CBSE MATH

Made Simple

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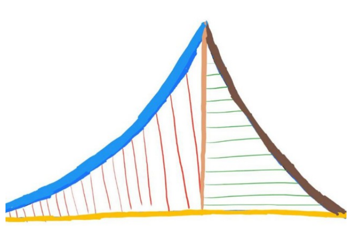

Contents

Introduction

This book links high school coordinate geometry to linear algebra and matrix analysis through solved problems.

Chapter 1

Vectors

1.1. 2023

1.1.1. 10

1.1.1 In what ratio, does x-axis divide the line segment joinin the points $\mathbf{A}(3,6)$ and $\mathbf{B}(-12,-3)$?

(a) $1:2$

(b) $1:4$

(c) $4:1$

(d) $2:1$

1.1.2 The distance between the point $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$ is

(a) $2\sqrt{10}$ units

(b) $4\sqrt{10}$ units

(c) $2\sqrt{20}$ units

(d) 0 units

1.1.3 If $(-5, 3)$ and $(5, 3)$ are two vetices of an equilateral triangle,then coordinates of the third vertex,given that origin lies inside the triangle $(take\sqrt{3} = 1.7)$

1.1.4 Show that the points $(-2, 3)$, $(8, 3)$ and $(6, 7)$ are the verices of right-angled triangle

1.1.5 If $\mathbf{Q} = (0, 1)$ is equidistant from $\mathbf{P} = (5, -3)$ and $\mathbf{R} = (x, 6)$, find the value of x.

1.1.6 The distance of the point $(-6, 8)$ from origin is :

(a) 6

(b) -6

(c) 8

(d) 10

1.1.7 The points $(-4, 0)$ $(4, 0)$ and $(0, 3)$ are the vertices of a :

(a) right triangle

(b) isosceles triangle

(c) equilateral triangle

(d) scalene triangle

1.1.2. 10

1. The area of the triangle formed by the line $\frac{x}{a} + \frac{y}{b} = 1$ with the coordinate axes is :

(a) ab

(b) $\frac{1}{2}ab$

(c) $\frac{1}{4}ab$

(d) $2ab$

2. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the fiel d for growing wheat and remaining for growing vegetables as shown in figure.2.1. In the field , there is a pole marked as O .

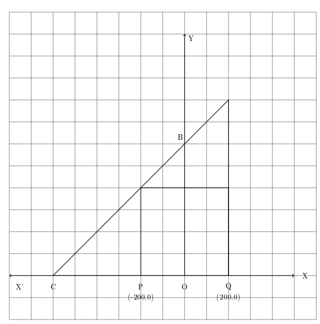

Figure 2.1: Image

Based on the above information,answer the following equations:

(a) Taking O as origin , coordinates of P are (-200,0) and of Q are (200,0). PQRS being a square, what are the coordinates of R and S?

(b) i. What is the area of square PQRS?

 ii. What is the length of diagonal PR in PQRS?

(c) If S divides CA in the ratio K:1,what is the value of K,where point A is (200,800)?

1.1.3. 12

1.1.1. Unit vector along **PQ**, where coordinates of **P** and **Q** respectively are (2,1,-1)and(4,4,-7), is

(a) $2\hat{i} + 3\hat{j} - 6\hat{k}$

(b) $-2\hat{i} - 3\hat{j} + 6\hat{k}$

(c) $-\frac{2\hat{i}}{7} - \frac{3\hat{j}}{7} + \frac{6\hat{k}}{7}$

(d) $\frac{2\hat{i}}{7} + \frac{3\hat{j}}{7} - \frac{6\hat{k}}{7}$

1.1.2. If in $\triangle$ABC, $\overrightarrow{BA}$=2$\overrightarrow{a}$ and $\overrightarrow{BC}$=3$\overrightarrow{b}$, then $\overrightarrow{AC}$ is

(a) $2\overrightarrow{a}$ + $3\overrightarrow{b}$

(b) $2\overrightarrow{a}$ - $3\overrightarrow{b}$

(c) $3\overrightarrow{b}$ - $2\overrightarrow{a}$

(d) $-2\overrightarrow{a}$ -$3\overrightarrow{b}$

1.1.3. Equation of line passing through origin and making 30°, 60° and 90°with x, y, z axes respectively is

(a) $\frac{2x}{\sqrt{3}}=\frac{y}{2}=\frac{z}{0}$

(b) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{0}$

(c) 2x=$\frac{2y}{\sqrt{3}}=\frac{z}{1}$

(d) $\frac{2x}{\sqrt{3}}=\frac{2y}{1}=\frac{z}{1}$

1.1.4. If $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ are three non-zero unequal vectors such that $\overrightarrow{a}.\overrightarrow{b} = \overrightarrow{a}.\overrightarrow{c}$, then find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ - $\overrightarrow{c}$.

1.1.5. If the equation of a line is

$$x = ay + b, z = cy + d, \tag{1.1.5.1}$$

then find the direction ratios of the line and a point on the line.

1.1.6. Using Integration, find the area of triangle whose vertices are (-1, 1), (0, 5) and (3, 2).

1.2. 2022

1.2.1. 10

1.2.1. The distance between the points $(0, 0)$ and $(a - b, a + b)$ is

(a) $2\sqrt{ab}$

(b) $\sqrt{2a^2 + ab}$

(c) $2\sqrt{a^2 + b^2}$

(d) $\sqrt{2a^2 + 2b^2}$

1.2.2. The value of m which makes the point $(0,0)$, $(2m,-4)$ and $(3,6)$ collinear, is ______

1.2.3. A circle has its center at $(4,4)$. If one end of a diameter is $(4,0)$, then find the coordinates of other end.

1.2.4. Find the area of the quadrilateral ABCD whose vertices are $\mathbf{A}(-4,-3)$, $\mathbf{B}(3,-1)$, $\mathbf{C}(0,5)$ and $\mathbf{D}(-4,2)$

1.2.5. If the points $\mathbf{A}(2,0)$, $\mathbf{B}(6,1)$, and $\mathbf{C}(p,q)$ form a triangle of area 12sq. units (positive only) and

$$2p + q = 10 \tag{1.2.5.1}$$

, then find the values of p and q.

1.2.2. 12

1.2.1. $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that

$$\left|2\overrightarrow{a} + 3\overrightarrow{b}\right| = \left|3\overrightarrow{a} - 2\overrightarrow{b}\right|. \tag{1.2.1.1}$$

Find the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

1.2.2. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\overrightarrow{a} = \hat{i} - \hat{j} + \hat{k} \tag{1.2.2.1}$$

and

$$\overrightarrow{b} = 2\hat{i} - \hat{j} - 3\hat{k} \tag{1.2.2.2}$$

then find the vector $\overrightarrow{c}$, given that

$$\overrightarrow{a} \times \overrightarrow{c} = \overrightarrow{b} \tag{1.2.2.3}$$

and

$$\overrightarrow{a} . \overrightarrow{c} = 4. \tag{1.2.2.4}$$

1.2.3. If

$$\left|\overrightarrow{a} \times \overrightarrow{b}\right|^2 + \left|\overrightarrow{a} . \overrightarrow{b}\right|^2 = 400 \tag{1.2.3.1}$$

and

$$\left|\overrightarrow{b}\right| = 5 \tag{1.2.3.2}$$

find the value of $|\overrightarrow{a}|$.

1.2.4. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a} . \overrightarrow{b} = 1 \tag{1.2.4.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k} \tag{1.2.4.2}$$

, then find $\left|\overrightarrow{b}\right|$

1.2.5. If

$$|\overrightarrow{a}| = 3, \left|\overrightarrow{b}\right| = 2\sqrt{3} \tag{1.2.5.1}$$

and

$$\overrightarrow{a} . \overrightarrow{b} = 6, \tag{1.2.5.2}$$

then find the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$.

1.2.6. $|\overrightarrow{a}| = 8, \left|\overrightarrow{b}\right| = 3$ and $\overrightarrow{a} . \overrightarrow{b} = 12\sqrt{3}$, then the value of $\left|\overrightarrow{a} \times \overrightarrow{b}\right|$ is

(a) 24

(b) 144

(c) 2

(d) 12

1.2.7. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.2.7.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.2.7.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.2.8. $\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}$ and $\overrightarrow{d}$ are four non-zeros vectors such that

$$\overrightarrow{a} \times \overrightarrow{b} = \overrightarrow{c} \times \overrightarrow{d} \tag{1.2.8.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{c} = 4\overrightarrow{b} \times \overrightarrow{d} \tag{1.2.8.2}$$

, then show that $(\overrightarrow{a} - 2\overrightarrow{d})$ is parallel to $(2\overrightarrow{b} - \overrightarrow{c})$ where

$$\overrightarrow{a} \neq 2\overrightarrow{d}, \overrightarrow{c} \neq 2\overrightarrow{b} \tag{1.2.8.3}$$

1.2.9. If

$$\overrightarrow{a} = \hat{i} + \hat{j} + \hat{k}, \overrightarrow{a}.\overrightarrow{b} = 1 \tag{1.2.9.1}$$

and

$$\overrightarrow{a} \times \overrightarrow{b} = \hat{j} - \hat{k}, \tag{1.2.9.2}$$

then find $\left|\overrightarrow{b}\right|$

1.2.10. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two vectors such that

$$\left|\overrightarrow{a} + \overrightarrow{b}\right| = \left|\overrightarrow{b}\right|, \tag{1.2.10.1}$$

then prove that $(\overrightarrow{a} + 2\overrightarrow{b})$ is perpendicular to $\overrightarrow{a}$.

1.2.11. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are unit vectors and θ is the angle between them , then prove that sin

$$\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.2.11.1}$$

1.2.12. If $\overrightarrow{a}$ and $\overrightarrow{b}$ are two unit vectors such that and θ is the angle between them, then prove that

$$sin\frac{\theta}{2} = \frac{1}{2}\left|\overrightarrow{a} - \overrightarrow{b}\right| \tag{1.2.12.1}$$

1.2.13. If

$$\overrightarrow{a} = 2\hat{i} + y\hat{j} + \hat{k} \tag{1.2.13.1}$$

and

$$\overrightarrow{b} = \hat{i} + 2\hat{j} + 3\hat{k} \tag{1.2.13.2}$$

are two vectors for which the vector $(\overrightarrow{a} + \overrightarrow{b})$ is perpendicular to the vector $(\overrightarrow{a} - \overrightarrow{b})$ then find all the possible values of y.

1.2.14. Write the projection of the vector $(\overrightarrow{b} + \overrightarrow{c})$ on the vector $\overrightarrow{a}$, where

$$\overrightarrow{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + 2\hat{j} - 2\hat{k} \tag{1.2.14.1}$$

and

$$\overrightarrow{c} = 2\hat{i} - \hat{j} + 4\hat{k}. \tag{1.2.14.2}$$

1.2.15. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + \hat{k}, \overrightarrow{b} = \hat{i} + \hat{j} - 2\hat{k} \tag{1.2.15.1}$$

and

$$\overrightarrow{c} = \hat{i} + 3\hat{j} - \hat{k} \tag{1.2.15.2}$$

and the projection of vector $\overrightarrow{c} + \lambda\overrightarrow{b}$ on vector $\overrightarrow{a}$ is $2\sqrt{6}$, find the value of λ.

1.2.16. If

$$\overrightarrow{a} = 2\hat{i} + \hat{j} + 3\hat{k}, \hat{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.2.16.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} + 2\hat{k} \tag{1.2.16.2}$$

, then find $\overrightarrow{a}.(\overrightarrow{b} \times \overrightarrow{c})$.

1.2.17. If

$$\overrightarrow{a} = 2\hat{i} - \hat{j} + 2\hat{k} \tag{1.2.17.1}$$

and

$$\overrightarrow{b} = 5\hat{i} - 3\hat{j} - 4\hat{k} \tag{1.2.17.2}$$

, then find the ratio $\dfrac{projectionofvector\overrightarrow{a}onvector\overrightarrow{b}}{projectionofvector\overrightarrow{b}onvector\overrightarrow{a}}$

1.2.18. Show that the three vectors $2\hat{i} - \hat{j} + \hat{k}, \hat{i} - 3\hat{j} - 5\hat{k}$, and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the vertices of a right-angled triangle. If

$$\overrightarrow{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{b} = -\hat{i} + 2\hat{j} + \hat{k} \tag{1.2.18.1}$$

and

$$\overrightarrow{c} = 3\hat{i} + \hat{j} \tag{1.2.18.2}$$

are such that the vector $(\overrightarrow{a} + \lambda\overrightarrow{b})$ is perpendicular to vector $\overrightarrow{c}$, then find the value of λ.

1.2.19. If $\overrightarrow{a}, \overrightarrow{b}$ and $\overrightarrow{c}$ are the position vectors of the points $\mathbf{A}(2,3,-4)$, $\mathbf{B}(3,-4,-5)$ and $\mathbf{C}(3,2,-3)$ and respectively, then $\left|\overrightarrow{a} + \overrightarrow{b} + \overrightarrow{c}\right|$ is equal to

(a) $\sqrt{113}$

(b) $\sqrt{185}$

(c) $\sqrt{203}$

(d) $\sqrt{209}$

1.2.20. Find the values λ, for which the distance of point $(2,1,\lambda)$ from plane

$$3x + 5y + 4z = 11 \tag{1.2.20.1}$$

is $2\sqrt{2}$ units.

1.2.21. Find the coordinates of the point where the line through $(3,4,1)$ crosses the ZX-plane

1.2.22. Using vectors, find the area of the triangle withvertices $\mathbf{A}(-1,0,-2)$, $\mathbf{B}(0,2,1)$ and $\mathbf{C}(-1,4,1)$

1.2.23. Using integration, find the area of triangle region whose vertices are $(2,0)$, $(4,5)$ and $(1,4)$.

1.2.24. If a line makes $60°$ and $45°$ angles with the positive directions of X-axis and z-axis respectively, then find the angle that it makes with the positive direction of y-axis. Hence, write the direction cosines of the line.

1.2.25. The Cartesian equation of a line AB is :

$$\frac{2x-1}{12} = \frac{y+2}{2} = \frac{z-3}{3} \tag{1.2.25.1}$$

.

1.2.26. Find the directions cosines of a line parallel to line AB.

1.2.27. Find the direction cosines of a line whose cartesian equation is given as

$$3x+1 = 6y-2 = 1-z. \tag{1.2.27.1}$$

1.2.28. A vector of magnitude 9 units in the direction of the vector $-2\hat{\imath}-\hat{\jmath}+2\hat{k}$ is ______

1.2.29. The two adajacent sides of a parallelogram are represented by $2\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$. Find the unit vectors parallel to its diagonals. Using the diagonal vectors, find the area of the parallelogram also.

1.2.30. The two adjacent sides of a parallelogram are represented by vectors

$2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to one of its diagonals. Also,find the area of the parallelogram.

1.2.31. If

$$\overrightarrow{a} = \overrightarrow{i} + 2\overrightarrow{j} + 3\overrightarrow{k} \tag{1.2.31.1}$$

and

$$\overrightarrow{b} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k} \tag{1.2.31.2}$$

represent two adjacent sides of a parallelogram, then find the unit vector parallel to the diagonal of the parallelogram.

1.3. 2021

1.3.1. 10

1.3.1.1. Find the distance between the points $\mathbf{A}(-\frac{7}{3}, 5)$ and $\mathbf{B}(\frac{2}{3}, 5)$.

1.3.1.2. Check whether 13cm, 12cm, 5cm can be the sides of a right triangle.

1.3.1.3. (a) If PL and PM are two tangents to a circle with centre $\mathbf{O}$ from an external point $\mathbf{P}$ and $PL = 4$ cm, find the length of OP, where radius of the circle is 3 cm.

