Macroeconomics Assignment

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Abstract

Abstract to be written here. The abstract should not be too long and should provide the reader with a good understanding what you are writing about. Academic papers are not like novels where you keep the reader in suspense. To be effective in getting others to read your paper, be as open and concise about your findings here as possible. Ideally, upon reading your abstract, the reader should feel he / she must read your paper in entirety.

Start writing about what you are planning to do, note that this is an assignment and what it is (introduction vibes)

Please give credit: I haved used an R package from Mati (2019)

1. Model Specification

Core RBC Foundations

Above could note be a heading on itself. Just a paragraph that it is a core rbc foundations and what that means.

Note that capital letters denote nominal amounts and lowercase denote real values.

$$C_t \;=\; P_t\, c_t, \qquad I_t \;=\; P_t\, i_t \tag{1.1}$$

$$\max_{\{C_t,\, h_t,\, M_t,\, B_t,\, K_t,\, I_t\}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{\left(\frac{C_t}{P_t} - \eta\, \frac{C_{t-1}}{P_{t-1}}\right)^{1-\theta}}{1-\theta} \;-\; \chi\, \frac{h_t^{1+\gamma}}{1+\gamma} \;+\; \psi\, \ln\!\left(\frac{M_t}{P_t}\right) \right] \tag{1.2}$$

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$$C_t \;+\; I_t \;+\; B_t \;+\; M_t \;\le\; R^B_{t-1}\,B_{t-1} \;+\; M_{t-1} \;+\; W_t\,h_t \;+\; R^k_t\,K_{t-1} \;+\; \Pi_t \;-\; P_t\,\tau_t \tag{1.3}$$

$$K_t \;=\; (1-\delta)\,K_{t-1} \;+\; I_t \;-\; \frac{\phi}{2}\left(\frac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1} \tag{1.4}$$

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2. Appendix

2.1. Household First Order Conditions

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \begin{cases} \dfrac{\left(\frac{C_t}{P_t} - \eta\frac{C_{t-1}}{P_{t-1}}\right)^{1-\theta}}{1-\theta} - \chi\dfrac{h_t^{1+\gamma}}{1+\gamma} + \psi\ln\left(\dfrac{M_t}{P_t}\right) \\ + \lambda_t\left[R^B_{t-1}B_{t-1} + M_{t-1} + W_t h_t + R^k_t K_{t-1} + \Pi_t - P_t\tau_t - C_t - I_t - B_t - M_t\right] \\ + \mu_t\left[(1-\delta)K_{t-1} + I_t - \dfrac{\phi}{2}\left(\dfrac{I_t}{K_{t-1}} - \delta\right)^2 K_{t-1} - K_t\right] \end{cases} \tag{2.1}$$

Key components:

λ_t: Lagrange multiplier for the nominal flow constraint (1.3)

μ_t: Lagrange multiplier for the capital accumulation constraint (1.4)

Constraints are embedded within the period-t terms of the infinite sum.

$$\frac{\partial\mathcal{L}}{\partial C_t} = 0 \quad\Rightarrow\quad \Big[(c_t - \eta c_{t-1})^{-\theta}/P_t - \lambda_t\Big] - \beta\mathbb{E}_t\Big[\eta\,(c_{t+1} - \eta c_t)^{-\theta}/P_t\Big] = 0,$$

Combining terms:

$$\frac{1}{P_t}\Big[(c_t - \eta c_{t-1})^{-\theta} - \beta\eta\,\mathbb{E}_t(c_{t+1} - \eta c_t)^{-\theta}\Big] - \lambda_t = 0,$$

Multiply by P_t:

$$(c_t - \eta c_{t-1})^{-\theta} - \beta\eta\,\mathbb{E}_t\big[(c_{t+1} - \eta c_t)^{-\theta}\big] - \lambda_t P_t = 0.$$

$$\boxed{\lambda_t P_t \;=\; (c_t - \eta c_{t-1})^{-\theta} \;-\; \beta\eta\,\mathbb{E}_t\big[(c_{t+1} - \eta c_t)^{-\theta}\big]} \tag{2.2}$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \quad \Rightarrow \quad \lambda_t W_t = \chi h_t^{\gamma}.$$

$$\boxed{\lambda_t W_t = \chi h_t^{\gamma}} \tag{2.3}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial M_t} &= \beta^{t-1}\left[\psi \cdot \frac{\partial}{\partial M_t}\left(\ln \frac{M_t}{P_t}\right) + \lambda_t \cdot \frac{\partial}{\partial M_t}(-M_t)\right] \\
&\quad + \beta^t \mathbb{E}_t\left[\lambda_{t+1} \cdot \frac{\partial}{\partial M_t}(M_t)\right] \\
&= \beta^{t-1}\left[\psi \cdot \frac{1}{M_t} - \lambda_t\right] + \beta^t \mathbb{E}_t\left[\lambda_{t+1}\right] \\
&= 0 \quad \Rightarrow \quad \psi \frac{1}{M_t} - \lambda_t + \beta \mathbb{E}_t \lambda_{t+1} = 0
\end{aligned}$$

$$\boxed{\frac{\psi}{M_t} = \lambda_t - \beta \mathbb{E}_t \lambda_{t+1}} \tag{2.4}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial B_t} &= \beta^{t-1}\left[\lambda_t \cdot \frac{\partial}{\partial B_t}(-B_t)\right] \\
&\quad + \beta^t \mathbb{E}_t\left[\lambda_{t+1} \cdot \frac{\partial}{\partial B_t}(R_t^B B_t)\right] \\
&= \beta^{t-1}\left[-\lambda_t\right] + \beta^t \mathbb{E}_t\left[\lambda_{t+1} R_t^B\right]
\end{aligned}$$

