Model Insight

New Implied Volatility Factors in the Barra Integrated Model

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Contents

Introducing New Implied Volatility Factors for the Barra Integrated Model (BIM)

At the heart of any valuation of a fixed income security, there is an interest rate model. For instruments with no embedded optionality, only the current term structure plays a role in valuation and risk analysis. In contrast, the uncertain character of future interest rates has a significant impact on the analysis of instruments with optionality; for example, callable bonds, mortgage pass-throughs, or explicit options like caps and swaptions.

The implied volatility factors of the Barra Integrated Model (BIM) use some of these instruments to infer the market's varying expectation of the volatility of the term structure, or alternatively, variation in the market price of risk for interest rate volatility. We can use these factors to forecast the risk of volatility-sensitive securities.

This Model Insight reviews the implied volatility factor model and introduces new implied volatility factor market coverage for Switzerland, Sweden, Canada and Australia. The complete list of markets covered by factors is:

Previously in BarraOne	New in BarraOne 3.7
USD	CHF
EUR	SEK
GBP	CAD
JPY	AUD

Understanding Implied Volatility Risk

In this section, we investigate what characteristics of an option will result in it having high risk due to implied volatility. We investigate three common assets exposed to implied volatility risk: swaptions, bonds with embedded options, and US agency backed MBS. We find that two principal causes will result in high risk due to implied volatility: The first main factor affecting the implied volatility risk is the moneyness of the option: Options that are at the money will have more implied volatility risk than options that are either in or out of the money. Options that are either way in or way out of the money have very little risk due to implied volatility. The second main factor that determines the amount of implied volatility risk is maturity: risk from implied volatility increases with maturity. Not only does implied volatility risk increase with maturity, but it increases faster than the increase in risk from interest rate moves so that at long maturities implied volatility risk can dominate interest rate risk.

Implied Volatility Risk of European Swaptions

A European swaption is an over-the-counter option that allows the holder to enter into an underlying swap contract on a specified date. The underlying asset is usually a plain vanilla swap with a maturity ranging from one to 30 years. A payer swaption gives the buyer the right to pay the fixed leg of the swap, while a receiver option gives the buyer the right to receive the fixed leg. Swaptions can be used to set a maximum rate on floating rate debt (or a minimum return on a floating rate investment), or to transform fixed-rate assets into floating rate assets when interest rates rise (and vice versa.) European swaptions in BarraOne are evaluated using a mean-reverting Gaussian model (MRG, also called Hull-White). In this section, we illustrate the risk characteristics of European swaptions while leaving aside the details of the evaluation.[1]

Because a swaption, if exercised, will lead to a sequence of cash flows, it has significant market risk arising from changes in interest rates. As a contract with a counterparty, swaptions also have spread risk which is modeled with the volatility of the swap curve. And, as an option, a swaption has market risk due to changes in the market implied volatility of interest rates.

The risk decomposition for a set of European payer swaptions on five-year par swaps with maturities ranging from two to twenty years is shown in Figure 1. Interest rate risk is significant at all swaption maturities, while spread risk makes a modest contribution to risk that declines with maturity. In contrast, implied volatility risk is modest at short maturities, but grows to become the dominant source of risk at long maturities.

To interpret this result, note that for short maturity swaptions, the most critical factor in the market price of the option is the current moneyness of the option, which depends on the shifting level of interest rates. Risk therefore comes from interest rate volatility. As maturity increases, however, future interest rates are increasingly difficult to predict from current rates and the future volatility of rates has a greater influence on the value. The risk from changes in interest rate volatility is described by the implied volatility risk, which increases with maturity.

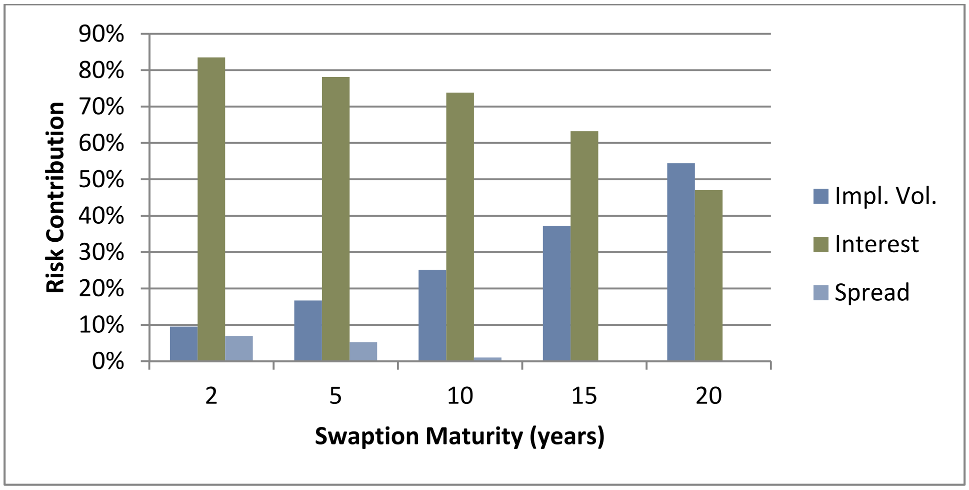
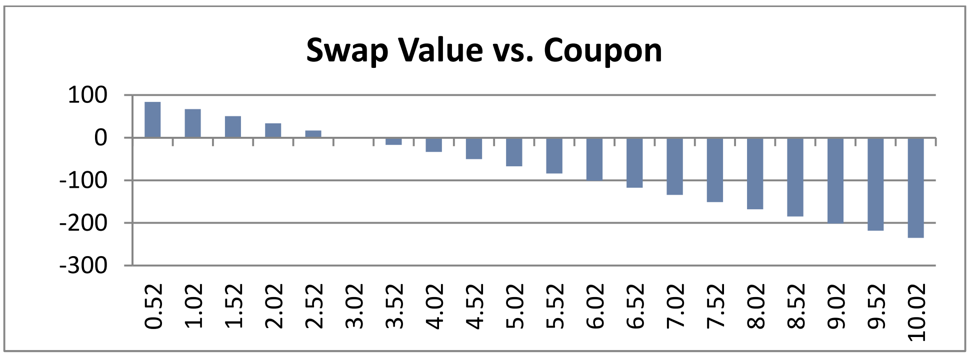


Figure 1: Risk Decomposition for European swaptions on five-year swaps. The decomposition uses the Correlation Risk Attribution scheme, in which correlation of risk factors is included in the decomposition and the decomposition sums to 100 percent.

