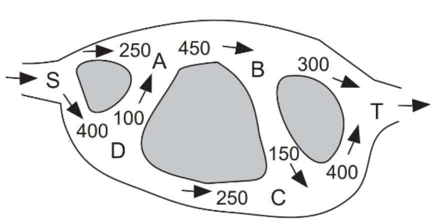
CSE 4131: ALGORITHM DESIGN 2

Assignment 1:

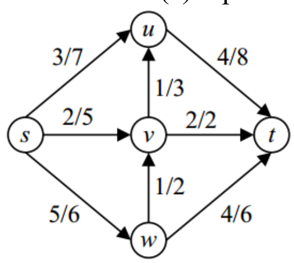
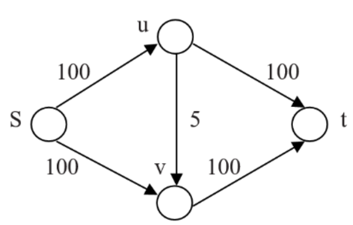
Submission due date: 15/03/2023

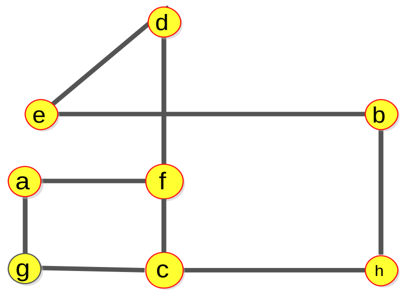
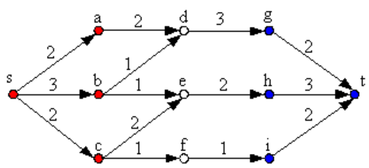
- Assignment scores/markings depend on neatness and clarity.
- Write your answers with enough detail about your approach and concepts used, so that the grader will be able to understand it easily. You should ALWAYS prove the correctness of your algorithms either directly or by referring to a proof in the book.
- The marking would be out of 100
- You are allowed to use only those concepts which are covered in the lecture class till date.
- Plagiarized assignments will be given a zero mark.

CO 1: to understand the Network flow problem and apply it to solve different real-world problems.

Sl. No.	Questions	PO	level
1.	Consider the maximum flow in a given network between two designated nodes s and t. For each of the following statements, either explain why it is true or provide a counterexample. a. If the capacity of every edge is even, then the value of the maximum flow must be even. b. If the capacity of every edge is even, then there is a maximum flow in which the flow on each edge is even. c. If the capacity of every edge is odd, then the value of the maximum flow must be odd. d. If the capacity of every edge is odd, then there is a maximum flow in which the flow on each edge is odd.	PO1,PO2, PO3	L1, L4
2.	Suppose you are given a directed graph G = (V, E), with a positive integer capacity C_e on each edge e, a designated source s ∈ V, and a designated sink t ∈ V. You are also given an integer maximum s-t flow in G, defined by a flow value f_e on each edge e. Now suppose we pick a specific edge e ∈ E and increase its capacity by one unit. Show how to find a maximum flow in the resulting capacitated graph in time $O(m + n)$, where m is the number of edges in G and n is the number of nodes.	PO1,PO2, PO3	L1, L4
3.	This diagram represents a road network. All vehicles enter at S and leave at T. The numbers represent the maximum flow rate in vehicles per hour in the direction from S to T. a. What is the maximum number of vehicles which can enter and leave the network every hour? b. Which single section of road could be improved to increase the traffic flow in the network?  	PO1,PO2, PO3	L5
4.	The Jungle Junket food truck produces a large variety of different lunch menu items. Unfortunately, they can only produce their foods in limited quantities, so they often run	PO1,PO2, PO3	L3,L4

	out of popular items, making customers sad. To minimize sadness, Jungle Junket is implementing a sophisticated lunch-ordering system. Customers text in their acceptable choices before lunch time. Then they can use an algorithm to preassign lunches to customers. Customers who do not get one of their choices should receive a Rs100 voucher. Jungle Junket would like to minimize the number of vouchers they give out. Give an efficient algorithm for Jungle Junket to assign lunches to customers. In general, suppose that, on a given day, Jungle Junket has produced *m* types of food items $b_1, \ldots,$ b_m, and the quantity of each type of food item b_j is exactly q_j . Suppose that *n* customers $a_1, \ldots, a_n$ text in their preferences, where each customer ai submits a set A_i of one or more acceptable lunch choices. The algorithm should assign each customer either one of his/her choices or a Rs100 voucher. It should minimize the number of vouchers. (Hint: Model this as a max flow problem.)		
5.	In this problem, you will design an algorithm that takes the following inputs: • A flow network F = (G, c), where G = (V, E) is a graph with source vertex s and target vertex t, and c is a capacity function mapping each directed edge of G to a nonnegative integer; • A maximum flow f for F ; and • A triple (u, v, r), where u and v are vertices of G and r is a nonnegative integer = c(u, v). The algorithm should produce a maximum flow for flow network F = (G, c), where c is identical to c except that c(u, v) = r. The algorithm should run in time $O(k \cdot (V + E))$, where $\lvert c(u,v) - r \rvert = k$. The algorithm should behave differently depending on whether $r > c(u,v)$ *or* $r < c(u,v)$. (a) Start by proving the following basic, general results about flow networks: 1. Increasing the capacity of a single edge (*u, v*) by 1 can result in an increase of at most 1 in the max flow. 2. Increasing the capacity of a single edge (*u, v*) by a positive integer *k* can result in an increase of at most *k* in the max flow. 3. Decreasing the capacity of a single edge (*u, v*) by 1 can result in a decrease of at most 1 in the max flow. 4. Decreasing the capacity of a single edge (*u, v*) by a positive integer *k* can result in a decrease of at most *k* in the max flow. (b) Suppose that $r > c(u,v)$. Describe your algorithm for this case in detail, prove that it works correctly, and analyze its time complexity (in terms of V , E, and k). (c) Suppose that $r < c(u,v)$. Describe your algorithm for this case in detail, prove that it works correctly, and analyze its time complexity.	PO1,PO2, PO3	L1,L4
6.	Consider the following network flow $G(V,E)$, where $V = \{s,a,b,c,d,t\}$ and capacities of the edges in E are $c(s,a) = 13$, $c(s,b) = 17, c(a,c) = 18$, $c(b,a) = 12$, $c(c,t) = 19$, $c(b,d) = 8$, $c(d,t) = 14$, $c(b,c) = 5$, $c(c,d) = 12$. Where s and t are the source and sink node respectively. a. Draw the above flow network. b. Show each residual graph. c. Compute the maximum flow of the above graph.	PO1,PO2, PO3	L5
7.	You are helping the medical consulting firm Doctors Without Weekends set up the work schedules of doctors in a large hospital. They've got the regular daily schedules mainly worked out. Now, however, they need to deal with all the special cases and, in particular, make sure that they have at least one doctor covering each vacation day. Here's how this works. There are k vacation periods (e.g., the week of Christmas, the July 4th weekend, the Thanksgiving weekend, . . .), each spanning several contiguous	PO1,PO2, PO3	L3,L4

