MODULE *DAG*

DiGraph operators taken from https://*github.com*/nano-o/TLA-Library/blob/master/*DiGraph.tla*

EXTENDS *FiniteSets*, *Naturals*, *Sequences*

The following two operators are from the Specifying Systems book, though I can't make *Path* work because of *Seq* being infinite and non-enumerable.

$Path(G) \triangleq$ The set of paths in G, where a path is represented as a sequence of nodes

$$\{p \in Seq(G.node) : \land\, p \neq \langle\rangle \\ \land\, \forall\, i \in 1 \,..\, (Len(p) - 1) : \langle p[i],\, p[i+1] \in G.edge\rangle\}$$

$SeqOf(set,\, n) \triangleq$

All sequences up to length n with all elements in set. Includes empty sequence.

$$\text{UNION } \{[1 \,..\, m \rightarrow set] : m \in 0 \,..\, n\}$$

$Contains(s,\, e) \triangleq$

TRUE iff the element $e \in ToSet(s)$.

$$\exists\, i \in 1 \,..\, Len(s) : s[i] = e$$

$SimplePath(G) \triangleq$

A simple path is a path with no repeated nodes.

$$\{p \in SeqOf(G.node,\, Cardinality(G.node)) : \\ \land\ \ p \neq \langle\rangle \\ \land\ \ Cardinality(\{p[i] : i \in \text{DOMAIN } p\}) = Len(p) \\ \land\ \ \forall\, i \in 1 \,..\, (Len(p) - 1) : \langle p[i],\, p[i+1]\rangle \in G.edge\}$$

$AreConnected(m,\, n,\, G) \triangleq$ True if there is a path from m to n in G

$$\exists\, p \in Path(G) : (p[1] = m) \land (p[Len(p)] = n)$$