


```
In [10]: import numpy as np

def gradient_descent(f, initial_point, learning_rate, iterations):
    point = initial_point.copy()
    history = []

    for _ in range(iterations):
        # Compute gradient of the function at the current point
        gradient = np.array([2 * point[0], 2 * point[1]])

        # Update point using gradient descent update rule
        point -= learning_rate * gradient

        # Calculate the value of the function at the updated point
        value = f(point[0], point[1])

        # Save the history of points and function values
        history.append((point.copy(), value))

    print("Values of points and objective function value are: {} and {}".format(
        history, value))

    return point, history # Make sure this line is inside the function

def objective_function(x, y):
    return x**2 + y**2 + 4

# Define initial point, Learning rate, and number of iterations
initial_point = np.array([1.0, 1.0])
learning_rate = 0.1
iterations = 100

# Run gradient descent
minimum_point, history = gradient_descent(objective_function, initial_point, learning_rate, iterations)

print("Minimum point (x, y):", minimum_point)
print("Minimum value of the function:", objective_function(*minimum_point))
```

Values of points and objective function value are: [0.8 0.8] and 5.28
Values of points and objective function value are: [0.64 0.64] and 4.8192
Values of points and objective function value are: [0.512 0.512] and 4.524288
Values of points and objective function value are: [0.4096 0.4096] and 4.33554432
Values of points and objective function value are: [0.32768 0.32768] and 4.2147483648
Values of points and objective function value are: [0.262144 0.262144] and 4.137438953472
Values of points and objective function value are: [0.2097152 0.2097152] and 4.08796093022208
Values of points and objective function value are: [0.16777216 0.16777216] and 4.056294995342131
Values of points and objective function value are: [0.13421773 0.13421773] and 4.036028797018964
Values of points and objective function value are: [0.10737418 0.10737418] and 4.023058430092137
Values of points and objective function value are: [0.08589935 0.08589935] and 4.014757395258967
Values of points and objective function value are: [0.06871948 0.06871948] and 4.009444732965739
Values of points and objective function value are: [0.05497558 0.05497558] and 4.006044629098073
Values of points and objective function value are: [0.04398047 0.04398047] and 4.003868562622767
Values of points and objective function value are: [0.03518437 0.03518437] and 4.0024758800785705
Values of points and objective function value are: [0.0281475 0.0281475] and 4.001584563250285
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Values of points and objective function value are: [0.01441152 0.01441152] and 4.000415383748683
Values of points and objective function value are: [0.01152922 0.01152922] and 4.000265845599157
Values of points and objective function value are: [0.00922337 0.00922337] and 4.00017014118346
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Values of points and objective function value are: [0.00193428 0.00193428] and 4.000007482888383
Values of points and objective function value are: [0.00154743 0.00154743] and 4.000004789048565
Values of points and objective function value are: [0.00123794 0.00123794] and 4.000003064991081

Values of points and objective function value are: [0.00099035 0.00099035] and 4.000001961594292

Values of points and objective function value are: [0.00079228 0.00079228] and 4.000001255420347

Values of points and objective function value are: [0.00063383 0.00063383] and 4.000000803469022

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Values of points and objective function value are: [0.00025961 0.00025961] and 4.000000134799733

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Values of points and objective function value are: [6.80564734e-05 6.80564734e-05] and 4.000000009263367

Values of points and objective function value are: [5.44451787e-05 5.44451787e-05] and 4.0000000059285545

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Values of points and objective function value are: [4.97323236e-10 4.97323236
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e-10] and 4.0
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e-10] and 4.0
Values of points and objective function value are: [2.54629497e-10 2.54629497
e-10] and 4.0
Values of points and objective function value are: [2.03703598e-10 2.03703598
e-10] and 4.0
Minimum point (x, y): [2.03703598e-10 2.03703598e-10]
Minimum value of the function: 4.0
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In []: