Macroeconomics Assignment

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Abstract

Abstract to be written here. The abstract should not be too long and should provide the reader with a good understanding what you are writing about. Academic papers are not like novels where you keep the reader in suspense. To be effective in getting others to read your paper, be as open and concise about your findings here as possible. Ideally, upon reading your abstract, the reader should feel he / she must read your paper in entirety.

Start writing about what you are planning to do, note that this is an assignment and what it is (introduction vibes)

Please give credit: I haved used an R package from Mati (2019)

1. Model Specification

Core RBC Foundations

Above could note be a heading on itself. Just a paragraph that it is a core rbc foundations and what that means.

Note that capital letters denote nominal amounts and lowercase denote real values.

1.1. Households

$$C_t = P_t c_t, \qquad I_t = P_t i_t \tag{1.1}$$

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$$\max_{\{C_t, h_t, M_t, B_t, K_t, I_t\}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{\left(\frac{C_t}{P_t} - \eta \frac{C_{t-1}}{P_{t-1}}\right)^{1-\theta}}{1-\theta} - \chi \frac{h_t^{1+\gamma}}{1+\gamma} + \psi \ln\!\left(\frac{M_t}{P_t}\right) \right] \tag{1.2}$$

$$C_t + I_t + B_t + M_t \le R_{t-1}^B B_{t-1} + M_{t-1} + W_t h_t + R_t^k K_{t-1} + \Pi_t - P_t \tau_t \tag{1.3}$$

$$K_t = (1-\delta) K_{t-1} + I_t - \frac{\phi}{2} \left(\frac{I_t}{K_{t-1}} - \delta \right)^2 K_{t-1} \tag{1.4}$$

From which we can derive the following equations, found in full form in Appendix 2.1.1

$$(c_t - \eta c_{t-1})^{-\theta} = \beta \mathbb{E}_t\!\left[R_t^B \frac{P_t}{P_{t+1}} (c_{t+1} - \eta c_t)^{-\theta} \right] \tag{1.5}$$

Interpretation: Marginal rate of substitution between current and future consumption equals the expected real bond return. Habit persistence (η) links today's utility to past and future consumption, and inflation (P_t/P_{t+1}) scales the real payoff on bonds.

$$\frac{W_t}{P_t} = \frac{\chi h_t^{\gamma}}{(c_t - \eta c_{t-1})^{-\theta} - \beta \eta \mathbb{E}_t\!\left[(c_{t+1} - \eta c_t)^{-\theta}\right]} \tag{1.6}$$

Interpretation: Habit formation reduces effective marginal utility of consumption, so stronger habits ($\eta \uparrow$) or more elastic labour supply ($\gamma \uparrow$) require a higher real wage to induce the same hours.

$$M_t = \frac{\psi}{\beta \mathbb{E}_t\!\left[(R_t^B - 1) \cdot \frac{(c_{t+1} - \eta c_t)^{-\theta} - \beta \eta \mathbb{E}_{t+1}\left[(c_{t+2} - \eta c_{t+1})^{-\theta}\right]}{P_{t+1}} \right]} \tag{1.7}$$

Interpretation: Higher expected nominal rates ($R_t^B - 1$) raise the opportunity cost of money, while habits and inflation expectations shape the curvature of demand. The denominator captures the liquidity premium adjusted for consumption dynamics.

$$
\begin{aligned}
q_t \;&\equiv\; 1 - \phi\left(\tfrac{I_t}{K_{t-1}} - \delta\right) \\
\frac{\beta\,\mathbb{E}_t[\lambda_{t+1}\,R_t^B]}{q_t} \;&=\; \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\Big(R_{t+1}^k + \frac{1}{q_{t+1}}\Big(1 - \delta + \tfrac{\phi}{2}\big[(I_{t+1}/K_t)^2 - \delta^2\big]\Big)\Big)\Big]
\end{aligned}
\tag{1.8}
$$

Interpretation: The bond-return-scaled discount factor divided by q_t equals expected return on capital plus adjustment-cost terms. If $I_t/K_{t-1} > \delta$, then $q_t > 1$ signals profitable expansion; disinvestment flips the sign. Adjustment costs (ϕ) create investment frictions.

