

# Multiscale Modelling of Granular Flows



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A thesis submitted for the degree of  
*Doctor of Philosophy*

King's College

Yet to be decided



I would like to dedicate this thesis to my loving parents ...



## **Declaration**

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other University. This dissertation is entirely the result of my own work and includes nothing which is the outcome of work done in collaboration, except where specified in the text. This dissertation contains less than 65,000 words, excluding table of contents, tables, figures, titles, footnotes, references and appendices and 150 figures.

Krishna Kumar Soundararajan

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## **Acknowledgements**

And I would like to acknowledge ...





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# Nomenclature

## Roman Symbols

$F$  complex function

## Greek Symbols

$\gamma$  a simply closed curve on a complex plane

$i$  unit imaginary number  $\sqrt{-1}$

$\pi \simeq 3.14\dots$

## Superscripts

$j$  superscript index

## Subscripts

$0$  subscript index

## Other Symbols

$\oint_{\gamma}$  integration around a curve  $\gamma$

## Acronyms / Abbreviations

*CIF* Cauchy's Integral Formula





# Chapter 1

## Background

### 1.1 What is Lorem Ipsum? Title with Math $\sigma$

Lorem Ipsum is simply dummy text of the printing and typesetting industry section 1.3. Lorem Ipsum [2] has been the industry's standard dummy text ever since the 1500s, when an unknown printer took a galley of type and scrambled it to make a type specimen book. It has survived not only five centuries, but also the leap into electronic typesetting, remaining essentially unchanged. It was popularised in the 1960s with the release of Letraset sheets containing Lorem Ipsum passages, and more recently with desktop publishing software like Aldus PageMaker including versions of Lorem Ipsum [1, 3, 4].

The most famous equation in the world:  $E^2 = (m_0c^2)^2 + (pc)^2$ , which is known as the **energy-mass-momentum** relation as an in-line equation.

A *L<sup>A</sup>T<sub>E</sub>X* class file is a file, which holds style information for a particular L<sup>A</sup>T<sub>E</sub>X.

$$CIF : \quad F_0^j(a) = \frac{1}{2\pi i} \oint_{\gamma} \frac{F_0^j(z)}{z-a} dz \quad (1.1)$$

### 1.2 Why do we use Lorem Ipsum?

It is a long established fact that a reader will be distracted by the readable content of a page when looking at its layout. The point of using Lorem Ipsum is that it has a more-

or-less normal distribution of letters, as opposed to using ‘Content here, content here’, making it look like readable English. Many desktop publishing packages and web page editors now use Lorem Ipsum as their default model text, and a search for ‘lorem ipsum’ will uncover many web sites still in their infancy. Various versions have evolved over the years, sometimes by accident, sometimes on purpose (injected humour and the like).

### 1.3 Where does it come from?

Contrary to popular belief, Lorem Ipsum is not simply random text. It has roots in a piece of classical Latin literature from 45 BC, making it over 2000 years old. Richard McClintock, a Latin professor at Hampden-Sydney College in Virginia, looked up one of the more obscure Latin words, *consectetur*, from a Lorem Ipsum passage, and going through the cites of the word in classical literature, discovered the undoubtable source. Lorem Ipsum comes from sections 1.10.32 and 1.10.33 of “*de Finibus Bonorum et Malorum*” (The Extremes of Good and Evil) by Cicero, written in 45 BC. This book is a treatise on the theory of ethics, very popular during the Renaissance. The first line of Lorem Ipsum, “*Lorem ipsum dolor sit amet..*”, comes from a line in section 1.10.32.

The standard chunk of Lorem Ipsum used since the 1500s is reproduced below for those interested. Sections 1.10.32 and 1.10.33 from “*de Finibus Bonorum et Malorum*” by Cicero are also reproduced in their exact original form, accompanied by English versions from the 1914 translation by H. Rackham

“*Lorem ipsum dolor sit amet, consectetur adipiscing elit, sed do eiusmod tempor incididunt ut labore et dolore magna aliqua. Ut enim ad minim veniam, quis nostrud exercitation ullamco laboris nisi ut aliquip ex ea commodo consequat. Duis aute irure dolor in reprehenderit in voluptate velit esse cillum dolore eu fugiat nulla pariatur. Excepteur sint occaecat cupidatat non proident, sunt in culpa qui officia deserunt mollit anim id est laborum.*”

Section 1.10.32 of “*de Finibus Bonorum et Malorum*”, written by Cicero in 45 BC: “*Sed ut perspiciatis unde omnis iste natus error sit voluptatem accusantium doloremque laudantium, totam rem aperiam, eaque ipsa quae ab illo inventore veritatis et quasi architecto beatae vitae dicta sunt explicabo. Nemo enim ipsam voluptatem quia voluptas sit aspernatur aut odit aut fugit, sed quia consequuntur magni dolores eos qui ratione voluptatem sequi nesciunt. Neque porro quisquam est, qui dolorem ipsum quia dolor*

sit amet, consectetur, adipisci velit, sed quia non numquam eius modi tempora incidunt ut labore et dolore magnam aliquam quaerat voluptatem. Ut enim ad minima veniam, quis nostrum exercitationem ullam corporis suscipit laboriosam, nisi ut aliquid ex ea commodi consequatur? Quis autem vel eum iure reprehenderit qui in ea voluptate velit esse quam nihil molestiae consequatur, vel illum qui dolorem eum fugiat quo voluptas nulla pariatur?"