(b) Find the distance between two parallel tangents of a cicle of radius 2.5 cm.

1.3.1.4. Find the coordinates of the points which divides the line segment joining the points $\mathbf{A}(7,-1)$ and $\mathbf{B}(-3,-4)$ in the ratio $2:3$.

1.3.1.5. To divide a line segment QP internally in the ratio $2:3$, we draw a ray QY such that $\angle$ PQY is acute. What will be the minimum number of points to be located at equal distances on the ray QY ?

1.3.1.6. Answer any four of the following questions :

(i) The point which divides the line segment joining the points $(7,-6)$ and $(3,4)$ in the ratio $1:2$ lies in

(A) I quadrant

(B) II quadrant

(C) III quadrant

(D) IV quadrant

(ii) If the $\mathbf{A}(1,2)$, $\mathbf{O}(0,0)$ and $\mathbf{C}(a,6)$ are collinear, then the value of a is

(A) 6

(B) $\frac{3}{2}$

(C) 3

(D) 12

(iii) The distance between the points $\mathbf{A}(0,6)$ and $\mathbf{B}(0,-2)$ is

(A) 6 units

(B) 8 units

(C) 4 units

(D) 2 units

(iv) If $(\frac{a}{3}, 4)$ is the mid-point of the line segment joining the points $(-6, 5)$ and $(-2, 3)$, then the value of 'a ' is

(A) -4

(B) 4

(C) -12

(D) 12

(v) What kind of triangle is formed with vertices $\mathbf{A}(0, 2)$, $\mathbf{B}(-3, 0)$ and $\mathbf{C}(3, 0)$?

(A) A right triangle

(B) An equilateral triangle

(C) An isosceles triangle

(D) A scalene triangle

1.3.1.7. (a) If the distance between the points $(k, -2)$ and $(3, -6)$ is 10 units, find the positive value of k.

(b) Find the length of the segment joining $\mathbf{A}(-6, 7)$ and $\mathbf{B}(-1, -5)$.Also, find the mid-point of AB.

1.3.1.8. A man goes 5 metres due to West and then 12 metres due North. How far is he from the starting point ?

1.3.1.9. Students of a school are standing in rows and columns in their school playground to celebrate their annual sports day. **A**, **B**, **C** and **D** are the positions of four students as shown in the figure.

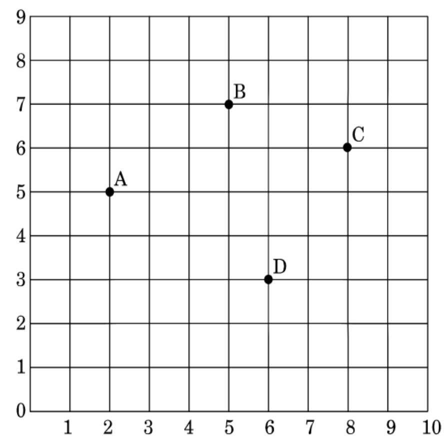

Figure 1.3.1.9.1: Based on the above, answer the following question :

(i) The figure formed by the points **A**, **B**, **C** and **D** is a

(A) sqaure

(B) parallelogram

(C) rhombus

(D) quadrilateral

(ii) If the sports teacher is sitting at the origin, then which of the four students is closest to him ?

(A) **A**

(B) **B**

(C) **C**

(D) **D**

(iii) The distance between **A** and **C** is

(A) $\sqrt{37}$ units

(B) $\sqrt{35}$ units

(C) 6 units

(D) 5 units

(iv) The coordinates of the mid-point of line segment AC are

(v) If a point $\mathbf{P}$ divides the line segment AD in the ratio $1:2$, then coordinates of $\mathbf{P}$ are

(A) $(\frac{8}{3}, \frac{8}{3})$

(B) $(\frac{10}{3}, \frac{13}{3})$

(C) $(\frac{13}{3}, \frac{10}{3})$

(D) $(\frac{16}{3}, \frac{11}{3})$

1.3.1.10. (a) Check whether the points $\mathbf{P}(5,-2)$, $\mathbf{Q}(6,4)$ and $\mathbf{R}(7,-2)$ are the vertices of an isosceles triangle PQR.

(b) Find the ratio in which $\mathbf{P}(4,5)$ divides the join of $\mathbf{A}(2,3)$ and $\mathbf{B}(7,8)$.

1.3.1.11. The coordinate of the three consecutive vertices of a parallelogram ABCD are $\mathbf{A}(1,3)$, $\mathbf{B}(-1,2)$, and $\mathbf{C}(2,5)$. Find the cordinates of the fourth vertex $\mathbf{D}$.

1.3.1.12. (a) If $\mathbf{P}(2,2)$, $\mathbf{Q}(-4,-4)$ and $\mathbf{R}(5,-8)$ are the vertices of a $\triangle$PQR, then find the length of the median through $\mathbf{R}$.

(b) Find the ratio in which y-axis divides the line segment joining the points $\mathbf{A}(5,-6)$ and $\mathbf{B}(-1,-4)$.Also, find the coordinates of the point of intersection.

1.3.1.13. (a) Find the ratio in which the line segment joining the points $\mathbf{A}(1,-5)$

and $\mathbf{B}(-4,5)$ is divided by the ax-axis. Also, find coordinates of the point of division.

(b) The points $\mathbf{A}(0,3)$, $\mathbf{B}(-2,a)$ and $\mathbf{C}(-1,4)$ are the vertices of a rigth triangle, right-angled at $\mathbf{A}$. Find the value of a.

1.3.2. 12

1. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are position vectors of the points A(2,3,-4), B(3,-4,-5) and C(3,2,-3) respectively, then $|\mathbf{a}+\mathbf{b}+\mathbf{c}|$ is equal to

   (a) $\sqrt{113}$

   (b) $\sqrt{185}$

   (c) $\sqrt{203}$

   (d) $\sqrt{209}$

2. Find the distance of the point (a,b,c) from the x-axis

3. If $\mathbf{a} = 2\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\mathbf{b} = 5\hat{\imath}-3\hat{\jmath}-4\hat{k}$, then find the ratio $\frac{\text{projection of vector } \mathbf{a} \text{ on } \mathbf{b}}{\text{projection of vector } \mathbf{b} \text{ on vector } \mathbf{a}}$

4. Let $\hat{a}$ and $\hat{b}$ be two unit vectors. If the vectors $\mathbf{c} = \hat{a} + 2\hat{b}$ and $\mathbf{d} = 5\hat{a} - 4\hat{b}$ are perpendicular to each other, then find the angle between the vectors $\hat{a}$ and $\hat{b}$.

5. Show that $|\mathbf{a}|\,\mathbf{b}$ + $|\mathbf{b}|\,\mathbf{a}$ is perpendicular to $|\mathbf{ab}|$- $|\mathbf{b}|\,\mathbf{a}$, for any two non-zero vectors $\mathbf{a}$ and $\mathbf{b}$.

6. Prove that three points A,B and C with position vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ respectively are collinear if and only if $(\mathbf{b}\times\mathbf{c})+(\mathbf{c}\times\mathbf{a})+(\mathbf{a}\times\mathbf{b}) = \mathbf{0}$.

Chapter 2

Linear Forms

2.1. 2023

2.1.1. 10

2.1.1. **Assertion (A):** Point $\mathbf{P}(0,2)$ is the point of intersection of $y - axis$ with the line $3x + 2y = 4$.

Reason (R): The distance of point $\mathbf{P}(0,2)$ from $x - axis$ is 2 units.

2.1.2. If the pair of equations $3x - y + 8 = 0$ and $6x - ry + 16 = 0$ represent coincident lines, then the value of 'r' is:

(a) $-\frac{1}{2}$

(b) $\frac{1}{2}$

(c) -2

(d) 2

2.1.3. The of linear equations $2x = 5y + 6$ and $15y = 6x - 18$ represents two lines which are:

(a) intersecting

(b) parallel

(c) coincident

(d) either intersecting or parallel

2.1.4. Find the equations of the diagonals of the parallelogram **PQRS** whose vertices are **P**(4,2,-6), **Q**(5,-3,1), **R**(12,4,5) and **S**(11,9,-2). Use these equations to find the point of intersection of diagonals.

2.1.5. A line l passes through point (-1,3,-2) and is perpendicular to both the lines $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$. Find the ctor equation of the line l. Hence, obtain its distance from origin.

2.1.2. 12

1. Equation of line passing through origin and making $30°, 60°$ and $90°$ with x, y, z axes respectively is

   (a) $\frac{2x}{\sqrt{3}} = \frac{y}{2} = \frac{z}{0}$

   (b) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{0}$

   (c) $2x = \frac{2y}{\sqrt{3}} = \frac{z}{1}$

   (d) $\frac{2x}{\sqrt{3}} = \frac{2y}{1} = \frac{z}{1}$

2. If the equation of a line is $x = ay + b, z = cy + d$,then find the direction ratios of the line and a point on the line.

3. (a) Find the equations of the diagonals of the parallelogram $PQRS$ whose vertices are $P(4,2,-6), Q(5,-3,1), R(12,4,5), S(11,9,-2)$. Use these equations to find the point of intersection of diagonals.

   (b) A line l passes through point $(-1,3,-2)$ and is perpendicular to both the lines $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and $\frac{x+2}{-3}=\frac{y-1}{2}=\frac{z+1}{5}$. Find the vector equation of the line l. Hence, obtain its distance from origin.

2.2. 2022

2.2.1. Solve the equations $x+2y=6$ and $2x-5y=12$ graphically.

2.2.2. Solve the following equations for x and y using cross-multiplication method:

$$(ax-by)+(a+4b)=0 \tag{2.2.2.1}$$

$$(bx+ay)+(b-4a)=0 \tag{2.2.2.2}$$

2.2.3. Find the co-ordinates of the point where the line $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}$ crosses the plane passing through the points $\left(\frac{7}{2},0,0\right),(0,7,0),(0,0,7)$.

2.2.4. Electrical transmission wires which are laid down in winters are stretched tightly to accommodate expansion in summers.

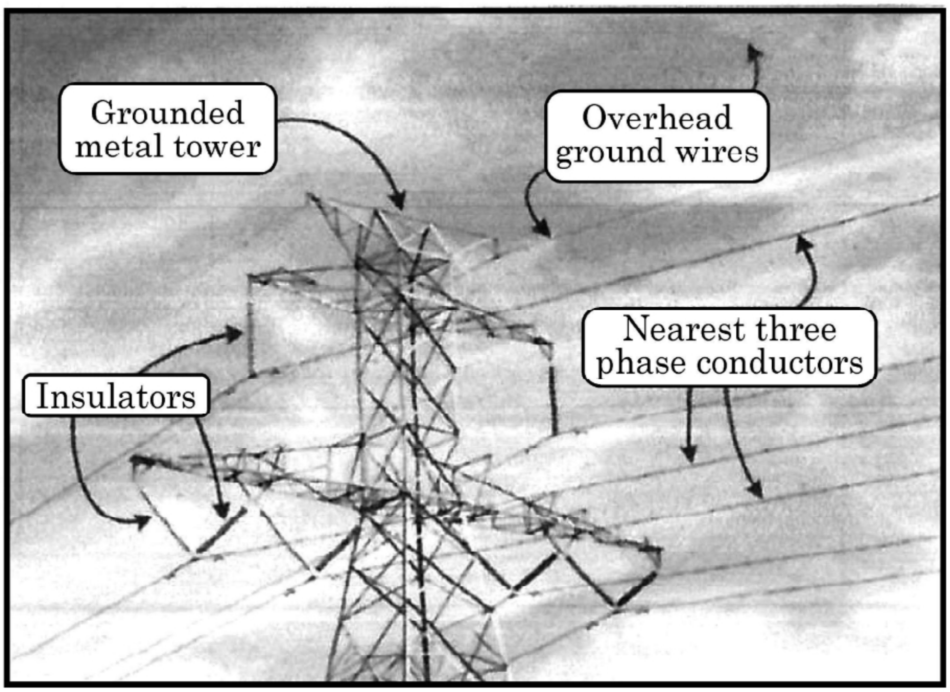

Figure 2.2.4.1: Electrical transmission wires connected to a transmission tower.

Two such wires in the figure 2.2.4.1 lie along the following lines:

$$l_1 : \frac{x+1}{3} = \frac{y-3}{-2} = \frac{z+2}{-1} \tag{2.2.4.1}$$

$$l_2 : \frac{x}{-1} = \frac{y-7}{3} = \frac{z+7}{-2} \tag{2.2.4.2}$$

Based on the given information, answer the following questions:

(a) Are the l_1 and l_2 coplanar? Justify your answer.

(b) Find the point of intersection of lines l_1 and l_2.

2.2.5. Write the cartesian equation of the line PQ passing through points

P$(2,2,1)$ and Q$(5,1,-2)$. Hence, find the y-coordinate of the point on the line PQ whose z-coordinate is -2.

2.2.6. Find the distance between the lines $x = \frac{y-1}{2} = \frac{z-2}{3}$ and $x+1 = \frac{y+2}{2} = \frac{z-1}{3}$.

2.2.7. Find the shortest distance between the following lines:

$$\mathbf{r} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k} + \lambda(\hat{\imath} - 2\hat{\jmath} + \hat{k}) \tag{2.2.7.1}$$

$$\mathbf{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(7\hat{\imath} - 6\hat{\jmath} + \hat{k}) \tag{2.2.7.2}$$

2.2.8. Two motorcycles A and B are running at a speed more than the allowed speed on the road (as shown in figure 2.2.8.1) represented by the following lines

$$\mathbf{r} = \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}) \tag{2.2.8.1}$$

$$\mathbf{r} = (3\hat{\imath} + 3\hat{\jmath}) + \mu(2\hat{\imath} + \hat{\jmath} + \hat{k}) \tag{2.2.8.2}$$

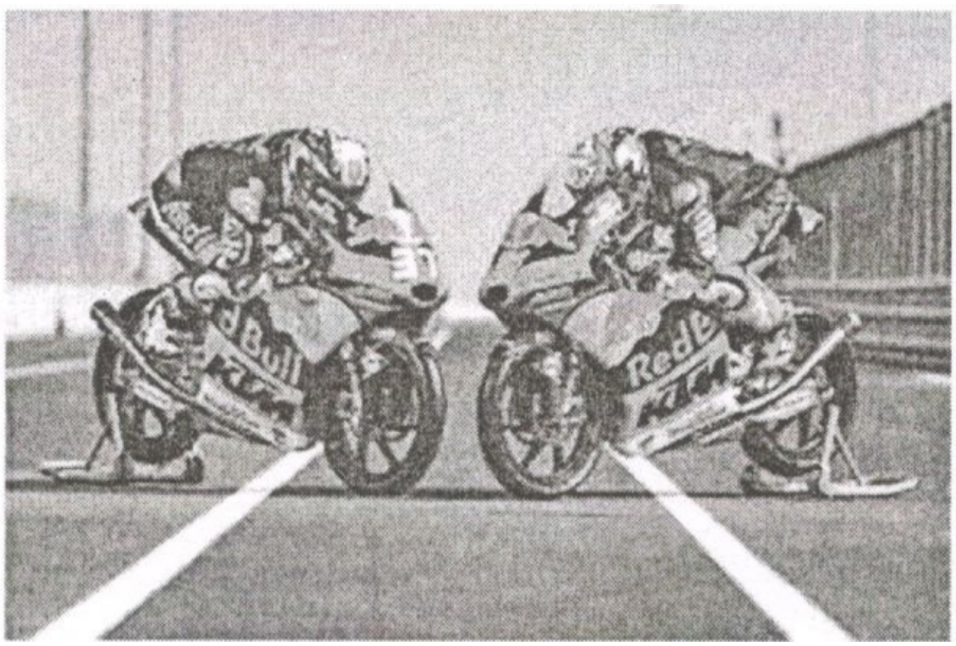

Figure 2.2.8.1: Two motorcycles moving along the road in a straight line.

