot price indices measure the price movement in a basket of commodities. Total return fully collateralized indices measure the return of a passive investment strategy. The strategy invests in a basket of commodity futures contracts that are rolled over into new commodity futures contracts on an ongoing basis. The term "fully collateralized" indicates that an amount equivalent to the notional investment in futures has been set aside as collateral. A commodity futures investment that is not fully collateralized implies the use of leverage.

Exposure to a fully collateralized total return commodity index results in return drivers beyond a pure exposure to the price fluctuations of the underlying commodities. A total return index can produce positive returns even when commodity prices are flat or declining. For spot price indices, inflation and idiosyncratic supply and demand factors are the most often-cited return drivers of commodity *prices*. For fully collateralized total return commodity indices, there are a number of other potential return drivers. These return drivers may include the return on the collateral, changes in commodity spot prices, an insurance or risk premium, a convenience yield, and a diversification/rebalancing return. Unlike a pure investment in the price of the underlying commodities, the purchase of a fully collateralized total return commodity index is an investment in a passive (rule-based) investment strategy.[9] **This paper focuses on the type of exposure to commodities produced by a fully collateralized total return commodity index.**

The three most widely used total return commodity future indices are the Goldman Sachs Commodities Index (GSCI), the Dow Jones-AIG Commodity Index (DJ-AIG), and the Reuters/Jefferies CRB Index (RJ-CRB). In addition to analyzing these three commercially available indices, we analyze the fully collateralized total return commodity index of Gorton and

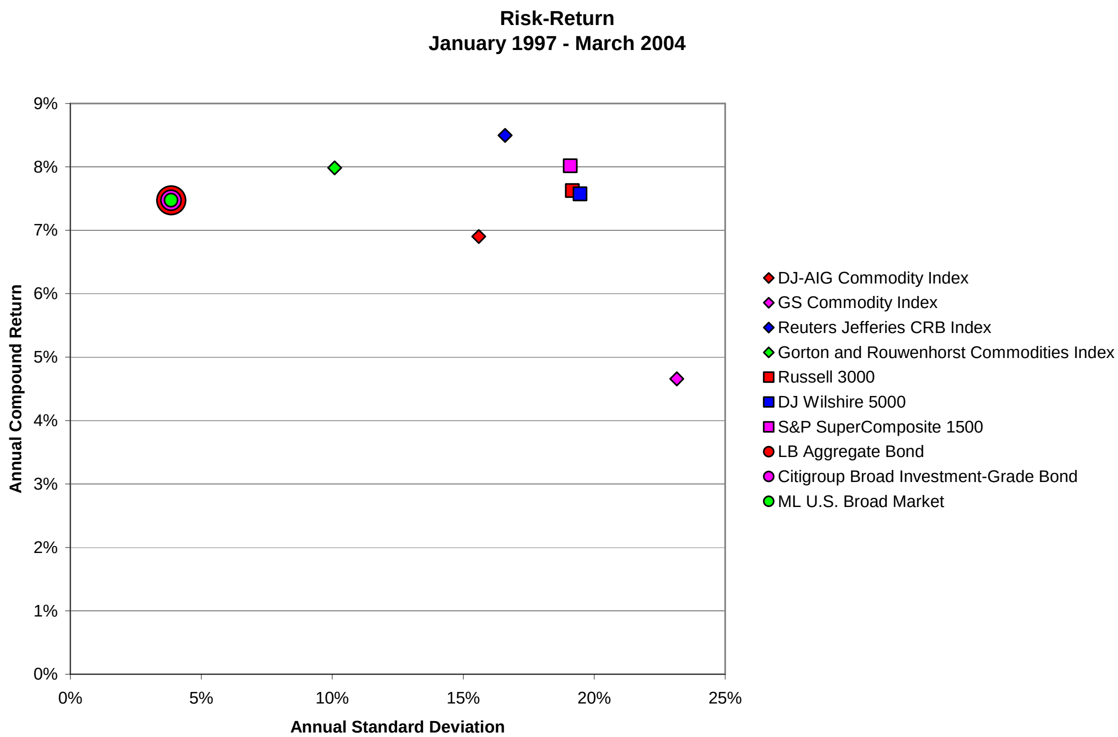
[8] Baierl, Cummisford, and Riepe [1999] develops a global hard asset index that is approximately 80% commodity-related equities and 20% collateralized futures. In an updated version of the study, between 1970 and 2004, the correlation of the Global Hard Asset Index with U.S. large stocks and international stocks was .29 and .36. The correlation of the commodity-related equity portion of the Global Hard Asset Index was a much higher .43 and .48, respectively. An additional method of exposure to commodities, a direct investment in oil and gas producing properties, is investigated in Chen and Pinsky [2003].

[9] Technically, even traditional equity indices, such as the Russell 3000, represent rule-based trading strategies. The implementation of these rule-based strategies is not conditional on skill.

Rouwenhorst [2005], which we will refer to as the Gorton and Rouwenhorst Commodity Index (GRCI). All four of these indices use U.S. Treasury bills as collateral. Even though we have narrowed our focus to total return indices, there are significant differences in the historical return characteristics of these indices, indicating that there is less agreement as to the appropriate way to define and measure the performance of the commodity asset class.

Figure 1 focuses on the common time period among 10 asset class proxies, January 1997 to March 2004.[10] In addition to the four commodities indices, Figure 1 includes three asset class proxies for U.S. stocks and three asset class proxies for U.S. bonds. Commodities, U.S. stocks, and U.S. bonds are represented by diamonds, squares, and circles, respectively. The three U.S. bond indices appear as a series of concentric circles; nearly identical risk and return characteristics place them on top of one another. Contrasting with the logical intra-asset class clustering of U.S. stock and U.S. bond market proxies, the commodity proxies do not display the same level of consistency. We examine the reasons for the disparity shortly.

Figure 1: Risk-Return (January 1997 – March 2004)

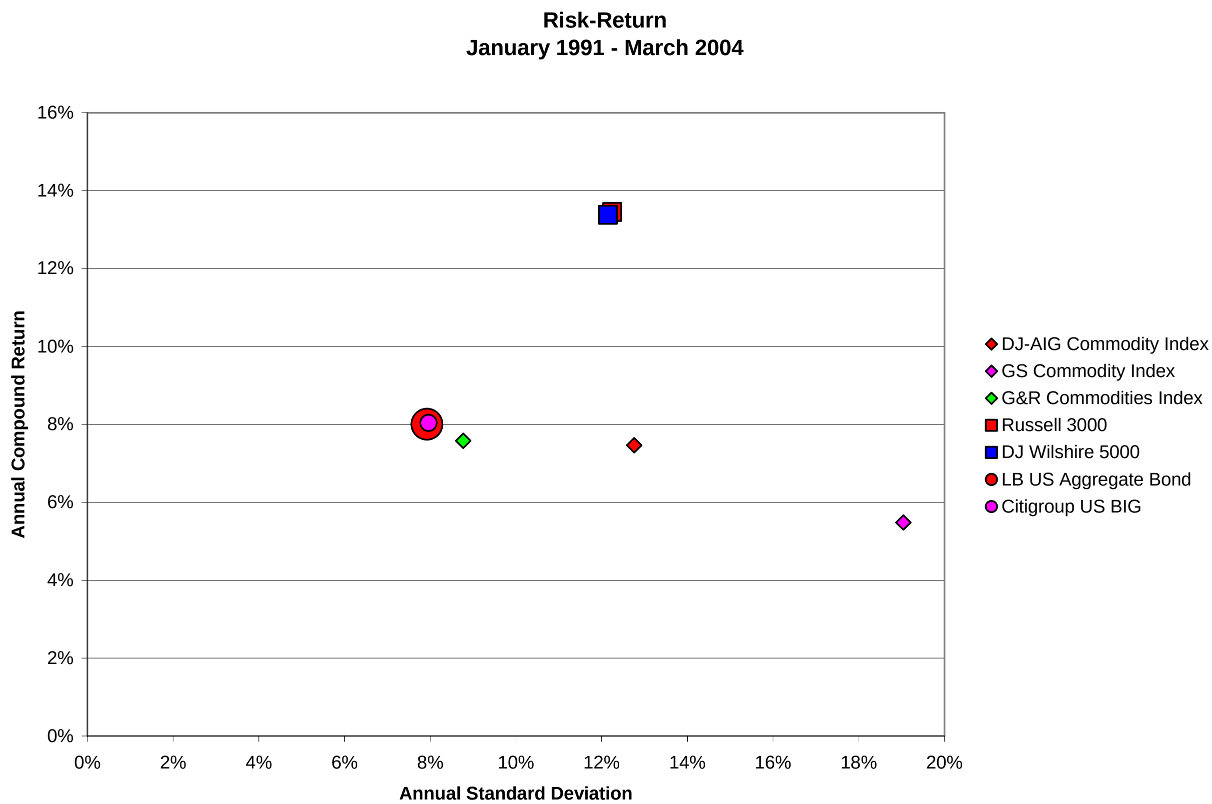


Based on annualized monthly data.

Figure 2 repeats the analysis, excluding the three asset class proxies with the shortest histories. This expands the common time period among the seven remaining asset classes by six years, from January 1991 to March 2004.

[10] The March 2004 end date corresponds with the available data on the Gorton and Rouwenhorst Commodity Index.

Figure 2: Risk-Return (January 1991 - March 2004)



Based on annualized monthly data.

Starting the analysis in January 1991 (the start date of the DJ-AIG Commodity Index) does not change the qualitative findings. The congruency between risk and return characteristics for the U.S. equity asset class proxies and the U.S. bond asset class proxies is very high, while there is considerable disparity between the risk and return characteristics for the three commodity indices.

Table 1 presents historical annual compounded returns and standard deviations for the 10 indices used in Figure 1, as well as the condensed set of indices used in Figure 2. Table 1 is segmented into three sections. The first section contains the statistics for the complete history for which we have data, the second section contains the statistics for the common period data from January 1997 to March 2004 used in Figure 1, and the third section contains the statistics for the common period data from January 1991 to March 2004 used in Figure 2.

Table 1: Historical Returns of Commodity, U.S. Stock, and U.S. Bond Indices

	Complete History	Compound Annual Return	Standard Deviation	Common Period 1 (Figure 1)	Compound Annual Return	Standard Deviation	Common Period 2 (Figure 2)	Compound Annual Return	Standard Deviation
DJ-AIG Commodity Index	1/91-10/05	7.62%	13.12%	1/97-3/04	6.90%	15.59%	1/91-3/04	7.46%	12.76%
GS Commodity Index	1/70-10/05	12.45%	21.35%	1/97-3/04	4.66%	23.15%	1/91-3/04	5.48%	19.04%
Reuters Jefferies CRB Index	1/94-11/05	13.03%	15.55%	1/97-3/04	8.50%	16.60%	1/91-3/04	N/A	N/A
G&R Commodities Index	7/59-3/04	10.80%	13.44%	1/97-3/04	7.98%	10.09%	1/91-3/04	7.58%	8.77%
Russell 3000	1/79-10/05	13.27%	17.51%	1/97-3/04	7.62%	19.16%	1/91-3/04	13.45%	12.25%
DJ Wilshire 5000	1/71-10/05	11.36%	17.67%	1/97-3/04	7.58%	19.46%	1/91-3/04	13.38%	12.15%
S&P SuperComposite 1500	1/95-10/05	11.50%	16.97%	1/97-3/04	8.02%	19.09%	1/91-3/04	N/A	N/A
LB US Aggregate Bond	1/76-11/05	8.75%	6.46%	1/97-3/04	7.47%	3.86%	1/91-3/04	8.00%	7.93%
Citigroup US BIG	1/80-10/05	9.30%	6.56%	1/97-3/04	7.47%	3.84%	1/91-3/04	8.04%	7.96%
ML U.S. Broad Market	1/97-10/05	6.43%	3.90%	1/97-3/04	7.47%	3.85%	1/91-3/04	N/A	N/A

Asset allocation studies focus on total return characteristics and how those total returns relate to the total returns of the other asset classes. The split between income return (e.g. dividends and interest) and capital appreciation returns differs across asset classes. Focusing on total returns enables us to compare the returns of different asset classes and, as the term indicates, it reflects the total return available to the investor. The convention of focusing on total returns applies to commodities as well; thus, we focus on fully collateralized total return commodity future indices rather than the spot price indices.

The two primary sources of return for a fully collateralized commodity total return index are the return on the collateral and the return associated with the passive rules-based trading strategy of rolling over futures positions (i.e. closing out maturing contracts and opening corresponding contracts with longer maturities). An investment in a commodity total return index is a bundled investment in the collateral and the passive commodity trading strategy. Later, when we estimate the forward-looking capital market assumption of commodities, we will study the return drivers associated with a passive commodity trading strategy and speculate on the likelihood of these drivers persisting in the future.

Regarding the collateral, just as we believe Treasury Inflation Protected Securities (TIPS) will become the *de facto* risk-free asset in the future, we believe TIPS will also become the preferred form of collateral for commodity futures. However, until that day is reached, we focus our analysis on the four total return indices, all of which are collateralized with U.S. Treasury bills.

Commodity Indices

Figure 1, Figure 2, and Table 1 above illustrate the performance disparity between the four commodity indices. Here, we take a closer look at some of the methodological differences that lead to return differences among the commodity indices.

The Goldman Sachs Commodities Index (GSCI) was first published in 1991, but the rules-based methodology was coupled with historical price data to create a history that begins on January 2, 1970, giving it the longest history of the commercially available commodity indices. The index tracks a hypothetical investment in a world production-weighted basket of nearby commodity futures contracts in which expiring contracts are rolled forward to the next corresponding nearby commodity futures contracts. It assumes that for every notional dollar invested in futures, an actual dollar is invested in 3-month U.S. Treasury bills as collateral. The weights are primarily based on delayed rolling 5-year averages of production *quantities*.[11] In 1970, the GSCI included five commodities. Today, the index includes 24 commodities. The 24 commodities can be organized into five sub-asset classes: Agriculture, Livestock, Energy, Industrial Metals, and Precious Metals. The rules governing the construction of the index are determined by an eight-member committee and are publicly available.

The Dow Jones-AIG Commodity Index (DJ-AIG) was created in 1998, with a backfilled history that starts on January 2, 1991. It also tracks a hypothetical investment in a basket of commodity futures contracts in which expiring contracts are rolled forward to the next set of corresponding nearby commodity futures contracts. It assumes that for every notional dollar invested in futures, an actual dollar is invested in 3-month U.S. Treasury bills. The weights are based on two factors: liquidity and world production *values*, where liquidity is the dominant factor. The index constructors believe that relying too heavily on production data understates the economic significance of store-of-value assets, such as gold and silver. The 19 commodities can be organized into the same five sub-asset classes: Agriculture, Livestock, Energy, Industrial Metals, and Precious Metals. Like the GSCI, the rules governing the construction of the index are determined by an oversight committee and are published. Two such important rules are that the minimum allowable weight for any single commodity in the index is 2%, and the maximum for any sub-asset class is 33% at the time of the annual reconstitution.

The original Commodity Research Bureau (CRB) index began in 1957. Since then, 10 revisions have taken place, the most recent occurring in May 2005. The current embodiment of the index is called the Reuters/Jefferies CRB Index (RJ-CRB). The current methodology was coupled with historical price data to create a simulated history back to 1994. It, too, tracks a hypothetical investment in a basket of commodity futures contracts in which expiring contracts are rolled forward to the next set of corresponding nearby commodity futures contracts. It assumes that for every notional dollar invested in futures, an actual dollar is invested in 3-month U.S. Treasury bills. The Reuters/Jefferies CRB Index contains 19 commodities arranged into four groups.

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[11] According to Goldman Sachs [2005], the delayed averages are used due to the time required to have accurate and complete production data. Using a five-year rolling window smoothes the effect of temporary production peaks and valleys.

Group I includes only petroleum products with a fixed weight of 33% in the index. The remaining three groups are based on liquidity.

The Gorton and Rouwenhorst Commodity Index (GRCI) is an equally weighted, fully collateralized commodity index. Unlike the other three indices, which use 3-month U.S. Treasury bills for collateral, the GRCI uses 30-day U.S. Treasury bills. From January 1978 to October 2005, the compounded annual returns of 3-month U.S. Treasury bills and 30-day U.S. Treasury bills were 6.13% and 6.32%, respectively. The difference of 19 basis points results in a slight downward collateral return bias for the GRCI relative to the other indices. In addition to the difference in collateral, the GRCI contains a larger number of commodities than the other indices and is equally weighted. At its inception, the index included nine commodities, which gradually increased to 36. As an equally weighted index, the performance of the index is more reflective of the performance of the average individual commodity, but it may not be as reflective of the broader commodities asset class.

Table 2 contains the most recently available components and weights of the four indices. For comparison purposes, we have attempted to group the individual commodities into one of five sub-asset classes: Agriculture, Livestock, Energy, Industrial Metals, and Precious Metals. A few observations stand out. Relative to the other indices, the Gorton and Rouwenhorst Commodity Index overweights Agriculture, Livestock, and both types of Metals. Relative to the other indices, the Goldman Sachs Commodities Index underweights Agriculture and both types of Metals and significantly overweights Energy. Based on the weights of the five sub-asset classes, the Dow Jones-AIG Commodity Index and Reuters/Jefferies CRB Index appear to be the most similar. We observe that in Figure 1 the risk-return plot points for these two indices were the closest of the four commodity indices. In addition to obvious differences in constituents and weighting schemes, rebalancing rules are a likely third cause for performance differences.

