**Handling Risk On/Risk Off Dynamics with Correlation Regimes and Correlation Networks**

Jochen Papenbrock
PPI AG, Frankfurt
Think Tank Firamis
E-Mail: j.papenbrock@gmx.de

Peter Schwendner
Zurich University of Applied Sciences
Center for Alternative Investments and Risk Management
E-Mail: peter.schwendner@zhaw.ch.

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*Abstract*: In this paper, we present a framework to detect distinct correlation regimes and to analyze the emerging state dependences for a multi-asset futures portfolio from 1998 to 2013. Since the financial crisis in 2008, these correlation regimes are significantly different than before. Cluster tracking shows that asset classes are less separated since 2008 than before. We identify distinct "risk on" and "risk off"-assets with the help of correlation networks. Apart from visualization, we quantify these observations using suitable metrics for the clusters and correlation networks.

Keywords: Regime switching, correlation regimes, clustering, correlation networks

JEL Classification: C32, C14, G11

INTRODUCTION

Since the global financial crisis, “correlations jump to one” has become the most widely used phrase in describing asset price behavior. This can be explained by a new market environment which oscillates between two regimes: “risk on” and “risk off” (hereafter referred to as RoRo). When risk appetite is high, commodities, equities, and high-yielding currencies tend to rally together. When risk appetite fades, those assets fall and there is a flight to safe-haven assets. This is in contrast to the pre-crisis dynamics wherein high correlations were only associated with market contractions (see Fengler and Schwendner [2004]).

As a result, in the period from mid-2007 to the beginning of 2013, most markets and asset classes were highly synchronized, despite the stock market recovery and bond rally from March 2009 until March 2013 that was accompanied by decreasing volatilities.

The long-term positive drift of equities and bond prices happened despite their negative short-term return correlation.

**Consequences of RoRo for the Investment Process**

This bipolar attitude toward risk of market participants causes difficulties in the conventional investment processes. Examples are the following:

1. Due to the extreme market coordination there is a lack of diversification potential so diversification fails where it is most needed. For example, after the Lehman default in September 2008, not only equities, but also commodities, credit and most hedge funds declined in value substantially or even became illiquid.

2. Since asset classes “have lost their identity”, Flood [2012], fundamental asset class evaluations are dominated by correlated movements triggered by macro events, Lee [2012]. Also, it is hard to add value since there is little differentiation within asset classes.
3. Parameter estimation of risk and expected return is a challenge more than ever.
4. Former relative price relations become ineffective. Factor sensitivities of individual stocks relevant for quantitative investment strategies become more and more similar, therefore decreasing the value of these models, Lee [2012].
5. Strategies betting on relative changes between assets struggle with the new market synchronization and with the stress in the money market, Khandani and Lo [2007]. On the other hand, increased correlations offer new opportunities for pair trading, Stopford [2012].
6. Fundamental forecasting is nearly impossible as macro events suddenly arise and have different regional input.

It becomes more difficult to have persistent success in investment strategies, and there will be a wider dispersion of potential portfolio outcomes and a higher degree of portfolio concentration.

**How to deal with RoRo?**

Using a simple correlation matrix approach for the identification of the RoRo dynamics has drawbacks. First, a correlation heatmap’s visual outcome can vary according to the ordering of assets/markets. Second, the short time window used to compute the correlation matrix would introduce statistical noise to the result (Laloux et al [1999], Plerou et al [2002], Fengler, Pilz and Schwendner [2007]).

An indirect method to deal with RoRo is estimating the driving factors of RoRo. According to Lee [2012], latest risk-based portfolio construction approaches estimate the factor structure, time the factor returns and dynamically change portfolio exposures. Carrying the argument to the extreme case, the only skill that should be rewarded when correlations go to one is factor timing. However, there are some difficulties with this method:

- The breadth of decisions is relatively limited and it has to be relied on the quality of factor information.
- It is not clear what the right number of factors is and what to do in time of crisis where the number of factors may even reduce. Also, if the risk factor approach is fully executed it converges to the asset class approach. However, specifics of asset classes have dissolved since Lehman, as we have learned.
- The question of finding the right factor timing is open.

Finding the risk factors statistically has the advantage that no assumptions about which variables correspond to which factors have to be made and factors are directly defined from the return time series instead. Other statistical approaches, HSBC [2010] assign the first principal component of the correlation matrix to the RoRo factor. The authors use the correlation between an individual asset and the first principal component to classify this asset:

1. If the correlation between the asset returns and the RoRo factor is high and positive, we have a typical risk-on asset.
2. If the correlation between the asset returns and the RoRo factor is high and negative, we have a typical risk-off asset.
3. If there is only low correlation, is is a RoRo neutral asset.

This analysis can be dynamic according to rolling correlation windows. Also, it uses the relevant parts of noisy correlation matrix. Finally, a breakdown of the usual relations could be identified indicating the end of the RoRo paradigm.

In the next section we introduce cluster-based identification of different correlation regimes. Conditional on each regime there is a typical conditional correlation structure and it is also known in which time each regime was valid. From the regimes and their timing we can deduce early warning signals, correlation stress test design, proper model calibration of portfolio optimization programs, and outlooks of correlation structures. Also, we identify a typical RoRo-regime and use network technologies to find the most typical Risk on or Risk off assets. This information together with the correlation regimes can be the basis for trading strategies based on the RoRo phenomenon and the specific recovery pattern of markets. The network based representation also gives insight into "the most similar" to a specific market. This is interesting for proxy hedge purposes. Finally, we analyze the cluster structure of each regime and track the merging, splitting, birth and death of the clusters in time. This could be the basis for pairs-like trading strategies but with pairs corresponding to clusters. Specifically, it can be traded on certain reversal trends in the correlation structure.

