CBSE MATH

Made Simple

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Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. Product vectors

1. $\vec{a}$ and $\vec{b}$ are two unit vectors such that

$$\left|2\vec{a} + 3\vec{b}\right| = \left|3\vec{a} - 2\vec{b}\right|. \tag{1.1}$$

Find the angle between $\vec{a}$ and $\vec{b}$.

2. If $\vec{a}$ and $\vec{b}$ are two vectors such that

$$\vec{a} = \hat{i} - \hat{j} + \hat{k} \tag{1.2}$$

and

$$\vec{b} = 2\hat{i} - \hat{j} - 3\hat{k} \tag{1.3}$$

then find the vector $\vec{c}$, given that

$$\vec{a} \times \vec{c} = \vec{b} \tag{1.4}$$

and

$$\vec{a}.\vec{c} = 4. \tag{1.5}$$

3.

$$If \left| \vec{a} \times \vec{b} \right|^2 + \left| \vec{a}.\vec{b} \right|^2 = 400 \tag{1.6}$$

and

$$\left| \vec{b} \right| = 5 \tag{1.7}$$

find the value of $|\vec{a}|$.

4. If

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a}.\vec{b} = 1 \tag{1.8}$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k} \tag{1.9}$$

, then find $\left| \vec{b} \right|$

5. If

$$|\vec{a}| = 3, \left| \vec{b} \right| = 2\sqrt{3} \tag{1.10}$$

and

$$\vec{a} \cdot \vec{b} = 6, \tag{1.11}$$

then find the value of $\left| \vec{a} \times \vec{b} \right|$.

6. $|\vec{a}| = 8, \left| \vec{b} \right| = 3$ and $\vec{a} \cdot \vec{b} = 12\sqrt{3}$, then the value of $\left| \vec{a} \times \vec{b} \right|$ is

   (a) 24

   (b) 144

   (c) 2

   (d) 12

7. If

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.12}$$

   and

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.13}$$

   , then find $\vec{a} \cdot (\vec{b} \times \vec{c})$.

8. $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are four non-zeros vectors such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$

   and

$$\vec{a} \times \vec{c} = 4\vec{b} \times \vec{d} \tag{1.14}$$

, then show that $(\vec{a} - 2\vec{d}$ is parallel to$(2\vec{b} - \vec{c})$ where

$$\vec{a} \neq 2\vec{d}, \vec{c} \neq 2\vec{b} \tag{1.15}$$

9. If

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{a}.\vec{b} = 1 \tag{1.16}$$

and

$$\vec{a} \times \vec{b} = \hat{j} - \hat{k}, \tag{1.17}$$

then find $\left|\vec{b}\right|$

10. If $\vec{a}$ and $\vec{b}$ are two vectors such that

$$\left|\vec{a} + \vec{b}\right| = \left|\vec{b}\right|, \tag{1.18}$$

then prove that $(\vec{a} + 2\vec{b})$ is perpendicular to $\vec{a}$.

11. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right| \tag{1.19}$$

12. If $\vec{a}$ and $\vec{b}$ are two unit vectors such that and θ is the ang le between

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them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.20}$$

1.2. Projection vectors

13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.21}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.22}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{1.23}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.24}$$

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15. If

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{1.25}$$

and

$$\vec{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.26}$$

and the projection of vector $\vec{c} + \lambda\vec{b}$ on vector $\vec{a}$ is $2\sqrt{6}$, find the value of λ.

16. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.27}$$

, then find $\vec{a}.(\vec{b} \times \vec{c})$.

17. If

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.28}$$

and

$$\vec{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.29}$$

, then find the ratio $\dfrac{projection of vector \vec{a} on vector \vec{b}}{projection of vector \vec{b} on vector \vec{a}}$

18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$

form the vertices of a right-angled triangle. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\vec{c} = 3\hat{i} + \hat{j} \tag{1.30}$$

are such that the vector $(\vec{a} + \lambda \vec{b})$ is perpendicular to vector $\vec{c}$, then find the value of λ.

