

MODULE *SASwap*

SASwap TLA+ specification (c) by Dmitry Petukhov (<https://github.com/dgpv>)
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EXTENDS *Naturals*, *Sequences*, *FiniteSets*, *TLC*

CONSTANT *PARTICIPANTS_IRRATIONAL* Can participants act irrational?

ASSUME *PARTICIPANTS_IRRATIONAL* ∈ BOOLEAN

CONSTANT *BLOCKS_PER_DAY*

More blocks per day means larger state space to check

ASSUME *BLOCKS_PER_DAY* ≥ 1

A transaction that has no deadline can be 'stalling',

i.e. not being sent while being enabled, for this number of days

CONSTANT *MAX_DAYS_STALLING*

More days allowed stalling means larger state space to check

ASSUME *MAX_DAYS_STALLING* ≥ 1

Is it possible for participants to send transactions

bypassing the *mempool* (give directly to the miner)

CONSTANT *STEALTHY_SEND_POSSIBLE*

When TRUE, the state space is increased dramatically.

ASSUME *STEALTHY_SEND_POSSIBLE* ∈ BOOLEAN

Operator to create transaction instances

$Tx(id, ss, by, to, via) \triangleq$

$[id \mapsto id, ss \mapsto ss, to \mapsto to, by \mapsto by, via \mapsto via]$

VARIABLE *blocks* $\langle \{Tx, \dots\}, \dots \rangle$

VARIABLE *next_block* $\{Tx, \dots\}$

VARIABLE *mempool* $\{Tx, \dots\}$

VARIABLE *shared_knowledge* $\{Tx, \dots\}$

VARIABLE *signers_map* $[participant \mapsto \{allowed_sig, \dots\}]$

VARIABLE *per_block_enabled* $\langle \{Tx, \dots\}, \dots \rangle$

$fullState \triangleq \langle blocks, next_block, signers_map, shared_knowledge, mempool, per_block_enabled \rangle$

$unchangedByMM \triangleq \langle blocks, signers_map, shared_knowledge, mempool \rangle$

A few generic operators

$$\begin{aligned} \text{Range}(f) &\triangleq \{f[x] : x \in \text{DOMAIN } f\} \\ \text{Min}(set) &\triangleq \text{CHOOSE } x \in set : \forall y \in set : x \leq y \\ \text{Max}(set) &\triangleq \text{CHOOSE } x \in set : \forall y \in set : x \geq y \end{aligned}$$

Various definitions that help to improve readability of the spec

$$\begin{aligned} \text{Alice} &\triangleq \text{"Alice"} \\ \text{Bob} &\triangleq \text{"Bob"} \\ \text{participants} &\triangleq \{\text{Alice}, \text{Bob}\} \\ \text{sigAlice} &\triangleq \text{"sigAlice"} \\ \text{sigBob} &\triangleq \text{"sigBob"} \\ \text{secretAlice} &\triangleq \text{"secretAlice"} \\ \text{secretBob} &\triangleq \text{"secretBob"} \\ \text{all_secrets} &\triangleq \{\text{secretAlice}, \text{secretBob}\} \\ \text{all_sigs} &\triangleq \{\text{sigAlice}, \text{sigBob}, \text{secretAlice}, \text{secretBob}\} \\ \text{tx_lock_A} &\triangleq \text{"tx_lock_A"} \\ \text{tx_lock_B} &\triangleq \text{"tx_lock_B"} \\ \text{tx_success} &\triangleq \text{"tx_success"} \\ \text{tx_refund_1} &\triangleq \text{"tx_refund_1"} \\ \text{tx_revoke} &\triangleq \text{"tx_revoke"} \\ \text{tx_refund_2} &\triangleq \text{"tx_refund_2"} \\ \text{tx_timeout} &\triangleq \text{"tx_timeout"} \\ \text{tx_spend_A} &\triangleq \text{"tx_spend_A"} \\ \text{tx_spend_B} &\triangleq \text{"tx_spend_B"} \\ \text{tx_spend_success} &\triangleq \text{"tx_spend_success"} \\ \text{tx_spend_refund_1_alice} &\triangleq \text{"tx_spend_refund_1_alice"} \\ \text{tx_spend_refund_1_bob} &\triangleq \text{"tx_spend_refund_1_bob"} \\ \text{tx_spend_revoke} &\triangleq \text{"tx_spend_revoke"} \\ \text{tx_spend_refund_2} &\triangleq \text{"tx_spend_refund_2"} \\ \text{tx_spend_timeout} &\triangleq \text{"tx_spend_timeout"} \\ \text{nLockTime} &\triangleq \text{"nLockTime"} \\ \text{nSequence} &\triangleq \text{"nSequence"} \\ \text{NoTimelock} &\triangleq [\text{days} \mapsto 0, \text{type} \mapsto \text{nLockTime}] \end{aligned}$$

If blocks per day are low, the absolute locks need to be shifted,
otherwise not all contract paths will be reachable

$$ABS_LK_OFFSET \triangleq \text{CASE } BLOCKS_PER_DAY = 1 \rightarrow 2 \\ \square \quad BLOCKS_PER_DAY = 2 \rightarrow 1 \\ \square \quad \text{OTHER} \rightarrow 0$$

The map of the transactions, their possible destinations and timelocks.
Adaptor signatures are modelled by an additional value in the required
signature set – *ss* . For modelling purposes, the secret acts as just another signature.

ds stands for “destinations”, and *lk* stands for “lock” (timelocks).

