Assignment-3

Chapter-3

Problem Set 3.1

(1) Among given vectors,

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$v_1^T v_2 = \begin{bmatrix} 1 & 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix}$$

$$= 4 - 8$$

$$= -4$$

$$v_1^T v_3 = \begin{bmatrix} 1 & 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$= \cancel{1} - \cancel{2} + \cancel{2} - \cancel{1} = 0$$

$$v_1^T v_4 = \begin{bmatrix} 1 & 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= 1 + 2 - 2 + 1$$

$$= 2$$

$$v_2^T v_3 = \begin{bmatrix} 4 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

$$= 4 - 4$$

$$= 0$$

$$v_2^T v_4 = \begin{bmatrix} 4 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= 4 + 4$$

$$= 8$$

$$v_3^T v_4 = \begin{bmatrix} 1 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \cancel{1} - \cancel{1} - 1 - 1$$

$$= -2$$

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$\therefore$ $\{v_1 \,\&\, v_3\}$ and $\{v_2 \,\&\, v_3\}$ are orthogonal pairs.

(2)

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{matrix} NR_2 \to R_2 - 2R_1 \\ NR_3 \to R_3 - 3R_1 \end{matrix}$$

U

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$ (By ~~Row~~ Column Exchange)

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{matrix} NR_2 \to R_2 - R_1 \\ NR_3 \to R_3 - R_1 \end{matrix}$$

R

So, $C(A^T) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

So, vector x orthogonal to row space of A, i.e., $C(A^T)$ will be,

$$x = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$(\because x^T . C(A^T) = 0)$

Now, $C(A) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

So, vector y orthogonal to the column space, i.e., $C(A)$ will be,

$$y = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \quad (\because y^T . C(A) = 0)$$

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For $N(A)$,

$$Ux = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow x + 2y + z = 0$$
$$\& \quad z = 0$$

So, one of the null space will be,

$$N(A) = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

So, vector z orthogonal to null space, i.e, $N(A)$ will be,

$$z = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad (\because z^T . N(A) = 0)$$

(7) According to Question,

$$x = (1,4,0,2)$$

So, $\|x\| = \sqrt{1^2 + 4^2 + 0^2 + 2^2}$

$= \sqrt{1+16+0+4}$

$= \sqrt{21}$

Also, $y = (2,-2,1,3)$

So, $\|y\| = \sqrt{4+4+1+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Inner product of x and y will be,

$$x^T y = \begin{bmatrix} 1 & 4 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 1 \\ 3 \end{bmatrix}$$

$= 2 - 8 + 6$

$= 0$

So, x & y are orthogonal.

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(9) As we know, orthogonal compliment of Row Space is Null Space.

So, $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix}$
U

So, $Ux = 0$

$\Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Pivot entries

$\Rightarrow x + 2z = 0$

$y + 2z = 0$

So, As z is the free variable, let us take $z = 1$.

So, for $z = 1$,
$x = -2$ & $y = -2$.

So, $N(A) = \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$ (Basis of Null Space)

Now, Basis of row space will be,

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} \right\}$$

So, to split $x = (3,3,3)$ into x_r & x_n.

$x = a x_r + b x_n$

$$\Rightarrow \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = a \begin{bmatrix} 1 \\ 1 \\ 4 \end{bmatrix} + b \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

Now, for $a = 1$ & $b = -1$, x can be split int x_r & x_n.

(10) According to question,

$x + 2y - z = 0$

So, $\begin{bmatrix} 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$

So, vector $\perp^{er}$ to P will be $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

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So, matrix A having P as its null space is given by,

$$A = \begin{bmatrix} 1 & 2 & -1 \end{bmatrix}$$

Now, to find the matrix that has P as its row space, we need to find the vectors that span P.