## Correlation Regimes and Correlation Networks for handling the RoRo

We use techniques which can detect changes in the correlation structure of several markets in very short times scales like few trading days. The resulting correlation structure switches are be the basis for finding several distinct correlation regimes. Similar to HSBC [2010], we have analyzed 25 different markets covering 4 different asset classes: government bond futures, equity index future, commodity futures and foreign exchange futures (see Table 1) in the time period 01/07/1998 to 31/01/2013 which covers several financial crises and recoveries. In contrast to HSBC, we rely purely on time-series of rolled futures contract instead of a mixture of futures and cash instruments. This makes the return computations across asset classes more comparable and the correlation computations more reliable. The influence of different currency denominations on futures returns is only of second order, as these instruments do not involve a funding component.

| Bloomberg Ticker | Longname | Asset Class | Color |
|---|---|---|---|
| TY1 Comdty | US T-Note | gov bond future | orange |
| G 1 Comdty | Gilt | gov bond future | orange |
| JB1 Comdty | JGB | gov bond future | orange |
| RX1 Comdty | Bund | gov bond future | orange |
| XM1 Comdty | Aussie bond | gov bond future | orange |
| CN1 Comdty | Canada bond | gov bond future | orange |
| SP1 Index | S&P500 | equity index future | green |
| Z 1 Index | FTSE | equity index future | green |
| NK1 Index | Nikkei | equity index future | green |
| VG1 Index | EuroSTOXX50 | equity index future | green |
| GX1 Index | DAX | equity index future | green |
| HI1 Index | Hang Seng | equity index future | green |
| BZ1 Index | Bovespa | equity index future | green |
| HG1 Comdty | Copper | commodity future | yellow |
| GC1 Comdty | Gold | commodity future | yellow |
| CL1 Comdty | Oil | commodity future | yellow |
| NG1 Comdty | Natural Gas | commodity future | yellow |
| S1 Comdty | Soybean | commodity future | yellow |
| W1 Comdty | Wheat | commodity future | yellow |
| BP1 Curncy | GBP | fx future | blue |
| JY1 Curncy | JPY | fx future | blue |
| EC1 Curncy | EUR | fx future | blue |
| AD1 Curncy | AUD | fx future | blue |
| CD1 Curncy | CAD | fx future | blue |
| RA1 Curncy | ZAR | fx future | blue |

**Table 1: Futures markets used to compute the correlation matrices**

We identify correlation regimes with the following six steps:

1. Split the data set of daily returns into 175 subsets each containing the daily returns for each of the 175 corresponding months. Each of the 175 matrices defines a fundamental correlation time slice of the markets.
2. Compute the 25x25 correlation matrices for each of the 175 months.
3. Transform the 175 correlation matrices to 175 correlation based distance matrices.
4. Filter each correlation distance matrix with a hierarchical clustering technique due to its high noise. That noise is created by the relatively short estimation windows, Laloux [1999].
5. Compute the pair-wise similarity between the filtered correlation distance matrices (similarity is measured in terms of their correlation structure). The result is a 175x175 matrix with pairwise similarities of all filtered correlation distance matrices.
6. Find discrete state clusters whereas each cluster contains very similar monthly time slices, in terms of their correlation structure. We use the well-known k-means clustering with k=5 clusters. The choice of 5 clusters was driven mainly by the idea to have a low number of clusters for simplicity on the one hand and to have economically meaningful clusters on the other. The latter is made sure by sanity checking the distribution and number of clusters in good correspondence to a “market fear” index.

Figure 1 shows the occurrence of the k=5 clusters in time, laid over the VIX index as black squares. The VIX index measures the implied volatility of S&P 500 index options and is often gauged as a “fear index”.

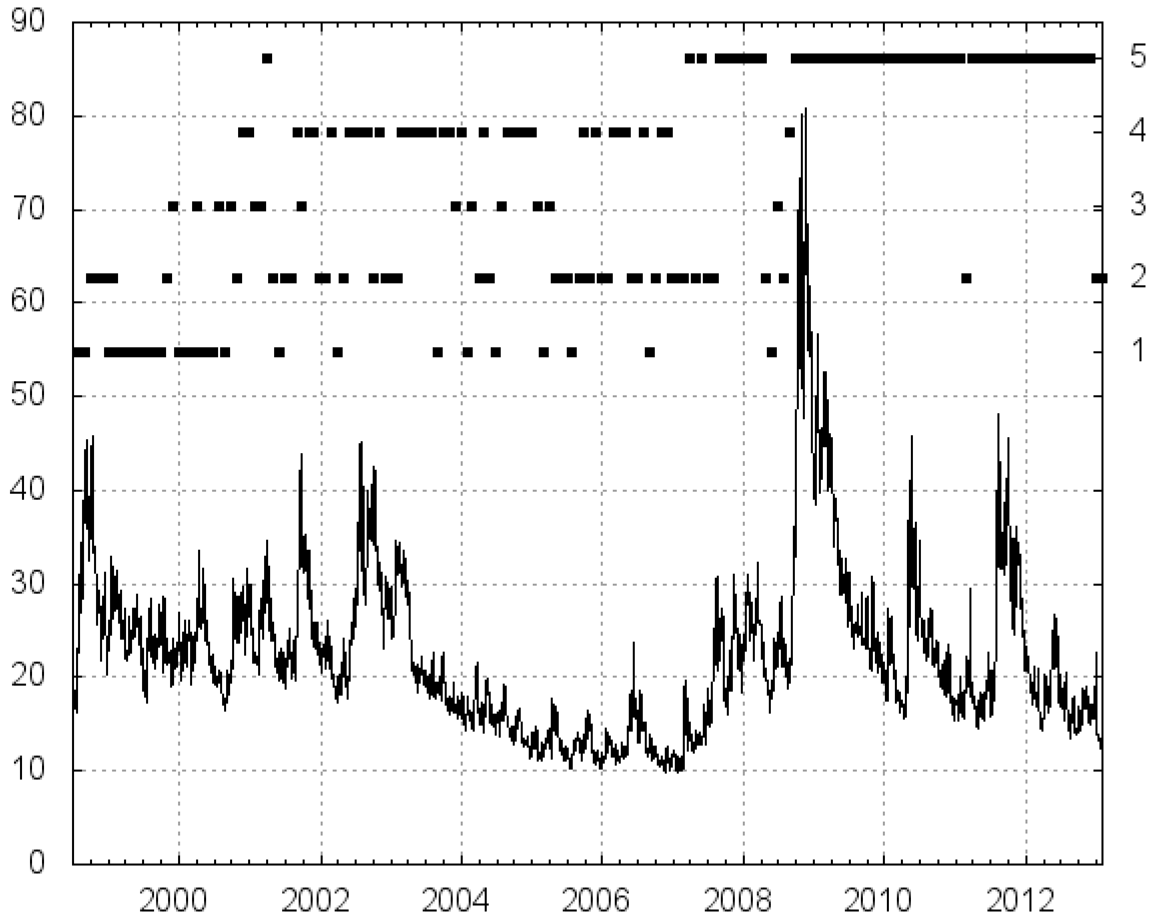



**Figure 1: the chart shows the VIX index from 01/07/1999 to 31/01/2013. The little squares each represent one of the 175 months covered by the period under investigation. The 5 different levels of the squares represent the 5 different correlation regimes, designated on the right hand side.**

This procedure is similar to Münnix et al [2012] who use the original noisy state correlations and then use a state distance/dissimilarity measure that strongly suppresses noise. Instead, we first filter the state correlations by extracting a hierarchical cluster structure and then use a state distance/dissimilarity measure that matches the hierarchical data structure.

In step 3 we choose a widespread correlation based distance. This is the necessary input for the filtering based on hierarchical clustering. The correlation based metric has found widespread use among practitioners of clustering in financial applications (see for example Lisi and Corazza [2008], Mantegna [1999], Tola et al [2008], Tumminello, Lillo and Mantegna [2010], Dose and Cincotti [2005]. The Pearson correlation coefficient is widely used as a measure of strength of linear dependence between two variables:

$$\rho_{ij} = \frac{\langle r_i(t,\Delta t) r_j(t,\Delta t)\rangle_T - \langle r_i(t,\Delta t)\rangle_T \langle r_j(t,\Delta t)\rangle_T}{\sqrt{\langle r_i^2(t,\Delta t)\rangle_T - \langle r_i(t,\Delta t)\rangle_T^2}\sqrt{\langle r_j^2(t,\Delta t)\rangle_T - \langle r_j(t,\Delta t)\rangle_T^2}} \qquad where\, i, j = 1,\dots,n$$

where n is the number of markets, i and j label the markets, T is the horizon, $r_i(t,\Delta t)$ is the return of market i in the subhorizon $[t,\Delta t]$.

However, the correlation coefficient of a pair of asset returns cannot be used as a distance because it does not fulfill the axioms that form a metric. A real metric can be designed using a function of the correlation coefficient $\rho$. It can be rigorously determined by a transformation of the correlation coefficient so that the distance between variables decreases if correlation between them increases (Gower [1966]):

$$d(i,j) = \sqrt{2(1-\rho_{ij})}$$

It can be shown that this distance fulfills the usual metric properties including the triangle relation (see e.g. Mantegna [1999]).

In step 4, the correlation distance matrices are input for a hierarchical clustering called average linkage (see Tola et al [2008]). Their study compares the following four procedures in terms of portfolio risk and weight concentration:

i. Markowitz basic estimation

ii. Random Matrix Theory

iii. Single Linkage Clustering

iv. Average Linkage Clustering

For the specific data set, Tola et al [2008] found that average linkage clustering is one of the successful filtering methods with respect to level of portfolio risk and the concentration of weights across a portfolio. For this reason we also use the average linkage clustering procedure to filter the original state correlations. As “state”, we denote the filtered or unfiltered correlation matrix computed in a non-overlapping window of all daily returns available in one month. The average linkage clustering for one of the correlation time slices has the dendrogrammatic representation shown in Figure 2.

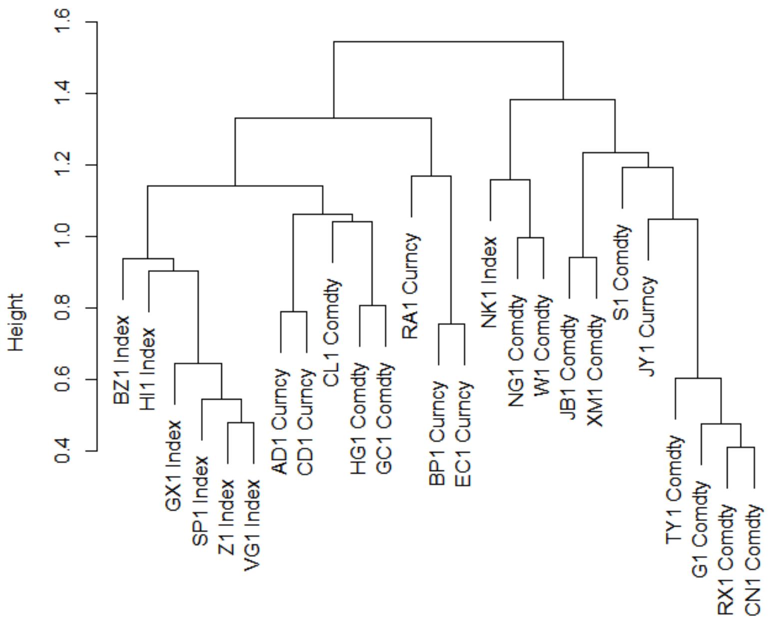



**Figure 2: Dendrogram output of a hierarchical clustering of one correlation matrix (= one state) computed in one month of daily data.**

The cophenetic distance between two markets that have been clustered is defined to be the intergroup dissimilarity at which the two markets are first combined into a single cluster (height on the y-axis). In comparison to the correlation distance matrix which has $n(n\text{-}1)/2$ distinct entries, the cophenetic distance matrix has only $n$-1 distinct entries.

It can be argued that a dendrogram is an appropriate summary of some data if the correlation between the original distances and the cophenetic distances is high. The cophenetic correlation coefficient is a measure of how faithfully a dendrogram preserves the original correlation distance matrix and it has the following formula

$$c_{st} = \frac{\sum_{i<j}(s_{ij} - \bar{s})(t_{ij} - \bar{t})}{\sqrt{\left[\sum_{i<j}(s_{ij} - \bar{s})^2\right]\left[\sum_{i<j}(t_{ij} - \bar{t})^2\right]}}$$

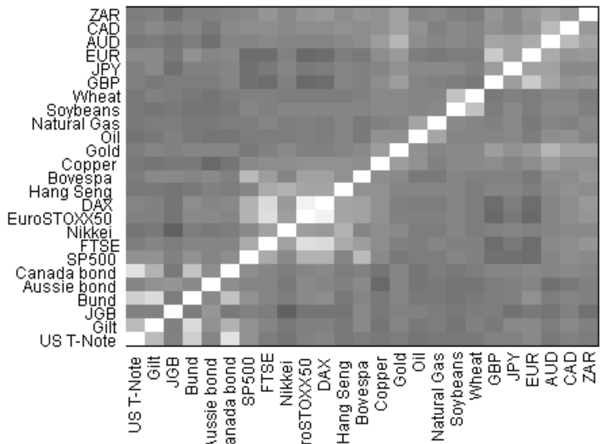
where $s_{ij}$ is a matrix entry of the original distance correlation matrix and $t_{ij}$ is an entry of the cophenentic distance matrix, and $\bar{s}$ being the average of the $s_{ij}$ and $\bar{t}$ being the average of the $t_{ij}$. Averaged over the 175 filtered time slices, the cophenetic correlation between the original correlations and the hierarchical cluster representation is 0.8360984, so we have a good approximation of the original data by the hierarchical cluster structure. In step 5, we have to compute a state similarity between the 175 filtered states and we also use the cophenetic correlation coefficient with the following definitions: $s_{ij}$ is a matrix entry of a cophenentic state matrix at time s and $t_{ij}$ is a matrix entry of a cophenentic state matrix at time $t$.