Only blockheight-based timelocks are modelled.

$$tx_map \triangleq [$$

‘Contract’ transactions – destinations are other transactions

$$\begin{aligned} tx_lock_A &\mapsto [ds \mapsto \{tx_success, tx_refund_1, tx_revoke, tx_spend_A\}, \\ &\quad ss \mapsto \{sigAlice\}], \\ tx_lock_B &\mapsto [ds \mapsto \{tx_spend_B\}, \\ &\quad ss \mapsto \{sigBob\}], \\ tx_success &\mapsto [ds \mapsto \{tx_spend_success\}, \\ &\quad ss \mapsto \{sigAlice, sigBob, secretBob\}], \\ tx_refund_1 &\mapsto [ds \mapsto \{tx_spend_refund_1_bob, tx_spend_refund_1_alice\}, \\ &\quad ss \mapsto \{sigAlice, sigBob, secretAlice\}, \\ &\quad lk \mapsto [days \mapsto ABS_LK_OFFSET + 1, type \mapsto nLockTime]], \\ tx_revoke &\mapsto [ds \mapsto \{tx_refund_2, tx_timeout, tx_spend_revoke\}, \\ &\quad ss \mapsto \{sigAlice, sigBob\}, \\ &\quad lk \mapsto [days \mapsto ABS_LK_OFFSET + 2, type \mapsto nLockTime]], \\ tx_refund_2 &\mapsto [ds \mapsto \{tx_spend_refund_2\}, \\ &\quad ss \mapsto \{sigAlice, sigBob, secretAlice\}, \\ &\quad lk \mapsto [days \mapsto 1, type \mapsto nSequence]], \\ tx_timeout &\mapsto [ds \mapsto \{tx_spend_timeout\}, \\ &\quad ss \mapsto \{sigAlice, sigBob\}, \\ &\quad lk \mapsto [days \mapsto 2, type \mapsto nSequence]], \end{aligned}$$

'Terminal' transactions – destinations are participants

$$\begin{aligned}
tx_spend_A & \mapsto [ds \mapsto \{Alice, Bob\}, \\
& \quad ss \mapsto \{sigAlice, sigBob\}], \\
tx_spend_B & \mapsto [ds \mapsto \{Alice, Bob\}, \\
& \quad ss \mapsto \{secretAlice, secretBob\}], \\
tx_spend_success & \mapsto [ds \mapsto \{Bob\}, \\
& \quad ss \mapsto \{sigBob\}], \\
tx_spend_refund_1_bob & \mapsto [ds \mapsto \{Bob\}, \\
& \quad ss \mapsto \{sigAlice, sigBob\}], \\
tx_spend_refund_1_alice & \mapsto [ds \mapsto \{Alice\}, \\
& \quad ss \mapsto \{sigAlice\}, \\
& \quad lk \mapsto [days \mapsto 1, type \mapsto nSequence]], \\
tx_spend_revoke & \mapsto [ds \mapsto \{Alice, Bob\}, \\
& \quad ss \mapsto \{sigAlice, sigBob\}], \\
tx_spend_refund_2 & \mapsto [ds \mapsto \{Alice\}, \\
& \quad ss \mapsto \{sigAlice\}], \\
tx_spend_timeout & \mapsto [ds \mapsto \{Bob\}, \\
& \quad ss \mapsto \{sigBob\}]
\end{aligned}$$

]

$$all_transactions \triangleq \text{DOMAIN } tx_map$$

first_transaction defined so that miner's actions do not need to refer to any contract-specific info, and can just refer to *first_transaction* instead.

$$first_transaction \triangleq tx_lock_A$$

$$ConfirmedTransactions \triangleq \{tx.id : tx \in \text{UNION } Range(blocks)\}$$

$$NextBlockTransactions \triangleq \{tx.id : tx \in next_block\}$$

$$\begin{aligned}
NextBlockConfirmedTransactions & \triangleq \\
& ConfirmedTransactions \cup NextBlockTransactions
\end{aligned}$$

$$MempoolTransactions \triangleq \{tx.id : tx \in mempool\}$$

$$SentTransactions \triangleq ConfirmedTransactions \cup MempoolTransactions$$

$EnabledTransactions \triangleq \{tx.id : tx \in \text{UNION } \text{Range}(per_block_enabled)\}$

$ContractTransactions \triangleq$
 $\{id \in all_transactions :$
 $\quad \forall d \in tx_map[id].ds : d \in all_transactions\}$

$TerminalTransactions \triangleq$
 $\{id \in all_transactions :$
 $\quad \forall d \in tx_map[id].ds : d \in participants\}$

ASSUME $\forall id \in all_transactions : \forall id \in TerminalTransactions$
 $\quad \forall id \in ContractTransactions$

In this contract each transaction has only one parent,
 so we can use simple mapping from dep_id to parent id

$dependency_map \triangleq$
 $[dep_id \in \text{UNION } \{tx_map[id].ds : id \in ContractTransactions\}$
 $\mapsto \text{CHOOSE } id \in ContractTransactions : dep_id \in tx_map[id].ds]$

Special destination for the case when funds will still be locked
 at the contract after the transaction is spent

$Contract \triangleq \text{"Contract"}$

$DstSet(id) \triangleq$
 IF $id \in ContractTransactions$ THEN $\{Contract\}$ ELSE $tx_map[id].ds$

The CASE statement has no 'OTHER' clause - only single dst is expected

$SingleDst(id) \triangleq \text{CASE } id \in ContractTransactions \rightarrow Contract$
 $\quad \square \quad Cardinality(tx_map[id].ds) = 1$
 $\quad \rightarrow \text{CHOOSE } d \in tx_map[id].ds : \text{TRUE}$

The set of transactions conflicting with the given transaction

$ConflictingSet(id) \triangleq$
 IF $id \in \text{DOMAIN } dependency_map$
 THEN $\{dep_id \in \text{DOMAIN } dependency_map :$
 $\quad dependency_map[dep_id] = dependency_map[id]\}$
 ELSE $\{id\}$

Transaction also conflicts with itself

ASSUME $\forall id \in all_transactions : id \in ConflictingSet(id)$

$ConfirmationHeight(id) \triangleq$

CHOOSE $bn \in \text{DOMAIN } blocks : \exists tx \in blocks[bn] : tx.id = id$

All the transactions the given transaction depends on.