Based on the following information, answer the following questions:

(a) Find the shortest distance between the given lines.

(b) Find a point at which the motorcycles may collide.

2.2.9. Find the shortest distance between the following lines

$$\mathbf{r} = (\lambda + 1)\hat{\imath} + (\lambda + 4)\hat{\jmath} - (\lambda - 3)\hat{k} \tag{2.2.9.1}$$

$$\mathbf{r} = (3 - \mu)\hat{\imath} + (2\mu + 2)\hat{\jmath} + (\mu + 6)\hat{k} \tag{2.2.9.2}$$

2.2.10. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z, \frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.2.10.1}$$

2.2.11. Find the equation of the plane passing through the points $(2, 1, 0), (3, -2, -2)$ and $(1, 1, 7)$. Also, obtain its distance from the origin.

2.2.12. The foot of a perpendicular drawn from the point $(-2, -1, -3)$ on a plane is $(1, -3, 3)$. Find the equation of the plane.

2.2.13. Find the cartesian and the vector equation of a plane which passes through the point $(3, 2, 0)$ and contains the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$.

2.2.14. The distance between the planes $4x-4y+2z+5 = 0$ and $2x-2y+z+6 = 0$ is

(a) $\frac{1}{6}$
(b) $\frac{7}{6}$
(c) $\frac{11}{6}$
(d) $\frac{16}{6}$

2.2.15. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r} \cdot (\hat{\imath} + 3\hat{\jmath}) + 6 = 0 \tag{2.2.15.1}$$

$$\mathbf{r} \cdot (3\hat{\imath} - \hat{\jmath} - 4\hat{k}) = 0 \tag{2.2.15.2}$$

which is at a unit distance from the origin.

2.2.16. If the distance of the point $(1,1,1)$ from the plane $x-y+z+\lambda=0$ is $\frac{5}{\sqrt{3}}$, find the value(s) of λ.

2.2.17. Find the distance of the point $(2,3,4)$ measured along the line $\frac{x-4}{3}=\frac{y+5}{6}=\frac{z+1}{2}$ from the plane $3x+2y+2z+5=0$.

2.2.18. Find the distance of the point $P(4,3,2)$ from the plane determined by the points $A(-1,6,-5), B(-5,-2,3)$ and $C(2,4,-5)$.

2.2.19. The distance of the line

$$\mathbf{r}=(\hat{\imath}-\hat{\jmath})+\lambda(\hat{\imath}+5\hat{\jmath}+\hat{k}) \tag{2.2.19.1}$$

from the plane

$$\mathbf{r}\cdot(\hat{\imath}-\hat{\jmath}+4\hat{k})=5 \tag{2.2.19.2}$$

is

(a) $\sqrt{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{3\sqrt{2}}$

(d) $\frac{-2}{3\sqrt{2}}$

2.2.20. Find a unit vector perpendicular to each of the vectors $(\mathbf{a}+\mathbf{b})$ and

$(\mathbf{a} - \mathbf{b})$ where

$$\mathbf{a} = \hat{\imath} + \hat{\jmath} + \hat{k} \tag{2.2.20.1}$$

$$\mathbf{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} \tag{2.2.20.2}$$

2.2.21. Find the distance of the point $(1, -2, 9)$ from the point of intersection of the line

$$\mathbf{r} = 4\hat{\imath} + 2\hat{\jmath} + 7\hat{k} + \lambda(3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}) \tag{2.2.21.1}$$

and the plane

$$\mathbf{r} \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}) = 10. \tag{2.2.21.2}$$

2.2.22. Find the area bounded by the curves $y = |x - 1|$ and $y = 1$, using integration.

2.2.23. Find the coordinates of the point where the line through $(4, -3, -4)$ and $(3, -2, 2)$ crosses the plane $2x + y + z = 6$.

2.2.24. Fit a straight line trend by the method of least squares and find the trend value for the year 2008 using the data from Table 2.2.24.1:

Table 2.2.24.1: Table showing yearly trend of production of goods in lakh tonnes

Year	Production (in lakh tonnes)
2001	30
2002	35
2003	36
2004	32
2005	37
2006	40

2.3. 2021

2.3.1. 10

2.3.1. If the graph of a pair of lines $x-2y+3=0$ and $2x-4y=5$ be drawn, that what type of lines are drawn ?

2.3.2. 12

1. If the two lines

$$L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2} \tag{1.1}$$

$$L_1 : x = 2, \frac{y}{-1} = \frac{z}{z-\alpha} \tag{1.2}$$

are perpendicular,then the value of α

(a) $\frac{2}{3}$

(b) 3

(c) 4

(d) $\frac{7}{3}$

2. Find the shortest distance between the following lines and hence write whether the lines are intersecting or not.

$$\frac{x-1}{2} = \frac{y+1}{3} = z \tag{2.1}$$

$$\frac{x+1}{5} = \frac{y-2}{1}, z = 2 \tag{2.2}$$

3. Find the equation of the plane through the line of intersection of the planes

$$\mathbf{r}.\,(i + 3j) + 6 = 0 \tag{3.1}$$

$$\mathbf{r}.\,(3i - j - 4k) = 0 \tag{3.2}$$

which is at a unit distance from the origin.

4. If segment of the line intercepted between the co-ordinate-axes is bi-

sected at the point $M\,(2,3)$, then the equation of this line is

$$2x + 3y = 13 \tag{4.1}$$

$$x + y = 5 \tag{4.2}$$

$$2x + y = 7 \tag{4.3}$$

$$3x + 2y = 12 \tag{4.4}$$

5. The equation of a line through $(2, -4)$ and parallel to x-axis is __________.

6. Find the equation of the median through vertex A of the triangle ABC, having vertices $A\,(2,5)$, $B\,(-4,9)$ and $C\,(-2,-1)$.

7. Solve the system of linear equations, using matrix method :

$$7x + 2y = 11 \tag{7.1}$$

$$4x - y = 2 \tag{7.2}$$

Chapter 3

Circles

3.1. 2023

3.1.1. 10

3.1.1. In the given figure Fig. 3.1.1.1, PQ is tangent to the circle centred at **O**. If $\angle AOB = 95^\circ$, then measure of $\angle ABQ$ will be

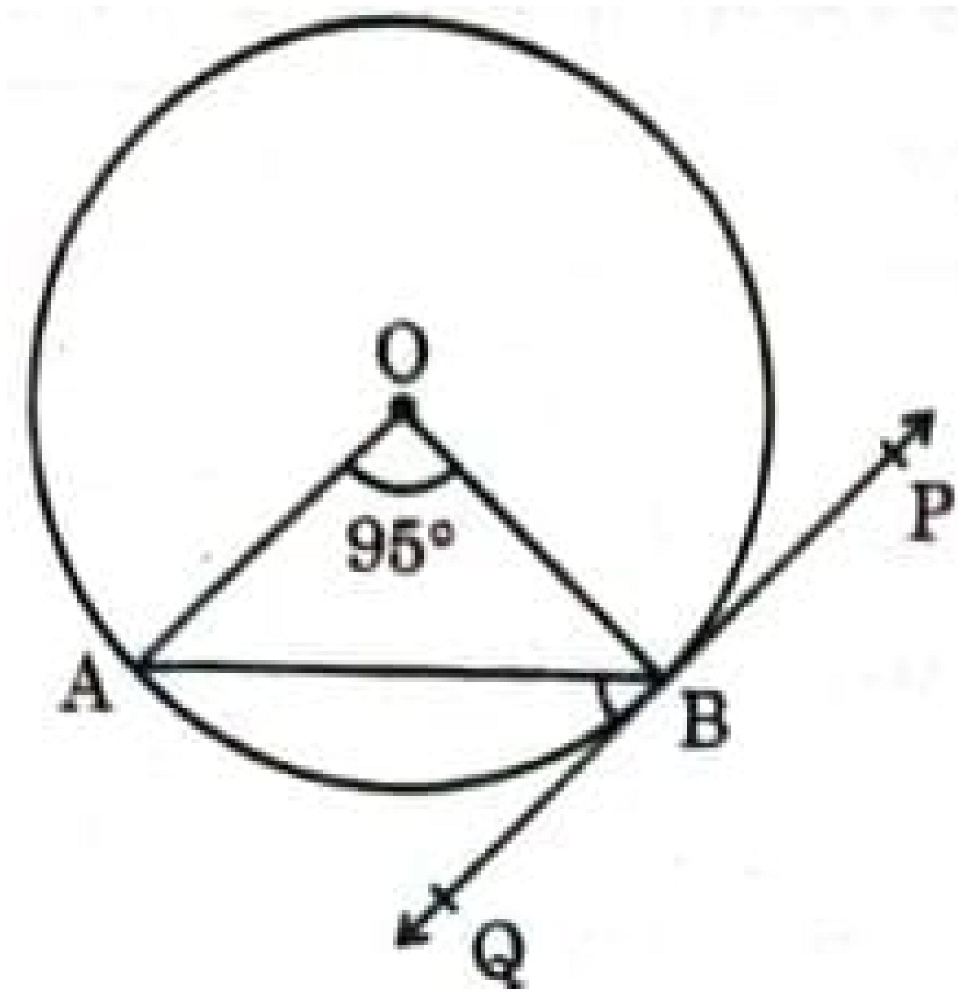

Figure 3.1.1.1:

(a) 47.5°

(b) 42.5°

(c) 85°

(d) 95°

3.1.2. (a) In the given figure Fig. 3.1.2.1, two tangents TP and TQ are drawn to be a circle with centre **O** from an external point **T**. Prove that $\angle PTQ = 2\angle OPQ$.

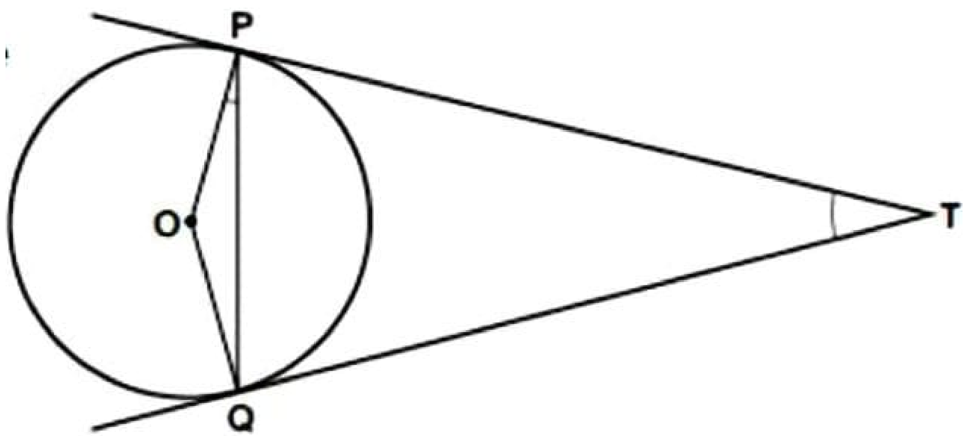

Figure 3.1.2.1:

(b) In the given figure Fig. 3.1.2.2, a circle is inscribed in a quadrilateral $ABCD$ in which $\angle B = 90°$. If $AD = 17cm, AB = 20cm$ and $DS = 3cm$, then find the radius of the circle.

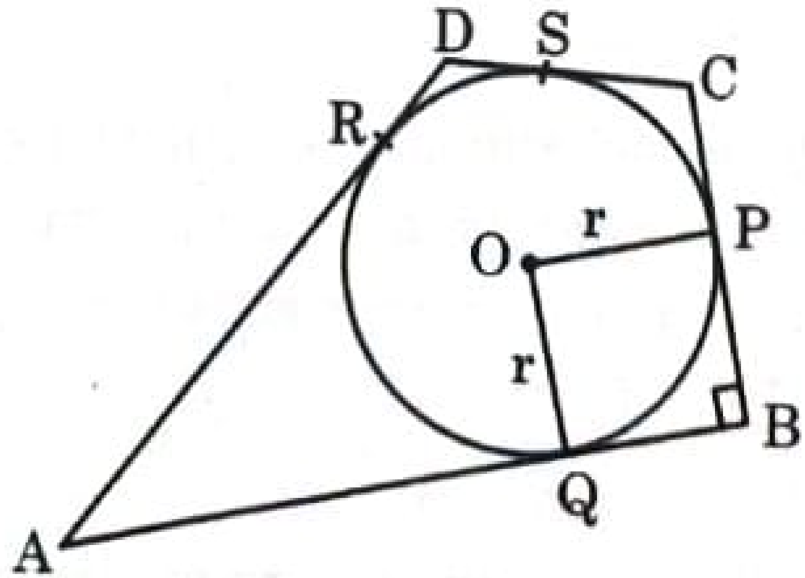

Figure 3.1.2.2:

3.1.3. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anti-clockwise around one and a half times through a cicrle as shown in Fig. 3.1.3.1 below, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.

Figure 3.1.3.1:

In the given figure Fig. 3.1.3.2, AB is one such tangent to a circle of radius 75 cm. Point **O** is centre of the circle and $\angle ABO = 30^{\circ}.PQ$ is parallel to OA.

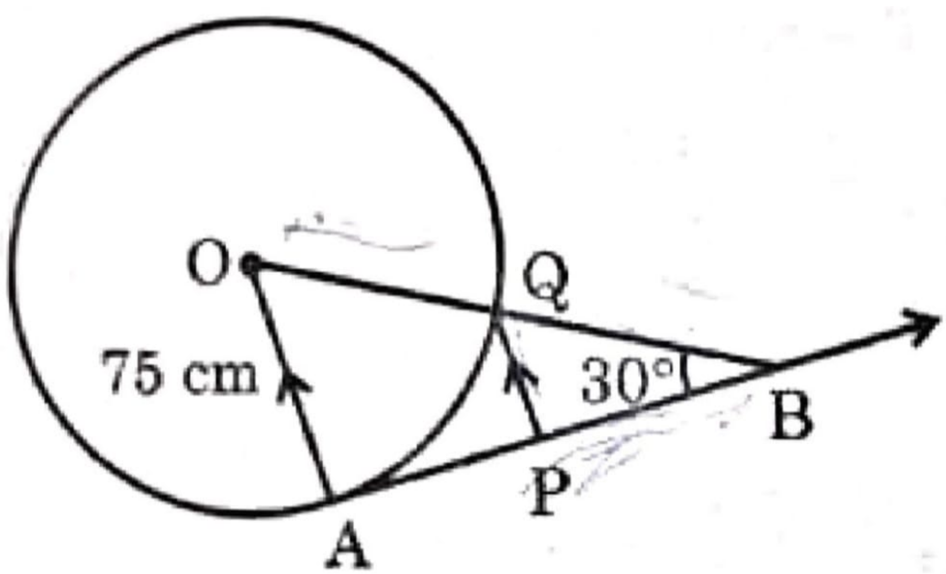

Figure 3.1.3.2:

Based on above information :

(a) find the length of AB.

(b) find the length of OB.

(c) find the length of AP.

(d) find the length of PQ.

