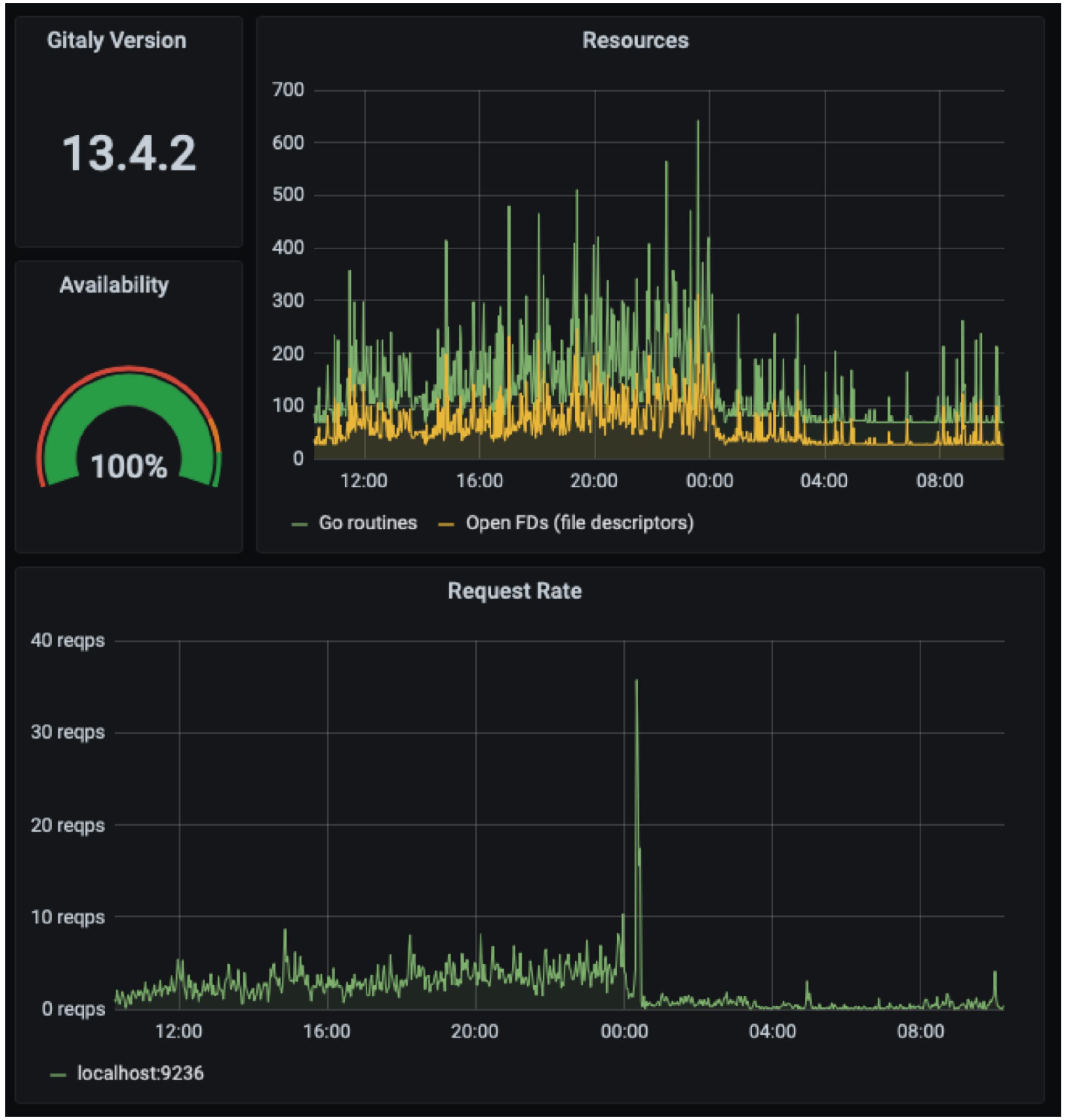
Graphs

Prof. Darrell Long

CSE 13S

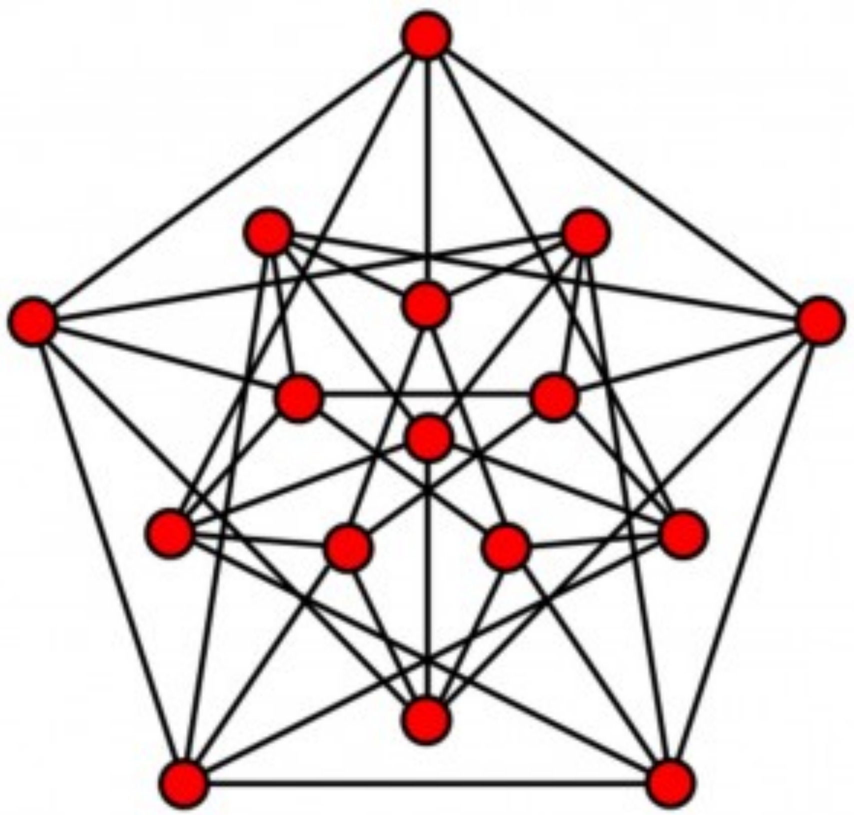
Is this a graph?

- Yes, this is commonly called a graph, but it is not what we are talking about here…
- We will call these depictions of numerical quanities *plots*.



Is this a graph?

- This is an example of what we will call a *graph*.
- There is an entire branch of mathematics dedicated to *graph theory*.
- Graphs of this type are used in mathematics, computer science, physics, and even sociology.



