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# CBSE MATH

## Made Simple

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# Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.



# Chapter 1

## Vectors

### 1.1. 2023

#### 1.1.1. 10

1.1.1 In what ratio, does  $x$ -axis divide the line segment joinin the points  $\mathbf{A}(3, 6)$  and  $\mathbf{B}(-12, -3)$  ?

(a) 1 : 2

(b) 1 : 4

(c) 4 : 1

(d) 2 : 1

1.1.2 The distance between the point  $(0, 2\sqrt{5})$  and  $(-2\sqrt{5}, 0)$  is

(a)  $2\sqrt{10}$  units

(b)  $4\sqrt{10}$  units

(c)  $2\sqrt{20}$  units

(d) 0 units



1.1.3 If  $(-5, 3)$  and  $(5, 3)$  are two vertices of an equilateral triangle, then coordinates of the third vertex, given that origin lies inside the triangle (*take*  $\sqrt{3} = 1.7$ )

1.1.4 Show that the points  $(-2, 3)$ ,  $(8, 3)$  and  $(6, 7)$  are the vertices of right-angled triangle

1.1.5 If  $\mathbf{Q} = (0, 1)$  is equidistant from  $\mathbf{P} = (5, -3)$  and  $\mathbf{R} = (x, 6)$ , find the value of  $x$ .

1.1.6 The distance of the point  $(-6, 8)$  from origin is :

- (a) 6
- (b)  $-6$
- (c) 8
- (d) 10

1.1.7 The points  $(-4, 0)$ ,  $(4, 0)$  and  $(0, 3)$  are the vertices of a :

- (a) right triangle
- (b) isosceles triangle
- (c) equilateral triangle
- (d) scalene triangle

## 1.1.2. 12

1.1.1. Unit vector along  $\mathbf{PQ}$ , where coordinates of  $\mathbf{P}$  and  $\mathbf{Q}$  respectively are  $(2, 1, -1)$  and  $(4, 4, -7)$ , is

(a)  $2\hat{i} + 3\hat{j} - 6\hat{k}$

(b)  $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(c)  $-\frac{2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$

(d)  $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

1.1.2. If in  $\triangle ABC$ ,  $\overrightarrow{BA}=2\vec{a}$  and  $\overrightarrow{BC}=3\vec{b}$ , then  $\overrightarrow{AC}$  is

(a)  $2\vec{a} + 3\vec{b}$

(b)  $2\vec{a} - 3\vec{b}$

(c)  $3\vec{b} - 2\vec{a}$

(d)  $-2\vec{a} - 3\vec{b}$

1.1.3. Equation of line passing through origin and making  $30^\circ$ ,  $60^\circ$  and  $90^\circ$  with x, y, z axes respectively is

(a)  $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

(b)  $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

(c)  $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

(d)  $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

1.1.4. If  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$  are three non-zero unequal vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ , then find the angle between  $\vec{a}$  and  $\vec{b} - \vec{c}$ .

1.1.5. If the equation of a line is

$$x = ay + b, z = cy + d, \quad (1.1.5.1)$$

then find the direction ratios of the line and a point on the line.

- 1.1.6. Using Integration, find the area of triangle whose vertices are  $(-1, 1)$ ,  $(0, 5)$  and  $(3, 2)$ .

## 1.2. 2022

- 1.2.1.  $\vec{a}$  and  $\vec{b}$  are two unit vectors such that

$$\left| 2\vec{a} + 3\vec{b} \right| = \left| 3\vec{a} - 2\vec{b} \right|. \quad (1.2.1.1)$$

Find the angle between  $\vec{a}$  and  $\vec{b}$ .

- 1.2.2. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that

$$\vec{a} = \hat{i} - \hat{j} + \hat{k} \quad (1.2.2.1)$$

and

$$\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k} \quad (1.2.2.2)$$

then find the vector  $\vec{c}$ , given that

$$\vec{a} \times \vec{c} = \vec{b} \quad (1.2.2.3)$$

and

$$\vec{a} \cdot \vec{c} = 4. \quad (1.2.2.4)$$

1.2.3.

$$\text{If } \left| \vec{a} \times \vec{b} \right|^2 + \left| \vec{a} \cdot \vec{b} \right|^2 = 400 \quad (1.2.3.1)$$

and

$$\left| \vec{b} \right| = 5 \quad (1.2.3.2)$$

find the value of  $\left| \vec{a} \right|$ .

1.2.4. If

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1 \quad (1.2.4.1)$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k} \quad (1.2.4.2)$$

, then find  $\left| \vec{b} \right|$

1.2.5. If

$$\left| \vec{a} \right| = 3, \left| \vec{b} \right| = 2\sqrt{3} \quad (1.2.5.1)$$

and

$$\vec{a} \cdot \vec{b} = 6, \quad (1.2.5.2)$$

then find the value of  $\left| \vec{a} \times \vec{b} \right|$ .

