# DATA COMPRESSION

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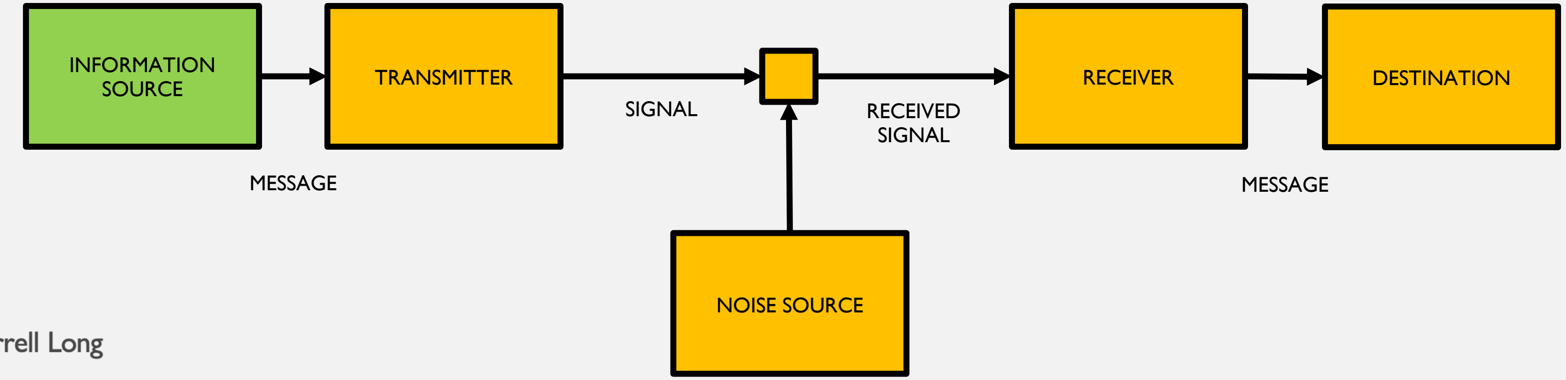
CSE 13S

# COMMUNICATION

- Claude Shannon, the "father of modern information theory," breaks down communication into five parts:
  1. Information source
  2. Transmitter
  3. Channel
  4. Receiver
  5. Destination

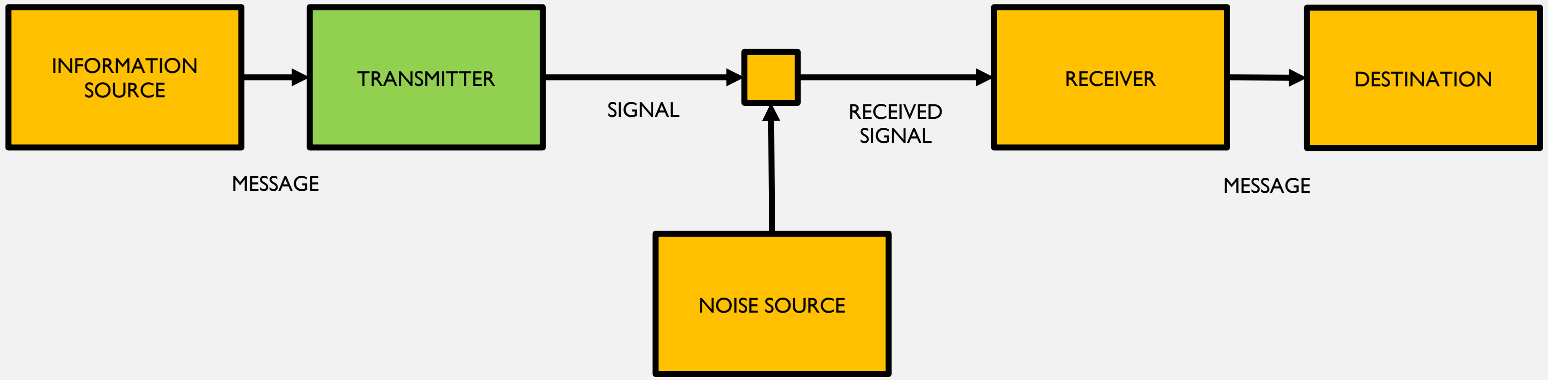
# INFORMATION SOURCE

- Produces a message, or a sequence of messages, to be communicated to the receiving terminal.
- A message takes on various forms:
  - A sequence of symbols like in a telegraph or teletype system.
  - A single function of time like radio.
  - A function of time and other variables like black/white television.
  - Two of more functions of time like sound transmission (several channels).
  - Several functions of variables like color television.



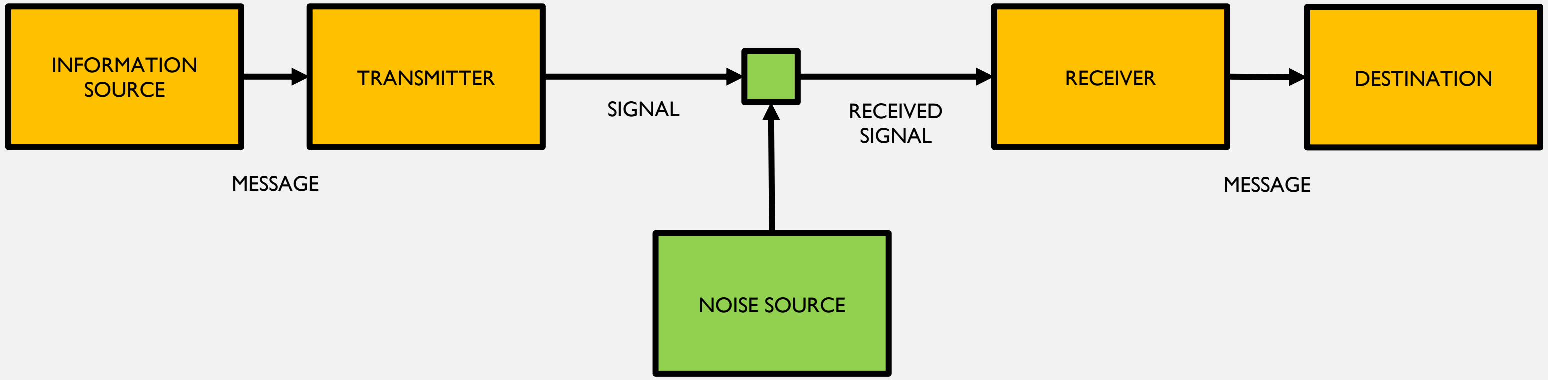
# TRANSMITTER

- Operates on the message to produce a signal suitable for transmission over the channel.
- This is where compression and/or encryption occurs.



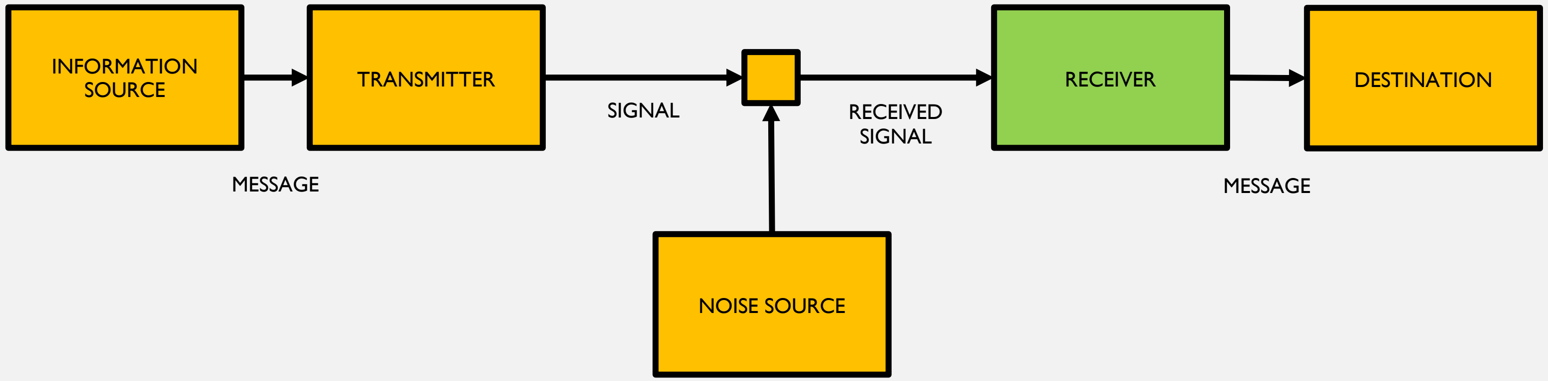
# CHANNEL

- The medium through which a signal is transmitted.
- Could be, but not limited to:
  - Radio frequencies.
  - Beams of light.
  - Coaxial cables.
  - Wires.
  - Fiber optics.