Components of a graph

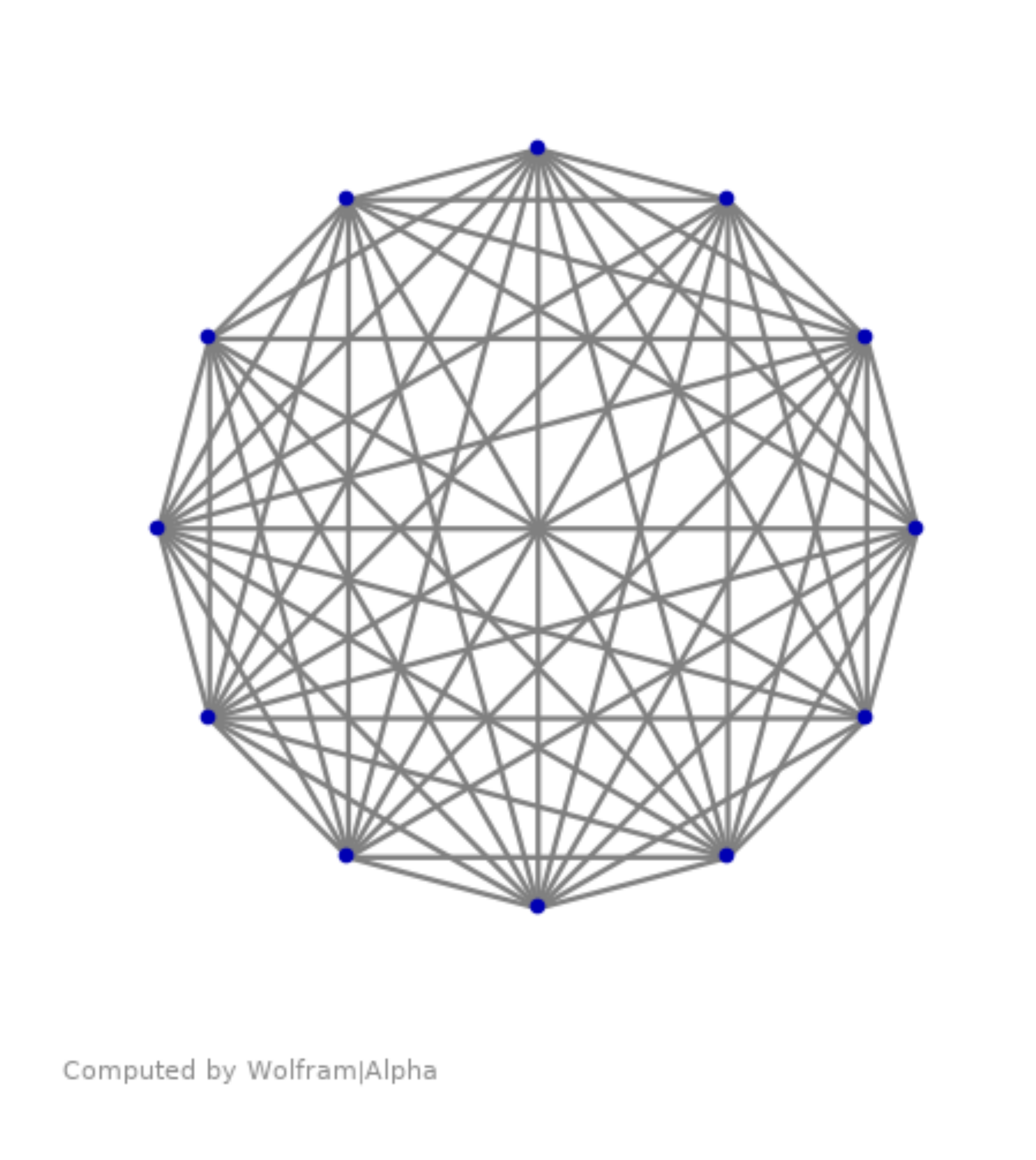
- Nodes
- Edges
- Weights

Directed and Undirected Graphs

- Edges may have a direction, $n_1 \rightarrow n_2$, and we call that a *directed graph*.
- Edges may have no direction (or both directions), $n_1 \leftrightarrow n_2$, and we call that an *undirected graph*.
- The edges may have *weights*, which represent capacity, strength, or cost.

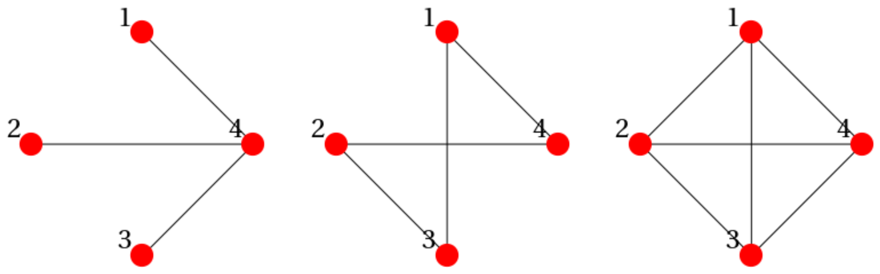
Representing a graph

- Adjacency Matrix
  - $n \times n$ matrix
  - Binary: edges present or absent
  - Weighted: $n \neq 0$
- Adjacency List
  - Column array for nodes
  - Linked list of edges from each node
  - May contain weights



Adjacency Matrix

- A non-zero entry in $M_{i,j}$ means there is an edge $n_i \rightarrow n_j$.
- A matrix that is symmetric around the diagonal represents an *undirected* graph.
- The entry can specify not only the existence of an edge, but also its weight.
- Requires $O(n^2)$ space.
    - Sparse matrix techniques can improve it.