Table 2: Commodity Index Constituents and Weights

Commodity	GSCI	DJ-AIG	RJ-CRB	GRCI
Agriculture				
Butter	--	--	--	2.78%
Cocoa	0.30%	--	5%	2.78%
Coffee	0.68%	2.93%	5%	2.78%
Corn	4.11%	5.87%	6%	2.78%
Cotton	1.74%	3.16%	5%	2.78%
Lumber	--	--	--	2.78%
Milk	--	--	--	2.78%
Oats	--	--	--	2.78%
Orange Juice	--	--	1%	2.78%
Rough Rice	--	--	--	2.78%
Soybean Meal	--	--	--	2.78%
Soybean Oil	--	2.77%	--	2.78%
Soybeans	3.01%	7.77%	6%	2.78%
Sugar	1.26%	2.97%	5%	2.78%
Wheat	--	4.77%	1%	2.78%
Wheat (Chicago Wheat)	3.87%	--	--	--
Wheat (Kansas Wheat)	1.41%	--	--	--
Sub-Total	16.38%	30.24%	34.00%	41.67%
Livestock				
Pork Bellies	--	--	--	2.78%
Lean Hogs	2.39%	4.35%	1%	2.78%
Live Cattle	3.74%	6.09%	6%	2.78%
Feeder Cattle	0.90%	--	--	2.78%
Sub-Total	7.03%	10.45%	7.00%	11.11%
Energy				
Coal	--	--	--	2.78%
Electricity	--	--	--	2.78%
Heating Oil	7.14%	3.85%	5%	2.78%
Gasoil	3.83%	--	--	--
Unleaded Reg Gas	7.90%	4.05%	5%	2.78%
Crude Oil	25.79%	12.78%	23%	2.78%
Brent Crude Oil	11.75%	--	--	--
Propane	--	--	--	2.78%
Natural Gas	10.29%	12.32%	6%	2.78%
Sub-Total	66.70%	33.00%	39.00%	19.44%
Industrial Metals				
Aluminum	3.31%	6.85%	6%	2.78%
Copper	2.42%	5.88%	6%	2.78%
Lead	0.31%	--	--	2.78%
Nickel	0.93%	2.66%	1%	2.78%
Zinc	0.57%	2.70%	--	2.78%
Tin	--	--	--	2.78%
Sub-Total	7.54%	18.09%	13.00%	16.67%
Precious Metals				
Gold	2.12%	6.22%	6%	2.78%
Silver	0.23%	2.00%	1%	2.78%
Platinum	--	--	--	2.78%
Palladium	--	--	--	2.78%
Sub-Total	2.35%	8.22%	7.00%	11.11%
Total	**100.00%**	**100.00%**	**100.00%**	**100.00%**

Sources: Goldman Sachs [2005], the Dow Jones Indexes [2005], and Reuters / Jeffries CRB Index [2005].

Moving forward, we focus primarily on a composite index based on the four commodity indices. We will call our index the Composite Commodity Index. It is an equally weighted, monthly rebalanced composite of the four other indices in existence over the applicable time period. In order to avoid the existence of a single index in the composite, the inception of our Composite Commodity Index coincides with the inception of the Goldman Sachs Commodity Index in 1970. Conclusions based on the Composite Commodity Index are not specific to any one index. The evolution of the weights of the Composite Commodity Index is presented in Table 3. Intuitively, had we included a plot point for the Composite Commodity Index for the common time period used in Figure 1, the plot point would be approximately in the center of the four other commodity indices, indicating that it is a collective representation of all of the commodity indices. While investors cannot directly invest in the Composite Commodity Index, the hope is that the performance characteristics of the Composite Commodity Index are close enough to the individual indices that the results of the study can be generalized, within reason, to the other indices.[12]

Table 3: Composite Commodity Index Weights

Dates	GSCI	DJ - AIG	RJ - CRB	GRCI
Jan 70 to Dec 90	50.0%	--	--	50.0%
Jan 91 to Jan 94	33.3%	33.3%	--	33.3%
Feb 94 to Mar 04	25.0%	25.0%	25.0%	25.0%
Mar 04 to Oct 05	33.3%	33.3%	33.3%	--

Asset Class Proxies for the Opportunity Set

Having identified an asset class proxy for commodities, we must also identify appropriate proxies for the other asset classes in our opportunity set, which includes U.S. Treasury bills (cash), TIPS, U.S. bonds, international bonds, U.S. stocks, and international stocks. When selecting asset class proxies, we attempt to select indices that are representative of the investable opportunities, are popular benchmarks for both active and passive implementation vehicles, and are commonly selected for formalizing a strategic asset allocation as a strategic policy benchmark. Unfortunately, most of the indices that meet these criteria have limited historical data.

In order to have historical time series data dating back to 1970, we appended the short data histories with data from highly correlated series with sufficient histories. For U.S. Treasury bills (cash), the Citigroup U.S. Domestic 3-Month T-Bill is appended with Ibbotson 30-Day Treasury Bill data. For TIPS, the Lehman Brothers Global Real U.S. TIPS index is appended with Ibbotson's Synthetic TIPS Index prior to October 1997.[13] For U.S. Bonds, the Lehman Brothers U.S. Aggregate Bond index is appended with an Ibbotson-created synthetic aggregate bond

[12] For those seeking to implement an asset allocation to commodities, there is a growing number of active and passive products benchmarked to commercially available indices.

[13] The Synthetic TIPS index is based on the methodology and series developed in Chen and Terrien [2003]. The Synthetic TIPS Index is an annual index; thus, when performing calculations based on annual data the calculations for 1997 are based on the Synthetic TIPS Index, rather than Lehman Brothers Global Real U.S. TIPS index.

index.[14] For International Bonds, the Citigroup World BIG xUS Index is appended with two series: the Citigroup Non-US Dollar World Government Bond Index and an Ibbotson-created international bond composite.[15] For U.S. Stocks, the Russell 3000 Index is appended with S&P 500 data. For International Stocks, an extension was not required as the MSCI EAFE index begins in 1970. While more sophisticated series-extending methods exist (and are often used at Ibbotson), for our purposes we felt simplicity was best and more transparent. Many of these changes are indicative of the changing nature of investors' perceptions of the different asset classes. For example, the changes in the proxy used for international bonds reflect the growing importance of corporate debt relative to sovereign debt. Table 4 contains the asset classes and the asset class proxies used in the rest of the study.

Table 4: Asset Classes and Asset Class Proxies

Asset Classes	Asset Class Proxies	Date
U.S. Treasury bills	Citigroup U.S. Domestic 3-Month T-Bill	Jan 1978 to Oct 2005
(Cash)	Ibbotson Associates 30-Day T-Bill	Jan 1970 to Dec 1977
TIPS	Lehman Brothers Global Real U.S. TIPS	Jan 1998 to Oct 2005
	Ibbotson Associates Synthetic TIPS	Jan 1970 to Dec 1997
U.S. Bonds	Lehman Brothers U.S. Aggregate Bond	Jan 1976 to Oct 2005
	Ibbotson Associates Synthetic U.S. Aggregate Bond	Jan 1970 to Dec 1975
International Bonds	Citigroup World BIG xUS Index	Jan 1999 to Oct 2005
	Citigroup Non-US Dollar World Gov't Bond Index	Jan 1985 to Dec 1998
	Ibbotson Associates World Bond xUSA Composite	Jan 1970 to Dec 1984
U.S. Stocks	Russell 3000	Jan 1979 to Oct 2005
	S&P 500	Jan 1970 to Dec 1978
International Stocks	MSCI EAFE	Jan 1970 to Oct 2005
Commodities	Composite Commodities Index	See Table 3

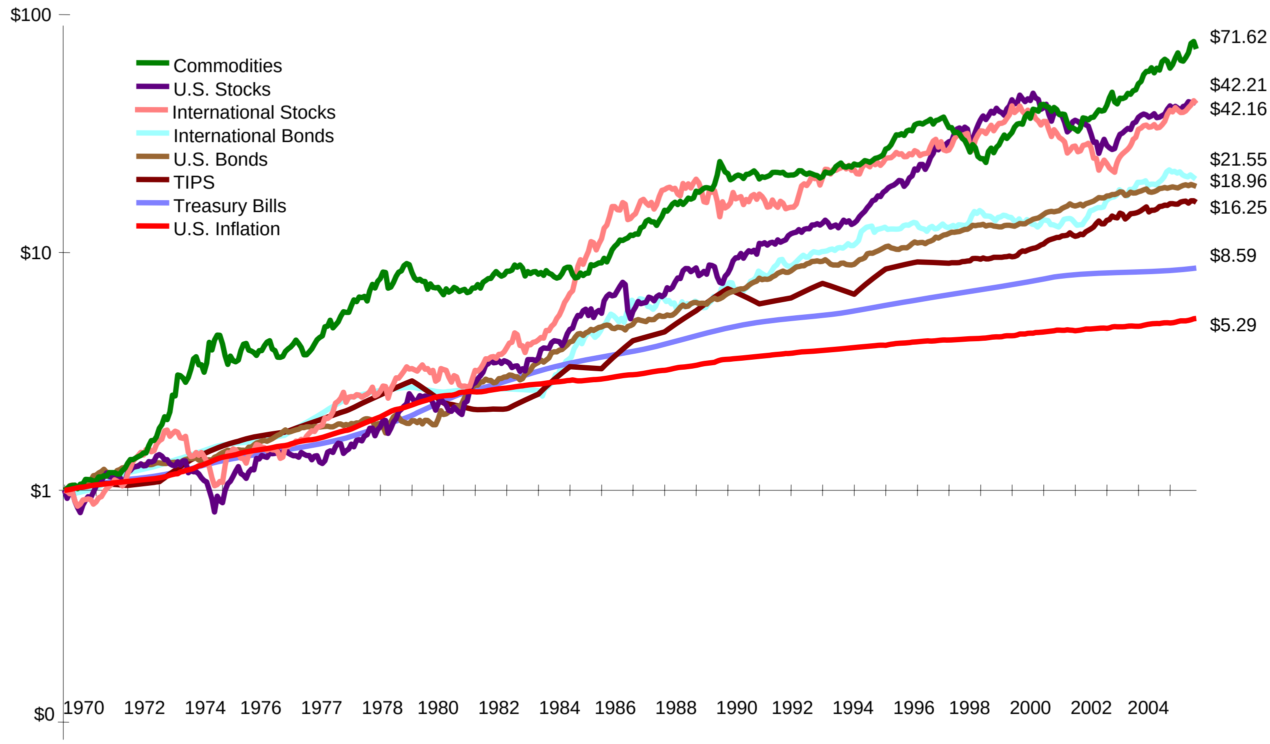
Historical Return and Risk

Using monthly return data, Figure 3 shows the growth of a $1 investment in the various asset classes starting in 1970. Figure 3 assumes that all cash flows generated from each asset class are reinvested and that no taxes or transaction costs are paid. These results are indicative of how a passive index investment with reinvested cash flows would have performed. Inflation is shown in red. The lines representing TIPS and International Bonds are based on annual data prior to 1998 and 1985, respectively. **Over this historical time period, commodities have been the top performing asset class in the opportunity set.**

[14] The synthetic aggregate bond index was created using the methodology explained in Coleman, Fisher, and Ibbotson [1992].

[15] Using individual country bond and bill data from 14 developed countries, we created an equally weighted international bond market proxy. Over the respective common time periods, the correlations of the Ibbotson-created international bond composite with the Citigroup World BIG xUS Index and Citigroup Non-US Dollar World Gov't Bond Index were .99 and .95, respectively.

Figure 3: Growth of a $1 Investment, January 1970 – October 2005



From Figure 3, we see that the growth of a $1 investment in TIPS, U.S. Bonds, and International Bonds approximately doubled the growth of a $1 investment in Treasury Bills. The increase in the two equity asset classes approximately doubled that of TIPS, U.S. Bonds, and International Bonds. Similar to the analysis of Kaplan and Lummer [1997], which focuses on the Goldman Sachs Commodity Index and a similar opportunity set, we find that commodities is the top performing asset class. The magnitude of the standout performance of commodities is highlighted in Figure 3.

Table 5 presents the historical returns and standard deviations for the asset classes using calendar-year data. The analysis looks at the entire holding period (1970 to 2004), a high inflation period (1970 to 1981), and a low inflation period (1982 to 2004). Over the entire time period, commodities produced the highest arithmetic *and* compounded returns, while the standard deviation was between the standard deviations of U.S. Stocks and International Stocks. This certainly supports the statement that commodities have equity-like returns. During the high inflation period, commodities were the top performing asset class by a wide margin and substantially outpaced inflation. During the low inflation period, commodities still produced double digit returns, although the compounded return was more bond-like than equity-like.

Table 5: Historical Returns and Standard Deviations

Asset Class	Entire Period 1970 – 2004 Arithmetic Annual Return	Entire Period 1970 – 2004 Compound Annual Return	Entire Period 1970 – 2004 Standard Deviation	High Inflation 1970 – 1981 Arithmetic Annual Return	High Inflation 1970 – 1981 Compound Annual Return	High Inflation 1970 – 1981 Standard Deviation	Low Inflation 1982 – 2004 Arithmetic Annual Return	Low Inflation 1982 – 2004 Compound Annual Return	Low Inflation 1982 – 2004 Standard Deviation
Treasury Bills	6.31	6.27	3.00	7.53	7.48	3.35	5.67	5.64	2.66
TIPS	8.88	8.25	11.74	7.86	7.31	10.84	9.41	8.74	12.39
U.S. Bonds	8.95	8.74	7.05	6.41	6.31	4.91	10.27	10.02	7.70
International Bonds	9.11	8.76	8.83	8.96	8.74	7.16	9.19	8.77	9.75
U.S. Stocks	12.60	11.22	17.23	9.07	7.36	19.58	14.44	13.29	16.02
International Stocks	13.25	11.09	22.45	12.18	10.24	21.30	13.81	11.53	23.47
Commodities	14.06	12.38	19.88	19.51	17.09	25.62	11.22	10.00	16.06
U.S. Inflation	4.78	4.74	3.18	7.96	7.91	3.43	3.12	3.12	1.18

Table 5 provides anecdotal evidence that commodity returns are linked to inflation. While realized inflation is one of the factors that explains the return of commodities, it does not account for the approximate 90% return in commodities that occurred between 2002 and the end of 2004 (see Figure 3).

Commodity Correlations with the Other Asset Classes

According to modern portfolio theory, the interaction of asset classes with each other provides diversification. Commodities are thought to have low correlations with traditional asset classes and to have the ability to provide positive return when traditional asset allocations are underperforming. Below, we examine the notion that commodities provide positive return when they are needed most for three typical asset allocations—a 100% U.S. stock investor, a 50% U.S. stock and 50% U.S. bond investor, and a 100% U.S. Bond investor.

Of the 35 years of annual data, there were eight years that U.S. stocks had negative total returns. Table 6 displays the average arithmetic return of the asset classes for the eight years in which U.S. stocks experience negative returns. During these eight years, commodities had the highest average arithmetic return. The crux of diversification is grouping together assets that do not always move in sync, which has the desired effect of smoothing a portfolio's total variability.

Table 6: Arithmetic Historical Returns for Eight Years of Negative U.S. Stock Returns

Asset Class	Arithmetic Annual Return
Treasury Bills	6.85
TIPS	11.47
U.S. Bonds	7.34
International Bonds	8.16
U.S. Stocks	-12.28
International Stocks	-11.49
Commodities	19.02
U.S. Inflation	6.27

Next, we repeated the above analysis, only this time we focus on the seven years in which a 50% U.S. stock and 50% U.S. bond portfolio had a negative return. Table 7 displays the average arithmetic return of the asset classes for the seven years in which a 50% U.S. stock and 50% U.S. bond portfolio experienced negative returns. During these seven years, commodities once again had the highest average arithmetic return.

Table 7: Arithmetic Historical Returns for Seven Years of Negative Returns for 50% U.S. Stock / 50% U.S. Bond Portfolio

Asset Class	Arithmetic Annual Return
Treasury Bills	6.45
TIPS	6.36
U.S. Bonds	5.03
International Bonds	8.27
U.S. Stocks	-12.22
International Stocks	-6.67
Commodities	16.47
U.S. Inflation	6.19

This time, we focus on the two years in which U.S. bonds had negative returns. Table 8 displays the average arithmetic return of the asset classes for the two years in which U.S. bonds experienced negative returns. During these two years, commodities again had the highest average arithmetic return.

Table 8: Arithmetic Historical Returns for Two Years of Negative U.S. Bond Returns

Asset Class	Arithmetic Annual Return
Treasury Bills	4.49
TIPS	-3.90
U.S. Bonds	-1.87
International Bonds	-1.98
U.S. Stocks	10.54
International Stocks	17.68
Commodities	20.97
U.S. Inflation	2.68

Tables 6–8 illustrate that for three typical types of portfolios—an all U.S. equity portfolio, a 50/50 U.S. stock / U.S. bond mix, and an all U.S. bond portfolio—commodities provided positive returns when they were needed most. **Historically, commodities act as a form of portfolio insurance and are excellent diversifiers.**

While Tables 6–8 provide intuitive evidence of the low correlation of commodities with traditional asset classes, Table 9 presents the correlation coefficients. The final row of Table 9 presents the average correlation coefficient of each of the asset classes with all of the other asset classes (inflation is excluded from the averages).

Table 9: Correlation Coefficients of Annual Total Returns, 1970–2004

Asset Class	Treasury Bills	TIPS	U.S. Bonds	International Bonds	U.S. Stocks	International Stocks	Commodities	U.S. Inflation
Treasury Bills	1.00	-0.08	0.23	-0.35	0.03	-0.12	-0.10	0.61
TIPS	-0.08	1.00	0.02	0.38	-0.10	-0.04	0.41	0.19
U.S. Bonds	0.23	0.02	1.00	0.14	0.24	-0.03	-0.32	-0.29
International Bonds	-0.35	0.38	0.14	1.00	0.03	0.40	0.15	-0.09
U.S. Stocks	0.03	-0.10	0.24	0.03	1.00	0.58	-0.24	-0.19
International Stocks	-0.12	-0.04	-0.03	0.40	0.58	1.00	-0.07	-0.20
Commodities	-0.10	0.41	-0.32	0.15	-0.24	-0.07	1.00	0.29
U.S. Inflation	0.61	0.19	-0.29	-0.09	-0.19	-0.20	0.29	1.00
Average Correlation (Excluding Inflation)	-0.06	0.10	0.05	0.12	0.09	0.12	-0.03	0.05

The average pair-wise correlation among the opportunity set is fairly low. Of the seven asset classes, Treasury Bills and Commodities are the only two asset classes with a negative average correlation to the other asset classes.

Ultimately, portfolios are used for consumption; thus, investors need to consider the ability of various asset classes to hedge against inflation and provide real purchasing power. Treasury Bills, TIPS, and Commodities are positively correlated with inflation, supporting the notion that TIPS and Commodities are real return assets. The correlation of TIPS with inflation may be understated by our use of a synthetic TIPS index prior to 1998. While positive, the correlation of 0.19 between TIPS and U.S. Inflation is surprisingly low, given that both are related to the Consumer Price Index (CPI). The correlations of TIPS and Commodities with *changes in inflation* were .55 and .60, respectively.[16] Practitioners frequently use equity asset classes to hedge against inflation, but our analysis shows that both U.S. Stocks and International Stocks are negatively correlated with inflation.

