Robot Kinematics and Dynamics

2.1 INTRODUCTION

Robot arm kinematics deals with the analytic study of the motion of a robot arm with respect to a fixed reference coordinate system as a function of time. The mechanical manipulator can be modelled as an open loop articulated chain with several rigid links connected in series by either 'revolute' or 'prismatic' joints driven by the actuators.

For a manipulator, if the position and orientation of the end-effector are derived from the given joint angles and link parameters, the scheme is called the *forward kinematics* problem. If, on the other hand, the joint angles and the different configuration of the manipulator are derived from the position and orientation of the end-effector, the scheme is called the *reverse kinematics* problem. Figure 2.1 illustrates the scheme of forward and reverse kinematics.

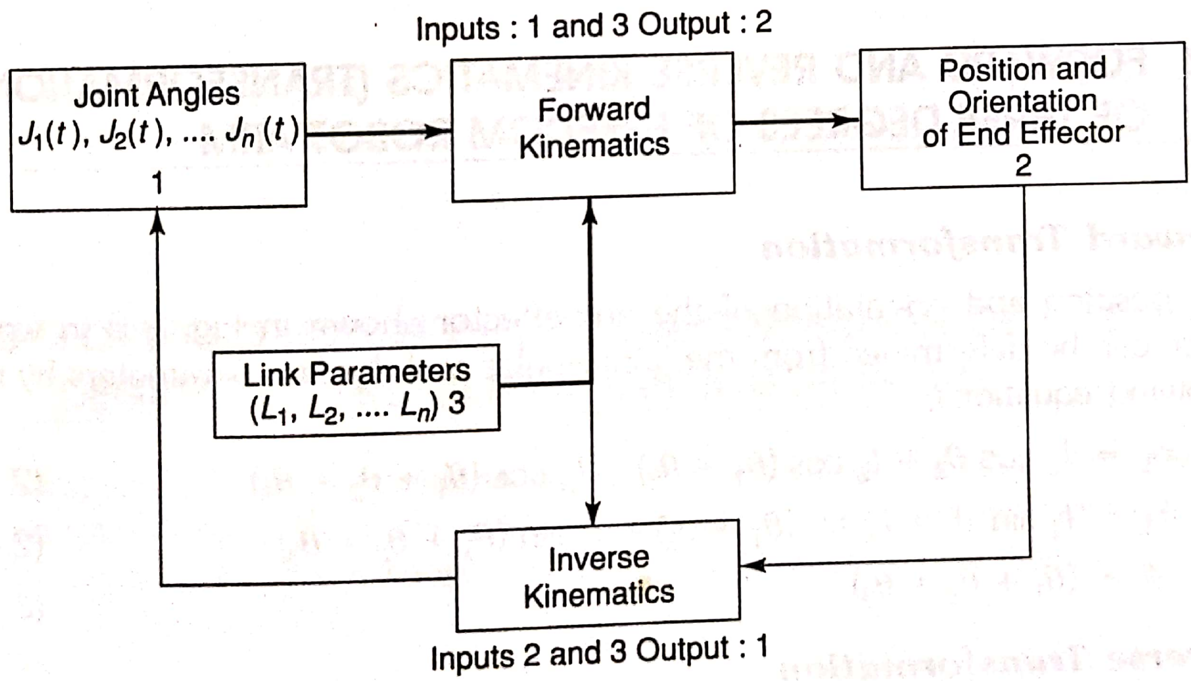

FIG. 2.1 *Forward and inverse kinematics scheme*

Representing the Position

Considering the revolute type of joint only, the position of the end-effector can be represented by the joint angles, θ_1, θ_2, ..., θ_n as,

$$P_{joint} = (\theta_1, \theta_2, \theta_3, \ldots \theta_n) \tag{2.1}$$

The position of the end-effector can also be defined in world space as,

$$P_{WORLD} = (x, y, z) \tag{2.2}$$

For a Revolute-Revolute (*R-R*) joint having 2 degrees of freedom, the schematic diagram of the links in 2-D, is shown in Fig. 2.2. l_1 and l_2 are the links. θ_1 and θ_2 are the angles of rotation.