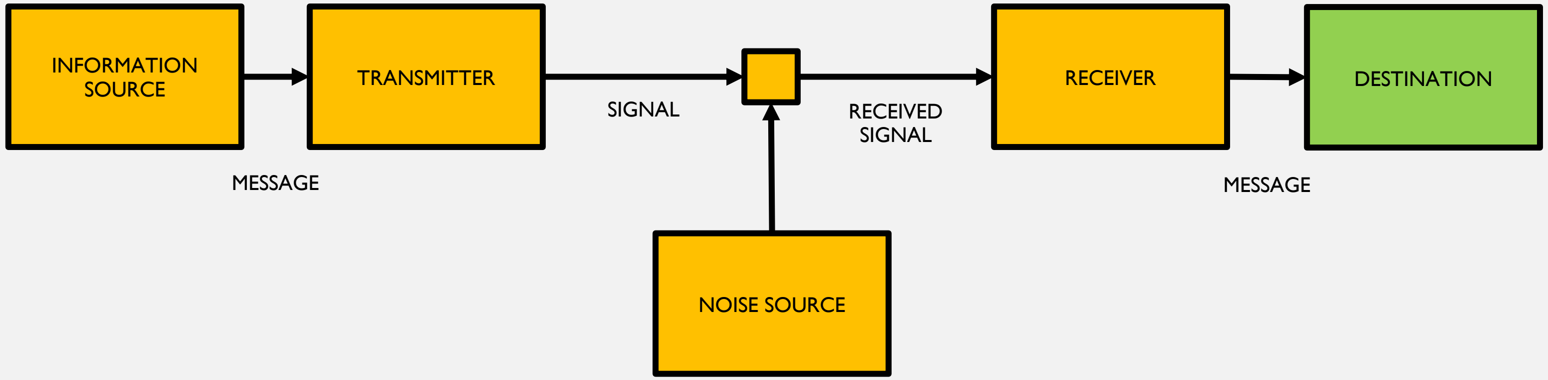
# RECEIVER

- Performs the inverse operation of that done by the transmitter.
- Decompression if the transmitter used compression, decryption if the transmitter used encryption.



# DESTINATION

- The intended target of the message.



# ENTROPY

- Defined by Shannon as a measure of uncertainty of the occurrence of an event.
- Assume a set of possible events with probabilities $p = (p_1, p_2, \cdots, p_n)$ such that $\sum_{i=1}^{n} p_i = 1$.
  - That is, the probability of event $e_i$ occurring is $p_i$ for all $1 \leq i \leq n$ .
- If such a measure exists, call it $H(p_1, p_2, \cdots, p_n)$, it can be required to have the following properties:
  1. $H$ is continuous in $p$
  2. If all $p_i$ equal, $p_i = \frac{1}{n}$, thus $H$ is a monotonically increasing function of $n$.
     - There is some amount of uncertainty that occurs given equally likely events.
     - The more events, the more uncertainty.
  3. If a choice is broken down into two successive choices, the original $H$ is the weighted sum of the individual values of $H$.
- Thus, $H = -\sum_{i=1}^{n} p_i \log_2(p_i)$
  - We call this entropy.

# WHAT DOES IT ALL MEAN?!

- Assume the following three messages:
  1. $AAAA$
  2. $AABC$
  3. $ABCD$
- Which message, if we pick a random symbol and guess what it is, has:
  - The greatest probability of correctly guessing the symbol?
  - The least probability of correctly guessing the symbol?
- In other words, which message has:
  - The least entropy?
  - The most entropy?

# BETTER UNDERSTANDING ENTROPY

- Consider message (1), which is $AAAA$.
  - $p(A) = 1$
- Using the formula for entropy we see that

$$\begin{aligned} H &= -\sum_{i=1}^{n} p_i \log_2(p_i) \\ &= -\,1 \log_2(1) \\ &= 0 \end{aligned}$$

- Which means the entropy for this message is 0.

# BETTER UNDERSTANDING ENTROPY

- Consider message (2), which is $AABC$.
  - $p(A) = \frac{1}{2}, p(B) = \frac{1}{4}, p(C) = \frac{1}{4}$
- Using the formula for entropy we see that

$$\begin{aligned} H &= -\sum_{i=1}^{n} p_i \log_2(p_i) \\ &= -\frac{1}{2}\log_2\left(\frac{1}{2}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) \\ &= \frac{3}{2} \end{aligned}$$

- Which means the entropy for this message is $\frac{3}{2}$.

# BETTER UNDERSTANDING ENTROPY

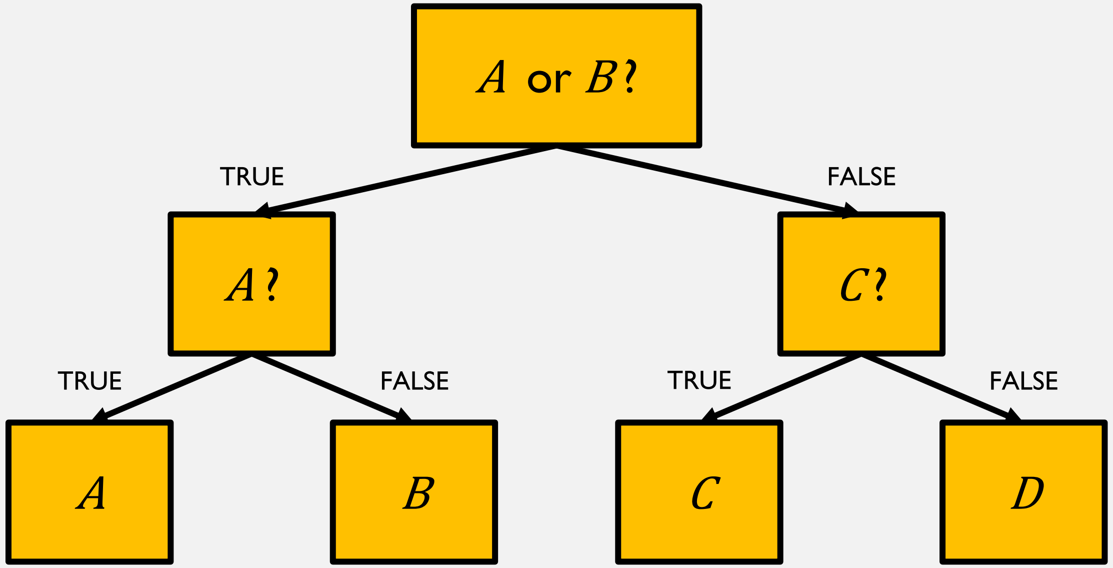
- Consider message (3), which is $ABCD$.
  - $p(A) = \frac{1}{4}, p(B) = \frac{1}{4}, p(C) = \frac{1}{4}, p(D) = \frac{1}{4}$
- Using the formula for entropy we see that

$$\begin{aligned} H &= -\sum_{i=1}^{n} p_i \log_2(p_i) \\ &= -\frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) - \frac{1}{4}\log_2\left(\frac{1}{4}\right) \\ &= 2 \end{aligned}$$

- Which means the entropy for this message is 2.

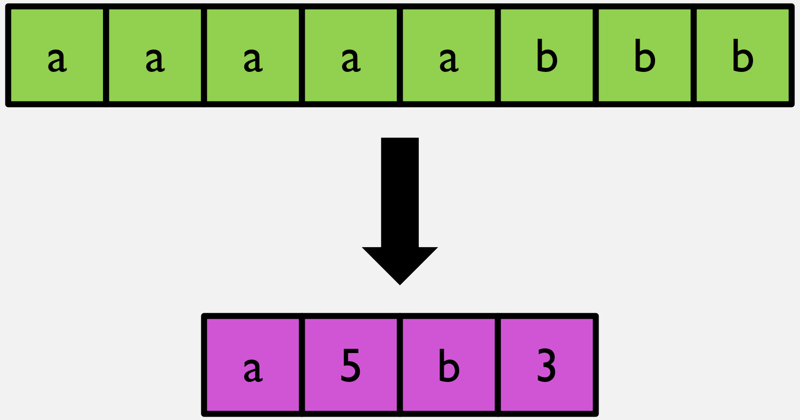
# GUESSING A SYMBOL

- We can think of entropy as the average number of questions needed to be asked to correctly guess what random symbol we picked from the message.
- Consider message (3), which is $ABCD$.
- The best way to guess a random symbol randomly selected out of this message is to simulate binary search.
  - Is the symbol $A$ or $B$?
    - If yes, then is it $A$?
    - Else, it must be $B$.
  - Else, the symbol must be either $C$ or $D$.
    - Is it $C$?
    - Else, it must be $D$.
- Thus the average number of questions asked is 2, which is exactly the entropy we calculated for this message!