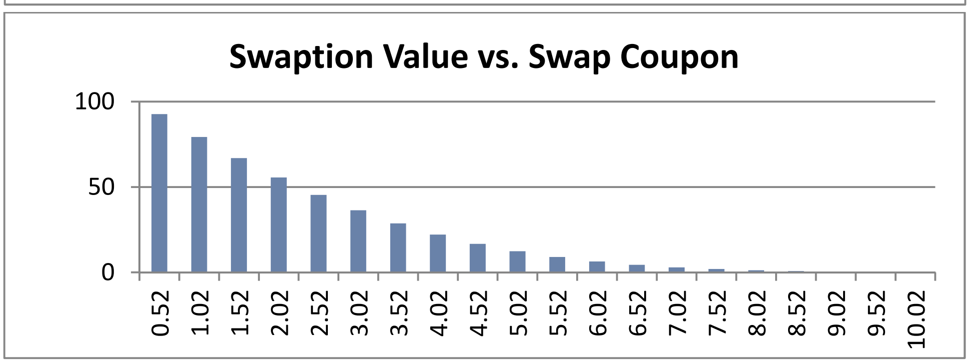
We analyzed the impact of coupon on risk for non-par swaptions by taking the par 15-year swaption from Figure 2 and varying the coupon. As shown in Figure 2A, the par coupon for the 15-year forward swap was 3.02 percent and the value of a swap on $1000 notional varies from around $100 to -$100 for coupons ranging from 0.52 percent to 10.02 percent. The value of European swaptions on these underliers ranges from 92 USD to practically zero, as shown in Figure 2B.

[1] For further details see the BarraOne Analytics Guide.

Figure 2C shows the risk of the funded swaptions.[2] The risk of the swaptions on low coupon swaps is similar to that of par coupon swap, while the risk falls off as the coupon increases. The fall off in risk is due to the declining value of the swaption, as the percentage risk of an un-funded swaption strictly increases with coupon (the dollar risk also declines). Figure 2D shows the relative contribution of interest rate and implied volatility in the risk of the swaption. For swaptions on the lowest coupon swaps, interest rate risk dominates the other sources of risk, accounting for more than 80 percent of the risk, but implied volatility increases its contribution to risk as the coupon increases. For the swaption on the par coupon swap implied volatility accounts for 37 percent of the risk (as in Figure 1), and implied volatility becomes the dominant source of risk for coupons above 5 percent. This result can be interpreted as follows: at the lowest coupon levels, the option will most likely be exercised and the main risk is to changes in the values of the swap cash flows due to changes in interest rates. For higher coupons, it becomes progressively less likely that the option will be exercised. Changes in the implied volatility of interest rates affect the likelihood of exercise and thus explain a larger relative portion of the risk at higher coupons.



A)



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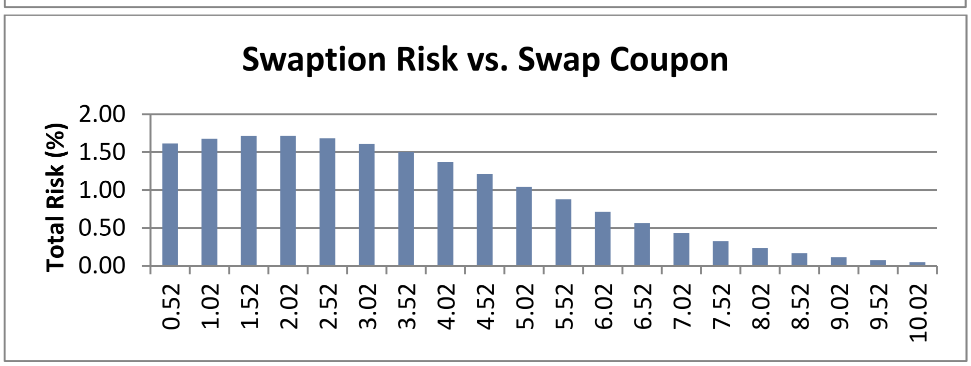

C)

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[2] A portfolio consisting of the swaption plus a cash position equivalent to the notional of the swap.

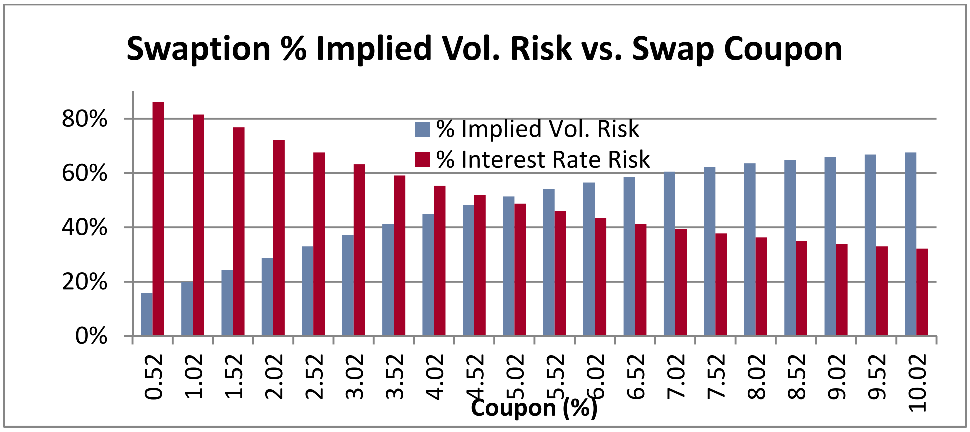
D) 

Figure 2: Risk characteristics of swaptions relative to coupon on $1000 notional.

