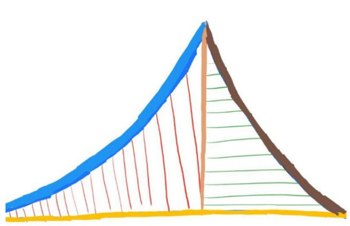
CBSE MATH

Made Simple

G. V. V. Sharma



Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

Chapter 2

Linear Forms

1. Solve the equations $x + 2y = 6$ and $2x - 5y = 12$ graphically.

2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 \tag{2.1}$$

$$(bx + ay) + (b - 4a) = 0 \tag{2.2}$$

3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2}, 0, 0\right)$, $(0, 7, 0)$, $(0, 0, 7)$.

4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

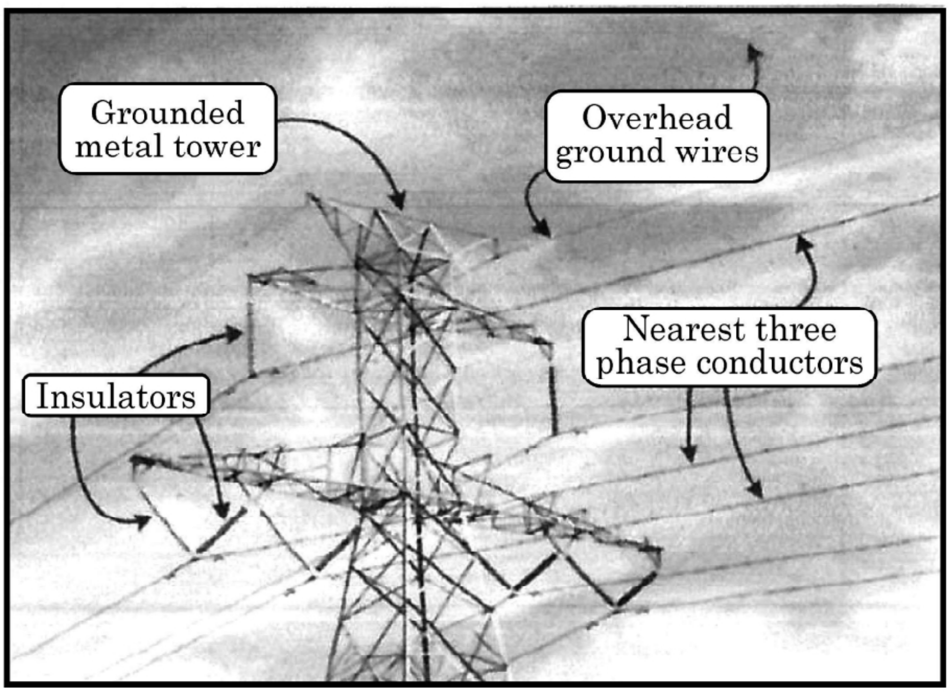


Figure 2.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.3}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.4}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

5. Write the cartesian equation of the line PQ passing through points P$(2, 2, 1)$ and Q$(5, 1, -2)$. Hence, find the y-coordinate of the point on

the line PQ whose z-coordinate is -2.

6. Find the distance between the lines $x = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ and $x+1 = \dfrac{y+2}{2} = \dfrac{z-1}{3}$.

7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \tag{2.5}$$

$$\mathbf{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \tag{2.6}$$

8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.2) represented by the following lines

$$\mathbf{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \tag{2.7}$$

$$\mathbf{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k}) \tag{2.8}$$

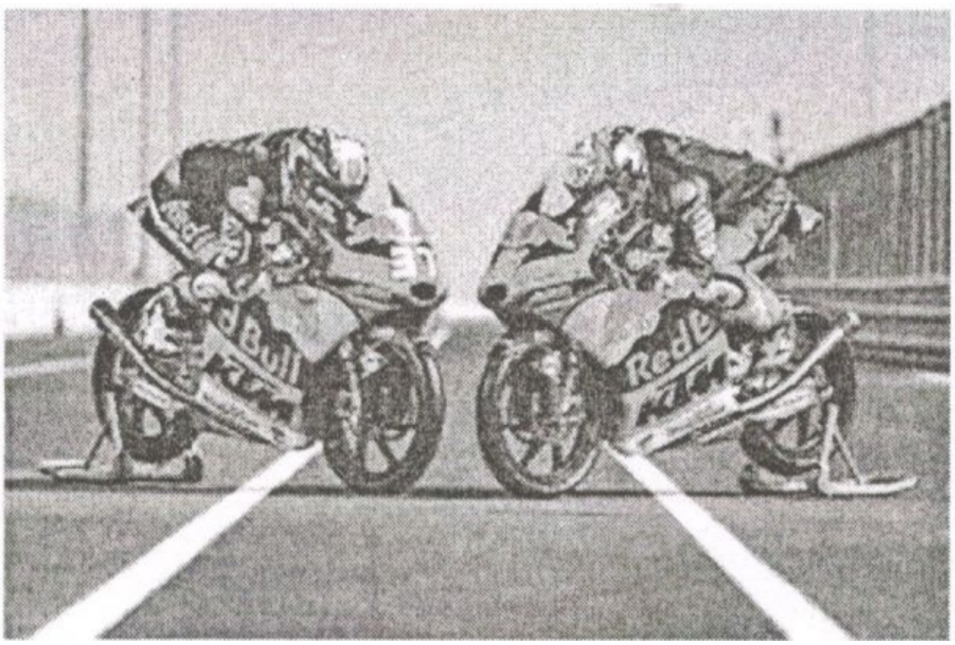


Figure 2.2: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{\imath} + (\lambda + 4)\hat{\jmath} - (\lambda - 3)\hat{k} \tag{2.9}$$

$$\mathbf{r} = (3 - \mu)\hat{\imath} + (2\mu + 2)\hat{\jmath} + (\mu + 6)\hat{k} \tag{2.10}$$