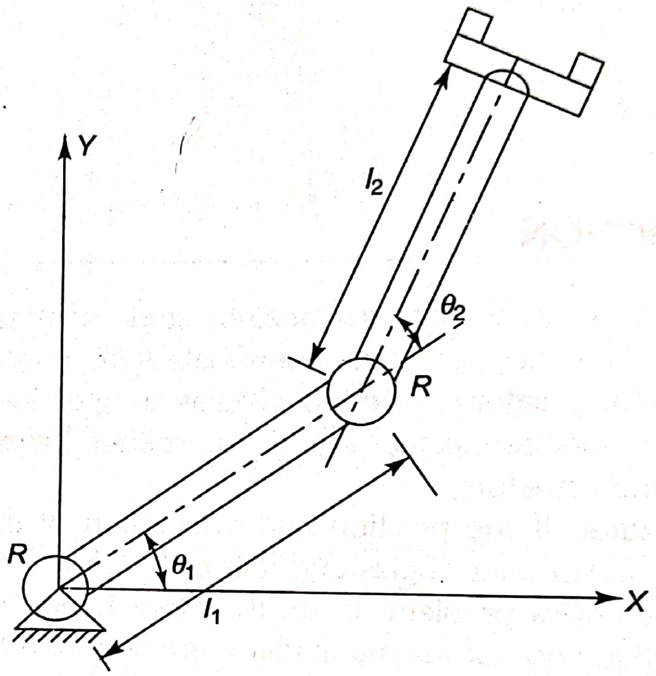

FIG. 2.2 *R-R type two DOF 2-D manipulator*

2.2 FORWARD AND REVERSE KINEMATICS (TRANSFORMATION) OF THREE DEGREES OF FREEDOM ROBOT ARM

Forward Transformation

The position and orientation of the end-effector shown in Fig. 2.3 in world space can be determined from the joint angles and the link parameters by the following equations,

$$x_3 = l_1 \cos \theta_1 + l_2 \cos (\theta_1 + \theta_2) + l_3 \cos (\theta_1 + \theta_2 + \theta_3) \tag{2.3}$$

$$y_3 = l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2) + l_3 \sin (\theta_1 + \theta_2 + \theta_3) \tag{2.4}$$

$$\phi = (\theta_1 + \theta_2 + \theta_3) \tag{2.5}$$

Reverse Transformation

The joint angles can also be determined from the end-effector position (x_3, y_3) and the orientation (ϕ), using reverse transformation in the following way

$$x_2 = x_3 - l_3 \cos \phi \tag{2.6}$$

$$y_2 = y_3 - l_3 \sin \phi \tag{2.7}$$

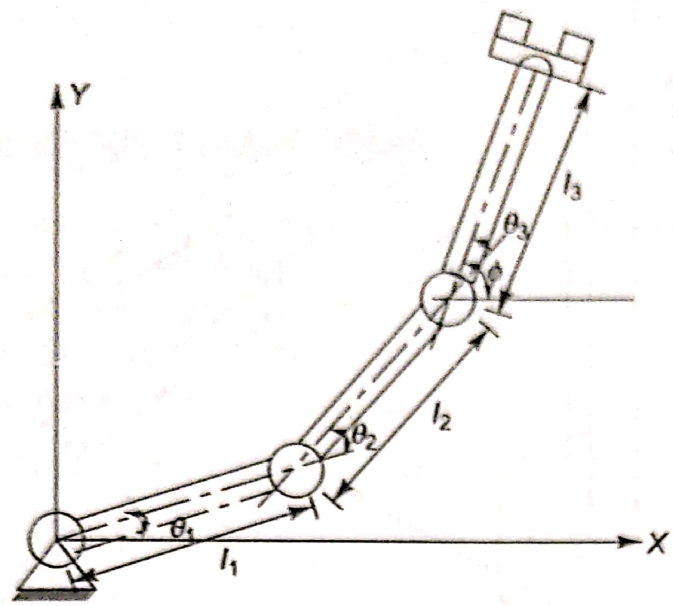

FIG. 2.3 *Three DOF 2-D manipulator*

From the given geometry,

$$x_2 = l_1 \cos \theta_1 + l_2 \cos \theta_1 \cos \theta_2 - l_2 \sin \theta_1 \sin \theta_2 \tag{2.8}$$

$$y_2 = l_1 \sin \theta_1 + l_2 \sin \theta_1 \cos \theta_2 + l_2 \cos \theta_1 \sin \theta_2 \tag{2.9}$$

Squaring and adding Eqs (2.8) and (2.9),

$$\cos \theta_2 = \frac{x_2^2 + y_2^2 - l_1^2 - l_2^2}{2l_1l_2} \tag{2.10}$$

Substituting the value of θ_2 in Eqs (2.8) and (2.9), we obtain the value of θ_1. Finally, the value of θ_3 can be obtained using the following relation:

$$\theta_3 = \phi - (\theta_i + \theta_2) \tag{2.11}$$

2.3 FORWARD AND REVERSE TRANSFORMATION OF A FOUR DEGREES OF FREEDOM MANIPULATOR IN 3-D

A 4-degrees of freedom manipulator in 3-D is illustrated in Fig. 2.4. Joint 1 allows rotation about the *z*-axis, joint 2 allows rotation about an axis perpendicular to the *z*-axis, joint 3 is a linear joint and joint 4 allows rotation about an axis parallel to the joint 2 axis.