Because each transaction can only have one dependency in our model,

all dependencies form a chain, not a tree.

RECURSIVE $DependencyChain(-)$

$DependencyChain(id) \triangleq$

IF $id \in \text{DOMAIN } dependency_map$

THEN $\{id\} \cup DependencyChain(dependency_map[id])$

ELSE $\{id\}$

All the transactions that depend on the given transaction.

Dependants form a tree, but the caller is interested in just a set.

RECURSIVE $AllDependants(-)$

$AllDependants(id) \triangleq$

LET $dependants \triangleq tx_map[id].ds \setminus participants$

IN IF $dependants = \{\}$

THEN $\{id\}$

ELSE $dependants \cup \text{UNION } \{AllDependants(d_id) : d_id \in dependants\}$

All transactions that cannot ever become valid because other, conflicting transactions were confirmed before them

$InvalidatedTransactions \triangleq$

UNION $\{\{c_id\} \cup AllDependants(c_id) : c_id \in$

UNION $\{ConflictingSet(id) \setminus \{id\} : id \in ConfirmedTransactions\}\}$

All transactions that is not yet sent/confirmed, and have a chance to be.

$RemainingTransactions \triangleq$

$((all_transactions \setminus ConfirmedTransactions) \setminus InvalidatedTransactions)$

$Timelock(id) \triangleq$ IF "lk" $\in \text{DOMAIN } tx_map[id]$ THEN $tx_map[id].lk$ ELSE $NoTimelock$

$UnreachableHeight \triangleq 2^{30} + (2^{30} - 1)$

Calculate the height at which the timelock for the given transaction expires, taking *BLOCKS_PER_DAY* and dependencies confirmation into account

```

TimelockExpirationHeight(id)  $\triangleq$ 
  LET lk  $\triangleq$  Timelock(id)
  IN CASE lk.type = nLockTime
    → lk.days * BLOCKS_PER_DAY
  □ lk.type = nSequence
    → IF dependency_map[id] ∈ ConfirmedTransactions
      THEN ConfirmationHeight(dependency_map[id])
        + lk.days * BLOCKS_PER_DAY
    ELSE UnreachableHeight

```

“Hard” deadline for transaction means that it is unsafe to publish the transaction after the deadline

```

Deadline(id)  $\triangleq$ 
  LET hs  $\triangleq$  { TimelockExpirationHeight(c_id) :
    c_id ∈ ConflictingSet(id) \ {id} }
    higher_hs  $\triangleq$  { h ∈ hs : h > TimelockExpirationHeight(id) }
  IN IF higher_hs = { }
    THEN UnreachableHeight
    ELSE Min(higher_hs)

```

“Soft” deadline for transaction means that after the deadline, mining the transaction will mean that it was ‘stalling’ for too long

```

SoftDeadline(id)  $\triangleq$ 
  LET dl  $\triangleq$  Deadline(id)
    h  $\triangleq$  TimelockExpirationHeight(id)
  IN IF dl = UnreachableHeight
    THEN IF id ∈ EnabledTransactions
      THEN (CHOOSE en ∈ DOMAIN per_block_enabled :
        ∃ tx ∈ per_block_enabled[en] : tx.id = id)
        + MAX_DAYS_STALLING * BLOCKS_PER_DAY
      ELSE IF h ≠ UnreachableHeight
        THEN h + MAX_DAYS_STALLING * BLOCKS_PER_DAY
        ELSE 0
    ELSE dl

```

$$\begin{aligned}
& \text{SigsAvailable}(id, sender, to) \triangleq \\
& \quad \text{LET } secrets_shared \triangleq \\
& \quad \quad \text{UNION } \{tx.ss \cap all_secrets : tx \in shared_knowledge\} \\
& \quad \quad sigs_shared \triangleq \\
& \quad \quad \quad \text{UNION } \{tx.ss : tx \in \{tx \in shared_knowledge : \wedge tx.id = id \\
& \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \wedge tx.to = to\}\} \\
& \quad \text{IN } sigs_shared \cup secrets_shared \cup signers_map[sender]
\end{aligned}$$

$$\begin{aligned}
& \text{DependencySatisfied}(id, ids) \triangleq \\
& \quad id \in \text{DOMAIN } dependency_map \Rightarrow dependency_map[id] \in ids
\end{aligned}$$

$$\begin{aligned}
& \text{IsSpendableTx}(tx, other_ids) \triangleq \\
& \quad \wedge \{\} = \text{ConflictingSet}(tx.id) \cap other_ids \\
& \quad \wedge \text{DependencySatisfied}(tx.id, other_ids) \\
& \quad \wedge tx.ss \subseteq \text{SigsAvailable}(tx.id, tx.by, tx.to) \\
& \quad \wedge \text{Len}(blocks) \geq \text{TimelockExpirationHeight}(tx.id)
\end{aligned}$$

