UNIVERSITY OF AMSTERDAM

VU VRIJE UNIVERSITEIT AMSTERDAM

Faculty of Science

Exam

Master Computational Science
Introduction to Computational Science

Final Exam

Date: 28 October 2021
Time: 09:00 to 12:00
Course examiner: Michael Lees

Number of pages: 3
Number of questions: 3
Maximum number of points to earn: 33
At each question is indicated how many points it is worth.

BEFORE YOU START

- Please wait until you are instructed to open the booklet.
- Check if your version of the exam is complete.
- Write down your name, student ID number, and if applicable the version number of the exam on each sheet that you hand in. Also number the pages.
- Your mobile phone has to be switched off and in the coat or bag. Your coat and bag must be under your table.
- Tools allowed: pen, pencil, ruler, calculator.

PRACTICAL MATTERS

- The first 30 minutes and the last 15 minutes you are not allowed to leave the room. not even to visit the toilet.
- You are obliged to identify yourself at the request of the examiner (or their representative) with a proof of your enrollment or a valid ID.
- During the examination it is not permitted to visit the toilet. unless the invigilator gives permission to do so.
- 15 minutes before the end, you will be warned that the time to hand in is approaching.
- If applicable, please fill out the evaluation form at the end of the exam.
- This exam paper may be taken with you afterwards.

Good luck!

(1) (a) Define the terms *verification* and *validation* in the context of modelling and simulation.
[2 marks]

(b) What is a discrete event model and how is it executed?
[2 marks]

(c) Descibe two ways to study natural systems and explain in which circumstances you would choose one or the other.
[2 marks]

(d) What is an endemic equilibrium?
[1 marks]

(e) Given the 2D non-linear system below.

$$\frac{dx}{dt} = 7x - 2x^2 - xy \tag{1}$$

$$\frac{dy}{dt} = 8y - 2y^2 - 2xy \tag{2}$$

(i) Find all the fixed points.
(ii) Determine what type of fixed point exists when $x = 0$ and $y \neq 0$
[4 marks]

(2) We will consider an SIR type model to model disease dynamics with quarantine behaviour. The model has four compartments: X is the succeptible population. Y_{nq} is the non-quarantined infected population, Y_q is the quarantined infected population and Z is the recovered population. X, Y, Z refer to absolute population numbers and not fractions. We assume a fixed population size of N and hence frequency dependent transmission.

A succeptible individual becomes infected and then moves to the non-quarantined infected group. After this time the individuals may recover. or may move to the quarantined group before recovering.

(a) Let us assume that both quarantined and non-quarantined individuals can infect others (i.e., non-perfect quarantine), with parameter β_q and β_{nq} respectively.

(i) Define the force of infection λ (moving from X to Y_{nq}) in terms of β_q. β_{nq}.Y_q and Y_{nq}
[1 marks]

(ii) In general we can write β as $\kappa \ln(1-c)$ where κ is the average contacts per unit time and c is the probability of successful disease transmission upon contact. With reference to the above, explain (with reasoning) if you would expect $\beta_q = \beta_{nq}$ or not.
[2 marks]

(b) Write down the set of ODEs that describe this SIR type model. Assume that people are moved from non-quarantined to quarantined with rate ρ and the recovery rates of quarantined and non-quarantined infected individuals are γ_q and γ_{nq} respectively.
[2 marks]

(c) Find the fixed points of this system (no need to classify). Is there an endemic state?
[2 marks]

(d) If your model does not have an endemic state then modify your model (equations) to add the endemic state. Explain why your original model or your modified model includes an endemic state.

[2 marks]

(e) Explain how you would add seasonal variation to your model, discuss if you would expect seasonal variation to impact quarantined and non-quarantined individuals differently.

[2 marks]

(3) (a) We can made model stochastics in epidemics by using *stochastic ODEs* or *discrete event* models. Give one advantage/benefit of each approach.

[1 marks]

(b) *Stochastic resonance* and *stochastic extinction* are two features of stochastic models of epidemics. Explain what both of these terms mean. and explain under what conditions they are (more) likely to occur.

[2 marks]

(c) Consider a Meta-population model with three subpopulations with a one-way coupling. meaning population 1 is coupled to population 2 and 3. but not vice-versa. Also assume that the populations are large enough to be described as fully deterministic. Formulate the SIR equations with demography for these coupled populations. You may assume equal birth/date rates and recovery rates in all populations.

[2 marks]

(d) Both Random (Erdős–Rényi) and Scale-Free (Barabási–Albert) networks can be used to model and understand disease spread. For an SIS model we can derive the epidemic threshold as:

$$\lambda_c^R = \frac{1}{\langle k \rangle + 1} \tag{3}$$

$$\lambda_c^{SF} = \frac{\langle k \rangle}{\langle k^2 \rangle} \tag{4}$$

Where λ_c^R is the epidemic threshold for random network and λ_c^{SF} is the epidemic threshold for scale-free networks. Explain what the epidemic threshold means for the SIS network model, and based on the above results describe an implication for disease spread on each type of network.

[2 marks]

(e) Consider vacination strategies in scale free networks and assume that a fraction g_c of the population is vaccinated at random.

(i) Explain in words how a successful or sufficient vaccination rate relates to the epidemic threshold.

[1 marks]

(ii) Derive an expression for the required random vaccination rate for scale free networks g_c in terms of λ_c^{SF}, assume the transmission term is $\beta\langle k \rangle$ and the recovery rate back to S is given by μ. You should show your reasoning and steps when deriving the expression.

[2 marks]

(iii) Based on the expression you derived for g_c. what can you say about random vaccination in scale free networks? Why?

[1 marks]