The representation of correlation regimes as in Figure 1 has many advantages:

1. If new data arrives, it can immediately be decided if their correlation structure is similar to any of the regimes or if there is a completely new regime emerging.
2. Regimes can be found which only occur in stressful periods whereas "stress" is defined as a high VIX index in our case. These regimes could be called "correlation at risk" or "correlation at stress" and they could be suitable for stress testing and for a parameterization of diversification models. For example, following the idea of Fabozzi and Focardi [2010], these regimes could be used to diversify trends/drifts using those correlation structures typical for crisis scenarios in well-known models like Markowitz portfolio construction. The latter approaches are known for their vanishing diversification properties in periods of crises. So a parameterization of Markowitz approaches with conditional correlations like "correlation at stress" could improve their modeling accuracy. Also, the use of conditional correlations could rejustify the normal distribution assumption which has to hold when employing Markowitz.
3. Any correlation-based model that has to be parameterized with historical data can benefit by the regimes as the input would not be most recent data (e.g. the data of the last 250 trading days) but the most similar data. In simple terms, model input would be the returns data whenever the same regime as the present one occurred in the past.
4. Another application is of forecasting nature with respect to future dependence structures. For a certain correlation regime in one step, there are probabilities for the transition to another (or the same) regime in the next time step.

Looking at the correlation regime evolution in time, it can be observed that between 07/1998 until the run on Northern Rock in September 2007 the regimes 1-4 are dominating. The Lehman fall marks a major reconstruction of correlations as after that event the regimes 1, 3 and 4 have almost disappeared and there are regimes 2 and 5 instead. Regime 5 seems to occur at almost all crisis events after Lehman. It is the stress regime. It is now interesting to see what typical correlation structure can be found behind the 5 regimes. Therefore we average the state correlations belonging to each regime and plot the 5 typical regime correlation matrices as heatmaps[1] (Figure 3).

[1] *"Risk on – risk off" – how a paradigm is born,* Currency Weekly, HSBC Global Research, 02 August 2010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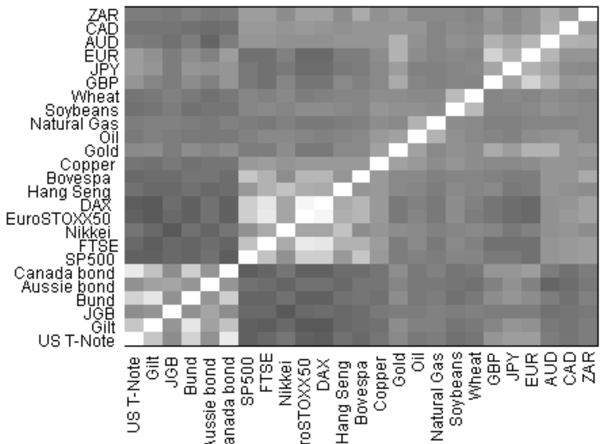
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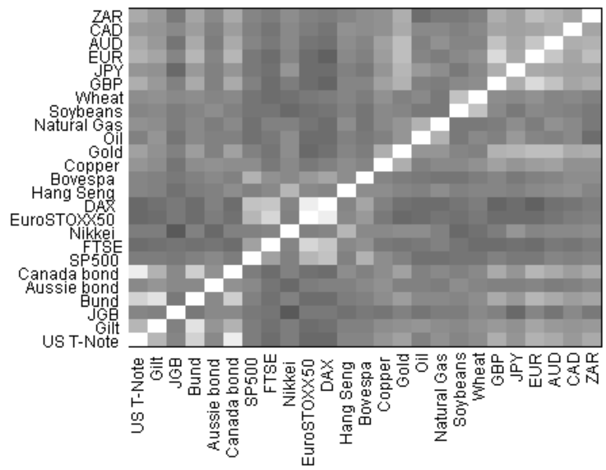
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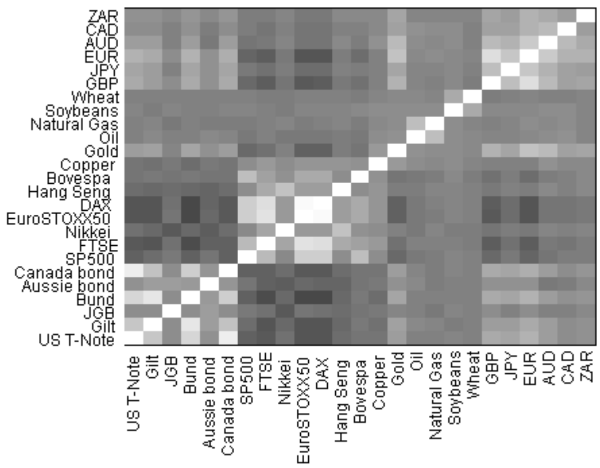
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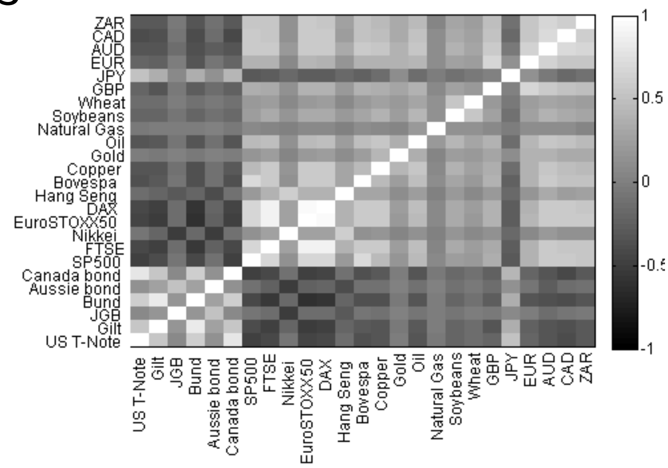
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**Figure 3: Heatmaps of the 5 regimes. Each heatmap represents the average correlation matrix of all unfiltered time slices belonging to this regime.**

White cells in the heat map stand for high correlation and black cells are anti-correlation. The individual assets are sorted in groups of their asset class (from left to right or bottom to top: government bond futures, equity index futures, commodity futures and FX futures). It can be observed that in the beginning most asset classes build their own cluster of high intra-correlation. The range of grey in the heat map implies there are many separate forces in markets that are driving different assets in a non-trivial way. This range of forces leads to many different behaviors and consequently to a large number of uncorrelated assets before the crisis. The heatmap of regime 5 is very extreme in terms of RoRo coupling. The visual inspection suggests that regime 1 and 2 are quite similar and regimes 3 and 4 are quite similar. However, looking at several statistics and later even at dynamic cluster analysis, it can be seen that there are still huge differences between the 5 regimes. Table 2 compares the 5 regimes in terms of typical average return and average correlation:

| | Regime 1 | Regime 2 | Regime 3 | Regime 4 | Regime 5 |
|---|---|---|---|---|---|
| Geometric return averaged over all assets and regime-specific time slices | 0.0057 | 0.0022 | -0.0081 | 0.0101 | 0.0015 |
| Average Absolute Correlation | 0.11 | 0.16 | 0.14 | 0.17 | 0.30 |

**Table 2: Average returns and correlations within the five regime correlation matrices.**

The highest and lowest returns are in regime 5 and 1. The strong coupling in the regime 5 leads to an average absolute correlation > 30%. The same value is <20% in all other regimes.