1.3. Position vectors

19. If $\vec{a}, \vec{b}$ and $\vec{c}$ are the position vectors of the points $\mathbf{A}(2, 3, -4)$, $\mathbf{B}(3, -4, -5)$ and $\mathbf{C}(3, 2, -3)$ and respectively, then $\left|\vec{a} + \vec{b} + \vec{c}\right|$ is equal to

    (a) $\sqrt{113}$

    (b) $\sqrt{185}$

    (c) $\sqrt{203}$

    (d) $\sqrt{209}$

1.4. Section formula

20. **A** circle has its center at $(4, 4)$. If one end ofa diameter is $(4, 0)$, then find the coordinates ofother end.

1.5. Plane vectors

21. Find the values λ, for which the distance of point $(2, 1, \lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.31}$$

    is $2\sqrt{2}$ units.

22. Find the coordinates of the point where the line through $(3, 4, 1)$ crosses the ZX-plane

1.6. Geometry vectors

23. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1, 0, -2)$, $\mathbf{B}(0, 2, 1)$ and $\mathbf{C}(-1, 4, 1)$

24. Using integration, find the area of triangle region whose vertices are $(2, 0)$, $(4, 5)$ and $(1, 4)$.

1.7. Distance formula

25. The distance between the points $(0, 0)$ and $(a - b, a + b)$ is

    (a) $2\sqrt{ab}$

    (b) $\sqrt{2a^2 + ab}$

    (c) $2\sqrt{a^2 + b^2}$

(d) $\sqrt{2a^2 + 2b^2}$

26. The value of m which makes the point $(0,0)$, $(2m, -4)$ and $(3,6)$ collinear, is _____

1.8. Direction vectors

27. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direct6on cosines of the line.

28. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \qquad (1.32)$$

.

29. Find the directions cosines of a line parallel to line AB.

30. Find the direction cosines of a line whose cartesian equation is given as

$$3x + 1 = 6y - 2 = 1 - z. \qquad (1.33)$$

31. A vector of magnitude 9 units in the direction of the vector $-2\hat{i} - \hat{j} + 2\hat{k}$ is _____

1.9. Diagonal vectors

32. The two adajacent sides of a parallelogram are represented by $2\hat{i} - 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

33. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\hat{i} - 2\hat{j} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

34. If

$$\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k} \tag{1.34}$$

and

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \tag{1.35}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

1.10. Area of triangle

35. Find the area of the quadrilateral $ABCD$ whose vertices are $\mathbf{A}(-4, -3)$, $\mathbf{B}(3, -1)$, $\mathbf{C}(0, 5)$ and $\mathbf{D}(-4, 2)$

36. If the points $\mathbf{A}(2, 0)$, $\mathbf{B}(6, 1)$, and $\mathbf{C}(p, q)$ form a triangle of area 12sq.

units (positive only) and

$$2p + q = 10, \tag{1.36}$$

then find the values of p and q.

Chapter 2

Linear Forms

2.0.1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2.0.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \qquad\qquad (2.0.2.1)$$

$$(bx + ay) + (b - 4a) = 0 \qquad\qquad (2.0.2.2)$$

2.0.3. Find the co-ordinates of the point where the line $\dfrac{x - 3}{-1} = \dfrac{y + 4}{1} = \dfrac{z + 5}{6}$ crosses the plane passing through the points $\left(\dfrac{7}{2}, 0, 0\right), (0, 7, 0), (0, 0, 7)$.

2.0.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.
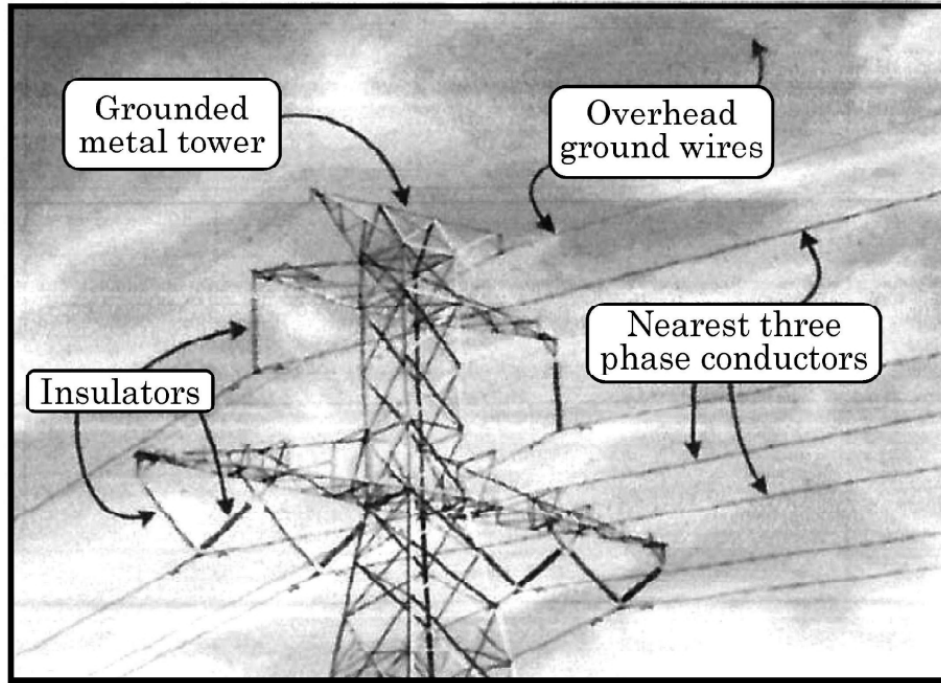
Figure 2.0.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.0.4.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.0.4.1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.0.4.2}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