Sending tx_spend_B does not actually expose secrets, because the secrets are used as keys, and $sigSecretBob$ would be exposed rather than $secretBob$. Instead of introducing $revealSecret < Alice | Bob >$, $sigSecret < Alice | Bob >$ we simply filter out signatures of tx_spend_B before placing into shared knowledge

$$\begin{aligned}
& \text{ShareKnowledge}(knowledge) \triangleq \\
& \quad \text{LET } knowledge_filtered \triangleq \\
& \quad \quad \{ \text{IF } tx.id \neq tx_spend_B \text{ THEN } tx \text{ ELSE } [tx \text{ EXCEPT } !.ss = \{\}] : \\
& \quad \quad \quad tx \in knowledge \} \\
& \quad \quad \quad shared_knowledge \text{ may not change here, callers need to check if they care} \\
& \quad \text{IN } shared_knowledge' = shared_knowledge \cup knowledge_filtered
\end{aligned}$$

$$\begin{aligned}
& \text{ShareTransactions}(ids, by) \triangleq \\
& \quad \text{LET } Ss(id) \triangleq (tx_map[id].ss \cap signers_map[by]) \setminus all_secrets \\
& \quad \quad txs \triangleq \{Tx(id, Ss(id), by, \text{SingleDst}(id), \text{"direct"}) : id \in ids\} \\
& \quad \text{IN } \wedge \text{ShareKnowledge}(txs) \\
& \quad \quad \wedge shared_knowledge' \neq shared_knowledge \quad \text{not a new knowledge} \Rightarrow \text{fail}
\end{aligned}$$

Txs enabled at the current cycle, used to update *per_block_enabled* vector
 $NewlyEnabledTxs \triangleq$

$$\{tx \in$$

$$\text{UNION}$$

$$\{ \text{UNION}$$

$$\{$$

$$\{$$

$$Tx(id, tx_map[id].ss, sender, to, \text{"enabled"}) : to \in DstSet(id)$$

$$\} : id \in RemainingTransactions$$

$$\} : sender \in participants$$

$$\} : \wedge \neg \exists etx \in \text{UNION } Range(per_block_enabled) : etx.id = tx.id$$

$$\wedge IsSpendableTx(tx, ConfirmedTransactions)$$

$$\}$$

$SendTransactionToMempool(id, sender, to) \triangleq$
 LET $tx \triangleq Tx(id, tx_map[id].ss, sender, to, \text{"mempool"})$
 IN $\wedge IsSpendableTx(tx, SentTransactions)$
 $\wedge Len(blocks) < Deadline(id)$
 $\wedge mempool' = mempool \cup \{tx\}$
 $\wedge ShareKnowledge(\{tx\})$

Give tx directly to miner, bypassing global *mempool*
 No *Deadline* check because information is not shared,
 and after the block is mined, there's no possible contention
 unless the block is orphaned. Orphan blocks are not modelled,
 and therefore there's no need for additional restriction
 as any state space restriction can possibly mask some other issue

$SendTransactionToMiner(id, sender, to) \triangleq$
 $\wedge STEALTHY_SEND_POSSIBLE$
 $\wedge \text{LET } tx \triangleq Tx(id, tx_map[id].ss, sender, to, \text{"miner"})$
 IN $\wedge IsSpendableTx(tx, NextBlockConfirmedTransactions)$
 $\wedge next_block' = next_block \cup \{tx\}$

■

$$\begin{aligned}
& \text{SendTransaction}(id, sender, to) \triangleq \\
& \quad \vee \wedge \text{SendTransactionToMempool}(id, sender, to) \\
& \quad \wedge \text{UNCHANGED } next_block \\
& \quad \vee \wedge \text{SendTransactionToMiner}(id, sender, to) \\
& \quad \wedge \text{UNCHANGED } \langle mempool, shared_knowledge \rangle \\
\\
& \text{SendSomeTransaction}(ids, sender) \triangleq \\
& \quad \text{LET } \text{SendSome}(filtered_ids) \triangleq \\
& \quad \quad \exists id \in filtered_ids : \\
& \quad \quad \exists to \in (\text{IF } id \in \text{ContractTransactions} \\
& \quad \quad \quad \text{THEN } \{ \text{Contract} \} \\
& \quad \quad \quad \text{ELSE } tx_map[id].ds \cap \{ sender \}) : \\
& \quad \quad \quad \text{SendTransaction}(id, sender, to) \\
& \quad \quad terminal_ids \triangleq ids \cap \text{TerminalTransactions} \\
& \quad \text{IN CASE } \text{PARTICIPANTS_IRRATIONAL} \\
& \quad \quad \rightarrow \text{SendSome}(ids) \quad \text{Irrational participants do no prioritization} \\
& \quad \square \text{ ENABLED } \text{SendSome}(terminal_ids) \\
& \quad \quad \rightarrow \text{SendSome}(terminal_ids) \quad \text{Can send terminal } tx \Rightarrow \text{do it immediately} \\
& \quad \square \text{ OTHER} \\
& \quad \quad \rightarrow \text{SendSome}(ids \setminus terminal_ids) \\
\\
& \text{HasCustody}(ids, participant) \triangleq \\
& \quad \exists id \in ids : \exists tx \in \text{UNION } \text{Range}(blocks) : tx.id = id \wedge tx.to = participant \\
\\
& \text{Sharing secrets or keys has to occur before deadline to send } tx_success \\
& \text{TooLateToShare} \triangleq \text{Len}(blocks) \geq \text{Deadline}(tx_success)
\end{aligned}$$

Participant actions

Transactions *Alice* initially shares signatures on

$$phase0_to_share_Alice \triangleq \{tx_revoke, tx_timeout\}$$

Transactions *Bob* initially shares signatures on

$$phase0_to_share_Bob \triangleq \{tx_refund_1, tx_revoke, tx_refund_2, tx_timeout\}$$