In the next step we use the same method for the state distances as before but this time just for the 5 typical regime correlation matrices. This allows us to compare how different the 5 regimes are in terms of their cophenetic correlation structure (Table 3).

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.94 | 0.79 | 0.82 | 0.62 |
| **2** | 0.94 | 1.00 | 0.71 | 0.81 | 0.70 |
| **3** | 0.79 | 0.71 | 1.00 | 0.90 | 0.36 |
| **4** | 0.82 | 0.81 | 0.90 | 1.00 | 0.39 |
| **5** | 0.62 | 0.70 | 0.36 | 0.39 | 1.00 |

**Table 3: Cophenetic correlations between the five regime correlation matrices.**

This analysis supports the visual observation that regimes 1 and 2 are similar and regimes 3 and 4 are similar. However, the amount if difference between the regimes now was quantified in a differentiated way.

### Cluster Tracking

It is now interesting to analyze how the markets group in each regime, how the grouping structure changes, if there are groups which always exist throughout the different regimes and if there are functional roles of certain markets/assets in the whole system.

To answer these questions we mine for distinct clusters in each regime based on the k-means clustering. Also, we use an algorithm for tracking the clusters in each regime to find the following evolving cluster structures:

1. Birth of a new cluster which has not been observed before

2. Death of an existing cluster

3. Merging of one or more clusters

4. Splitting of a cluster

5. Contraction of a cluster

6. Growth of a cluster.

We suggest a procedure similar to Fenn et al [2012] and focus on a market-centric cluster analysis. They outline that many approaches require defining core markets of clusters which seems overly restrictive. In order to avoid these and other ambiguities, instead of tracking whole clusters we identify clusters from the perspective of individual nodes. In this way, the temporal dynamics of functional market roles can be studied.

A market's identity is known in all 5 regimes as well as its cluster membership in each regime is known. Thus, we can track the cluster evolution from the perspective of individual markets. We investigate the persistence through time of a market's cluster by defining a cluster autocorrelation. For a market $i$ with cluster $c_i(s)$ in regime $s$, and cluster $c_i(t)$ in regime $t$ the autocorrelation is defined by

$$a_i^{st} = \frac{c_i(s) \cap c_i(t)}{c_i(s) \cup c_i(t)}$$

Using the k-mean clustering algorithm (2:15 clusters whereas clustering quality measure is average silhouette width), we split each regime correlation matrix into discrete clusters. We use the market-centric cluster tracking version which does not require us to determine which cluster in each regime represents the descendant of a cluster in another regime. Figure 4 tracks the clusters and labels each cluster with the amount of market constituents.

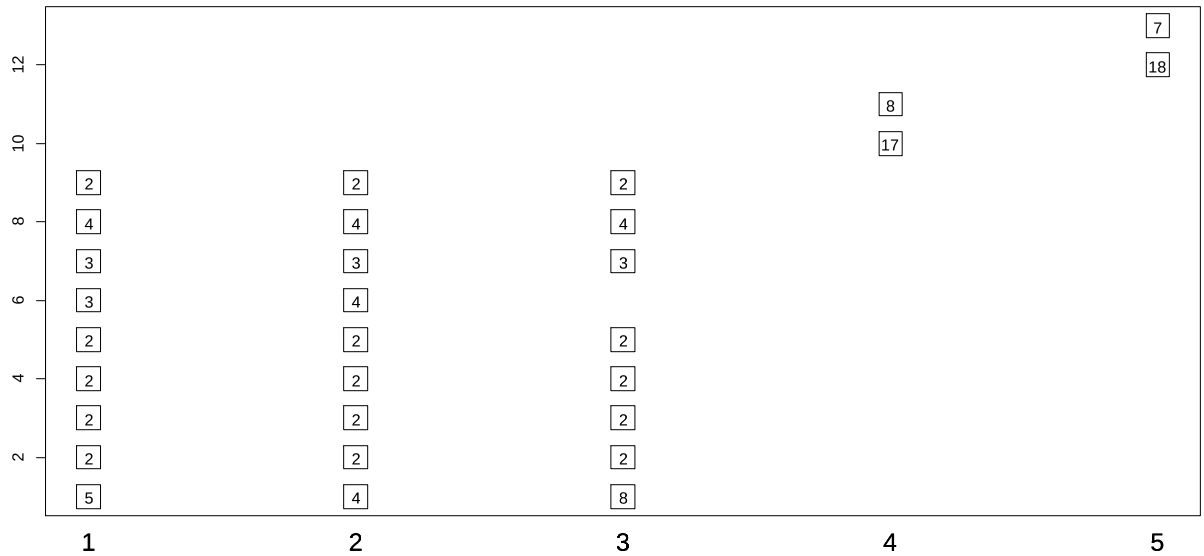


**Figure 4: Discrete Clusters tracked through the 5 regimes. On the x-axis there are the five regimes and on the y-axis there is the cluster number. A square marks the occurrence of a specific cluster in a certain regime. The numbers in the squares indicate the number of markets contained in a specific cluster in a specific regime. It can be seen that no cluster is contained in all regimes. Clusters 1-5 and 7-9 only occur in regimes 1-3. Clusters 10 and 11 only occur in regime 4 and clusters 12 and 13 occur only in regime 5. In regime 4 and 5, the clusters are of way larger size than in regimes 1-3.**

It was expected that after Lehman there is a strong RoRo dominance as seen in regime 5. It is remarkable to see that even before the crisis there is a bipolar regime 4. But the two clusters in regime 4 and in regime 5 are very different ones which is first expressed in the relatively low cophenetic correlation of 0.4 in table 3 and second in the cluster tracking which indicates all clusters are different ones.