Based on this historical correlation analysis, commodities are most closely linked to TIPS, which provides a logical basis for the fact these two asset classes are often bundled together. A potential concern with highly correlated assets within the mean-variance optimization framework is that they will be viewed as reasonable substitutes for one another, with the optimizer strongly favoring the asset class with more attractive risk-return characteristics.

Above, Table 9 presents the correlations over the entire historical time period. Below in Figure 4, Panels A through G graph the rolling ten-year correlations of commodities with the other six asset classes and inflation. The dashed lines of each panel represent the correlation over the entire time period. Figure 4 highlights the time-varying nature of correlations. Treating all observations equally and in the absence of additional information, the best estimates of the future long-term correlations are the entire time period correlations presented in Figure 4.

[16] The principal, and thus the amount of the interest payments on TIPS, is linked to the U.S. All Urban Consumer Price Index (CPI-U). Consistent with our analysis, Greer [2000] finds that commodities are more highly correlated to changes in inflation than the level of inflation. Similarly, Gorton and Rouwenhorst [2005] finds that commodities are more highly correlated with unexpected inflation and changes in inflation than they are to the level of inflation.

Figure 4: Rolling Ten-Year Correlations of Commodities with Other Asset Classes

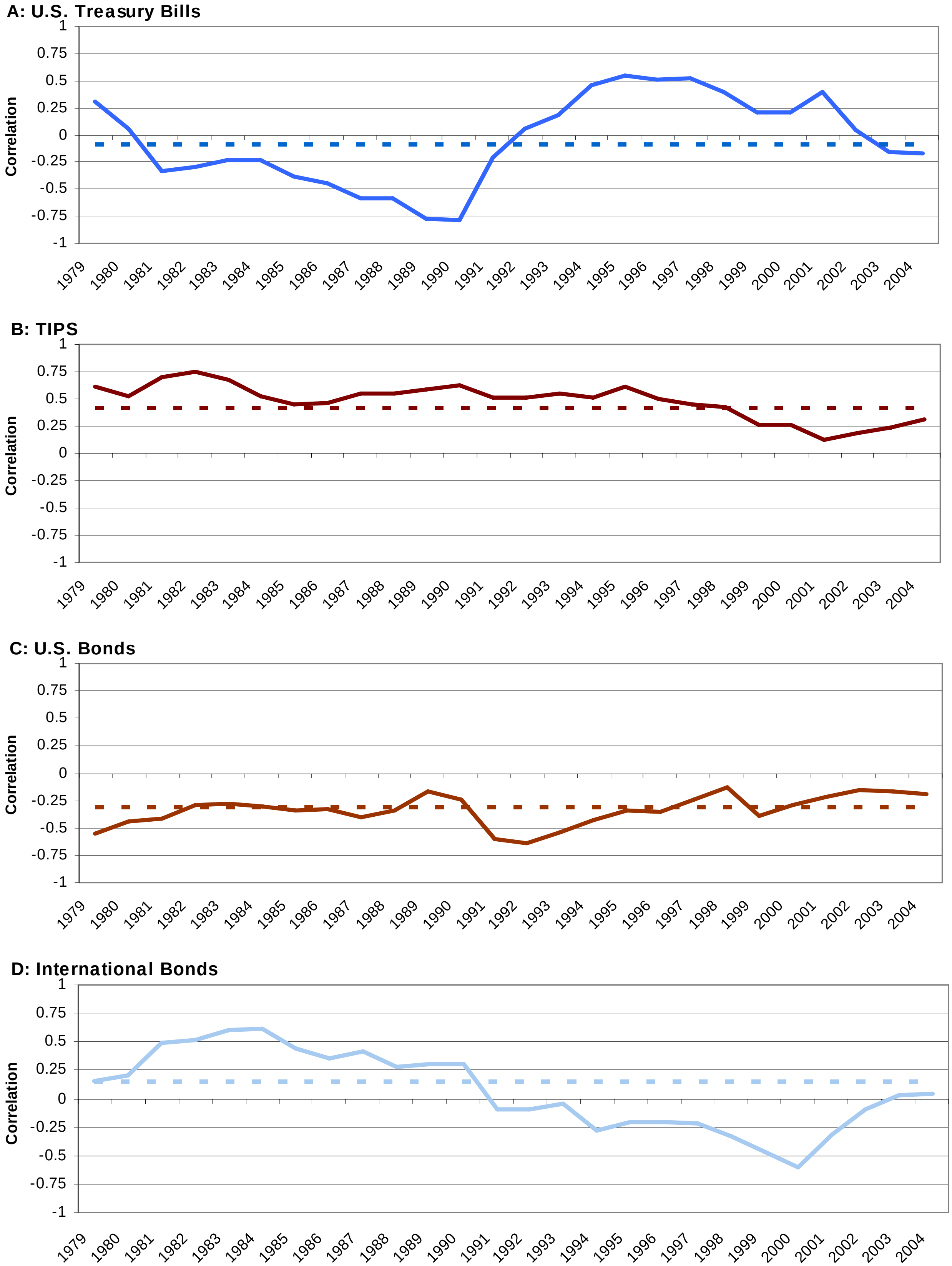
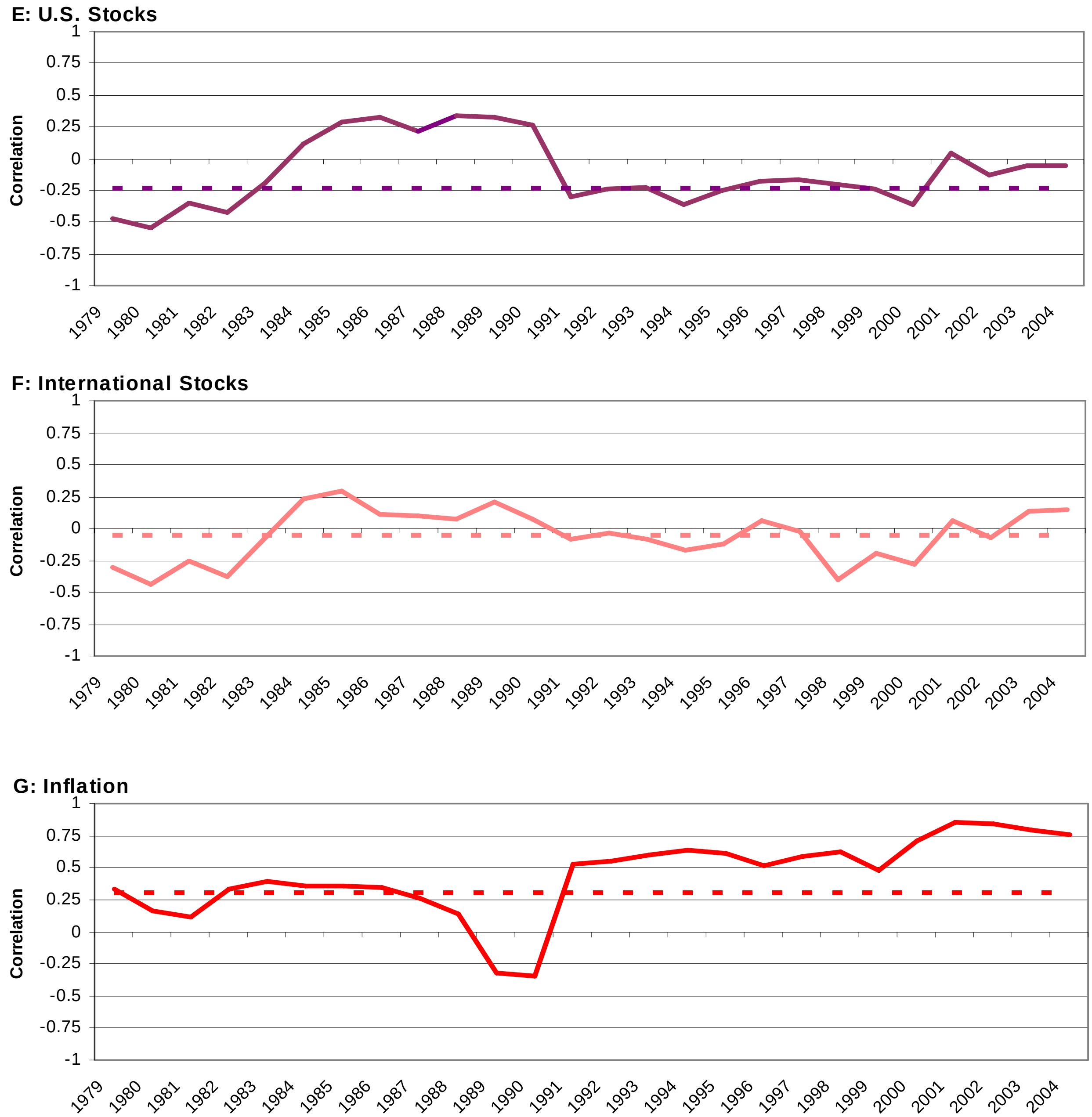


Figure 4 (continued): Rolling Ten-Year Correlations of Commodities with Other Asset Classes



In this section, we defined a commodity asset class proxy, the Composite Commodity Index, which enabled us to study the historical return characteristics of commodities. Our analysis supports the claims that commodities have low correlations to traditional stocks and bonds, produce high returns, hedge against inflation, and provide diversification through superior returns when they are needed most. In Section 2, we will use the historical returns, standard deviations, and correlations developed in this section to determine the asset allocations that would have been optimal in the *past*. Finally, the time-varying nature of the historical returns, standard deviations, and correlations that we have observed motivates the robust asset allocation procedures used in Section 3 to develop forward-looking asset allocations.

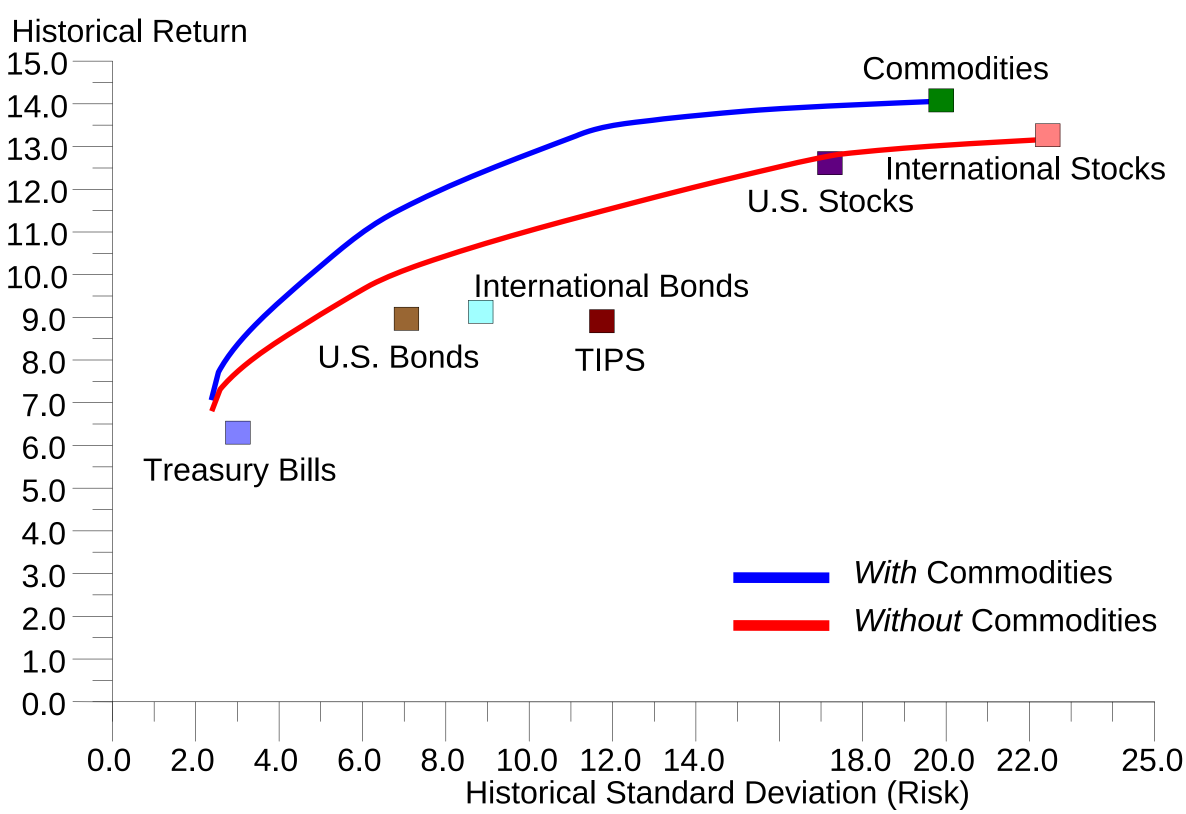
Section 2: Historical Efficient Frontiers with and without Commodities

The tool of choice for developing strategic asset allocations is Harry Markowitz's mean-variance optimization. Mean-variance optimization requires three inputs—returns, standard deviations, and correlations. Based on these three inputs, an efficient frontier is constructed in which each point on the frontier maximizes the return per unit of risk.

Efficient frontiers that are based on historical inputs provide an unbiased analysis of the strategic asset allocations that would have been optimal in the *past*. Two historical efficient frontiers demonstrate the historical effects of including commodities in a strategic asset allocation. The returns and standard deviations are based on the arithmetic returns and standard deviations of the entire time period (1970 – 2004) reported in Table 5, while the correlations are taken from Table 9.

Figure 5 compares a historical efficient frontier *with* commodities to a historical efficient frontier *without* commodities. The historical efficient frontier *with* commodities is superior to the historical efficient frontier *without* commodities. Including commodities in the opportunity set improved the risk-return tradeoff over the entire historical efficient frontier. Over the common standard deviation range of approximately 2.4% to 19.8%, the average improvement in historical return at each of the risk levels was approximately 133 basis points, with a maximum of 188 basis points!

Figure 5: Historical Efficient Frontier *with* and *without* the Commodities Asset Class



Efficient frontier graphs, such as Figure 5, can mask the relatively large differences in the underlying asset allocations. Figures 6 and 7 are efficient frontier area graphs. Efficient frontier area graphs display the asset allocations of the efficient frontier across the entire risk spectrum. Conceptually, the efficient frontier area graph is similar to a standard asset allocation pie chart that shows the asset allocation that corresponds to a particular spot on the efficient frontier, except the efficient frontier area graph displays all of the asset allocations on the efficient frontier. It is helpful to look at the efficient frontier graphs and the efficient frontier area graphs together. The efficient frontier graph displays standard deviation on the horizontal axis and the asset allocations on the vertical axis.

Figure 6 displays the asset allocations for the historical efficient frontier *with* commodities, and Figure 7 displays the asset allocations for the historical efficient frontier *without* commodities. In Figure 6, the allocation to commodities is significant across the entire historical efficient frontier, with allocations ranging from 3% to 100%.

Inclusion of commodities in the opportunity set substantially changed the asset allocation of the historical efficient frontier. TIPS were excluded from the efficient asset allocations. The amount allocated to International Bonds, U.S. Stocks, and International Stocks was significantly reduced. Commodities are part of the minimum variance asset allocation and gradually represent the entire asset allocation as the asset class with the highest historical return in the opportunity set.

When commodities are included in the opportunity set, historical capital market assumptions coupled with the traditional mean-variance optimization framework result in very large allocations to commodities across the entire risk spectrum. Additionally, at equivalent risk levels, the inclusion of the commodities in the opportunity set significantly increased returns. **Historically, commodities have played a significant role in strategic asset allocation.**

Knowing that what was optimal in the past will almost certainly not be optimal in the future, we turn our attention to future expectations.

