Route Planning

Student

Francesco Lorenzo Carone

LOGISTICS PROJECT

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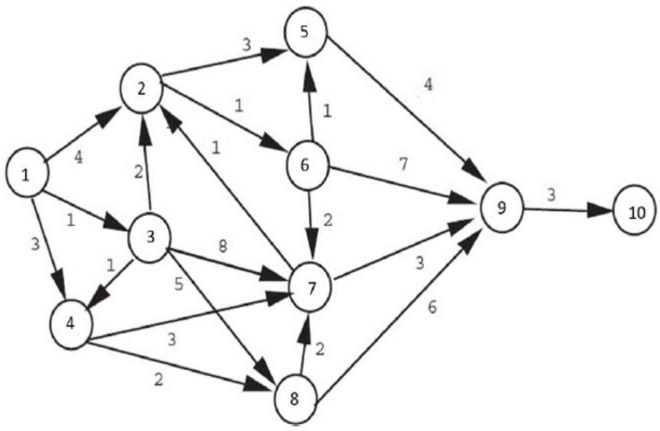
Capitolo 1

Route Planning

1.1 The Problem

Consider the graph in the figure, representing a street network, where a traveling cost is associated with each arc. The RoutePlan Agency wants to determine a directed path in this graph, from node 1 to node 10, having minimum traveling cost. Due to traffic congestion motivations, however, the determined path must include the arc (7,2) but not the arc (7,9).



If the determined path includes the arc (5,9), then a discount equal to the 15 % of the total path cost can be obtained

1.2 Model Classification

This problem is a transportation problem, in particular this is the shortest path problem with a Starting node and an End node, with that we have some other constraints like the arc that must or must not be included in the path, or in the second part of the problem the discount of the total cost.

To model this problem we need :

$$x_{i,j} \forall_{i,j} \in A \tag{1.1}$$

a decision variable that is equal to 1 if the i,j arc is in the path and 0 otherwise.

$$c_{i,j} \forall_{i,j} \in A \tag{1.2}$$

cost along (i,j)

Flow conservation constraints

The constraints are based on this assumption about the balance of the nodes:

$$b_i = \begin{cases} -1, & \text{if i is the Source} \\ 1, & \text{if i is the Destination} \\ 0, & \text{otherwise} \end{cases} \tag{1.3}$$

also we refer to the Backward star of i with positive and Forward Star with negative variables

Observations

With the minimization of the total cost we have also to include in the first part the arc (7,2) in the path and must not include the arc (7,9) to do this we are going to add 2 constraints, and also for the second part we have to modify the objective function to include the potential discount

1.3 A Mathematical Model Solution

1.3.1 Question 1:

Formulate the RoutePlan Agency problem in terms of ILP and implement the proposed ILP model by means of the modelling language AMPL, and solve it using the optimization solver CPLEX.

ILP Model

Decision variable:

$$\mathbf{x_{i,j}} = \begin{cases} 1, & \text{if (i,j) is in the path} \\ 0, & \text{otherwise} \end{cases}$$

Objective Function:

$$\min \sum_{(i,j)\in \mathrm{A}} \mathrm{C}[i,j] \cdot X[i,j] :$$

$$\begin{aligned}\min\,[4\cdot x_{1,2}+1\cdot x_{1,3}+3\cdot x_{1,4}+3\cdot x_{2,5}+1\cdot x_{2,6}\\ +2\cdot x_{3,2}+1\cdot x_{3,4}+8\cdot x_{3,7}+5\cdot x_{3,8}+3\cdot x_{4,7}\\ +2\cdot x_{4,8}+4\cdot x_{5,9}+1\cdot x_{6,5}+2\cdot x_{6,7}+7\cdot x_{6,9}\\ +1\cdot x_{7,2}+3\cdot x_{7,9}+2\cdot x_{8,7}+6\cdot x_{8,9}+3\cdot x_{9,10}]\end{aligned} \tag{1.4}$$