As we will later see in the network analysis, Natural Gas has a special role and populates a network community of its own which indicates it has nothing to do with RoRo dynamics. Table 4 shows the degree of accordance in asset classes and intrinsic clusters:

| Regime | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Adjusted Rand Index (reference partition is asset class) | 0.37 | 0.35 | 0.46 | 0.32 | 0.28 |

**Table 4: Adjusted Rand index for the five regimes**

The adjusted Rand index for regime 3 is 0.46 which means that its discrete clustering is quite similar to the partition of asset classes. As expected, bipolar regime 5 has relatively low accordance with the asset classes.

With this analysis, it is quite clear how to get rid of the RoRo influence: either a diversification program similar to Markowitz is implemented and parameterized with the correlation matrix of a crisis regime. This should increase diversification if the market changes to this regime in the future. The other way is to conduct a finer grained clustering of times slices in even more clusters and find the regime where RoRo occurs the least. The correlation structure of this regime delivers input of how assets classes are truly correlated without the RoRo influence.

Implementing the RoRo dynamics in active management, however, requires an additional analysis which has to find those specific markets during a period of RoRo behavior. Using both the correlation regimes as well as correlation networks, we could set up a process to detect the timing and the markets relevant for actively trading the RoRo regime. In the following section we will outline some background of correlation networks.

**Network Analysis**

There are many types of networks in finance: threshold networks, causal networks, influence networks, partial networks, bipartite networks and many others. A wide range of financial assets and markets have been investigated with networks, including equities, currencies, commodities, bonds, and interest rates.

The class of topological filter networks in finance goes back to Mantegna [1999]. In a system with $n$ markets/assets, the approach selects $n$-1 correlations from the $n(n\text{-}1)/2$ distinct entries of a correlation matrix in order to construct a spanning tree, a network with no cycles. When constructing the spanning tree only the highest (and therefore highly significant) possible correlations are chosen. Out of all possible spanning trees this is the one called Minimal Spanning Tree (MST) as the highest correlations translate into minimal distances (a correlation of one is a distance of 0) from which the tree is constructed. Such network approaches consider the interaction of several markets/assets as a complex system. These are composed of many interacting elements and can exhibit numerous forms of 'emergent' collective dynamics without the need for any external organizing principle. Such dynamics typically cannot be explained by studying the constituent parts in isolation, so a complex system must be analyzed as a whole. Networks reflected by graphs provide a tractable framework and controlled environment for the quantitative analysis of many complex systems by distilling them to their key dependence structure. They just focus on the most important 'backbone' dependence structure and reduce complexity smoothing the way for more sophisticated analyses. The elements of the system are represented as the network's nodes and the important interactions between them are links that connect the nodes. Indeed, networks naturally reflect the variety of elements of the system by their set of vertices and by their edges the plurality of the interrelations between elements. Networks can be analyzed by network statistics on a global, local and dynamic scale. These relations and changes can be visualized so that subsequent analyses can focus on the relevant structures. For examples, central or decoupled assets are easily identified. On the other hand, networks are not easily invertible like a principle component approach.

Coming back to our analysis we generate the MSTs of the five correlation regimes and color them by asset class code (Figure 5).

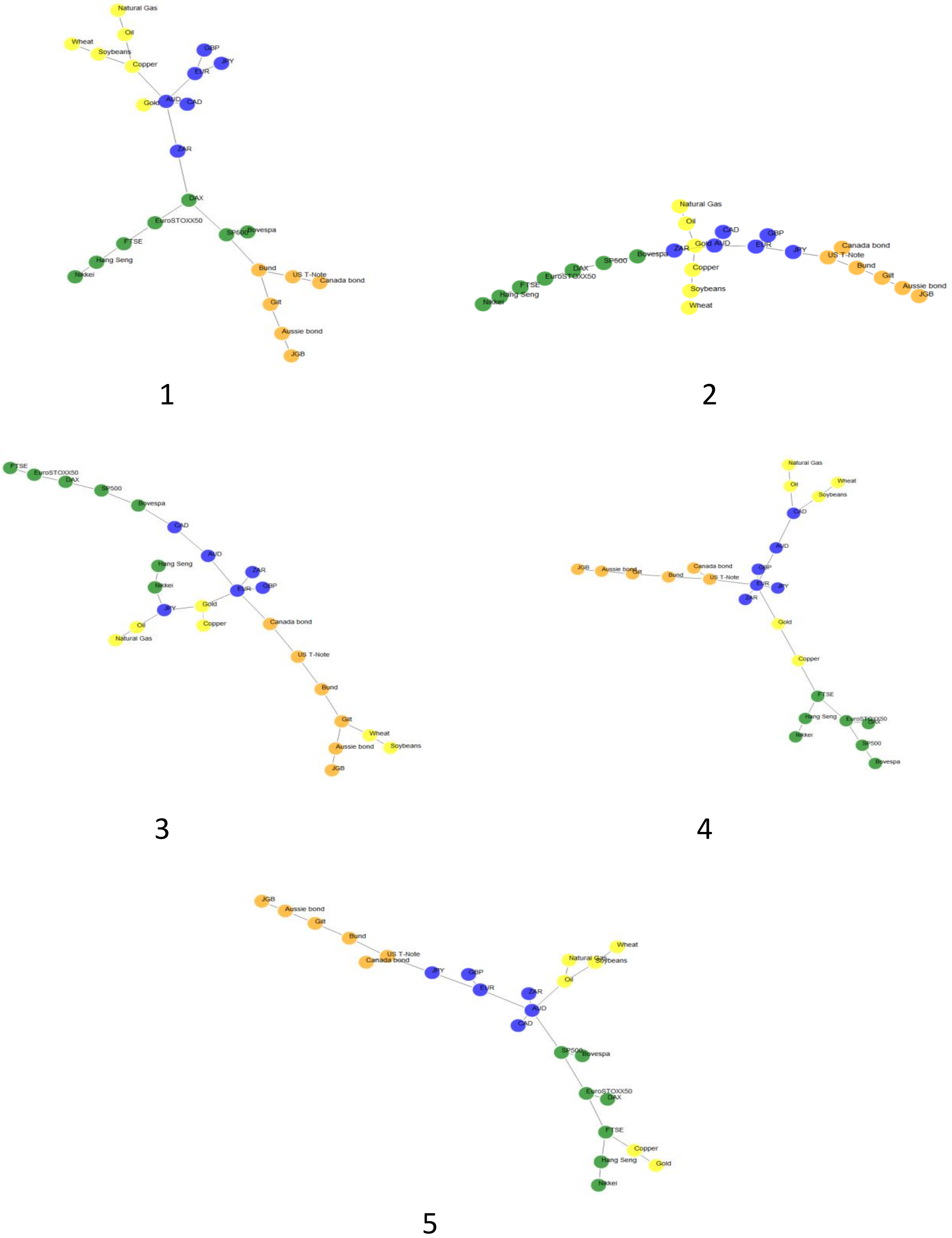


**Figure 5: Minimum Spanning Trees (MST) generated from the five regimes.**

In order to compare the 5 MSTs of the five regimes we use the edge survival measure between regime s and t:

$$ES(s,t) = \frac{1}{N-1}\left|E^s \bigcap E^t\right|,$$

where $E^s$ refers to the set of edges of the MST of regime s, $E^t$ refers to the set of edges of the MST of regime t, and the $\bigcap$ operation intersects the two MSTs. So it is simply the fraction of surviving edges to all edges. Table 5 shows all combinations of survival ratios for the five regimes.