10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.11}$$

1. If the distance of the point $(1, 1, 1)$ from the plane $x - y + z + \lambda = 0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2. Find the distance of the point $(2, 3, 4)$ measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane $3x + 2y + 2z + 5 = 0$.

3. Find the distance of the point $P(4, 3, 2)$ from the plane determined by the points $A(-1, 6, -5), B(-5, -2, 3)$ and $C(2, 4, -5)$.

4. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k}) \tag{2.12}$$

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5 \tag{2.13}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

5. Find a unit vector perpendicular to each of the vectors $(\mathbf{a}+\mathbf{b})$ and $(\mathbf{a}-\mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.14}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.15}$$

1. Find the equation of the plane passing through the points $(2,1,0), (3,-2,-2)$ and $(1,1,7)$. Also, obtain its distance from the origin.

2. The foot of a perpendicular drawn from the point $(-2,-1,-3)$ on a plane is $(1,-3,3)$. Find the equation of the plane.

3. Find the cartesian and the vector equation of a plane which passes through the point $(3,2,0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

4. The distance between the planes $4x-4y+2z+5=0$ and $2x-2y+z+6=0$ is

   (a) $\frac{1}{6}$
   (b) $\frac{7}{6}$
   (c) $\frac{11}{6}$
   (d) $\frac{16}{6}$

5. Find the equation of the plane through the line of intersection of the

planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.16}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.17}$$

which is at a unit distance from the origin.

1. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.18}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.19}$$

2. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

3. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

4. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1:

Table 2.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

Chapter 3

Intersection of Conics

1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

9. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration

Chapter 4

Tangent And Normal

1. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

3. In Figure 4.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

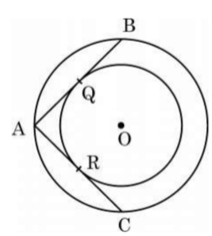


Figure 4.1: Two concentric circles with **O** as centre

4. In Figure 4.2, if a circle touches the side QR of ΔPQR at **S** and extended sides PQ and PR at **M** and **N**, respectively,

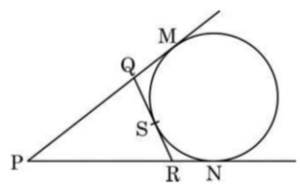


Figure 4.2: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

5. In Figure 4.3, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

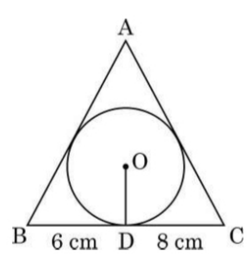


Figure 4.3: Circle with **O** as center circumscribed in triangle ABC

6. In Figure 4.4, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

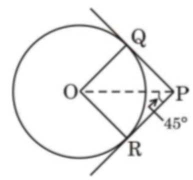


Figure 4.4: Two tangents drawn from point **P** to a circle whose centre is **O**

7. In Figure 4.5, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.

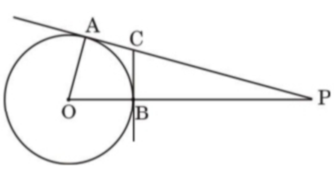


Figure 4.5: Two tangents drawn from point **C** to a circle whose centre is **O**

8. In Figure 4.6, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

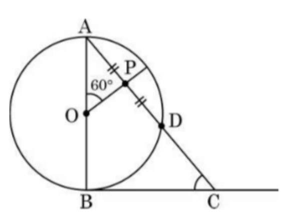


Figure 4.6: Tangent BC is drawn from point **C** to a circle whose centre is **O**

9. In Figure 4.7, XAY is a tangent to the circle centered at **O**. If $\angle ABO =$

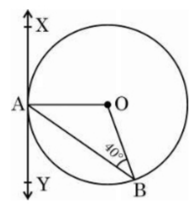


Figure 4.7: The line XAY is tangent to the circle centered at **O**

$60°$,then find $m\angle BAY$ and $m\angle AOB$.

10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

11. In Figure 4.8, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.

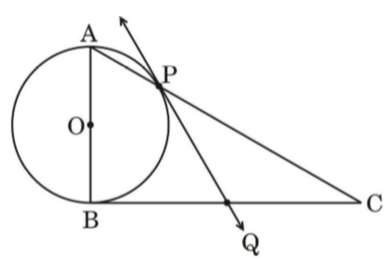


Figure 4.8: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

4.1. Construction