Flow Conservation Constraints:

$$\begin{aligned}-x_{1,2}-x_{1,3}-x_{1,4}&=-1\\ x_{1,2}+x_{3,2}+x_{7,2}-x_{2_5}-x_{2,6}&=0\\ x_{1,3}-x_{3,2}-x_{3,4}-x_{3,7}-x_{3,8}&=0\\ x_{1,4}+x_{3,4}-x_{4,7}-x_{4,8}&=0\\ x_{2,5}+x_{6,5}-x_{5,9}&=0\\ x_{2,6}-x_{6,5}-x_{6,7}-x_{6,9}&=0\\ x_{3,7}+x_{4,7}+x_{6,7}+x_{8,7}-x_{7,9}&=0\\ x_{3,8}+x_{4,8}-x_{8,7}-x_{8,9}&=0\\ x_{5,9}+x_{6,9}+x_{7,9}+x_{8,9}-x_{9,10}&=0\\ x_{9,10}&=1\end{aligned}$$

Additional Constraints:

$$\begin{aligned}x_{7,2}&=1\\ x_{7,9}&=0\end{aligned}$$

Optimal Solution
After implementing the model in AMPL the optimal solution is :

$$\begin{array}{lllll} x_{1,2}=0, & x_{1,3}=1, & x_{1,4}=0, & x_{2,5}=0, & x_{2,6}=1,\\ x_{3,2}=0, & x_{3,4}=1, & x_{3,7}=0, & x_{3,8}=0, & x_{4,7}=1,\\ x_{4,8}=0, & x_{5,9}=1, & x_{6,5}=1, & x_{6,7}=0, & x_{6,9}=0,\\ x_{7,2}=1, & x_{7,9}=0, & x_{8,7}=0, & x_{8,9}=0, & x_{9,10}=1\end{array}$$

with a total cost of 15.

1.3.2 Question 2:

if the determined path includes the arc (5,9), then a discount equal to the 15% of the total path cost can be obtained. Propose an ILP model for this variant of the problem, modify the file .mod accordingly, and solve it via CPLEX.

ILP Model

The model is the same as before but we have to change the objective function to include the discount

Objective Function:

$$\min \sum_{(i,j)\in \mathrm{A}} (1-0.15\cdot x_{5,9})\cdot \mathrm{C}[i,j]\cdot X[i,j]\,:$$

$$
\begin{aligned}
\min\,(1-0.15\cdot x_{5,9})\cdot[4\cdot x_{1,2}+1\cdot x_{1,3}+3\cdot x_{1,4}+3\cdot x_{2,5}+1\cdot x_{2,6}\\
+2\cdot x_{3,2}+1\cdot x_{3,4}+8\cdot x_{3,7}+5\cdot x_{3,8}+3\cdot x_{4,7}\\
+2\cdot x_{4,8}+4\cdot x_{5,9}+1\cdot x_{6,5}+2\cdot x_{6,7}+7\cdot x_{6,9}\\
+1\cdot x_{7,2}+3\cdot x_{7,9}+2\cdot x_{8,7}+6\cdot x_{8,9}+3\cdot x_{9,10}]
\end{aligned}
\tag{1.5}
$$

with this function, if $x_{5,9}$ is equal to 1 we have the discount, if not we just use the same function of the first question.

Optimal Solution

After implementing the model in AMPL the optimal solution is :

$$
\begin{array}{lllll}
x_{1,2}=0, & x_{1,3}=1, & x_{1,4}=0, & x_{2,5}=0, & x_{2,6}=1,\\
x_{3,2}=0, & x_{3,4}=1, & x_{3,7}=0, & x_{3,8}=0, & x_{4,7}=1,\\
x_{4,8}=0, & x_{5,9}=1, & x_{6,5}=1, & x_{6,7}=0, & x_{6,9}=0,\\
x_{7,2}=1, & x_{7,9}=0, & x_{8,7}=0, & x_{8,9}=0, & x_{9,10}=1
\end{array}
$$

so the same as before with a total cost of 12.75. so the only difference is in the total cost of the path