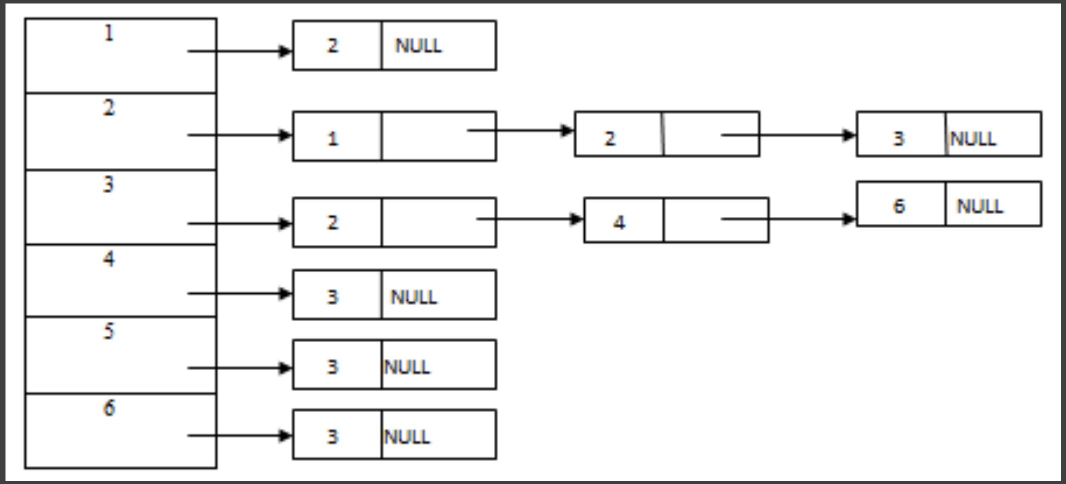


$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

Computed by Wolfram|Alpha

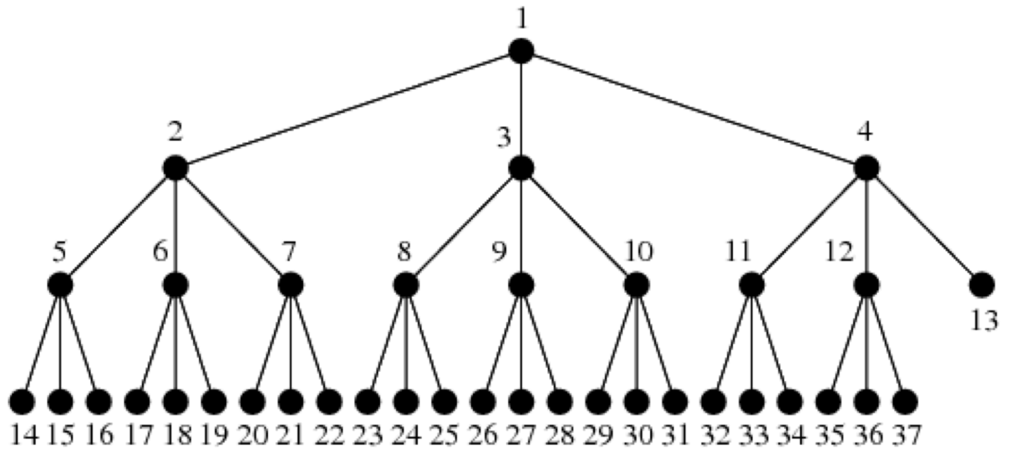
Adjacency List

- Each node is represented as an entry in a column vector.
    - Each entry is the head of a linked list.
- The list elements contain:
    - The destination node, and
    - The *weight* of the edge.
- Why would you prefer this over an adjacency matrix?
    - An adjacency matrix is $O(n^2)$ space,
    - An adjacency list will be more space efficient for sparse graphs.