~~$x + 2y - z = 0$~~

~~So, $z = x + 2y$~~

~~For $x = 1$, $y = 1$,~~

~~$z = 3$~~

So, $P = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x + 2y - z = 0 \right\}$

$$\Rightarrow P = \left\{ \begin{bmatrix} -2y + z \\ y \\ z \end{bmatrix} : x = -2y + z \right\}$$

$$\Rightarrow P = \left\{ \begin{bmatrix} -2y + z \\ y \\ z \end{bmatrix} : y, z \in \mathbb{R} \right\}$$

$$\Rightarrow P = \left\{ y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} : y, z \in \mathbb{R} \right\}$$

So, P is spanned by $\left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\therefore$ Matrix B that has P as its row space will be,

$$B = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

(12) Let us assume,

$x - y \perp^{r}$ to $x + y$

So, $(x-y)^T (x+y) = 0$

$\Rightarrow (x^T - y^T)(x+y) = 0$

$\Rightarrow x^T x + \cancel{x^T y} - \cancel{y^T x} - y^T y = 0$

$\Rightarrow \|x\|^2 - \|y\|^2 = 0 \quad (\because x^T x = \|x\|)$

$\Rightarrow \|x\|^2 = \|y\|^2$

$\Rightarrow \|x\| = \|y\|$ (Proved)

($\because$ norms are always non-negative)

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(18) According to the Question,

Since V & W are orthogonal.

let us consider $x \in V \cap W$, so x belongs to both V & W.

So, $x^T x = 0$ ($\because$ V & W are $\perp$er)

So, $||x||^2 = 0$

$\Rightarrow ||x|| = 0$

$\Rightarrow \boxed{x = 0}$

So, V & W intersect at $x=0$.

So, $\boxed{V \cap W = \{0\}}$.

Problem Set 3.2

(1)

(a) $a = \begin{bmatrix} \sqrt{y} \\ \sqrt{x} \end{bmatrix}$, $b = \begin{bmatrix} \sqrt{x} \\ \sqrt{y} \end{bmatrix}$

According to Schwarz inequality,

$$|a^T b| \le \|a\| \|b\|$$

$$\Rightarrow 2\sqrt{xy} \le \sqrt{x+y}\,\sqrt{x+y}$$

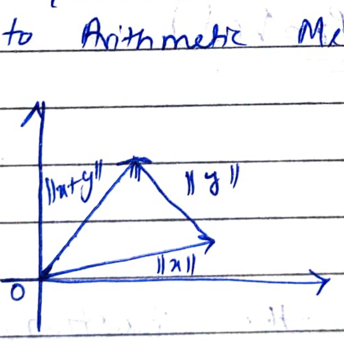
$$\Rightarrow 2\sqrt{xy} \le x+y$$

$$\Rightarrow \sqrt{xy} \le \frac{1}{2}(x+y)$$

So, GM $\le$ AM

$\therefore$ Geometric Mean is less than or equal to Arithmetic Mean.

(b)



So, Given, $\|x+y\| \le \|x\| + \|y\|$

$$\Rightarrow \|x+y\|^2 \le (\|x\| + \|y\|)^2$$

$(\because \|x\|^2 = x^T x)$

$$\Rightarrow (x+y)^T (x+y) \le x^T x + y^T y + 2\|x\|\|y\|$$

$$\Rightarrow \cancel{x^T x} + x^T y + y^T x + \cancel{y^T y} \le \cancel{x^T x} + \cancel{y^T y} + 2\|x\|\|y\|$$

$$\Rightarrow |x^T y| \le \|x\| \|y\|$$ (Schwarz Inequality)

(2) According to the Question,

$$P^2 = \left(\frac{aa^T}{a^Ta}\right)^2$$

$$\Rightarrow P^2 = \frac{aa^Taa^T}{a^Taa^Ta}$$

$$\Rightarrow P^2 = \frac{a\|a\|^2a^T}{\|a\|^2a^Ta}$$

$$\Rightarrow P^2 = \frac{aa^T}{a^Ta}$$

$$\Rightarrow \boxed{P^2 = P}$$ (Proved)

(5)