Implied Volatility Risk of Bonds with Embedded Options

Another common asset class exposed to implied volatility risk is bonds with embedded options, such as embedded calls and puts. The value and risk exposures of bonds containing embedded options are evaluated using numerical methods for solving partial differential equations. BarraOne uses appropriate methods for evaluation based on the MRG (Hull-White) model.[1] In this section, we explain the risk exposures of such bonds while leaving aside details of the evaluation. To illustrate implied volatility risk in optionable bonds, we analyzed bonds from the Merrill Lynch Bank of America Global Broad Market Corporate index. As of March 2013, this index contains 10,045 bonds, and of these 1,141 (11.4 percent) have embedded options. As shown in Figure 3, the largest number of such bonds is in the US dollar bond market, where 14 percent of all bonds in the index contain an embedded option and carry some degree of risk from implied volatility. Of those bonds with embedded options 97 percent contain embedded calls and 3 percent contain embedded puts.

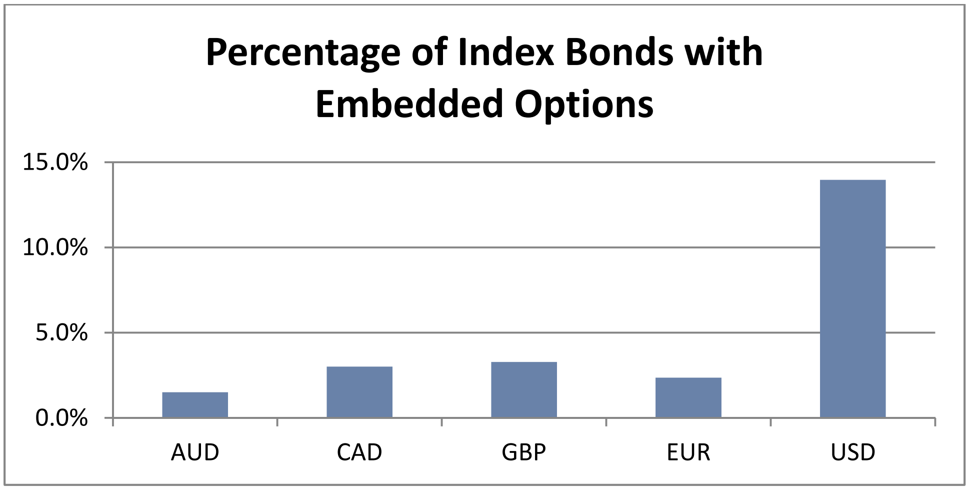


Figure 3: Percentage of bonds, by market, in the ML BofA Global Broad Market Corporate Index containing embedded options.

For those bonds with embedded options we compared the risk from implied volatility to the interest rate risk from term structure level changes. We found that only a small number of bonds with embedded options have a significant risk from implied volatility. As shown in Figure 4, 91 percent of all of the index bonds with embedded options have implied volatility risk that is less than 1 percent the amount of the interest rate structure risk. At the same time, some bonds receive a larger amount of risk from implied volatility: implied volatility risk relative to interest rate risk ranges up to around 45 percent.

What accounts for the fact that many corporate bonds with embedded options receive such a small fraction of their term structure risk from implied volatility? And what characterizes those bonds that receive more risk from implied volatility? To address these questions we analyzed these bonds along the dimensions of option value and time to maturity.

The first aspect of an embedded option bond that determines the amount of implied volatility risk is the value of the embedded option. To illustrate the impact on implied volatility risk, the value of the embedded option is plotted versus the ranking in amount of implied volatility risk in Figure 5 (i.e., those bonds with the most implied volatility risk are plotted on the left, those bonds with the least implied volatility risk are plotted on the right).

The individual option values are highly variable, but the smoothed curve shows a distinctive “J” shape bonds with significant implied volatility risk have a *moderate* option value, in the range of around five to 20; bonds with little implied volatility risk have very little option value; but bonds with the *least* implied volatility risk of all tend to have very *high* option value.