	days. Let Dj be the set of days included in the jth vacation period; we will refer to the union of all these days, ∪jDj, as the set of all vacation days. There are n doctors at the hospital, and doctor i has a set of vacation days Si when he or she is available to work. (This may include certain days from a given vacation period but not others; so, for example, a doctor may be able to work the Friday, Saturday, or Sunday of Thanksgiving weekend, but not the Thursday.) Give a polynomial-time algorithm that takes this information and deter- mines whether it is possible to select a single doctor to work on each vacation day, subject to the following constraints. ● For a given parameter c, each doctor should be assigned to work at most c vacation days total, and only days when he or she is available. ● For each vacation period j, each doctor should be assigned to work at most one of the days in the set Dj. (In other words, although a particular doctor may work on several vacation days over the course of a year, he or she should not be assigned to work two or more days of the Thanksgiving weekend, or two or more days of the July 4th weekend, etc.) The algorithm should either return an assignment of doctors satisfying these constraints or report (correctly) that no such assignment exists.		
8.	A graph has a unique minimum cut if there is only one cut that whose weight is the minimum. Design an algorithm that finds if a graph has a unique minimum cut.	PO1,PO2, PO3	L1,L4
9.	a. Draw the residual graph for the given graph, in each edge f (e)/c(e), f (e) represents the flow and c(e) represents the capacity in that edge.  b. Given a graph G, determine the maximum and minimum number of augmentations possible for the Ford-Fulkerson algorithm to compute the maximum flow from a source vertex s to a sink vertex t (path may repeat)  	PO1,PO2, PO3	L5
10.	ITER, SOA requires an algorithm to schedule mid-semester exams for their courses each semester. There are n courses offered, r available rooms, and m time slots for exams. Given arrays $E[1...n]$ and $S[1...r]$, $E[i]$ represents the number of students enrolled in $i-th$ course and $S[j]$ represents the number of seats in each room respectively, an exam for course i can only be scheduled in room j. If the number of enrolled students for that course is less than or equal to the number of seats available in the room ($E[i] \leq S[j]$). It's assumed that no two courses have overlapping enrollments. The goal is to develop an efficient algorithm for ITER to assign a room and a time slot to each course, or report if no such assignment is possible.	PO1,PO2, PO3	L1,L4
11.	a) You have given a graph G with 8 vertices and 9 edges. Check the graph G is bipartite or not? If yes, then find the maximum matching of the graph $G(V, E)$ and if no then explain why? b) Define the definition of perfect matching in a graph with a suitable example? If the graph G is a bipartite graph and has a maximum matching, then justify that the matching formed in G is perfect matching?	PO1,PO2, PO3	L5

	c) c) Show that $\|f\|=\|M\|$, if possible of the above example, where f and M denotes the maximum flow in flow networks and maximum matching in bipartite graph. 		
12.	a) Run the Ford Fulkerson to find the Maximum flow. Show each residual graph. b) Show the minimum-cut and find the capacity of minimum cuts. c) Justify it with Ford-Fulkerson algorithm (time complexity). 	PO1,PO2, PO3	L5
13.	Consider the following network flow $G(V,E)$, where $V=\{s,u,v,y,z,w,t\}$ and capacities of the edges in E are $c(s,u)=18$, $c(u,v)=12, c(v,t)=6$, $c(s,y)=13$, $c(y,z)=6$, $c(z,t)=10$, $c(u,w)=11$, $c(w,v)=3$, $c(y,w)=4$, $c(w,z)=5$, Where s and t are the source and sink node respectively. a. Draw the above flow network. b. Show each residual graph. c. Compute the maximum flow of the above graph. d. If set $A=\{s,u,y,w\}$ and $B=\{v,z,t\}$, then what will be the capacity of the cut set (A,B).	PO1,PO2, PO3	L5