1.2. Production

While the RBC model features a single representative firm under perfect competition, we now introduce two layers of firms to capture monopolistic competition and nominal rigidities:

1. **Final goods producer** (perfectly competitive aggregator)
2. **Intermediate goods producers** (monopolistically competitive with sticky prices)

1.2.1. Final Goods Producer

(Perfectly competitive, zero profits)
Role: Aggregates differentiated inputs into final output.

Key equations:

CES Production Function:

$$
Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{1.9}
$$

Demand for Good j:

$$
Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t \tag{1.10}
$$

Aggregate Price Index:

$$
P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{1.11}
$$

Equations 1.10 and 1.11 are derived from 1.9 in Appendix 2.2.1.

Economic Intuition:

- ϵ: Elasticity of substitution (lower ϵ implies more market power)
- Demand for j decreases with its relative price $\frac{P_t(j)}{P_t}$

1.2.2. Intermediate Goods Producers

(Monopolistic competitors, indexed by $j \in [0, 1]$)

1.2.3. Technology and Factor Inputs

Production Function (same as RBC):

$$Y_t(j) = A_t K_t(j)^{\alpha} h_t(j)^{1-\alpha}$$

Assumptions:

- Identical technology A_t (common TFP shock)
- Firms rent capital $K_t(j)$ and labor $h_t(j)$ from households
- Competitive factor markets: prices R_t^k, W_t

1.2.4. Price-Setting Friction

Rotemberg Price Adjustment Cost:

$$\text{AdjCost}_t(j) = \frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t$$

Intuition:

- Convex cost for price changes (relative to past price)

- ψ: Adjustment cost parameter (higher $\rightarrow$ more stickiness)
- Scaling by Y_t ensures cost grows with output

1.3. Firm Optimization and Equilibrium

1.3.1. Profit Maximization Problem

Objective: Choose $P_t(j)$ to maximize discounted real profits:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \left[\underbrace{\frac{P_t(j)}{P_{t+s}} Y_{t+s}(j)}_{\text{Revenue}} - \underbrace{\mathrm{MC}_{t+s} Y_{t+s}(j)}_{\text{Cost}} - \underbrace{\mathrm{AdjCost}_{t+s}(j)}_{\text{Price adjustment cost}} \right]$$

Where:

- $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$: Household stochastic discount factor
- MC_t: Real marginal cost (defined next)

1.3.2. Key Equilibrium Concepts

Marginal Cost:

$$\mathrm{MC}_t = \frac{1}{A_t} \left(\frac{R_t^k}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha}$$

Intuition: Inverse of TFP times Cobb-Douglas cost

Symmetric Equilibrium: > All firms face identical conditions, so $P_t(j) = P_t$, $Y_t(j) = Y_t$.

1.3.3. New Keynesian Phillips Curve (NKPC)

Core NKPC Equation:

$$\psi(\Pi_t - 1)\Pi_t = \epsilon \left(1 - \frac{\mathrm{MC}_t}{\mu} \right) + \beta\psi \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right]$$

Definitions:

- $\Pi_t \equiv \frac{P_t}{P_{t-1}}$: Gross inflation
- $\mu \equiv \frac{\epsilon}{\epsilon-1}$: Desired markup

Interpretation:

- LHS: Current inflation cost

- RHS:
  - $\epsilon(1 - \mathrm{MC}_t/\mu)$: Markup gap

  - Expectation term: Forward-looking inflation pressure

1.4. Resource Constraint

Final Output Allocation:

$$Y_t = C_t + I_t + \frac{\psi}{2}(\Pi_t - 1)^2 Y_t$$

Interpretation: Price adjustment costs are a deadweight loss.

Here's a concise explanation of how the firm section extends a simple RBC model, integrating monopolistic competition and Rotemberg price rigidity.

1.5. From RBC to New Keynesian Framework

In a basic RBC model, you have a single representative firm with perfect competition and flexible prices. We extend this in two key ways:

1. **Monopolistic Competition**
   Instead of one firm, we now have:
   - A *final goods producer* that aggregates differentiated intermediate goods (from firms $j \in [0, 1]$) into final output Y_t using a CES technology. This creates downward-sloping demand curves for each firm j:
   $$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} Y_t$$

where $\epsilon > 1$ governs market power (higher ϵ = more competition).