Figure 6: Historical Efficient Frontier Area Graph *with* Commodities

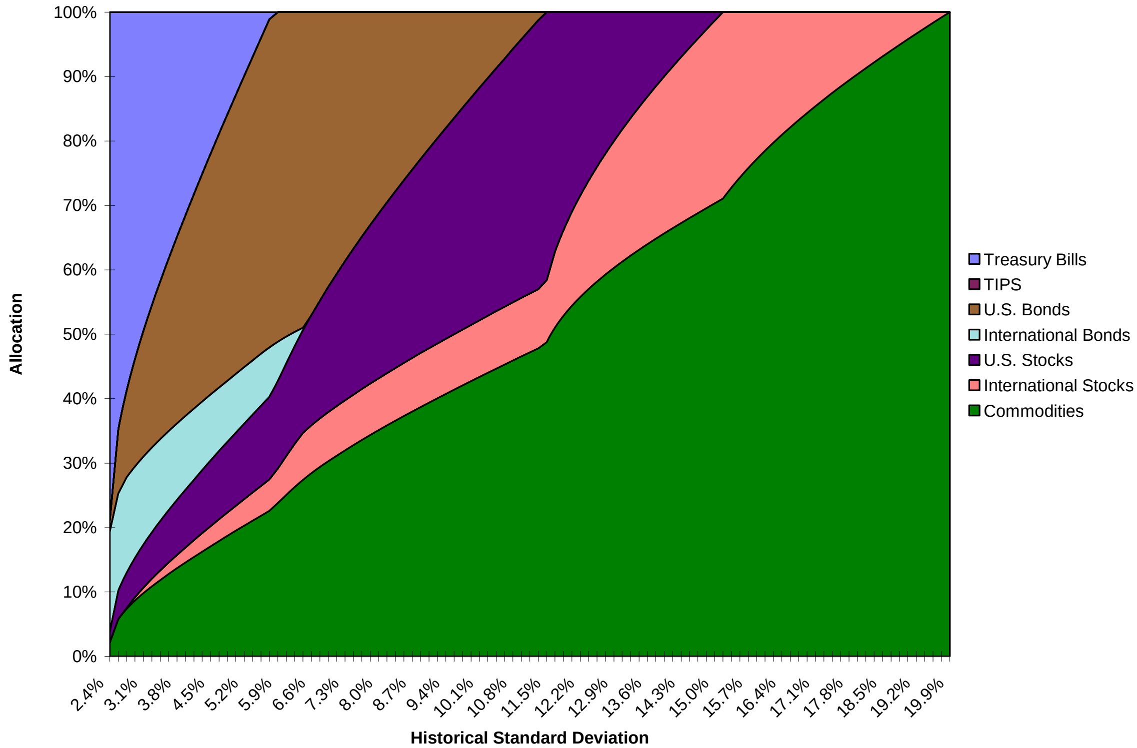
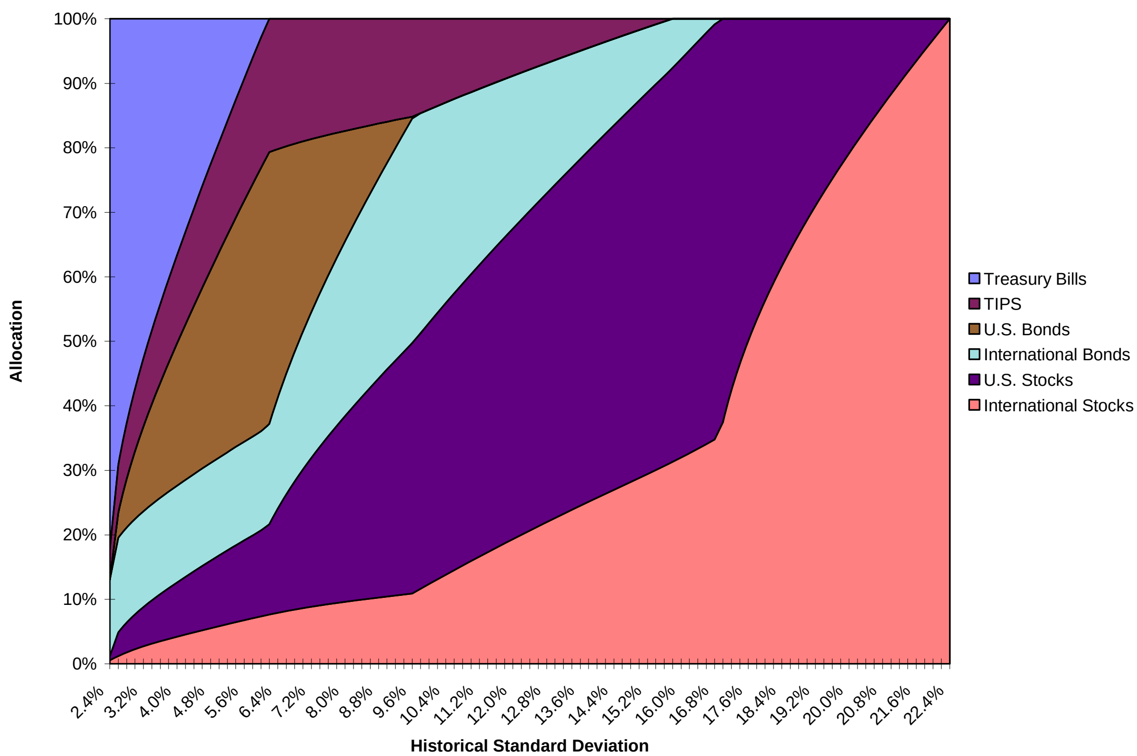


Figure 7: Historical Efficient Frontier Area Graph *without* Commodities



Section 3: Forward-Looking Efficient Frontiers with and without Commodities

Forward-looking efficient frontiers flow from forward-looking capital market assumptions. As such, professional judgment and estimation error begin to play a large role. Of the three inputs required to create an efficient frontier, returns are by far the most important, and unfortunately, the least stable. Chopra and Ziemba [1993] estimated that at a moderate risk tolerance level, mean-variance optimization is 11 times more sensitive to estimation error in returns relative to estimation error in risk (variance) and mean-variance optimization is 2 times more sensitive to estimation error in risk (variance) relative to estimation error in covariances (which also applies to correlations). It is well known that mean-variance optimization is very sensitive to the estimates of returns, standard deviations, and correlations (see Michaud [1989] and Best and Grauer [1991]).

In an effort to overcome these issues, all of the forward-looking efficient frontiers are constructed using resampled mean-variance optimization. Resampled mean-variance optimization combines traditional mean-variance optimization and multivariate Monte Carlo simulation. With the exception of no short selling and the usual budget constraint, we use an unconstrained optimization. As such, the resulting asset allocations do not incorporate typical client-driven constraints, such as constraints on international asset classes reflecting the home-bias of many U.S. investors.[17] Cognizant of the importance of return estimates and the sensitivity of traditional mean-variance optimization, we develop forward-looking efficient frontiers *with* and *without* commodities.

The Commodity Return Puzzle

The ever-present disclaimer that "past performance is no guarantee of future performance" is, of course, true and encourages us to study the nature of commodity returns. More specifically, we need to understand the fundamental drivers of historical performance and then estimate the likelihood that these return drivers will persist in the future and remain relevant. In order to justify a strategic investment in commodities, we must believe that commodities offer an inherent return to a long-only investor.

Between January 1970 and December 2004, the compounded annual return on collateral was 6.27%; yet the compounded return on our Composite Commodity Index was 12.38%, resulting in a historical premium of 6.11% above U.S. Treasury bills. What caused this historical premium?

Erb and Harvey [2005, p. 4] finds that the average excess compounded return of 36 commodity futures was close to zero; yet, an equally weighted portfolio of commodity futures has a statistically significant excess compounded return of about 4.5%. They ask, "how can a portfolio have 'equity-like' returns when the average return of the portfolio's constituents is zero?" In a response to Erb and Harvey [2005], Gorton and Rouwenhorst [2006] questions aspects of the Erb

[17] The high cost of information and high transaction fees that may have justified a home-biased asset allocation in the past have now largely disappeared and consequently investors should be more willing to invest internationally.

and Harvey [2005] analysis, especially the relative importance of the different sources of return. A rebuttal from Erb and Harvey is forthcoming.

The unexplained historical return premium, the dramatic difference between the average individual commodity return and an equally weighted portfolio of commodities, and the disagreement over the importance of the different sources of return between Erb and Harvey [2005] and Gorton and Rouwenhorst [2005, 2006] form what we call the *commodity return puzzle*.

Greer's Commodity Futures Pricing Model

Greer [2005] consolidates a number of commodity pricing theories into a commodities futures pricing model, in which the return drivers include *expected* inflation, an insurance (risk) premium, a convenience yield, diversification/rebalancing return, and expectational variance, on which we will elaborate below.

Expected Inflation - While a number of market prognosticators declared that the war with inflation was won, the events of the last two years remind us of the dangers of disinflation and how quickly inflation and inflation fears can return. Even if we do not revisit long periods of double-digit inflation, inflation is, in fact, normal. The relatively tight link between commodities and inflation should provide some aspect of a real return, or inflation-adjusted return. Inflation affects the return on collateral and provides the inherent return to a spot price commodities index. Ever-increasing demand for commodities should continue to result in long-run price increases for commodities.

Insurance (Risk) Premium - Keynes [1930] presents the theory of normal backwardation, in which a number of market participants seek to hedge their future price risk associated with various commodities. To bear this price risk, speculators demand the expectation of return. As a result, it is common for far off contracts to trade at a discount beyond what all market participants (both hedgers and speculators) believe the true price of the commodity will be.[18] In other words, hedgers are willing to pay a risk or insurance premium to speculators. Producers are more often hedgers than the processors who are buyers of the commodity, since producers typically have higher inventories and higher fixed costs, thus producers have a greater need for the price insurance. On average, the true price of the commodity is realized as the futures contract nears expiration and the long-only speculator realizes the risk (insurance) premium.

Convenience Yield - Related to the risk or insurance premium is a *convenience yield*, which at times enhances the return to a long commodities investor by driving up the futures price as the contracts near expiration. A convenience yield, or Kaldor's convenience yield as it is sometimes called (see Kaldor [1938]), occurs when the futures price is bid up by market participants (typically manufacturers) seeking to guarantee the supply of commodities. Manufacturers are willing to pay for the convenience of guaranteeing their supplies.

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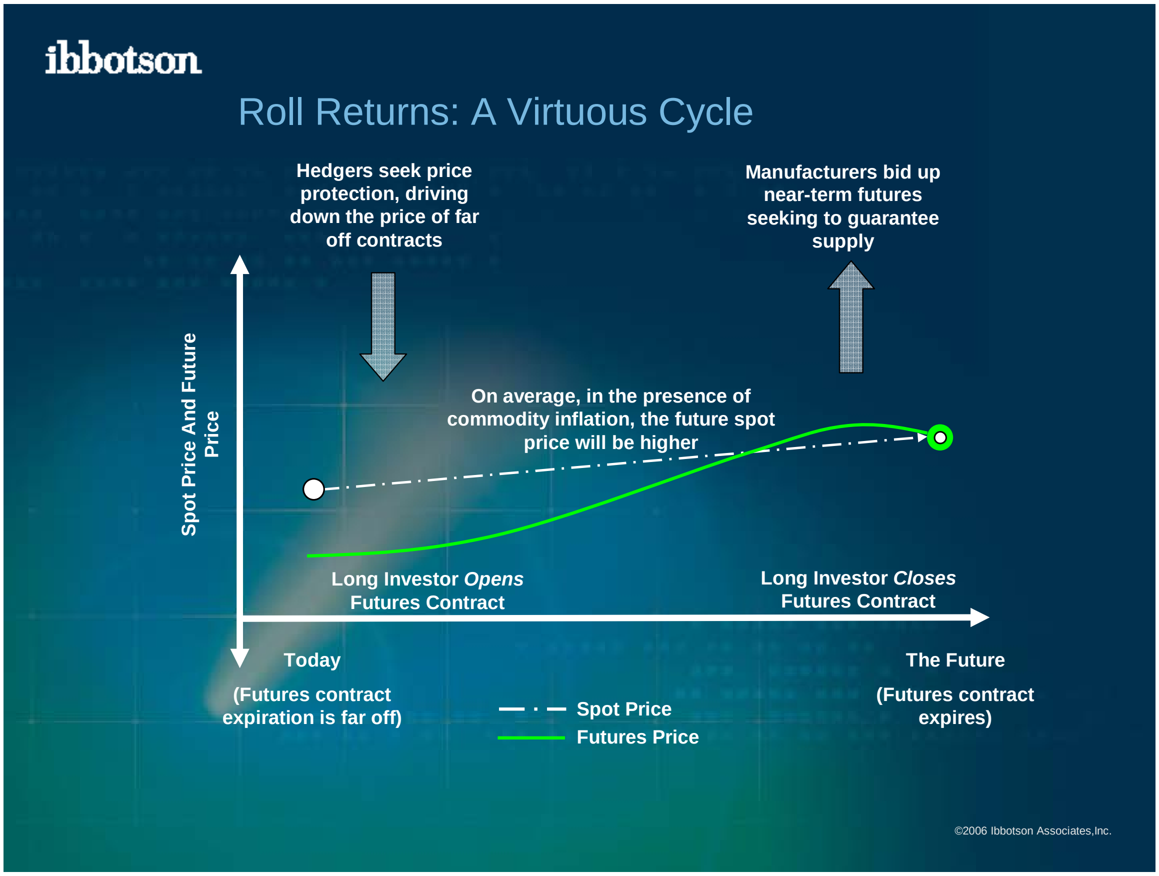
[18] Because the expectations of the market participants are unobservable, it is impossible to prove the theory of normal backwardation and the existence of an insurance premium.

Collectively, the actions of the market participants that cause the risk (insurance) premium and the convenience yield lead to what is referred to as a roll return. The interaction between hedgers and speculators depresses the price of far off contracts, which corresponds to the time frame when long investors enter into a futures contract. As futures contracts near expiration, manufacturers may bid up futures contracts as they seek to guarantee their supply, which corresponds to the time frame when long investors close out a futures contract. In other words, the long investor buys low and sells high. The roll return is inherent in the strategy of a total return commodity index and is depicted Figure 8. Erb and Harvey [2005] states that over reasonably long time periods, "roll returns, and not spot returns, have been the driver of investment success." We believe long-term increases in spot prices may contribute to roll returns. Thus, segmenting excess returns into spot returns and roll returns is problematic, since the calculated roll return only captures unexpected increases in spot prices.

In Figure 8, the current spot price, represented by the larger white (left-side) dot, is below the expected future spot price, represented by the darker (right-side) dot. While we have graphed the expected future spot price in Figure 8, the expected future spot price is unobservable. In the presence of higher expected spot prices, the expected spot price increases over time. This need not be the case. For commodities with seasonal price movement, it is not uncommon for the current spot price to be above the expected future spot price (a condition that does not occur for financial futures). On the right side of the graph, the white dot is on top of the darker dot, indicating that the spot price and futures contract converge at expiration.

Figure 8: Roll Returns



The more important relationship is the typical futures price path represented by the darker (solid) line that culminates with the futures contract expiration. Regardless of the current spot price, Keynes' normal backwardation suggests that the farther we are from the expiration of a futures contract, the more price risk speculators are taking on, and thus, speculators will demand a larger "discount" relative to the expected *unobservable* future spot price, in order to take on this price risk. It follows that contracts that are farther from expiration will trade at a discount relative to contracts that are closer to expiration. This typically results in a downward-sloping term structure for futures contacts.

The increased popularity of investing in fully collateralized commodity futures may reduce the size of the insurance (risk) premium paid by hedgers to speculators as the number of speculators relative to hedgers increases and is one of the factors motivating the sensitivity analysis performed later in the analysis.

Diversification/Rebalancing Return - The next element of Greer's commodities pricing model is a diversification/rebalancing return. We treat diversification return and rebalancing return as separate but related return drivers. Booth and Fama [1992] used the term "diversification return" to describe the difference between a portfolio's compounded return and the weighted average compound return of the portfolio's components. When the variability of an asset's return is high, so is the difference between arithmetic returns and geometric (compounded) returns. This is referred to as variance drain. Forming a portfolio of uncorrelated assets with high standard deviations results in a portfolio whose variability is considerably less than the standard deviations of the individual assets. This is the nature of diversification and Markowitz's framework, but it is especially dramatic given the relatively high standard deviations and low correlations among individual commodities.

The ability of low correlations to reduce the overall standard deviation of a portfolio is illustrated using an Evans-Archer diversification graph. Figure 9 illustrates the reduction in a portfolio's standard deviation as a function of the average constituent's standard deviation, the average pair-wise correlation, and the number of available assets.[19] Based on Gorton and Rouwenhorst [2005] individual commodity data, the average commodity had an annualized standard deviation of approximately 30% and an average pair-wise correlation with each other of 0.1. For comparison purposes, over a similar time, the current 30 components of the Dow Jones Industrial Index had an annualized standard deviation of 27% and an average pair-wise correlation with each other of 29%.[20]

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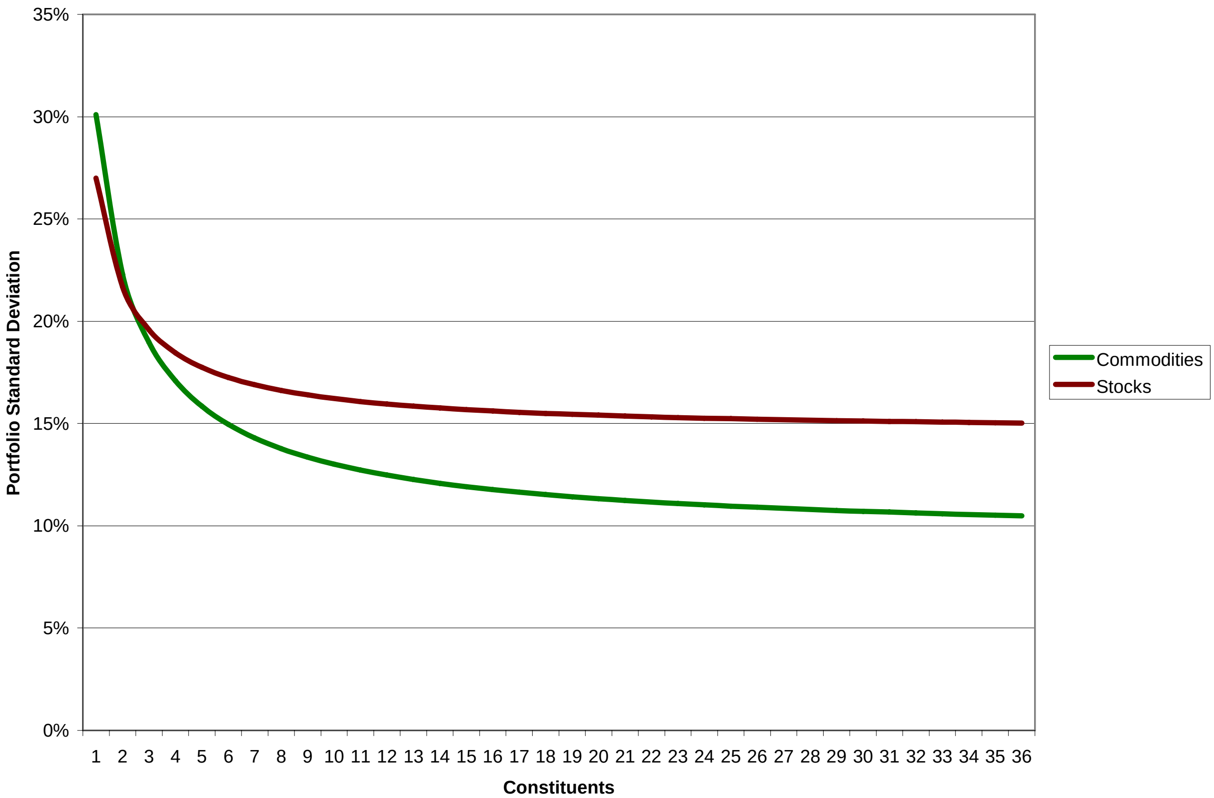
[19] More precisely, the formula is $\sigma_P = \sqrt{\frac{1}{n}\bar{\sigma}^2 + \frac{n-1}{n}\bar{\rho}\bar{\sigma}\bar{\sigma}}$, where σ_P, n, $\bar{\rho}$, $\bar{\sigma}$ represent the portfolio standard deviation, the number of constituents, the average pair-wise correlation, and the average constituent standard deviation, respectively.

[20] Consistent with Appendix 3 of Gorton and Rouwenhorst [2005], the annualized standard deviations and correlations are based on monthly data.