2.0.5. Write the cartesian equation of the line PQ passing through points

P(2, 2, 1) and Q(5, 1, −2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.0.6. Find the distance between the lines $x = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ and $x + 1 = \dfrac{y+2}{2} = \dfrac{z-1}{3}$.

2.0.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k}) \qquad (2.0.7.1)$$

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \qquad (2.0.7.2)$$

2.0.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.0.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \qquad (2.0.8.1)$$

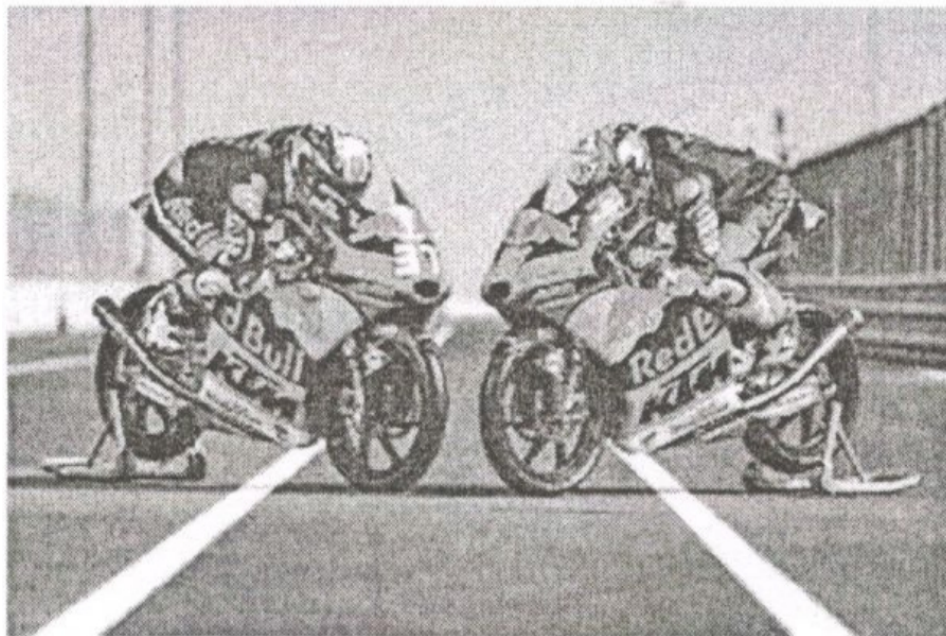$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k}) \qquad (2.0.8.2)$$

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Figure 2.0.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

2.0.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k} \qquad (2.0.9.1)$$

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k} \qquad (2.0.9.2)$$

2.0.10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \qquad (2.0.10.1)$$

2.0.11. Find the equation of the plane passing through the points $(2, 1, 0), (3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.

2.0.12. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

2.0.13. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

2.0.14. The distance between the planes $4x - 4y + 2z + 5 = 0$ and $2x - 2y + z + 6 = 0$ is

(a) $\frac{1}{6}$

(b) $\frac{7}{6}$

(c) $\frac{11}{6}$

(d) $\frac{16}{6}$

2.0.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0 \qquad (2.0.15.1)$$

$$\mathbf{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0 \qquad (2.0.15.2)$$

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which is at a unit distance from the origin.

