Algorithms

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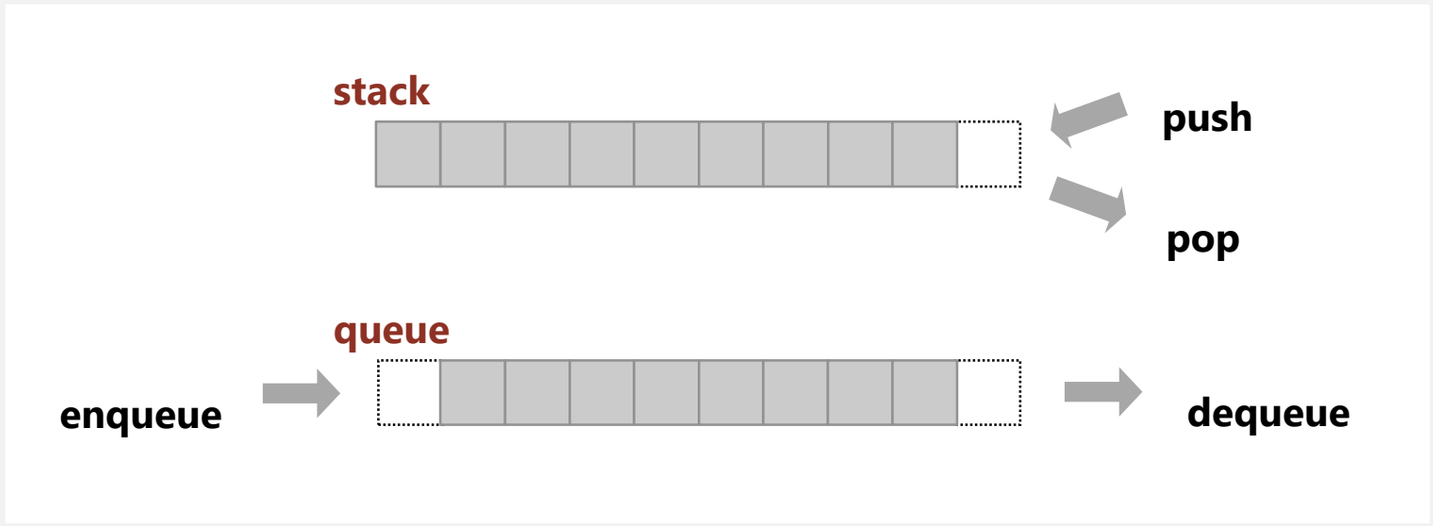
1.3 Bags, Queues, and Stacks

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators*
- *applications*

Stacks and queues

Fundamental data types.

- Value: collection of objects.
- Operations: insert, remove, iterate, test if empty.
- Intent is clear when we insert.
- Which item do we remove?



Stack. Examine the item most recently added. ← LIFO = "last in first out"

Queue. Examine the item least recently added. ← FIFO = "first in first out"

Client, implementation, interface

Separate interface and implementation.

Ex: stack, queue, bag, priority queue, symbol table, union-find, ...

Benefits.

- Client can't know details of implementation $\Rightarrow$
  client has many implementation from which to choose.
- Implementation can't know details of client needs $\Rightarrow$
  many clients can re-use the same implementation.
- Design: creates modular, reusable libraries.
- Performance: use optimized implementation where it matters.

Client: program using operations defined in interface.

Implementation: actual code implementing operations.

Interface: description of data type, basic operations.

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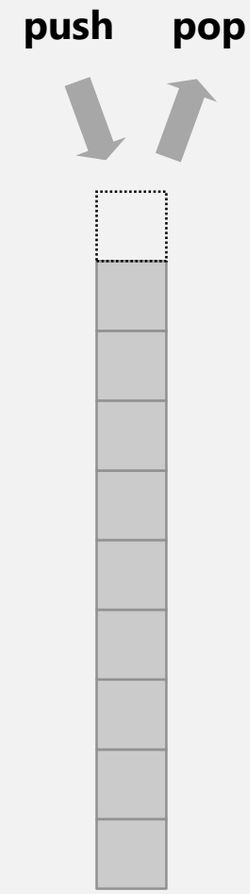
Stack API

Warmup API. Stack of strings data type.

public class StackOfStrings	
StackOfStrings()	*create an empty stack*
void push(String item)	*insert a new string onto stack*
String pop()	*remove and return the string most recently added*
boolean isEmpty()	*is the stack empty?*
int size()	*number of strings on the stack*

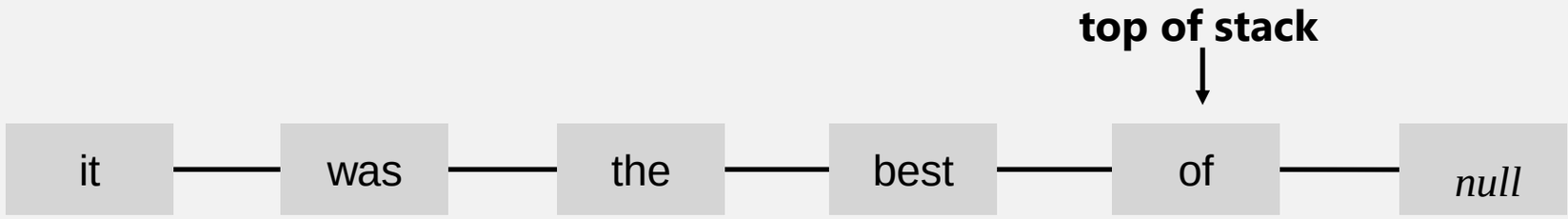


Warmup client. Reverse sequence of strings from standard input.

How to implement a stack with a linked list?

A. Can't be done efficiently with a singly-linked list.

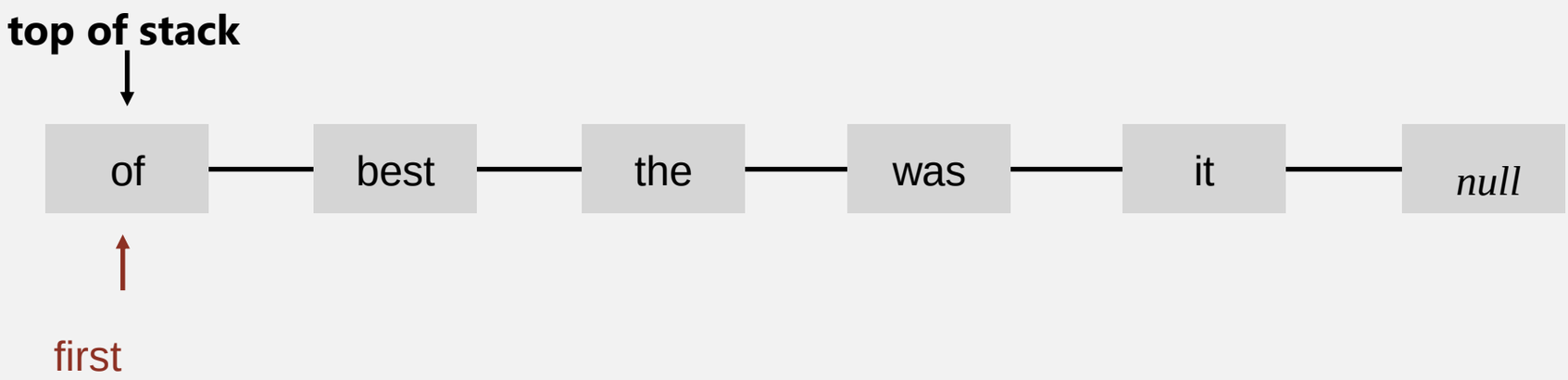
B.



C.

Stack: linked-list implementation

- Maintain pointer first to first node in a singly-linked list.
- Push new item before first.
- Pop item from first.