Figure 9: Evans-Archer Diversification



In this example, the magnitude of the diversification benefit of holding a portfolio of commodities rather than a single commodity is significantly greater than the diversification benefit of holding a portfolio of stocks rather than a single stock. The low average pair-wise correlation of the index constituents is a unique attribute of the commodities asset class. The average pair-wise correlation of the constituents of a typical equity index or bond index is far greater. The commodities asset class is a collection of economically unique assets, while equity and bond asset classes are a collection of economically similar assets.

Perhaps more surprising than the diversification return is that the simple act of rebalancing contributes to returns. Simulating 10,000 45-year histories for 40 securities with continuously compounded means of 0% and standard deviations of 30%, Erb and Harvey [2005, p. 5] found that an equally-weighted rebalanced portfolio outperformed an initially equally-weighted buy-and-hold portfolio by 50 basis points per year. Rebalancing is a contrarian strategy that sells the winners to buy the losers. Rebalancing typically results in a more diversified portfolio and thus a portfolio with a lower standard deviation.[21] All else equal, a lower standard deviation decreases the difference between the geometric return and the arithmetic return. By lowering the standard deviation, rebalancing provides a boost to the diversification return. This return boost is even more significant when returns are mean-reverting.

[21] Over time, the disparity between winners and losers in buy-and-hold strategies becomes large and, in most cases, leads to concentrated portfolios, which are intuitively less diversified, and hence, have higher standard deviations.

Erb and Harvey [2005] describes diversification return as turning water into wine. They find that the diversification return for a buy-and-hold portfolio is close to zero, while for a rebalanced, equally weighted portfolio, such as the Gorton and Rouwenhorst Commodity Index, the diversification return ranges from 3% to 4.5%. Erb and Harvey [2005] concludes that rebalancing is the most reliable source of return for long-term investments in commodities.[22] Gorton and Rouwenhorst [2005] finds that *annual* rebalancing *increased* return while *monthly* rebalancing *decreased* returns. This debate over the relative importance of return drivers remains unsettled and is highlighted in Chernoff [2005] and Sullivan [2006].

The importance of diversification and rebalancing as return drivers coupled with the information in the Evans-Archer diversification graph provides further insight to the likely cause of the diverse risk and return characteristics of the four commodity indices depicted in Figure 1. The index weighting schemes and rebalancing rules are the most likely reasons for the risk-return disparity.

Expectational Variance - The final element of Greer's commodity pricing model is expectational variance. In all likelihood, the unobservable expected future price of a futures contract will not match the actual future spot price; sometimes it will be too low and other times it will be too high. In other words, reality will vary around our ex ante expectations. These ex ante expectations include an implicit commodity-specific inflation forecast. *Unexpected* inflation, commodity-specific shocks, and supply and demand all affect whether or not the unobservable expected future price is realized. Believers of the efficient market hypothesis would expect, on average, for the difference between the ex ante expectations and the ex post future price to be zero. Greer [2005] speculates that significant supply disruptions are more likely than supply gluts; thus, expectational variance, while not a long-term source of return, may create positive (and beneficial) skewness in the realized returns.

We believe most of these return drivers are likely to persist in the future and contribute to an inherent return for commodities. Inflation is an element of a normal economy. The actions and motives of market participants should continue to contribute to roll returns. Individual commodities are likely to continue to have high standard deviations and low pair-wise correlations with each other. Rebalancing and diversification should continue to be relevant fundamental return drivers. Finally, the bias for positive skewness associated with expectational variance is likely to continue.

Extending the Capital Asset Pricing Model to Commodities Futures

The most famous of all asset pricing models is the Sharpe-Lintner-Mossin-Treynor CAPM. The CAPM, developed separately in Sharpe [1964], Lintner [1965], Mossin [1966], and Treynor [1961, 1962], provides the framework for separating the return of any asset into two components: the portion that is correlated with the market and the portion that is uncorrelated with the market. According to the CAPM, the expected return of any asset is a function of the asset's contribution

[22] Erb and Harvey [2005] provides an excellent exploration of diversification returns.

to market risk. Other names for market risk include systematic risk, benchmark risk, correlated risk, non-diversifiable risk, and beta risk. This single risk factor is represented by the unobservable, all-inclusive market portfolio that should include all tradable assets, such as traditional capital assets, as well as non-tradable assets, such as human capital (see Roll [1977]). The Capital Asset Pricing Model is not limited to *capital* assets as its name suggests.

The idiosyncratic nature of individual commodity returns relative to traditional definitions of the market portfolio presents real challenges to systematic or market risk-based pricing models. Questions regarding the applicability of the CAPM to commodity futures, which are not capital assets, are understandable. If we are to have any hope of success in applying the CAPM to commodities, we must develop a reasonably accurate working version of the hypothetical market portfolio—a version that includes commodities.

Unfortunately, unlike stocks and bonds, commodities do not have an observable market capitalization. In fact, the market capitalization of a futures index is zero, as long positions and short positions perfectly offset one another. If commodities had an observable market capitalization, we would conclude that the role of commodities in the hypothetical, all-encompassing market portfolio of the CAPM is proportional to their market capitalization. Likewise, when selecting a total return commodity index, we would prefer an index based on market capitalization weights.

In the absence of observable market capitalizations, we must think outside the box in order to extend the CAPM to commodities. Literature from the three commercially available commodity return indices states their desire to use weights that reflect the *economic significance* of commodities. Our desire is the same.

Commodities are the raw materials (including semi-finished goods) that build, power, and feed society. The world could not function without commodities. Commodities are mined (metals), grown (agricultural products), and, to some degree, processed (oil). The relative importance, amount, and price of individual commodities evolve with society’s preferences.

We will start with what we know. Table 10 contains estimates of the market capitalizations for the asset classes in the opportunity set. Table 10 represents one possible definition of the market portfolio, albeit a definition that does not yet include commodities. The market capitalization of the global portfolio of stocks and bonds in our opportunity set is approximately $48.8 trillion.

Table 10: Market Capitalization Estimates

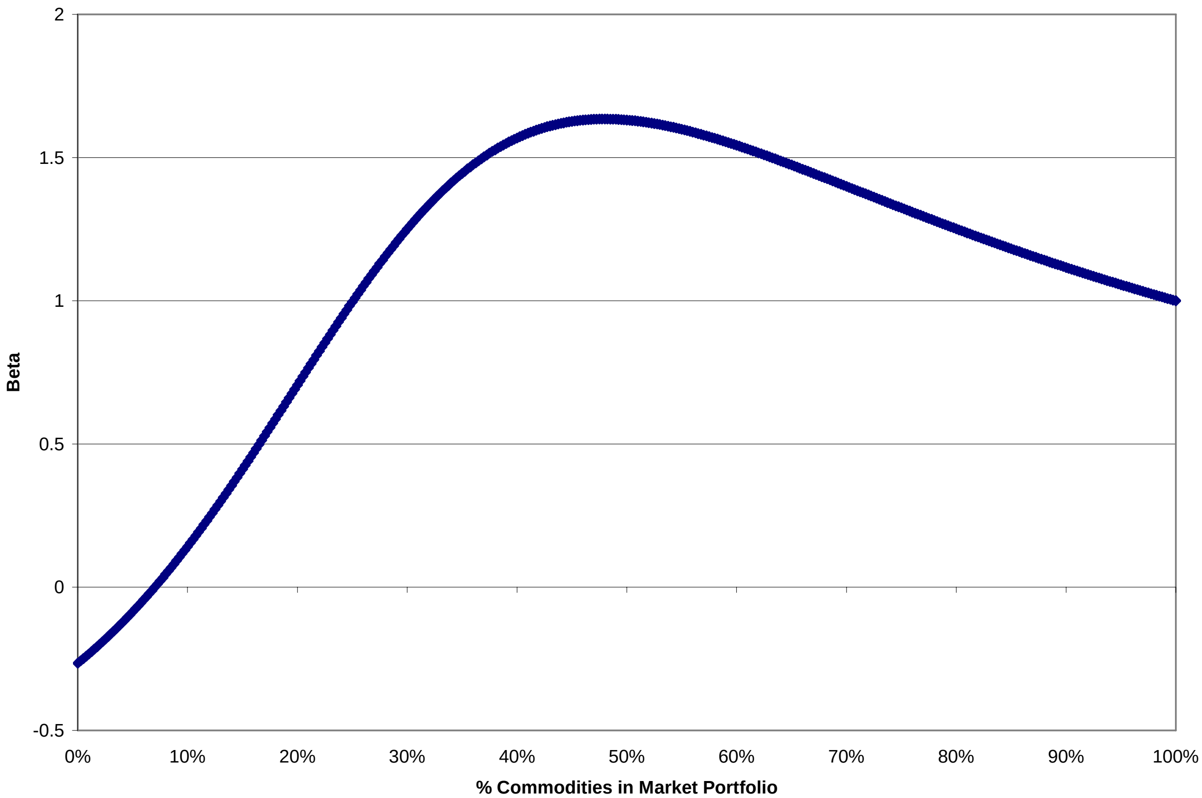
Asset Class	Market Capitalization Estimate (In Billions)	Weight in Global Market Portfolio
Treasury Bills	$450	0.9%
TIPS	$800	1.6%
U.S. Bonds	$8,171	16.7%
International Bonds	$13,024	26.7%
U.S. Stocks	$13,502	27.7%
International Stocks	$12,874	26.4%
Total	$48,821	100.0%

If commodities are indeed an asset class, then they belong in the market portfolio. The question isn't "if?" but, "how much?" Holding the weights of the asset classes in Table 10 constant, relative to each other, and using the historical covariance matrix (a combination of historical standard deviations and correlations), we can infer the market capitalization required for the CAPM to produce positive return estimates for commodities.

Figure 10 graphs the implied CAPM beta of commodities as a function of the percentage of commodities in the market portfolio. The implied CAPM beta is on the vertical axis, and the percentage of commodities in the market portfolio is on the horizontal axis. The market capitalization estimates of the capital assets are held constant at $48.8 trillion. In Figure 10, the market capitalization estimate for commodities is slowly increased, increasing the percentage of commodities in the growing total market portfolio. For example, when the market capitalization estimate for commodities is $5.4 trillion, the total market capitalization of the market portfolio increases to $54.2 trillion; thus, the percentage of commodities in the market portfolio is 10%. As the market capitalization estimate for commodities grows large, commodities eventually dominate the market portfolio. As this happens, the beta of commodities, relative to a commodities-dominated definition of the market portfolio, approaches one.

For commodities to have a positive beta, and thus a positive CAPM expected return beyond the risk-free rate, nearly 7.2% of the market portfolio must consist of commodities. This corresponds to an approximate market capitalization of commodities of $3.8 trillion. As the percentage of commodities in the market portfolio increases toward 50%, the beta of commodities increases to nearly 1.65.

Figure 10: Implied Beta of Commodities



In addition to calculating the beta of commodities relative to the increasingly commodity-centric definition of the market portfolio, we also calculated the expected Sharpe Ratio and variance of the market portfolio. Coincidentally, the expected Sharpe Ratio of the market portfolio is maximized and the variance of the market portfolio is minimized when commodities represent approximately 25% of the market portfolio.

One method for approximating the market capitalization of commodities is to use aggregate annual production data for commodities, which is approximately $2 trillion. At a minimum, the $2 trillion in aggregate annual production data can serve as an absolute floor for the market capitalization estimate of commodities, which corresponds to a minimum role in the market capitalization-weighted portfolio of slightly greater than 4%. A potential improvement to the estimate is obtained by applying standard valuation metrics to the $2 trillion annual production figure. It is unclear whether or not annual production figures should be treated more like earnings or more like sales. For U.S. equities, the current approximate price-to-earnings ratio and price-to-sales ratios are 18 and 1.5, respectively. Applying these metrics leads to market capitalization estimates of $36 trillion and $3 trillion.

In a CAPM context, empirically, the high historical returns suggest that commodities play a significant role in the market portfolio. We move forward with the assumption that commodities represent 20% of the market portfolio, which corresponds to a market capitalization of $12.2 trillion. This is more than the approximate 7.2% required for the CAPM model to lead to a positive excess return estimate and more than the 3.8% that corresponds to the $3 trillion obtained by apply a price-to-sales ratio of 1.5. However, it is less than the 25% that corresponds to the market portfolio variance minimizing position, and substantially less than the 42% that corresponds to the $36 trillion estimate obtained by applying a price-to-earnings ratio of 18 to annual production data. One could certainly argue with this assumption; however, this assumption provides us with the economist's proverbial can opener that allows us to proceed. The role of commodities in the market portfolio deserves considerably more research.[23] Later, we forecast commodity returns using a model that is independent of an estimate of the market capitalization of commodities.

Table 11 contains an estimate of the market capitalizations of the asset classes in the opportunity set, the weights of the market portfolio, the betas of the asset classes relative to the market portfolio, the reverse-optimized CAPM excess returns, and the corresponding total returns.

Table 11: Market Capitalization Estimates

Asset Class	Market Capitalization Estimate (In Billions)	Weight in Market Portfolio	Beta Relative to Market Portfolio	CAPM Excess Return	CAPM Total Return
Treasury Bills	$ 450	0.7%	-0.05	-0.2%	4.39%
TIPS	$ 800	1.3%	0.29	1.4%	6.05%
U.S. Bonds	$ 8,171	13.4%	0.07	0.3%	4.94%
International Bonds	$ 13,024	21.3%	0.53	2.6%	7.17%
U.S. Stocks	$ 13,502	22.1%	1.29	6.3%	10.90%
International Stocks	$ 12,874	21.1%	2.13	10.3%	14.96%
Commodities	$ 12,205	20.0%	0.71	3.4%	8.05%
Total	$ 61,026	100.0%	1	4.9%	9.5%

We have calibrated the CAPM model so that U.S. Stocks have an expected return of 10.9%, which is based on a long-term risk-free rate of 4.62% and a Russell 3000 premium of 6.28%. This corresponds to a global market portfolio premium of 4.9%. There are noticeable differences between the historical returns and the CAPM returns. With the exception of International Stocks, the CAPM model produced estimates of expected return that are lower than the historical arithmetic returns.[24]

Figure 11 presents forward-looking efficient frontiers *with* and *without* commodities, based on the CAPM model of expected returns and the historical standard deviations and correlations used earlier.[25] In the absence of additional information, we assume that the historical standard deviations and correlations remain the best estimates of forward-looking standard deviations and correlations.

[23] We are not alone in our endeavor to approximate the role of commodities in a working version of the market portfolio. In private correspondence with Kurt Winkelmann of Goldman Sachs, we have learned that they too plan to investigate the role of commodities in the global market portfolio.
[24] Our International Bond and International Stock asset class proxies are unhedged. Using hedged asset class proxies would result in expected returns for these asset classes that are closer to the expected returns of U.S. Bonds and U.S. Stocks, respectively.
[25] For the CAPM efficient frontier *without* commodities, we continued to use the CAPM returns reported in Table 11 and simply omitted commodities from the opportunity set. Alternatively, we could have calculated a new set of CAPM returns based on an opportunity set that did not include commodities.

In Figure 11, the efficient frontier *with* commodities dominates the efficient frontier *without* commodities. Including commodities in the opportunity set improved the risk-return tradeoff over the entire historical efficient frontier, with the exception of the maximum return asset allocation. Over the common standard deviation range of approximately 2.4% to 22.4%, the average improvement in return at each of the risk levels was approximately 35 basis points, with a maximum of 55 basis points.

Figure 11: Forward-Looking Efficient Frontier *with* and *without* the Commodities Asset Class based on CAPM Expected Return

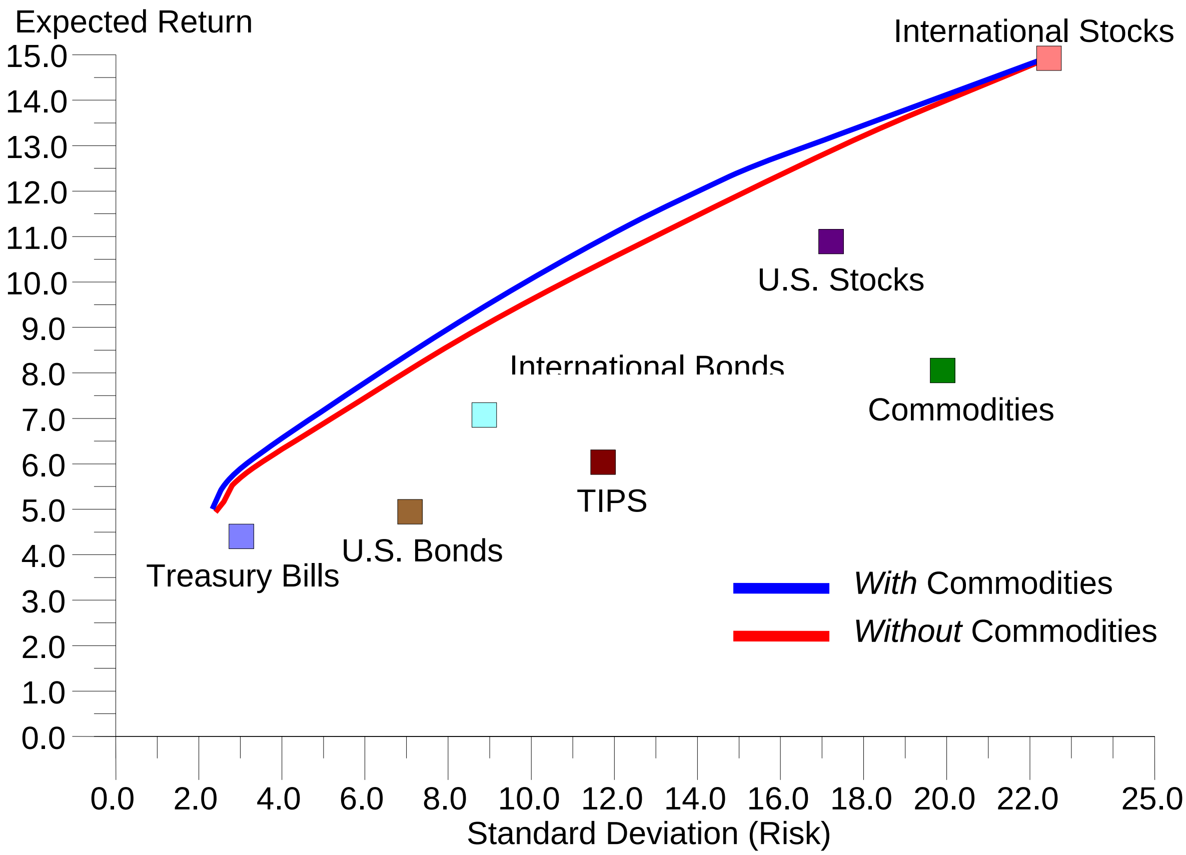


Figure 12 displays the asset allocations for the efficient frontier *with* commodities, and Figure 13 displays the asset allocations for the efficient frontier *without* commodities. The allocation to commodities is significant across the majority of the efficient frontier, with allocations ranging from 0% to nearly 25%. When using CAPM expected returns, the resulting asset allocation to commodities is tightly linked to the role of commodities in the market portfolio. The role, or proportion, of commodities in the market portfolio affects the beta of commodities relative to the evolving definition of the market portfolio, and hence, the CAPM expected returns. Depending on the size of the estimation error of the market capitalization of commodities, the asset allocations depicted in Figure 12 may be dramatically misstated.