2. **Intermediate Firms with Sticky Prices**
   Each firm j produces with Cobb-Douglas technology:

$$Y_t(j) = A_t K_t(j)^\alpha h_t(j)^{1-\alpha}$$

   but faces two new frictions:

   - **Market Power**: They set $P_t(j) > \mathrm{MC}_t$, where

$$\mathrm{MC}_t = \frac{1}{A_t}\left(\frac{R_t^k}{\alpha}\right)^\alpha \left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}$$

   - **Rotemberg Price Rigidity**: Changing prices incurs a quadratic cost:

$$\mathrm{AdjCost}_t = \frac{\psi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t$$

     This penalizes large price changes (ψ controls stickiness).

1.6. Price Setting Dynamics

Firms maximize **discounted real profits**:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t}\left[\frac{P_t(j)}{P_{t+s}} Y_{t+s}(j) - \mathrm{MC}_{t+s} Y_{t+s}(j) - \mathrm{AdjCost}_{t+s}\right]$$

- $\beta^s \frac{\lambda_{t+s}}{\lambda_t}$: Household's stochastic discount factor (from your Euler equation).
- In symmetric equilibrium ($P_t(j) = P_t$, $Y_t(j) = Y_t$), this yields the **New Keynesian Phillips Curve**:

$$\psi(\Pi_t - 1)\Pi_t = \epsilon\left(1 - \frac{\mathrm{MC}_t}{\mu}\right) + \beta\psi\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}(\Pi_{t+1} - 1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]$$

  where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ is inflation, and $\mu \equiv \frac{\epsilon}{\epsilon-1}$ is the markup.

This equation links inflation to marginal costs (real side) and expected inflation (forward-looking term).

1.7. Key Implications for RBC Foundations

- **Real Variables**: Production still uses K_{t-1} and h_t (as in RBC), but now inputs are rented from households at competitive rates R_t^k and W_t.
- **New Nominal Rigidity**: The adjustment cost $\psi(\Pi_t - 1)^2 Y_t$ appears in the resource constraint, acting like a "friction tax" on output.
- **Monetary Policy Transmission**: Interest rates (from your bond Euler equation) now affect real activity via inflation dynamics.

This structure preserves RBC foundations while adding nominal rigidities and imperfect competition — essential for analyzing monetary policy. The only new state variable is P_{t-1} (for inflation dynamics), maintaining tractability.

Final Good Producer The final good Y_t is produced by combining intermediate goods $Y_t(j)$:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{1.12}$$

Profit maximization yields demand for each intermediate good:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t \tag{1.13}$$

and the aggregate price index:

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{1.14}$$

Intermediate Goods Producers Each firm j produces with Cobb-Douglas technology:

$$Y_t(j) = A_t K_t(j)^{\alpha} h_t(j)^{1-\alpha} \tag{1.15}$$

Minimizing costs yields nominal marginal cost (common to all firms):

$$\mathrm{MC}_t = \frac{1}{A_t} \left(\frac{R_t^k}{\alpha} \right)^{\alpha} \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \tag{1.16}$$

Firms face Rotemberg (1982) price adjustment costs. The real profit function is:

$$\Pi_t(j) = \underbrace{\frac{P_t(j)}{P_t} Y_t(j)}_{\text{real revenue}} - \underbrace{\mathrm{MC}_t \cdot Y_t(j)}_{\text{real cost}} - \underbrace{\frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t}_{\text{adjustment cost}} \tag{1.17}$$

Price Setting Firms maximize discounted future profits:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{t+s}}{\lambda_t} \Pi_{t+s}(j) \tag{1.18}$$

subject to demand (1.13). In symmetric equilibrium ($P_t(j) = P_t$, $Y_t(j) = Y_t$), we obtain the Rotemberg Phillips Curve:

$$\psi(\Pi_t - 1)\Pi_t = \epsilon \left(1 - \frac{\mathrm{MC}_t}{\mu} \right) + \beta\psi \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] \tag{1.19}$$

where $\Pi_t \equiv P_t/P_{t-1}$ and $\mu \equiv \epsilon/(\epsilon - 1)$.