3.1.4. In the given figure Fig. 3.1.4.1, the quadrilateral $PQRS$ circumscribes a circle. Here $PA + CS$ is equal to :

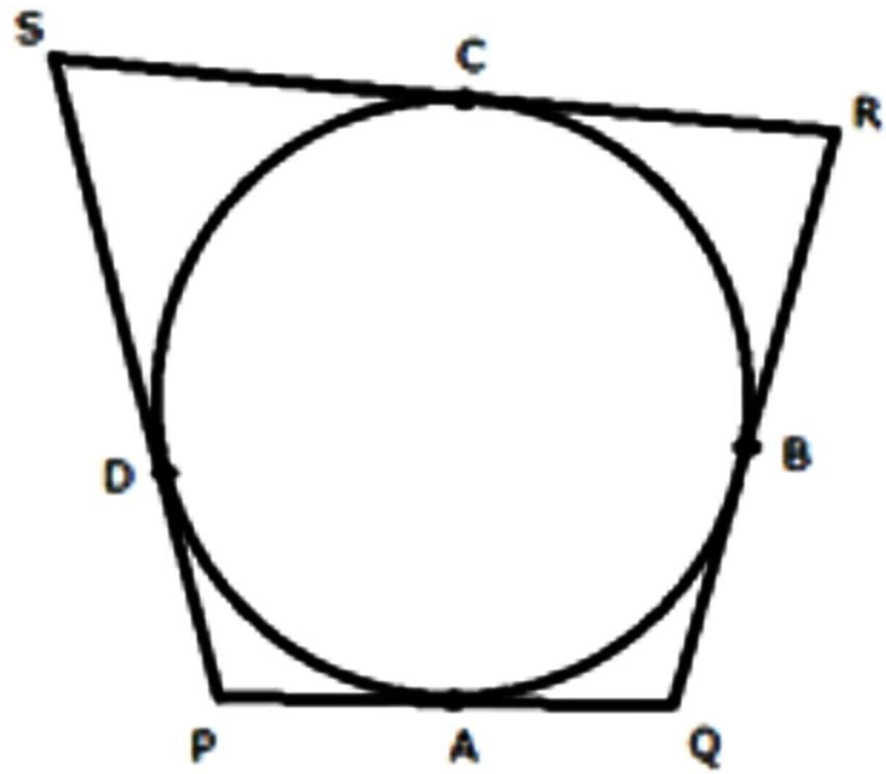

Figure 3.1.4.1:

(a) QR

(b) PR

(c) PS

(d) PQ

3.1.5. In the given figure Fig. 3.1.5.1, **O** is the centre of the circle. AB and AC are tangents drawn to the circle from point **A**. If $\angle BAC = 65°$, then find the measure of $\angle BOC$.

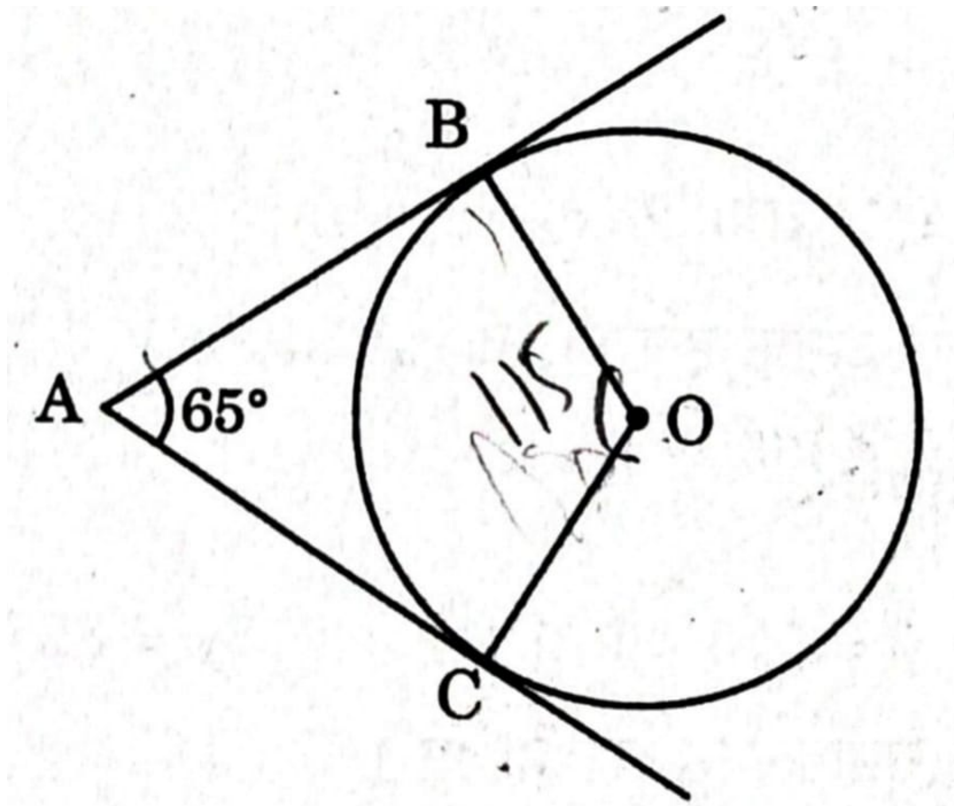

Figure 3.1.5.1:

3.1.6. In the given figure Fig. 3.1.6.1, **O** is the centre of the circle and QPR is the tangent to it at **P**. Prove that $\angle QAP + \angle APR = 90°$.

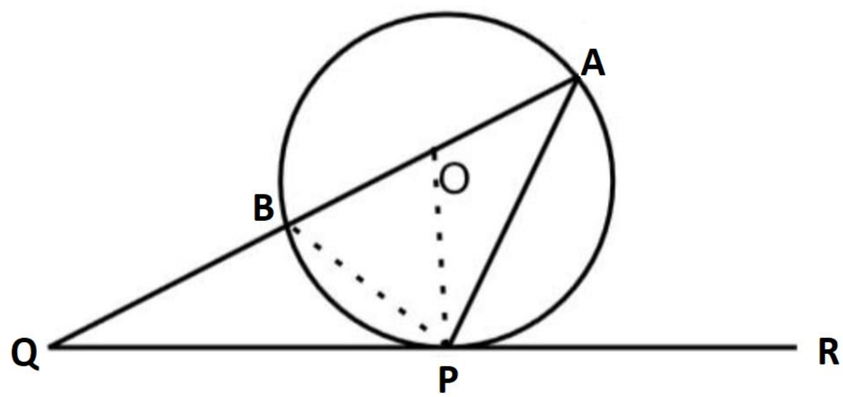

Figure 3.1.6.1:

3.1.7. In the given figure Fig. 3.1.7.1, TA is a tangent to the circle with centre **O** such that $OT = 4cm, \angle OTA = 30°$, then length of TA is :

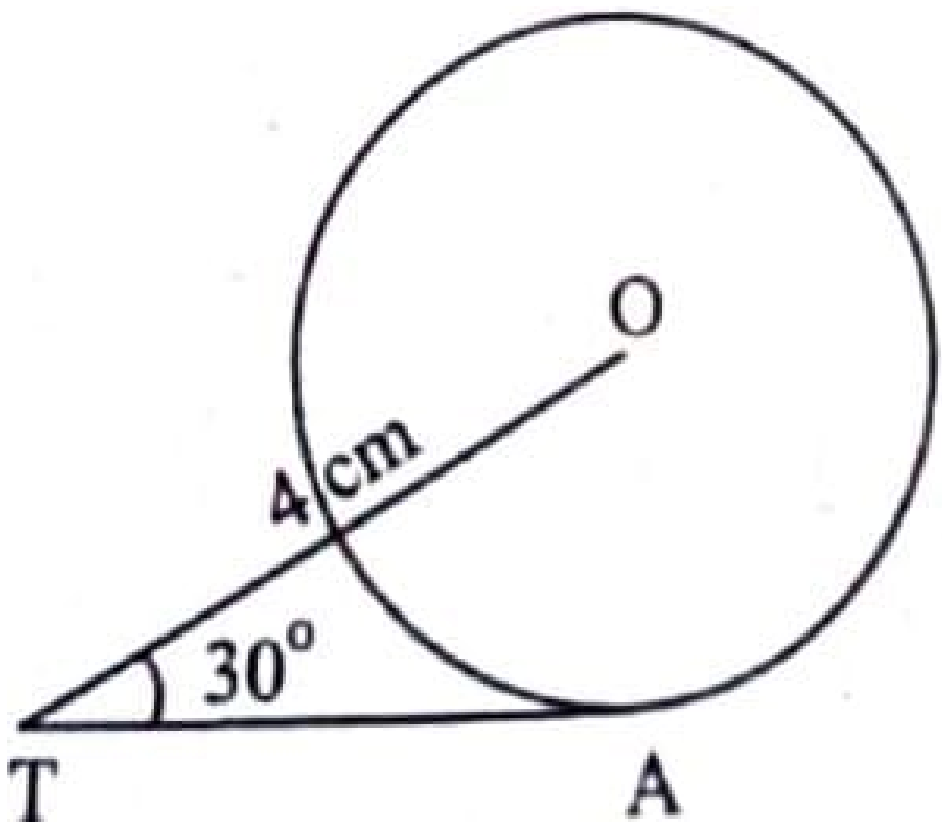

Figure 3.1.7.1:

(a) $2\sqrt{3}cm$

(b) $2cm$

(c) $2\sqrt{2}cm$

(d) $\sqrt{3}cm$

3.1.8. In the given figure Fig. 3.1.8.1, PT is a tangent at **T** to the circle with centre **O**. If $\angle TPO = 25°$, then x is equal to :

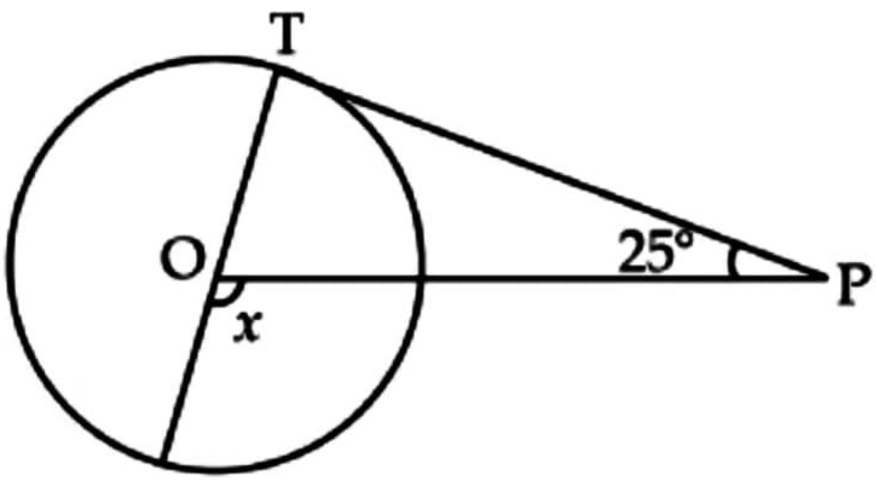

Figure 3.1.8.1:

(a) 25°

(b) 65°

(c) 90°

(d) 115°

3.1.9. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

3.2. 2022

3.2.1. Draw a circle of radius 2.5 cm. Take a point **P** outside the circle at a distance of 7 cm from the center. Then construct a pair of tangents to the circle from point **P**.

3.2.2. Write the steps of construction for constructing a pair of tangents to a circle of radius 4 cm from a point **P**, at a distance of 7 cm from its center **O**.

3.2.3. In Figure 3.2.3.1, there are two concentric circles with centre **O**. If ARC and AQB are tangents to the smaller circle from the point **A** lying on the larger circle, find the length of AC, if $AQ = 5$ cm.

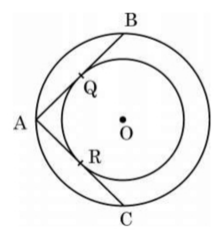

Figure 3.2.3.1: Two concentric circles with **O** as centre

3.2.4. In Figure 3.2.4.1, if a circle touches the side QR of ΔPQR at **S** and extended sides PQ and PR at **M** and **N**, respectively,

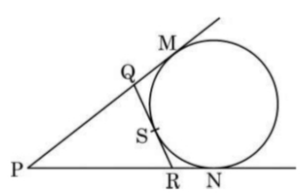

Figure 3.2.4.1: Two tangents are drawn from point **P** to the circle

prove that $PM = \frac{1}{2}(PQ + QR + PR)$

3.2.5. In Figure 3.2.5.1, a triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact **D** are of lengths 6 cm and 8 cm respectively. If the area of ΔABC is 84 cm^2, find the lengths of sides AB and AC.

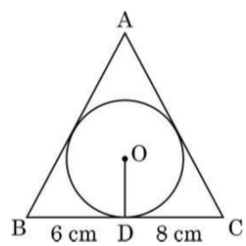

Figure 3.2.5.1: Circle with **O** as center circumscribed in triangle ABC

3.2.6. In Figure 3.2.6.1, PQ and PR are tangents to the circle centered at **O**. If $\angle OPR = 45°$, then prove that $ORPQ$ is a square.

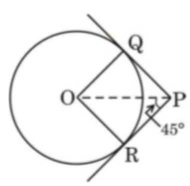

Figure 3.2.6.1: Two tangents drawn from point **P** to a circle whose centre is **O**

3.2.7. In Figure 3.2.7.1, **O** is the centre of a circle of radius 5 cm. PA and BC are tangents to the circle at **A** and **B** respectively. If OP is 13 cm, then find the length of tangents PA and BC.

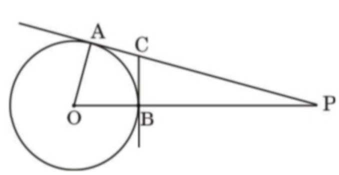

Figure 3.2.7.1: Two tangents drawn from point **C** to a circle whose centre is **O**

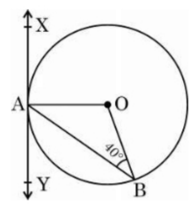

Figure 3.2.9.1: The line XAY is tangent to the circle centered at **O**

3.2.8. In Figure 3.2.8.1, AB is diameter of a circle centered at **O**. BC is tangent to the circle at **B**. If OP bisects the chord AD and $\angle AOP = 60°$, then find $m\angle C$.

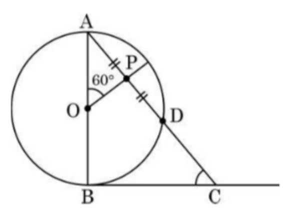

Figure 3.2.8.1: Tangent BC is drawn from point **C** to a circle whose centre is **O**

3.2.9. In Figure 3.2.9.1, XAY is a tangent to the circle centered at **O**. If $\angle ABO = 60°$,then find $m\angle BAY$ and $m\angle AOB$.

3.2.10. Two concentric circles are of radii 4cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

3.2.11. In Figure 3.2.11.1, a triangle ABC with $\angle B = 90°$ is shown. Taking AB as diameter, a circle has been drawn intersecting AC at point **P**. Prove that the tangent drawn at point **P** bisects BC.

