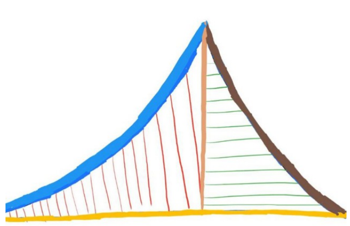
CBSE MATH

Made Simple

G. V. V. Sharma



Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2022

1.1.1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that

$$\left|2\overrightarrow{a}+3\overrightarrow{b}\right|=\left|3\overrightarrow{a}-2\overrightarrow{b}\right|. \tag{1.1.1.1}$$

Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

1.1.2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\overrightarrow{a}=\hat{i}-\hat{j}+\hat{k} \tag{1.1.2.1}$$

and

$$\overrightarrow{b}=2\hat{i}-\hat{j}-3\hat{k} \tag{1.1.2.2}$$

then find the vector $\overrightarrow{c}$, given that

$$\overrightarrow{a}\times\overrightarrow{c}=\overrightarrow{b} \tag{1.1.2.3}$$

and

$$\overrightarrow{a}.\overrightarrow{c} = 4. \tag{1.1.2.4}$$

1.1.3.

$$If \left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 + \left|\overrightarrow{a}.\overrightarrow{b}\right|^2 = 400 \tag{1.1.3.1}$$

and

$$\left|\overrightarrow{b}\right| = 5 \tag{1.1.3.2}$$

find the value of $|\overrightarrow{a}|$.

1.1.4. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.1.4.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k} \tag{1.1.4.2}$$

, then find $\left|\overrightarrow{b}\right|$

1.1.5. If

$$|\overrightarrow{a}| = 3, \left|\overrightarrow{b}\right| = 2\sqrt{3} \tag{1.1.5.1}$$

and

$$\overrightarrow{a}.\overrightarrow{b} = 6, \tag{1.1.5.2}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

1.1.6. $|\overrightarrow{a}| = 8, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a}.\overrightarrow{b} = 12\sqrt{3}$, then the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is

(a) 24

(b) 144

(c) 2

(d) 12

1.1.7. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.1.7.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.1.7.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.1.8. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that $\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$ and

$$\overrightarrow{a} \times \overrightarrow{c} = 4\overrightarrow{b} \times \overrightarrow{d} \tag{1.1.8.1}$$

, then show that $(\overrightarrow{a} - 2\overrightarrow{d}$ is parallel to$(2\overrightarrow{b} - \overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b} \tag{1.1.8.2}$$

1.1.9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.1.9.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{1.1.9.2}$$

then find $\left|\overrightarrow{b}\right|$

1.1.10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|, \tag{1.1.10.1}$$

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

1.1.11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and θ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.1.11.1}$$

1.1.12. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and θ is the ang le between them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.1.12.1}$$

1.1.13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.1.13.1}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.1.13.2}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

1.1.14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{1.1.14.1}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.1.14.2}$$

1.1.15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{1.1.15.1}$$

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.1.15.2}$$

and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of λ.

1.1.16. If $\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.1.16.1}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.1.17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.1.17.1}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.1.17.2}$$

, then find the ratio $\frac{projection of vector \overrightarrow{a} on vector \overrightarrow{b}}{projection of vector \overrightarrow{b} on vector \overrightarrow{a}}$

1.1.18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If $\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{1.1.18.1}$$

are such that the vector $(\overrightarrow{a} + \lambda \overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of λ.

1.1.19. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points $\mathbf{A}(2,3,-4)$, $\mathbf{B}(3,-4,-5)$ and $\mathbf{C}(3,2,-3)$ and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to

(a) $\sqrt{113}$

(b) $\sqrt{185}$

(c) $\sqrt{203}$

(d) $\sqrt{209}$

1.1.20. **A** circle has its center at $(4,4)$. If one end ofa diameter is $(4,0)$, then find the coordinates ofother end.

1.1.21. Find the values λ, for which the distance of point $(2,1,\lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.1.21.1}$$

is $2\sqrt{2}$ units.