2.0.16. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\dfrac{5}{\sqrt{3}}$, find the value(s) of λ.

2.0.17. Find the distance of the point $(2, 3, 4)$ measured along the line $\dfrac{x - 4}{3} = \dfrac{y + 5}{6} = \dfrac{z + 1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

2.0.18. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5)$, $B(-5, -2, 3)$ and $C(2, 4, -5)$.

2.0.19. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \qquad (2.0.19.1)$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \qquad (2.0.19.2)$$

is

(a) $\sqrt{2}$

(b) $\dfrac{1}{\sqrt{2}}$

(c) $\dfrac{1}{3\sqrt{2}}$

(d) $\dfrac{-2}{3\sqrt{2}}$

2.0.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and

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$(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{i} + \hat{j} + \hat{k} \qquad\qquad (2.0.20.1)$$

$$\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k} \qquad\qquad (2.0.20.2)$$

2.0.21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k}) \qquad\qquad (2.0.21.1)$$

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10. \qquad\qquad (2.0.21.2)$$

2.0.22. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

2.0.23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

2.0.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.0.24.1:

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Table 2.0.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

Chapter 3

Intersection of Conics

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\dfrac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \dfrac{1}{4}$, $y = 0$ and $x = 1$, using integration.

6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\dfrac{32}{3}$ sq. units, then find the positive value of m, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

9. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration

Chapter 4

Tangent And Normal

4.1. Construction

1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

3. In Figure 4.1.24.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.
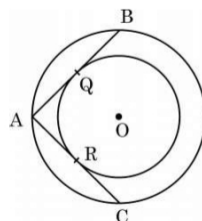
Figure 3.1: Two concentric circles with **O** as centre

4. In Figure 4.1.24.2, if a circle touches the side QR of $\triangle PQR$ at **S** and extended sides PQ and PR at **M** and **N**, respectively,
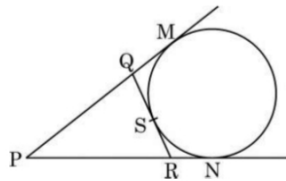
Figure 4.1: Two tangents are drawn from point **P** to the circle

prove that $PM = \dfrac{1}{2}(PQ + QR + PR)$

5. In Figure 4.1.24.3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of $\triangle ABC$ is 84 cm^2, find the lengths of sides AB and AC.
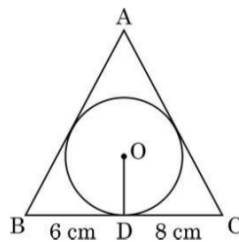
Figure 5.1: Circle with **O** as center circumscribed in triangle ABC

6. In Figure 4.1.24.4, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.
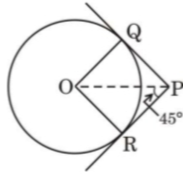
Figure 6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

7. In Figure 4.1.24.5, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.
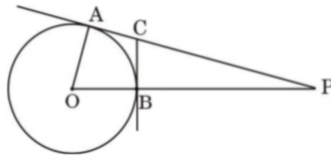

Figure 7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

8. In Figure 4.1.24.6, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.
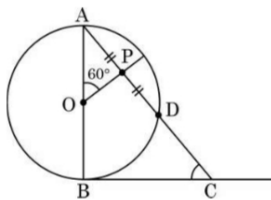

Figure 8.1: Tangent BC is drawn from point **C** to a circle whose centre is **O**

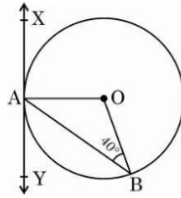9. In Figure 4.1.24.7, XAY is a tangent to the circle centered at **O**. If

Figure 9.1: The line XAY is tangent to the circle centered at **O**

$\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

11. In Figure 4.1.24.8, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.
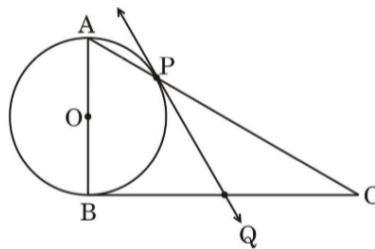

Figure 11.1: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

4.2. Properties

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

2. The slope of the normal to the curve $y = 2x^2 + 3sinx$ at $x = 0$ is

_____.