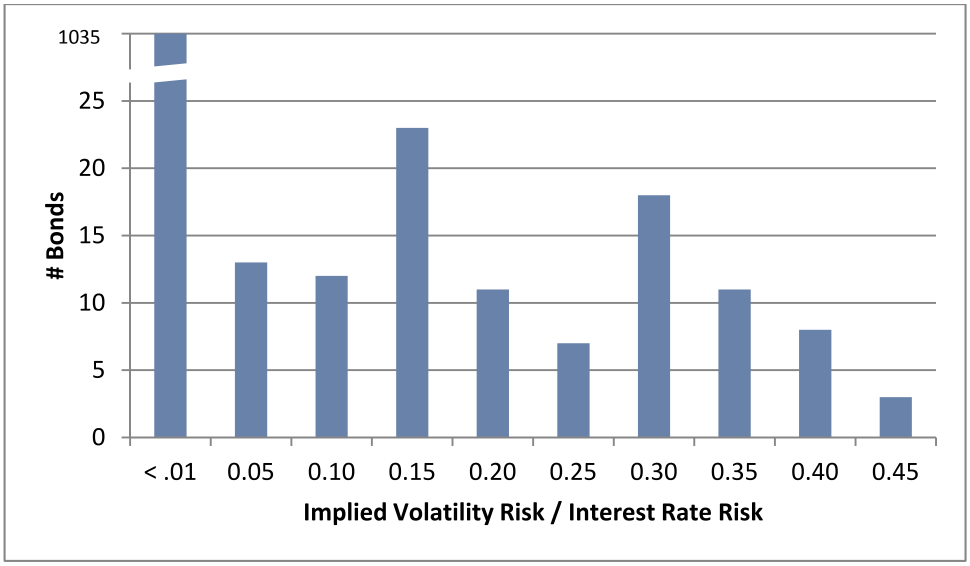


Figure 4: Histogram showing the risk from implied volatility relative to the interest rate risk for bonds having embedded options in the ML BofA Global Broad Market Corporate Index.

Although this relationship is complex, it is fairly intuitive: risk due to changes in the volatility of interest rates is greatest when an option is likely to be exercised, but it is not certain. That is, the option is at the money, or moderately in or out. In that case, changes in the volatility of rates may decisively affect the likelihood of option exercise. But if an option is extremely unlikely to be exercised in the current rate environment, and thus has little value, changes in the volatility of rates will not immediately make it more likely to be exercised. At the same time, if an option is certain to be exercised at current rates, (i.e., it is far in the money and has high value), then changes in rate volatility will also have little impact on the value and thus incur little risk.

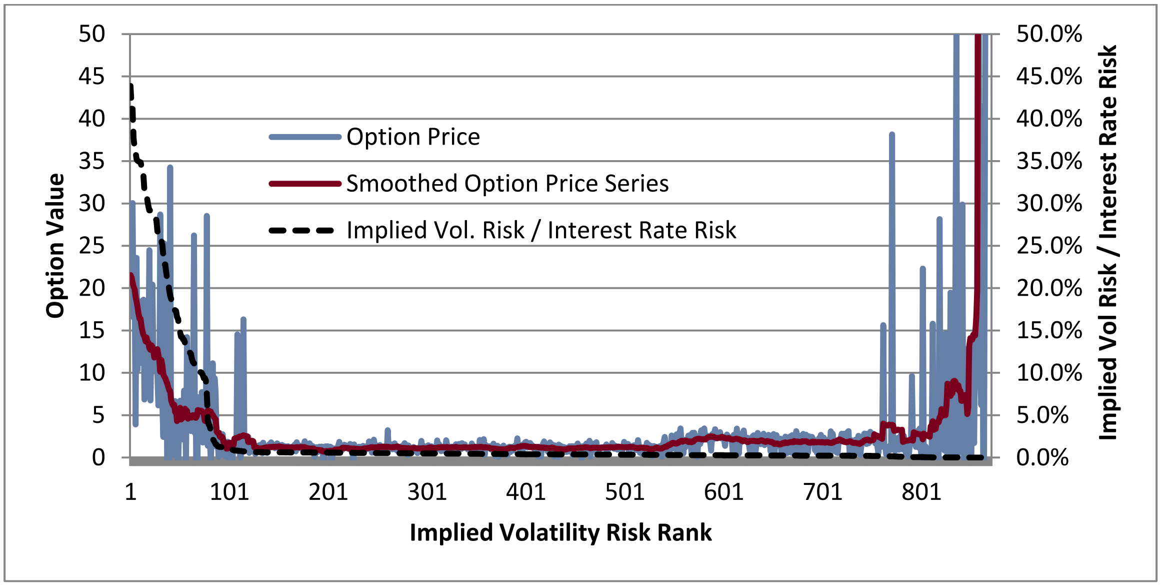


Figure 5: Price for callable bond options in the ML BofA Global Broad Market Corporate Index plotted in rank order of the amount of risk from implied volatility.

For bonds with a significant amount of implied volatility risk (implied volatility risk > 1 percent of interest rate risk), the option value explains a significant amount of the variation in implied volatility risk, as illustrated in Figure 6A. A regression shows that the option value explains 31 percent of the variability in the amount of implied volatility risk.

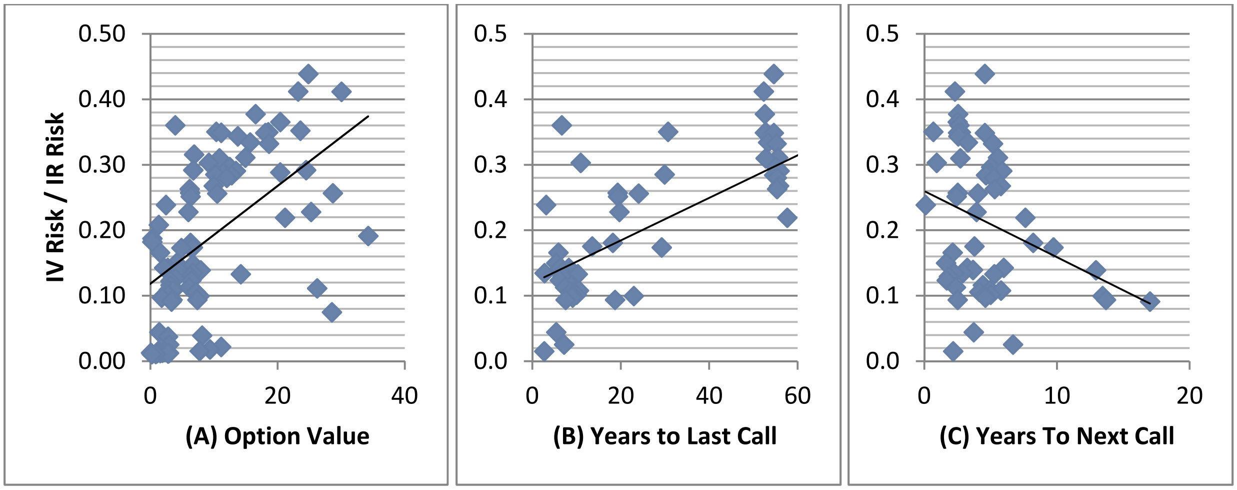


Figure 6: Explaining the variation in the implied volatility risk of callable bonds in the ML BofA Global Broad Market Corporate Index. Only bonds with implied volatility risk / Term structure risk greater than 1 percent are included in the analysis. A) Option Value; regression r-squared=0.31; B) Years to last call date; regression r-squared=0.47; C) Years to next call date; regression r-squared=0.09.

