Assignment -2

Problem Set 2.1

(1)

(a) Let the subset be,

$S = \{(x,0) : x \in Z\}$ in xy-plane.

Here, in addition & subtraction,
let two points be, $(x,0)$ & $(y,0)$,
So, $(x,0) \pm (y,0) = (x \pm y, 0) \in S$

Hence subset S is closed in addition & subtraction.

Now, for α = any irrational no.,

$\alpha(x,0)$
$= (\alpha x, 0)$, here $\alpha x \notin Z$.

So, it is not closed for scalar multiplication.

(b) Let,
$S = \{(x,0), (0,y) : x, y \in R\}$

So, for any $\alpha \in R$, it is closed under scalar multiplication, but under addition,

$(x,0) + (0,y) = (x,y) \notin S$

So, it is not closed under vector addition.

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(4) According to the question

(a) Let $a = (1,0,1,0,\dots) \in W$
$b = (0,1,0,1,\dots) \in W$

So, $a+b = (1,1,1,1,\dots) \notin W$

So, W isn't a subspace of V.

(b) Let $a = (x_1, x_2, 0,0,0\dots) \in W$
$b = (x_1, x_2, x_3, 0,0,\dots) \in W$

So, $\alpha a = (\alpha x_1, \alpha x_2, 0,0,0\dots) \in W$

$a+b = (2x_1, 2x_2, x_3, 0,0,0,\dots) \in W$

So, W is a subspace of V.

(c) Let,

$a = (3, 2, 1, 0, -1, -2, \dots) \in W$
$b = (4, 3, 2, 1, 0, -1, -2, \dots) \in W$

Now,

$-\alpha a = (-\alpha 3, -\alpha 2, -\alpha, 0, \alpha, 2\alpha, \dots) \notin W$

So, it is not a subspace of V.

(d) let x be a member of this set, such that

$l(x) = \lim_{j \to \infty} x_j$ exists, for $\epsilon > 0$ there

exist $|l(x) - x_j| < \epsilon$.

Now for constant α,

if $\alpha = 0$,

$\alpha x = (0, 0, \ldots)$, converges to limit 0.

else if $\alpha \neq 0$,

$l(cx) = \lim_{j \to \infty} c x_j$ exists and ~~converges~~

converges to $c\,l(x)$. Hence it is closed under scalar multiplication.

Now for addition,

$$l(x+y) = \lim_{j\to\infty} x_j + y_j$$ exists, and

converges to $l(x) + l(y)$.

So, it is also closed under vector addition.

Hence, it is a subspace of V.

(e) Let,

$$x = (x_1, x_2, x_3, \ldots) \in W$$
$$y = (y_1, y_2, y_3, \ldots) \in W$$

$\alpha x = (\alpha x_1, \alpha x_2, \alpha x_3, \ldots) \in W$ as it is also in A.P.

Now,

$x + y = (x_1 + y_1, x_2 + y_2, \ldots)$, we know adding two APs also gives an AP.

So, $x+y \in W$.

Hence, W is a subspace of V.

(f) let,

$x = (1, 2, 4, 8..)$
for, $x_1 = 1$ & $k = 2$

$y = (1, 3, 9, 27...)$ for $x_1 = 1$, $k = 3$.

Now, $x + y = (2, 5, 13, 35, ...)$

As per GP, third element ~~th~~ should be

$(2.5)^2 \times 2 = 12.5$ instead of 13.

Hence, $x + y \notin W$.

So, it is not a subspace of V.

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(a) Here rules 7 and 8 are broken.

(b) Here all eight rules are satisfied so it is a vector space under the vector addition and scalar multiplication operations defined above, with 1 as the zero vector.

(c) Here, 1 and 2 rules are not satisfied.

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(8) Here,

$$A x = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

is in the form of,

$$Ax = 0,$$

So, it forms,

(b) a line
(d) a subspace
(e) a nullspace

(24)

(a)
$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & b_1 \\ 2 & 8 & 4 & b_2 \\ -1 & -4 & -2 & b_3 \end{array}\right]$$

$NR_2 \rightarrow R_2 - 2R_1$

$NR_3 \rightarrow R_3 + R_1$

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & b_1 \\ 0 & 0 & 0 & b_2 - 2b_1 \\ 0 & 0 & 0 & b_3 + b_1 \end{array}\right]$$

The given ~~given~~ equation is solvable only if,

$$b_2 - 2b_1 = 0 \quad \text{and} \quad b_3 + b_1 = 0$$

in other words,

$$b \text{ vector} = \begin{bmatrix} b_1 \\ 2b_1 \\ -b_1 \end{bmatrix} = b_1 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

on right hand side.

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$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So, $x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

Here, $z = 0$,

$x + 2y = b_1$
$b_2 = 0$
$b_3 = 0$

So, $C(A) = x$-axis.

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}$$

So, $x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

So, $x = b_1$

$2y = b_2$

$b_3 = 0$

So, it lies on x-y plane. $(z=0)$

$$C = \begin{bmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$x\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} + y\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

So, $x = b_1$

$2x = b_2$

$0 = b_3$

So, it lies on x-y plane. $(z=0)$

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$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Here, Solution exist for all values of b_1, b_2 & b_3.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Here, solution exists only if $b_3 = 0$.