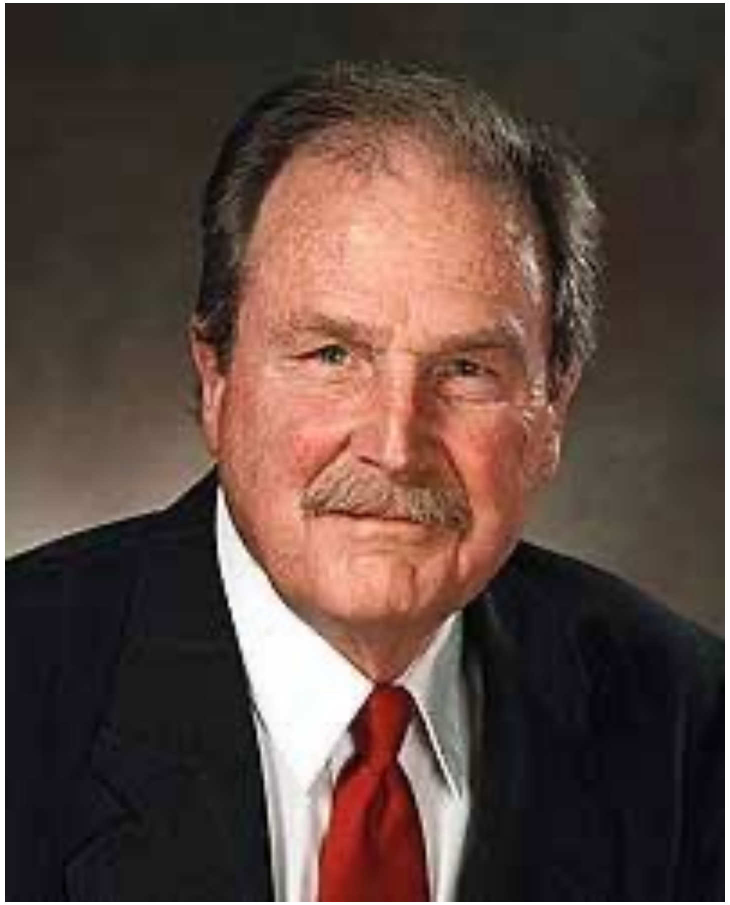
# RUN-LENGTH CODING

- Idea: Split up a message into sequences of identical symbols.
  - These sequences are referred to as runs.
  - Runs are represented as a single instance of the repeated value followed by a repetition count.
- Great for files containing long runs of repeated data.



# HUFFMAN CODING

- Developed by David A. Huffman.
  - Distinguished member of the UCSC faculty.
  - One of the founders of UCSC's Computer Science department.
- Idea: Assign each symbol a unique bit-string code.
  - Generate a histogram of unique symbols in data.
  - The more frequently a symbol appears, the shorter its code.
- So how does it work?



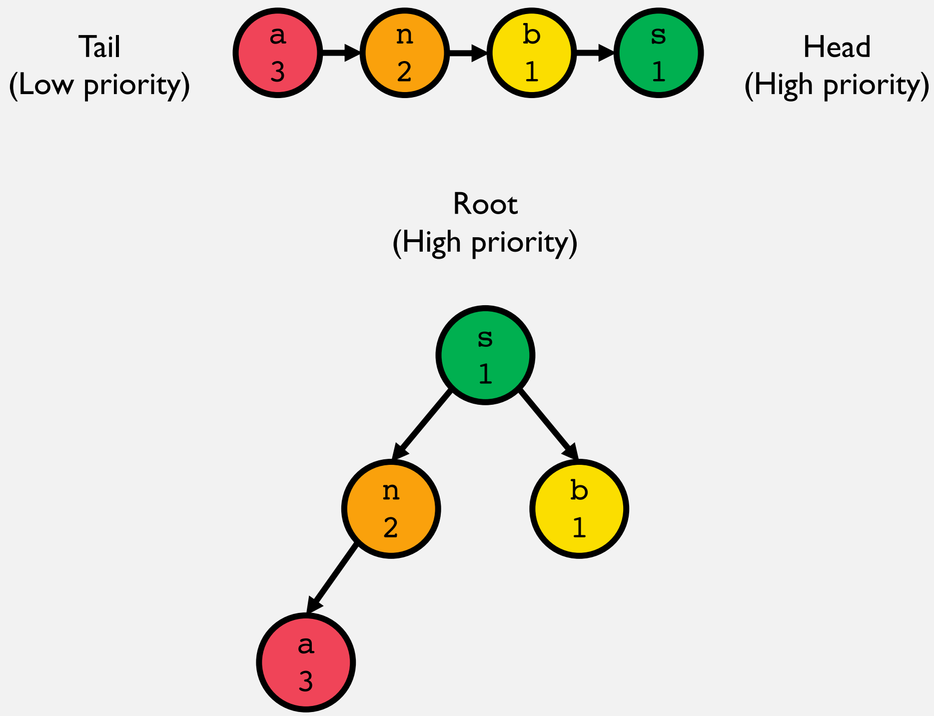
# CONSTRUCTING A HISTOGRAM

- We'll compress the message `"bananas"`.
- First, we create a histogram of the message's unique symbol and their frequencies.
- We assume that each symbol is an ASCII character.
- We can represent this histogram as an array with 256 indices.

| Symbol | Frequency |
|---|---|
| 0 | 0 |
| … | |
| a = 97 | 3 |
| b = 98 | 1 |
| … | 0 |
| n = 110 | 2 |
| … | |
| s = 115 | 1 |
| 255 | 0 |

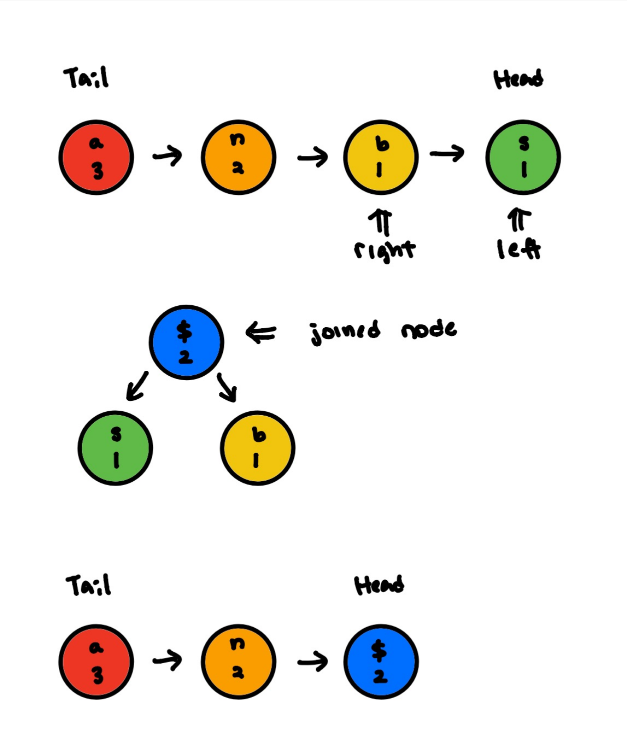
# PRIORITY QUEUING

- Each symbol with a non-zero frequency is added to a priority queue as a node.
- The lower the frequency, the higher the priority.
- In a priority queue, the only guarantee is that the highest priority element is first.
  - The order of the rest is not well defined.
  - We can provide this guarantee by:
    1. Sorting the queue.
    2. Using a heap.



# BUILDING A HUFFMAN TREE

- We will create a Huffman tree using the nodes in the priority queue.
- While the queue has more than one node,
  1. Dequeue a node. This will be the left child node.
  2. Dequeue another node. This will be the right child node.
  3. Create a parent node for the left and right child nodes. The frequency of the parent node is the sum of its children's frequencies.
  4. Enqueue the parent node.
- The last node in the queue is the root of the tree.