1.2.6.  $|\vec{a}| = 8, |\vec{b}| = 3$  and  $\vec{a} \cdot \vec{b} = 12\sqrt{3}$ , then the value of  $\left| \vec{a} \times \vec{b} \right|$  is

(a) 24

(b) 144

(c) 2

(d) 12

1.2.7. If

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \quad (1.2.7.1)$$

and

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k} \quad (1.2.7.2)$$

, then find  $\vec{a} \cdot (\vec{b} \times \vec{c})$ .

1.2.8.  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  are four non-zero vectors such that  $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$  and

$$\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d} \quad (1.2.8.1)$$

, then show that  $(\vec{a} - 2\vec{d})$  is parallel to  $(2\vec{b} - \vec{c})$  where

$$\vec{a} \neq 2\vec{d}, \vec{c} \neq 2\vec{b} \quad (1.2.8.2)$$

1.2.9. If

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a} \cdot \vec{b} = 1 \quad (1.2.9.1)$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}, \quad (1.2.9.2)$$

then find  $|\vec{b}|$

1.2.10. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that

$$|\vec{a} + \vec{b}| = |\vec{b}|, \quad (1.2.10.1)$$

then prove that  $(\vec{a} + 2\vec{b})$  is perpendicular to  $\vec{a}$ .

1.2.11. If  $\vec{a}$  and  $\vec{b}$  are unit vectors and  $\theta$  is the angle between them, then prove that  $\sin$

$$\frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}| \quad (1.2.11.1)$$

1.2.12. If  $\vec{a}$  and  $\vec{b}$  are two unit vectors such that and  $\theta$  is the angle between them, then prove that

$$\sin \frac{\theta}{2} = \frac{1}{2} |\vec{a} - \vec{b}| \quad (1.2.12.1)$$

1.2.13. If

$$\vec{a} = 2\hat{i} + y\hat{j} + \hat{k} \quad (1.2.13.1)$$

and

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (1.2.13.2)$$

are two vectors for which the vector  $(\vec{a} + \vec{b})$  is perpendicular to the vector  $(\vec{a} - \vec{b})$  then find all the possible values of y.

1.2.14. Write the projection of the vector  $(\vec{b} + \vec{c})$  on the vector  $\vec{a}$ , where

$$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k} \quad (1.2.14.1)$$

and

$$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \quad (1.2.14.2)$$

1.2.15. If

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j} - 2\hat{k} \quad (1.2.15.1)$$

and

$$\vec{c} = \hat{i} + 3\hat{j} - \hat{k} \quad (1.2.15.2)$$

and the projection of vector  $\vec{c} + \lambda \vec{b}$  on vector  $\vec{a}$  is  $2\sqrt{6}$ , find the value of  $\lambda$ .

1.2.16. If  $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ ,  $\hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k} \quad (1.2.16.1)$$

, then find  $\vec{a} \cdot (\vec{b} \times \vec{c})$ .

1.2.17. If

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \quad (1.2.17.1)$$

and

$$\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \quad (1.2.17.2)$$

, then find the ratio  $\frac{\text{projection of vector } \vec{a} \text{ on vector } \vec{b}}{\text{projection of vector } \vec{b} \text{ on vector } \vec{a}}$

1.2.18. Show that the three vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$ , and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  form the vertices of a right-angled triangle. If  $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$  and

$$\vec{c} = 3\hat{i} + \hat{j} \quad (1.2.18.1)$$

are such that the vector  $(\vec{a} + \lambda \vec{b})$  is perpendicular to vector  $\vec{c}$ , then find the value of  $\lambda$ .

1.2.19. If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are the position vectors of the points  $\mathbf{A}(2, 3, -4)$ ,



$\mathbf{B}(3, -4, -5)$  and  $\mathbf{C}(3, 2, -3)$  and respectively, then  $\left| \vec{a} + \vec{b} + \vec{c} \right|$  is equal to

(a)  $\sqrt{113}$

(b)  $\sqrt{185}$

(c)  $\sqrt{203}$

(d)  $\sqrt{209}$

1.2.20. A circle has its center at  $(4, 4)$ . If one end of a diameter is  $(4, 0)$ , then find the coordinates of the other end.

1.2.21. Find the values  $\lambda$ , for which the distance of point  $(2, 1, \lambda)$  from plane

$$3x + 5y + 4z = 11 \quad (1.2.21.1)$$

is  $2\sqrt{2}$  units.

1.2.22. Find the coordinates of the point where the line through  $(3, 4, 1)$  crosses the ZX-plane

1.2.23. Using vectors, find the area of the triangle with vertices  $\mathbf{A}(-1, 0, -2)$ ,  $\mathbf{B}(0, 2, 1)$  and  $\mathbf{C}(-1, 4, 1)$

1.2.24. Using integration, find the area of triangle region whose vertices are  $(2, 0)$ ,  $(4, 5)$  and  $(1, 4)$ .

1.2.25. The distance between the points  $(0, 0)$  and  $(a - b, a + b)$  is

(a)  $2\sqrt{ab}$

(b)  $\sqrt{2a^2 + ab}$

(c)  $2\sqrt{a^2 + b^2}$

(d)  $\sqrt{2a^2 + 2b^2}$

1.2.26. The value of  $m$  which makes the point  $(0, 0)$ ,  $(2m, -4)$  and  $(3, 6)$  collinear, is \_\_\_\_\_

1.2.27. If a line makes  $60^\circ$  and  $45^\circ$  angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

1.2.28. The Cartesian equation of a line  $AB$  is :

$$\frac{2x - 1}{12} = \frac{y + 2}{2} = \frac{z - 3}{3} \quad (1.2.28.1)$$

.