Stack pop: linked-list implementation

inner class

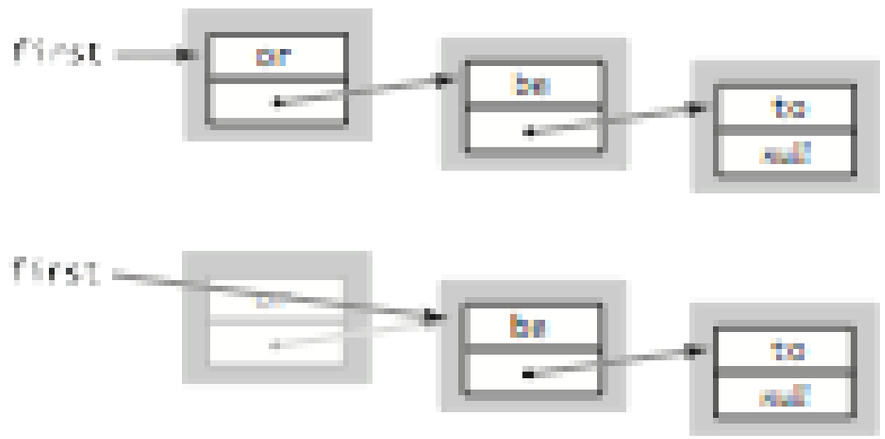
```
private class Node
{
   String item;
   Node next;
}
```

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



return saved item

```
return item;
```

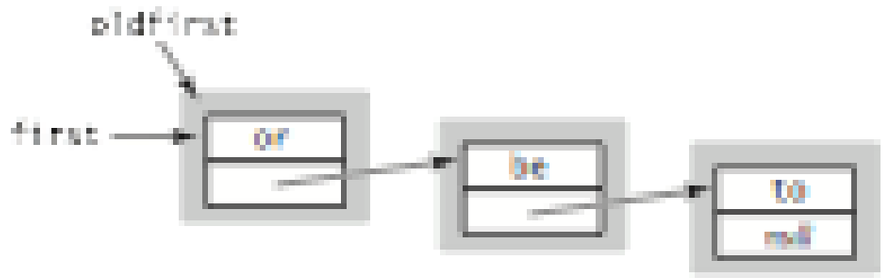
Stack push: linked-list implementation

inner class

```
private class Node
{
   String item;
   Node next;
}
```

save a link to the list

```
Node oldfirst = first;
```

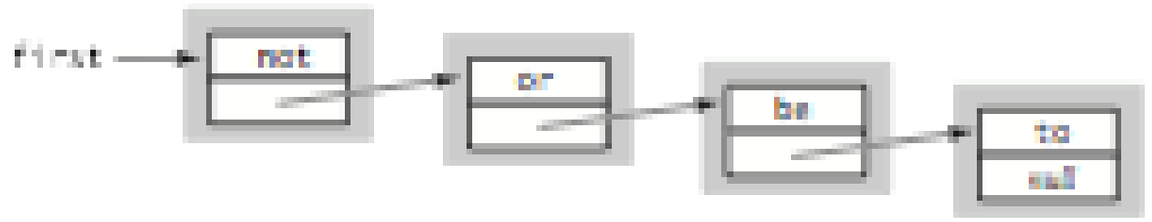


create a new node for the beginning

```
first = new Node();
```

set the instance variables in the new node

```
first.item = "not";
first.next = oldfirst;
```



Stack: linked-list implementation in Java

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
```

private inner class

(access modifiers for instance variables don't matter)

Stack: linked-list implementation performance

Proposition. Every operation takes constant time in the worst case.

Proposition. A stack with N items uses $\sim 40\,N$ bytes.

inner class

```
private class Node
{
   String item;
   Node next;
}
```

16 bytes (object overhead)

8 bytes (inner class extra overhead)

8 bytes (reference to String)

8 bytes (reference to Node)

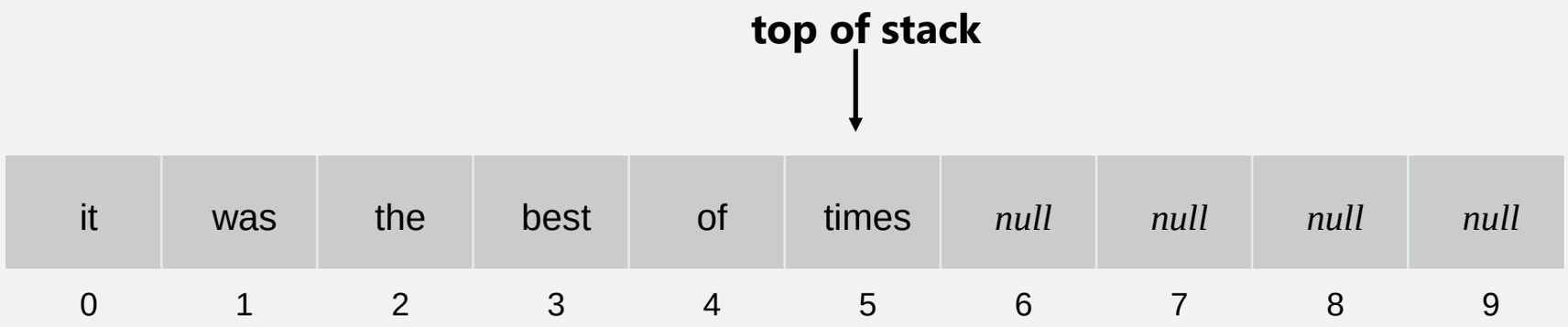
40 bytes per stack node

Remark. This accounts for the memory for the stack (but not the memory for strings themselves, which the client owns).

How to implement a fixed-capacity stack with an array?

A. Can't be done efficiently with an array.

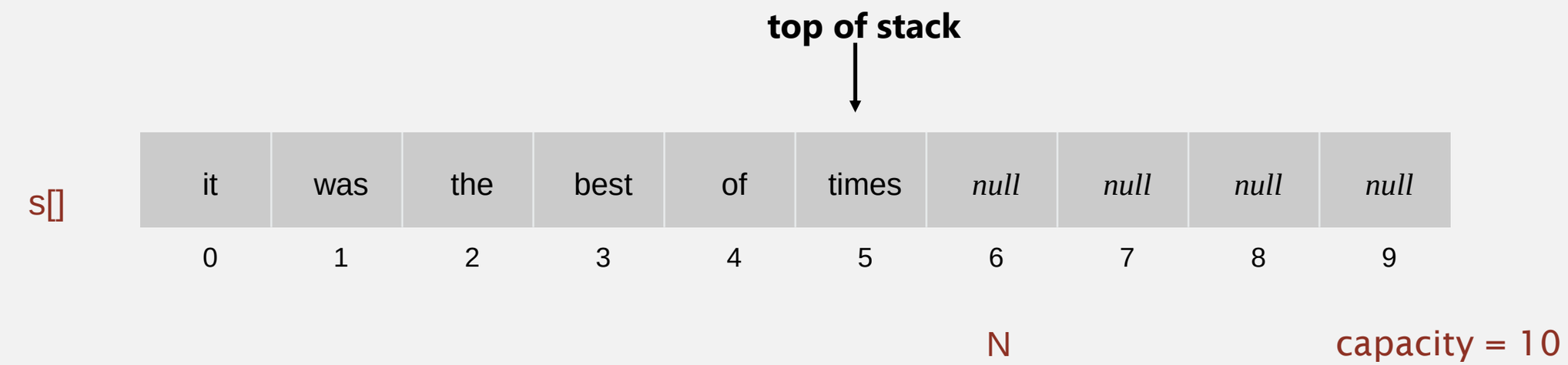
B.



C.

Fixed-capacity stack: array implementation

- Use array s[] to store N items on stack.
- push(): add new item at s[N].
- pop(): remove item from s[N-1].



Defect. Stack overflows when N exceeds capacity. [stay tuned]

Fixed-capacity stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public FixedCapacityStackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

a cheat
(stay tuned)

use to index into array;
then increment N

decrement N;
then use to index into array

Stack considerations

Overflow and underflow.

- Underflow: throw exception if pop from an empty stack.
- Overflow: use resizing array for array implementation. [stay tuned]

Null items. We allow null items to be inserted.

Loitering. Holding a reference to an object when it is no longer needed.