What other factors explain differences in the amount of implied volatility risk for callable bonds? The example of par swaptions having different maturities (see Figure 1) suggests one factor: the lifetime of the option. As illustrated in Figure 6B, the number of years until the *last* call date is an important variable in explaining the implied volatility risk of callable bonds. Regression shows that the years remaining until the last call explains 47 percent of the variability in implied volatility risk for those bonds where the implied volatility risk is at least 1 percent of the interest rate risk. A final factor, illustrated in Figure 6C, is the time until the next call date. Although the explanatory power is weak (regression R-squared = 9 percent), it does show that a long delay before the next call date tends to reduce the risk from implied volatility.

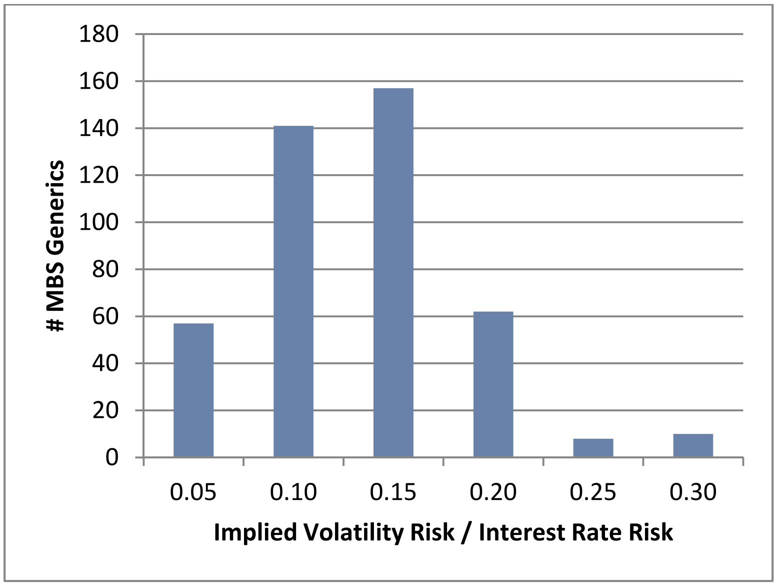


Figure 7: Histogram showing the risk from implied volatility relative to the interest rate risk for MBS generics in the Merrill Lynch Bank of America Mortgage Master Index, as of April 2013.

Implied Volatility Risk of US Agency Mortgage Backed Securities

A third asset class exposed to implied volatility risk is US agency mortgage backed securities (MBS), which contain a prepayment option for the borrower. We analyzed implied volatility risk in the Merrill Lynch Mortgage Master Index to demonstrate the main factors affecting the level of implied volatility risk. As shown in Figure 7, the level of implied volatility rate risk is more normally distributed in MBS than in bonds with embedded options, although still skewed. In contrast to bonds with embedded options, the majority of MBS have a non-trivial level of implied volatility risk. The level of implied volatility risk is primarily affected by the moneyness of the option and the remaining maturity in a similar way to swaptions and bonds. As shown in Figure 8A, the moneyness of the prepayment option can be measured by the coupon for fixed coupon MBS generics with original terms of 30 years. At any given time there is only a limited variety in the coupon rate and pre-payment terms, leading to the more normal distribution of implied volatility risk in comparison to bonds with embedded options. Figure 8B shows the impact of maturity on implied volatility risk – as for swaptions and bonds with embedded options, increasing time to maturity increases the risk of implied volatility.[3] An MBS close to maturity has very little risk from implied volatility. Figure 9 shows the impact of maturity in another way, plotting implied volatility risk versus the original terms of the mortgages: a longer original term is associated with more risk from implied volatility.

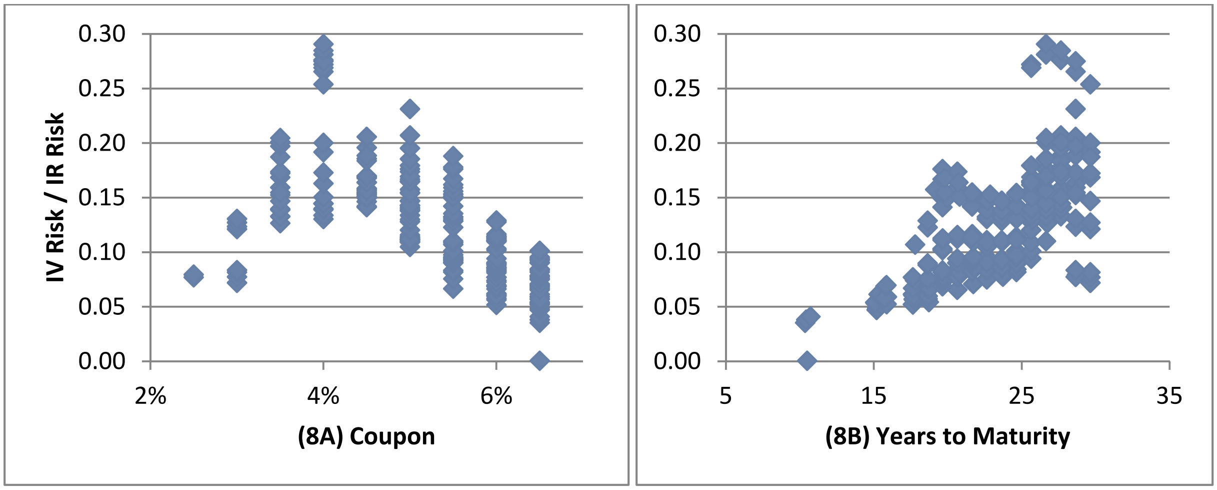


Figure 8: Risk from implied volatility relative to the interest rate risk for MBS 30-year generics in the Merrill Lynch Bank of America Mortgage Master Index, plotted versus coupon (a) and maturity (b), as of April 2013.