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| **1** | 1.00 | 0.83 | 0.63 | 0.67 | 0.71 |
| **2** | 0.83 | 1.00 | 0.67 | 0.71 | 0.79 |
| **3** | 0.63 | 0.67 | 1.00 | 0.71 | 0.63 |
| **4** | 0.67 | 0.71 | 0.71 | 1.00 | 0.79 |
| **5** | 0.71 | 0.79 | 0.63 | 0.79 | 1.00 |

**Table 5: Matrix of tree survival rates of the 5 regimes**

The results are similar to those in Table 3 as again, regimes 1/2 are similar and 3/4. It is interesting to see that this analysis interprets regime 5 to be similar to regime 4 and also regime 2.

In the next step we just focus in the crisis regime 5 and cut those edges with absolute correlation level lower than 0.24 as these are the weakest links in the MST. This is a different analysis than just doing a flat clustering of regime 5 as done before.

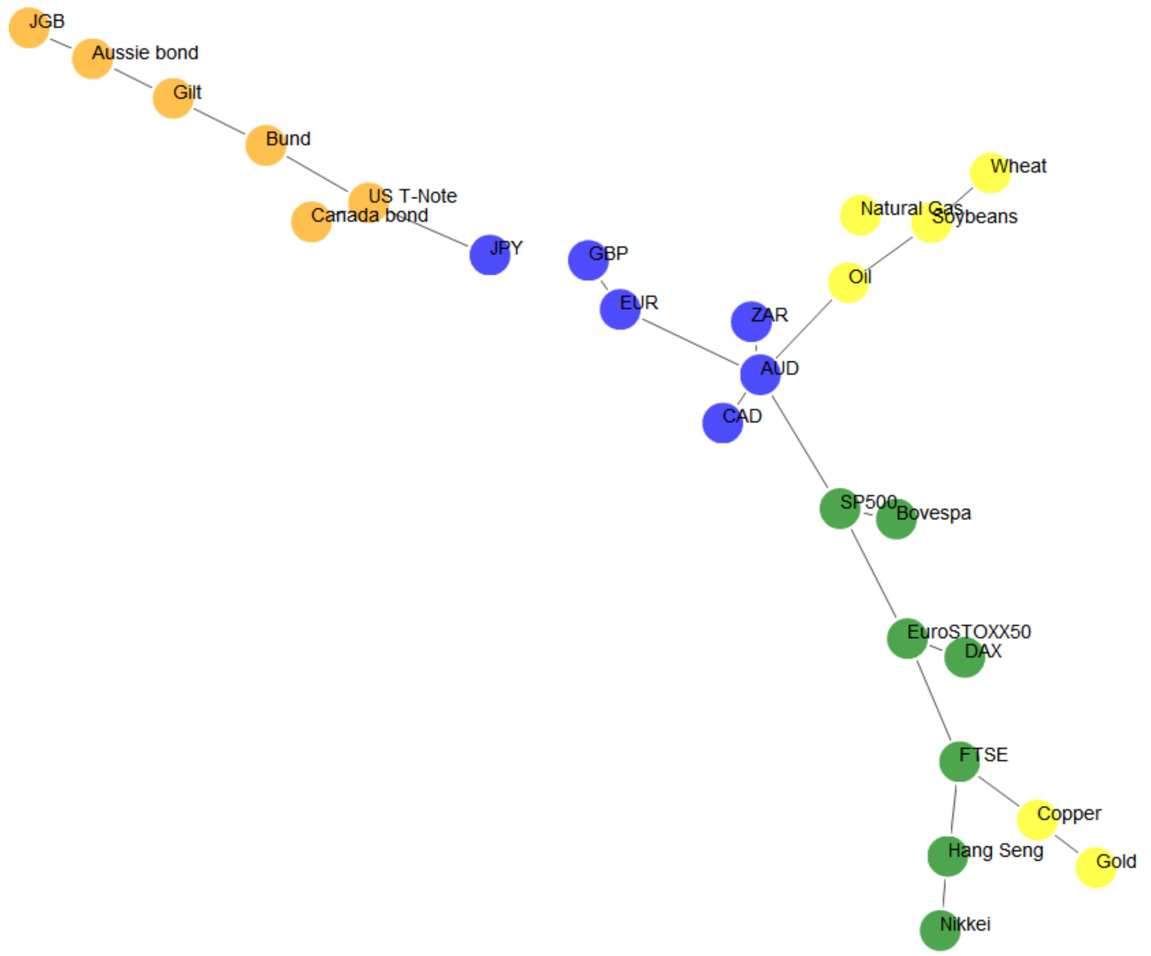


**Figure 6: This graph shows the MST of regime 5, which is the crisis RoRo regime. Nodes are colored by asset class membership. The weakest links (|correlation| < 0.24) are cut.**

The results of the cut-off (shown in Figure 6) are two branches with numerous nodes and one single node: Natural Gas. One of the branches is mostly populated with assets commonly known as risk on and the other branch is characterized by typical risk off asset. The centroid of the risk on assets is AUD and the centroid of the risk off asset is the US T-Note future.

**Summary**

We analyzed the “Risk on, Risk off” dynamics of a multi-asset futures portfolio from July 1998 to February 2013 with the following steps:

1. We partitioned the time axis into equidistant time slices. For each time slice, we calculated the correlation matrix and filtered it using average linkage clustering to reduce noise.
2. We computed similarity matrices from the filtered correlation matrices and perform k-means clustering to detect correlation regimes. With $k$=5, we found four distinct regimes (1-4) appearing before the Lehman default, and two regimes (2 and 5) appearing afterwards. Comparing the timing of their appearance with spikes of the VIX, we assigned regimes 1 and 2 to market recoveries and regimes 3-5 to contractions. Regime 5 is a specific crisis regime that occurs only “post Lehman”.
3. For each of the five regimes, we calculated the average correlation matrix. The “post-Lehman” regime correlation matrix for regime 5 shows markedly higher absolute correlations than the others.
4. We derived clusters from each of the five regime correlation matrices using $k$-means and quantified the accordance of the intrinsic clusters with asset classes. We found far less asset class separation in the post-Lehman period.
5. A network analysis using Minimum Spanning Trees (MST) for the five regime correlations was quantified using edge survival ratios. We found similarities between regimes 1 and 2.