1.1.22. Find the coordinates of the point where the line through $(3,4,1)$ crosses the ZX-plane

1.1.23. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1,0,-2)$, $\mathbf{B}(0,2,1)$ and $\mathbf{C}(-1,4,1)$

1.1.24. Using integration, find the area of triangle region whose vertices are $(2,0)$, $(4,5)$ and $(1,4)$.

1.1.25. The distance between the points $(0,0)$ and $(a-b, a+b)$ is

(a) $2\sqrt{ab}$

(b) $\sqrt{2a^2+ab}$

(c) $2\sqrt{a^2+b^2}$

(d) $\sqrt{2a^2+2b^2}$

1.1.26. The value of m which makes the point $(0,0)$, $(2m,-4)$and $(3,6)$ collinear, is _____

1.1.27. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direct6on cosines of the line.

1.1.28. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \tag{1.1.28.1}$$

.

1.1.29. Find the directions cosines of a line parallel to line AB.

1.1.30. Find the direction cosines of a line whose cartesian equation is given as

$$3x+1 = 6y-2 = 1-z. \tag{1.1.30.1}$$

1.1.31. A vector of magnitude 9 units in the direction of the vector $-2\hat{\imath}-\hat{\jmath}+2\hat{k}$ is ______

1.1.32. The two adajacent sides of a parallelogram are represented by $2\hat{\imath}-4\hat{\jmath}-5\hat{k}$ and $\hat{\imath}+2\hat{\jmath}+3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

1.1.33. The two adjacent sides of a parallelogram are represented by vectors $2\hat{\imath}-4\hat{\jmath}+5\hat{k}$ and $\hat{\imath}-2\hat{\jmath}-3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

1.1.34. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{1.1.34.1}$$

and

$$\overrightarrow{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} \tag{1.1.34.2}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

1.1.35. Find the area of the quadrilateral $ABCD$ whose vertices are $\mathbf{A}(-4,-3)$, $\mathbf{B}(3,-1)$, $\mathbf{C}(0,5)$ and $\mathbf{D}(-4,2)$

1.1.36. If the points $\mathbf{A}(2,0)$, $\mathbf{B}(6,1)$, and $\mathbf{C}(p,q)$ form a triangle of area 12sq. units (positive only) and

$$2p+q=10, \tag{1.1.36.1}$$

then find the values of p and q.

Chapter 2

Linear Forms

2.1. 2022

2.1.1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2.1.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{2.1.2.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{2.1.2.2}$$

2.1.3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, $(0, 0, 7)$.