1914 translation by H. Rackham: "But I must explain to you how all this mistaken idea of denouncing pleasure and praising pain was born and I will give you a complete account of the system, and expound the actual teachings of the great explorer of the truth, the master-builder of human happiness. No one rejects, dislikes, or avoids pleasure itself, because it is pleasure, but because those who do not know how to pursue pleasure rationally encounter consequences that are extremely painful. Nor again is there anyone who loves or pursues or desires to obtain pain of itself, because it is pain, but because occasionally circumstances occur in which toil and pain can procure him some great pleasure. To take a trivial example, which of us ever undertakes laborious physical exercise, except to obtain some advantage from it? But who has any right to find fault with a man who chooses to enjoy a pleasure that has no annoying consequences, or one who avoids a pain that produces no resultant pleasure?"

Section 1.10.33 of "de Finibus Bonorum et Malorum", written by Cicero in 45 BC: "At vero eos et accusamus et iusto odio dignissimos ducimus qui blanditiis praesentium voluptatum deleniti atque corrupti quos dolores et quas molestias excepturi sint occaecati cupiditate non provident, similique sunt in culpa qui officia deserunt mollitia animi, id est laborum et dolorum fuga. Et harum quidem rerum facilis est et expedita distinctio. Nam libero tempore, cum soluta nobis est eligendi optio cumque nihil impedit quo minus id quod maxime placeat facere possimus, omnis voluptas assumenda est, omnis dolor repellendus. Temporibus autem quibusdam et aut officiis debitis aut rerum necessitatibus saepe eveniet ut et voluptates repudiandae sint et molestiae non recusandae. Itaque earum rerum hic tenetur a sapiente delectus, ut aut reiciendis voluptatibus maiores alias consequatur aut perferendis doloribus asperiores repellat."

1914 translation by H. Rackham: "On the other hand, we denounce with righteous indignation and dislike men who are so beguiled and demoralized by the charms of pleasure of the moment, so blinded by desire, that they cannot foresee the pain and trouble that are bound to ensue; and equal blame belongs to those who fail in their duty

## **Background**

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through weakness of will, which is the same as saying through shrinking from toil and pain. These cases are perfectly simple and easy to distinguish. In a free hour, when our power of choice is untrammelled and when nothing prevents our being able to do what we like best, every pleasure is to be welcomed and every pain avoided. But in certain circumstances and owing to the claims of duty or the obligations of business it will frequently occur that pleasures have to be repudiated and annoyances accepted. The wise man therefore always holds in these matters to this principle of selection: he rejects pleasures to secure other greater pleasures, or else he endures pains to avoid worse pains."

# Chapter 2

## My Second Chapter

### 2.1 Reasonably Long Section Title

The SI Units for dynamic viscosity is  $\text{N s m}^{-2}$ . I'm going to randomly include a picture fig. 2.1.

If you have trouble viewing this document contact Krishna [kks32@cam.ac.uk](mailto:kks32@cam.ac.uk).



Fig. 2.1 This is just a long figure caption for the minion in Despicable Me from Pixar

## **Enumeration**

1. The first topic is dull
2. The second topic is duller
  - (a) The first subtopic is silly
  - (b) The second subtopic is stupid
3. The third topic is dullest

## **itemize**

- The first topic is dull
- The second topic is duller
  - The first subtopic is silly
  - The second subtopic is stupid
- The third topic is dullest

## **description**

**The first topic** is dull

**The second topic** is duller

**The first subtopic** is silly

**The second subtopic** is stupid

**The third topic** is dullest

## 2.2 Hidden Section

**Lorem ipsum dolor sit amet**, *consectetur adipiscing elit*. In magna nisi, aliquam id blandit id, congue ac est. Fusce porta consequat leo. Proin feugiat at felis vel consectetur. Ut tempus ipsum sit amet congue posuere. Nulla varius rutrum quam. Donec sed purus luctus, faucibus velit id, ultrices sapien. Cras diam purus, tincidunt eget tristique ut, egestas quis nulla. Curabitur vel iaculis lectus. Nunc nulla urna, ultrices et eleifend in, accumsan ut erat. In ut ante leo. Aenean a lacinia nisl, sit amet ullamcorper dolor. Maecenas blandit, tortor ut scelerisque congue, velit diam volutpat metus, sed vestibulum eros justo ut nulla. Etiam nec ipsum non enim luctus porta in in massa. Cras arcu urna, malesuada ut tellus ut, pellentesque mollis risus. Morbi vel tortor imperdiet arcu auctor mattis sit amet eu nisi. Nulla gravida urna vel nisl egestas varius. Aliquam posuere ante quis malesuada dignissim. Mauris ultrices tristique eros, a dignissim nisl iaculis nec. Praesent dapibus tincidunt mauris nec tempor. Curabitur et consequat nisi. Quisque viverra egestas risus, ut sodales enim blandit at. Mauris quis odio nulla. Cras euismod turpis magna, in facilisis diam congue non. Mauris faucibus nisl a orci dictum, et tempus mi cursus.