Market Clearing Aggregate production:

$$Y_t = A_t K_{t-1}^{\alpha} h_t^{1-\alpha} \tag{1.20}$$

Resource constraint (adjustment costs reduce output):

$$Y_t = C_t + I_t + \underbrace{\frac{\psi}{2}(\Pi_t - 1)^2 Y_t}_{\text{price adjustment cost}} \tag{1.21}$$

Factor markets clear:

$$\int_0^1 h_t(j)dj = h_t \tag{1.22}$$

$$\int_0^1 K_t(j)dj = K_{t-1} \tag{1.23}$$

2. Appendix

2.1. Households

Define Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \Big\{ \underbrace{\frac{\left(\frac{C_t}{P_t} - \eta \frac{C_{t-1}}{P_{t-1}}\right)^{1-\theta}}{1-\theta}}_{\text{Consumption utility}} - \underbrace{\chi \frac{h_t^{1+\gamma}}{1+\gamma}}_{\text{Labor disutility}} + \underbrace{\psi \ln\left(\frac{M_t}{P_t}\right)}_{\text{Money utility}}$$

$$+ \underbrace{\lambda_t \left[R_{t-1}^B B_{t-1} + M_{t-1} + W_t h_t + R_t^k K_{t-1} + \Pi_t - P_t \tau_t - C_t - I_t - B_t - M_t\right]}_{\text{Nominal flow constraint}} + \underbrace{\mu_t \left[(1-\delta) K_{t-1} + I_t - \frac{\phi}{2}\left(\frac{I_t}{K_{t-1}}\right.\right.}_{\text{Capital accumulatio}}$$

2.1.1. First Order Conditions

FOC w.r.t. Consumption :

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0$$

$$\left[(c_t - \eta c_{t-1})^{-\theta}/P_t - \lambda_t\right] - \beta\, \mathbb{E}_t\left[\eta\,(c_{t+1} - \eta c_t)^{-\theta}/P_t\right] = 0$$

Combine terms over $1/P_t$

$$\frac{1}{P_t}\left[(c_t - \eta c_{t-1})^{-\theta} - \beta\eta\, \mathbb{E}_t[(c_{t+1} - \eta c_t)^{-\theta}]\right] - \lambda_t = 0$$

Multiply by P_t

$$(c_t - \eta c_{t-1})^{-\theta} - \beta\eta\, \mathbb{E}_t[(c_{t+1} - \eta c_t)^{-\theta}] - \lambda_t P_t = 0$$

$$\boxed{\lambda_t P_t = (c_t - \eta c_{t-1})^{-\theta} - \beta\eta\, \mathbb{E}_t\left[(c_{t+1} - \eta c_t)^{-\theta}\right]} \tag{2.1}$$

FOC w.r.t. Labour :

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0$$

$$-\chi h_t^{\gamma} + \lambda_t W_t = 0$$

Rearrange

$$\lambda_t W_t = \chi h_t^{\gamma}$$

$$\boxed{\lambda_t W_t = \chi h_t^\gamma} \tag{2.2}$$

FOC w.r.t. Real Money Balances:

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial M_t} = 0\\
&\quad \beta^{t-1}[\psi/M_t - \lambda_t] + \beta^t \mathbb{E}_t[\lambda_{t+1}] = 0\\
&\text{Divide by } \beta^{t-1} \text{ and rearrange}\\
&\quad \psi/M_t - \lambda_t + \beta\, \mathbb{E}_t[\lambda_{t+1}] = 0
\end{aligned}$$

$$\boxed{\frac{\psi}{M_t} = \lambda_t - \beta\, \mathbb{E}_t[\lambda_{t+1}]} \tag{2.3}$$

FOC w.r.t. Bonds (2.4)):

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial B_t} = 0\\
&\quad -\beta^{t-1}\lambda_t + \beta^t \mathbb{E}_t[\lambda_{t+1} R_t^B] = 0\\
&\text{Divide by } \beta^{t-1} \text{ and simplify}\\
&\quad -\lambda_t + \beta\, \mathbb{E}_t[\lambda_{t+1} R_t^B] = 0
\end{aligned}$$

$$\boxed{\lambda_t = \beta\, \mathbb{E}_t[\lambda_{t+1} R_t^B]} \tag{2.4}$$

FOC w.r.t. Capital :