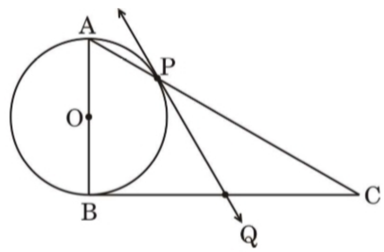

Figure 3.2.11.1: PQ is tangent to the circle centered at **O**. AB is the diameter and $\angle B = 90°$

3.2.12. Find the equation of tangent to the curve $y = x^2 + 4x + 1$ at the point $(3, 22)$.

3.3. 2021

3.3.1. 10

1. A quadrilateral $ABCD$ is drawn to circumscribe a circle (see Figure-1). Prove that $AB + CD = AD + BC$.

2. Draw a pair of tangents to a circle of radius $4cm$ which are inclined to each other at an angle of $45°$.

3. A point **T** is $13cm$ away from the centre of a circle. The length of the tangent drawn from **T** to the circle is $12cm$. Find the radius of the circle.

4. Two tangents TP and PQ are drawn to a circle with centre **O** from an external point **T**. Prove that $\angle PTQ = 2\angle OPQ$.

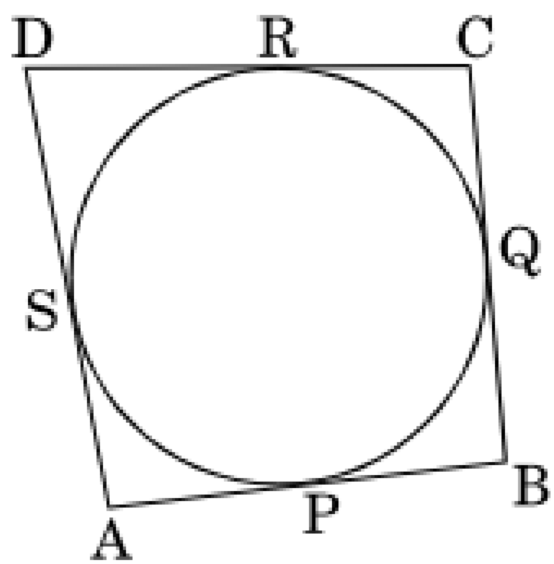

Figure 1.1:

5. PQ is a tangent to a circle with centre **O** at the point **P** on the circle. If $\triangle OPQ$ is an isosceles triangle, then find $\angle OQP$.

6. Two concentric circles have radii $10cm$ and $6cm$. Find the length of the chord of the larger circle which touches the smaller circle.

7. If tangents PA and PB from an external point **P** to a circle with centre **O** are inclined to each other at an angle of $70°$, then find $\angle POA$.

8. ABC is right triangle, right-angled at **B** with $BC = 6cm$ and $AB = 8cm$. A circle with centre **O** and radius r cm has been inscribed in $\triangle ABC$ as shown in the figure. Find the value of r.

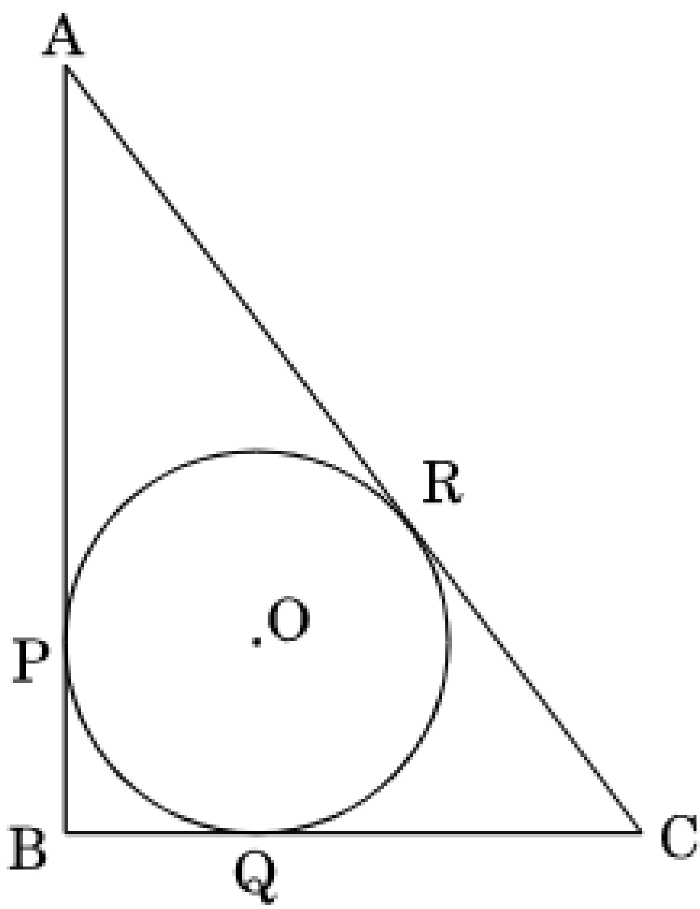

Figure 8.1:

9. Draw a circle of radius $5cm$. From a point $8cm$ away from its centre, construct a pair of tangents to the circle.

10. In the given figure, PT and PS are tangents to a circle with centre **O**, from a point **P**, such that $PT = 4cm$ and $\angle TPS = 60°$. Find the length of the chord TS. Also, find the radius of the circle.

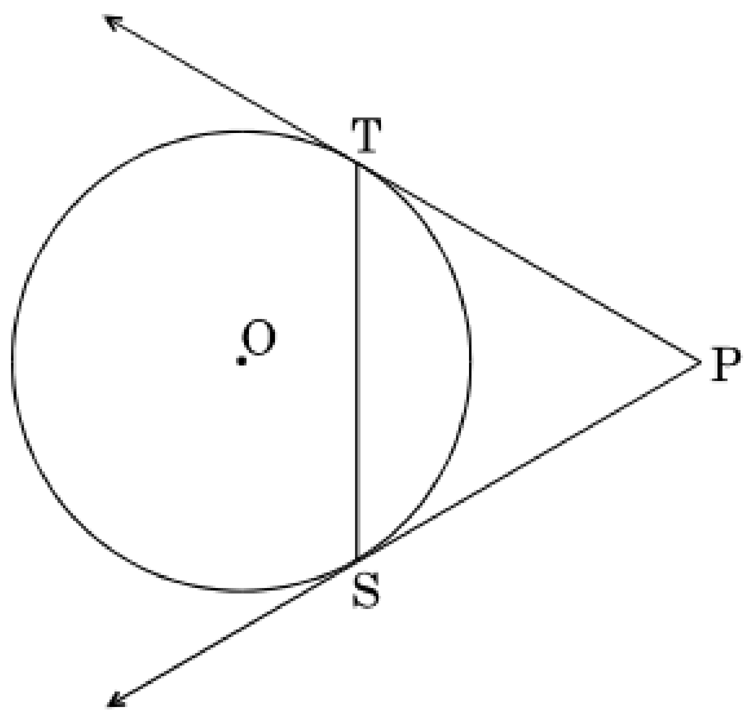

Figure 10.1:

11. (a) In a right triangle ABC, right-angled at **B**, $BC = 6cm$ and $AB = 8cm$. A circle is inscribed in the $\triangle ABC$. Find the radius of the incircle.

    (b) Two circles touch externally at **P** and AB is a common tangent, touching one circle at **A** and the other at **B**. Find the measure of $\angle APB$.

12. From an external point **P**, tangents PQ and PR are drawn to a circle with centre **O**, touching the circle at **Q** and **R**. If $\angle QOR = 140°$, find the measure of $\angle QPR$.

13. A circle touches all the sides of a quadrilateral $ABCD$. Prove that $AB + CD = DA + BC$.

14. Write the steps of construction of a circle of diameter $6cm$ and drawing of a pair of tangents to the circle from a point $5cm$ away from the centre.

Chapter 4

Intersection of Conics

4.1. 2022

4.1.1. Using integration, find the area of the region enclosed by the curve $y = x^2$, the x-axis and the ordinates $x = -2$ and $x = 1$.

4.1.2. Using integration, find the area of the region enclosed by line $y = \sqrt{3}x$ semi-circle $y = \sqrt{4 - x^2}$ and x-axis in first quadrant.

4.1.3. Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$.

4.1.4. If the area of the regin bounded by the curve $y^2 = 4ax$ and the line $x = 4a$ is $\frac{256}{3}$ sq. units, then using integration, find the value of a, where $a > 0$.

4.1.5. Find the area of the region enclosed by the curves $y^2 = x$, $x = \frac{1}{4}$, $y = 0$ and $x = 1$, using integration.

4.1.6. If the area of the region bounded bythe line $y = mx$ and the curve $x^2 = y$ is $\frac{32}{3}$ sq. units, then find the positive value of m, using integration.

4.1.7. If the area between the curves $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, then find the value of a, using integration.

4.1.8. Find the area bounded by the ellipse $x^2 + 4y^2 = 16$ and the ordinates $x = 0$ and $x = 2$, using integration.

4.1.9. Find the area of the region $\{(x, y) : x^2 \leq y \leq x\}$, using integration

4.2. 2021

4.2.1. 12

1. The point at which the normal to the curve

$$y = x + \frac{1}{x}, x > 0 \tag{1.1}$$

is perpendicular to the line

$$3x - 4y - 7 = 0 \tag{1.2}$$

(a) $\left(2, \frac{5}{2}\right)$

(b) $\left(\pm 2, \frac{5}{2}\right)$

(c) $\left(-\frac{1}{2}, \frac{5}{2}\right)$

(d) $\left(\frac{1}{2}, \frac{5}{2}\right)$

2. The points on the curve

$$\frac{x^2}{9} + \frac{y^2}{16} = 1 \tag{2.1}$$

at which the tangents are parallel to y-axis are:

(a) $(0, \pm 4)$

(b) $(\pm 4, 0)$

(c) $(\pm 3, 0)$

(d) $(0, \pm 3)$

3. For which value of m is the line

$$y = mx + 1 \tag{3.1}$$

a tangent to the curve

$$y^2 = 4x \tag{3.2}$$

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) 3

Chapter 5

Probability

5.1. 2021

5.1.1. 10

5.1.1. During the lockdown period,many families got bored of watching TV all the time.Out of these families, one family of 6 members decided to play a card game.17 cards numbered 1,2,3,4,..,17 are put in a box and mixed thoroughly.One card is drawn by one member at random and other family members bet for the chances of drawing the number either prime,odd or even etc.

Figure 5.1.1.1: Family of six

Based on the above,answer the following questions:

(i) The first member of the family draws a card at random and another member bets that it is an even prime number.What is the probability of his winning the bet?

(A) $\frac{2}{17}$

(B) $\frac{3}{17}$

(C) $\frac{1}{17}$

(D) $\frac{4}{17}$

(ii) The second member of the family draws a card at random and some other member bets that it is an even number. What is the probability of his winning the bet ?

(A) $\frac{7}{17}$

(B) $\frac{8}{17}$

(C) $\frac{9}{17}$

(D) $\frac{10}{17}$

(iii) What is the probability that the number on the card drawn at random is divisible by 5 ?

(A) $\frac{5}{17}$

(B) $\frac{4}{17}$

(C) $\frac{3}{17}$

(D) $\frac{2}{17}$

(iv) What is the probability that the number on the card drawn at random is a multiple of 3 ?

(A) $\frac{5}{17}$

(B) $\frac{6}{17}$

(C) $\frac{7}{17}$

(D) $\frac{8}{17}$

5.1.2. (a) Two different coins are tossed simultaneously. Write all the possible outcomes.

(b) A die is thrown once. Write the probability of getting a number less than 7.

5.1.3. If the probability of occurrence of event E,$\Pr(E)$=0.99, what is the probability of non-occurrence of the event E,$\Pr(notE)$?

5.1.4. (a) A bag contains 5 white balls and 7 red balls.A ball is drawn at random from the bag.What is the probability that it is either a white or a red ball?

(b) Two coins are tossed together once.What is the probability of getting at least one head?

5.1.5. Cards marked with numbers 1,2,3,4, ..., 100 are placed in a bag and mixed together throughly. A card is randomly drawn from the bag.Find the probability that the numbers on the card is

(i) an even number,

(ii) a 2-digit number,

(iii) a perfect square.

5.1.6. (a) How many outcomes are possible when three dice are thrown together?

(b) if $\Pr(E)$=0.015, then find $\Pr(notE)$.

5.1.7. During summer break, Harish wanted to play with his friends but it was too hot outside, so he decided to play some indoor game with his friends. He collects 20 identical Icards and writes the numbers 1 to 20 on them (one number on one card). He puts them in a box. He and his friends make a bet for the chances of drawing various cards out of the box. Ench was given a chance to tell the probability of picking one card out of the box.

Based on the above,answer the following questions:

(i) The probability that the number on the card drawn is an odd prime number,is

(A) $\frac{3}{5}$

(B) $\frac{2}{5}$

(C) $\frac{9}{20}$

(D) $\frac{7}{20}$

(ii) The probability that the number on the card drawn is a composite number is

(A) $\frac{11}{20}$

(B) $\frac{3}{5}$

(C) $\frac{4}{5}$

(D) $\frac{1}{2}$

(iii) The probability that the number on the card drawn is a multiple of 3,6 and 9 is

(A) $\frac{1}{20}$

(B) $\frac{1}{20}$

(C) $\frac{3}{20}$

(D) 0

(iv) The probability that the number on the card drawn is a multiple of 3 and 7is

(A) $\frac{3}{10}$

(B) $\frac{1}{10}$

(C) 0

(D) $\frac{2}{5}$

(v) If all cards having odd numbers written on them are removed from the box and then one card is drawn from the remaining cards, the probability of getting a card having a prime number is

(A) $\frac{1}{20}$

(B) $\frac{1}{10}$

(C) 0

(D) $\frac{1}{5}$

5.1.8. (a) In a single throw of a pair of dice,find the probability that both dice have the same number.

(b) A card is drawn from a well-shuffled pack of 52 cards.Find the probability that it is not an ace.

5.1.2. 12

1. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively.If both try to solve the question independently, the probability that the question is solved is

   (a) $\frac{7}{15}$

   (b) $\frac{8}{15}$

   (c) $\frac{2}{15}$

   (d) $\frac{14}{15}$

2. From a pack of 52 cards, 3 cards are drawn at random (without replacement).The probability that they are two red cards and one black card is__________.

3. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

4. Find the probability distribution of the number of successes in two tosses of a die, when a success is defined as "number greater than 5".

5. A bag contains 5 red and 4 black balls, a second bag contains 3 red and 6 black balls. One of the two bags is selected at random and two balls are drawn at random (without replacement), both of which are found to be red. Find the probability that these two balls are drawn from the second bag.

6. An unbiased die is thrown. What is the probability of getting an odd number or a multiple of 3 ?

   (a) $\frac{3}{4}$

   (b) $\frac{1}{2}$

   (c) $\frac{2}{3}$

   (d) $\frac{1}{3}$

7. A card is drawn from an ordinary pack of 52 cards and a gambler bets that it is a heart or a king card. What are the odds against his winning

this bet ?

(a) 4 : 9

(b) 1 : 4

(c) 4 : 1

(d) 9 : 4

8. In a lottery of 25 tickets, numbered 1 to 25, two tickets are drawn simultaneously. Find the probability that none of the tickets has prime number.

9. If E_1 and E_2 are two events, where E_1 is a subset of E_2, then evaluate $P(E_2 \mid E_1)$.

10. Two dice are thrown simultaneously. Find the probability of getting a multiple of 3 on one dice and a multiple of 2 on the other dice.

11. An urn contains 4 white, 7 green and 9 blue balls. If two balls are drawn at random, find the probability that the drawn balls are of the same colour.

5.2. 2023

5.2.1. 10

5.2.1. Probability of happining of an event is denoted by p and probability of non-happening of the event is denoted by q. Relation between p and q is

(a) $p+q=1$

(b) $p=1$, $q=1$

(c) $p=q$-1

(d) $p+q+1=0$

5.2.2. A girl calculates that the probability of her winning the first prize in a lottery is 0.08. If 6000 tickets are sold, how many tickets has she bought ?

