

LEAN Project Circle

1 Content in file CCPosition.lean

In this file, we define the relative positions between circles. Definitions:

(defn) $\text{separated}_{of_circle_circle} : Prop \rightarrow Prop \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{return a proposition that } r_1 + r_2 \leq |O_1 O_2| < r_1 + r_2$. (defn) $\text{intersected}_{of_circle_circle} : Prop \rightarrow Prop \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{return a proposition that the distance between two circle centers is less than the sum of two radii } r_1 + r_2 < |O_1 O_2| < r_1 + r_2$. (defn) $\text{circumscribed}_{of_circle_circle} : Prop \rightarrow Prop \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{return a proposition that the distance between two circle centers is equal to the sum of two radii } r_1 + r_2 = |O_1 O_2| = r_1 + r_2$. (defn) $\text{included}_{of_circle_circle} : Prop \rightarrow Prop \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{return a proposition that } r_1 - r_2 \leq |O_1 O_2| < r_1 + r_2$. (defn) $\text{inscribed}_{of_circle_circle} : Prop \rightarrow Prop \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{return a proposition that } r_1 - r_2 = |O_1 O_2| < r_1 + r_2$.

Theorems : $\text{separated}_{circles_zero_intersection} \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{if they are separated, then they have no intersection}$. $\text{intersected}_{circles_two_intersection} \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{if they intersect each other, then they have exactly two common points}$. $\text{circumscribed}_{circles_one_intersection} \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{if they are circumscribed, then they have exactly one common point}$. $\text{included}_{circles_no_intersection} \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{if } \omega_2 \text{ is included in } \omega_1, \text{ then they have no intersection}$. $\text{inscribed}_{circles_one_intersection} \rightarrow \text{Giventwo circles } (\omega_1 \omega_2 : CircleP), \text{if } \omega_1 \text{ is inscribed with } \omega_2, \text{ then they have exactly one common point}$.

2 Content in file LCPosition.lean

In this file, we define the relative position between lines and circles and rays and circles. What's more, when discuss position between rays and circles, we want give a criterion whether the underlying lines have common points with the circles and give constructions to the TWO intersections (may be the same point).

2.1 position of lines and circles

Definitions : (defn) $\text{line_tangent_to_circle} : Prop \rightarrow Prop \rightarrow \text{Given a line and a circle, return a proposition that the line is tangent to the circle}$. (defn) $\text{line_secant_to_circle} : Prop \rightarrow Prop \rightarrow \text{Given a line and a circle, return a proposition that they have exactly two common points}$. (defn) $\text{line_disjoint_from_circle} : Prop \rightarrow Prop \rightarrow \text{Given a line and a circle, return a proposition that they have no common points}$. (defn) $\text{line_inside_circle} : Prop \rightarrow Prop \rightarrow \text{Given a line and a circle, return a proposition that the line is inside the circle}$.

Prop—Given a line and a circle, return a proposition that the line has at least a common point with the circle if and only if one of the following conditions holds:—Given a line and a circle, the line is tangent to the circle if and only if the distance from the center of the circle to the line is equal to the radius.
 —Given a line and a circle, the line is secant to the circle if and only if the distance from the center of the circle to the line is less than the radius.
 —Given a line and a circle, the line is disjoint from the circle if and only if the distance from the center of the circle to the line is greater than the radius.