Figure 12: Forward-Looking (CAPM) Efficient Frontier Area Graph *with* Commodities

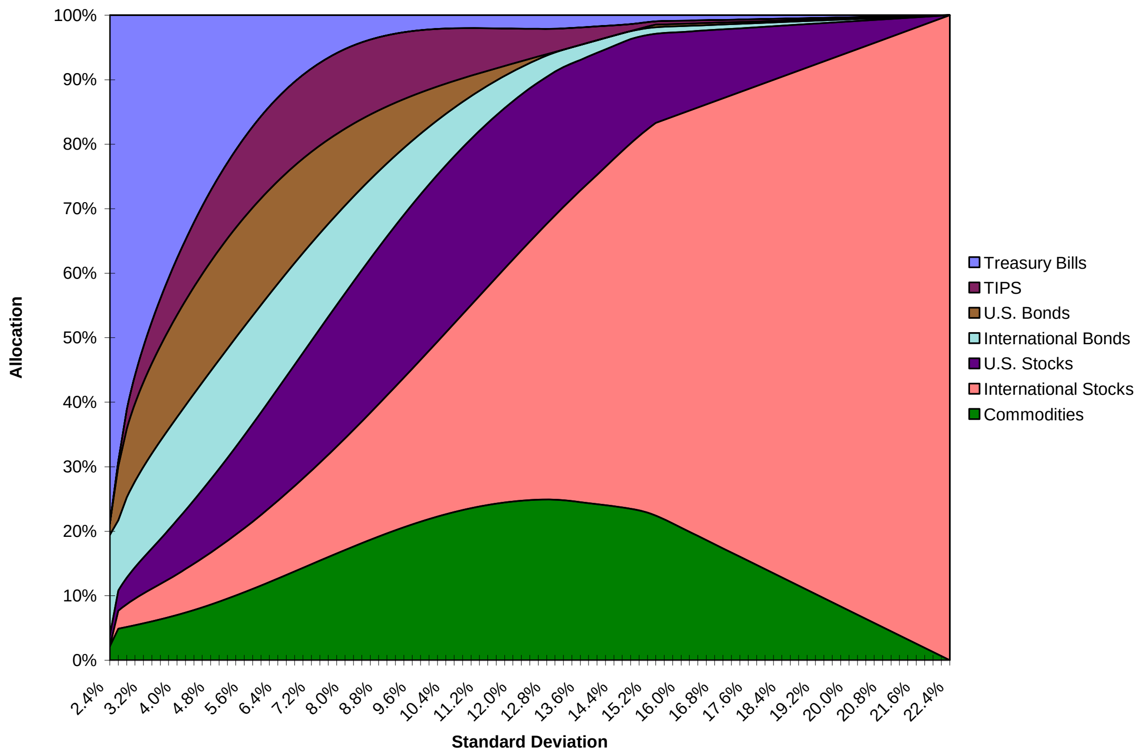
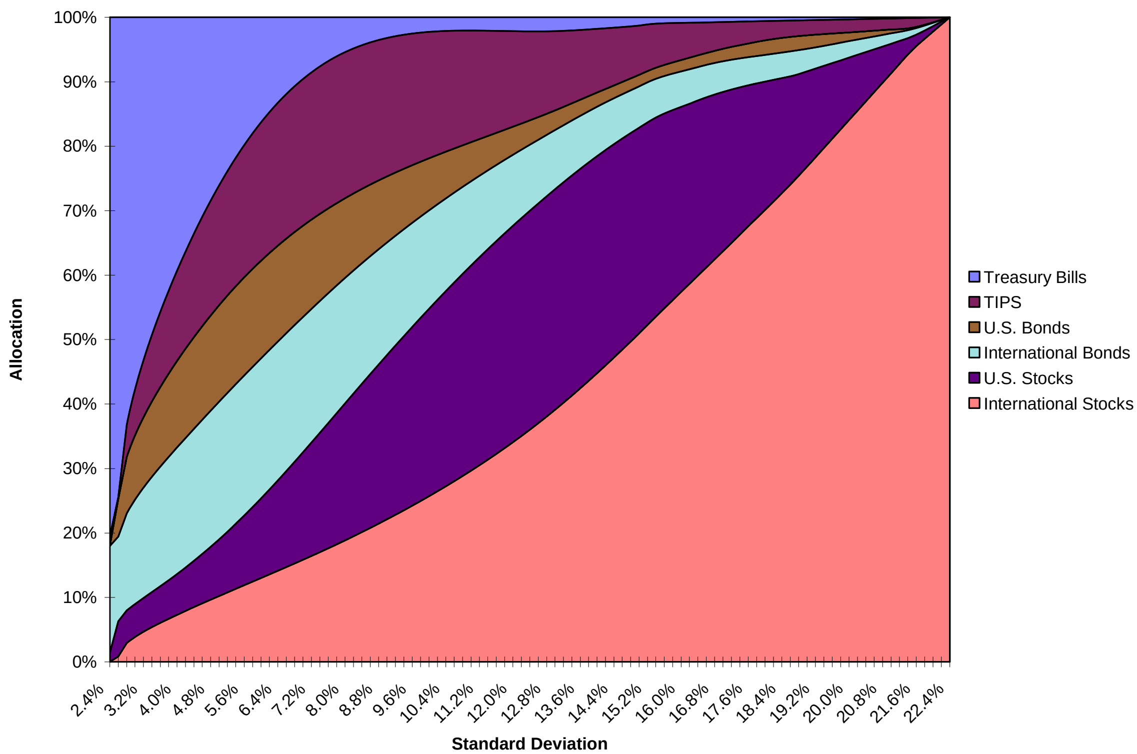


Figure 13: Forward-Looking (CAPM) Efficient Frontier Area Graph *without* Commodities



To gain insights into the expected improvement in the forward-looking efficient frontier that results from including commodities in the opportunity set, we identified three asset allocations on the efficient frontiers that correspond with conservative, moderate, and aggressive asset allocations. The standard deviations of these asset allocations are 5%, 10%, and 15%, respectively. Table 12 contains representative conservative, moderate, and aggressive asset allocations, as well as several summary statistics. Of the three representative model asset allocations, the moderate asset allocation with a standard deviation of 10% is most closely aligned to the average investor's 60% stock / 40% bond asset allocation.

Table 12: Asset Allocations and Summary Statistics

	Conservative		**Moderate**		**Aggressive**	
Asset Class	*With* Commodities	*Without* Commodities	*With* Commodities	*Without* Commodities	*With* Commodities	*Without* Commodities
Treasury Bills	24.78%	29.33%	2.20%	5.06%	1.25%	4.06%
TIPS	11.43%	17.77%	9.31%	19.07%	0.90%	7.50%
U.S. Bonds	17.06%	14.32%	5.79%	7.90%	0.09%	1.74%
International Bonds	16.91%	20.42%	8.95%	14.87%	1.15%	6.27%
U.S. Stocks	12.15%	8.40%	25.77%	28.16%	15.46%	31.08%
International Stocks	8.54%	9.76%	26.09%	24.94%	57.90%	49.35%
Commodities	9.13%	0.00%	21.89%	0.00%	23.24%	0.00%
Expected Return	7.15%	6.89%	10.04%	9.61%	12.40%	11.92%
Standard Deviation	**5.00%**	**5.00%**	**10.00%**	**10.00%**	**15.00%**	**15.00%**
Sharpe Ratio	0.506	0.453	0.542	0.499	0.518	0.486

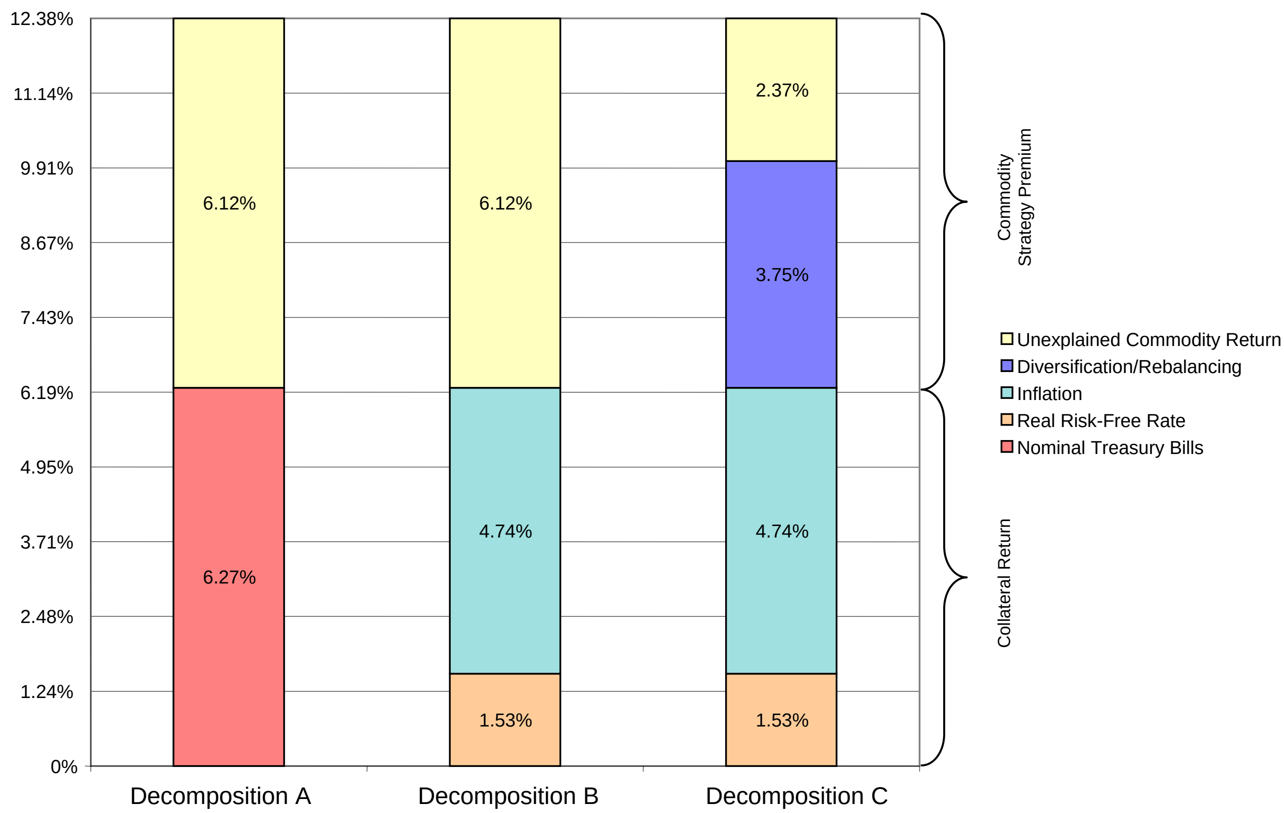
Based on these forward-looking efficient frontiers, asset allocations that include commodities have better risk-adjusted performance (as measured by the Sharpe Ratio) than asset allocations without the commodities.[26] For the conservative and moderate asset allocations, the allocations to commodities were primarily sourced by decreases in the asset allocation to fixed income asset classes. For the aggressive asset allocation, the allocation to commodities appears to be sourced by all of the asset classes except International Stocks. Even when returns are forecast using the CAPM, including commodities in the opportunity can potentially improve the reward-to-risk ratio in conservative, moderate, and aggressive asset allocations by including commodities.

Can we produce better forward-looking expected return forecasts and hence better forward-looking asset allocations? We would like to think so.

[26] The Sharpe Ratio is a return-to-risk measure. It is the ratio of the excess return over the risk-free rate to the standard deviation, first presented in Sharpe [1966].

Ibbotson and Chen [2003] presents a series of possible decompositions of compounded annual equity returns. In a similar spirit, we decompose compounded annual commodity returns. The original building blocks of Ibbotson and Sinquefield [1976b] are inflation, the real risk-free rate, and the equity risk premium. Commodity returns can be broken down into three similar components: inflation, the real risk-free rate, and a yet to be explained or defined premium. The real risk-free rate is inferred from inflation and the nominal risk-free rate. These starting decompositions are the first two stacked area graphs (A and B) in Figure 14. The nominal risk-free rate is the return on U.S. Treasury bills, which is also the return on the collateral.

Figure 14: Historical Return Decomposition



These starting decompositions account for the effect of inflation on the return of the collateral. The challenge is to decompose the historical compounded annual risk premium of 6.12%. We cannot pinpoint the exact source of the unexplained historical premium of 6.12%; however, we suspect that the various aspects of Greer's commodity pricing model are at work. We believe the unexplained historical excess return is an inherent return from investing in a fully collateralized commodity index that we call the "commodity strategy premium." We use the term commodity strategy premium to distinguish it from Keynes' insurance risk premium and to emphasize that the premium is the result of a specific investment strategy.

Erb and Harvey [2005] would likely attribute the historical excess return to diversification/rebalancing. Depending on the average pair-wise correlation assumption of the individual commodities futures coupled with an average individual commodity futures standard deviation assumption of 30%, Erb and Harvey [2005, p. 40] would attribute approximately 3% to 4.5% of the historical premium to diversification/rebalancing for an equally weighted rebalanced portfolio.[27] Splitting the difference between 3% and 4.5% leads to a sizable diversification/rebalancing premium of 3.75% and leaves an unexplained commodity return premium of 2.37%. This is depicted in decomposition C of Figure 14. In contrast, Gorton and Rouwenhorst [2005] would likely attribute most of the historical excess return to an insurance premium resulting from Keynes' normal backwardation. We believe diversification/rebalancing, Keynes' normal backwardation, and the other elements of Greer's commodity pricing model all contribute to the historical excess return.

The commodity strategy premium measures the return above the return on collateral. Without a robust method for estimating the forward-looking commodity strategy premium, we use the historical commodity strategy premium for a forward-looking estimate of 6.12%. Later, in a sensitivity analysis, we reduce this to a far more conservative figure of 2%. In a forward-looking context, the building blocks methodology attempts to maximize the use of applicable historical data and couple it with current market expectations. In the case of commodities, the commodity strategy premium is based on historical data, while the expected collateral return and inflation estimates are based on current market expectations and current forecasts. Table 13 summarizes one possible forecast for commodity returns. Interestingly, this forecast is reasonably consistent with the CAPM forecast of 8.05% presented in Table 11, and perhaps more importantly, it is independent of any market capitalization assumptions and suggests that the market capitalization estimate used to arrive at the 8.05% return may be conservative.

Table 13: Commodity Returns

Return Drivers	Returns
Expected Collateral Return (U.S. Treasury Bills)	2.87%
Commodity Strategy Premium	6.12%
Commodity Returns	8.99%

Table 14: Building Block Return Estimates

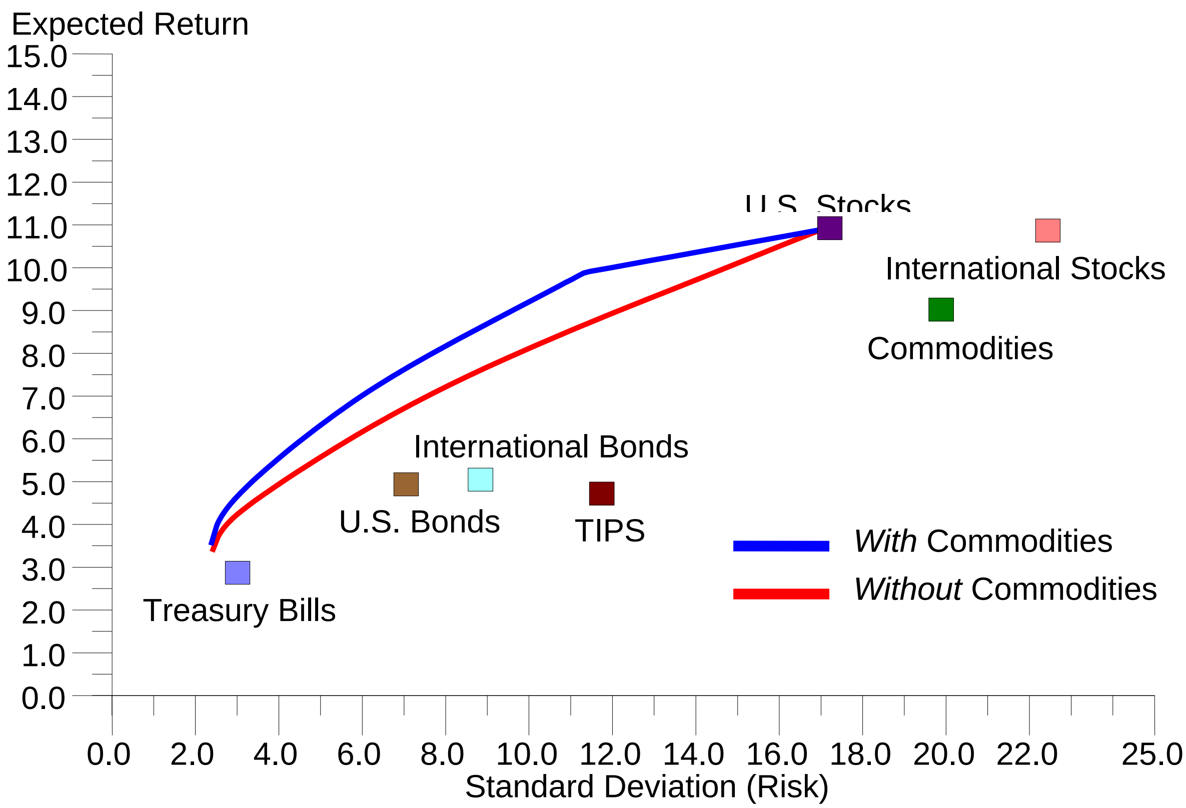
Asset Class	Estimated Total Return
Treasury Bills	2.87%
TIPS	4.73%
U.S. Bonds	4.94%
International Bonds	5.05%
U.S. Stocks	10.90%
International Stocks	10.86%
Commodities	8.99%

[27] Using the same inputs used to calculate the Evans-Archer diversification graph, the formula for approximating the diversification return of an equally weighted rebalanced portfolio is $r_d = \frac{1}{2}\left(1 - \frac{1}{n}\right)\overline{\sigma}^2\left(1 - \overline{\rho}\right)$, where r_d, n, $\overline{\rho}$, $\overline{\sigma}$ represent the diversification return, the number of constituents, the average pair-wise correlation, and the average constituent standard deviation, respectively.