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial K_t} = 0\\
&\quad -\beta^{t-1}\mu_t + \beta^t \mathbb{E}_t\big[\lambda_{t+1} R_{t+1}^k + \mu_{t+1}(1 - \delta + \tfrac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2))\big] = 0\\
&\text{Divide by } \beta^{t-1} \text{ and solve}\\
&\quad \mu_t = \beta\, \mathbb{E}_t\big[\lambda_{t+1} R_{t+1}^k + \mu_{t+1}(1 - \delta + \tfrac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2))\big]
\end{aligned}$$

$$\boxed{\mu_t = \beta\, \mathbb{E}_t\big[\lambda_{t+1} R_{t+1}^k + \mu_{t+1}(1 - \delta + \tfrac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2))\big]} \tag{2.5}$$

FOC w.r.t. Investment :

$$\frac{\partial \mathcal{L}}{\partial I_t} = 0$$
$$\beta^{t-1}\big[-\lambda_t + \mu_t(1 - \phi(\tfrac{I_t}{K_{t-1}} - \delta))\big] = 0$$

Divide by β^{t-1} and isolate

$$\lambda_t = \mu_t\big(1 - \phi(\tfrac{I_t}{K_{t-1}} - \delta)\big)$$

$$\boxed{\lambda_t = \mu_t\big(1 - \phi(\tfrac{I_t}{K_{t-1}} - \delta)\big)} \tag{2.6}$$

2.1.2. Household Final Equations

Consumption Euler Equation

Combines consumption–habit dynamics with bond returns (from (2.4) and (2.1)) :

Start with FOC for Bonds

$$\lambda_t = \beta\,\mathbb{E}_t[\lambda_{t+1} R_t^B] \quad \text{(Equation 2.4)}$$

Substitute λ_t and λ_{t+1} from FOC for Consumption

$$\lambda_t = \frac{(c_t - \eta\, c_{t-1})^{-\theta}}{P_t} \quad \text{(from Equation 2.1 rearranged)}$$
$$\lambda_{t+1} = \frac{(c_{t+1} - \eta\, c_t)^{-\theta}}{P_{t+1}} \quad \text{(time-shifted)}$$

$textCombineresults$

$$\frac{(c_t - \eta\, c_{t-1})^{-\theta}}{P_t} = \beta\,\mathbb{E}_t\left[R_t^B \cdot \frac{(c_{t+1} - \eta\, c_t)^{-\theta}}{P_{t+1}}\right]$$

Clear denominator

$$(c_t - \eta\, c_{t-1})^{-\theta} = \beta\,\mathbb{E}_t\left[R_t^B \cdot \frac{P_t}{P_{t+1}} \cdot (c_{t+1} - \eta\, c_t)^{-\theta}\right]$$

$$\boxed{(c_t - \eta\, c_{t-1})^{-\theta} \;=\; \beta\, \mathbb{E}_t\Big[R_t^B\, \frac{P_t}{P_{t+1}}\, (c_{t+1} - \eta\, c_t)^{-\theta}\Big]} \tag{2.7}$$

Interpretation: Marginal rate of substitution between current and future consumption equals the expected real bond return. Habit persistence (η) links today's utility to past and future consumption, and inflation (P_t/P_{t+1}) scales the real payoff on bonds.

Labour Supply

Real wage equals the marginal rate of substitution between leisure and consumption (from (2.1) and (2.2)) :

Start with FOC for Hours Worked

$\lambda_t W_t = \chi\, h_t^{\gamma}$ (Equation 2.2)

Solve for λ_t

$$\lambda_t = \frac{\chi\, h_t^{\gamma}}{W_t}$$

Equate to FOC of Consumption expression

$$\frac{\chi\, h_t^{\gamma}}{W_t} = \frac{(c_t - \eta\, c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t[(c_{t+1} - \eta\, c_t)^{-\theta}]}{P_t}$$

Solve for real wage (W_t/P_t)

$$\frac{W_t}{P_t} = \frac{\chi\, h_t^{\gamma}}{(c_t - \eta\, c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t[(c_{t+1} - \eta\, c_t)^{-\theta}]}$$

$$\boxed{\frac{W_t}{P_t} \;=\; \frac{\chi\, h_t^{\gamma}}{(c_t - \eta\, c_{t-1})^{-\theta} \;-\; \beta\, \eta\, \mathbb{E}_t\big[(c_{t+1} - \eta\, c_t)^{-\theta}\big]}} \tag{2.8}$$

Interpretation: Habit formation reduces effective marginal utility of consumption, so stronger habits ($\eta \uparrow$) or more elastic labour supply ($\gamma \uparrow$) require a higher real wage to induce the same hours.

