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Product Insight

Portfolio Optimization with Custom Factor Weight

Fine-tuning the risk aversion to different systematic factors

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Introduction

Portfolio managers often have different risk aversions for stocks from different regions, industries, and investment styles. In a standard mean-variance portfolio optimization framework, managers may define a single risk aversion parameter, which is applied evenly across all sources of systematic risk. This paper demonstrates how managers can use the Custom Factor Weight feature in the Barra Aegis System to express different levels of risk aversion to different factors. We will first discuss broadly how mangers can adopt such an approach to fine-tune their portfolio construction process and align it with their view of the market. We will then use a case study to demonstrate the benefits of this optimization framework when a manager has a strong conviction on a specific alpha signal. Lastly, we will provide details on the mathematical formulation of this methodology and explain its effect on both common factor risk and specific risk, and address some frequently asked questions.

Why the Need for Custom Factor Weight in the Optimization Process

The trade-off between risk factors and alpha signals is an important consideration during any optimization process. Applying Custom Factor Weight allows users to manage this trade-off effectively, and we discuss several cases where such an approach can be incorporated into the investment process.

1. Assign Different Risk Aversion Parameters to Different Risk Factors

Traditionally, managers would only use a single risk aversion parameter to quantify the amount of risk they were willing to accept. This parameter would typically be applied to all factors equally. By expressing different weights for different factors, which is equivalent to setting different risk aversion values for different factors, the approach effectively provides managers with the flexibility to express their different risk aversions for stocks from different regions, industries and investment styles.

2. Strengthen Bets on Alpha Factors with Higher Conviction Levels

In a standard portfolio optimization, the optimizer is looking for the optimal tradeoff between the expected portfolio return, as predicted by the alpha factors, and the portfolio risk as forecasted by a risk model. It is common that some factors, such as Value and Momentum, will appear to be both alpha and risk factors.

When alpha factors overlap with risk factors, the bets on alpha factors will be "pulled back" by the corresponding risk factors during the optimization. If managers have different confidence levels on the forecasting power of these alpha factors, they may want their views to be expressed differently based on the quality of the alpha factors they have determined. This is effectively equivalent to setting a lower risk aversion to a particular factor.

For example, if a manager is more confident on alpha factor 1 than alpha factor 2 while both appear in the risk model, the manager may want to reduce the weight on factor 1 relative to factor 2, which is likely to tilt the optimized portfolio more towards factor 1. If the manager's original assessment that factor 1 has a higher forecasting power than factor 2 was correct, this procedure may help the manager improve the portfolio's performance.

3. Suppress Certain Factors from the Covariance Matrix

When alpha factors overlap with risk factors, some portfolio managers prefer to remove the shared factors from the risk model in order to mitigate the alpha-risk misalignment effect. This new feature allows portfolio managers to remove or suppress what they deem to be an alpha factor from the risk model without manipulating the risk model data directly.

A Use Case Illustration

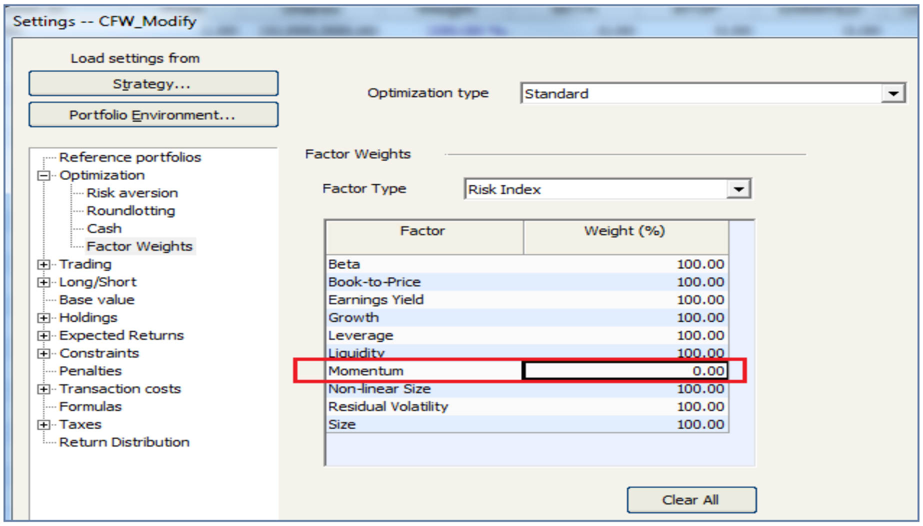
We will use a simple case study to illustrate the benefit of using this new feature when institutional investors have different confidence levels on their alpha factors.

