Example Preparation & Sample Exam Questions

This document is designed to help prepare for exam. First I highlight the topics that are covered in the exam (NOTE: Module 8 – ABM is not included). Below are indicative example questions. Note that you may find the questions to be "harder" or "easier" in the exam, but these give a good indication of expectation.

Revision Guideline (Topic Overview)

Module One : Introduction to Modelling & Simulation

- M&S cycle (with examples)
- Definitions: Model, System, Experiment, Simulation
- Ways to study systems
- Dangers of Simulation
- Why do we Model (other reasons)
- Why Simulate vs. Analytics
- Mathematical Models: Continuous, Discrete, Discrete Event

Module Two : Introduction to Infectious Disease

- Definitions: Disease, Infectious Disease, Pathogen, communicable, infection, vector, endemic, epidemic, pandemic
- Classifications of Infectious Diseases
- Modes of Transmission
- Primary/Secondary Infection
- Reproductive rate
- Spreading Phenomena
- R_0

Module Three : Modelling Infectious Diseases

- Why we model infectious diseases
- Definitions: Acute vs Chronic
- SIR Framework (and variants)
- I→R transition
- Naïve SIR and Solution
- System Dynamics/Stock Flow
- Phase plots
- S→I transition (mass vs density)
- Freq dependent transmission derivation
- SIR equations (without demography)
- Epidemic Threshold & Burnout
- SIR with demography
- Fixed Points
- Stability Analysis
- Oscillatory Dynamics
- Numerical Solvers

- SIR Variants (basics) & Residence Times

Module Four : Temporal Forcing

- Oscillations – what are they and where do they come from
- Seasonal effects & Soper's Argument
- Variable B(t)
- Harmonic & Sub-harmonic Resonance.
- Bifurcation Diagrams
- Vaccination (how to add to SIR)

Module Five : Stochastic Models

- Modeling Considerations (revisit)
- What are stochastic models (vs deterministic)
- Why do we need stochastic models
- Ways to add stochastics
- 5 key features in stochastic models
- Types of Noise
- How to add process Noise to SIR
- Scaled Noise vs constant noise (why)
- Relationship to population size
- Discrete event approach (what is it)
- Poisson processes
- Gillespie's Algorithms
- Stochastic Extinction
- Branching Processes
- Critical Community Size
- Persistence

Module Six : Meta Population Models

- Ways to introduce spatial models (Riley – Science)
- Important concepts
- Meta Population Models (force of infection)
- Deterministic and Stochastic subpops combined – consequences.
- Commuters
- Coarse Graining (overview)
- Gleam example

Module Seven : Networks & Epidemics

- Networks & Complex Systems
- Basic Measures: Degree distribution, Clustering Coefficient, Path length
- Centrality (Degree, betweenness, closeness)
- Random Networks – definition and degree distribution
- Scale Free Networks
- Barabasi Albert Model
- Network Robustness (BA vs ER)

- Spreading on Networks
- SIS and SI Model
- Degree Block Approximation
- Early time behaviour of SI
- Vanishing Epidemic
- Epidemic Threshold on Scale Free
- Vaccination on Networks

Module Eight: Agent-based Models & Epidemics

- **Not in Exam**

Sample Exam

The format of the written exam will be the same – 3 questions all worth equal points.
The first question will be shorter more direct questions, whereas the second two questions will be longer questions related to a topic.

Question 1 [10 points]

1 What is a Discrete Event Model and how is it simulated?
2 What is a stable fixed point in non-linear dynamics?
3 What is epidemic burnout in SIR without demography?
4 Describe and explain the modelling and simulation cycle with the aid of an example.

Question 2 [10 points]

Consider the SIR model with demography with birth - and death rate μ. The SIR equations, where all symbols have their standard meaning, are

$$\frac{dS}{dt} = \mu - \beta SI - \mu S;$$
$$\frac{dI}{dt} = +\beta SI - \gamma I - \mu I;$$
$$\frac{dR}{dt} = +\gamma I - \mu R.$$

Consider now the case of paediatric vaccination, where a fraction p of newborns are vaccinated and therefore protected from infection.

1 Adopt the standard SIR model from above to include this paediatric vaccination. (1 point)

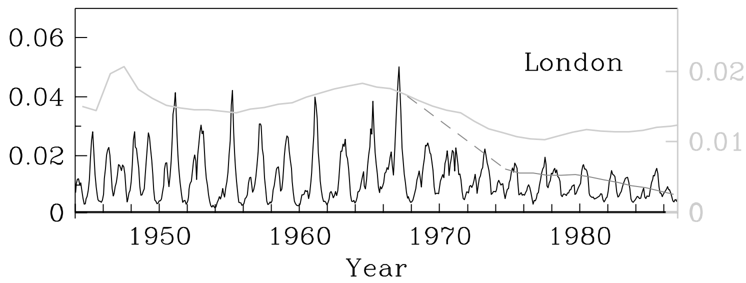
By a change of variable, $S = S'(1-p)$, $I = I'(1-p)$, $R = R'(1-p) - p$, and writing your model from (2) in terms of S', I', R', you find that it has exactly the same form as the standard SIR equations, but with one important modification.

2 Proof that with this change of variables you can indeed bring the model with paediatric vaccination in exactly the same form as the standard SIR equations (2 points).
3 What is the major difference between the model from (2) and the standard SIR model? And what does this mean for the dynamics of the model? (2 points)
4 The basic reproductive ratio for measles is 17. What fraction of the infant population should be vaccinated to prevent measles from spreading in the population. Explain why? (2 points)

Childhood diseases, for which paediatric vaccination is applied, are off course characterised by a strong seasonal forcing effect.

5 Mention at least three possible features of the SIR dynamics in the presence of seasonal forcing? (1point)

Finally, consider dynamic variability in childhood disease incidence in real data. The graph below is based on Figure 5.16 from Keeling and Rohani, Case reports for measles in London 1944 to 1988. The black line demonstrates weekly reported cases, with the gray line depicting the per capita birth rate. The dashed grey line demonstrates effective birth rate, correcting for vaccination, that started in 1968.



6 Describe the types of dynamics that you observe, and relate this back to what you know about SIR models with seasonal forcing, and what you discussed w.r.t. vaccination. (2 points)

Question 3 [10 points]

1 What is stochastic extinction, and when is this most likely to happen? (2 points)

In an invading scenario, where a single infected individual enters a fully susceptible population, stochastic extinction can also occur.

2 What is meant with stochastic extinction in this scenario, and what is the probability of this actually happening (no need to derive of proof this)? (1 points)

Next assume additional periodic forcing, which will lead to periodic outbreaks in large enough populations.

3 What happens with the observed dynamics in smaller and smaller populations. You may assume a scenario including immigrants. Take into account the concept of Critical Community Size. (2 points)

4 What is a metapopulation model in infectious disease modelling? (1 point)

Consider two subpopulations with a one-way coupling, meaning population 1 is coupled to population, but not vice-versa. Also assume that population 1 is large enough the be described as fully deterministic.

5 Assuming that population 2 is also deterministic, formulate the SIR equations with demography for these coupled populations. You may assume equal birth/date rates and recovery rates in both populations (1 point)

For these two coupled deterministic models, now assume that an infection is introduced in population 1, and that population 2 is fully susceptible. Also assume that this infection has a basic reproductive ratio that allows it to spread and cause an epidemic

6 Explain in words the dynamics in this coupled model. (1 point)

Finally assume that population 2 is too small to be considered fully deterministic. Replace the model population 2 now with a stochastic SIR model.

7 What do you observe now? Take the strength of coupling between population 1 and 2 into account in your discussion (2 points)