Money Demand

Opportunity cost of holding money vs. bonds (from (2.3), (2.4) and (2.1)) :

Combine FOC for Money and Bonds

$$\frac{\psi}{M_t} = \lambda_t - \beta\,\mathbb{E}_t[\lambda_{t+1}] \quad \text{(Equation 2.3)}$$

$$\lambda_t = \beta\,\mathbb{E}_t[\lambda_{t+1} R_t^B] \quad \text{(Equation 2.4)}$$

Substitute λ_t into FOC of money

$$\frac{\psi}{M_t} = \beta\,\mathbb{E}_t[\lambda_{t+1} R_t^B] - \beta\,\mathbb{E}_t[\lambda_{t+1}]$$

$$\frac{\psi}{M_t} = \beta\,\mathbb{E}_t\left[\lambda_{t+1}(R_t^B - 1)\right]$$

Substitute λ_{t+1} from FOC of Consumption

$$\lambda_{t+1} = \frac{(c_{t+1} - \eta\,c_t)^{-\theta} - \beta\,\eta\,\mathbb{E}_{t+1}[(c_{t+2} - \eta\,c_{t+1})^{-\theta}]}{P_{t+1}}$$

Solve for M_t

$$M_t = \frac{\psi}{\beta\,\mathbb{E}_t\left[(R_t^B - 1)\cdot \frac{(c_{t+1} - \eta\,c_t)^{-\theta} - \beta\,\eta\,\mathbb{E}_{t+1}[(c_{t+2} - \eta\,c_{t+1})^{-\theta}]}{P_{t+1}}\right]}$$

$$\boxed{M_t \;=\; \frac{\psi}{\beta\,\mathbb{E}_t\Big[(R_t^B - 1)\;\cdot\;\frac{(c_{t+1} - \eta\,c_t)^{-\theta}\;-\;\beta\,\eta\,\mathbb{E}_{t+1}[(c_{t+2} - \eta\,c_{t+1})^{-\theta}]}{P_{t+1}}\Big]}} \tag{2.9}$$

Interpretation: Higher expected nominal rates $(R_t^B - 1)$ raise the opportunity cost of money, while habits and inflation expectations shape the curvature of demand. The denominator captures the liquidity premium adjusted for consumption dynamics.

Capital Euler Equation Defines Tobin's q and links required returns on capital to bond returns (from (2.6), (2.5) and (2.4)) :

Define Tobin's q from FOC for Investment

$\lambda_t = \mu_t q_t \quad \text{where} \quad q_t \equiv 1 - \phi\left(\frac{I_t}{K_{t-1}} - \delta\right)$

Rearrange FOC for Capital

$\mu_t = \beta\,\mathbb{E}_t\Big[\lambda_{t+1} R^k_{t+1} + \mu_{t+1}\Big(1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]\Big)\Big]$

Substitute $\mu_t = \lambda_t/q_t$ and $\mu_{t+1} = \lambda_{t+1}/q_{t+1}$

$\dfrac{\lambda_t}{q_t} = \beta\,\mathbb{E}_t\Big[\lambda_{t+1} R^k_{t+1} + \dfrac{\lambda_{t+1}}{q_{t+1}}\left(1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]\right)\Big]$

Factor λ_{t+1}

$\dfrac{\lambda_t}{q_t} = \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\left(R^k_{t+1} + \dfrac{1}{q_{t+1}}\left(1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]\right)\right)\Big]$

Substitute FOC for Bonds ($\lambda_t = \beta\,\mathbb{E}_t[\lambda_{t+1} R^B_t]$)

$\dfrac{\beta\,\mathbb{E}_t[\lambda_{t+1} R^B_t]}{q_t} = \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\left(R^k_{t+1} + \dfrac{1}{q_{t+1}}\Gamma_{t+1}\right)\Big]$

where $\Gamma_{t+1} \equiv 1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]$

$$\boxed{\begin{aligned} q_t \;&\equiv\; 1 - \phi\left(\tfrac{I_t}{K_{t-1}} - \delta\right) \\ \frac{\beta\,\mathbb{E}_t[\lambda_{t+1}\,R^B_t]}{q_t} \;&=\; \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\Big(R^k_{t+1} + \frac{1}{q_{t+1}}\Big(1 - \delta + \tfrac{\phi}{2}\big[(I_{t+1}/K_t)^2 - \delta^2\big]\Big)\Big)\Big] \end{aligned}} \tag{2.10}$$

Interpretation: The bond-return-scaled discount factor divided by q_t equals expected return on capital plus adjustment-cost terms. If $I_t/K_{t-1} > \delta$, then $q_t > 1$ signals profitable expansion; disinvestment flips the sign. Adjustment costs (ϕ) create investment frictions.