1.2.29. Find the direction cosines of a line parallel to line  $AB$ .

1.2.30. Find the direction cosines of a line whose cartesian equation is given as

$$3x + 1 = 6y - 2 = 1 - z. \quad (1.2.30.1)$$

1.2.31. A vector of magnitude 9 units in the direction of the vector  $-2\hat{i} - \hat{j} + 2\hat{k}$  is \_\_\_\_\_

- 1.2.32. The two adjacent sides of a parallelogram are represented by  $2\hat{i} - 4\hat{j} - 5\hat{k}$  and  $\hat{i} + 2\hat{j} + 3\hat{k}$ . Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.
- 1.2.33. The two adjacent sides of a parallelogram are represented by vectors  $2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\hat{i} - 2\hat{j} - 3\hat{k}$ . Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.
- 1.2.34. If

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k} \quad (1.2.34.1)$$

and

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \quad (1.2.34.2)$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

- 1.2.35. Find the area of the quadrilateral  $ABCD$  whose vertices are  $\mathbf{A}(-4, -3)$ ,  $\mathbf{B}(3, -1)$ ,  $\mathbf{C}(0, 5)$  and  $\mathbf{D}(-4, 2)$
- 1.2.36. If the points  $\mathbf{A}(2, 0)$ ,  $\mathbf{B}(6, 1)$ , and  $\mathbf{C}(p, q)$  form a triangle of area 12sq. units (positive only) and

$$2p + q = 10, \quad (1.2.36.1)$$

then find the values of  $p$  and  $q$ .

## Chapter 2

# Linear Forms

## 2.1. 2022

2.1.1. Solve the equations  $x + 2y = 6$  and  $2x - 5y = 12$  graphically.

2.1.2. Solve the following equations for  $x$  and  $y$  using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \quad (2.1.2.1)$$

$$(bx + ay) + (b - 4a) = 0 \quad (2.1.2.2)$$

2.1.3. Find the co-ordinates of the point where the line  $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$  crosses the plane passing through the points  $\left(\frac{7}{2}, 0, 0\right), (0, 7, 0), (0, 0, 7)$ .

2.1.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

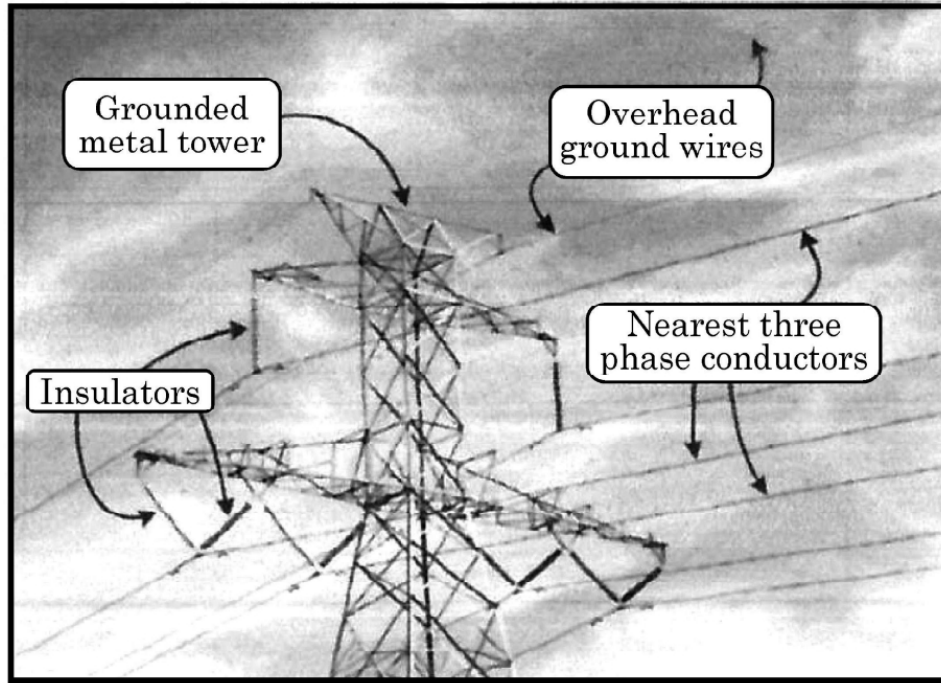


Figure 2.1.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1.4.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \quad (2.1.4.1)$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \quad (2.1.4.2)$$

Based on the given information, answer the following questions:

- (a) Are the  $l_1$  and  $l_2$  coplanar? Justify your answer.
- (b) Find the point of intersection of lines  $l_1$  and  $l_2$ .

2.1.5. Write the cartesian equation of the line PQ passing through points

P(2, 2, 1) and Q(5, 1, -2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.1.6. Find the distance between the lines  $x = \frac{y-1}{2} = \frac{z-2}{3}$  and  $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$ .

2.1.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \quad (2.1.7.1)$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \quad (2.1.7.2)$$

2.1.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.1.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad (2.1.8.1)$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \quad (2.1.8.2)$$

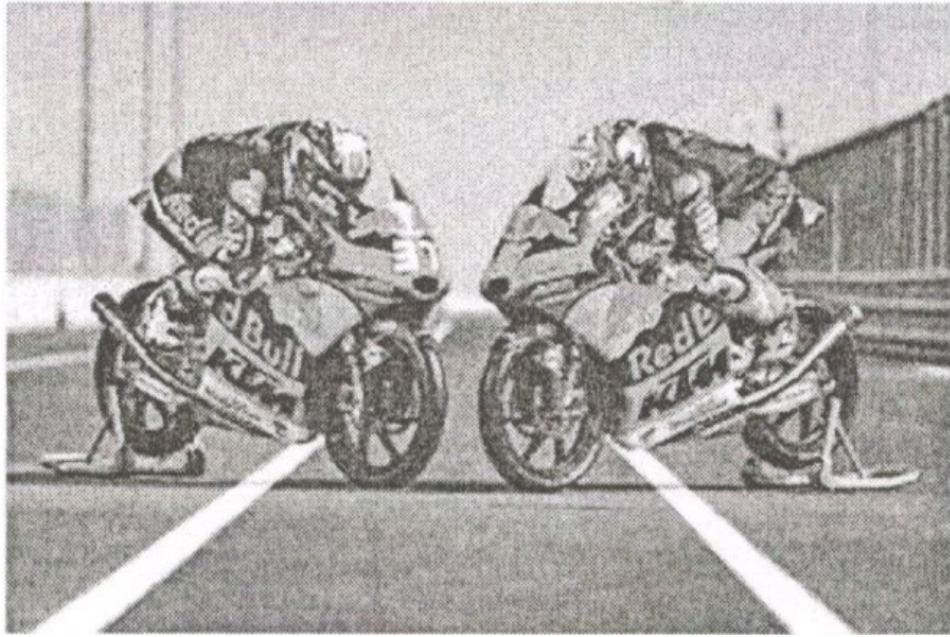


Figure 2.1.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

- (a) Find the shortest distance between the given lines.
- (b) Find a point at which the motorcycles may collide.

2.1.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \quad (2.1.9.1)$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \quad (2.1.9.2)$$

2.1.10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \quad (2.1.10.1)$$

2.1.11. Find the equation of the plane passing through the points  $(2, 1, 0)$ ,  $(3, -2, -2)$  and  $(1, 1, 7)$ . Also, obtain its distance from the origin.

2.1.12. The foot of a perpendicular drawn from the point  $(-2, -1, -3)$  on a plane is  $(1, -3, 3)$ . Find the equation of the plane.

2.1.13. Find the cartesian and the vector equation of a plane which passes through the point  $(3, 2, 0)$  and contains the line  $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ .

2.1.14. The distance between the planes  $4x-4y+2z+5=0$  and  $2x-2y+z+6=0$  is

- (a)  $\frac{1}{6}$
- (b)  $\frac{7}{6}$
- (c)  $\frac{11}{6}$
- (d)  $\frac{16}{6}$

2.1.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0 \quad (2.1.15.1)$$

$$\mathbf{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0 \quad (2.1.15.2)$$



which is at a unit distance from the origin.

2.1.16. If the distance of the point  $(1, 1, 1)$  from the plane  $x - y + z + \lambda = 0$  is  $\frac{5}{\sqrt{3}}$ , find the value(s) of  $\lambda$ .

2.1.17. Find the distance of the point  $(2, 3, 4)$  measured along the line  $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$  from the plane  $3x + 2y + 2z + 5 = 0$ .

2.1.18. Find the distance of the point  $P(4, 3, 2)$  from the plane determined by the points  $A(-1, 6, -5)$ ,  $B(-5, -2, 3)$  and  $C(2, 4, -5)$ .

2.1.19. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \quad (2.1.19.1)$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \quad (2.1.19.2)$$

is

- (a)  $\sqrt{2}$
- (b)  $\frac{1}{\sqrt{2}}$
- (c)  $\frac{1}{3\sqrt{2}}$
- (d)  $\frac{-2}{3\sqrt{2}}$

2.1.20. Find a unit vector perpendicular to each of the vectors  $(\mathbf{a} + \mathbf{b})$  and

$(\mathbf{a} - \mathbf{b})$  where

$$\mathbf{a} = \hat{i} + \hat{j} + \hat{k} \quad (2.1.20.1)$$

$$\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k} \quad (2.1.20.2)$$

2.1.21. Find the distance of the point  $(1, -2, 9)$  from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \quad (2.1.21.1)$$

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10. \quad (2.1.21.2)$$

2.1.22. Find the area bounded by the curves  $y = |x - 1|$  and  $y = 1$ , using integration.

2.1.23. Find the coordinates of the point where the line through  $(4, -3, -4)$  and  $(3, -2, 2)$  crosses the plane  $2x + y + z = 6$ .

2.1.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1.24.1:

Table 2.1.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

## Chapter 3

# Intersection of Conics

### 3.1. 2022

3.1.1. Using integration, find the area of the region enclosed by the curve  $y = x^2$ , the x-axis and the ordinates  $x = -2$  and  $x = 1$ .

3.1.2. Using integration, find the area of the region enclosed by line  $y = \sqrt{3}x$  semi-circle  $y = \sqrt{4 - x^2}$  and x-axis in first quadrant.

3.1.3. Using integration, find the area of the smaller region enclosed by the curve  $4x^2 + 4y^2 = 9$  and the line  $2x + 2y = 3$ .

3.1.4. If the area of the region bounded by the curve  $y^2 = 4ax$  and the line  $x = 4a$  is  $\frac{256}{3}$  sq. units, then using integration, find the value of  $a$ , where  $a > 0$ .

3.1.5. Find the area of the region enclosed by the curves  $y^2 = x$ ,  $x = \frac{1}{4}$ ,  $y = 0$  and  $x = 1$ , using integration.

3.1.6. If the area of the region bounded by the line  $y = mx$  and the curve  $x^2 = y$  is  $\frac{32}{3}$  sq. units, then find the positive value of  $m$ , using integration.

- 3.1.7. If the area between the curves  $x = y^2$  and  $x = 4$  is divided into two equal parts by the line  $x = a$ , then find the value of  $a$ , using integration.
- 3.1.8. Find the area bounded by the ellipse  $x^2 + 4y^2 = 16$  and the ordinates  $x = 0$  and  $x = 2$ , using integration.
- 3.1.9. Find the area of the region  $\{(x, y) : x^2 \leq y \leq x\}$ , using integration

## Chapter 4

# Tangent And Normal

### 4.1. 2022

4.1.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

4.1.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

4.1.3. In Figure 4.1.3.1, there are two concentric circles with centre **O**. If  $ARC$  and  $AQB$  are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of  $AC$ , if  $AQ = 5$  cm.

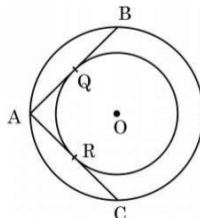


Figure 4.1.3.1: Two concentric circles with **O** as centre

- 4.1.4. In Figure 4.1.4.1, if a circle touches the side  $QR$  of  $\triangle PQR$  at **S** and extended sides  $PQ$  and  $PR$  at **M** and **N**, respectively,

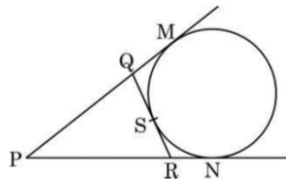


Figure 4.1.4.1: Two tangents are drawn from point **P** to the circle

prove that  $PM = \frac{1}{2}(PQ + QR + PR)$

- 4.1.5. In Figure 4.1.5.1, a triangle  $ABC$  is drawn to circumscribe a circle of radius 4 cm such that the segments  $BD$  and  $DC$  into which  $BC$  is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of  $\triangle ABC$  is  $84 \text{ cm}^2$ , find the lengths of sides  $AB$  and  $AC$ .

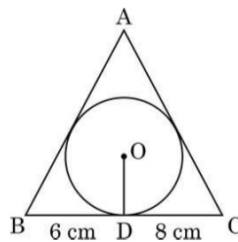


Figure 4.1.5.1: Circle with **O** as center circumscribed in triangle  $ABC$

- 4.1.6. In Figure 4.1.6.1,  $PQ$  and  $PR$  are tangents to the circle centered at **O**. If  $\angle OPR = 45^\circ$ , then prove that  $ORPQ$  is a square.

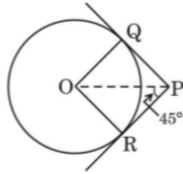


Figure 4.1.6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

4.1.7. In Figure 4.1.7.1, **O** is the centre of a circle of radius 5 cm.  $PA$  and  $BC$  are tangents to the circle at **A** and **B** respectively. If  $OP$  is 13 cm, then find the length of tangents  $PA$  and  $BC$ .

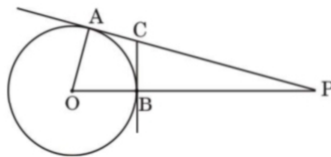


Figure 4.1.7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

4.1.8. In Figure 4.1.8.1,  $AB$  is diameter of a circle centered at **O**.  $BC$  is tangent to the circle at **B. If  $OP$  bisects the chord  $AD$  and  $\angle AOP = 60^\circ$ , then find  $m\angle C$ .**

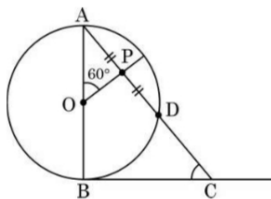


Figure 4.1.8.1: Tangent  $BC$  is drawn from point **C** to a circle whose centre is **O**



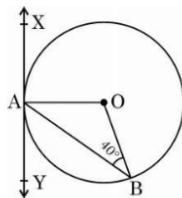


Figure 4.1.9.1: The line  $XAY$  is tangent to the circle centered at  $\mathbf{O}$

4.1.9. In Figure 4.1.9.1,  $XAY$  is a tangent to the circle centered at  $\mathbf{O}$ . If  $\angle ABO = 60^\circ$ , then find  $m\angle BAY$  and  $m\angle AOB$ .