(a) 40

(b) 240

(c) 480

(d) 750

5.2.3. In a group of 20 people, 5 can't swim. If one person is selected at random, then the probability that he/sh can swim, is

(a) $\frac{3}{4}$

(b) $\frac{1}{3}$

(c) 1

(d) $\frac{1}{4}$

5.2.4. A bag contain 4 red, 3 blue and 2 yellow balls. One ball is drawn at random from the bag. Find the probability that drawn ball is

(a) red

(b) yellow

5.2.5. A bag contain 100 cards numbered 1 to 100.Acard is drawn at random from the b. What is the probability that the number on the card is a perfect cube ?

(a) $\frac{1}{20}$

(b) $\frac{3}{50}$

(c) $\frac{1}{25}$

(d) $\frac{7}{100}$

5.2.6. If three coins are tossed simultaneously, what is the probability of getting a most one trail ?

(a) $\frac{3}{8}$

(b) $\frac{4}{8}$

(c) $\frac{5}{8}$

(d) $\frac{7}{8}$

5.2.7. Two dics are thrown together. The probability of getting the difference of numbers on their upper faces equals to 3 is :

(a) $\frac{1}{9}$

(b) $\frac{2}{9}$

(c) $\frac{1}{6}$

(d) $\frac{1}{12}$

5.2.8. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that the card drawn is not an ace is :

(a) $\frac{1}{13}$

(b) $\frac{9}{13}$

(c) $\frac{4}{13}$

(d) $\frac{12}{13}$

5.2.9. **Assertion (A) :** The probability that a leap year has 53 Students is $\frac{2}{7}$.

Reason (R) : The probability that a non-leap year has 53 Sundays is $\frac{5}{7}$.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

5.2.2. 12

5.2.1. If A and B are two events such that

$$\Pr(A|B) = 2 \times \Pr(B|A) \Pr(A) + \Pr(B) = \frac{2}{3} \tag{5.2.1.1}$$

then $\Pr(B)$ is equal to

(a) $\frac{2}{9}$

(b) $\frac{7}{9}$

(c) $\frac{4}{9}$

(d) $\frac{5}{9}$

5.2.2. (a) Two balls are drawn at random one by one with replacement from an urn containing equal number of red balls and green balls. Find the probability distribution of number of red balls. Also, find the mean of the random variable.

(b) A and B throw a die alternately till one of them gets '6' and wins the game. Find their respective probabilities of winning, if A starts the game first.

5.2.3. Recent studies suggest that roughly 12% of the world population is left handed.

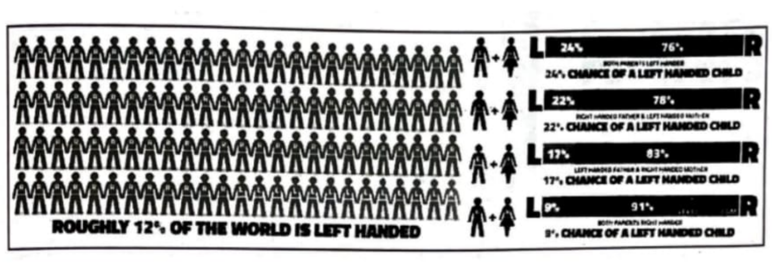

Figure 1: chance of left hand, depending upon parents

Figure 5.2.3.1: chance of left hand, depending upon parents

Depending upon the parents, the chances of having a left handed child are as follows :

(a) When both father and mother are left handed : Chances of left handed child is 24%.

(b) When father is right handed and mother is left handed : Chances of left handed child is 22%.

(c) when father is left handed and mother is right handed : Chances of left handed child is 17%.

(d) When both father nd mother are right handed : Chances of left handed child is 9%.

Assuming that $\Pr(A) = \Pr(B) = \Pr(C) = \Pr(D) = \frac{1}{4}$ and L denotes the event that child is left handed. Based on the above information, answer the following questions :

(a) Find $\Pr(L|C)$

(b) Find $\Pr\left(\overline{L}|A\right)$

(c) Find $\Pr(A|L)$

(d) Find the probability that a randomly selected child is left handed given that exactly one of the parent is left handed.

5.3. 2022

5.3.1. 12

5.3.1. Let A and B be two events such that $P(A) = \frac{5}{8}$, $P(B) = \frac{1}{2}$ and $P(A|B) = \frac{3}{4}$. Find the value of $P(B|A)$.

5.3.2. Two balls are drawn at random from a bag containing 2 red balls and 3 blue balls, without replacement. Let the variables X denotes the number of red balls. Find the probabillity distribution of X.

5.3.3. A card from a pack of 52 playing cards is lost. From the remaining cards, 2 cards are drawn at random without replacement, and are found to be both aces. Find the probability that lost card being an ace.

5.3.4. Probabilities of A and B solving a specific problem are $\frac{2}{3}$ and $\frac{3}{5}$, respectively. If both of them try independently to solve the problem, then find the probability that the problem is solved.

5.3.5. A pair of dice is thrown. It is given that the sum of numbers appearing on both dice is an even number. Find the probability that the number apprearing on at least one die is 3.

5.3.6. At the start of a cricket match, a coin is tossed and the team winning the toss has the opportunity to choose to bat or bowl. such a coin is unbaised with equal probabilities of getting head and tail Fig. 5.3.6.1

.

Based on the above information, answer the following question:

Figure 5.3.6.1: Toss before the match

(a) If such a coin is tossed 2 times, then find the probability distibution of numbers of tails.

(b) Find the probability of getting at least one head in three tosses of such a coin.

5.3.7. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of spade cards.

5.3.8. A pair of dice is thrown and the sum of the numbers appearing on the dice is observed to be 7. Find the probability that the number 5 has appeared on at least one die.

5.3.9. The probability that A hits the target is $\frac{1}{3}$ and the probability that B hits it, is $\frac{2}{5}$. If both try to hit the target independently, find the probabillity that the target is hit.

5.3.10. A shopkeeper sells three types of flower seeds A_1 , A_1 , A_3. They are sold in the form of a mixture, where the proportions of these seeds are 4 : 4 : 2, respectively. The germinaton rates of the three types of seeds are 45%, 60% and 35% respectively Fig. 5.3.10.1.

Figure 5.3.10.1: Three types of flowers

Based on the above information :

(a) Calculate the probability that a randomly chosen seed will germinate.

(b) Calculate the probability that the seed is of type A_2, given that a randomly choosen seed germinates.

5.3.11. Three friends A, B and C got their photograph clicked. Find the probability that B is standing at the central position, given that A is standing at the left corner.

5.3.12. In a game of Archery, each ring of the Archery target is valued. The centremost ring is worth 10 points and rest of the rings are alloted points 9 to 1 in sequential order moving outwards.Archer A is likely to earn 10 points with a probability of 0.8 and Archer B is likely the earn 10 points with a probability of 0.9 Fig. 5.3.12.1.

Based on the above innformation, answer the following questions :

(a) exactly one of them earns 10 points .

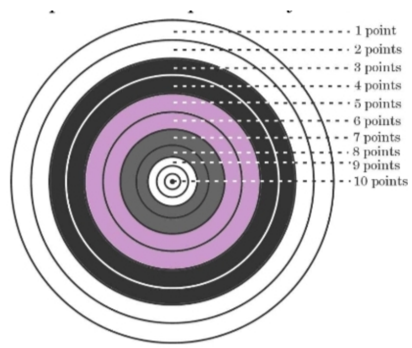

Figure 5.3.12.1: centermost ring

(b) both of them earn 10 point.

5.3.13. Event A and B are such that

$$P(A) = \frac{1}{2}, P(B) = \frac{7}{12} \tag{5.3.13.1}$$

and

$$P(\bar{A} \cup \bar{B}) = \frac{1}{4} \tag{5.3.13.2}$$

Find whether the events A and B are independent or not.

5.3.14. A box B_1 contain 1 white ball and 3 red balls. Another box B_2 contains 2 white balls and 3 red balls. If one ball is drawn at random from each of the boxes B_1 and B_2, then find the probability that the two balls drawn are of the same colour.

5.3.15. Let X be random variable which assumes values x_1, x_2, x_3, x_4 such that

$$2P(X = x_1) = 3P(X = x_2) = P(X = x_3) = 5P(X = x_4). \quad (5.3.15.1)$$

Find the probability distribution of X.

5.3.16. There are two boxes, namely box-I and box-II. Box-I contains 3 red and 6 black balls. Box-II contains 5 red and 5 black balls. One of the two boxes , is selected at random and a ball is drawn at random. The ball drawn is found to be red. Find the probability that this red ball comes out from box-II.

5.3.17. In a toss of three different coins, find the probability of comming up of three heads, if it is known that at least one head comes up.

5.3.18. A laboratory blood text is 98% effective in detecting a certain disease when it is fact, present. However, the text also yeilds a false positive result for 0.4% of the healthy person tested. From a large population, it is given that 0.2% of the population actually has the diseases.
Based on the above, answer the following questtion :

(a) one person, from the population, is taken at random and given the test. Find the probabiliy of his getting a positive test result.

(b) what is the probability that the person actually has the disease, given that his test result is positive ?

5.3.19. Two cards are drawn from a well-shuffled pack of playing cards one-by-one with replacement. The probability that the first card is a king and the second card is a queen is

(a) $\frac{1}{13} + \frac{1}{13}$

(b) $\frac{1}{13} \times \frac{4}{51}$

(c) $\frac{4}{52} \times \frac{3}{51}$

(d) $\frac{1}{13} \times \frac{1}{13}$

5.3.20. For two events A and B if P(A) $= \frac{4}{10}, PB = \frac{8}{10}$ and $P(B|A) = \frac{6}{10}$ then find $P(A \cup B)$.

5.3.21. Bag I contain 4 red and 3 black balls. Bag II contains 3 red and 5 black balls. One of two bags is selected at random and a ball is drawn from the bag, which if found to be red. Find the probability that the ball is drawn from bag II.

5.3.22. Two cards are drawn successively without replacement from a well-shuffled pack of 52 cards. Find the probability distribution of the number of aces and hence find its mean.

5.3.23. The probability of solving a specific question independently by A and B are $\frac{1}{3}$ and $\frac{1}{5}$ respectively . If both try to solve the question independently, the probability that the question is solved is

(a) $\frac{7}{15}$

(b) $\frac{8}{15}$

(c) $\frac{2}{15}$

(d) $\frac{14}{15}$

5.3.24. A card is picked at random from a pack of 52 playing cards. Given that the picked up card is a queen, the probability of it being a queen of spades is _____.

5.3.25. A bag contains 19 tickets, numbered 1 to 19. A ticket is drawn at random and then another ticket is drawn without replacing the first one in the bag. Find the probability distribution of the number of even numbers on the ticket.

5.3.26. Find the probability distribution of the numbers of successes in two tosses of a die, when a success is defined as number greater than 5.

5.3.27. Ten cartoons are taken at random from an automatic packing machine. The mean net weight of the ten carton is 11.8 kg and standard deviation is 0.15 kg. Does the sample mean differ significantly from the intended mean of 12 kg ? [Given that for d.f. = 9, $t_{0.05} = 2.26$]

5.3.28. A Coin is tossed twice. The following table5.3.28.2 shows the probability distribution of numbers of tails:

X	0	1	2
P(X)	K	6K	9K

Table 5.3.28.2: Table shows the probability distribution of numbers of tails

(a) Find the value of K.

(b) Is the coin tossed biased or unbaised? Justify your answer.

5.3.29. If X is a random variable with probability distribution as given below

5.3.29.2:

X	0	1	2
P(X)	K	4K	K

Table 5.3.29.2: table shows the proability distribution

The value of K and the mean of the distribution respectively are

(a) $\frac{1}{7}, 1$

(b) $\frac{1}{6}, 2$

(c) $\frac{1}{6}, 1$

(d) $1, \frac{1}{6}$

5.3.30. The random variable X has a probability function P(x) as defined below, where K is some number :

$$P(X) = \begin{cases} K, & \text{if } x = 0 \\ 2K, & \text{if } x = 1 \\ 3K, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases} \tag{5.3.30.1}$$

Find:

(a) The value of K.

(b) $P(X < 2), P(X \leq 2), P(X \geq 2)$.

5.3.31. Two rotten apples are mixed with 8 fresh apples. Find the probability distribution of number of rotten apples, if two apples are drawn at random, one-by-one without replacement.

Chapter 6

Construction

6.0.1. In the given figure, XZ is parallel to BC. $AZ = 3$cm, $ZC = 2$cm, $BM = 3$cm and $MC = 5$cm. Find the length of XY.

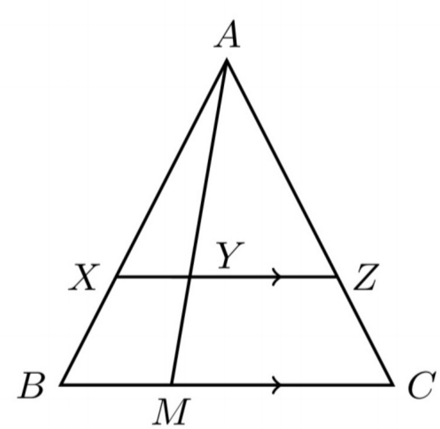

Figure 1: Isosceles Triangle

Figure 6.0.1.1: Isosceles Triangle

6.0.2. In the given figure, $DE \parallel BC$. If $AD = 2$units, $DB = AE = 3$units and $EC = x$units, then find the value of x is:

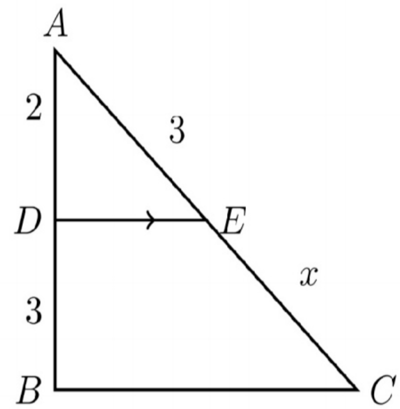

Figure 2: Right Angle Triangle

Figure 6.0.2.1: Right Angle Triangle

(a) 2

(b) 3

(c) 5

(d) $\frac{9}{2}$

6.0.3. In the given figure, ΔABC and ΔDBC are on te same base BC. If AD intersects BC at **O**, prove that $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$.

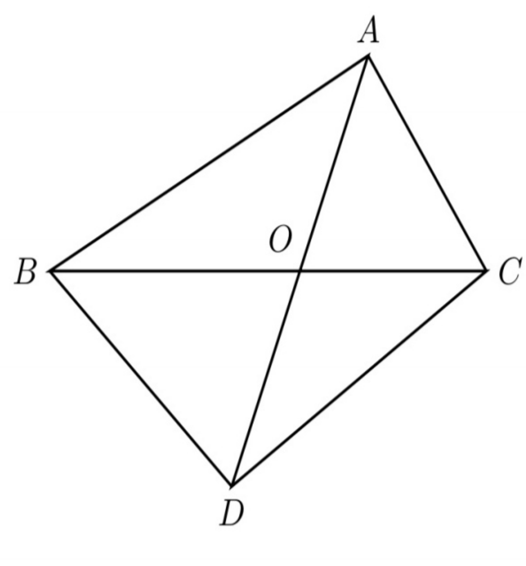

Figure 3: Triangles with same base

Figure 6.0.3.1: Triangles with same base

6.1. 2022

6.1.1. 10

6.1.1. In figure,Fig. 6.1.1.1 BN and CM are medians of a $\triangle$ ABC right-angled at A. Prove that

$$4(BN^2 + CM^2) = 5BC^2 \tag{6.1.1.1}$$

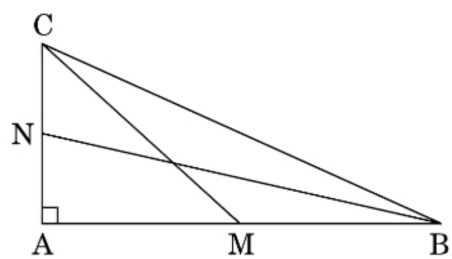

Figure 6.1.1.1: Right-angled triangle

6.1.2. **CaseStudy − 1** :

KiteFestival

Kite festival is celebrated in many countries at different times of the year. in India, every year 14th January is celebrated as international kite Day. on his day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below Fig. 6.1.2.1 , three kites flying together.