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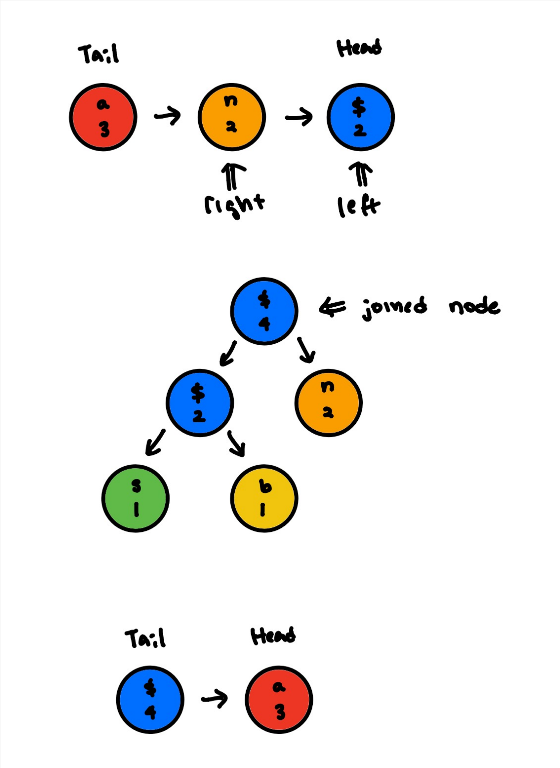

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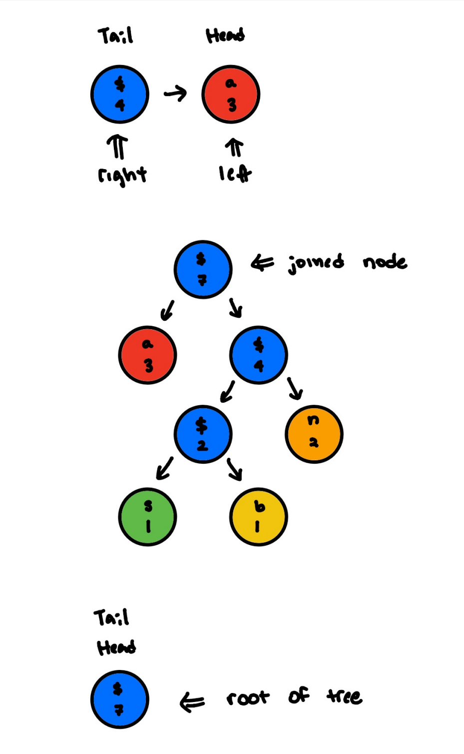

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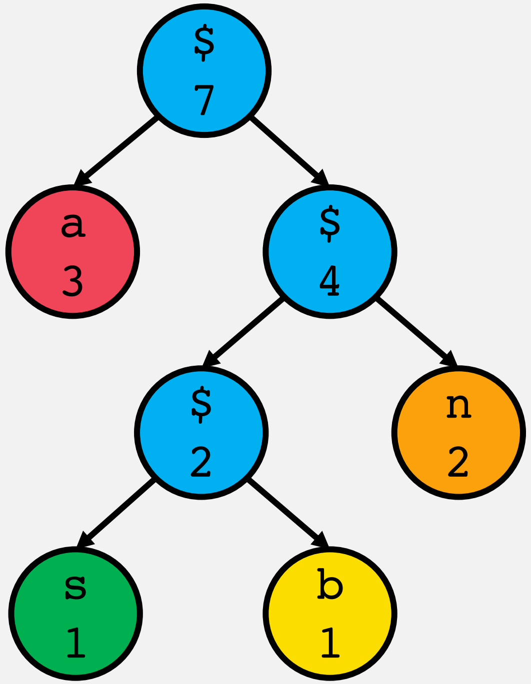
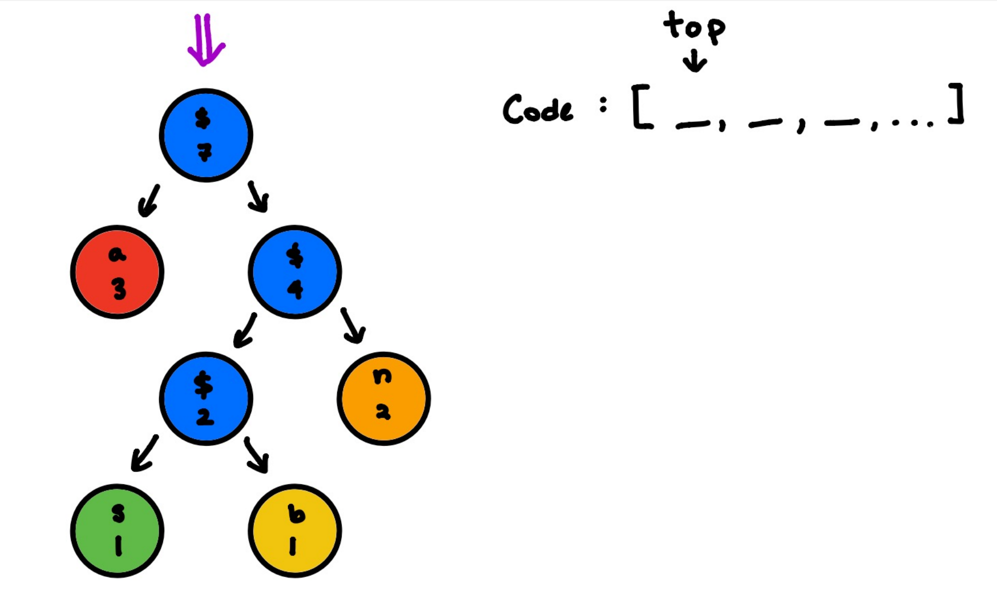


# BUILDING THE CODE TABLE

- We now populate a table of codes.
  - Like the histogram there are 256 indices, and each index stores a binary code.
- We will utilize a stack of bits to keep track of the code.
- To build these codes, we perform a *post-order traversal.*
  - If we walk down to the left child, we push a 0 to the bit-stack.
  - If we walk down to the right child, we push a 1 to the bit-stack.
  - If we reach a leaf node, the code for the node's symbol is the code in the bit-stack.
- A code can be, at maximum, 256 bits long!



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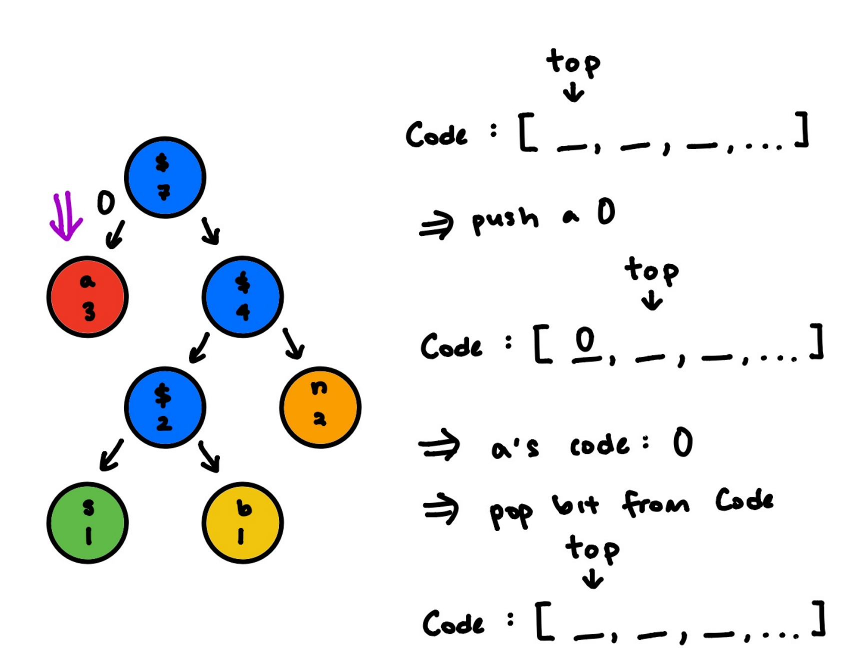

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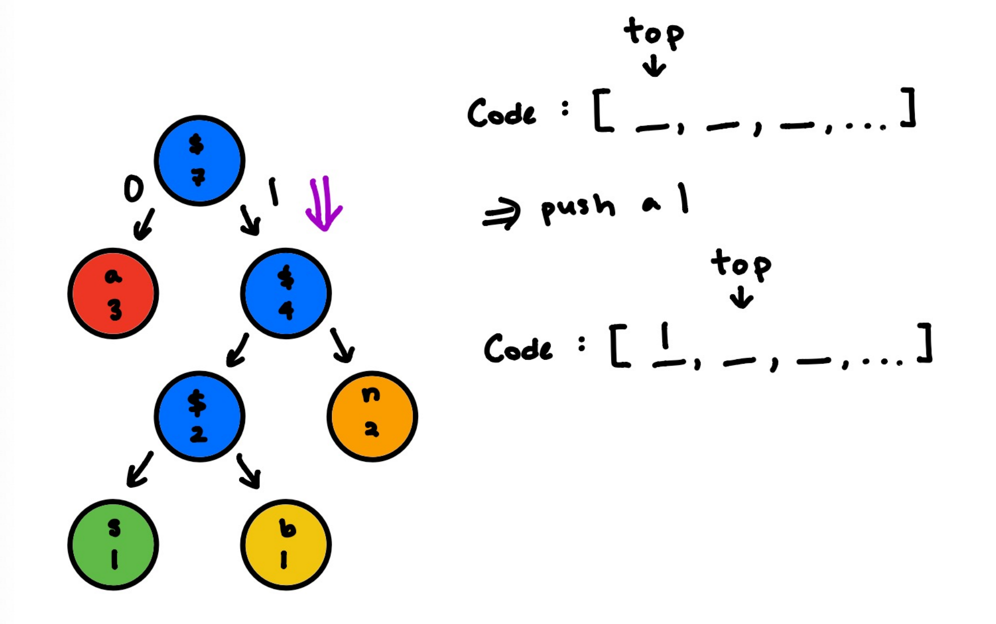