Conditions to divide the contract execution into phases according to original spec

$$Phase_3_cond \triangleq tx_lock_B \in ConfirmedTransactions$$

$$Phase_2_cond \triangleq tx_lock_A \in ConfirmedTransactions$$

$$Phase_1_cond \triangleq$$

$$\wedge \forall id \in phase0_to_share_Alice :$$

$$\exists tx \in shared_knowledge : tx.id = id \wedge sigAlice \in tx.ss$$

$$\wedge \forall id \in phase0_to_share_Bob :$$

$$\exists tx \in shared_knowledge : tx.id = id \wedge sigBob \in tx.ss$$

$$InPhase_3 \triangleq$$

$$\wedge Phase_3_cond$$

$$InPhase_2 \triangleq$$

$$\wedge Phase_2_cond$$

$$\wedge \neg Phase_3_cond$$

$$InPhase_1 \triangleq$$

$$\wedge Phase_1_cond$$

$$\wedge \neg Phase_2_cond$$

$$\wedge \neg Phase_3_cond$$

$$InPhase_0 \triangleq$$

$$\wedge \neg Phase_1_cond$$

$$\wedge \neg Phase_2_cond$$

$$\wedge \neg Phase_3_cond$$

Helper operators to declutter the action expressions

$$NoSending \triangleq \text{UNCHANGED } \langle mempool, next_block \rangle$$

$$NoKeysShared \triangleq \text{UNCHANGED } signers_map$$

$$NoKnowledgeShared \triangleq \text{UNCHANGED } shared_knowledge$$

$$\begin{aligned}
& AliceAction \triangleq \\
& \text{LET } Send(ids) \triangleq SendSomeTransaction(ids, Alice) \\
& \quad Share(ids) \triangleq ShareTransactions(ids, Alice) \\
& \quad SafeToSend(id) \triangleq \\
& \quad \text{CASE } PARTICIPANTS_IRRATIONAL \\
& \quad \quad \rightarrow \text{TRUE} \quad \text{Unsafe txs are OK for irrational Alice} \\
& \quad \quad \square \quad id = tx_refund_1 \quad \text{Do not send refund_1 if tx_success was shared} \\
& \quad \quad \quad \rightarrow tx_success \notin \{tx.id : tx \in shared_knowledge\} \\
& \quad \quad \square \quad secretAlice \in tx_map[id].ss \\
& \quad \quad \quad \text{Once Alice received secretBob, should never send out secretAlice} \\
& \quad \quad \quad \rightarrow \forall secretBob \notin signers_map[Alice] \\
& \quad \quad \quad \quad \vee id = tx_spend_B \quad \text{unless this is a transaction to get B} \\
& \quad \quad \quad \quad \quad \text{which does not in fact expose secrets} \\
& \quad \quad \square \quad \text{OTHER} \rightarrow \text{TRUE} \\
& \text{IN } \vee \wedge InPhase_0 \\
& \quad \wedge Share(phase0_to_share_Alice) \\
& \quad \wedge NoSending \wedge NoKeysShared \\
& \vee \wedge InPhase_1 \\
& \quad \wedge Send(\{tx_lock_A\}) \\
& \quad \wedge NoKeysShared \\
& \vee \wedge InPhase_2 \quad \text{Just waiting for Bob to lock B} \\
& \quad \wedge Send(\{id \in RemainingTransactions : SafeToSend(id)\}) \\
& \quad \wedge NoKeysShared \\
& \vee \wedge InPhase_3 \\
& \quad \wedge \vee \wedge secretBob \in signers_map[Alice] \quad \text{Bob gave Alice his secret} \\
& \quad \quad \wedge sigAlice \notin signers_map[Bob] \quad \text{Alice did not yet gave Bob her key} \\
& \quad \quad \wedge \neg TooLateToShare \\
& \quad \quad \wedge signers_map' = [signers_map \quad \text{Give Alice's key to Bob} \\
& \quad \quad \quad \text{EXCEPT } ![Bob] = @ \cup \{sigAlice\}] \\
& \quad \quad \wedge NoSending \wedge NoKnowledgeShared \\
& \vee \wedge tx_refund_1 \notin SentTransactions \\
& \quad \wedge \neg TooLateToShare \\
& \quad \wedge Share(\{tx_success\}) \quad \text{refund_1 not sent yet, can share} \\
& \quad \wedge NoSending \wedge NoKeysShared \\
& \vee \wedge Send(\{id \in RemainingTransactions : SafeToSend(id)\}) \\
& \quad \wedge NoKeysShared
\end{aligned}$$

$$\begin{aligned}
& \text{BobAction} \triangleq \\
& \text{LET } \text{Send}(ids) \triangleq \text{SendSomeTransaction}(ids, \text{Bob}) \\
& \quad \text{Share}(ids) \triangleq \text{ShareTransactions}(ids, \text{Bob}) \\
& \quad tx_success_sigs \triangleq \text{SigsAvailable}(tx_success, \text{Bob}, \text{Contract}) \\
& \text{IN } \vee \wedge \text{InPhase_0} \\
& \quad \wedge \text{Share}(\text{phase0_to_share_Bob}) \\
& \quad \wedge \text{NoSending} \wedge \text{NoKeysShared} \\
& \vee \wedge \text{InPhase_1} \quad \text{Just waiting for Alice to lock A} \\
& \quad \wedge \text{NoSending} \wedge \text{NoKnowledgeShared} \wedge \text{NoKeysShared} \\
& \vee \wedge \text{InPhase_2} \\
& \quad \wedge \text{Send}(\{tx_lock_B\}) \\
& \quad \wedge \text{NoKeysShared} \\
& \vee \wedge \text{InPhase_3} \\
& \quad \wedge \vee \wedge sigAlice \in tx_success_sigs \\
& \quad \quad \text{If Bob already knows secretAlice, he doesn't need to share secretBob} \\
& \quad \wedge secretAlice \notin tx_success_sigs \\
& \quad \wedge secretBob \notin signers_map[Alice] \\
& \quad \wedge \neg \text{TooLateToShare} \\
& \quad \wedge signers_map' = [signers_map \quad \text{Give secretBob to Alice} \\
& \quad \quad \text{EXCEPT } ![Alice] = @ \cup \{secretBob\}] \\
& \quad \wedge \text{NoSending} \wedge \text{NoKnowledgeShared} \\
& \vee \wedge \text{Send}(\text{RemainingTransactions}) \\
& \quad \wedge \text{NoKeysShared}
\end{aligned}$$

$$\begin{aligned}
& \text{MempoolMonitorActionRequired} \triangleq \\
& \quad \exists tx \in \text{mempool} : \wedge \text{Len}(\text{blocks}) + 1 = \text{Deadline}(tx.id) \\
& \quad \wedge tx.id \notin \text{NextBlockTransactions}
\end{aligned}$$