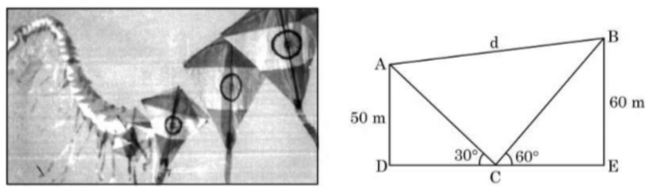

Figure 6.1.2.1: kites flying to gether

In Fig. 6.1.2.1, the angles of elevation of two kites (point C) are found to be °30 and °60 respectively. Taking

$$AD = 50m \tag{6.1.2.1}$$

and

$$BE = 60m \tag{6.1.2.2}$$

find

(a) The length of string used (take them straight) for kites A and B as shown in the figure.

(b) The distance 'd' between these two kites

6.2. 2021

6.2.1. 10

6.2.1. (a) **D** and **E** are points on the sides CA and CB respectively of a triangle ABC, right-angled at **C**.

Prove that $AE^2 + BD^2 = AB^2 + DE^2$

(b) Diagonals of a trapezium $ABCD$ with $AB \parallel DC$ intersect each other at the point **O**. If $AB = 2CD$, find the ratio of the areas of triangles AOB and COD.

6.2.2. Write the steps of construction of drawing a line segment $AB = 4.8$ cm and finding a point **P** on it suchthat $AP = \frac{1}{4}AB$.

6.2.3. Answer any $four$ of the following questions :

(a) Given $\triangle ABC$ $\triangle PQR$. If $\frac{AB}{PQ} = \frac{1}{3}$, than $\frac{ar(\triangle ABC)}{ar(\triangle PQR)}$ is

i. $\frac{1}{3}$

ii. 3

iii. $\frac{2}{3}$

iv. $\frac{1}{9}$

(b) The length of an altitude of n equilateral triangle of side 8 cm is

i. 4 cm

ii. $4\sqrt{3}$ cm

iii. $\frac{8}{3}$ cm

iv. 12 cm

(c) In $\triangle PQR, PQ = 6\sqrt{3}$ cm, $PR = 12cm$ and $QR = 6cm$. The measure of angle **Q** is

i. $120°$

ii. $60°$

iii. $90°$

iv. $45°$

(d) If $\triangle ABC$ $\triangle PQR$ and $\angle B = 46°$ and $\angle R = 69°$, then the measure of $\angle A$ is

i. $65°$

ii. $111°$

iii. $44°$

iv. $115°$

(e) **P** and **Q** are the points on the sides AB and AC respectively of a $\triangle ABC$ such that $PQ \parallel BC$. If $AP \parallel PB = 2 : 3$ and $AQ = 4cm$ then AC is equal to

i. 6 cm

ii. 8 cm

iii. 10 cm

iv. 12 cm

6.2.4. Answer any $four$ of the following questions :

(a) ABC and BDE are two equilateral triangles such that **D** is the mid-point of BC. The ratio of the areas of the triangles ABC and BDE is

i. 2 : 1

ii. 1 : 2

iii. 4 : 1

iv. 1 : 4

(b) In $\triangle ABC, AB = 4\sqrt{3}$ cm, $AC = 8cm$ and $BC = 4cm$. The angle **B** is

i. 120°

ii. 90°

iii. 60°

iv. 45°

(c) The perimeters of two similar triangles are 35 cm and 21 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of the second triangle is

i. 5.4 cm

ii. 4.5 cm

iii. 5.6 cm

iv. 15 cm

(d) In a $\triangle ABC$, **D** and **E** are points on the sides AB and Ac respectively such that $DE \parallel BC$ and $AD : DB = 3 : 1$. If $AE = 3.3cm$, then AC is equal to

i. 4 cm

ii. 1.1 cm

iii. 4.4 cm

iv. 5.5 cm

(e) In the isosceles triangle ABC, if $AC = BC$ and $AB^2 = 2AC^2$, then $\angle C$ is equal to

i. 30°

ii. 45°

iii. 60°

iv. 90°

Chapter 7

Optimization

7.1. 2023

1. The objective function $Z = ax + by$ of an LLP has maximum value 42 at (4,6) and minimum value 19 at (3,2).Which of the following is true?

    (a) $a = 9, b = 1$

    (b) $a = 5, b = 2$

    (c) $a = 3, b = 5$

    (d) $a = 5, b = 3$

2. The corner point of the feasible region of a linear programming problem are (0,4), (8,0)and $(\frac{20}{3},\frac{4}{3})$.if $Z = 30x + 24y$ is the objective fnction, then (maximum value of Z-minimum value of Z) is equal to

    (a) 40

    (b) 96

    (c) 120

    (d) 136

3. Solve the following linear programming problem graphically :

$$Maximum : Z = x + 2y$$

$$subjecttoconstraints : x + 2y \geq 100,$$

$$2x - y \leq 0,$$

$$2x + y \leq 200,$$

$$x \geq 0, y \geq 0.$$

4. Engine displacement is the measure of the cylinder volume swept by all the pistons engine.The piston move inside the cylinder bore

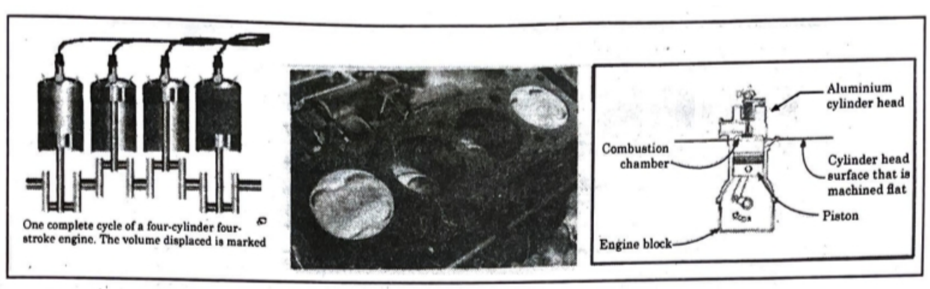

Figure 4.1: Engine

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi cm^2$

Based on the above information,answer the following questions:

(a) if the radius of cylinder is r cm and height is h cm,then write the volme V of cylinder in terms of radius r.

(b) Find $\frac{dV}{dr}$.

(c) i. Find the radius of cylinder when its volume is maximum.

  ii. For maximum volume,$h > r$.State true or false and justify.

7.2. 2021

7.2.1. 12

1. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires $3g$ of silver and $1g$ of gold, while that of type B requires $1g$ of silver and $2g$ of gold. The company can use at the most $9g$ of silver and $8g$ of gold. If each unit of type A brings a profit of ₹ 120 and that of type B ₹ 150, then find the number of units of each type that the company should produce to maximise profit. Formulate the above LPP and solve it graphically. Also, find the maximum profit.

2. Find the maximum value of $7x + 6y$ subject to the constrains:

$$x + y \geq 2 \tag{2.1}$$

$$2x + 3y \leq 6 \tag{2.2}$$

$$x \geq 0 \text{and} y \geq 0 \tag{2.3}$$

3. A window is in the form of a rectangular mounted by a semi-circular opening.The total perimeter of the window to admit maximum light through the whole opening.

4. Divide the number 8 into two positive numbers such that the sum of the cube of one and the square of the other is maximum.

5. Find the maximum and the minimum values of

$$z = 5x + 2y \tag{5.1}$$

subject to the constrains:

$$-2x - 3y \leq -6 \tag{5.2}$$

$$x - 2y \leq 2 \tag{5.3}$$

$$6x + 4y \leq 24 \tag{5.4}$$

$$-3x + 2y \leq 3 \tag{5.5}$$

$$x \geq 0, y \geq 0 \tag{5.6}$$

6. A furniture dealer deals in only two items : chairs and tables. He has ₹ 5,000 to invest and a space to store at most 60 pieces. A table costs him ₹ 250 and a chair ₹ 50. He sells a table at a profit of ₹ 50 and a chair at a profit of ₹ 15. Assuming that he can sell all the items he buys, how should he invest his money in order that he may maximize his profit ? Formulate the above as a linear programming problem.

7. The least value of the function

$$f(x) = 2\cos(x) + x \tag{7.1}$$

in the closed interval $\left[0, \frac{\pi}{2}\right]$ is:

(a) 2

(b) $\frac{\pi}{6} + \sqrt{3}$

(c) $\frac{\pi}{2}$

(d) The least value does not exist.

8. A linear programming problem is as follows: Minimize

$$Z = 30x + 50y \tag{8.1}$$

subject to the constrains,

$$3x + 5y \geq 15 \tag{8.2}$$

$$2x + 3y \leq 18 \tag{8.3}$$

$$x \geq 0, y \geq 0 \tag{8.4}$$

In the feasible region, the minimum value of Z occurs at

(a) a unique point

(b) no point

(c) infinitely many points

(d) two points only

9. The area of a trapezium is defined by function f and given by

$$f(x) = (10 + x)\sqrt{100 - x^2} \tag{9.1}$$

, then the area when it is maximised is:

(a) $75cm^2$

(b) $7\sqrt{3}cm^2$

(c) $75\sqrt{3}cm^2$

(d) $5cm^2$

10. For an objective function

$$Z = ax + by \tag{10.1}$$

,where $a, b > 0$;the corner points of the feasible region determined by a set of constrains (linear inequalities) are $(0, 20)$, $(10, 10)$, $(30, 30)$, and $(0, 40)$.The condition on a and b such that the maximum Z occurs at the points $(30, 30)$ and $(0, 40)$ is:

(a) $b - 3a = 0$

(b) $a = 3b$

(c) $a + 2b = 0$

(d) $2a - b = 0$

11. In a linear programming problem, the constrains on the decision variables x and y are $x - 3y \geq 0, y \geq 0, 0 \leq x \leq 3$.The feasible region

(a) is not in the first quadrant

(b) is bounded in the first quadrant

(c) is unbounded in the first quadrant

(d) does not exist

12. Based on the given shaded region in figure 12.1 as the feasible region in the graph, at which point(S) is the objective function

$$Z = 3x + 9y \tag{12.1}$$

maximum?

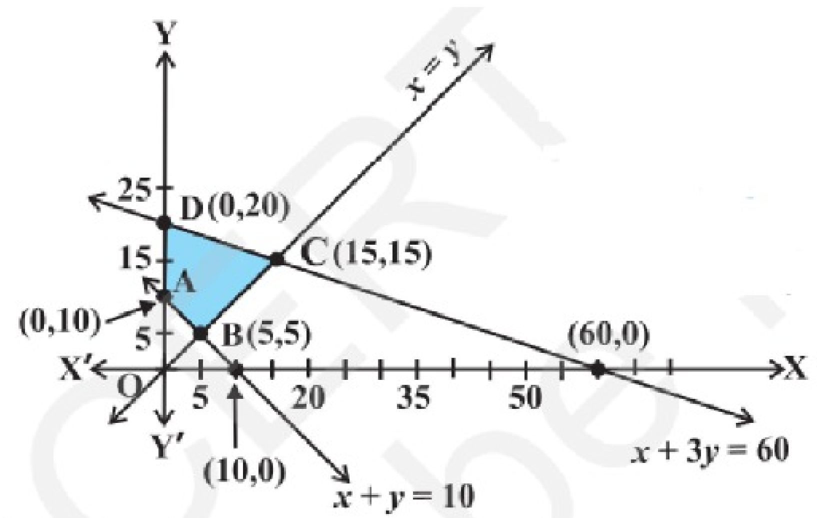

Figure 12.1: Optimization graph

(a) point B

(b) point C

(c) point D

(d) every point on the line segment CD

13. In figure 13.1, the feasible region for a LPP is shaded.The objective function

$$Z = 2x - 3y \tag{13.1}$$

,will be minimum at:

(a) $(4, 10)$

(b) $(6, 8)$

(c) $(0, 8)$

(d) $(6, 5)$

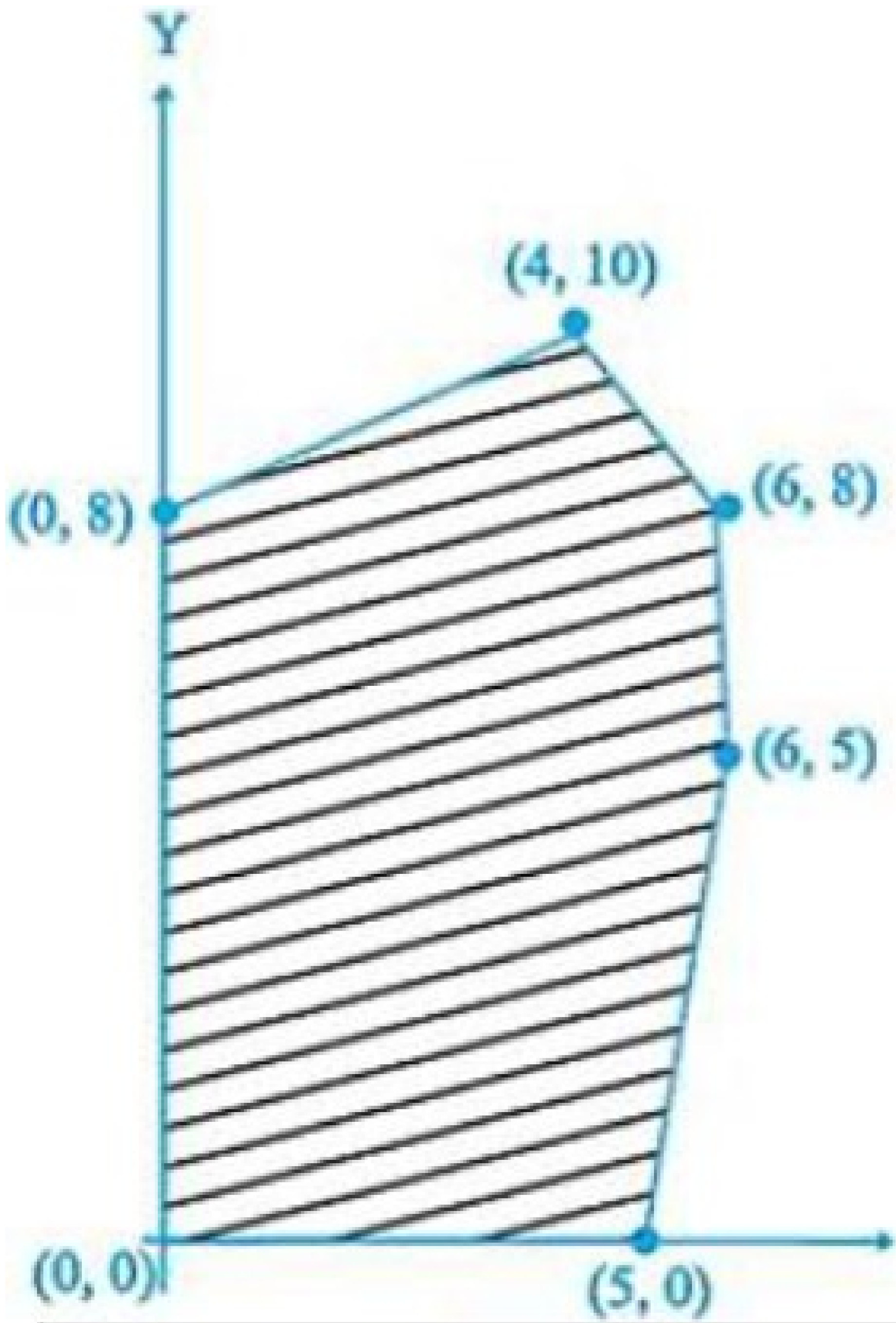

Figure 13.1: Optimization graph

Chapter 8

Algebra

8.1. 2023

8.1.1. 10

1. If one zero of the polynomial

$$p(x) = 6x^2 + 37x - (k - 2) \tag{1.1}$$

   is reciprocal of the other, then find the value of k?