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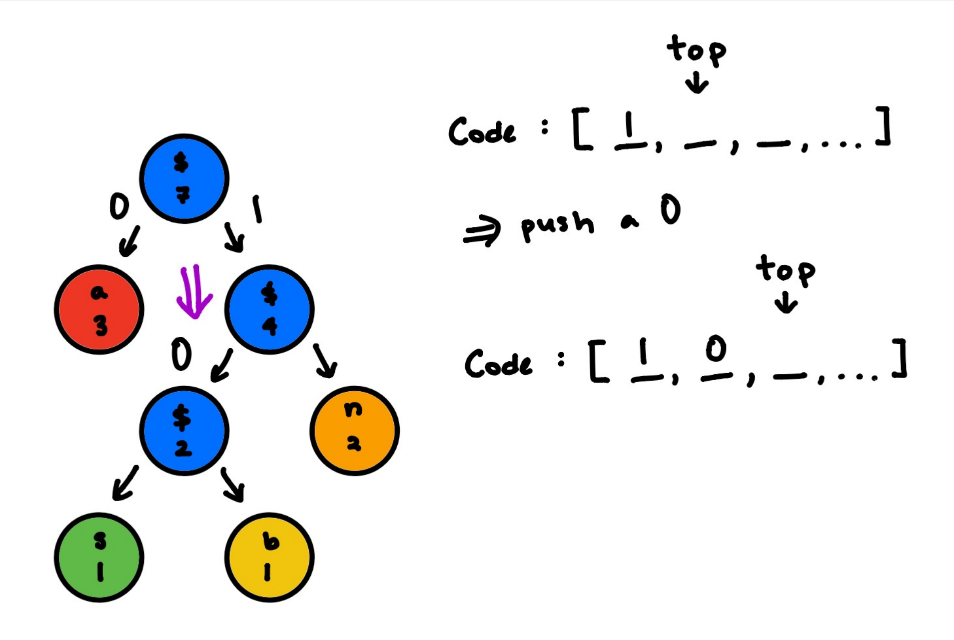

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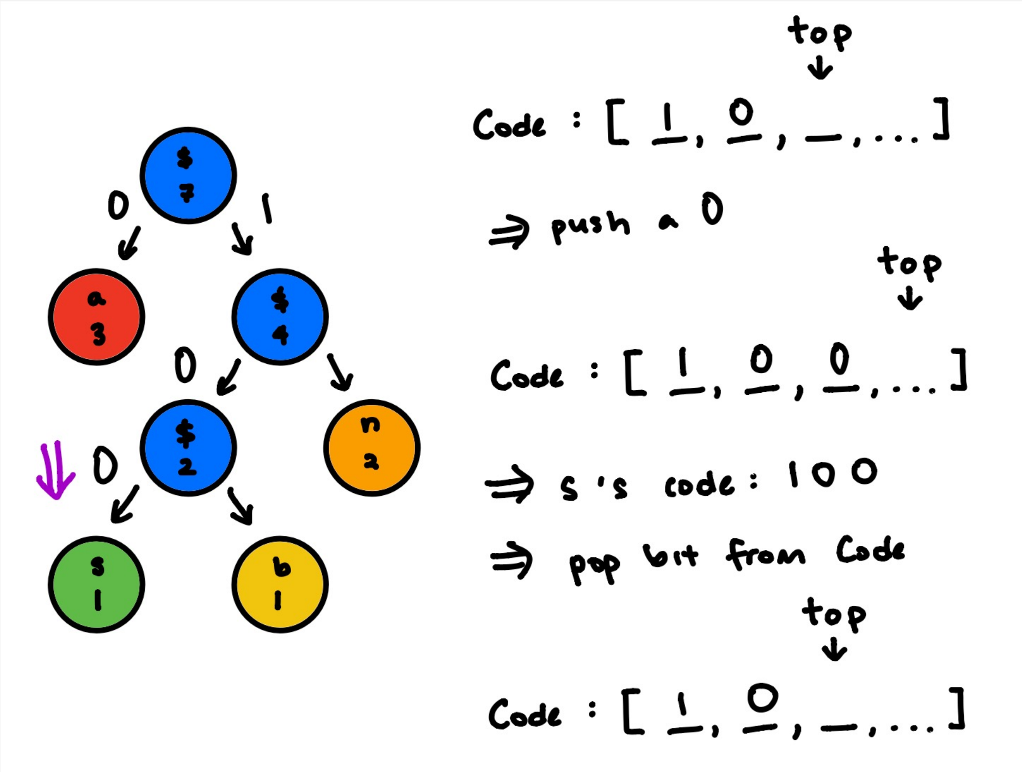

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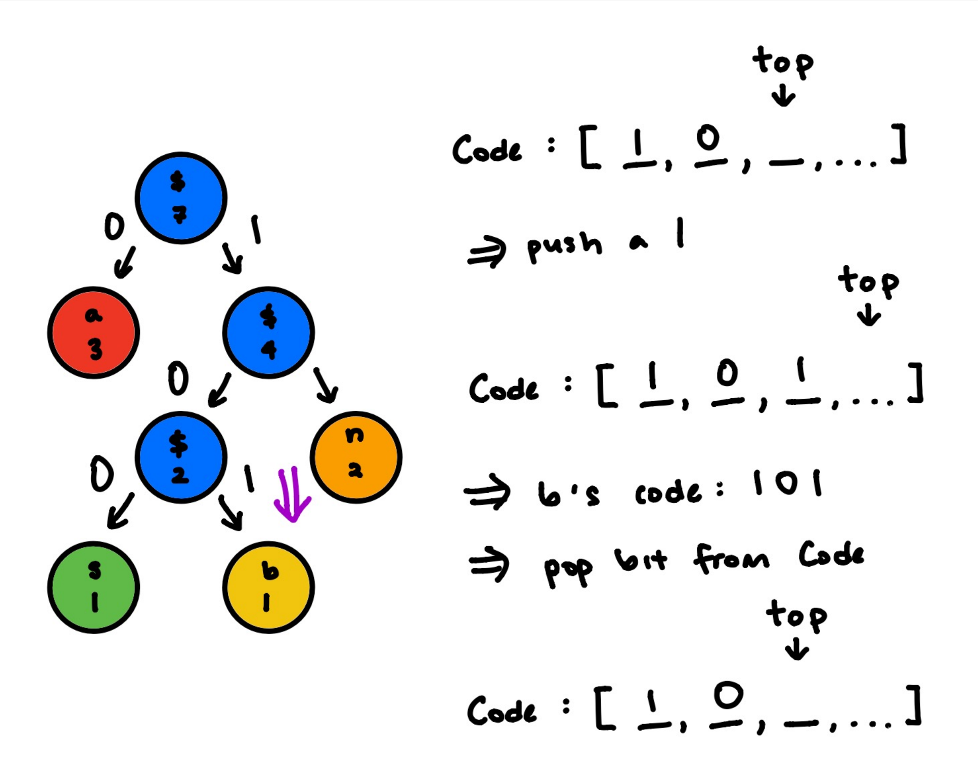

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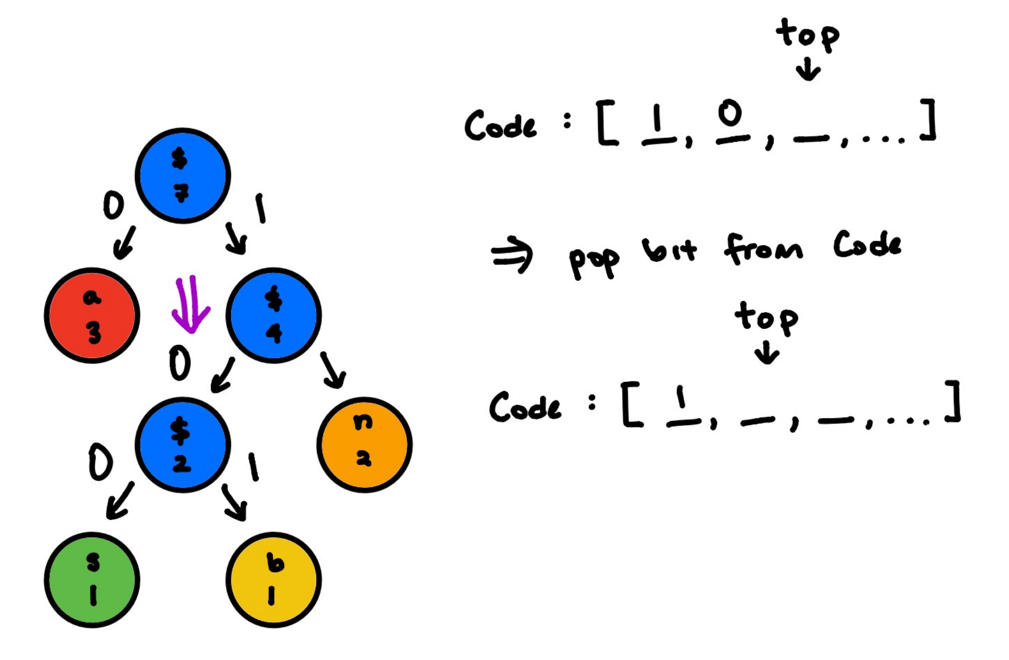