Table 14 contains a complete set of expected returns estimated using the building blocks approach.

The resulting efficient frontiers *with* and *without* commodities are presented in Figure 15. Once again, the efficient frontier that includes commodities in the opportunity set dominates the efficient frontier *without* commodities. Over the common standard deviation range of approximately 2.4% to 17.2%, the average improvement in return at each of the risk levels was approximately 77 basis points, with a maximum of 123 basis points.

Figure 15: Forward-Looking Efficient Frontier *with* and *without* the Commodities Asset Class based on Building Block Expected Return



As before, we create asset allocation area graphs that display the evolution of the asset allocation across the respective efficient frontiers (see Figures 16 and 17). In Figure 16, the allocation to commodities is significant across the majority of the efficient frontier, with allocations ranging from 0% to 31%. The allocations to commodities of Figure 16 and Figure 12 (the CAPM analysis) are extremely consistent with one another.

Again, we identified three asset allocations from the efficient frontiers that correspond with conservative, moderate, and aggressive asset allocations. The standard deviations of these asset allocations are 5%, 10%, and 15%, respectively. Table 15 reports the conservative, moderate, and aggressive asset allocations, as well as several summary statistics.

Figure 16: Forward-Looking (Building Block) Efficient Frontier Area Graph *with* Commodities

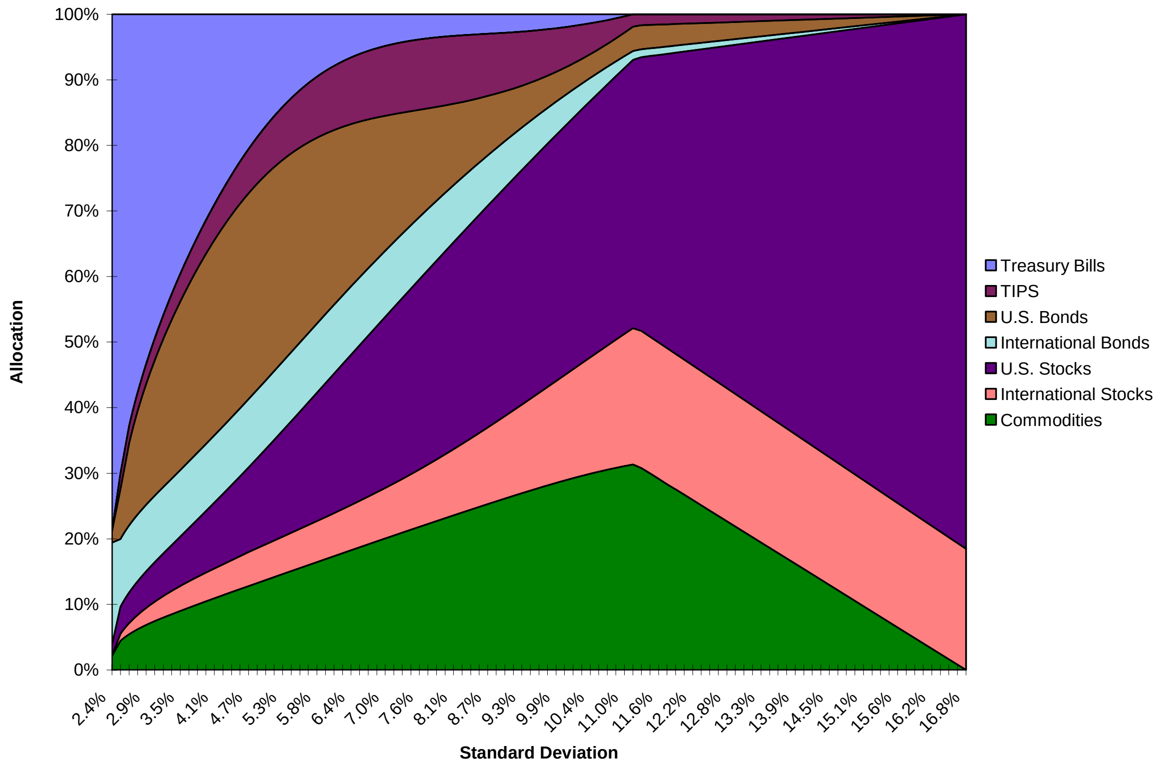


Figure 17: Forward-Looking (Building Block) Efficient Frontier Area Graph *without* Commodities

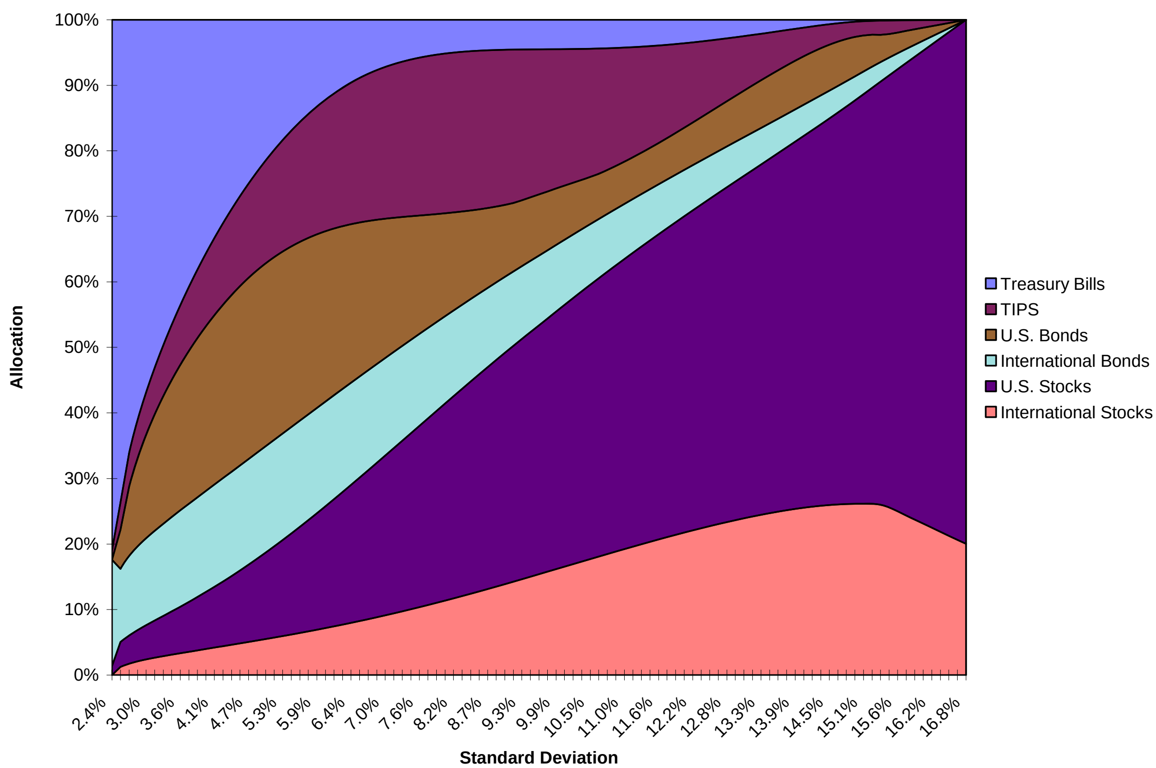


Table 15: Asset Allocations and Summary Statistics

Asset Class	Conservative With Commodities	Conservative Without Commodities	Moderate With Commodities	Moderate Without Commodities	Aggressive With Commodities	Aggressive Without Commodities
Treasury Bills	16.70%	21.32%	1.99%	4.46%	0.00%	0.23%
TIPS	7.54%	15.78%	6.26%	20.93%	0.41%	2.24%
U.S. Bonds	31.30%	27.91%	4.25%	8.34%	0.91%	5.75%
International Bonds	10.48%	16.24%	4.75%	9.99%	0.30%	3.56%
U.S. Stocks	14.69%	13.24%	37.89%	39.80%	71.70%	62.08%
International Stocks	5.48%	5.51%	16.02%	16.48%	19.06%	26.14%
Commodities	13.81%	0.00%	28.85%	0.00%	7.61%	0.00%
Expected Return	6.35%	5.60%	9.27%	8.16%	10.65%	10.18%
Standard Deviation	**5.00%**	**5.00%**	**10.00%**	**10.00%**	**15.00%**	**15.00%**
Sharpe Ratio	0.346	0.196	0.465	0.355	0.402	0.371

The risk-adjusted performance as measure by the Sharpe Ratio improved when commodities were included in the opportunity set. Relative to the CAPM-based asset allocations, the building blocks-based asset allocations are less international. For the conservative and moderate asset allocations, the allocations to commodities were primarily sourced by decreases in the asset allocation to fixed income asset classes. For the aggressive asset allocation, the allocation to commodities appears to be sourced by all of the asset classes except U.S. Stocks. The improvements in the Sharpe Ratios were most pronounced for the conservative and moderate asset allocations.

The Black-Litterman Model: Combining the CAPM with Building Blocks

In our third analysis, we combine the CAPM expected return estimate with building block return estimates using the Black-Litterman model. The Black-Litterman model uses a Bayesian approach to combine the subjective views of an investor regarding the expected returns of one or more assets with CAPM expected returns to form a new, mixed estimate of expected returns. In our case, the subjective views are the complete set of expected returns estimated using the building blocks methodology. Using the Black-Litterman model, we combine the CAPM expected return of Table 11 with the expected returns estimated using the building blocks methodology from Table 14 to form what can be thought of as a complex, weighted average of the two sets of expected returns.[28] Table 16 presents the combined expected returns.

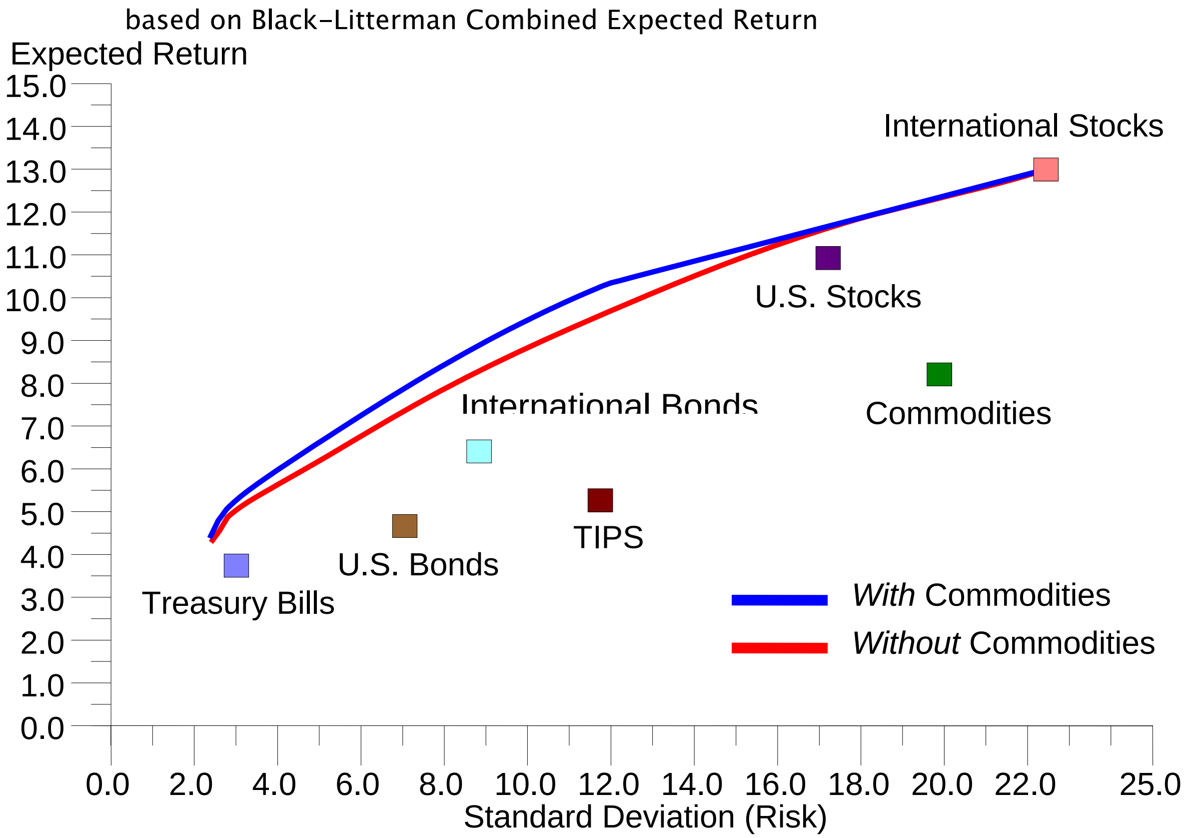
[28] More specifically, we treat each of the building blocks' expected returns as an absolute view and specify a confidence level of 50% in each view using the method described in Idzorek [2005].

Table 16: Black-Litterman Combined Return Estimates

Asset Class	Combined Total Return
Treasury Bills	3.74%
TIPS	5.28%
U.S. Bonds	4.64%
International Bonds	6.37%
U.S. Stocks	10.90%
International Stocks	13.00%
Commodities	8.18%

The resulting efficient frontiers *with* and *without* commodities are presented in Figure 18. Once again, the efficient frontier that includes commodities in the opportunity set dominates the efficient frontier *without* commodities. Over the common standard deviation range of approximately 2.4% to 22.4%, the average improvement in return at each of the risk levels was approximately 36 basis points, with a maximum of 70 basis points.

Figure 18: Forward-Looking Efficient Frontier *with* and *without* the Commodities Asset Class



As before, we create asset allocation area graphs that display the evolution of the asset allocation across the respective efficient frontiers (see Figures 19 and 20). In Figure 19, the allocation to commodities is significant across the majority of the efficient frontier, with allocations ranging from 0% to nearly 28% (the same range as the building blocks-based asset allocations of Figure 16).

Figure 19: Forward-Looking (Combined) Efficient Frontier Area Graph *with* Commodities

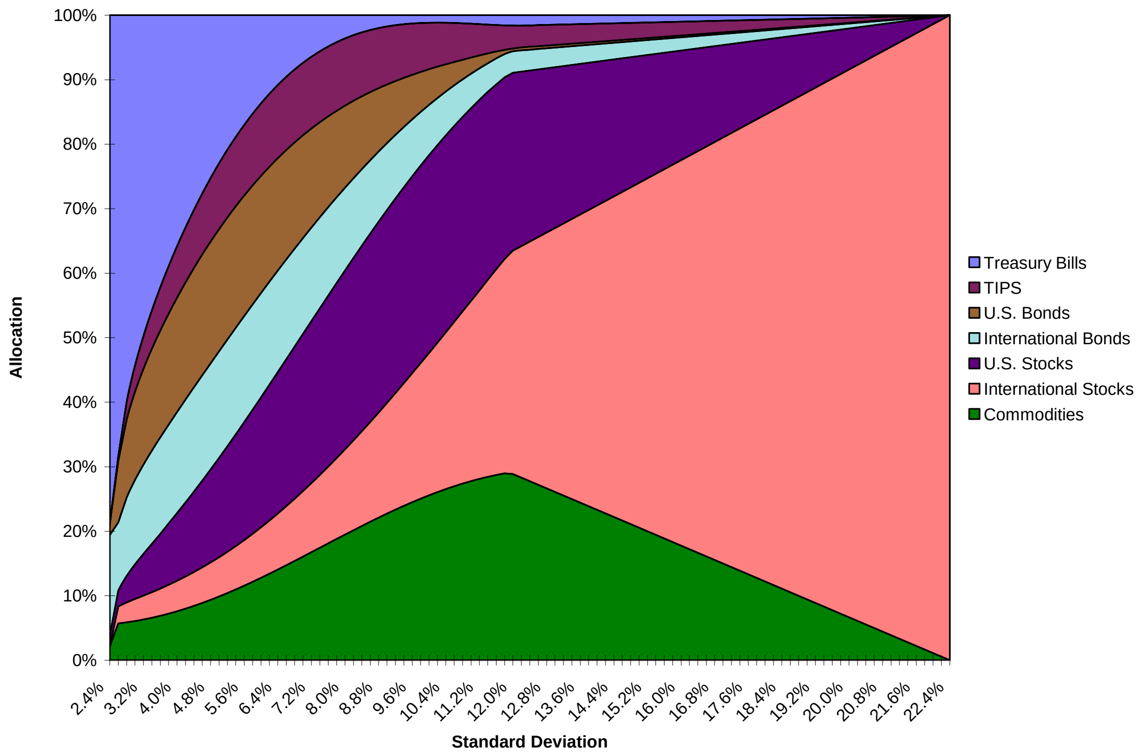


Figure 20: Forward-Looking (Combined) Efficient Frontier Area Graph *without* Commodities

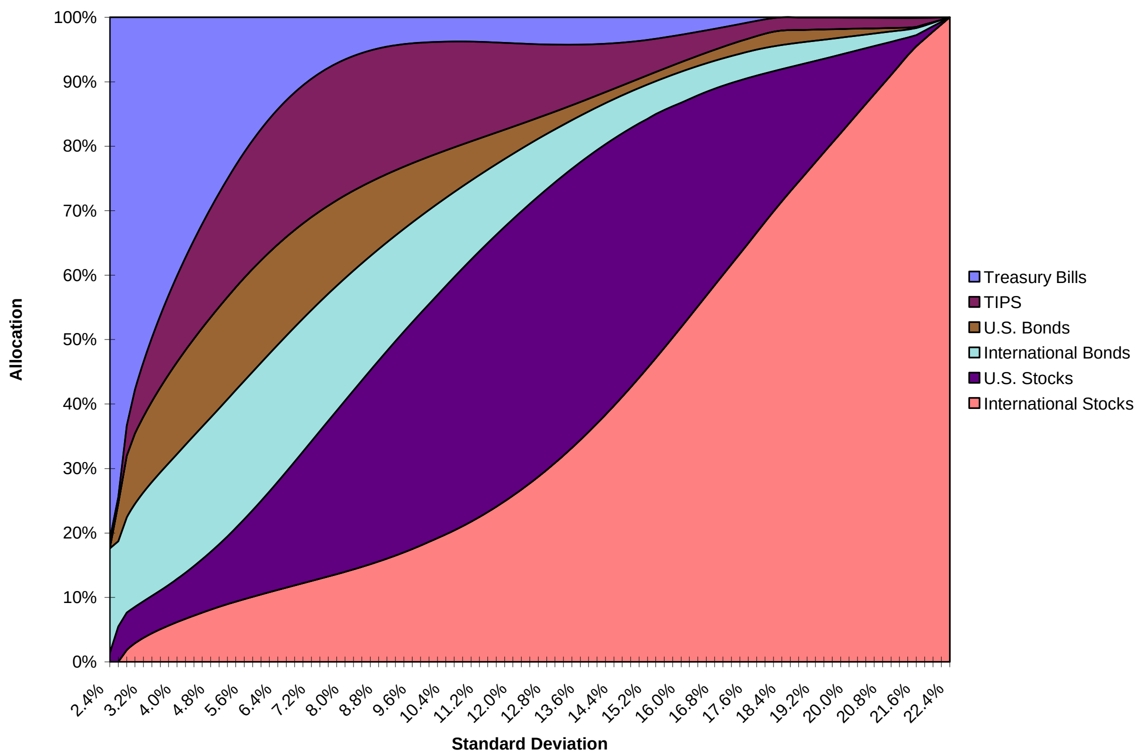


Table 17 reports the conservative, moderate, and aggressive asset allocations, as well as several summary statistics.

