CBSE MATH

Made Simple

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Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems. $\,$

Chapter 1

Vectors

1.1. 2023

- 1.1.1 In what ratio, does x-axis divide the line segment joinin the points $\mathbf{A}(3,6)$ and $\mathbf{B}(-12,-3)$?
 - (a) 1:2
 - (b) 1:4
 - (c) 4:1
 - (d) 2:1
- 1.1.2 The distance between the point $(0,2\sqrt{5})$ and $(-2\sqrt{5},0)$ is
 - (a) $2\sqrt{10}$ units
 - (b) $4\sqrt{10}$ units
 - (c) $2\sqrt{20}$ units
 - (d) 0 units
- 1.1.3 If (-5,3) and (5,3) are two vetices of an equilateral triangle, then coordinates of the third vertex, given that origin lies inside the triangle

 $(take\sqrt{3} = 1.7)$

1.1.4 Show that the points (-2,3), (8,3) and (6,7) are the verices of right-angled triangle

1.1.5 If $\mathbf{Q} = (0,1)$ is equidistant from $\mathbf{P} = (5,-3)$ and $\mathbf{R} = (x,6)$, find the value of x.

1.1.6 The distance of the point (-6,8) from origin is :

- (a) 6
- (b) -6
- (c) 8
- (d) 10

1.1.7 The points (-4,0) (4,0) and (0,3) are the vertices of a:

- (a) right triangle
- (b) isosceles triangle
- (c) equilateral triangle
- (d) scalene triangle

1.2. 2022

1.2.1. \overrightarrow{a} and \overrightarrow{b} are two unit vectors such that

$$\left| 2\overrightarrow{a} + 3\overrightarrow{b} \right| = \left| 3\overrightarrow{a} - 2\overrightarrow{b} \right|.$$
 (1.2.1.1)

Find the angle between \overrightarrow{a} and \overrightarrow{b} .

1.2.2. If \overrightarrow{d} and \overrightarrow{b} are two vectors such that

$$\overrightarrow{a} = \hat{i} - \hat{j} + \hat{k} \tag{1.2.2.1}$$

and

$$\overrightarrow{b} = 2\hat{i} - \hat{j} - 3\hat{k} \tag{1.2.2.2}$$

then find the vector \overrightarrow{c} , given that

$$\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \tag{1.2.2.3}$$

and

$$\overrightarrow{a}.\overrightarrow{c} = 4. \tag{1.2.2.4}$$

1.2.3.

$$If \left| \overrightarrow{a} \times \overrightarrow{b} \right|^2 + \left| \overrightarrow{a} \cdot \overrightarrow{b} \right|^2 = 400 \tag{1.2.3.1}$$

and

$$\left|\overrightarrow{b}\right| = 5\tag{1.2.3.2}$$

find the value of $|\overrightarrow{a}|$.

1.2.4. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.2.4.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k} \tag{1.2.4.2}$$

, then find $\left|\overrightarrow{b}\right|$

1.2.5. If

$$|\overrightarrow{a}| = 3, |\overrightarrow{b}| = 2\sqrt{3}$$
 (1.2.5.1)

and

$$\overrightarrow{a}.\overrightarrow{b} = 6, \tag{1.2.5.2}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

1.2.6. $|\overrightarrow{a}| = 8$, $|\overrightarrow{b}| = 3$ and $|\overrightarrow{a}| = 12\sqrt{3}$, then the value of $|\overrightarrow{a}| \times |\overrightarrow{b}|$ is

- (a) 24
- (b) 144
- (c) 2
- (d) 12

1.2.7. If

$$\vec{d} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k}$$
 (1.2.7.1)

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.2.7.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b}\times\overrightarrow{c}).$

1.2.8. \overrightarrow{d} , \overrightarrow{b} , \overrightarrow{c} and \overrightarrow{d} are four non-zeros vectors such that $\overrightarrow{d} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d}$ and

$$\overrightarrow{d} \times \overrightarrow{c} = 4 \overrightarrow{b} \times \overrightarrow{d} \tag{1.2.8.1}$$

, then show that $(\overrightarrow{d}-2\overrightarrow{d}$ is parallel to (2 $\overrightarrow{b}-\overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b}$$
 (1.2.8.2)

1.2.9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.2.9.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{1.2.9.2}$$

then find $\left|\overrightarrow{b}\right|$

1.2.10. If \overrightarrow{a} and \overrightarrow{b} are two vectors such that

$$\left| \overrightarrow{a} + \overrightarrow{b} \right| = \left| \overrightarrow{b} \right|,$$
 (1.2.10.1)

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to \overrightarrow{a} .