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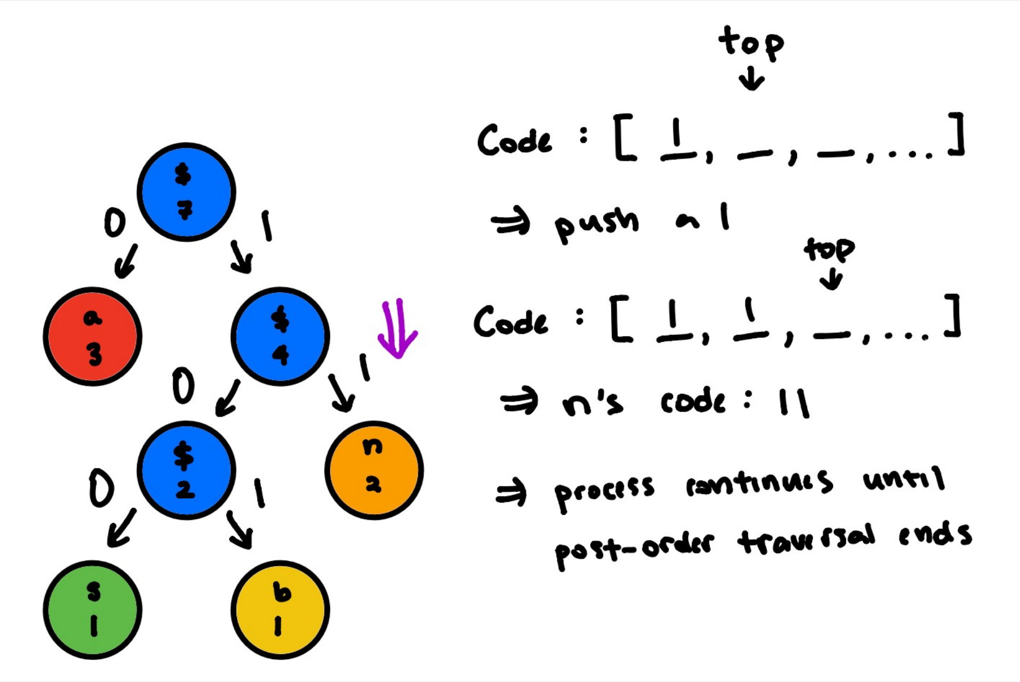

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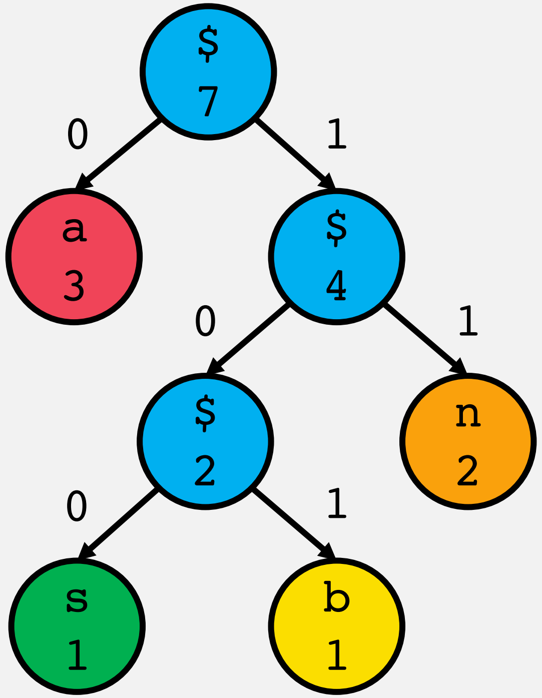
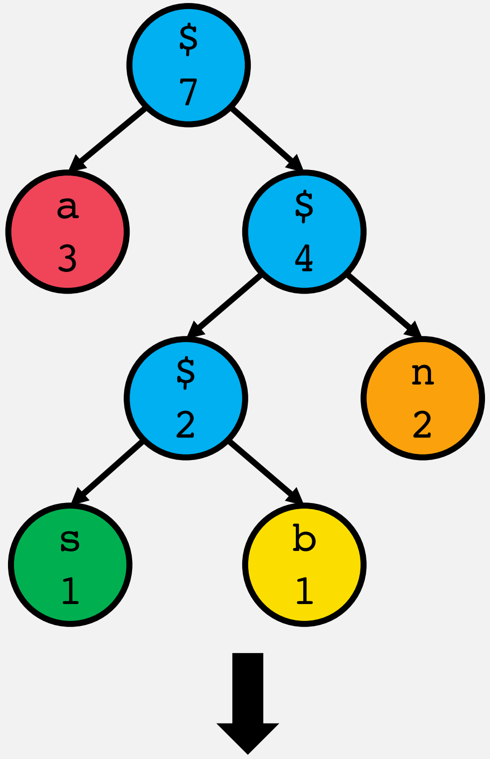



| Symbol | Code |
|---|---|
| 0 | |
| … | |
| a = 97 | 0 |
| b = 98 | 101 |
| … | |
| n = 110 | 11 |
| … | |
| s = 115 | 100 |
| 255 | |

# DUMPING THE TREE

- Some versions of Huffman coding emit the constructed tree.
  - The tree is dumped to the encoding through a post-order traversal.
- Performing a post-order traversal of the tree,
  - If a leaf node is reached, output an `'L'` followed by the node's symbol.
  - If an interior node's children have been traversed, output an `'I'` to indicate an interior node.
- The number of symbols representing the tree dump is $(3 \times \text{leaves}) - 1$.




`LaLsLbILnII`

# EMITTING CODES

- For each symbol in the message, output its code.
- We use the constructed code table for fast symbol to code translation.
- It's a function – table: symbol → code.

| Symbol | Code |
|---|---|
| 0 | |
| … | |
| 97 | 0 |
| 98 | 101 |
| … | |
| 110 | 11 |
| … | |
| 115 | 100 |
| 255 | |

bananas

↓

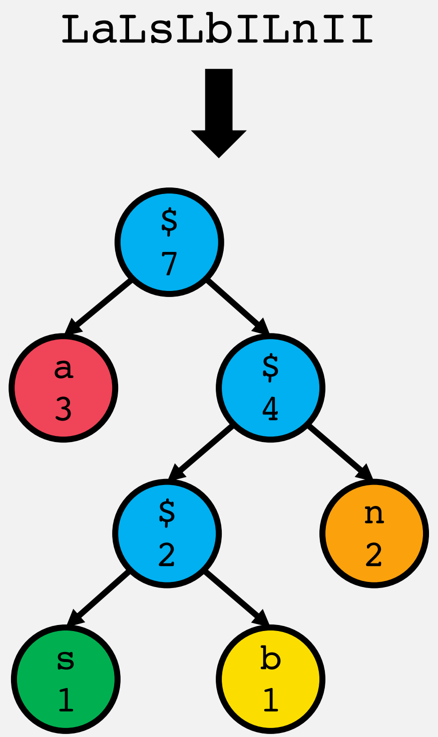
[101] [0] [11] [0] [11] [0] [100]

↓

1010110110100

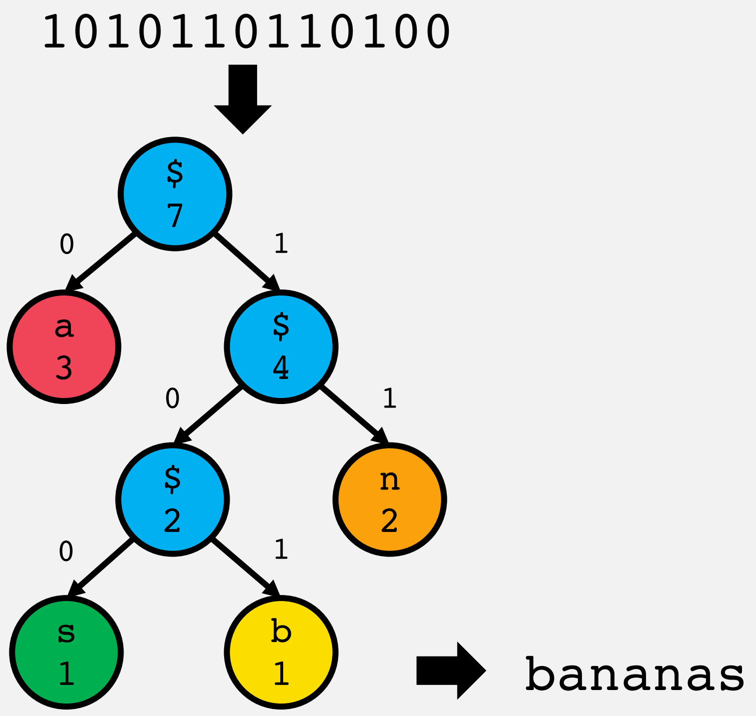
# RECONSTRUCTING THE TREE

- To decompress the compressed message, we first reconstruct the tree from its dump.
  - Recall that the dump was performed through post-order traversal.
- Initialize a stack of nodes.
- While all the symbols of the dump haven't been processed,
  - If the current symbol is an 'L', then the next symbol in the dump is the leaf's symbol.
    - Put this symbol into a node and push it onto the node-stack.
  - If the current symbol is an 'I', then an interior node has been reached.
    - Pop the stack for the right child, then pop again for the left child.
    - Create the parent node for these children and push it onto the node-stack.