Optimization Settings

In this case study, the manager uses the Barra China Equity Model (CNE5) and chooses MSCI China A Index as the benchmark. The manager wants to build an enhanced index tracking portfolio with respect to MSCI China A Index and only likes stocks with strong recent performance (positive Momentum) and small-cap companies (negative Size). To be more precise, the manager forecasts the Momentum factor with a +6% return and the Size factor with a -6% return to express its views. Risk aversion is set to be 0.5 and a stock's weight is enforced to deviate from its weight in the benchmark by at most 5%.

The above strategy is defined as the Base Case. The manager then changes the factor weight for Momentum to zero as the manager is very confident on the outperformance of the Momentum factor. That modified strategy is defined as the Modified Case.

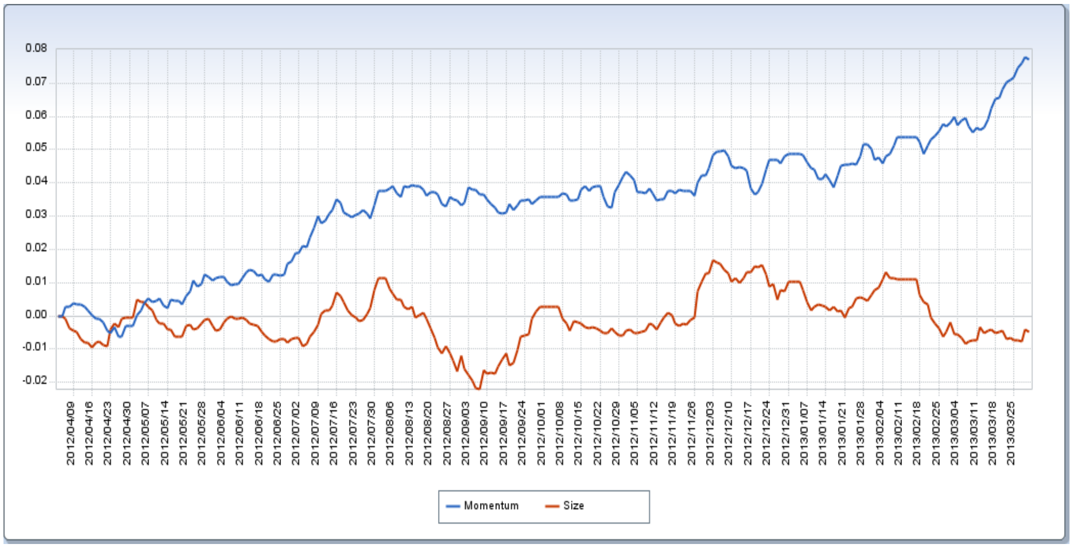
Figure 1: Setting the Factor Weight for Momentum to Zero



Backtest Result

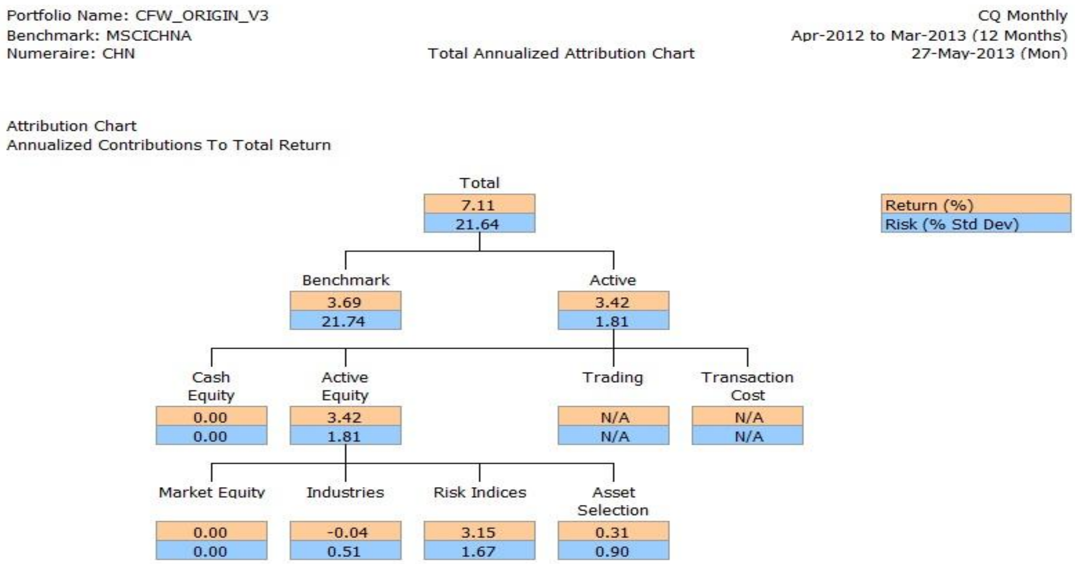
Based on the settings described above, a backtest is run on both the Base Case and Modified Case from 2012 April to 2013 Mar (12 months), which was a period where the Momentum factor outperformed strongly while the Size factor's behavior was muted. The results will allow the manager to confirm whether the optimizer increases the bet on the momentum factor as desired.