4.1.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

4.1.11. In Figure 4.1.11.1, a triangle  $ABC$  with  $\angle B = 90^\circ$  is shown. Taking  $AB$  as diameter, a circle has been drawn intersecting  $AC$  at point  $\mathbf{P}$ . Prove that the tangent drawn at point  $\mathbf{P}$  bisects  $BC$ .

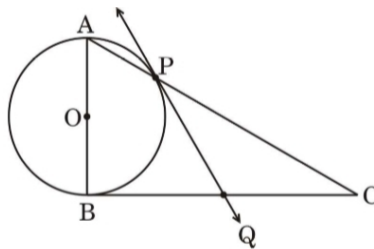


Figure 4.1.11.1:  $PQ$  is tangent to the circle centered at  $\mathbf{O}$ .  $AB$  is the diameter and  $\angle B = 90^\circ$

4.1.12. Find the equation of tangent to the curve  $y = x^2 + 4x + 1$  at the point  $(3, 22)$ .

## Chapter 5

# Probability

### 5.1. 2023

#### 5.1.1. 10

5.1.1. Probability of happening of an event is denoted by  $p$  and probability of non-happening of the event is denoted by  $q$ . Relation between  $p$  and  $q$  is

(a)  $p+q=1$

(b)  $p=1, q=1$

(c)  $p=q-1$

(d)  $p+q+1=0$

5.1.2. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought ?

(a) 40

(b) 240

(c) 480

(d) 750

5.1.3. In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/sh can swim, is

(a)  $\frac{3}{4}$

(b)  $\frac{1}{3}$

(c) 1

(d)  $\frac{1}{4}$

5.1.4. A bag contain 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that drawn ball is

(a) red

(b) yellow

5.1.5. A bag contain 100 cards numbered 1 to 100. A card is drawn at random from the bag. What is the probability that the number on the card is a perfect cube ?

(a)  $\frac{1}{20}$

(b)  $\frac{3}{50}$

(c)  $\frac{1}{25}$

(d)  $\frac{7}{100}$

5.1.6. If three coins are tossed simultaneously, what is the probability of getting at most one tail ?

(a)  $\frac{3}{8}$

(b)  $\frac{4}{8}$

(c)  $\frac{5}{8}$

(d)  $\frac{7}{8}$

5.1.7. Two dics are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is :

(a)  $\frac{1}{9}$

(b)  $\frac{2}{9}$

(c)  $\frac{1}{6}$

(d)  $\frac{1}{12}$

5.1.8. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is :

(a)  $\frac{1}{13}$

(b)  $\frac{9}{13}$

(c)  $\frac{4}{13}$

(d)  $\frac{12}{13}$

5.1.9. **Assertion (A) :** The probability that a leap year has 53 Students is  $\frac{2}{7}$ .

**Reason (R) :** The probability that a non-leap year has 53 Sundays is  $\frac{5}{7}$ .

- (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
- (b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).
- (c) Assertion (A) is true but Reason (R) is false.
- (d) Assertion (A) is false but Reason (R) is true.

## 5.1.2. 12

5.1.1. If A and B are two events such that

$$\Pr(A|B) = 2 \times \Pr(B|A) \Pr(A) + \Pr(B) = \frac{2}{3} \quad (5.1.1.1)$$

then  $\Pr(B)$  is equal to

- (a)  $\frac{2}{9}$
- (b)  $\frac{7}{9}$
- (c)  $\frac{4}{9}$
- (d)  $\frac{5}{9}$

5.1.2. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

(b) A and B throw a die alternately till one of them gets '6' and

wins the game. Find their respective probabilities of winning, if A starts the game first.

5.1.3. Recent studies suggest that roughly 12% of the world population is left handed.

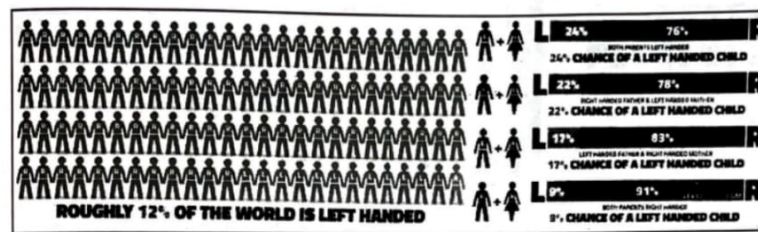


Figure 1: chance of left hand, depending upon parents

Figure 5.1.3.1: chance of left hand, depending upon parents

Depending upon the parents, the chances of having a left handed child are as follows :

- When both father and mother are left handed : Chances of left handed child is 24%.
- When father is right handed and mother is left handed : Chances of left handed child is 22%.
- when father is left handed and mother is right handed : Chances of left handed child is 17%.
- When both father nd mother are right handed : Chances of left handed child is 9%.

Assuming that  $\Pr(A) = \Pr(B) = \Pr(C) = \Pr(D) = \frac{1}{4}$  and L denotes the event that child is left handed. Based on the above information, answer the following questions :

- (a) Find  $\Pr(L|C)$
- (b) Find  $\Pr(\bar{L}|A)$
- (c) Find  $\Pr(A|L)$
- (d) Find the probability that a randomly selected child is left handed given that exactly one of the parent is left handed.

## 5.2. 2022

1. Let A and B be two events such that  $P(A) = \frac{5}{8}$ ,  $P(B) = \frac{3}{4}$ ,  $P(A/B) = \frac{3}{4}$ . Find the value of  $P(B/A)$ .
2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variable X denotes the number of red balls. Find the probability distribution of X.
3. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.
4. probabilities of A and B solving a specific problem are  $\frac{2}{3}$  and  $\frac{3}{5}$ , respectively. If both of them try independently to solve the problem,

then find the probability that the problem is solved.

5. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number appearing on at least one die is 3.
6. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. such a coin is unbiased with equal probabilities of getting head and tail. Based on

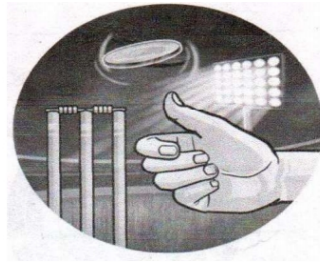


Figure 6.1: Toss before the match

the above information, answer the following question:

- (a) If such a coin is tossed 2 times, then find the probability distribution of numbers of tails.
  - (b) Find the probability of getting at least one head in three tosses of such a coin.
7. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.
  8. A pair of dice is thrown and the sum of the numbers appearing on the



dice is observed to be 7. Find the probability that the number 5 has appeared on atleast one die.

9. The probability that **A** hits the target is  $\frac{a}{3}$  and the probability that **B** hits it, is  $\frac{2}{5}$ . If both try to hit the target independently, find the probability that the target is hit.
10. A shopkeeper sells three of flower seeds **A1,A2,A3**. They are sold in the form of a mixture, where the proportions of these seeds are **4 : 4 : 2**, respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively. Based on the above information :



Figure 10.1: Three types of flowers

- (a) Calculate the probability that a randomly chosen seed will germinate;
- (b) Calculate the probability that the seed is of type A2, given that a randomly chosen seed germinates.
- (c) Three friends **A**, **B**, and **C** got their photograph clicked. Find the probability that **B** is standing at the central position, given

that **A** is standing at the left corner.

11. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are allotted points **9** to **1** in sequential order moving outwards. Archer **A** is likely to earn 10 points with a probability of 0.8 and Archer **B** is likely to earn 10 points with a probability of 0.9. Based on the above information,

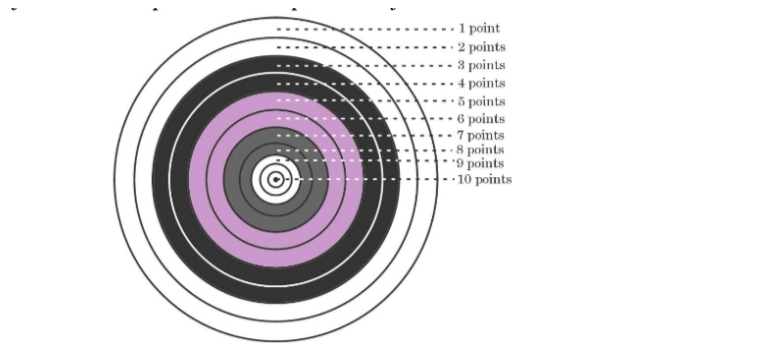


Figure 11.1: centermost ring

mation, answer the following questions :

- (a) exactly one of them earns 10 points .
- (b) both of them earn 10 point.
12. Event **A** and **B** are such that

$$p(A) = \frac{1}{2}, p(B) = \frac{7}{12} \text{ and } p(\bar{A} \cup \bar{B}) = \frac{1}{4} \quad (12.1)$$

Find whether the events **A** and **B** are independent or not.

13. A box  $B_1$  contain 1 white ball and 3 red balls. Another box  $B_2$  contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes  $B_1$  and  $B_2$ , then find the probability that the two balls drawn are of the same colour.
14. Let  $X$  be random variable which assumes values  $x_1, x_2, x_3, x_4$  such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4). \quad (14.1)$$

Find the probability distribution of  $X$ .

15. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes, is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.
16. In a toss of three different coins, find the probability of coming up of three heads, if it is known that at least one head comes up.
17. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.
18. A laboratory blood test is 98% effective in detecting a certain disease when it is fact, present. However, the test also yields a false positive result for 0.4% of the healthy person tested. From a large population,

it is given that 0.2% of the population actually has the diseases. Based on the above, answer the following question :

- (a) one person, from the population, is taken at random and given the test. Find the probability of his getting a positive test result.
- (b) what is the probability that the person actually has the disease, given that his test result is positive ?

19. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

- (a)  $\frac{1}{13} + \frac{1}{12}$
- (b)  $\frac{1}{13} \times \frac{4}{51}$
- (c)  $\frac{4}{52} \times \frac{3}{51}$
- (d)  $\frac{1}{13} \times \frac{1}{13}$

20. For two events A and B if  $p(A) = \frac{4}{10}$ ,  $PB = \frac{8}{10}$  and  $P(B-A) =$  then find  $P(A \cup B)$ .

21. Bag I contain 4 red and 3 black balls Bag II contains 3 red and 5 black balls. One of two bags is selected at random and a ball is drawn from the bag, which is found to be red. Find the probability that the ball is drawn from bag II.

22. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the num-

ber of aces and hences find its mean.

23. The probability of solving a specific question independently by A and B are  $\frac{1}{3}$  and  $\frac{1}{5}$  respectively . If both try to solve the question independently, the probability that the question is solved is

- (a)  $\frac{7}{15}$
- (b)  $\frac{8}{15}$
- (c)  $\frac{2}{15}$
- (d)  $\frac{14}{15}$

24. A card is packed at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability equation  $x \frac{dy}{dx} - y = \log x$  is \_\_\_\_\_.

25. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

26. Find the probability distribution of the numbers of successes in two tosses of a die, when a success is defined as "number greater than 5".

27. Ten cartoons are taken at random from an automatic packing machine. The mean net weight of the ten carton is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg ? [Given that for  $d.f. = 9, t_{0.05} = 2.26$ ]

28. coin is tossed twice. The following table shows the probability distribution of numbers of tails:

Table 28.1: Table shows the probability distribution of numbers of tails

X	0	1	2
P(X)	K	6K	9K

29. If X is a random variable with probability distribution as given below:

Table 29.1: table shows the probability distribution

X	0	1	2
P(X)	K	4K	K

30. The random variable X has a probability function P(x) as defined

$$\text{below, where K is some number : } P(X) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$



## Chapter 6

# Construction

### 6.1. 2022

1. In figure-4, BN and CM are medians of a  $\triangle ABC$  right-angled at A.

Prove that  $4(BN^2 + CM^2) = 5BC^2$

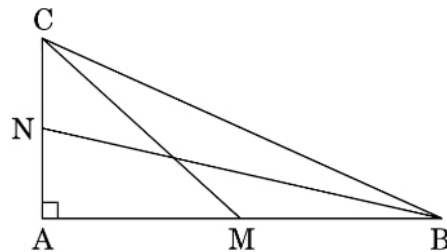


Figure 1.1: Right-angled triangle

2. **CaseStudy – 1 :**

#### KiteFestival

Kite festival is celebrated in many countries at different times of the year. In India, every year 14th January is celebrated as international kite Day. On this day many people visit India and participate in the



festival by flying various kinds of kites. The picture given below , three kites flying together.

In Fig. 5, the angles of elevation of two kites (point C) are found to

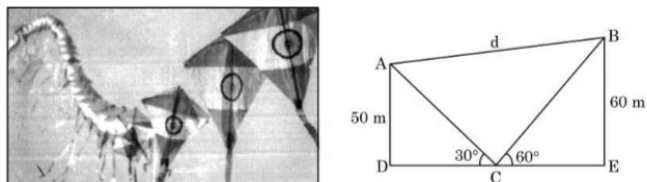


Figure 2.1: kites flying together

be  $30^\circ$  and  $60^\circ$  respectively. Taking  $AD = 50$  m and  $BE = 60$  m, find

- the length of string used (take them straight) for kites A and B as shown in the figure.
- the distance 'd' between these two kites

## Chapter 7

# Optimization

### 7.1. 2023

1. The objective function  $Z = ax + by$  of an LLP has maximum value 42 at (4,6) and minimum value 19 at (3,2). Which of the following is true?

(a)  $a = 9, b = 1$

(b)  $a = 5, b = 2$

(c)  $a = 3, b = 5$

(d)  $a = 5, b = 3$

2. The corner point of the feasible region of a linear programming problem are (0,4), (8,0) and  $(\frac{20}{3}, \frac{4}{3})$ . if  $Z = 30x + 24y$  is the objective function, then ( maximum value of Z - minimum value of Z ) is equal to

(a) 40

(b) 96

(c) 120

(d) 136

3. Solve the following linear programming problem graphically :

$$\text{Maximum : } Z = x + 2y$$

$$\text{subject to constraints : } x + 2y \geq 100,$$

$$2x - y \leq 0,$$

$$2x + y \leq 200,$$

$$x \geq 0, y \geq 0.$$

4. Engine displacement is the measure of the cylinder volume swept by all the pistons engine. The piston move inside the cylinder bore



Figure 4.1: Engine

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area  $75\pi cm^2$

Based on the above information, answer the following questions:

- (a) if the radius of cylinder is  $r$  cm and height is  $h$  cm, then write the volume  $V$  of cylinder in terms of radius  $r$ .
- (b) Find  $\frac{dV}{dr}$ .
- (c) i. Find the radius of cylinder when its volume is maximum.  
ii. For maximum volume,  $h > r$ . State true or false and justify.