1.2.11. If \overrightarrow{a} and \overrightarrow{b} are unit vectors and θ is the angle between them , then prove that \sin

$$\frac{\theta}{2} = \frac{1}{2} \left| \overrightarrow{a} - \overrightarrow{b} \right| \tag{1.2.11.1}$$

1.2.12. If \overrightarrow{a} and \overrightarrow{b} are two unit vectors such that and θ is the angle between them, then prove that

$$\sin\frac{\theta}{2} = \frac{1}{2} \left| \overrightarrow{a} - \overrightarrow{b} \right| \tag{1.2.12.1}$$

1.2.13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.2.13.1}$$

and

$$\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.2.13.2}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the

vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

1.2.14. Write the projection of the vector $(\overrightarrow{b}+\overrightarrow{c})$ on the vector \overrightarrow{a} , where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$(1.2.14.1)$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.2.14.2}$$

1.2.15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k}$$
 (1.2.15.1)

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.2.15.2}$$

and the projection of vector $\overrightarrow{c} + \lambda \overrightarrow{b}$ on vector \overrightarrow{a} is $2\sqrt{6}$, find the value of λ .

1.2.16. If $\overrightarrow{a}=2\hat{i}+\hat{j}+3\hat{k}, \hat{b}=-\hat{i}+2\hat{j}+\hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.2.16.1}$$

, then find $\overrightarrow{a}.(\overrightarrow{b}\times\overrightarrow{c}).$

1.2.17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.2.17.1}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.2.17.2}$$

, then find the ratio $\frac{projection of vector \overrightarrow{d} \ on vector \overrightarrow{b}}{projection of vector \overrightarrow{b} \ on vector \overrightarrow{d}}$

1.2.18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If $\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{1.2.18.1}$$

are such that the vector $(\overrightarrow{a} + \lambda \overrightarrow{b})$ is perpendicular to vector \overrightarrow{c} , then find the value of λ .

- 1.2.19. If \overrightarrow{a} , \overrightarrow{b} and \overrightarrow{c} are the position vectors of the points $\mathbf{A}(2,3,-4)$, $\mathbf{B}(3,-4,-5)$ and $\mathbf{C}(3,2,-3)$ and respectively, then $|\overrightarrow{a}+\overrightarrow{b}+\overrightarrow{c}|$ is equal to
 - (a) $\sqrt{113}$
 - (b) $\sqrt{185}$
 - (c) $\sqrt{203}$
 - (d) $\sqrt{209}$

- 1.2.20. **A** circle has its center at (4,4). If one end of a diameter is (4,0), then find the coordinates of other end.
- 1.2.21. Find the values λ , for which the distance of point $(2,1,\lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.2.21.1}$$

is $2\sqrt{2}$ units.

- 1.2.22. Find the coordinates of the point where the line through (3,4,1) crosses the ZX-plane
- 1.2.23. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1,0,-2)$, $\mathbf{B}(0,2,1)$ and $\mathbf{C}(-1,4,1)$
- 1.2.24. Using integration, find the area of triangle region whose vertices are (2,0), (4,5) and (1,4).
- 1.2.25. The distance between the points (0,0) and (a-b,a+b) is
 - (a) $2\sqrt{ab}$
 - (b) $\sqrt{2a^2 + ab}$
 - (c) $2\sqrt{a^2+b^2}$
 - (d) $\sqrt{2a^2 + 2b^2}$
- 1.2.26. The value of m which makes the point (0,0) , (2m,-4)and (3,6) collinear, is _____
- 1.2.27. If a line makes 60° and 45° angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the

positive direction of y-axis. Hence, write the direct6on cosines of the line.

1.2.28. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \tag{1.2.28.1}$$

.

- 1.2.29. Find the directions cosines of a line parallel to line AB.
- 1.2.30. Find the direction cosines of a line whose cartesian equation is given as

$$3x + 1 = 6y - 2 = 1 - z.$$
 (1.2.30.1)

- 1.2.31. A vector of magnitude 9 units in the direction of the vector $-2\hat{i}-\hat{j}+2\hat{k}$ is _____
- 1.2.32. The two adajacent sides of a parallelogram are represented by $2\hat{i} 4\hat{j} 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.
- 1.2.33. The two adjacent sides of a parallelogram are represented by vectors $2\hat{i} 4\hat{j} + 5\hat{k}$ and $\hat{i} 2\hat{j} 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also, find the area of the parallelogram.