We update *next_block* directly rather than having to deal with fees and prioritization. What we want to model is the behavior of participants where once they have sent the transaction, they do anything possible to meet the deadline set by the protocol to confirm the transaction. Failure to do so before the deadline is out of scope, even though it could be caused by some unexpected *mempool* behavior.

Exact *mempool* behavior is too low-level and is better modelled separately to check that high-level constraints can be met. Although if we were to have more complex model where the amounts available for each participant are tracked, it might make sense to include the fees and *mempool* behavior into the model of the contract to catch the cases when participants just can't bump fees anymore, for example.

We could just not model the *mempool* monitoring, and constrain state space such that states with late *txs* are invalid, to express that we don't care about the cases when participants fail to get their *txs* confirmed in time. But maybe there could be some interesting behaviors to be modelled if more elaborate monitor action is implemented

$$\begin{aligned}
& \text{MempoolMonitorAction} \triangleq \\
& \quad \text{LET } tx \triangleq \text{CHOOSE } tx \in \text{mempool} : \text{Len}(\text{blocks}) + 1 = \text{Deadline}(tx.id) \\
& \quad \quad \text{txs_to_bump} \triangleq \{tx\} \cup \{dptx \in \text{mempool} : \\
& \quad \quad \quad \wedge tx.id \in \text{DOMAIN } \text{dependency_map} \\
& \quad \quad \quad \wedge dptx.id = \text{dependency_map}[tx.id] \\
& \quad \quad \quad \wedge dptx.id \notin \text{NextBlockTransactions}\} \\
& \quad \text{IN } \text{next_block}' = \\
& \quad \quad \{nbtx \in \text{next_block} : \text{conflicting } txs \text{ are expunged from } \text{next_block} \\
& \quad \quad \quad \{\} = \text{DependencyChain}(nbtx.id) \cap \\
& \quad \quad \quad \text{UNION } \{\text{ConflictingSet}(bmptx.id) : bmptx \in \text{txs_to_bump}\}\} \\
& \quad \quad \cup \{[bmptx \text{ EXCEPT } !.via = \text{"fee-bump"}] : bmptx \in \text{txs_to_bump}\}
\end{aligned}$$

Miner action

$$\begin{aligned}
& \text{IncludeTxIntoBlock} \triangleq \\
& \quad \wedge \exists tx \in \text{mempool} : \\
& \quad \quad \wedge \{ \} = \text{ConflictingSet}(tx.id) \cap \text{NextBlockConfirmedTransactions} \\
& \quad \quad \wedge \text{DependencySatisfied}(tx.id, \text{NextBlockConfirmedTransactions}) \\
& \quad \quad \wedge \text{next_block}' = \text{next_block} \cup \{ tx \} \\
& \quad \wedge \text{UNCHANGED} \langle \text{blocks}, \text{mempool}, \text{shared_knowledge} \rangle \\
\\
& \text{CanMineEmptyBlock} \triangleq \\
& \quad \wedge \text{first_transaction} \in \text{ConfirmedTransactions} \\
& \quad \wedge \text{LET } \text{soft_dls} \triangleq \{ \text{SoftDeadline}(id) : id \in \text{RemainingTransactions} \} \\
& \quad \quad \text{IN } \text{soft_dls} \neq \{ \} \wedge \text{Len}(\text{blocks}) + 1 < \text{Max}(\text{soft_dls}) \\
\\
& \text{MineTheBlock} \triangleq \\
& \quad \text{IF } \text{next_block} = \{ \} \\
& \quad \quad \text{THEN } \wedge \text{CanMineEmptyBlock} \\
& \quad \quad \quad \wedge \text{blocks}' = \text{Append}(\text{blocks}, \{ \}) \\
& \quad \quad \quad \wedge \text{UNCHANGED} \langle \text{mempool}, \text{next_block}, \text{shared_knowledge} \rangle \\
& \quad \quad \text{ELSE } \wedge \text{blocks}' = \text{Append}(\text{blocks}, \text{next_block}) \\
& \quad \quad \quad \wedge \text{mempool}' = \\
& \quad \quad \quad \quad \{ tx \in \text{mempool} : \text{conflicting txs are expunged from mempool} \\
& \quad \quad \quad \quad \{ \} = \text{DependencyChain}(tx.id) \cap \\
& \quad \quad \quad \quad \quad \text{UNION } \{ \text{ConflictingSet}(nbtx.id) : nbtx \in \text{next_block} \} \} \\
& \quad \quad \quad \wedge \text{next_block}' = \{ \} \\
& \quad \quad \quad \wedge \text{ShareKnowledge}(\text{next_block} \setminus \text{mempool}) \\
\\
& \text{MinerAction} \triangleq \text{IncludeTxIntoBlock} \vee \text{MineTheBlock}
\end{aligned}$$

Auxiliary action for soft-deadline tracking

$$\begin{aligned}
& \text{UpdateEnabledPerBlock} \triangleq \\
& \quad \text{per_block_enabled}' = \\
& \quad \quad \text{IF } \text{Len}(\text{per_block_enabled}) < \text{Len}(\text{blocks}) + 1 \\
& \quad \quad \quad \text{THEN } \text{Append}(\text{per_block_enabled}, \text{NewlyEnabledTxes}) \\
& \quad \quad \quad \text{ELSE } [\text{per_block_enabled} \text{ EXCEPT } ![\text{Len}(\text{blocks}) + 1] = @ \cup \text{NewlyEnabledTxes}]
\end{aligned}$$

High-level contract spec

First, the 'unnatural' cases.