```
public String pop()
{  return s[--N];  }
```

loitering

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   return item;
}
```

this version avoids "loitering":

garbage collector can reclaim memory for an object only if no outstanding references

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Stack: resizing-array implementation

Problem. Requiring client to provide capacity does not implement API!

Q. How to grow and shrink array?

First try.

- push(): increase size of array s[] by 1.
- pop(): decrease size of array s[] by 1.

Too expensive.

- Need to copy all items to a new array, for each operation.
- Array accesses to insert first N items = $N + (2 + 4 + \ldots + 2(N-1)) \sim N^2$.

infeasible for large N

1 array access per push

2(k-1) array accesses to expand to size k (ignoring cost to create new array)

Challenge. Ensure that array resizing happens infrequently.

Stack: resizing-array implementation

Q. How to grow array?

A. If array is full, create a new array of twice the size, and copy items.

"repeated doubling"

```
public ResizingArrayStackOfStrings()
{  s = new String[1];  }

public void push(String item)
{
   if (N == s.length) resize(2 * s.length);
   s[N++] = item;
}

private void resize(int capacity)
{
   String[] copy = new String[capacity];
   for (int i = 0; i < N; i++)
      copy[i] = s[i];
   s = copy;
}
```

Arra

$+ 8 + \ldots + N) \sim 3N.$

1 array access per push

k array accesses to double to size k (ignoring cost to create new array)

Stack: resizing-array implementation

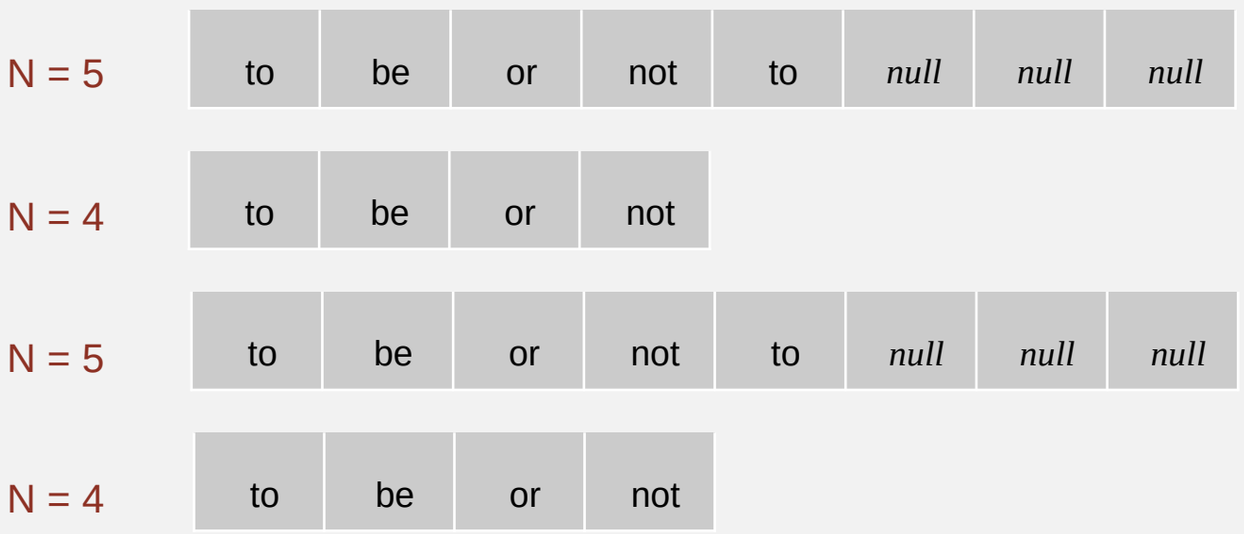
Q. How to shrink array?

First try.

- push(): double size of array s[] when array is full.
- pop(): halve size of array s[] when array is one-half full.

Too expensive in worst case.

- Consider push-pop-push-pop-... sequence when array is full.
- Each operation takes time proportional to N.



Stack: resizing-array implementation

Q. How to shrink array?

Efficient solution.

- push(): double size of array s[] when array is full.
- pop(): halve size of array s[] when array is one-quarter full.

```
public String pop()
{
   String item = s[--N];
   s[N] = null;
   if (N > 0 && N == s.length/4) resize(s.length/2);
   return item;
}
```

Invariant. Array is between 25% and 100% full.

Stack resizing-array implementation: performance

Amortized analysis. Starting from an empty data structure, average running time per operation over a worst-case sequence of operations.

Proposition. Starting from an empty stack, any sequence of M push and pop operations takes time proportional to M.

	best	worst	amortized
construct	1	1	1
push	1	N	1
pop	1	N	1
size	1	1	1

doubling and halving operations

order of growth of running time for resizing stack with N items

Stack resizing-array implementation: memory usage

Proposition. Uses between $\sim 8\,N$ and $\sim 32\,N$ bytes to represent a stack with N items.

- $\sim 8\,N$ when full.
- $\sim 32\,N$ when one-quarter full.

```
public class ResizingArrayStackOfStrings
{
   private String[] s;
   private int N = 0;
   …
}
```

← 8 bytes × array size

Remark. This accounts for the memory for the stack (but not the memory for strings themselves, which the client owns).

Stack implementations: resizing array vs. linked list

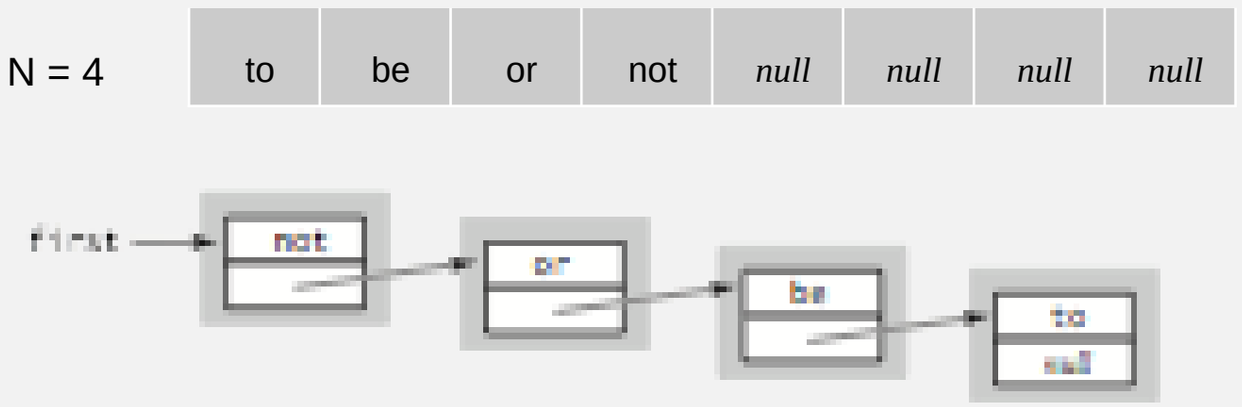
Tradeoffs. Can implement a stack with either resizing array or linked list; client can use interchangeably. Which one is better?

Linked-list implementation.

- Every operation takes constant time in the worst case.
- Uses extra time and space to deal with the links.

Resizing-array implementation.

- Every operation takes constant amortized time.
- Less wasted space.