1.2.34. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{1.2.34.1}$$

and

$$\vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k} \tag{1.2.34.2}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram

- 1.2.35. Find the area of the quadrilateral ABCD whose vertices are $\mathbf{A}(-4, -3)$, $\mathbf{B}(3, -1)$, $\mathbf{C}(0, 5)$ and $\mathbf{D}(-4, 2)$
- 1.2.36. If the points $\mathbf{A}(2,0)$, $\mathbf{B}(6,1)$, and $\mathbf{C}(p,q)$ form a triangle of area 12sq. units (positive only) and

$$2p + q = 10, (1.2.36.1)$$

then find the values of p and q.

Chapter 2

Linear Forms

2.1. 2022

- 2.1.1. Solve the equations x + 2y = 6 and 2x 5y = 12 graphically.
- 2.1.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax - by) + (a + 4b) = 0 (2.1.2.1)$$

$$(bx + ay) + (b - 4a) = 0 (2.1.2.2)$$

- 2.1.3. Find the co-ordinates of the point where the line $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2},0,0\right), (0,7,0), (0,0,7)$.
- 2.1.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.



Figure 2.1.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.1.4.1 lie along the following lines:

$$l_1: \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1}$$
 (2.1.4.1)

$$l_2: \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.1.4.2}$$

Based on the given information, answer the following questions:

- (a) Are the l_1 and l_2 coplanar? Justify your answer.
- (b) Find the point of intersection of lines l_1 and l_2 .
- 2.1.5. Write the cartesian equation of the line PQ passing through points

P(2,2,1) and Q(5,1,-2). Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.1.6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

2.1.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \lambda(\hat{i} - 2\hat{j} + \hat{k})$$
 (2.1.7.1)

$$\mathbf{r} = (-\hat{i} - \hat{j} - \hat{k}) + \mu(7\hat{i} - 6\hat{j} + \hat{k}) \tag{2.1.7.2}$$

2.1.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.1.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{i} + 2\hat{j} - \hat{k}) \tag{2.1.8.1}$$

$$\mathbf{r} = (3\hat{i} + 3\hat{j}) + \mu(2\hat{i} + \hat{j} + \hat{k})$$
 (2.1.8.2)



Figure 2.1.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

- (a) Find the shortest distance between the given lines.
- (b) Find a point at which the motorcycles may collide.
- 2.1.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{i} + (\lambda + 4)\hat{j} - (\lambda - 3)\hat{k}$$
 (2.1.9.1)

$$\mathbf{r} = (3 - \mu)\hat{i} + (2\mu + 2)\hat{j} + (\mu + 6)\hat{k}$$
 (2.1.9.2)

2.1.10. Find the shortest distance between the following lines and hence write

whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2$$
 (2.1.10.1)

- 2.1.11. Find the equation of the plane passing through the points (2,1,0), (3,-2,-2) and (1,1,7). Also, obtain its distance from the origin.
- 2.1.12. The foot of a perpendicular drawn from the point (-2, -1, -3) on a plane is (1, -3, 3). Find the equation of the plane.
- 2.1.13. Find the cartesian and the vector equation of a plane which passes through the point (3,2,0) and contains the line $\frac{x-3}{1}=\frac{y-6}{5}=\frac{z-4}{4}$.
- 2.1.14. The distance between the planes 4x-4y+2z+5=0 and 2x-2y+z+6=0 is
 - (a) $\frac{1}{6}$
 - (b) $\frac{7}{6}$
 - (c) $\frac{11}{6}$
 - (d) $\frac{16}{6}$
- 2.1.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{i} + 3\hat{j}) + 6 = 0 \tag{2.1.15.1}$$

$$\mathbf{r} \cdot (3\hat{i} - \hat{j} - 4\hat{k}) = 0 \tag{2.1.15.2}$$

which is at a unit distance from the origin.

- 2.1.16. If the distance of the point (1,1,1) from the plane $x-y+z+\lambda=0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ .
- 2.1.17. Find the distance of the point (2,3,4) measured along the line $\frac{x-4}{3} = \frac{y+5}{6} = \frac{z+1}{2}$ from the plane 3x + 2y + 2z + 5 = 0.
- 2.1.18. Find the distance of the point P(4,3,2) from the plane determined by the points A(-1,6,-5), B(-5,-2,3) and C(2,4,-5).
- 2.1.19. The distance of the line

$$\mathbf{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 5\hat{j} + \hat{k})$$
 (2.1.19.1)

from the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + 4\hat{k}) = 5$$
 (2.1.19.2)

is

- (a) $\sqrt{2}$
- (b) $\frac{1}{\sqrt{2}}$
- (c) $\frac{1}{3\sqrt{2}}$
- (d) $\frac{-2}{3\sqrt{2}}$
- 2.1.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a} + \mathbf{b})$ and