Figure 2: Monthly Cumulative Returns for Momentum and Size factors from Apr 2012 – Mar 2013



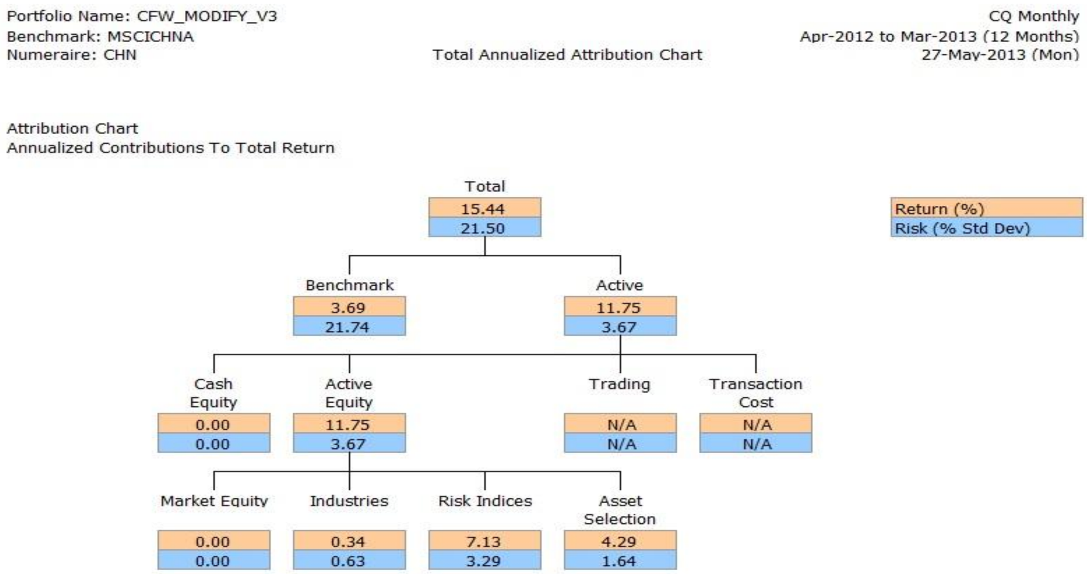
Based on the results, the Base Case delivered a 3.42% active return with 1.81% tracking error over the 12-month test period (annualized), which gave an information ratio of 1.89.

Figure 3: Performance Attribution of the Backtest Result of Base Case



In contrast, the Modified Case had a 11.75% active return with 3.67% active risk over the same test period, which translated to an information ratio of 3.2.

Figure 4: Performance Attribution of the Backtest Result of Modified Case



As we can see from the performance attribution charts above, the increase of the active return in the Modified Case was mainly driven by Risk Indices and Asset Selection. Let's now check how style factor exposures have changed in the Modified Case and the impact on the factor return contribution.

Table 1: Comparison of Average Active Exposure and Return Contribution

	Original Case		Modified Case	
Source of Return	Average Active Exposure	Return Contribution	Average Active Exposure	Return Contribution
Beta	-0.04	-0.16	-0.12	-0.64
Momentum	0.42	3.52	0.92	8.21
Size	-0.32	0.07	-0.17	0.12
Earnings Yield	0.15	-0.25	0.02	-0.19
Residual Volatility	-0.16	0.16	0.08	0.03
Growth	0.01	0.03	0.14	0.31
Book-to-Price	0.02	0	-0.15	0.07
Leverage	0.05	-0.09	-0.04	-0.13
Liquidity	-0.09	0.9	-0.01	0.14
Non-linear Size	0.22	-1.04	0.15	-0.8
Total		**3.15**		**7.13**