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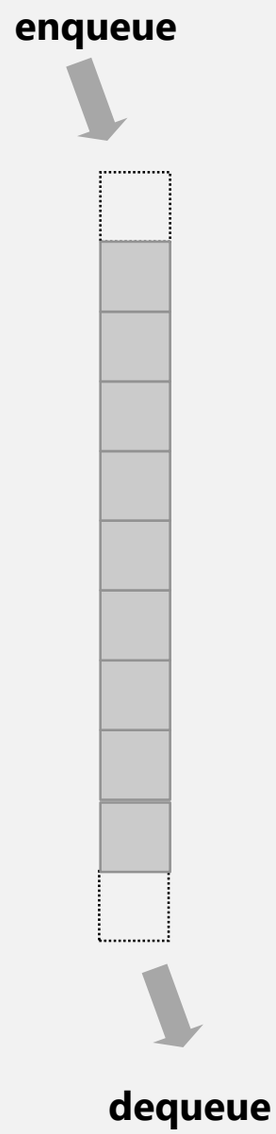
Queue API

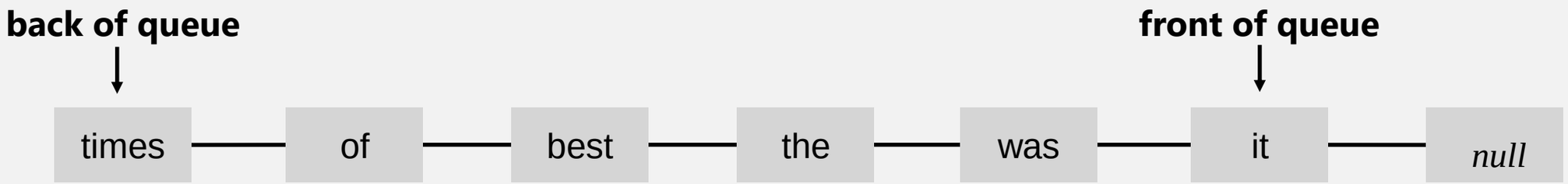
public class QueueOfStrings	
QueueOfStrings()	*create an empty queue*
void enqueue(String item)	*insert a new string onto queue*
String dequeue()	*remove and return the string least recently added*
boolean isEmpty()	*is the queue empty?*
int size()	*number of strings on the queue*



How to implement a queue with a linked list?

A. Can't be done efficiently with a singly-linked list.

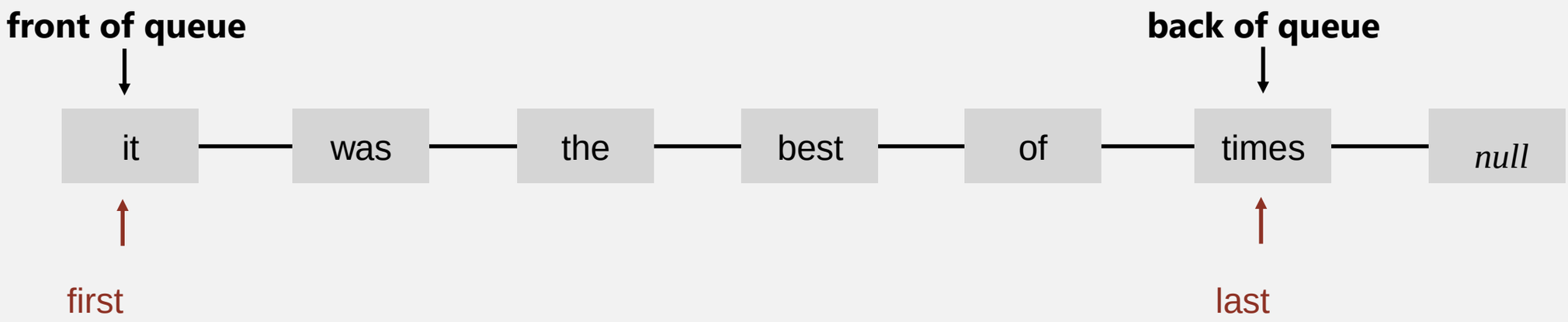
B.



C.

Queue: linked-list implementation

- Maintain one pointer first to first node in a singly-linked list.
- Maintain another pointer last to last node.
- Dequeue from first.
- Enqueue after last.



Queue dequeue: linked-list implementation

inner class

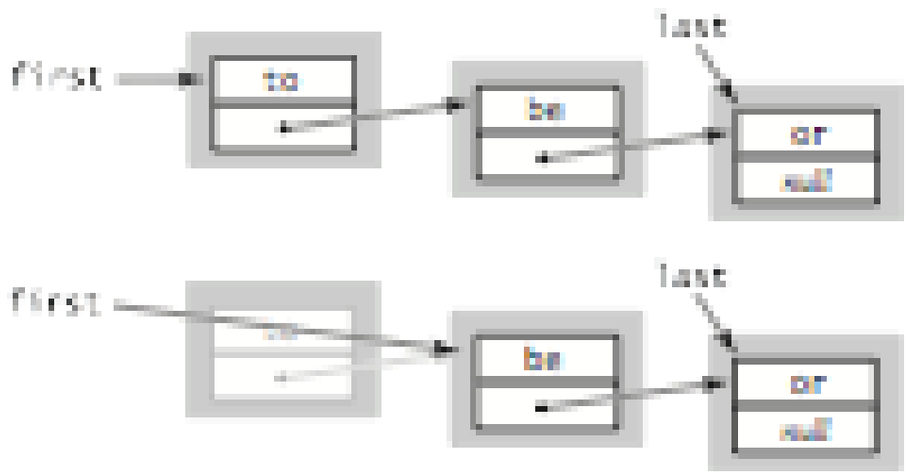
```
private class Node
{
   String item;
   Node next;
}
```

save item to return

```
String item = first.item;
```

delete first node

```
first = first.next;
```



return saved item

```
return item;
```

Remark. Identical code to linked-list stack pop().

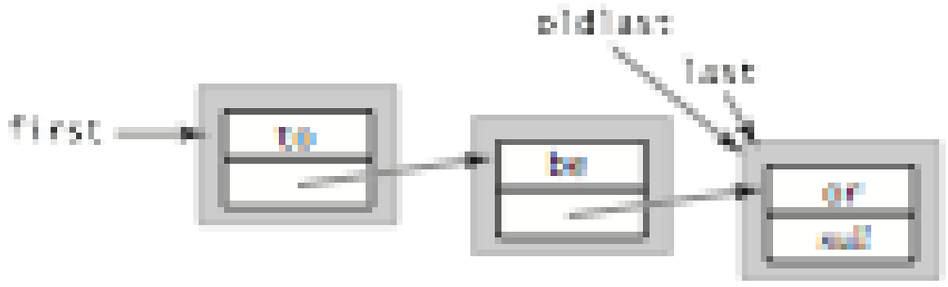
Queue enqueue: linked-list implementation

inner class

```
private class Node
{
   String item;
   Node next;
}
```

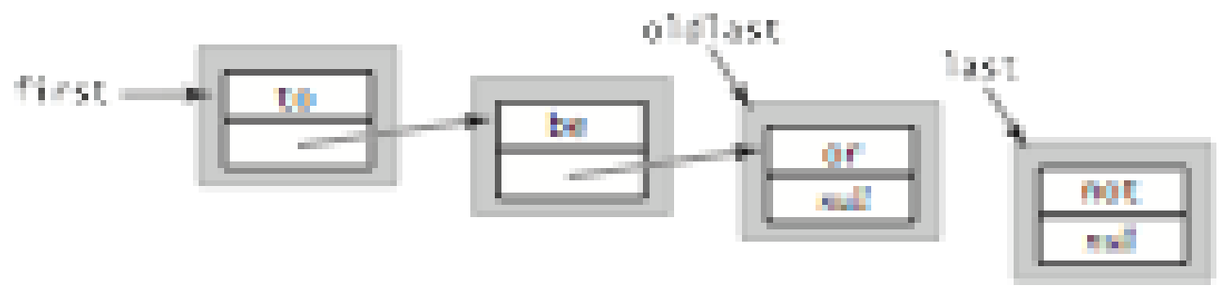
save a link to the last node

```
Node oldlast = last;
```



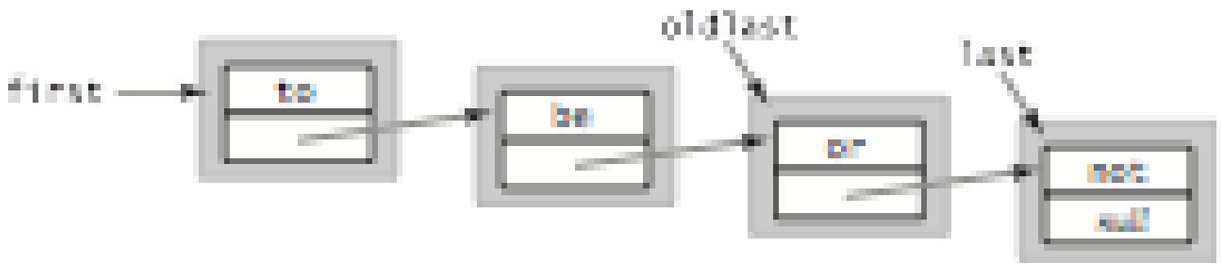
create a new node for the end

```
last = new Node();
last.item = "not";
```