Trees

- Trees are form of *acyclic* graph.
  - Acyclic means that if you follow the edges, there are *no loops* (cycles).
- You will often hear the term *DAG*, which stands for:
  - <u>D</u>irected
  - <u>A</u>cyclic
  - <u>G</u>raph



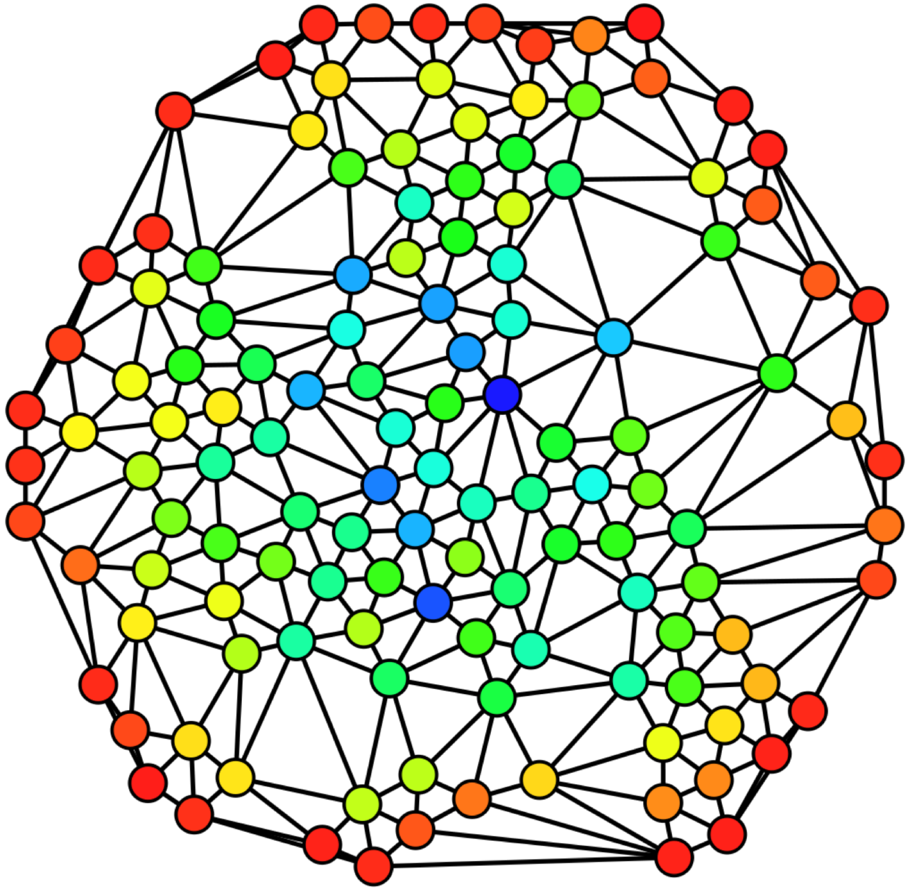
Basic Graph Algorithms

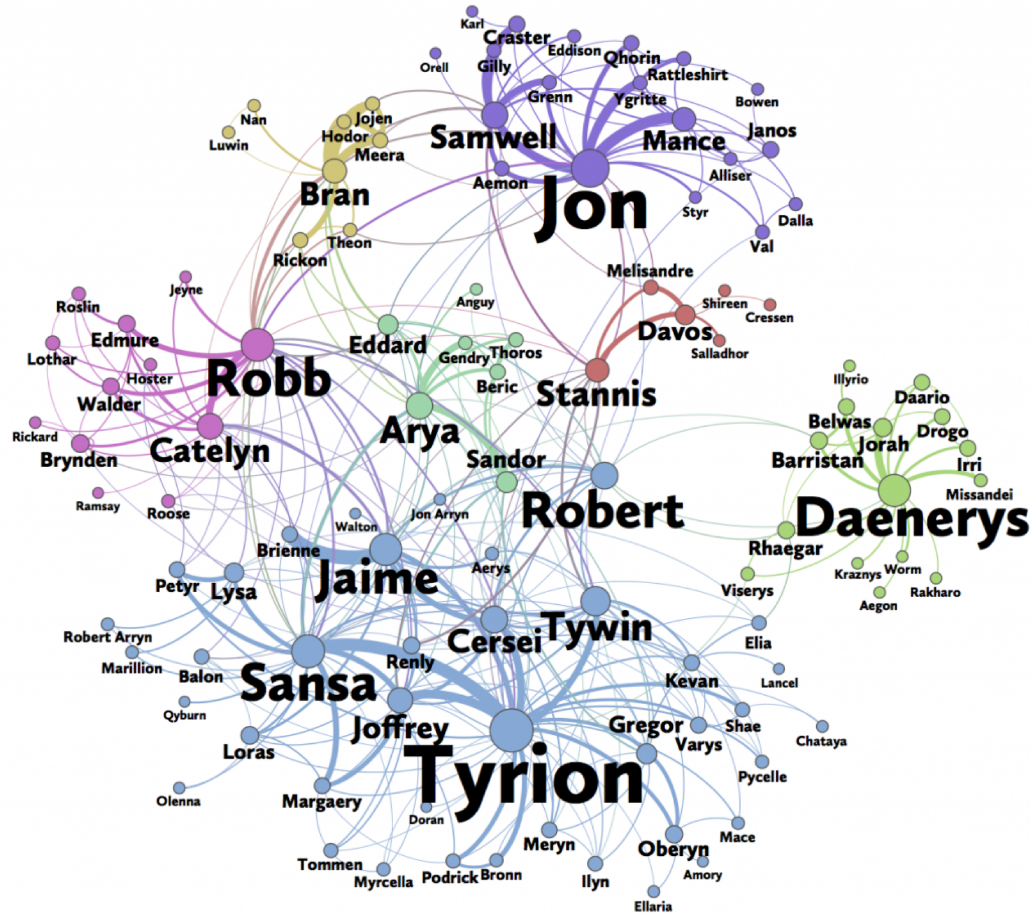
- Depth-first search
  - Based on recursion, or
  - A stack
- Breadth-first search
  - Based on a queue