2.2. Production

2.2.1. Final Good Producer

Derivation of Intermediate Goods Demand and Aggregate Price Index

Final goods producer's profit:

$$\Pi_t = P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj$$

$$\text{subject to } Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

Substitute production function into profit:

$$\Pi_t = P_t \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(j) Y_t(j) dj$$

First-order condition for $Y_t(j)$:

$$\frac{\partial \Pi_t}{\partial Y_t(j)} = P_t \cdot \frac{\epsilon}{\epsilon - 1} \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} \cdot \frac{\epsilon - 1}{\epsilon} Y_t(j)^{-\frac{1}{\epsilon}} - P_t(j) = 0$$

$$\Rightarrow P_t \cdot Y_t^{\frac{1}{\epsilon}} Y_t(j)^{-\frac{1}{\epsilon}} = P_t(j)$$

Rearrange to obtain demand curve:

$$Y_t(j) = \left(\frac{P_t}{P_t(j)} \right)^{\epsilon} Y_t$$

Substitute demand into production function:

$$Y_t = \left(\int_0^1 \left[\left(\frac{P_t}{P_t(j)} \right)^{\epsilon} Y_t \right]^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$= Y_t \left(\int_0^1 \left(\frac{P_t}{P_t(j)} \right)^{\epsilon-1} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

Simplify to obtain price index:

$$1 = \left(\int_0^1 \left(\frac{P_t}{P_t(j)} \right)^{\epsilon - 1} dj \right)^{\frac{\epsilon}{\epsilon - 1}}$$

$$\Rightarrow P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj$$

$$\Rightarrow P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$$

$$\boxed{\begin{gathered} Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t \\ \\ P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \end{gathered}} \tag{2.11}$$

3. Old Stuff

Components breakdown:

Expenditures:

Consumption: $P_t c_t$

Investment: $P_t i_t$

Bonds: B_t

Money holdings: M_t

Income sources:

Bond returns: $(1 + i_{t-1})B_{t-1}$

Money carryover: M_{t-1}

Labor income: $W_t h_t$

Capital returns: $R_t^k K_{t-1}$ (Key addition missing in Sims)

Firm profits: Π_t

Net transfers: $-P_t \tau_t$

The utility function above (**??**) must be maximised subject to some sort of flow constraint. Note that the flow budget is undefined as of now because i am unsure if captial shows up there (which it should), bonds must too, hours worked and consumption (taxes too surely?)

$$K_t = (1-\delta)K_{t-1} + i_t - \frac{\phi}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1} \tag{3.1}$$

Key Improvements over Sims Capital integration:

Explicit rental rate R_t^k for capital services

Physical capital stock K_t in accumulation process

Convex adjustment costs ($\phi > 0$)

Real money balances:

Maintains money-in-utility (MIU) specification

Consistent with Walsh (2010) framework

Habit persistence:

$c_t - \eta c_{t-1}$ with $\eta \in (0,1)$

Generates consumption inertia matching SA data

3.1. Firm Sector with Nominal Rigidities

Production function with capital:

$$Y_t(i) = A_t K_t(i)^\alpha H_t(i)^{1-\alpha}, \quad \alpha \in (0,1) \tag{3.2}$$

Cost minisations:

$$\min_{K_t(i), H_t(i)} R_t^k K_t(i) + W_t H_t(i) \quad \text{s.t.} \quad Y_t(i) = A_t K_t(i)^\alpha H_t(i)^{1-\alpha} \tag{3.3}$$

3.1.1. Perfect competion final goods firl

CES aggreagation:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{3.4}$$

demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{3.5}$$

3.1.2. Pricing setting (calvo)

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \lambda_{t+k} MC_{t+k} P_{t+k}^{\epsilon} Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \lambda_{t+k} P_{t+k}^{\epsilon-1} Y_{t+k}} \tag{3.6}$$