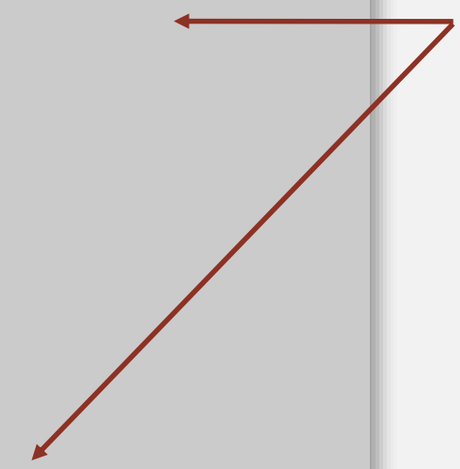
link the new node to the end of the list

```
oldlast.next = last;
```



Queue: linked-list implementation in Java

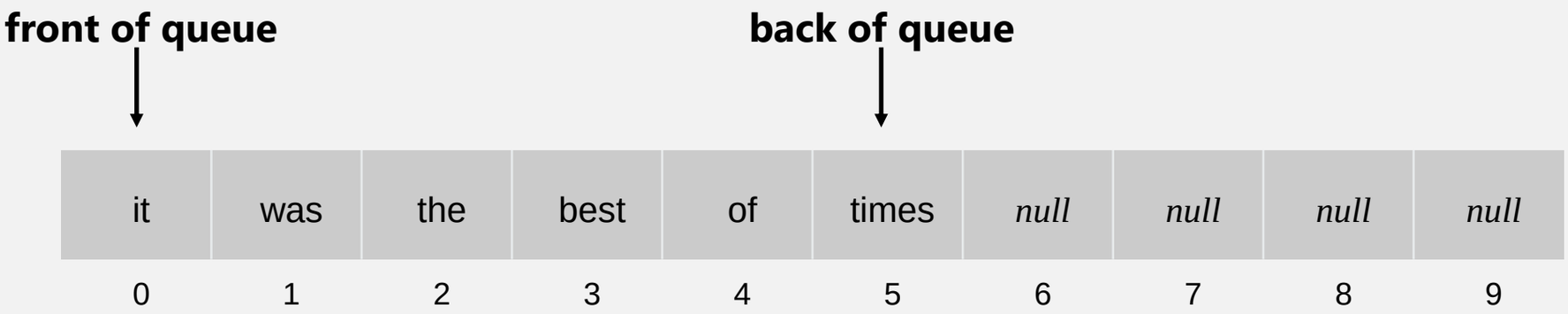
```
public class LinkedQueueOfStrings
{
   private Node first, last;

   private class Node
   {  /* same as in LinkedStackOfStrings */  }

   public boolean isEmpty()
   {  return first == null;  }

   public void enqueue(String item)
   {
      Node oldlast = last;
      last = new Node();
      last.item = item;
      last.next = null;
      if (isEmpty()) first = last;
      else           oldlast.next = last;
   }

   public String dequeue()
   {
      String item = first.item;
      first       = first.next;
      if (isEmpty()) last = null;
      return item;
```



How to implement a fixed-capacity queue with an array?

A. Can't be done efficiently with an array.

B.



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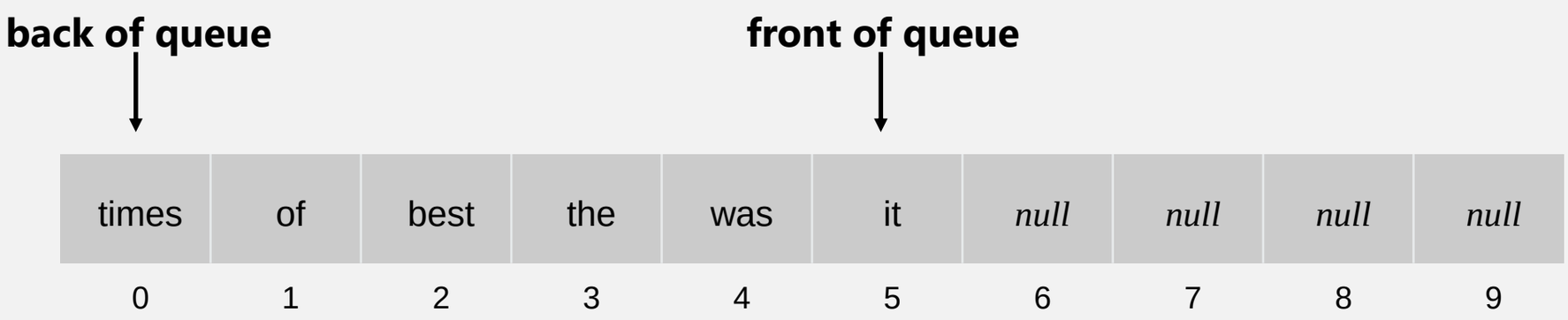
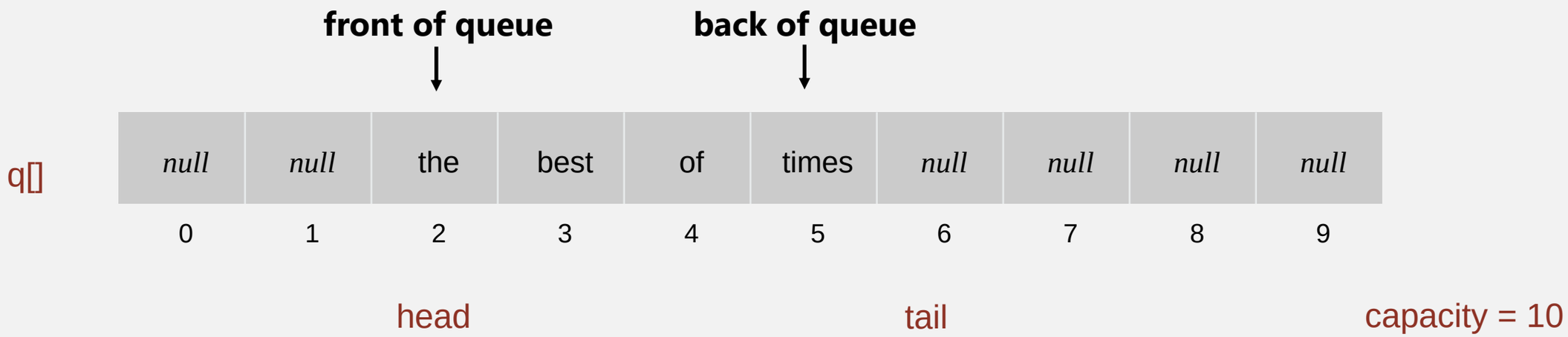


Queue: resizing-array implementation

- Use array q[] to store items in queue.
- enqueue(): add new item at q[tail].
- dequeue(): remove item from q[head].
- Update head and tail modulo the capacity.
- Add resizing array.



Q. How to resize?

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Parameterized stack

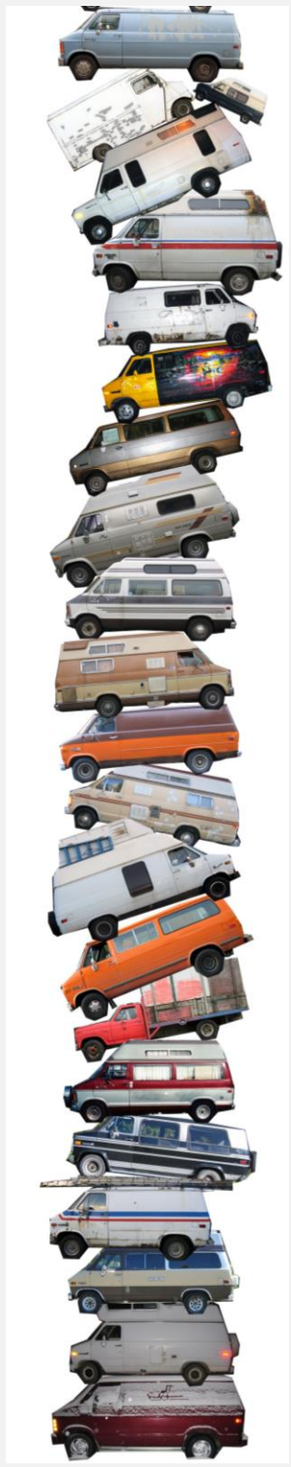
We implemented: StackOfStrings.

