CSE 4131: ALGORITHM DESIGN 2

Assignment 2:

Submission due date: 10/04/2023

- Assignment scores/markings depend on neatness and clarity.
- Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily. You should ALWAYS prove the correctness of your algorithms either directly or by referring to a proof in the book.
- The marking would be out of 100
- You are allowed to use only those concepts which are covered in the lecture class till date.
- Plagiarized assignments will be given a zero mark.

CO 2: Distinguish between computationally tractable and intractable problems, define and relate class P, NP, NPH, NPC, PSPACE and PSPACE complete and extension limit of Tractibility.

Sl. No.	Questions	PO	level
1.	a. Define the class NPH (NP-Hard). How does it differ from NPC? b. Provide examples of NP-Hard problems that are not necessarily in NP. Explain why they are classified as NP-Hard. c. Discuss the relationship between NP-Hard problems and computational hardness. How are they relevant in practical computing scenarios?	PO2,PO4	L1, L2
2.	a. Define the class NPC (NP-Complete). What are the defining properties of an NP-Complete problem? b. Provide examples of well-known NP-Complete problems and explain why they are considered as such. c. Explain the concept of polynomial-time reduction. How is it used to prove a problem is NP-Complete?	PO2,PO4	L1, L2
3.	Given a decision vertex-cover problem instances $G(V, E, k)$, where $V = \{1,2,3,4,5,6,7\}$ and $E = \{(1,2), (1,3), (2,3), (1,4), (2,5), (3,6), (4,6), (4,5), (5,7), (6,7)\}$, $k = 3$. Find the decision set-cover problem instances (i.e., V, subsets of V and k). And give a possible solution.	PO2,PO4	L5
4.	A new telecom company is trying to establish its offices in a large city. The company wants to open offices at various (traffic) junctions of the road to cover maximum percentage of consumers. The local govt. authority has put a restriction that the company cannot open offices at adjacent junctions and has to leave at least two junctions gap between two offices. Now, our goal is to find the maximum number of offices that can be opened by the company in the given city. a. Define the decision version of the problem b. Show that, the given problem is a NP-Complete problem. (Hint: Show the reduction of a NP complete problem to the given problem and justify the hardness).	PO2,PO4	L2,L4
5.	Draw a graph $G(V, E)$ from the given information where V is the set of all vertices, $E = \{(v_1, v_2), (v_1, v_9), (v_2, v_3), (v_2, v_8), (v_3, v_5), (v_4, v_5), (v_5, v_6), (v_6, v_7), (v_7, v_8), (v_7, v_9)\}$ set of all edges. a. Can a Vertex Cover of size 3 possible for the given graph? Justify your answer. b. Can a Vertex Cover of size 4 possible for the given graph? Justify your answer. c. Why can't an undirected Complete Graph with 8 vertices have a vertex cover of size 3?	PO2,PO4	L2,L4

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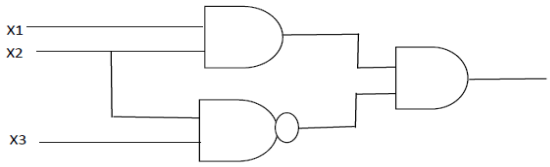
<tr><td>6.</td><td>Given a Boolean circuit as follows
 

Convert the following circuit to CNF (Reduction of Circuit Satisfiability Problem to CNF SAT)</td><td>PO2,PO4</td><td>L5</td></tr>
<tr><td>7.</td><td>Considering the context of planning problems,
<table><tr><td>1</td><td>2</td><td>3</td></tr><tr><td></td><td>4</td><td>6</td></tr><tr><td>7</td><td>5</td><td>8</td></tr></table>(Initial State)
<table><tr><td>1</td><td>2</td><td>3</td></tr><tr><td>4</td><td>5</td><td>6</td></tr><tr><td>7</td><td>8</td><td></td></tr></table>(Goal State)
a. Represent the Initial Configuration of the given instance of 8-puzzle game as a set of conditions.
b. Show the sequence configurations with feasible operations that can reach the Goal Configuration from the Initial Configuration.
c. Why are planning problems like the 15-puzzle game said to be in PSPACE?</td><td>PO2,PO4</td><td>L4,L5</td></tr>
<tr><td>8.</td><td>a. Write the 3-SAT expression for the given QSAT expression “$\exists x_1 \forall x_2 \exists x_3 \forall x_4 \exists x_5\ \varphi(x) = (x_1 \vee \neg x_3 \vee x_4) \wedge (\neg x_2 \vee x_3 \vee \neg x_5) \wedge (\neg x_1 \vee x_3 \vee \neg x_2)$” using appropriate quantifiers.
b. Is there a satisfying assignment for the given Q-SAT “$\exists x_1 \forall x_2 \exists x_3\ \varphi(x) = (x_1 \vee \neg x_3 \vee x_2) \wedge (\neg x_2 \vee x_3 \vee \neg x_1) \wedge (\neg x_1 \vee \neg x_3 \vee \neg x_2)$.
c. Why Q-SAT $\in$ PSPACE?</td><td>PO2,PO4</td><td>L5</td></tr>
<tr><td>9.</td><td>An Eulerian cycle is a tour that visits every edge in a graph exactly once. An Eulerian subgraph is a subset of the edges and vertices of a graph that has an Eulerian cycle. Prove that the problem of finding the number of edges in the largest Eulerian subgraph of a graph is NP-hard.</td><td>PO2,PO4</td><td>L4</td></tr>
<tr><td>10.</td><td>a. Reduce a 3-SAT formula to an independent set problem. (Explain with an example).
b. Suppose, the 3-SAT problem has k clauses, then it is satisfiable if and only if the corresponding graph has an independent set of size k. Now, give an example to show that 3-SAT is not satisfiable if and only if the corresponding graph does not have any independent set of size k.</td><td>PO2,PO4</td><td>L2,L5</td></tr>
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