2.1.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

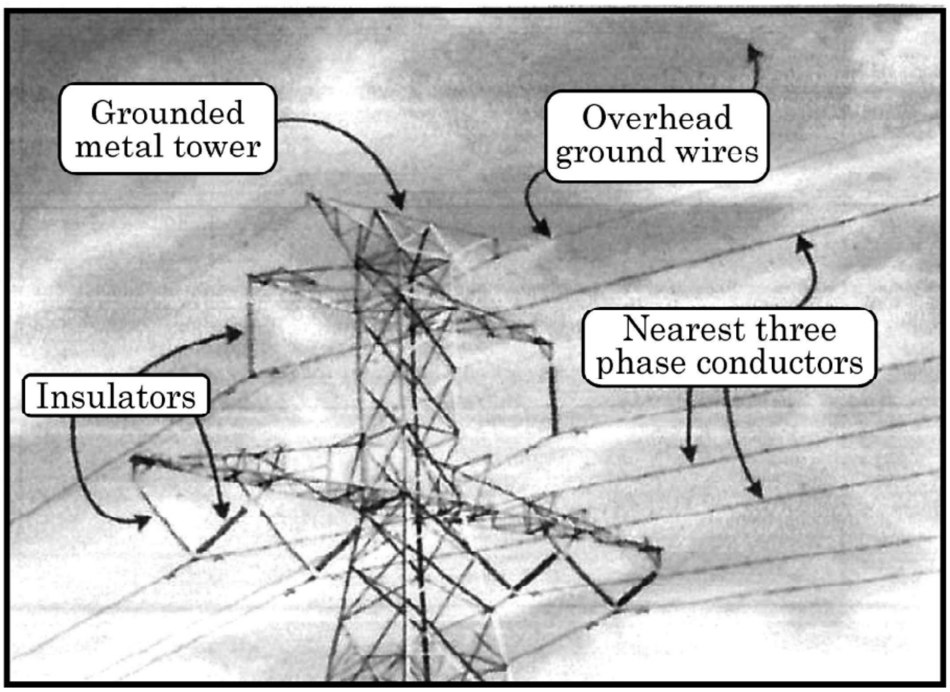


Figure 2.1.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1.4.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.1.4.1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.1.4.2}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

2.1.5. Write the cartesian equation of the line PQ passing through points

$\mathrm{P}(2,2,1)$ and $\mathrm{Q}(5,1,-2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.1.6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

2.1.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \tag{2.1.7.1}$$

$$\mathbf{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \tag{2.1.7.2}$$

2.1.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.1.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \tag{2.1.8.1}$$

$$\mathbf{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k}) \tag{2.1.8.2}$$

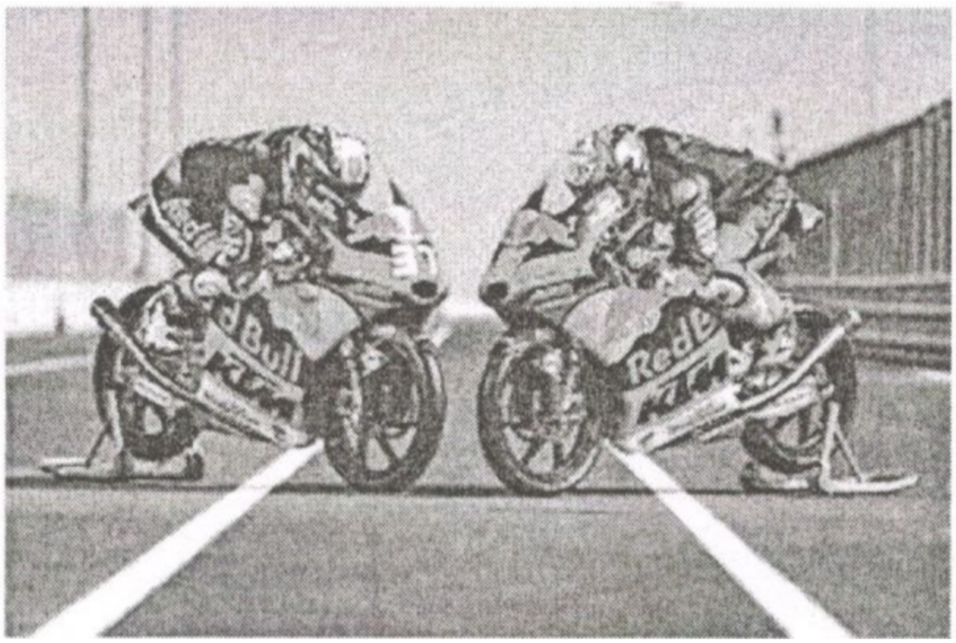


Figure 2.1.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

2.1.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{\imath} + (\lambda + 4)\hat{\jmath} - (\lambda - 3)\hat{k} \tag{2.1.9.1}$$

$$\mathbf{r} = (3 - \mu)\hat{\imath} + (2\mu + 2)\hat{\jmath} + (\mu + 6)\hat{k} \tag{2.1.9.2}$$

2.1.10. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.1.10.1}$$

2.1.11. Find the equation of the plane passing through the points $(2, 1, 0), (3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.

2.1.12. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

2.1.13. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

2.1.14. The distance between the planes $4x-4y+2z+5 = 0$ and $2x-2y+z+6 = 0$ is

(a) $\frac{1}{6}$

(b) $\frac{7}{6}$

(c) $\frac{11}{6}$

(d) $\frac{16}{6}$

2.1.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.1.15.1}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.1.15.2}$$

which is at a unit distance from the origin.