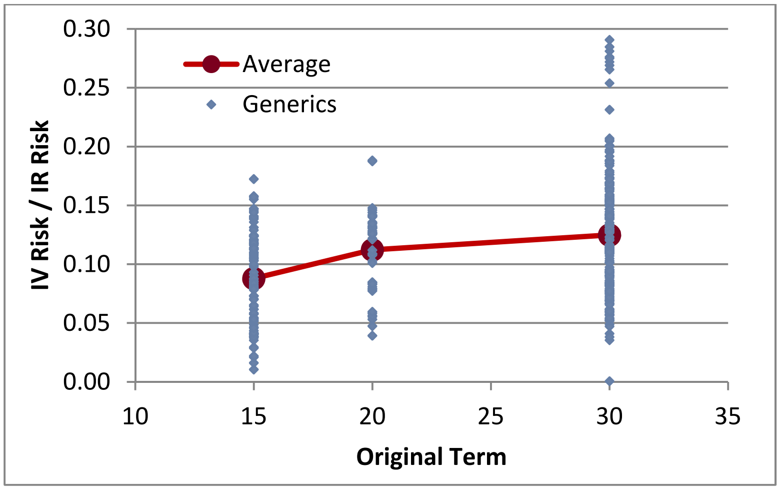


Figure 9: Risk from implied volatility relative to the interest rate risk for MBS generics in the Merrill Lynch Bank of America Mortgage Master Index, plotted by original term.

Considering the implied volatility risk versus the mortgage coupon shown in Figure 8A, it should be noted that the distribution of implied volatility risk is not even approximately linear in the coupon but has a peak at 4 percent. For comparison, Figure 10A shows the same data from January 2004, at which

[3] Note that at the time of this writing, older mortgages have a higher coupon and are also closer to maturity, so Figure 8B actually reflects *both* maturity and moneyness effects.

point the implied volatility risk peaks at around 7.75 percent. In both cases, implied volatility risk peaks *near* the current coupon, which for April 2013 is around 2.5 percent, and in January 2004 was around 5 percent. One reason the peak in implied volatility risk is not *at* the current coupon is that there is a seasoning effect in pre-payment behavior; new mortgages that are near the current coupon may be at the money in terms of the coupon, but mortgage holders usually cannot pre-pay in the first six months of a new mortgage regardless of the coupon. Other reasons the peak in implied volatility risk is not at the current coupon is that borrowers required a significantly positive incentive to refinance and that for many borrowers to get the best rate requires an upfront fee ("points") that makes such borrowers less likely to prepay.

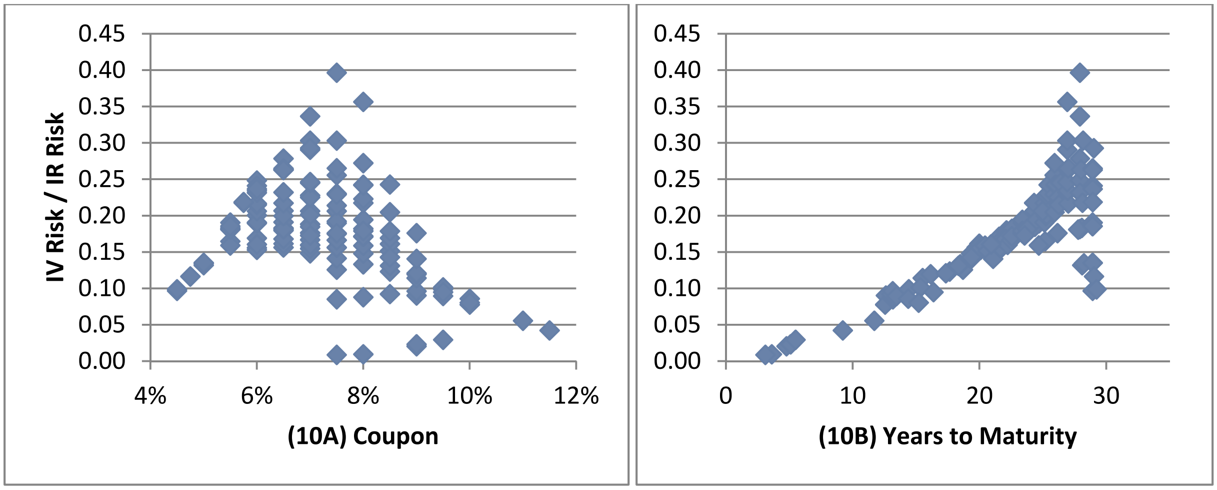


Figure 10: Risk from implied volatility relative to the interest rate risk for MBS 30-year generics in the Merrill Lynch Bank of America Mortgage Master Index, plotted versus coupon (left) and maturity (right), as of January 2004.