# DECODING BIT-BY-BIT

- To decode the binary codes, we walk the tree.
- Read in each bit from the input.
  - If the bit is a 0, walk down to the left child.
  - If the bit is a 1, walk down to the right child.
  - If a leaf node is reached, output its symbol and restart from the root of the tree.

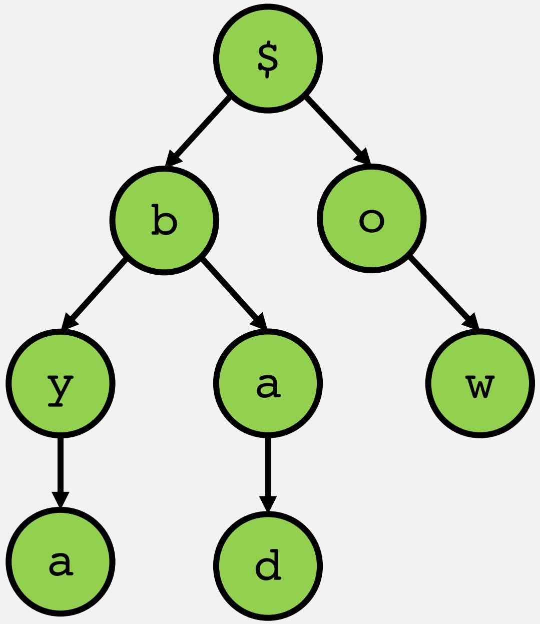


# LEMPEL-ZIV CODING (LZ78)

- Developed by Abraham Lempel and Jacob Ziv.
- An adaptive dictionary coder.
  - Dictionaries aren't output as part of encoding.
  - Dictionaries are built incrementally as encoded data is processed.
- Idea: If a message isn't uniformly random, it is likely to contain recurring patterns.
  - Use a dictionary to store seen patterns and give them unique codes.
- Terminology :
  - Word – a sequence of bytes.
  - Code – an unsigned 16-bit integer that denotes a word.
  - Pair – a code followed by a symbol.

# A DICTIONARY OF WORDS

- The best-case scenario for LZ78 is a message with words with long, common prefixes.
  - Which means, the larger the codes, the longer the words they denote.
- To store these words, we use a trie.
  - Named because it is an efficient information re-*trie*-val data structure.
  - Also called a prefix tree.
  - The trie node representing the end of a word stores the code for the word.
- Minimizes redundancies when storing many words with common prefixes.



# LZ78 ENCODING

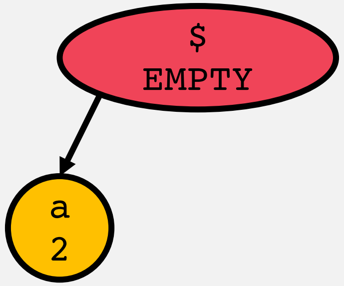
- The message to encode is “`abababa`”.
- Initialize the trie to just the root node.
  - It will be given the code `EMPTY = 1` since the root denotes the empty word.
- We will need to keep track of the current node, which is colored: .
- Nodes that are added to the trie at any step are colored: .
- All other uninteresting nodes are colored: .

# LZ78 ENCODING

- Message: “ababab a”.
- The first symbol, and thus the current symbol, to consider is ‘a’.
- Check if the current node has the child ‘a’.

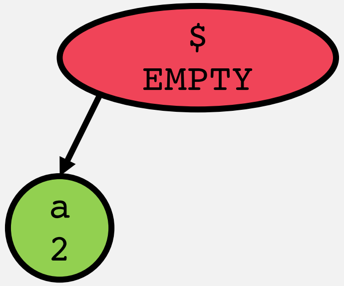
# LZ78 ENCODING

- Since it doesn't, we add the child.
  - The code for the child is the next unused code.
  - In this case, the code is 2.
- Each time we come across a word that doesn't exist in the trie, we output a pair.
  - The pair is made from the current code and symbol.
  - So we output `(EMPTY, 'a')`.
- We have just added a new word to the trie, so now we reset back to the root.



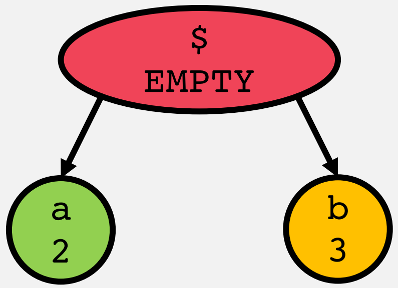
# LZ78 ENCODING

- Message: "abababa".
- The current symbol to consider is 'b'.
- Check if the current node has the child 'b'.



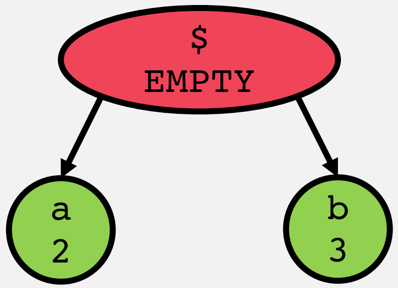
# LZ78 ENCODING

- Since it doesn't, we add the child.
  - The code for the child is the next unused code.
  - In this case, the code is 3.
- We output the pair `(EMPTY, 'b')`.
- We have just added a new word to the trie, so now we reset back to the root.



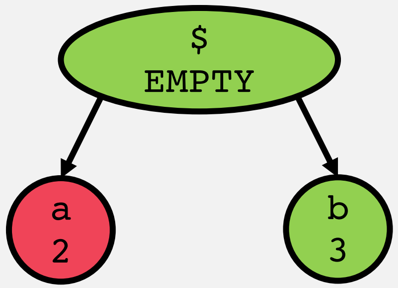
# LZ78 ENCODING

- Message: "ab<u>a</u>baba".
- The current symbol to consider is 'a'.
- Check if the current node has the child 'a'.



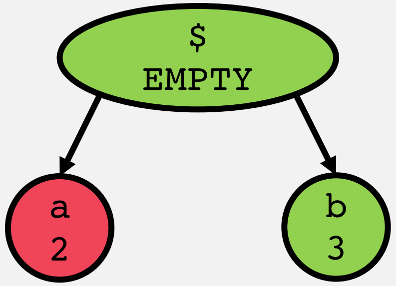
# LZ78 ENCODING

- Since it does, we simply step down the trie.
  - The current node is now the child node containing `'a'`.
- Since we haven't added anything, no pair is output.



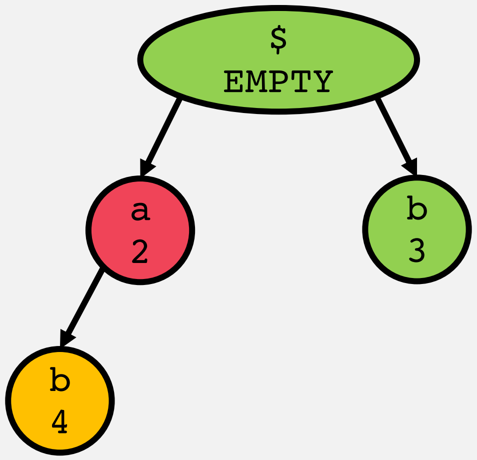
# LZ78 ENCODING

- Message: "`ababab̲aba`".
- The current symbol to consider is `'b'`.
- Check if the current node has the child `'b'`.



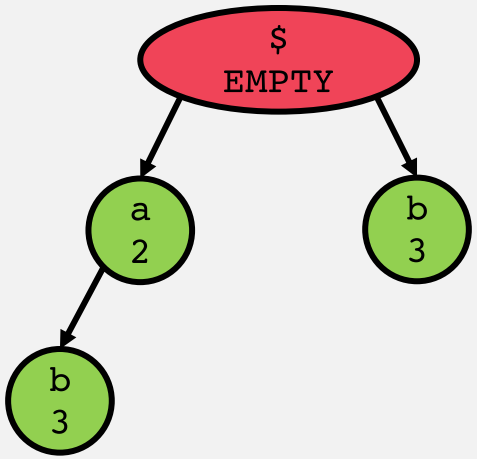
# LZ78 ENCODING

- Since it doesn't, we add the child.
  - The code for the child is the next unused code.
  - In this case, the code is 4.
- We output the pair `(2, 'b')`.
- We have just added a new word to the trie, so now we reset back to the root.