For all transactions defined by the original spec to be covered by the model, we need to also model the case where *Alice* misbehaves by sending transactions containing her secret after she gave *tx_success* to *Bob*. This behavior also enables *Bob* to misbehave by failing to punish *Alice* s misbehavior, which results in *Bob* losing *B*.

The following four actions are needed to express all that.

$$\begin{aligned} AliceLostByMisbehaving &\triangleq \\ &\wedge HasCustody(\{tx_spend_B\}, Bob) \\ &\wedge HasCustody(\{tx_spend_refund_1_bob\}, Bob) \\ BobLostByBeingLateOnRefund_1 &\triangleq \\ &\wedge HasCustody(\{tx_spend_B\}, Alice) \\ &\wedge HasCustody(\{tx_spend_refund_1_alice\}, Alice) \\ BobLostByBeingLateOnRefund_2 &\triangleq \\ &\wedge HasCustody(\{tx_spend_B\}, Alice) \\ &\wedge HasCustody(\{tx_spend_refund_2\}, Alice) \\ SwapUnnaturalEnding &\triangleq \\ &\vee AliceLostByMisbehaving \\ &\vee BobLostByBeingLateOnRefund_1 \\ &\vee BobLostByBeingLateOnRefund_2 \end{aligned}$$

■
The normal, 'natural' cases.

$$\begin{aligned}
\text{SwapSuccessful} &\triangleq \\
&\wedge \text{HasCustody}(\{tx_spend_B\}, Alice) \\
&\wedge \vee \text{HasCustody}(\{tx_spend_A, tx_spend_success, \\
&\quad tx_spend_timeout, tx_spend_revoke\}, Bob) \\
&\vee \wedge \text{PARTICIPANTS_IRRATIONAL} \\
&\quad \wedge \text{HasCustody}(\{tx_spend_refund_1_bob\}, Bob)
\end{aligned}$$

$$\begin{aligned}
\text{SwapAborted} &\triangleq \\
&\wedge \text{HasCustody}(\{tx_spend_A, tx_spend_refund_1_alice, tx_spend_refund_2\}, Alice) \\
&\wedge \vee \text{HasCustody}(\{tx_spend_B\}, Bob) \\
&\quad \vee tx_lock_B \notin \text{SentTransactions}
\end{aligned}$$

$$\begin{aligned}
\text{SwapTimedOut} &\triangleq \\
&\wedge tx_spend_timeout \in \text{ConfirmedTransactions} \\
&\quad \text{Alice can't claim } tx_spend_B \text{ on timeout} \\
&\wedge secretBob \notin signers_map[Alice] \\
&\wedge secretBob \notin \text{UNION } \{tx.ss : tx \in shared_knowledge\}
\end{aligned}$$

All possible endings of the contract

$$\begin{aligned}
\text{ContractFinished} &\triangleq \vee \text{SwapSuccessful} \\
&\quad \vee \text{SwapAborted} \\
&\quad \vee \text{SwapTimedOut} \\
&\quad \vee \text{PARTICIPANTS_IRRATIONAL} \wedge \text{SwapUnnaturalEnding}
\end{aligned}$$

Actions in the contract when it is not yet finished. Separated into
dedicated operator to be able to test ENABLED *ContractAction*

$$\begin{aligned}
\text{ContractAction} &\triangleq \\
&\vee AliceAction \quad \quad \quad \wedge \text{UNCHANGED } blocks \\
&\vee BobAction \quad \quad \quad \wedge \text{UNCHANGED } blocks \\
&\vee \text{IF } MempoolMonitorActionRequired \\
&\quad \text{THEN } MempoolMonitorAction \wedge \text{UNCHANGED } unchangedByMM \\
&\quad \text{ELSE } MinerAction \quad \quad \quad \wedge \text{UNCHANGED } signers_map
\end{aligned}$$