Modeling Implied Volatility Risk

Estimating the Risk Factors

The basic idea is to calibrate a stochastic interest rate model to match observed market prices of interest rate options. Then the variation over time of the calibration constants gives rise to new risk factors. More specifically, in each covered market, Barra's implied volatility factor is based on the log return of the market-implied volatility of the 10-year yield.

In BarraOne, the basic term structure model is a mean-reverting Gaussian model (MRG, also called Hull-White). It assumes that increments of the short rate are normally distributed with reversion to a long-term mean. The inputs are the initial term structure, the volatility σ of the short rate, and the mean-reversion parameter κ. One of the computational advantages of the MRG model is that it admits a closed form for the price of European swaptions.[4]

[4] See, for example, D Brigo and F. Mercurio, Interest Rate Models – Theory and Practice, Springer 2006.

The initial term structure is the LIBOR/swap curve, which is readily available from a variety of data sources. The parameters σ and κ, which determine the model's term structure of volatility, are fit to the market prices of swaptions with various tenors and expiries. The MRG model can be used to calculate the volatility of all spot rates and yields. For forward rates, the term structure of volatility is exponential. It follows that the term t spot rate volatility has the form:

$$\sigma \frac{1 - e^{-\kappa t}}{\kappa t}$$

Yield volatilities are determined as well, but do not have such a simple expression. The yield volatility is calculated using the relationship from Ito's Lemma:

$$Y(r,t,T)\sigma_{Y(r,t,T)} = r\sigma_r(r,t)\frac{\partial Y(r,t,T)}{\partial r}$$

The derivative $\partial Y/\partial r$ is evaluated analytically and σ_r is the fitted short rate volatility from the model.

Note that the market-implied volatility parameters are used only to specify the new volatility factor returns. The parameters delivered to the application and used elsewhere in the risk model are based on a best fit of the MRG model to the historical covariance matrix of spot rate changes.

Calculating Exposures

Exposure of a security to the appropriate implied volatility factor is determined by numerical differentiation, analogously to effective duration. If we denote the factor by X, the exposure of a security is the percentage change in price per unit increase of X. In practice, one computes this derivative in two stages, using the natural parameter σ of the model, as

$$D_{IV} = \frac{1}{P}\frac{\partial P}{\partial X} = \frac{1}{P}\frac{\partial P}{\partial \sigma}\left(\frac{\partial X}{\partial \sigma}\right)^{-1}$$

The factor X is the logarithm of the 10-year yield volatility. This was chosen to capture volatility of the portion of the term structure relevant for most optionable bonds and MBSs, as well as for analytic tractability.

Implied Volatility Model Results

Factor Volatilities and Exposures

Figure 11 shows the annualized volatilities of the implied volatility factors, including how the factors spiked during the market turmoil of 2008-2009. Most of these volatilities have nearly returned to pre-crisis levels, using the 52-week half life of the Barra Integrated Model (BIM) version 301L. Notable exceptions are Japan and Canada. In Japan implied volatility only responded in a muted way to the events of 2008-2009, but it has gradually increased since then in response to Japanese central bank actions and market conditions. In Canada, the response to 2008 market events was significant but less than in the larger markets; however, Canadian implied volatility factor volatility is at a peak at the time of this publication. This resulted from numerous factors beginning in late 2012, including the weakening of the Canadian dollar on weak growth forecasts and the departure of Canada's central bank governor,[5] which pushed Canadian implied volatilities to their lowest levels in years (the movement *increases* the implied volatility factor volatility).

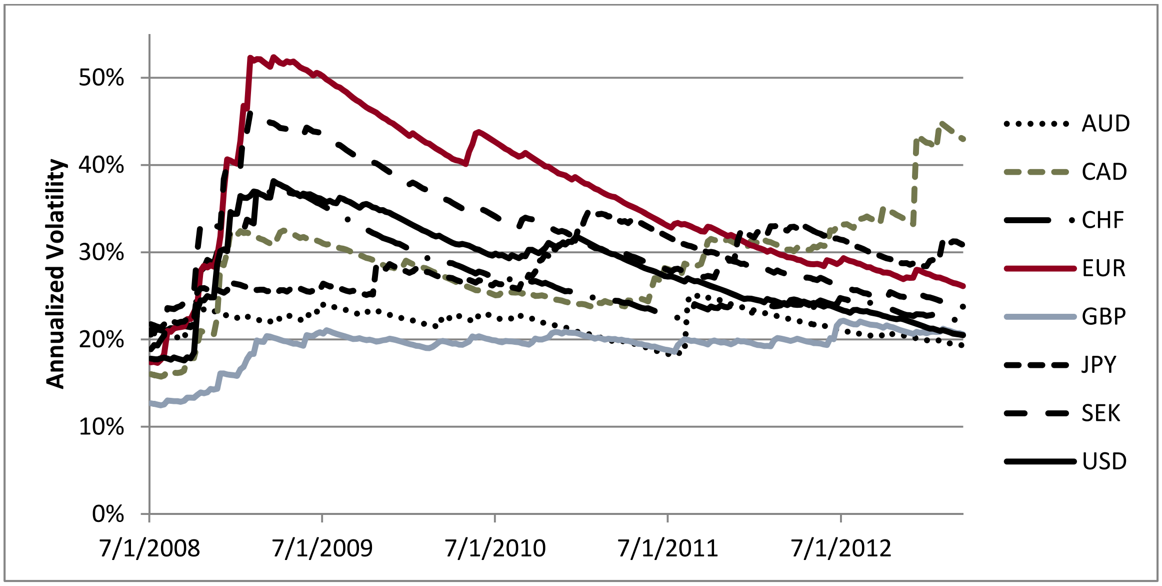


Figure 11: Volatility of the Implied Volatility Factors. Volatility is calculated using weekly returns and a 52-week half life.