From the table above, we can see that the average active exposure to Momentum factor increased from 0.42 in the Base Case to 0.92 in the Modified Case, while the active exposure to Size had dropped slightly from -0.32 to -0.17. This is expected, as the optimizer ignored the risk contribution from the Momentum factor and any factor correlation involving Momentum. Therefore, the optimized portfolio will have a larger tilt towards Momentum and the exposure to the rest of the factors will be adjusted accordingly based on their correlation with the Momentum factor. Since the Momentum factor had a strong performance over the test period, the return contribution from the Momentum factor was increased from 3.52% in the Base Case to 8.21% in the Modified Case, which resulted in a 4.69% improvement in the portfolio return. It is worth noting also that the active risk for the Modified Case was increased as well, although the information ratio was still significantly higher for the Modified Case.

How the Optimization Process is Modified by Custom Factor Weight

Let's take a look at the mathematical formulation of a portfolio optimization problem in Aegis and examine the impact of introducing the custom factor weight feature.

In a standard portfolio optimization problem in Aegis, we use the Mean-Variance framework to maximize the expected portfolio return adjusted by the risk forecast and transaction costs, subject to a set of user-defined constraints. Mathematically, this problem is represented as follows:

$$\text{maximize } h' * r - \lambda * (h' * X * F * X' * h + \theta_{\frac{AS}{CF}} * h' * D * h)$$

$$\text{subject to:} \quad \text{any constraints}$$

$$\text{e.g} \quad \text{total turnover} \leq 20\%$$

Where:

h' is a vector of the stock weights in the optimized portfolio

r is a vector of the forecast stock returns

X is a matrix of stock exposures to risk factors

F is the common factor covariance matrix

D is the specific risk matrix

λ is the risk aversion ratio

$\theta_{AS/CF}$ is the ratio of asset selection to common factor risk

With the new Custom Factor Weight feature, users have the flexibility to assign different weights to different factors in the common factor covariance matrix. Mathematically, the objective function now becomes:

$$\text{maximize } h' * r - \lambda * (h' * X * W * F * W' * X' * h + \theta_{\frac{AS}{CF}} * h' * D * h)$$

Where:

W is a diagonal matrix of $\text{diag}(w_1 \ldots w_i \ldots w_n)$

w_i is the weight for factor i.

There are a number of frequently asked questions about this new feature. We consider two questions in detail.

Does It Affect both Common and Specific Risk

Clearly, this new feature will only affect the common factor covariance matrix. In the augmented optimization problem, the weight matrix *W* is multiplied only by the common factor covariance matrix *F*. It has no impact on the specific risk matrix *D*.

Does It Affect Factor Correlations

To illustrate how the common factor risk matrix is affected by *W*, let's consider a covariance matrix with only two factors:

$$F = \begin{bmatrix} \sigma_1^2 & \rho^2 \\ \rho^2 & \sigma_2^2 \end{bmatrix}$$

Where:

σ_1, σ_2 are the standard deviation of factor 1, 2

ρ is the correlation between factor 1 and 2

When we multiply *F* by *W*, we get:

$$W * F * W' = \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix} * \begin{bmatrix} \sigma_1^2 & \rho^2 \\ \rho^2 & \sigma_2^2 \end{bmatrix} * \begin{bmatrix} w_1 & 0 \\ 0 & w_2 \end{bmatrix}'$$

$$= \begin{bmatrix} (w_1\sigma_1)^2 & w_1 w_2 \rho^2 \\ w_1 w_2 \rho^2 & (w_2\sigma_2)^2 \end{bmatrix}$$

Hence, both the factor risk and factor correlation are affected by our chosen weights in the resulting covariance matrix.

Summary

Custom Factor Weight is a powerful new feature in Barra Aegis, which allows users to customize the weights of individual risk factors in the covariance matrix during optimization. In this paper, we discussed how portfolio managers can derive value from this new feature to assign different risk aversion levels to different risk factors as well as to adjust bets on alpha factors when users have different confidence levels on them. We examined a sample use case, where manager had a strong confidence in the outperformance of the Momentum factor. During the sample period, the Momentum factor did indeed outperform and adjusting the weight placed on its risk in optimization helped improve the performance of the optimized portfolio significantly.

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[1] *As of September 30, 2012, as published by eVestment, Lipper and Bloomberg on January 31, 2013* *Apr 2013*