Introducing a correlation cutoff level for the regime 5 resulted in two distinct MSTs, one for the risk-on and the other for the risk-off assets, respectively. The proposed steps not only confirm the observation of higher correlations in the post-Lehman period, but also allow to assign distinct correlation regimes for specific time slices and to follow the time evolution of the regime classification. The correlation network of the mean correlation matrix within a specific regime shows the tightest couplings between markets. The clustering techniques therefore do not only result in an appealing visualization of correlation matrices, but also offer a quantitative framework to analyze the asset dynamics for applications in risk management, trading and portfolio management.

## References


Chatterjee, R. Rex. “Dictionaries Fail: The Volcker Rule's Reliance on Definitions Renders it Ineffective and a New Solution is Needed to Adequately Regulate Proprietary Trading.”, Brigham Young University International Law and Management Review (2011).

Dose C, S. Cincotti. “Application to index and enhanced-index tracking portfolio.” Physica A 355 No. 1, (2005).

Economist: “Taming a wild beast: Explaining the oil-price rally.” http://www.economist.com/node/13871138, June 18th, 2009.

Fabozzi, F.J., S.M. Focardi. “Diversification: Should We Be Diversifying Trends?” The Journal of Portfolio Management Summer 2010, Vol. 36, No. 4: pp. 1-4, DOI: 10.3905/jpm.2010.36.4.001

Fengler M.R., P. Schwendner. “Quoting Multiasset Equity Options in the Presence of Errors from Estimating Correlations.” Journal of Derivatives 11,4 (2004).

Fengler, M.R., K. Pilz, P. Schwendner: “Basket Volatility and Correlation.”, In I. Nelken, ed., Volatility as an Asset Class, RISK Books (2007).

Fenn, D., M. A. Porter, P. J. Mucha, M. McDonald, S. Williams, N. F. Johnson, N. S. Jones (2012). “Dynamical clustering of exchange rates.”, Quantitative Finance, 12:10, 1493-1520

Flood, Chris. „Investors ignore flak over volatility ETPs.“ Financial Times, http://www.ft.com/cms/s/0/939016de-8d4e-11e1-9798-00144feab49a.htm, April 29th, 2012.

Gower J.D. “Some distance properties of latent root and vector methods used in multivariate analysis.” Biometrika, 53 (1966), 325-338.

Guerrera, Francesco. „Seeking a Wizard: Time to Embrace Market Risk, or To Run From It?“ Wall Street Journal, June 11th, 2012

Haas, M., Mittnik, S. „Gering korrelierte Anlageklassen - Diversifikationsmodell der Vergangenheit?“ Studienreihe des Bayerischen Finanz Zentrum e.V., München (2009).

HSBC: „Risk on – Risk off: the full story.” Currency Quant Special, HSBC Global Research, November 2010.

Kenett Dror Y, M. Tumminello, A. Madi, G. Gur-Gershgoren, R. N. Mantegna, E. Ben-Jacob. „Dominating clasp of the financial sector revealed by partial correlation analysis of the stock market.“ PloS one 5 (12), e15032 (2010).

Khandani, A.E., A. W. Lo. “What happened to the quants in August 2007?” Journal of Investment Management, Vol. 5, No. 4, (2007).

Laloux L, P. Cizeau, M. Potters, J-P. Bouchaud. “Random matrix theory.” Risk 12 69 (1999).

Lauricella, T., J. Cheng. “After Fed, Yield Moves to the Fore.” Wall Street Journal , September 13, 2012.

Lee, W. “Risk on/Risk off.” The Journal of Portfolio Management, Vol. 38, No. 3 (2012), p. 28-39.

Lisi F., M. Corazza. “Clustering financial data for mutual fund management.” Mathematical and Statistical Methods in Insurance and Finance, Springer (2008).

Mantegna, R.N. “Hierarchical structure in financial markets.” Euro. Phys. J. (B 11) (1999), p. 193–197.

Münnix M.C., T. Shimada, R. Schäfer, F. Leyvraz, T. H. Seligman, T. Guhr, and H. E. Stanley. "Identifying States of a Financial Market." Nature Scientific Reports 2, 644 (2012).

Noland, Douglas: “Roro and the broken markets.” http://www.financialsense.com/contributors/doug-noland/roro-broken-markets , April 21st, 2012

Onnela, J.-P., A. Chakraborti, K. Kaski, J. Kertész, J. “Dynamic asset trees and portfolio analysis.“ European Physical Journal B 30, 285 (2002).

Onnela, J.-P., A. Chakraborti, K. Kaski, J. Kertesz. „Dynamic asset trees and Black Monday“ Physica A 324, 247 (2003).

Onnela, J.-P., A. Chakraborti, K. Kaski, J. Kertesz, A. Kanto. „Asset trees and asset graphs in financial markets“ Physica Scripta T106, 48 (2003).

Parker, Robert. "Death of 'risk on, risk off' behaviour." Financial Times, http://www.ft.com/cms/s/0/86df76f2-d284-11e1-abe7-00144feabdc0.html, July 24, 2012

Plerou, V., P. Gopikrishnan, B. Rosenow, N. Amaral, T. Guhr, H.E. Stanley. "Random Matrix Approach to Cross Correlations in Financial Data.", Physical Review E, 65 (2002), pp 1–18.

Stopford, John. "Diversification in a correlated World." Investec (2012).

Tang, K., W. Xiong. "Index Investment and the Financialization of Commodities." Financial Analysts Journal 68, 6 (2012).

Tola V., F. Lillo, M. Gallegati and R. N. Mantegna. "Cluster analysis for portfolio optimization." Journal of Economic Dynamics and Control, vol. 32, issue 1 (2008), p. 235-258

Tumminello M., T. Aste, T. Di Matteo, R. N. Mantegna. “A tool for filtering information in complex systems.“ Proceedings of the National Academy of Sciences of the United States of America, 102, 30 (2005).

Tumminello, M., F. Lillo, R. N Mantegna. “Correlation, hierarchies, and networks in financial markets.” Journal of Economic Behavior & Organization 75, 1 (2010).