 $(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{i} + \hat{j} + \hat{k} \tag{2.1.20.1}$$

$$\mathbf{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{2.1.20.2}$$

2.1.21. Find the distance of the point (1, -2, 9) from the point of intersection of the line

$$\mathbf{r} = 4\hat{i} + 2\hat{j} + 7\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$$
 (2.1.21.1)

and the plane

$$\mathbf{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 10.$$
 (2.1.21.2)

- 2.1.22. Find the area bounded by the curves y=|x-1| and y=1, using integration.
- 2.1.23. Find the coordinates of the point where the line through (4, -3, -4) and (3, -2, 2) crosses the plane 2x + y + z = 6.
- 2.1.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.1.24.1:

Table 2.1.24.1: Table showing yearly trend of production of goods in lakh tonnes $\,$

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

Chapter 3

Intersection of Conics

3.1. 2022

- 3.1.1. Using integration, find the area of the region enclosed by the curve $y=x^2$, the x-axis and the ordinates x=-2 and x=1.
- 3.1.2. Using integration, find the area of the region enclosed by line $y=\sqrt{3}x$ semi-circle $y=\sqrt{4-x^2}$ and x-axis in first quadrant.
- 3.1.3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line 2x + 2y = 3.
- 3.1.4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line x = 4a is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where a > 0.
- 3.1.5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, y = 0 and x = 1, using integration.
- 3.1.6. If the area of the region bounded by the line y=mx and the curve $x^2=y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

- 3.1.7. If the area between the curves $x=y^2$ and x=4 is divided into two equal parts by the line x=a, then find the value of a, using integration.
- 3.1.8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates x = 0 and x = 2, using integration.
- 3.1.9. Find the area of the region $\{(x,y): x^2 \leq y \leq x\}$, using integration

Chapter 4

Tangent And Normal

4.1. 2022

- 4.1.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.
- 4.1.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.
- 4.1.3. In Figure 4.1.3.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if AQ = 5 cm.



Figure 4.1.3.1: Two concentric circles with **O** as centre

4.1.4. In Figure 4.1.4.1, if a circle touches the side QR of ΔPQR at ${\bf S}$ and extended sides PQ and PR at ${\bf M}$ and ${\bf N}$, respectively,

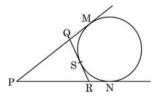


Figure 4.1.4.1: Two tangents are drawn from point \mathbf{P} to the circle

prove that
$$PM = \frac{1}{2}(PQ + QR + PR)$$

4.1.5. In Figure 4.1.5.1, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact \mathbf{D} are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2 , find the lengths of sides AB and AC.

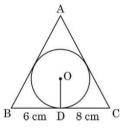


Figure 4.1.5.1: Circle with \mathbf{O} as center circumscribed in triangle ABC

4.1.6. In Figure 4.1.6.1, PQ and PR are tangents to the circle centered at \mathbf{O} . If $\angle OPR = 45^{\circ}$, then prove that ORPQ is a square.

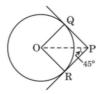


Figure 4.1.6.1: Two tangents drawn from point ${\bf P}$ to a circle whose centre is ${\bf O}$

4.1.7. In Figure 4.1.7.1, \mathbf{O} is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at \mathbf{A} and \mathbf{B} respectively. If OP is 13 cm, then find the length of tangents PA and BC.

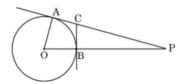


Figure 4.1.7.1: Two tangents drawn from point ${\bf C}$ to a circle whose centre is ${\bf O}$

4.1.8. In Figure 4.1.8.1, AB is diameter of a circle centered at \mathbf{O} . BC is tangent to the circle at \mathbf{B} . If OP bisects the chord AD and $\angle AOP = 60^{\circ}$, then find $m\angle C$.

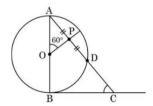


Figure 4.1.8.1: Tangent BC is drawn from point ${\bf C}$ to a circle whose centre is ${\bf O}$



Figure 4.1.9.1: The line XAY is tangent to the circle centered at **O**

- 4.1.9. In Figure 4.1.9.1, XAY is a tangent to the circle centered at **O**. If $\angle ABO = 60^{\circ}$, then find $m\angle BAY$ and $m\angle AOB$.
- 4.1.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.
- 4.1.11. In Figure 4.1.11.1, a triangle ABC with $\angle B = 90^{\circ}$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point \mathbf{P} . Prove that the tangent drawn at point \mathbf{P} bisects BC.

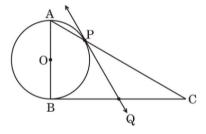


Figure 4.1.11.1: PQ is tangent to the circle centered at ${\bf O}$. AB is the diameter and $\angle B=90^\circ$

4.1.12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point (3, 22).