We also want: StackOfURLs, StackOfInts, StackOfVans, ….

Attempt 1. Implement a separate stack class for each type.

- Rewriting code is tedious and error-prone.
- Maintaining cut-and-pasted code is tedious and error-prone.

@#$*! most reasonable approach until Java 1.5.



Parameterized stack

We implemented: StackOfStrings.

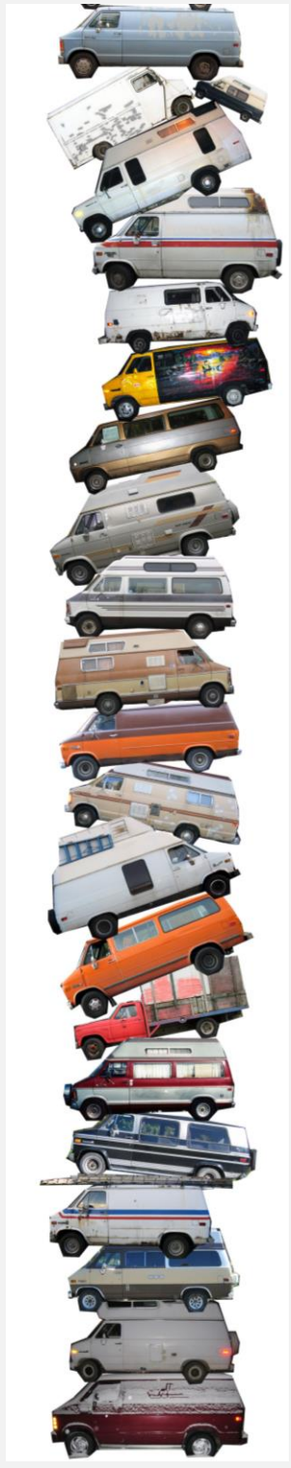
We also want: StackOfURLs, StackOfInts, StackOfVans, ….

Attempt 2. Implement a stack with items of type Object.

- Casting is required in client.
- Casting is error-prone: run-time error if types mismatch.

```
StackOfObjects s = new StackOfObjects();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = (Apple) (s.pop());
```

run-time error



Parameterized stack

We implemented: StackOfStrings.

We also want: StackOfURLs, StackOfInts, StackOfVans, ….

Attempt 3. Java generics.

- Avoid casting in client.
- Discover type mismatch errors at compile-time instead of run-time.

type parameter

```
Stack<Apple> s = new Stack<Apple>();
Apple  a = new Apple();
Orange b = new Orange();
s.push(a);
s.push(b);
a = s.pop();
```

compile-time error

Guiding principles. Welcome compile-time errors; avoid run-time errors.

Generic stack: linked-list implementation

```
public class LinkedStackOfStrings
{
   private Node first = null;

   private class Node
   {
      String item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(String item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public String pop()
   {
      String item = first.item;
      first = first.next;
      return item;
   }
```

```
public class Stack<Item>
{
   private Node first = null;

   private class Node
   {
      Item item;
      Node next;
   }

   public boolean isEmpty()
   {  return first == null;  }

   public void push(Item item)
   {
      Node oldfirst = first;
      first = new Node();
      first.item = item;
      first.next = oldfirst;
   }

   public Item pop()
   {
      Item item = first.item;
      first = first.next;
      return item;
   }
```

generic type name

Generic stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public ..StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it should be

```
public class FixedCapacityStack<Item>
{
   private Item[] s;
   private int N = 0;

   public FixedCapacityStack(int capacity)
   {  s = new Item[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

@#$*! generic array creation not allowed in Java

Generic stack: array implementation

```
public class FixedCapacityStackOfStrings
{
   private String[] s;
   private int N = 0;

   public ..StackOfStrings(int capacity)
   {  s = new String[capacity];  }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(String item)
   {  s[N++] = item;  }

   public String pop()
   {  return s[--N];  }
}
```

the way it is

```
public class FixedCapacityStack<Item>
{
   private Item[] s;
   private int N = 0;

   public FixedCapacityStack(int capacity)
   {  s = (Item[]) new Object[capacity]; }

   public boolean isEmpty()
   {  return N == 0;  }

   public void push(Item item)
   {  s[N++] = item;  }

   public Item pop()
   {  return s[--N];  }
}
```

the ugly cast

Unchecked cast

```
% javac FixedCapacityStack.java
Note: FixedCapacityStack.java uses unchecked or unsafe operations.
Note: Recompile with -Xlint:unchecked for details.

% javac -Xlint:unchecked FixedCapacityStack.java
FixedCapacityStack.java:26: warning: [unchecked] unchecked cast
found   : java.lang.Object[]
required: Item[]
        a = (Item[]) new Object[capacity];
                     ^
1 warning
```

Q. Why does Java make me cast (or use reflection)?

Short answer. Backward compatibility.

Long answer. Need to learn about type erasure and covariant arrays.

Generic data types: autoboxing

Q. What to do about primitive types?

Wrapper type.

- Each primitive type has a wrapper object type.
- Ex: Integer is wrapper type for int.

Autoboxing. Automatic cast between a primitive type and its wrapper.

```
Stack<Integer> s = new Stack<Integer>();
s.push(17);       // s.push(Integer.valueOf(17));
int a = s.pop();  // int a = s.pop().intValue();
```

Bottom line. Client code can use generic stack for any type of data.

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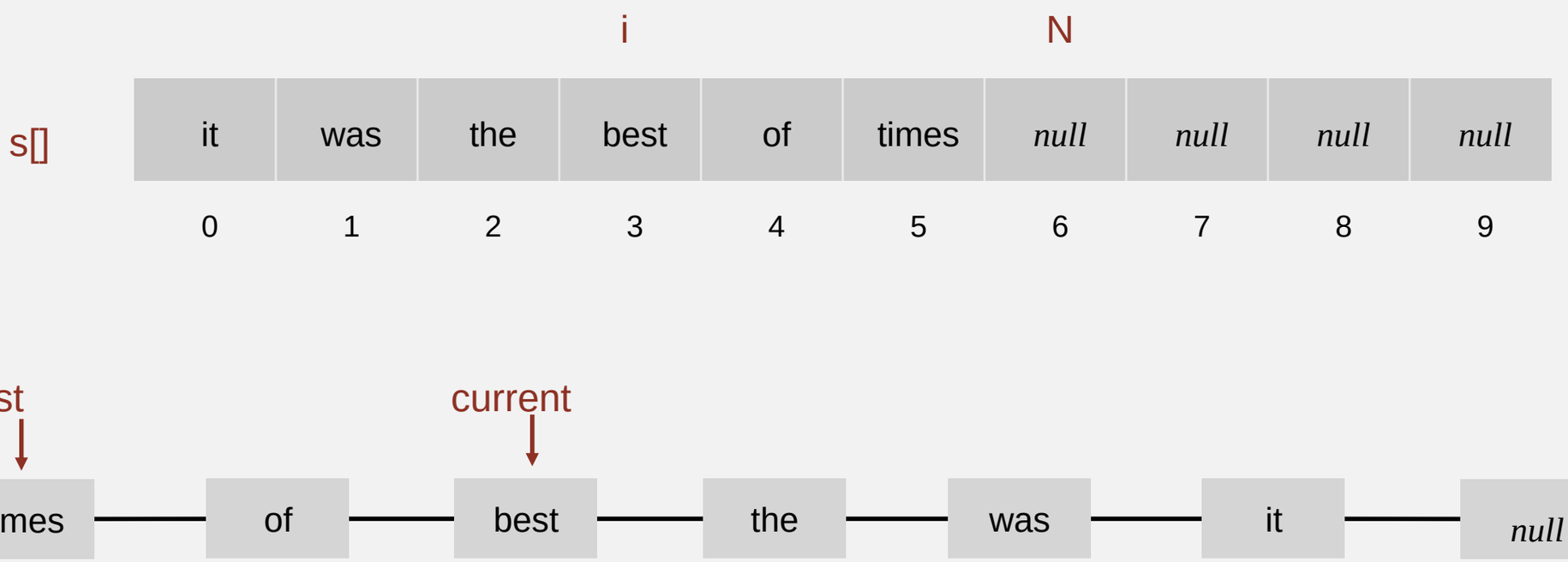
1.3 Bags, Queues, and Stacks

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators*
- *applications*

Iteration

Design challenge. Support iteration over stack items by client, without revealing the internal representation of the stack.



Java solution. Make stack implement the java.lang.Iterable interface.

Iterators

Q. What is an Iterable ?

A. Has a method that returns an Iterator.

Q. What is an Iterator ?

A. Has methods hasNext() and next().

Q. Why make data structures Iterable ?

A. Java supports elegant client code.

java.lang.Iterable interface