Let

θ_1 = angle of rotation of joint 1 (base rotation)

θ_2 = angle of rotation of joint 2 (elevation angle)

l = length of the linear joint 3 (extension) (a combination of l_2 and l_3)

θ_4 = angle of rotation of joint 4 (pitch angle)

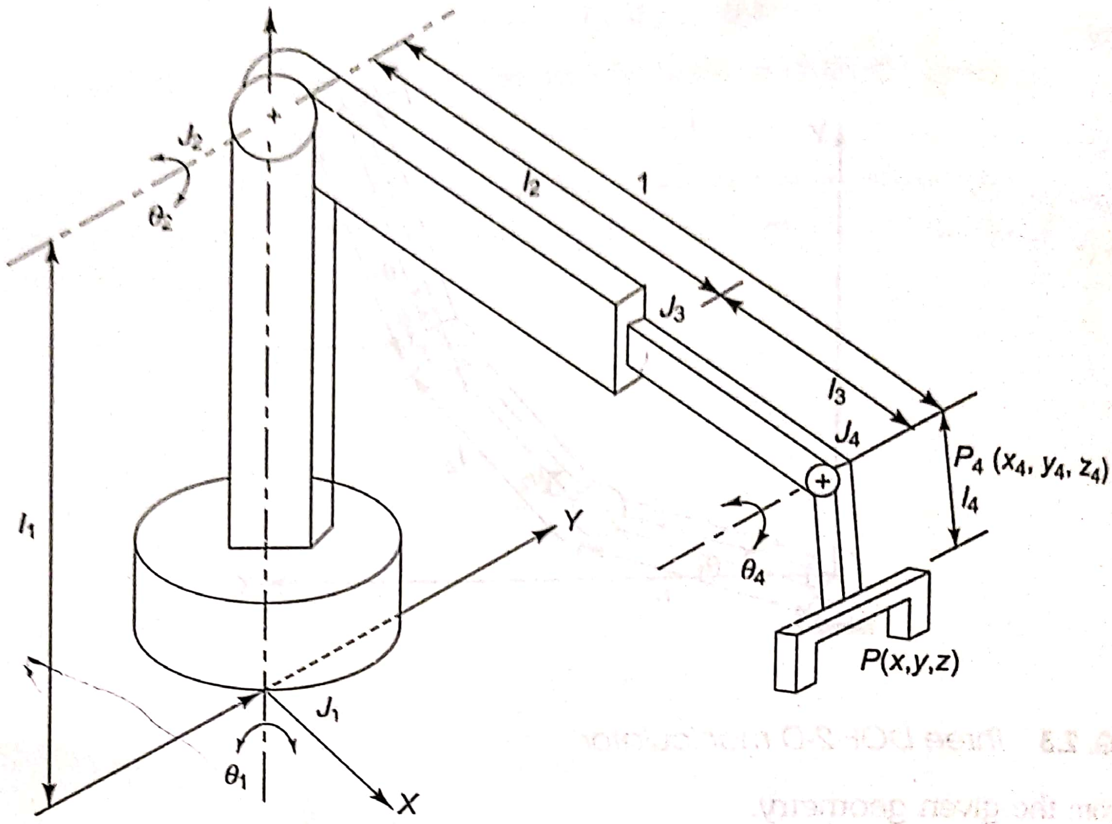

FIG. 2.4 *Four DOF 3-D manipulator*

Forward Transformation

The position of the end-effector P in world space is given by

$$x = (l \cos \theta_2 + l_4 \cos \theta_4) \times \cos \theta_1 \tag{2.12}$$

$$y = (l \cos \theta_2 + l_4 \cos \theta_4) \times \sin \theta_1 \tag{2.13}$$

$$z = l_1 + l \sin \theta_2 + l_4 \sin \theta_4 \tag{2.14}$$

Reverse Transformation

If the pitch angle (θ_4) and the world coordinates (x, y, z) of the point P are given, the joint positions can be determined in the following way:

Let the coordinate of the joint 4 be (x_4, y_4, z_4).

Then,

$$x_4 = x - \cos \theta_1 (l_4 \cos \theta_4) \tag{2.15}$$

$$y_4 = y - \sin \theta_1 (l_4 \cos \theta_4) \tag{2.16}$$

$$z_4 = z - l_4 \sin \theta_4$$

Now the values of θ_1, θ_2 and l can be found by

$$\cos \theta_1 = \frac{y_4}{l} \tag{2.17}$$

$$\sin \theta_2 = \frac{z_4 - l_1}{l} \tag{2.18}$$

$$l = \left[x_4^2 + y_4^2 + (z_4 - l_1)^2\right]^{1/2} \tag{2.19}$$

2.4 HOMOGENEOUS TRANSFORMATIONS

The use of homogeneous transformations is a general method for solving the kinematic equations of a robot manipulator with many joints. A generalized transformation is now described by a single matrix that combines the effects of translation and rotation.

Rotation matrices (4 × 4) are defined as

$$\text{Rot}(x,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.20}$$

$$\text{Rot}(y,\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.21}$$

$$\text{Rot}(z,\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.22}$$

Here, Rot (x, θ), Rot (y, θ) and Rot (z, θ) indicate a rotation of an angle θ around x, y, or z axis respectively and they can be multiplied with the position vector to find a new point that results from rotating a given point about its axis (x, y or z) through an angle θ.

The translation matrix (4 × 4) is also defined as:

$$\text{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.23}$$

The translations a, b, c are respectively along the x, y, and z-axis.

Transformation matrices are often used to describe the location of one coordinate system (coordinate frame) relative to another. The origin of the second frame may then be found as the transformation of the point.

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$