Price index dynamics:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \tag{3.7}$$

devidenet distribution:

$$\Pi_t = \int_0^1 \Big[P_t(i) Y_t(i) - W_t H_t(i) - R_t^k K_t(i) \Big] \, di \tag{3.8}$$

Aggregate equivalent:

$$\Pi_t = P_t Y_t - W_t H_t - R_t^k K_t \tag{3.9}$$

$Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$ Cobb-Douglas

Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$

3.1.3. Nominal Rigidities

Probability $\theta = 0.75$ of price non-adjustment

Phillips Curve derivation: $\pi_t = \beta E_t \pi_{t+1} + \kappa m c_t$

Dividend specification: $\Pi_t = Y_t - w_t h_t - r_t^k k_t$

3.2. Government Sector

Fiscal rule: $T_t = \tau Y_t$ (lump-sum taxes)

Monetary authority: - Taylor Rule: $R_t = \rho R_{t-1} + (1-\rho)[\phi_\pi \pi_t + \phi_y \hat{Y}_t] + \varepsilon_t^r$

-Money Growth Rule: $\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_t^m$

3.3. Exogenous Processes

- TFP shock: $\ln A_t = (1 - \rho_A)\ln A_{ss} + \rho_A \ln A_{t-1} + \varepsilon_t^A$
- Monetary policy shocks ($\varepsilon_t^r, \varepsilon_t^m$

3.4. Equilibrium and Model Closure

- Output Gap: $\hat{Y}_t = Y_t - Y_t^n$ (natural rate output)
- Market clearing conditions
- Determinacy Requirements: Blanchard-Kahn conditions for policy rules

4. Solution Strategy

4.1. Steady State Derivation

4.2. Log-Linearization Techniques

4.3. Determinacy Analysis

5. The Steady State

Taylor principle verification ($\phi_\pi > 1$)

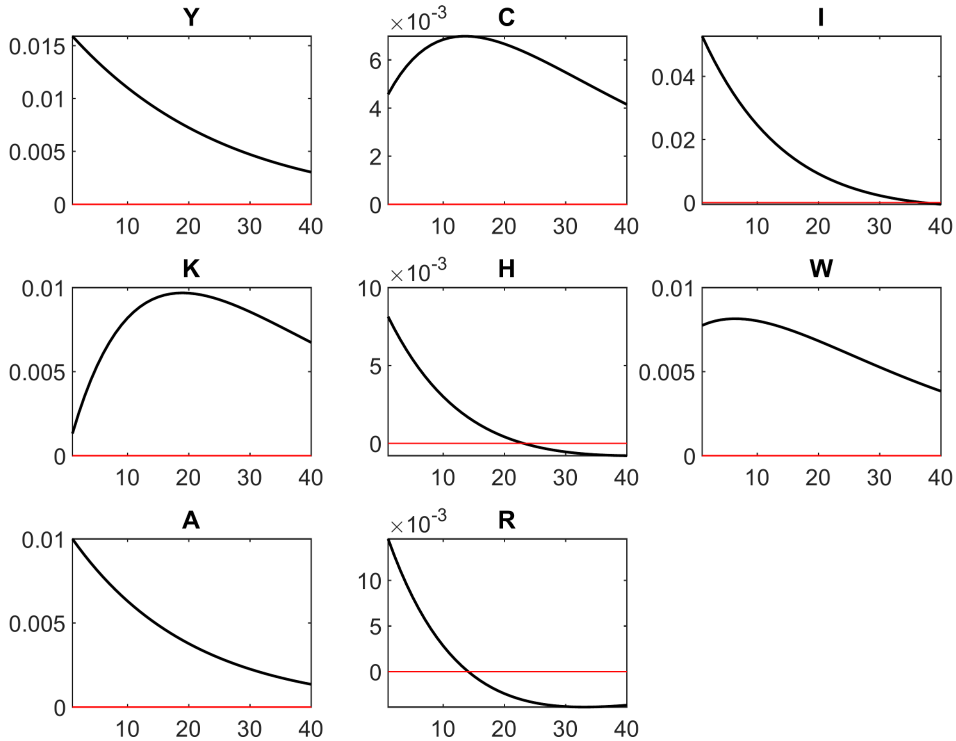

Figure 5.1: image

6. Parameterization

6.1. Calibration Table

6.2. Data Alignment

7. Quantitative Analysis

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## [1] "works"
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References

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