2. Find the value of $'p'$ for which one root of the quadratic equation

$$px^2 - 14x + 18 = 0 \tag{2.1}$$

   is 6 times the other?

3. (a) prove that

$$\frac{\sin A - 2\sin^3 A}{2\cos^3 A - \cos A} = \tan A \tag{3.1}$$

(b)

$$\sec A(1 - \sin A)(\sec A + \tan A) = 1 \tag{3.2}$$

4. Which of the following quadratic equations has sum of its roots as 4?

   (a) $2x^2 - 4x + 8 = 0$

   (b) $-x^2 + 4x + 4 = 0$

   (c) $\sqrt{2x^2} - \frac{4}{\sqrt{2}}x + 1 = 0$

   (d) $4x^2 - 4x + 4 = 0$

5. if one zero of the polynomial

$$6x^2 + 37x - (k - 2) \tag{5.1}$$

   is reciprocal of the other,then what is the value of k?

   (a) -4

   (b) -6

   (c) 6

   (d) 4

6. The zeroes of the polynomial

$$p(x) = x^2 + 4x + 3 \tag{6.1}$$

   are given by:

(a) 1,3

(b) -1,3

(c) 1,-3

(d) -1,-3

7. If α and β are the zeroes of the quadratic polynomial $p(x) = x^2 - ax - b$, then the value of $\alpha^2 + \beta^2$ is:

(a) $a^2 - 2b$

(b) $a^2 + 2b$

(c) $b^2 - 2a$

(d) $b^2 + 2a$

8. The below is the Assertion and Reason based question. Two statements are given, one labelled as Assertion(A) and the other is labelled as Reason(R). Select the correct answer to these questions from the codes (a),(b),(c) and (d) as given below.

(a) Both Assertion(A) and Reason(R) are true and Reason(R) is the correct explanation of the Assertion(A).

(b) Both Assertion(A) and Reason(R) are true, but Reason(R) is not the correct explanation of the Assertion(A).

(c) Assertion(A) is true, but Reason(R) is false.

(d) Assertion(A) is false, but Reason(R) is true.

Assertion(A): The polynomial $p(x) = x^2 + 3x + 3$ has two real zeroes.

Reason(R): A quadratic polynomial can have at most two real zeroes.

9. (a) If

$$4\cot^2 45^\circ - \sec^2 60^\circ + \sin^2 60^\circ + p = \frac{3}{4}, \tag{9.1}$$

then find the value of p.

(b) If

$$\cos A + \cos^2 A = 1, \tag{9.2}$$

then find the value of

$$\sin^2 A + \sin^4 A. \tag{9.3}$$

10. Prove that:

$$\left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right) = \frac{1}{\tan\theta + \cot\theta} \tag{10.1}$$

11. The value of k for which the pair of equations $kx = y+2$ and $6x = 2y+3$ has infinitely many solutions,

(a) is $k = 3$

(b) does not exist

(c) is $k = -3$

(d) is $k = 4$

12. If $2\tan A = 3$, then the value of $\frac{4sinA+3\cos A}{4\sin A-3\cos A}$ is

   (a) $\frac{7}{\sqrt{13}}$

   (b) $\frac{1}{\sqrt{13}}$

   (c) 3

   (d) does not exist

13. If α, β are the zeroes of a polynomial $p(x) = x^2 + x - 1$, then $\frac{1}{\alpha} + \frac{1}{\beta}$ equals to

   (a) 1

   (b) 2

   (c) -1

   (d) $\frac{-1}{2}$

14. $\left(\sec^2\theta - 1\right)\left(\csc^2\theta - 1\right)$ is equal to:

   (a) -1

   (b) 1

   (c) 0

   (d) 2

15. The roots of equation

$$x^2 + 3x - 10 = 0 \tag{15.1}$$

are:

(a) $(2, -5)$

(b) $(-2, 5)$

(c) $(2, 5)$

(d) $(-2, -5)$

16. If α , β are zeroes of the polynomial $x^2 - 1$,then value of $(\alpha + \beta)$ is:

(a) 2

(b) 1

(c) 1

(d) 0

17. If α, β are the zeroes of the polynomial

$$p(x) = 4x^2 - 3x - 7 \tag{17.1}$$

,then $\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$ is equal to:

(a) $\frac{7}{3}$

(b) $\frac{-7}{3}$

(c) $\frac{3}{7}$

(d) $\frac{-3}{7}$

18. Find the sum and product of the roots of the quadratic equation

$$2x^2 - 9x + 4 = 0 \tag{18.1}$$

19. Find the discriminant of the quadratic equation

$$4x^2 - 5 = 0 \tag{19.1}$$

and hence comment on the nature of roots of the equation.

20. Evaluate $2\sec^2\theta + 3\csc^2\theta - 2\sin\theta\cos\theta$ if

$$\theta = 45^\circ \tag{20.1}$$

21. If

$$\sin\theta - \cos\theta = 0 \tag{21.1}$$

,then find the value of $\sin^4\theta + \cos^4\theta$.

Chapter 9

Geometry

9.1. 2023

9.1.1. 10

1. What is the area of a semi-circle of diameter $'d'$?

   (a) $\frac{1}{16}\pi d^2$

   (b) $\frac{1}{4}\pi d^2$

   (c) $\frac{1}{8}\pi d^2$

   (d) $\frac{1}{2}\pi d^2$

2. Governing council of a local public development authority of Dehradun decided to build an adventurous playground on the top of a hill, which will have adequate space for parking.

   After survey, it was decided to build rectangular playground, with a semi-circular area allotted for parking at one end of the playground as shown in Fig. 2.1. The length and breadth of the rectangular playground are 14 units and 7 units, respectively. There are two quadrants

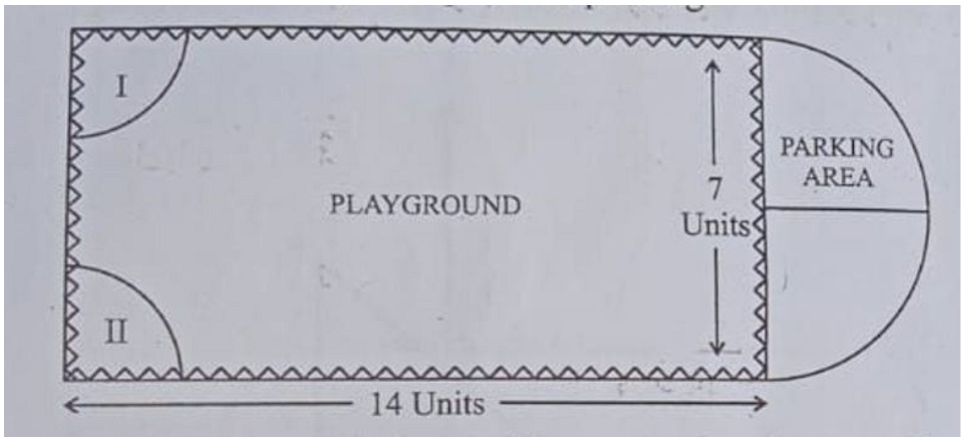

Figure 2.1:

of radius 2 units on one side for special seats.

Based on the above information, answer the following questions:

(a) What is the total perimeter of the parking area?

(b) What is the total area of parking and the two quadrants?

(c) What is the ratio of area of playground to the area of parking area?

(d) Find the cost of fencing the playground and parking area at the rate of ₹ 2 per unit.

Chapter 10

Differentiation

10.1. 2023

10.1.1. 12

1. If $\tan\left(\frac{x+y}{x-y}\right) = k$,then $\dfrac{dy}{dx}$ is equal to

   (a) $\frac{-y}{x}$

   (b) $\frac{y}{x}$

   (c) $\sec^2\left(\frac{y}{x}\right)$

   (d) $-\sec^2\left(\frac{y}{x}\right)$

2. **Assertion(A) :**Maximum value of $\left(\cos^{-1}\right)^2$ is π^2.

   Reason(R): Range of the principle value branch of $\cos^{-1} x$ is $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$.

3. If $y = \sqrt{ax+b}$, prove that $y\left(\dfrac{d^2y}{dx^2}\right) + \left(\dfrac{dy}{dx}\right)^2 = 0$

4. If the circumference of circle is increasing at the constant rate, prove that rate of change of area of circle is directly proportional to its radius.

5. Engine displacement is the measure of the cylinder volume swept by all the pistons of a piston engine.The piston moves inside the cylinder bore

Figure 5.1:

The cylinder bore in the form of circular cylinder open at the top is to be made from a metal sheet of area $75\pi\ cm^2$.

Based on the above information , answer the following questions:

(i) If the radius of cylinder is r cm and height is h cm, then write the volume V of cylinder in terms of radius r.

(ii) Find $\frac{dv}{dr}$

(iii) (a) Find the radius of cylinder when its volume is maximum.

(b) For maximum volume, $h > r$.State true or false and justify.

6. The use of electric vehicles will curb air pollution in the long run.

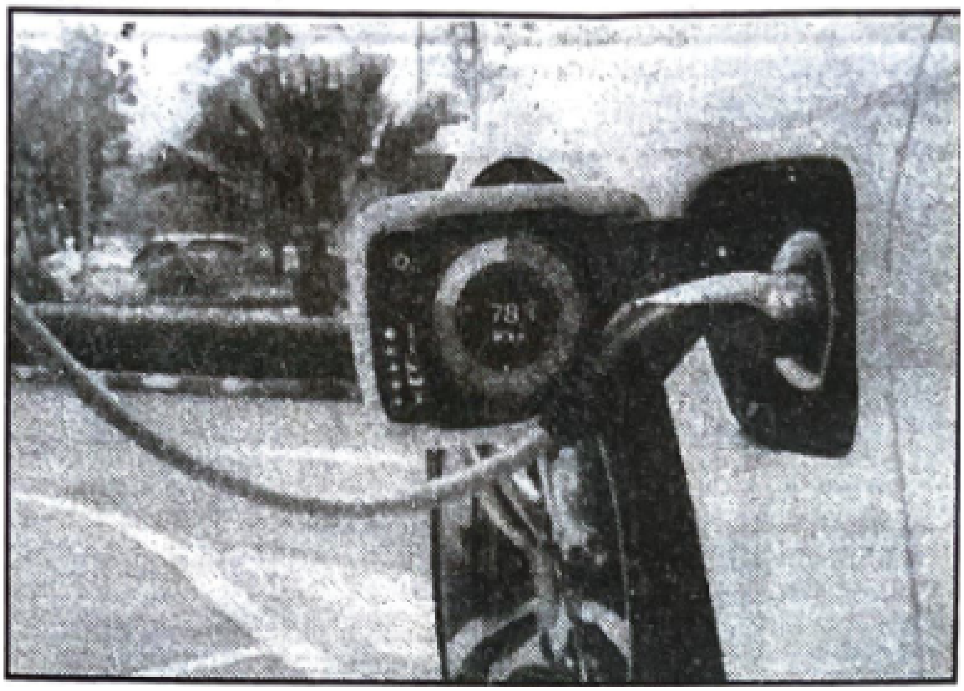

Figure 6.1:

The use of electric vehicles is increasing every year and estimated electric vehicles in use at any time t is given by the function V :

$$V(t) = \frac{1}{5}t^3 - \frac{5}{2}t^2 + 25t - 2 \tag{6.1}$$

Where t represents the time and t=1,2,3... corresponds to year 2001,2002,2003... respectively. Based on the above information, answer the following questions :

(i) Can the above function be used to estimate number of vehicles in the year 2000 ? Justify.

(ii) Prove that the function $V(t)$ is an increasing function.

Chapter 11

Functions

11.1. 2023

11.1.1. 12

1. The function $f(x) = x\,|x|$ is

   (a) continuous and differentiable at $x = 0$.

   (b) continuous but not differentiable at $x = 0$.

   (c) differentiable but not continuous at $x = 0$.

   (d) neither differentiable nor continuous at $x = 0$.

2. If

$$f(x) = \begin{cases} ax + b & 0 < x \le 1 \\ 2x^2 - x & 1 < x < 2 \end{cases} \tag{2.1}$$

   is a differentiable function is (0,2), then find the values of a and b.

3. A function $f : [-4, 4] \to [0, 4]$ is given by $f(x) = \sqrt{16 - x^2}$. Show that f is an onto function but not a one-one function.Further,find all possible values of 'a' for which $f(a) = \sqrt{7}$.

Chapter 12

Integrations

12.1. 2023

12.1.1. 12

1. If

$$\frac{d}{dx}\left(f(x)\right) = 2x + \frac{3}{x} \tag{1.1}$$

   and $f(1) = 1$,then $f(x)$is

   (a) $x^2 + 3\log|x| + 1$

   (b) $x^2 + 3\log|x|$

   (c) $2 - \frac{3}{x^2}$

   (d) $x^2 + 3\log|x| - 4$

2. The integral factor of the differential equation

$$\left(1 - y^2\right)\frac{dx}{dy} + yx = ay, (-1 < y < 1) \tag{2.1}$$

is

(a) $\frac{1}{y^2-1}$

(b) $\frac{1}{\sqrt{y^2-1}}$

(c) $\frac{1}{1-y^2}$

(d) $\frac{1}{\sqrt{1-y^2}}$

3. Anti derivative of $\frac{\tan(x)-1}{\tan(x)+1}$ with respect to x is:

   (a) $\sec^2(\frac{\pi}{4}-x)+c$

   (b) $-\sec^2(\frac{\pi}{4}-x)+c$

   (c) $\log\mid\sec(\frac{\pi}{4}-x)\mid+c$

   (d) $-\log\mid\sec(\frac{\pi}{4}-x)\mid+c$

4. Evaluate $\int_{\log\sqrt{2}}^{\log\sqrt{3}}\left(\frac{1}{(e^x+e^{-x})(e^x-e^{-x})}\right)dx$

5. (a) Find the general solution of the differential equation:

$$\left(xy-x^2\right)dy=y^2dx \tag{5.1}$$

   (b) Find the general solution of the differential equation:

$$\left(x^2+1\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4} \tag{5.2}$$

6. (a) Evaluate $\int_{-1}^{1}\left|x^4-x\right|dx$

   (b) Find $\int e^x\left(\frac{\sin^{-1}x}{(1-x^2)^{\frac{3}{2}}}\right)dx$

7. Find $\int e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$