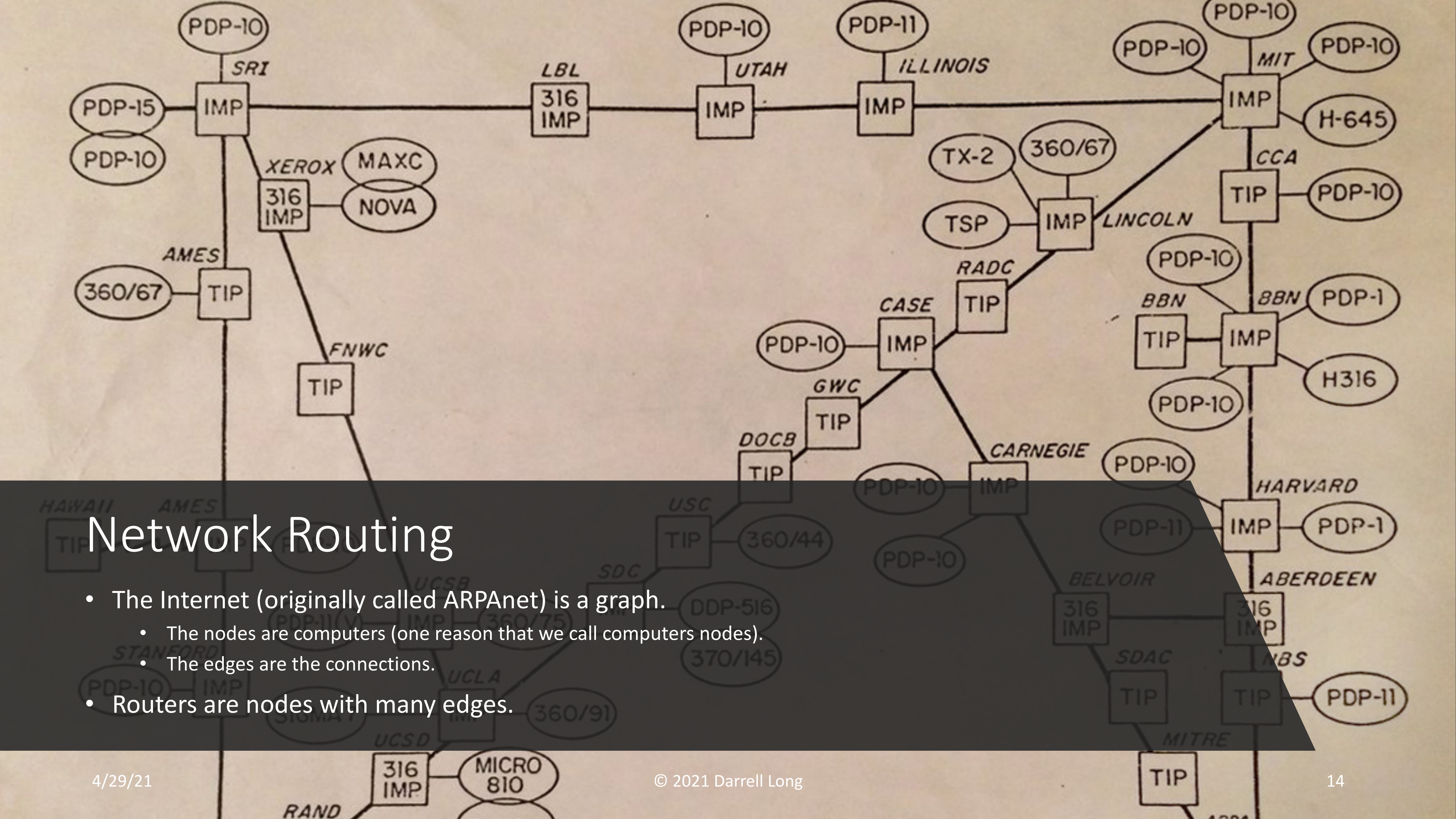
Graph problems

- Shortest path
- Hamiltonian path
  - The Traveling Salesman problem

Social Networks




Karl
Craster
Eddison
Qhorin
Orell
Gilly
Rattleshirt
Grenn
Ygritte
Bowen
Nan
Jojen
Hodor
Samwell
Mance
Janos
Luwin
Meera
Aemon
Alliser
Bran
Jon
Styr
Dalla
Theon
Val
Rickon
Melisandre
Jeyne
Anguy
Shireen
Cressen
Roslin
Davos
Edmure
Eddard
Thoros
Salladhor
Lothar
Gendry
Hoster
Robb
Beric
Stannis
Illyrio
Walder
Daario
Belwas
Rickard
Arya
Drogo
Jorah
Catelyn
Barristan
Irri
Brynden
Sandor
Missandei
Robert
Daenerys
Ramsay
Roose
Jon Arryn
Walton
Brienne
Rhaegar
Aerys
Kraznys
Worm
Petyr
Lysa
Jaime
Viserys
Aegon
Rakharo
Tywin
Robert Arryn
Cersei
Elia
Renly
Marillion
Balon
Sansa
Kevan
Lancel
Qyburn
Gregor
Shae
Joffrey
Varys
Chataya
Loras
Tyrion
Pycelle
Olenna
Margaery
Doran
Mace
Meryn
Oberyn
Tommen
Podrick
Bronn
Amory
Myrcella
Ilyn
Ellaria



Network Routing

- The Internet (originally called ARPAnet) is a graph.
  - The nodes are computers (one reason that we call computers nodes).
  - The edges are the connections.
- Routers are nodes with many edges.