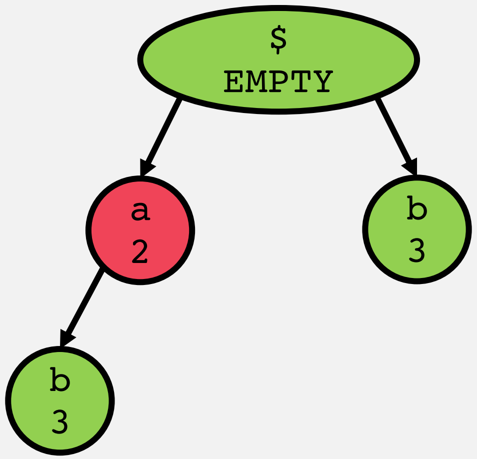
# LZ78 ENCODING

- Message: "ababab<u>a</u>ba".
- The current symbol to consider is 'a'.
- Check if the current node has the child 'a'.



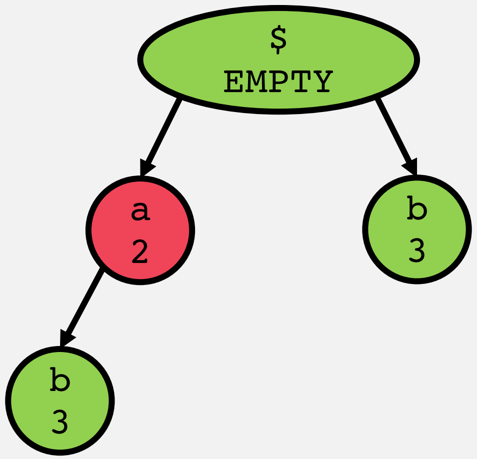
# LZ78 ENCODING

- Since it does, we simply step down the trie.
  - The current node is now the child node containing `'a'`.
- Since we haven't added anything, no pair is output.



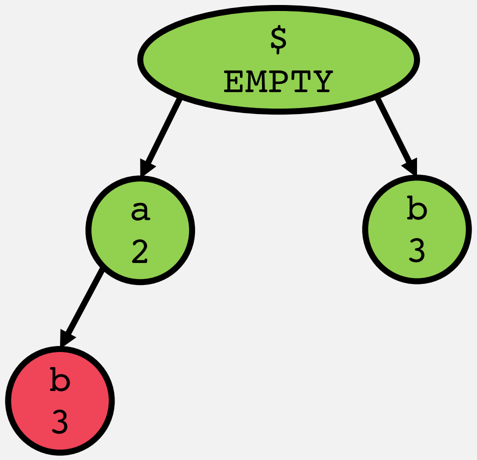
# LZ78 ENCODING

- Message: "abababa".
- The current symbol to consider is 'b'.
- Check if the current node has the child 'b'.



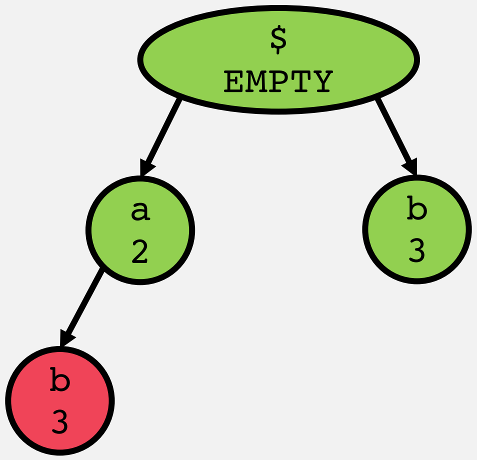
# LZ78 ENCODING

- Since it does, we simply step down the trie.
  - The current node is now the child node containing `'b'`.
- Since we haven't added anything, no pair is output.



# LZ78 ENCODING

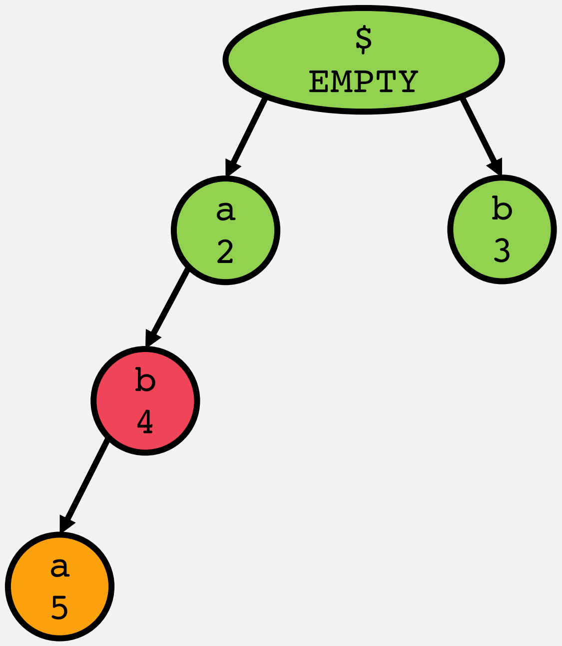
- Message: "`abababa`".
- The last symbol to consider is '`a`'.
- Check if the current node has the child '`a`'.



# LZ78 ENCODING

- Since it doesn't, we add the child.
  - The code for the child is the next unused code.
  - In this case, the code is 5.
- We output the pair `(4, 'a')`.
- The message has now been compressed to the following pairs, in order:

1. `(EMPTY, 'a')`
2. `(EMPTY, 'b')`
3. `(2, 'b')`
4. `(4, 'a')`



# LZ78 DECODING

- To decompress, we decode the pairs, filling out a dictionary as we go.
  - The key is a code, and the value the word that the code denotes.
- Like with the trie used in compression, the empty word is added initially to the dictionary.
  - Compression and decompression must agree on the starting set of words and codes.

| Code | Word |
|---|---|
| EMPTY | “” |
| | |
| | |
| | |
| | |

# LZ78 DECODING

- Pairs:
  1. `(EMPTY, ‘a’)` ←
  2. `(EMPTY, ‘b’)`
  3. `(2, ‘b’)`
  4. `(4, ‘a’)`
- We append `‘a’` to the word denoted by `EMPTY`.
  - This yields the new word: `"a"`.
  - We add this word to the dictionary, giving it the next unused code of 2.
- Output the word.
  - Current output: `“a”`.

| Code | Word |
|---|---|
| `EMPTY` | `“”` |
| 2 | `"a"` |
| | |
| | |
| | |

# LZ78 DECODING

- Pairs:
  1. `(EMPTY, ‘a’)`
  2. `(EMPTY, ‘b’)` ←
  3. `(2, ‘b’)`
  4. `(4, ‘a’)`
- We append `‘b’` to the word denoted by `EMPTY`.
  - This yields the new word: `"b"`.
  - We add this word to the dictionary, giving it the next unused code of 3.
- Output the word.
  - Current output: `“ab”`.

| Code | Word |
|---|---|
| EMPTY | “” |
| 2 | "a" |
| 3 | “b” |
| | |
| | |

# LZ78 DECODING

- Pairs:
  1. `(EMPTY, ‘a’)`
  2. `(EMPTY, ‘b’)`
  3. `(2, ‘b’)` ←
  4. `(4, ‘a’)`
- We append `‘b’` to the word denoted by `2`.
  - This yields the new word: `”ab”`.
  - We add this word to the dictionary, giving it the next unused code of 4.
- Output the word.
  - Current output: `“abab”`.

| Code | Word |
|---|---|
| EMPTY | “” |
| 2 | ”a” |
| 3 | “b” |
| 4 | “ab” |
| | |

# LZ78 DECODING

- Pairs:
  1. `(EMPTY, ‘a’)`
  2. `(EMPTY, ‘b’)`
  3. `(2, ‘b’)`
  4. `(4, ‘a’)` ←
- We append `‘a’` to the word denoted by `4`.
  - This yields the new word: `”aba”`.
  - We add this word to the dictionary, giving it the next unused code of 5.
- Output the word.
  - Current output: `“abababa”`.

| Code | Word |
| --- | --- |
| EMPTY | “” |
| 2 | ”a” |
| 3 | “b” |
| 4 | “ab” |
| 5 | “aba” |

# SUMMARY

- Two lossless compression algorithms:
  1. Huffman Coding
  2. LZ78
- Knowing that messages exhibit varying levels of entropy, it is good to pick your tools wisely.
  - Would it be good to use LZ78 for messages with high entropy?
  - Would it be better to use Huffman over LZ78 for message with low entropy?
- Compression algorithms have their limits.
  - They can't hope to compress messages consisting of uniform randomness.