It is worth noting that the implied volatility factors have large volatilities compared to the standard fixed income factors, but typical asset exposures are much smaller. For comparison, typical fixed income factors (i.e., factors for interest rates and spreads) have annualized volatility in the range of a fraction of a percent up to a few percentage points. Exposures to standard fixed income factors are on the order of typical bond durations, e.g., 1-10. But in the sample of bonds with embedded options described above (in the section "Implied Volatility Risk of Bonds with Embedded Options") the average absolute implied volatility exposure is only 0.005; the standard deviation of the absolute implied volatility exposure is 0.016 and the maximum absolute implied volatility exposure in the entire sample is 0.13. The different scale for implied volatility factor volatilities is a result of defining the factors simply from the underlying analytics.

Note that the implied volatility exposure of a *callable* bond is negative, as the embedded option increases in value with increasing implied volatility, to the detriment of the bond holder. Similarly for an MBS, the exposure is negative due to the prepayment option held by the mortgage holder. Bonds with embedded puts have positive exposure to the implied volatility factors, as the value to the bond holder increases with increasing implied volatility.

[5] See e.g. http://www.bloomberg.com/news/2012-11-26/canadian-dollar-falls-from-two-week-high-as-risk-appetite-ebbs.html and http://www.bloomberg.com/news/2013-01-25/canadian-dollar-falls-to-6-month-low-before-price-data.html .

Factor Correlations

An important characteristic of implied volatility factors is their correlation with interest rate factors, as assets with implied volatility exposure usually have significant interest rate exposure as well. Table 1 shows the correlation of implied volatility factors and interest rate level factors in each market. The implied volatility factors tend to be negatively correlated with interest rate factors in their respective markets, or else not significantly correlated at all. This says that a positive return from rates (corresponding to a drop in rates) tends to be accompanied by a negative return in implied volatility (a decrease in implied volatility so the option values decrease). Since our implied volatility factor is based on an absolute, rather than relative, volatility, this accords with standard market intuition: as the yield curve drops, so does the absolute level of volatility. Note that in these situations the relative level of volatility can (and often will) rise.

Market	Correlation of Interest Rate and Implied Volatility Factors	Market	Correlation of Interest Rate and Implied Volatility Factors
AUD	-0.002	GBP	-0.203
CAD	-0.268	JPY	-0.557
EUR	0.040	SEK	0.048
CHF	-0.012	USD	-0.321

Table 1: Correlation of Implied Volatility and Interest Rate Factors, as of 2/28/2013.

Table 2 shows the correlation of implied volatility factors between markets. These correlations tend to be lower than other types of fixed income factors. For comparison, the correlations between interest rate factors in the same markets are shown in Table 3. The conventional interest rate factors in Table 3 mostly have correlations between 0.5 and 0.7, with Japan the only exception in being somewhat less correlated with the other markets. In contrast, the implied volatility factors typically have correlations of around 0.3. The highest correlation between implied volatility factors is between GBP and USD (0.47) and the Euro (0.43). The implied volatility factors for AUD, CAD and CHF have particularly weak correlations with those in other markets, while the Euro, GBP, JPY, SEK and USD are somewhat more correlated as a block.

	AUD	CAD	CHF	EUR	GBP	JPY	SEK	USD
	1	0.03	0.05	0.10	0.09	0.06	0.07	0.09
	0.03	1	0.07	0.12	0.20	0.16	0.10	0.20
CHF	0.05	0.07	1	0.21	0.22	0.15	0.16	0.21
EUR	0.10	0.12	0.21	1	0.43	0.29	0.33	0.39
GBP	0.09	0.20	0.22	0.43	1	0.38	0.33	0.47
JPY	0.06	0.16	0.15	0.29	0.38	1	0.23	0.34
SEK	0.07	0.10	0.16	0.33	0.33	0.23	1	0.31
USD	0.09	0.20	0.21	0.39	0.47	0.34	0.31	1

Table 2: Correlation of Implied Volatility Factors in different Markets, as of 2/28/2013.

	AUD	CAD	CHF	EUR	GBP	JPY	SEK	USD
AUD	1	0.67	0.56	0.56	0.59	0.39	0.64	0.63
CAD	0.67	1	0.61	0.60	0.69	0.45	0.65	0.74
CHF	0.56	0.61	1	0.63	0.60	0.39	0.65	0.62
EUR	0.56	0.60	0.63	1	0.65	0.36	0.75	0.59
GBP	0.59	0.69	0.60	0.65	1	0.45	0.71	0.67
	0.39	0.45	0.39	0.36	0.45	1	0.39	0.44
	0.64	0.65	0.65	0.75	0.71	0.39	1	0.65
	0.63	0.74	0.62	0.59	0.67	0.44	0.65	1

Table 3: Correlation of interest rate level factors in different Markets, as of 2/28/2013.

Conclusion

Implied volatility factors capture the risk to interest rate options due to changes in the volatility of market implied interest rate volatility. In the options that we analyzed, we found that while implied volatility risk is small for short maturity options and those that are far out of the money, this risk takes on increasing importance and can even dominate interest rate risk for a long maturity option that is *at* the money. The moneyness of the option and remaining time to maturity also explained much of the variation in implied volatility risk for the bonds with embedded options and the mortgage backed securities with borrower pre-payment options. Due to how they are defined, the implied volatility factors have relatively high volatility compared to other fixed income factors, while assets have comparatively much smaller exposures. In the time period that we analyzed, we found that returns to implied volatility are weakly correlated across markets and tended to be negatively correlated with returns to interest rates. The BarraOne implied volatility factors can model this risk for the major developed markets and are a key component of the Barra Integrated Model for identifying such market risk.

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[1] *As of September 30, 2012, as published by eVestment, Lipper and Bloomberg on January 31, 2013* *Apr 2013*