```
public interface Iterable<Item>
{
   Iterator<Item> iterator();
}
```

java.util.Iterator interface

```
public interface Iterator<Item>
{
   boolean hasNext();
   Item next();          <-- optional; use at your own risk
   void remove();
}
```

"foreach" statement (shorthand)

```
for (String s : stack)
   StdOut.println(s);
```

equivalent code (longhand)

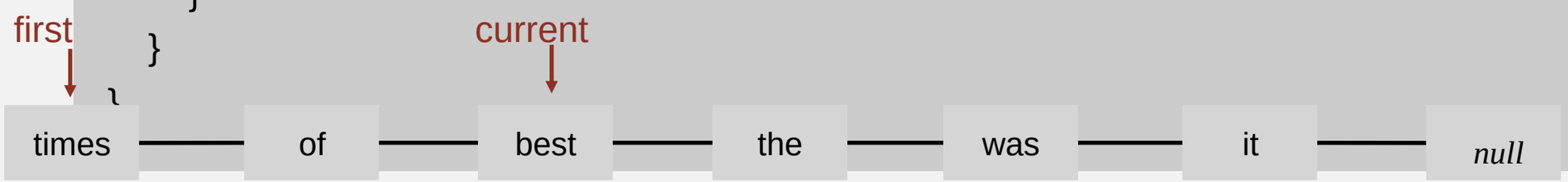
```
Iterator<String> i = stack.iterator();
while (i.hasNext())
{
   String s = i.next();
   StdOut.println(s);
}
```

Stack iterator: linked-list implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
   ...

   public Iterator<Item> iterator() { return new ListIterator(); }

   private class ListIterator implements Iterator<Item>
   {
      private Node current = first;

      public boolean hasNext() {  return current != null;  }
      public void remove()     {  /* not supported */      }
      public Item next()
      {
         Item item = current.item;
         current   = current.next;
         return item;
      }
   }
}
```

throw UnsupportedOperationException

throw NoSuchElementException
if no more items in iteration



Stack iterator: array implementation

```
import java.util.Iterator;

public class Stack<Item> implements Iterable<Item>
{
   …

   public Iterator<Item> iterator()
   { return new ReverseArrayIterator(); }

   private class ReverseArrayIterator implements Iterator<Item>
   {
      private int i = N;

      public boolean hasNext() {  return i > 0;       }
      public void remove()     {  /* not supported */  }
      public Item next()       {  return s[--i];      }
   }
}
```

i N

s[]	it	was	the	best	of	times	*null*	*null*	*null*	*null*
	0	1	2	3	4	5	6	7	8	9

Iteration: concurrent modification

Q. What if client modifies the data structure while iterating?

A. A fail-fast iterator throws a java.util.ConcurrentModificationException.

concurrent modification

```
for (String s : stack)
   stack.push(s);
```

Q. How to detect?

A.

- Count total number of push() and pop() operations in Stack.
- Save counts in *Iterator subclass upon creation.
- If, when calling next() and hasNext(), the current counts do not equal the saved counts, throw exception.

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1.3 Bags, Queues, and Stacks

- *stacks*
- *resizing arrays*
- *queues*
- *generics*
- *iterators*
- *applications*

Java collections library

List interface. java.util.List is API for an sequence of items.

public interface List<Item> implements Iterable<Item>		
	List()	*create an empty list*
boolean	isEmpty()	*is the list empty?*
int	size()	*number of items*
void	add(Item item)	*append item to the end*
Item	get(int index)	*return item at given index*
Item	remove(int index)	*return and delete item at given index*
boolean	contains(Item item)	*does the list contain the given item?*
Iterator<Item>	iterator()	*iterator over all items in the list*
	...	

Implementations. java.util.ArrayList uses resizing array;
java.util.LinkedList uses linked list.

caveat: only some operations are efficient

Java collections library

java.util.Stack.

- Supports push(), pop(), and iteration.
- Extends java.util.Vector, which implements java.util.List interface from previous slide, including get() and remove().
- Bloated and poorly-designed API (why?)

Java 1.3 bug report (June 27, 2001)

> The iterator method on java.util.Stack iterates through a Stack from the bottom up. One would think that it should iterate as if it were popping off the top of the Stack.

status (closed, will not fix)

> It was an incorrect design decision to have Stack extend Vector ("is-a" rather than "has-a"). We sympathize with the submitter but cannot fix this because of compatibility.

Java collections library

java.util.Stack.

- Supports push(), pop(), and iteration.
- Extends java.util.Vector, which implements java.util.List interface from previous slide, including get() and remove().
- Bloated and poorly-designed API (why?)

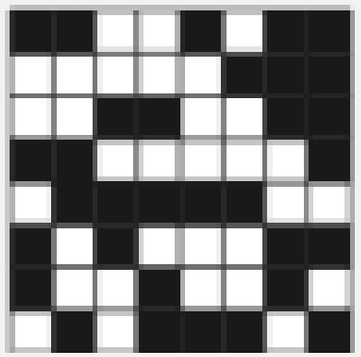
java.util.Queue. An interface, not an implementation of a queue.

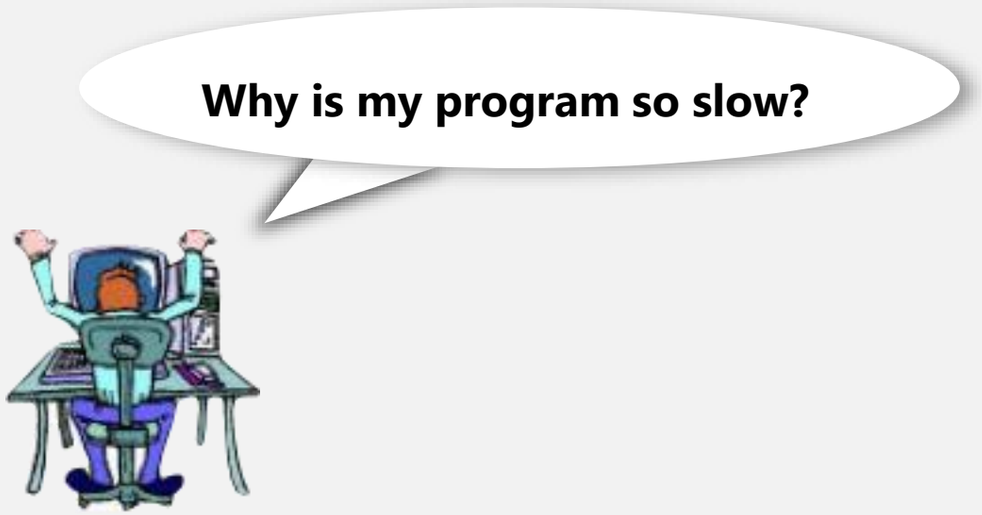
Best practices. Use our implementations of Stack, Queue, and Bag.