Etiam elementum tristique lacus, sit amet eleifend nibh eleifend sed <sup>1</sup>. Maecenas dapibus augue ut urna malesuada, non tempor nibh mollis. Donec sed sem sollicitudin, convallis velit aliquam, tincidunt diam. In eu venenatis lorem. Aliquam non augue porttitor tellus faucibus porta et nec ante. Proin sodales, libero vitae commodo sodales, dolor nisi cursus magna, non tincidunt ipsum nibh eget purus. Nam rutrum tincidunt arcu, tincidunt vulputate mi sagittis id. Proin et nisi nec orci tincidunt auctor et porta elit. Praesent eu dolor ac magna cursus euismod. Integer non dictum nunc.

---

<sup>1</sup>My footnote goes blah blah blah! ...

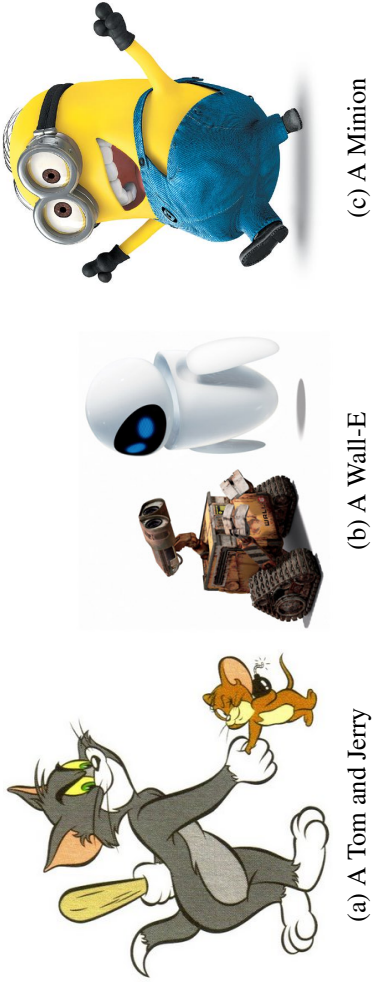


Fig. 2.2 Best Animations

## Subplots

I can cite Wall-E (see fig. 2.2b) and Minions in despicable me ( fig. 2.2c) or I can cite the whole figure as fig. 2.2



# **Chapter 3**

## **My Third Chapter**

### **3.1 First Section of the Third Chapter**

And now I begin my third chapter here ...

And now to cite some more people Read [5]

#### **3.1.1 First Subsection in the First Section**

...and some more

#### **3.1.2 Second Subsection in the First Section**

...and some more ...

##### **First subsub section in the second subsection**

...and some more in the first subsub section otherwise it all looks the same doesn't it?  
well we can add some text to it ...

#### **3.1.3 Third Subsection in the First Section**

...and some more ...

## My Third Chapter

---

### First subsub section in the third subsection

...and some more in the first subsub section otherwise it all looks the same doesn't it? well we can add some text to it and some more and some more and some more and some more and some more and some more and some more ...

### Second subsub section in the third subsection

...and some more in the first subsub section otherwise it all looks the same doesn't it? well we can add some text to it ...

## 3.2 Second Section of the Third Chapter

and here I write more ...

Now we can refer to the table using table 3.1.

Table 3.1 Table with Borders

1	2	3
4	5	6
7	8	9

# **Appendix A**

# **Appendix A**

and here I put a bit of postamble ...



## **Appendix B**

## **Appendix B**

and here I put some more postamble ...



# References

- [1] Abramovich, Y. A., Aliprantis, C. D., and Burkinshaw, O. (1995). Another characterization of the invariant subspace problem. *Operator Theory in Function Spaces and Banach Lattices*. The A.C. Zaanen Anniversary Volume, *Operator Theory: Advances and Applications*, 75:15–31. Birkhäuser Verlag.
- [2] Aupetit, B. (1991). *A Primer on Spectral Theory*. Springer-Verlag, New York.
- [3] Conway, J. B. (1990). *A Course in Functional Analysis*. Springer-Verlag, New York, second edition.
- [4] Ljubič, J. I. and Macaev, V. I. (1965). On operators with a separable spectrum. *Amer. Math. Soc. Transl. (2)*, 47:89–129.
- [5] Read, C. J. (1985). A solution to the invariant subspace problem on the space  $l_1$ . *Bull. London Math. Soc.*, 17:305–317.





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