# Deducing *get_y* formulas from StableSwap invariant

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## 1 Introduction

This paper is trying to fill the gap between StableSwap whitepaper and actual code of Curve contracts. While whitepaper covers basic principles, it may not be obvious how contract functions work just looking at code.

Here we explaining (rather basic) math behind the *get_y* function. But this paper should also be helpful when trying to understand how *get_D* and *get_y_D* functions work.

## 2 Math behind *get_y* function

Suppose we have $n$ coins with amounts $x_1, \ldots, x_n$. Let $S$ be equal to $\sum_{k=1}^{n} x_k$ and $P$ equal to $\prod_{k=1}^{n} x_k$. Let's write StableSwap invariant:

$$a \cdot n^n \cdot S + d = a \cdot n^n \cdot d + \frac{d^{n+1}}{n^n \cdot P}$$

where $a$ - amplification coefficient, $d$ - total amount of coins when they have an equal price.

We know index $i$ of amount that changed to value $x$. No other parameters have changed and StableSwap invariant is preserved. We want to find a new amount with a given index $j$. Let's denote this value $x_j$ as $y$. So for $S$ and $P$ we can write following:

$$S = x_1 + x_2 + \ldots + x_{i-1} + x + x_{i+1} + \ldots + x_{j-1} + y + x_{j+1} + \ldots + x_n$$

$$P = x_1 \cdot x_2 \cdot \ldots \cdot x_{i-1} \cdot x \cdot x_{i+1} \cdot \ldots \cdot x_{j-1} \cdot y \cdot x_{j+1} \cdot \ldots \cdot x_n$$

Let sum of all known terms in $S$ be $s$ and product of all known factors in $P$ be $p$. So we can rewrite $S$ and $P$:

$$S = s + y$$

$$P = p \cdot y$$

Now we can substitute these values into StableSwap invariant:

$$a \cdot n^n \cdot (s + y) + d = a \cdot n^n \cdot d + \frac{d^{n+1}}{n^n \cdot p \cdot y}$$

It's obvious that $y > 0$, $a > 0$ and $n > 0$. So we can rewrite previous equation as a quadratic equation with respect to $y$:

$$y^2 + \left(s + \frac{d}{a \cdot n^n} - d\right) y = \frac{d^{n+1}}{a \cdot n^{2n} \cdot p}$$

Let's introduce variable $ann$ that equals to $a \cdot n^n$. With $ann$ in mind rewrite previous equation:

$$y^2 + \left(s + \frac{d}{ann} - d\right) y = \frac{d^{n+1}}{ann \cdot n^n \cdot p}$$

Let's introduce $b$ and $c$ such that:

$$b = s + \frac{d}{ann} - d$$

$$c = \frac{d^{n+1}}{ann \cdot n^n \cdot p}$$

Now we can rewrite our quadratic equation as:

$$y^2 + by = c$$

To solve this equation numerically using fixed-point iteration method let's rewrite it as:

$$y = \frac{y^2 + c}{2y + b}$$