War story (from Assignment 1)

Generate random open sites in an N-by-N percolation system.

- Jenny: pick (i, j) at random; if already open, repeat.
  Takes $\sim c_1 N^2$ seconds.
- Kenny: create a java.util.ArrayList of N^2 closed sites.
  Pick an index at random and delete.
  Takes $\sim c_2 N^4$ seconds.





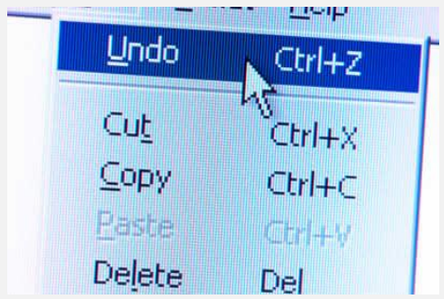
Kenny

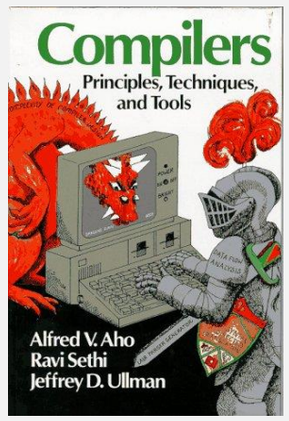
Lesson. Don't use a library until you understand its API!

This course. Can't use a library until we've implemented it in class.

Stack applications

- Parsing in a compiler.
- Java virtual machine.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Implementing function calls in a compiler.
- ...





Function calls

How a compiler implements a function.

- Function call: push local environment and return address.
- Return: pop return address and local environment.

Recursive function. Function that calls itself.

Note. Can always use an explicit stack to remo

p = 192, q = 24

gcd (192, 24)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

p = 216, q = 192

gcd (216, 192)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

p = 24, q = 0

gcd (24, 0)

```
static int gcd(int p, int q) {
   if (q == 0) return p;
   else return gcd(q, p % q);
}
```

Arithmetic expression evaluation

Goal. Evaluate infix expressions.

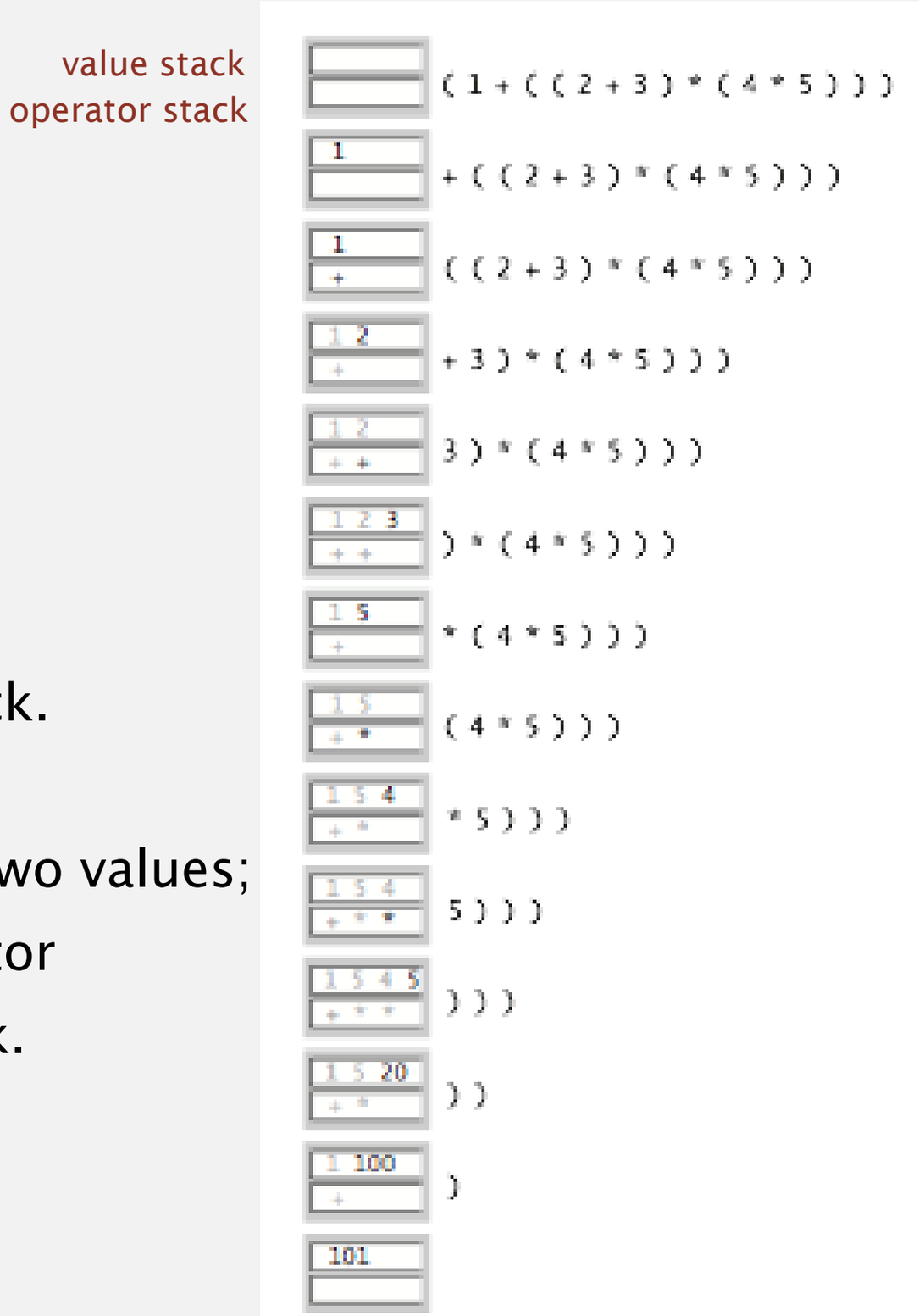
```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

operand operator

Two-stack algorithm. [E. W. Dijkstra]

- Value: push onto the value stack.
- Operator: push onto the operator stack.
- Left parenthesis: ignore.
- Right parenthesis: pop operator and two values; push the result of applying that operator to those values onto the operand stack.

Context. An interpreter!



Dijkstra's two-stack algorithm demo

value stack

operator stack

infix expression

(fully parenthesized)

(	1	+	(	(	2	+	3	)	*	(	4	*	5	)	)	)

operand

operator

Arithmetic expression evaluation

```
public class Evaluate
{
   public static void main(String[] args)
   {
      Stack<String> ops  = new Stack<String>();
      Stack<Double> vals = new Stack<Double>();
      while (!StdIn.isEmpty()) {
         String s = StdIn.readString();
      if      (s.equals("("))               ;
      else if (s.equals("+"))    ops.push(s);
      else if (s.equals("*"))    ops.push(s);
      else if (s.equals(")"))
      {
         String op = ops.pop();
         if      (op.equals("+")) vals.push(vals.pop() + vals.pop());
         else if (op.equals("*")) vals.push(vals.pop() * vals.pop());
      }
      else vals.push(Double.parseDouble(s));
   }
   StdOut.println(vals.pop());
}
```

```
% java Evaluate
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
101.0
```

Correctness

Q. Why correct?

A. When algorithm encounters an operator surrounded by two values within parentheses, it leaves the result on the value stack.

```
( 1 + ( ( 2 + 3 ) * ( 4 * 5 ) ) )
```

as if the original input were:

```
( 1 + ( 5 * ( 4 * 5 ) ) )
```

Repeating the argument:

```
( 1 + ( 5 * 20 ) )
( 1 + 100 )
101
```

Extensions. More ops, precedence order, associativity.

Stack-based programming languages

Observation 1. Dijkstra's two-stack algorithm computes the same value if the operator occurs after the two values.

```
( 1 ( ( 2 3 + ) ( 4 5 * ) * ) + )
```

Jan Lukasiewicz

Observation 2. All of the parentheses are redundant!

```
1 2 3 + 4 5 * * +
```

Bottom line. Postfix or "reverse Polish" notation.

Applications. Postscript, Forth, calculators, Java virtual machine, ...