(a) ~~Acco~~ According to the Question,

$$a = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

So, $P_1 = \frac{aa^T}{a^Ta}$

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$$\Rightarrow P_1 = \frac{\begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 3 \end{bmatrix}}{\begin{bmatrix} 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}}$$

$$\Rightarrow P_1 = \frac{\begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix}}{10}$$

$$\Rightarrow P_1 = \begin{bmatrix} 1/10 & 3/10 \\ 3/10 & 9/10 \end{bmatrix}.$$

Now, the vector b ler to a will be,

$$b = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \qquad (\because \underline{b^T a = 0})$$

So, $P_2 = \dfrac{b b^T}{b^T b}$

$$\Rightarrow P_2 = \frac{\begin{bmatrix} -3 \\ 1 \end{bmatrix} \begin{bmatrix} -3 & 1 \end{bmatrix}}{\begin{bmatrix} -3 & 1 \end{bmatrix} \begin{bmatrix} -3 \\ 1 \end{bmatrix}}$$

$$\Rightarrow P_2 = \begin{bmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{bmatrix}$$

(b) $P_1 + P_2 = \begin{bmatrix} 1/10 & 3/10 \\ 3/10 & 9/10 \end{bmatrix} + \begin{bmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{bmatrix}$

$\Rightarrow P_1 + P_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$\therefore \boxed{P_1 + P_2 = I}$

Now, $P_1 P_2 = \begin{bmatrix} 1/10 & 3/10 \\ 3/10 & 9/10 \end{bmatrix} \begin{bmatrix} 9/10 & -3/10 \\ -3/10 & 1/10 \end{bmatrix}$

$\Rightarrow P_1 P_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\therefore$ $P_1 P_2$ gives a zero vector.

8. According to the question,

let $a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$

So, $P = \dfrac{aa^T}{a^Ta}$

$= \dfrac{\begin{bmatrix} a_1^2 & a_1a_2 & \cdots & a_1a_n \\ a_2a_1 & a_2^2 & \cdots & a_2a_n \\ \vdots & \vdots & & \vdots \\ a_na_1 & a_na_2 & \cdots & a_n^2 \end{bmatrix}}{a^Ta}$

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Now, Trace of $P = \frac{a_1^2}{a^Ta} + \frac{a_2^2}{a^Ta} + \cdots + \frac{a_n^2}{a^Ta}$

$$= \frac{a_1^2 + a_2^2 + \cdots + a_n^2}{a^Ta}$$

$$= \frac{\|a\|^2}{a^Ta}$$

$$= \frac{a^Ta}{a^Ta}$$

$$= 1$$

$\therefore$ Trace of $P = 1$ (Proved)

11. According to the Question,

$a = (1,1,1)$ & $b = (2,4,4)$

So, $\hat{x} = \frac{a^Tb}{a^Ta} = \frac{\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}2\\4\\4\end{bmatrix}}{\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}}$

$$= \frac{10}{3}$$

So, $10/3$ times of a is ~~close~~ closest to b.

That means, $(10/3, 10/3, 10/3)$ is the closest ~~to~~ to b.

Now, closest point to a through b will be,

$$p = \frac{b^T a}{b^T b} b$$

$$= \frac{\begin{bmatrix} 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} 2 & 4 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}} \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$= \frac{10}{36} \begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{20}{36} \\ \frac{40}{36} \\ \frac{40}{36} \end{bmatrix}$$

$$= \begin{bmatrix} 5/9 \\ 10/9 \\ 10/9 \end{bmatrix}$$

$\therefore$ $(5/9, 10/9, 10/9)$ is closest to a through b.

8

(19)

(a) According to the Question,

$$b = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \quad \& \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

So, $p = \dfrac{a^T b}{a^T a} a$

$$\Rightarrow p = \frac{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow p = \frac{5}{3} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\Rightarrow p = \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix}$$

Now, $e = b - p$

$$\Rightarrow e = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} - \begin{bmatrix} 5/3 \\ 5/3 \\ 5/3 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

Now, $e^T a = \begin{bmatrix} -2/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= -2/3 + 2/3$

$= 0$

$\therefore$ $e \perp a$ (checked)

Problem Set 3.3

(1) According to Question,

$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

Normal Equations:

$A^T A \hat{x} = A^T b$ —— (1)