2.1.16. If the distance of the point $(1,1,1)$ from the plane $x-y+z+\lambda=0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2.1.17. Find the distance of the point $(2,3,4)$ measured along the line $\frac{x-4}{3}=\frac{y+5}{6}=\frac{z+1}{2}$ from the plane $3x+2y+2z+5=0$.

2.1.18. Find the distance of the point $P(4,3,2)$ from the plane determined by the points $A(-1,6,-5), B(-5,-2,3)$ and $C(2,4,-5)$.

2.1.19. The distance of the line

$$\mathbf{r}=(\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+5\hat{\jmath}+\hat{k}) \tag{2.1.19.1}$$

from the plane

$$\mathbf{r}\cdot(\hat{\imath}-\hat{\jmath}+4\hat{k})=5 \tag{2.1.19.2}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

2.1.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a}+\mathbf{b})$ and

$(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.1.20.1}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.1.20.2}$$

2.1.21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.1.21.1}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.1.21.2}$$

2.1.22. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

2.1.23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

2.1.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1.24.1:

Table 2.1.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

Chapter 3

Intersection of Conics

3.1. 2022

3.1.1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

3.1.2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3.1.3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

3.1.4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

3.1.5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

3.1.6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

3.1.7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

3.1.8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

3.1.9. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration

Chapter 4

Tangent And Normal

4.1. 2022

4.1.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

4.1.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

4.1.3. In Figure 4.1.3.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

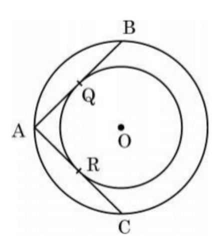


Figure 4.1.3.1: Two concentric circles with **O** as centre

4.1.4. In Figure 4.1.4.1, if a circle touches the side QR of ΔPQR at **S** and extended sides PQ and PR at **M** and **N**, respectively,

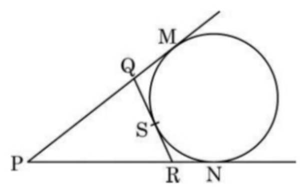


Figure 4.1.4.1: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

4.1.5. In Figure 4.1.5.1, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

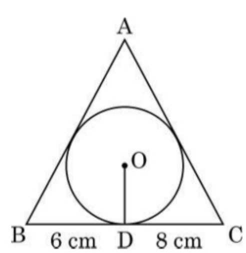


Figure 4.1.5.1: Circle with **O** as center circumscribed in triangle ABC

4.1.6. In Figure 4.1.6.1, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

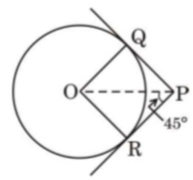


Figure 4.1.6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

4.1.7. In Figure 4.1.7.1, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.

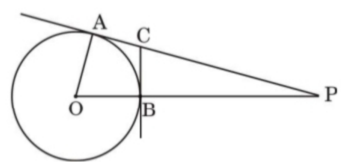


Figure 4.1.7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

4.1.8. In Figure 4.1.8.1, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

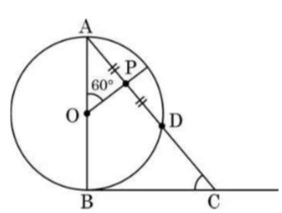


Figure 4.1.8.1: Tangent BC is drawn from point **C** to a circle whose centre is **O**

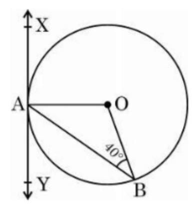


Figure 4.1.9.1: The line XAY is tangent to the circle centered at **O**

4.1.9. In Figure 4.1.9.1, XAY is a tangent to the circle centered at **O**. If $\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

4.1.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

4.1.11. In Figure 4.1.11.1, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.

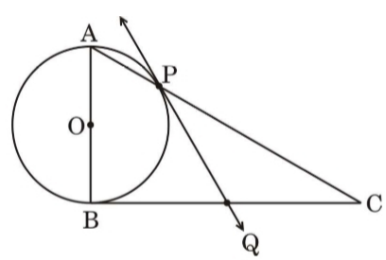


Figure 4.1.11.1: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

4.1.12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.