$$\begin{aligned}
& \text{TypeOK} \triangleq \\
& \text{LET } TxConsistent(tx, vias) \triangleq \wedge tx.id \in all_transactions \\
& \quad \wedge tx.ss \subseteq tx_map[tx.id].ss \\
& \quad \wedge tx.to \in DstSet(tx.id) \\
& \quad \wedge tx.by \in participants \\
& \quad \wedge tx.via \in vias \\
& AllSigsPresent(tx) \triangleq tx.ss = tx_map[tx.id].ss \\
& SigConsistent(sig) \triangleq \wedge sig.id \in all_transactions \\
& \quad \wedge sig.s \in all_sigs \\
& \quad \wedge sig.ds \subseteq participants \\
& \quad \quad \cup \text{DOMAIN } dependency_map \\
& \text{IN } \wedge \forall tx \in \text{UNION } Range(blocks) : \\
& \quad \vee \wedge TxConsistent(tx, \{ "mempool", "miner", "fee-bump" \}) \\
& \quad \wedge AllSigsPresent(tx) \\
& \quad \vee Print(\langle \sim \text{TypeOK blocks}, tx \rangle, \text{FALSE}) \\
& \wedge \forall tx \in \text{UNION } Range(per_block_enabled) : \\
& \quad \vee \wedge TxConsistent(tx, \{ "enabled" \}) \\
& \quad \wedge AllSigsPresent(tx) \\
& \quad \vee Print(\langle \sim \text{TypeOK blocks}, tx \rangle, \text{FALSE}) \\
& \wedge \forall tx \in next_block : \\
& \quad \vee \wedge TxConsistent(tx, \{ "mempool", "miner", "fee-bump" \}) \\
& \quad \wedge AllSigsPresent(tx) \\
& \quad \vee Print(\langle \sim \text{TypeOK next_block}, tx \rangle, \text{FALSE}) \\
& \wedge \forall tx \in mempool : \\
& \quad \vee \wedge TxConsistent(tx, \{ "mempool" \}) \\
& \quad \wedge AllSigsPresent(tx) \\
& \quad \vee Print(\langle \sim \text{TypeOK mempool}, tx \rangle, \text{FALSE}) \\
& \wedge \forall tx \in shared_knowledge : \\
& \quad \vee TxConsistent(tx, \{ "mempool", "miner", "fee-bump", "direct" \}) \\
& \quad \vee Print(\langle \sim \text{TypeOK shared_knowledge}, tx \rangle, \text{FALSE}) \\
& \wedge \forall p \in \text{DOMAIN } signers_map : \\
& \quad \vee p \in participants \wedge \forall sig \in signers_map[p] : sig \in all_sigs \\
& \quad \vee Print(\langle \sim \text{TypeOK signers_map}, p \rangle, \text{FALSE})
\end{aligned}$$

$ConsistentPhase \triangleq$
 $\text{LET } phases \triangleq \langle InPhase_0, InPhase_1, InPhase_2, InPhase_3 \rangle$
 $\text{IN } Cardinality(\{i \in \text{DOMAIN } phases : phases[i]\}) = 1$

$OnlyWhenParticipantsAreRational \triangleq$
 $PARTICIPANTS_IRRATIONAL$
 $\Rightarrow Assert(\text{FALSE}, \text{"Not applicable when participants are not rational"})$

$NoConcurrentSecretKnowledge \triangleq$
 $\wedge OnlyWhenParticipantsAreRational$
 $\wedge \text{LET } SecretsShared \triangleq$
 $(all_secrets \cap \text{UNION } \{tx.ss : tx \in shared_knowledge\})$
 $\cup (\{secretBob\} \cap signers_map[Alice])$
 $\cup (\{secretAlice\} \cap signers_map[Bob])$
 $\text{IN } Cardinality(SecretsShared) \leq 1$

$NoUnexpectedTransactions \triangleq$
 $\wedge OnlyWhenParticipantsAreRational$
 $\wedge tx_spend_refund_1_bob \notin SentTransactions$

$NoConflictingTransactions \triangleq$
 $\text{LET } ConflictCheck(txs) \triangleq$
 $\text{LET } ids \triangleq \{tx.id : tx \in txs\}$
 $\text{IN } \wedge Cardinality(ids) = Cardinality(txs)$
 $\wedge \forall id \in ids : ConflictingSet(id) \cap ids = \{id\}$
 $\text{IN } \wedge ConflictCheck(\text{UNION } Range(blocks) \cup next_block)$
 $\wedge ConflictCheck(\text{UNION } Range(blocks) \cup mempool)$

$NoSingleParticipantTakesAll \triangleq$
 $\wedge OnlyWhenParticipantsAreRational$
 $\wedge \forall p \in participants :$
 $\text{LET } txs_to_p \triangleq \{tx \in \text{UNION } Range(blocks) : tx.to = p\}$
 $\text{IN } Cardinality(\{tx.id : tx \in txs_to_p\}) \leq 1$

$TransactionTimelocksEnforced \triangleq$
 $\wedge \forall tx \in mempool : Len(blocks) \geq TimelockExpirationHeight(tx.id)$
 $\wedge STEALTHY_SEND_POSSIBLE$
 $\Rightarrow \forall tx \in next_block : Len(blocks) \geq TimelockExpirationHeight(tx.id)$

■ $ExpectedStateOnAbortOrTimeout \triangleq$
 $SwapAborted \vee SwapTimedOut$
 $\Rightarrow \text{LET } ids_left \triangleq \text{IF ENABLED } ContractAction \text{ THEN } \{tx_lock_B\} \text{ ELSE } \{\}$
 $\text{IN } RemainingTransactions \subseteq \{tx_spend_B\} \cup ids_left$

$ExpectedStateOnSuccess \triangleq$
 $SwapSuccessful \Rightarrow \wedge \neg \text{ENABLED } ContractAction$
 $\wedge RemainingTransactions = \{\}$
 $\wedge mempool = \{\}$
 $\wedge next_block = \{\}$

Can use this invariant to check if certain state can be reached.

If the CounterExample invariant is violated, then the state has been reached.

$CounterExample \triangleq \text{TRUE} \wedge \dots$

Temporal properties

$ContractEventuallyFinished \triangleq \Diamond ContractFinished$

Init & Next

$Init \triangleq$
 $\wedge blocks = \langle \rangle$
 $\wedge per_block_enabled = \langle \rangle$
 $\wedge next_block = \{\}$
 $\wedge mempool = \{\}$
 $\wedge shared_knowledge = \{\}$
 $\wedge signers_map = [Alice \mapsto \{sigAlice, secretAlice\},$
 $Bob \mapsto \{sigBob, secretBob\}]$

$Next \triangleq \vee \wedge ContractAction$
 $\wedge UpdateEnabledPerBlock$
 $\vee ContractFinished \wedge \text{UNCHANGED } fullState$

$Spec \triangleq Init \wedge \Box [Next]_{fullState} \wedge \text{WF}_{fullState}(Next)$