Table 17: Asset Allocations and Summary Statistics

Asset Class	Conservative		Moderate		Aggressive	
	With Commodities	*Without* Commodities	*With* Commodities	*Without* Commodities	*With* Commodities	*Without* Commodities
Treasury Bills	22.77%	27.34%	1.17%	3.89%	1.07%	3.68%
TIPS	10.24%	17.64%	7.21%	17.80%	2.29%	5.93%
U.S. Bonds	18.93%	15.74%	5.53%	8.31%	0.29%	1.43%
International Bonds	16.54%	21.07%	7.80%	14.62%	2.20%	5.52%
U.S. Stocks	15.48%	9.69%	30.65%	36.79%	18.13%	39.57%
International Stocks	6.11%	8.52%	22.05%	18.58%	57.05%	43.88%
Commodities	9.93%	0.00%	25.59%	0.00%	18.97%	0.00%
Expected Return	6.62%	6.19%	9.48%	8.83%	11.26%	10.88%
Standard Deviation	**5.00%**	**5.00%**	**10.00%**	**10.00%**	**15.00%**	**15.00%**
Sharpe Ratio	0.400	0.314	0.486	0.421	0.442	0.418

Consistent with both the CAPM and the building block-based analyses, the risk-adjusted performance as measured by the Sharpe Ratio improved when commodities were included in the opportunity set. For the conservative and moderate asset allocations, the allocations to commodities were primarily sourced by decreases in the asset allocation to fixed income asset classes. For the aggressive asset allocation, the allocation to commodities appears to be sourced by all of the asset classes except International Stocks. The improvements in the Sharpe Ratios were most pronounced for the conservative and moderate asset allocations.

The sizable allocations to commodities are especially noteworthy given the dramatic decrease in the expected return of commodities relative to the historical return of commodities. Of all of the asset classes, relative to historical returns, commodities received the largest reduction. The difference between the arithmetic historical return and the forecast return for commodities was over 6%. Had we continued to use equity-like return estimates for commodities, we would anticipate far larger asset allocations to commodities similar to the allocations of the historical efficient frontier that included commodities in the opportunity set (Figure 6).

We can make several general observations regarding the three forward-looking optimizations. Including commodities in the opportunity set improved the risk-return relationship of the asset allocations. All but the most extreme asset allocations associated with the ends of the efficient frontiers contained significant allocations to commodities. Including commodities in the opportunity set enabled the optimizer to increase the total equity exposure and decrease the total fixed income exposure while maintaining a target expected risk.

A Sensitivity Analysis with Low Expected Commodity Returns

Thus far, we have constructed three sets of forward-looking efficient frontiers based on three different sets of forward-looking capital market assumptions. Next, we repeat all three optimizations with one major change: the expected return of commodities is reduced to 2% above the return on collateral (Treasury Bills). The different return forecasting models produced different estimates of expected return on Treasury Bills, and hence the reduced return figures are different for each of the return forecasting models. For all of the models of expected return, the 2% reduction represents a dramatic decrease in the expected return of commodities. These changes are summarized in Table 18.

Table 18: Reduced Return Forecasts for Commodities

CAPM		Building Block		Black-Litterman	
Original Return	Reduced Return	Original Return	Reduced Return	Original Return	Reduced Return
8.05%	6.39%	8.99%	4.87%	8.18%	5.74%

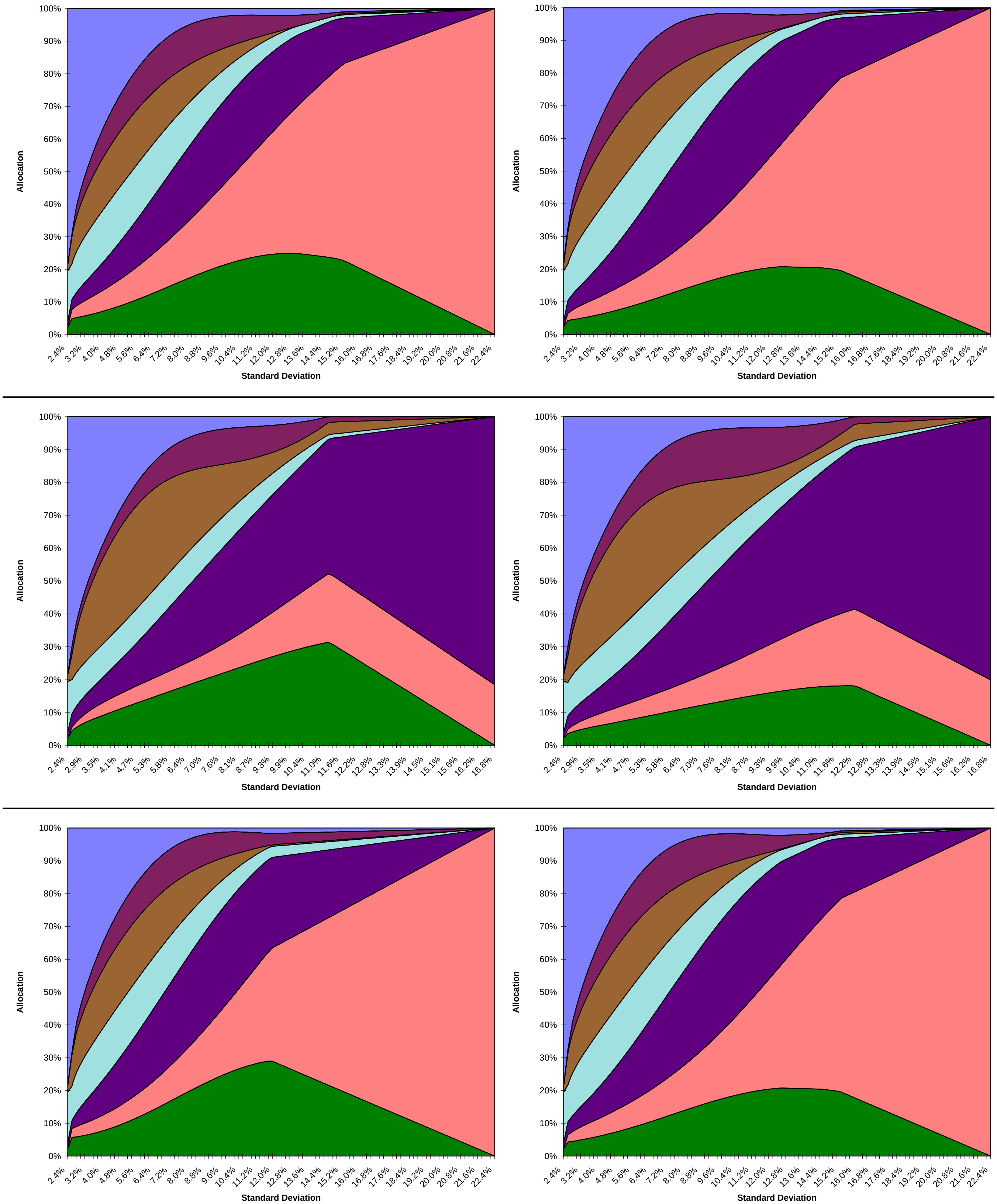
The reduction in the CAPM return from 8.05% to 6.39% could be explained by reducing the estimate of the market capitalization of commodities from $12.9 trillion to approximately $10.7 trillion. The reduction in the building block return from 8.99% to 4.87% could be explained by a lower forward-looking commodity strategy premium. The reduction in the Black-Litterman return from 8.18% to 5.74% could be explained as a combination of the previous two factors.

Figure 21 contains three panels in which each panel contains two asset allocation area graphs. The asset allocation area graphs on the left contain the original asset allocations (Figures 12, 16 and 19, respectively) and the asset allocation area graphs on the right contain the asset allocations based on reduced return expectations. Given the significant decreases in the return forecasts for commodities in Table 18, the allocations to commodities are surprisingly significant. **This suggests that allocations to commodities do not depend on continued high returns. Low correlations to traditional asset classes drive the relatively large allocations to commodities.**

In the three panels of Figure 21, the average allocation to commodities decreased from 15.0% to 12.1%, 17.6% to 11.1%, and 15.5% to 12.6%, respectively. The results of this sensitivity analysis lead us to conclude that commodities should play a significant role in the strategic asset allocation of most investors.

Figure 21: Sensitivity Analysis – Asset Allocation Comparison

Asset Allocations Based on *Original* Returns | Asset Allocations Based on *Reduced* Returns



** See Figure 19 for a legend.*

Surplus Optimization and Future Research

While our study has focused on asset-only optimization methods, it is worth noting that surplus optimization methods are receiving renewed interest.[29] Surplus optimization is an extension of the traditional asset-only approach to determining an optimal asset allocation in which the mean-variance optimizer is constrained to hold an asset class (or combination of asset classes) representing the liability short. Surplus optimization is considered the preferred approach when a portfolio of assets exists to fund a liability. Liabilities are typically modeled as a combination of TIPS, long-term nominal bonds, and perhaps a small allocation to equities. The liability model attempts to capture the systematic exposures embedded in the liability. When surplus optimization is used, the optimal asset allocations typically contain larger amounts of assets that help hedge the liability. This may include larger than normal asset-only based allocations to real return asset classes, such as commodities and TIPS. Thus, in a typical surplus optimization case, we would expect even higher allocations to commodities than allocations already presented in an asset-only optimization setting, although this ultimately depends on the liability model and the capital market assumptions.

In a similar vein, as higher moment portfolio selection models are developed and used in practice, they may shed additional light on the role of commodities in a strategic asset allocation. Gorton and Rouwenhorst [2005] documents significant skewness and kurtosis in commodity returns.

Future research is needed on two fronts. First, a greater understanding of the commodity return puzzle and the related commodity strategy premium are needed. Second, the role of commodities in the market portfolio needs to be defined.

Conclusion

We analyzed the role of commodities as represented by a fully collateralized total return index in a strategic asset allocation setting. Rather than use a single commodity index, we formed an equally weighted composite of four other total return commodity indices that we call the Composite Commodity Index. Commodities, as proxied by the Composite Commodity Index, produce the highest historical return of the asset classes that made up our opportunity set, which included six other asset classes: U.S. Treasury bills, TIPS, U.S. bonds, international bonds, U.S. stocks, and international stocks.

In addition to impressive historical returns, commodities had the lowest average correlation to the other asset classes; yet, the positive correlation to inflation supports the idea that commodities result in real inflation-adjusted returns. For three typical types of portfolios (an all U.S. equity portfolio, a 50/50 U.S. stock / U.S. bond mix, and an all U.S. bond portfolio), commodities provided positive returns when they were needed most. We also demonstrated that the correlation of commodities with other assets classes evolves over time.

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[29] See, for example, Waring [2004a, 2004b] or Siegel [2004].

Including commodities in the opportunity set resulted in a superior historical efficient frontier, which included large allocations to commodities. Over the common standard deviation range, the average improvement in historical returns at each of the risk levels was approximately 133 basis points.

We investigated the commodity return puzzle and introduced Greer's commodity pricing model. We looked at several possible decompositions of historical commodity returns and concluded that commodities have an inherent, non-skill-based return. Commodity returns come from collateral and commodity strategy premium, which likely stems from the various elements of Greer's commodities pricing model. An insurance premium, a convenience yield, and a diversification/rebalance return all contribute to the commodity strategy premium.

We developed an expanded working version of the hypothetical market portfolio that includes commodities. Doing so enabled us to create a set of expected returns based on the CAPM, a model that most practitioners believe does not apply to commodities. Under what can only be characterized as a very conservative commodity expected return estimate relative to historical commodity returns, we created forward-looking efficient frontiers *with* and *without* commodities included in the opportunity set. Using CAPM expected returns, the inclusion of commodities in the opportunity set improved the efficient frontier. Over the common standard deviation range of approximately 2.4% to 22.4%, the average improvement in return at each of the risk levels was approximately 35 basis points. Allocations to commodities ranged from 0% to nearly 25%.

Using expected returns based on the building blocks methodology led to similar results. These results were independent of market capitalization estimates. Over the common standard deviation range of approximately 2.4% to 17.2%, the average improvement in return at each of the risk levels was approximately 77 basis points. Allocations to commodities ranged from 0% to 31%.

In our third analysis, using the Black-Litterman model, CAPM expected returns were combined with expected returns based on the building blocks methodology. Once again, including commodities in the opportunity set led to an efficient frontier above an efficient frontier in which commodities were excluded from the opportunity set. Over the common standard deviation range of approximately 2.4% to 22.4%, the average improvement in return at each of the risk levels was approximately 36 basis points. Allocations to commodities ranged from 0% to nearly 28%.

In a sensitivity analysis, the three forward-looking optimizations were repeated under the assumption that the expected return on commodities will be 2% above the expected return of collateral. Despite the dramatic reductions in the expected returns of commodities, commodities continued to play a significant role in the forward-looking strategic asset allocations.

No matter which set of returns was used, including commodities in the opportunity set improved the risk-return characteristics of the efficient frontier. Furthermore, commodities played an important and significant role in the strategic asset allocations. Given the inherent return of commodities, there seems to be little risk that commodities will dramatically underperform the other asset classes on a risk-adjusted basis over any reasonably long time period. If anything, the risk is that commodities will continue to produce equity-like returns, in which case, the forward-looking strategic allocations to commodities are too low.

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The Russell 3000 Index is an unmanaged index generally representative of the U.S. market for large domestic stocks as determined by total market capitalization, which represents approximately 98% of the investable US equity market.

The Wilshire 5000 Index is an unmanaged market-weighted index that represents the total dollar value of all common stocks in the U.S. for which daily pricing information is available. The index includes only US-headquartered and US-traded issues, including common stocks, REITs, and limited partnerships, and excluding bulletin board issues.

Lehman Brothers Aggregate Index (LBAG) is an unmanaged index, considered generally representative of investment-grade fixed income securities issued within the U.S.

The Citigroup Broad Investment-Grade (BIG) Bond Index is market-capitalization weighted index and includes fixed-rate Treasury, Government-sponsored, mortgage, asset-backed and investment-grade issues (BBB- or Baa3) with a maturity of one year or longer.

The Citigroup World Broad Investment-Grade (BIG) xUS Index is a market-capitalization weighted index that tracks the performance of international (ex. U.S.) investment grade bonds.

The Citigroup Non-US Dollar World Gov't Bond Index t is a market-capitalization weighted index that tracks the performance of government bonds from approximately 22 developed countries, excluding U.S. government bonds.

The ML US Broad Market Index tracks the performance of US dollar-denominated investment grade Government and Corporate public debt issued in the US Domestic bond market, including collateralized products such as Mortgage Pass-Through and Asset Backed securities.

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The Lehman Brothers Global Real U.S. TIPS Index is an unmanaged market index comprised of all U.S. Treasury Inflation Protected Securities rated investment grade (Baa3 or better), have at least one year to final maturity, and at least $250 million par amount outstanding. Performance data for this index prior to 10/97 represents returns of the Lehman Inflation Notes Index.

The Ibbotson Associates 30-Day Treasury Bill Index tracks the performance of a one-bill portfolio containing, at the beginning of the month, the bill having the shortest maturity not less than one month.

Ibbotson Associates Synthetic TIPS Index is a synthetic inflation-indexed bond series based on historical inflation and Treasury bond yield data. It is based on the methodology developed in Chen and Terrien [2003].

The Ibbotson Associates Synthetic U.S. Aggregate Bond Index synthetic aggregate bond index that is representative of investment-grade fixed income securities issued in the U.S. It was created using the methodology explained in Coleman, Fisher, and Ibbotson [1992].

The Ibbotson Associates World Bond xUSA Composite Index is an equally weighted composite of bond and bill performance data from 14 developed countries. It is a measures the performance of international (ex. U.S.) bonds.

The Morgan Stanley Capital International (“MSCI”) Europe, Australasia, Far East Index ("EAFE") is an unmanaged index of over 900 companies, and is a generally accepted benchmark for major overseas markets. Index weightings represent the relative capitalizations of the major overseas markets included in the index on a U.S. dollar adjusted basis. The index is calculated separately; without dividends, with gross dividends reinvested and estimated tax withheld, and with gross dividends reinvested, in both U.S. Dollars and local currency.

The Standard & Poor’s 500 Composite Index (S&P 500) is an unmanaged index of U.S. companies with market capitalizations in excess of $4 billion. It is generally representative of the U.S. stock market.

It is not possible to invest directly in an unmanaged index.