So, $A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$A^T b = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now, $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \hat{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$\Rightarrow \hat{x} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \left(\because \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \right)$$

$$\Rightarrow \hat{x} = \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$$

Now, $p = A\hat{x}$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$$

$$\Rightarrow \boxed{p = \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix}}$$

Now, $e = b - p$

$$\Rightarrow e = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}$$

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Now, to check $e \perp^{er} A$,

$$e^T A = \begin{bmatrix} 2/3 & 2/3 & -2/3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$\therefore$ e is $\perp^{er}$ to A. (checked)

(2) Given,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} u \\ v \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$$

Now, $Ax - b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

$$= \begin{bmatrix} u - 1 \\ v - 3 \\ u + v - 4 \end{bmatrix}$$

So, $E^2 = \| Ax - b \|^2$

$\Rightarrow E^2 = (u-1)^2 + (v-3)^2 + (u+v-4)^2$

Now, to minimize E^2,

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$$\frac{\partial E^2}{\partial u} = 0$$

$\Rightarrow 2(u-1) + 2(u+v-4) = 0$

$\Rightarrow 4u + 2v = 10$ —— (1)

Also, $\frac{\partial E^2}{\partial v} = 0$

$\Rightarrow 2(v-3) + 2(u+v-4) = 0$

$\Rightarrow 2u + 4v = 14$ —— (2)

From (1) & (2),

$4u + 2v = 10 \Rightarrow 2u + v = 5$

$2u + 4v = 14 \Rightarrow u + 2v = 7$

So, $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ ——→ (3)

So, $A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$A^T b = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

So, $A^T A \hat{x} = A^T b$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \hat{x} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ ——→ (4)

So, $\hat{x} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\Rightarrow \hat{x} = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

$\Rightarrow \hat{x} = \frac{1}{3} \begin{bmatrix} 3 \\ 9 \end{bmatrix}$

$\Rightarrow \hat{x} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

Now, $p = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\Rightarrow p = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = b$

Here, $p = b$ because $b \in C(A)$.

So, $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (Ans)

(4) Given,

$$A_x = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 9 \end{bmatrix}$$

b

So, $C - D = 4$, $t = -1$
$C = 5$, $t = 0$
$C + D = 9$, $t = 1$

Let the best straight line be $C + Dt$.

Minimize $E^2 = (C-D-4)^2 + (C-5)^2 + (C+D-9)^2$

So, $\frac{\partial E^2}{\partial C} = 0$

$\Rightarrow 2(C-D-4) + 2(C-5) + 2(C+D-9) = 0$

$\Rightarrow \boxed{C = 6}$

Similarly, $\frac{\partial E^2}{\partial D} = 0$

$\Rightarrow -2(C-D-4) + 2(C+D-9) = 0$

$\Rightarrow \boxed{D = 5/2}$

$\therefore$ The best straight line is $6 + \frac{5}{2}t$.

So, $t = -1$,

$p_1 = 6 - \frac{5}{2} = \frac{7}{2}$

$t = 0$,

$p_2 = 6$

$t = 1$,

$p_3 = 6 + \frac{5}{2} = \frac{17}{2}$

So, proj. b onto $C(A) = \begin{bmatrix} 7/2 \\ 6 \\ 17/2 \end{bmatrix}$

(12) Given,

$a_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ & $a_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$

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So, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$

Projection Matrix,

$P = A(A^T A)^{-1} A^T$

So, $A^T A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

Now, $(A^T A)^{-1} = \frac{1}{6}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

Then, $A(A^T A)^{-1} = \frac{1}{6}\begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$

$= \frac{1}{6}\begin{bmatrix} 3 & 2 \\ 0 & 2 \\ 3 & -2 \end{bmatrix}$

Finally, $P = \frac{1}{6}\begin{bmatrix} 3 & 2 \\ 0 & 2 \\ 3 & -2 \end{bmatrix} A^T$

$\Rightarrow P = \begin{bmatrix} 5/6 & 1/3 & 1/6 \\ 1/3 & 1/3 & -1/3 \\ 1/6 & -1/3 & 5/6 \end{bmatrix}$