Setting the derivative equal to zero and simplifying:

$$\begin{aligned}
-\beta^{t-1}\lambda_t + \beta^t\mathbb{E}_t\left[\lambda_{t+1}R_t^B\right] &= 0 \\
\beta^{t-1}\left(-\lambda_t + \beta\mathbb{E}_t\left[\lambda_{t+1}R_t^B\right]\right) &= 0 \\
-\lambda_t + \beta\mathbb{E}_t\left[\lambda_{t+1}R_t^B\right] &= 0 \quad \text{(divide through by } \beta^{t-1} \neq 0\text{)}
\end{aligned}$$

Final Euler equation for bonds:

$$\boxed{\lambda_t = \beta\mathbb{E}_t\left[\lambda_{t+1}R_t^B\right]} \tag{2.5}$$

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial K_t} &= \beta^{t-1}(-\mu_t) + \beta^t\,\mathbb{E}_t\Big[\lambda_{t+1}R_{t+1}^k + \mu_{t+1}\Big((1-\delta) + \frac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2)\Big)\Big], \\
0 &= -\mu_t + \beta\,\mathbb{E}_t\Big[\lambda_{t+1}R_{t+1}^k + \mu_{t+1}\Big(1-\delta + \frac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2)\Big)\Big], \\
\mu_t &= \beta\,\mathbb{E}_t\Big[\lambda_{t+1}R_{t+1}^k + \mu_{t+1}\Big(1-\delta + \frac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2)\Big)\Big].
\end{aligned}$$

$$\boxed{\mu_t = \beta\,\mathbb{E}_t\Big[\lambda_{t+1}R_{t+1}^k + \mu_{t+1}\Big(1-\delta + \tfrac{\phi}{2}((I_{t+1}/K_t)^2 - \delta^2)\Big)\Big]} \tag{2.6}$$

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial I_t} &= \beta^{t-1}\left[-\lambda_t + \mu_t\left(1 - \phi\left(\frac{I_t}{K_{t-1}} - \delta\right)\right)\right] \\
&= 0
\end{aligned}$$

$$\boxed{\lambda_t = \mu_t\left(1 - \phi\left(\frac{I_t}{K_{t-1}} - \delta\right)\right)} \tag{2.7}$$

new formatting of wack

Consumption Euler Equation

Combines consumption–habit dynamics with bond returns (from (2.5) and (2.2)) :

Step 1: Start with FOC for Bonds

$\lambda_t = \beta\,\mathbb{E}_t[\lambda_{t+1} R_t^B]$ (Equation 2.5)

Step 2: Substitute λ_t and λ_{t+1} from FOC for Consumption

$$\lambda_t = \frac{(c_t - \eta\, c_{t-1})^{-\theta}}{P_t} \quad \text{(from Equation 2.2 rearranged)}$$

$$\lambda_{t+1} = \frac{(c_{t+1} - \eta\, c_t)^{-\theta}}{P_{t+1}} \quad \text{(time-shifted)}$$

Step 3: Combine results

$$\frac{(c_t - \eta\, c_{t-1})^{-\theta}}{P_t} = \beta\,\mathbb{E}_t\left[R_t^B \cdot \frac{(c_{t+1} - \eta\, c_t)^{-\theta}}{P_{t+1}}\right]$$

Step 4: Clear denominator

$$(c_t - \eta\, c_{t-1})^{-\theta} = \beta\,\mathbb{E}_t\left[R_t^B \cdot \frac{P_t}{P_{t+1}} \cdot (c_{t+1} - \eta\, c_t)^{-\theta}\right]$$

$$\boxed{(c_t - \eta\, c_{t-1})^{-\theta} \;=\; \beta\,\mathbb{E}_t\Big[R_t^B\, \frac{P_t}{P_{t+1}}\, (c_{t+1} - \eta\, c_t)^{-\theta}\Big]} \tag{2.8}$$

Interpretation: Marginal rate of substitution between current and future consumption equals the expected real bond return. Habit persistence (η) links today's utility to past and future consumption, and inflation (P_t/P_{t+1}) scales the real payoff on bonds.

Labour Supply

Real wage equals the marginal rate of substitution between leisure and consumption (from (2.2) and (2.3)) :

Step 1: Start with FOC for Hours Worked

$\lambda_t W_t = \chi\, h_t^{\gamma}$ (Equation 2.3)

Step 2: Solve for λ_t

$\lambda_t = \dfrac{\chi\, h_t^{\gamma}}{W_t}$

Step 3: Equate to FOC of Consumption expression

$\dfrac{\chi\, h_t^{\gamma}}{W_t} = \dfrac{(c_t - \eta\, c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t[(c_{t+1} - \eta\, c_t)^{-\theta}]}{P_t}$

Step 4: Solve for real wage (W_t/P_t)

$\dfrac{W_t}{P_t} = \dfrac{\chi\, h_t^{\gamma}}{(c_t - \eta\, c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t[(c_{t+1} - \eta\, c_t)^{-\theta}]}$

$$\boxed{\frac{W_t}{P_t} = \frac{\chi\, h_t^{\gamma}}{(c_t - \eta\, c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t\big[(c_{t+1} - \eta\, c_t)^{-\theta}\big]}} \tag{2.9}$$

Interpretation: Habit formation reduces effective marginal utility of consumption, so stronger habits ($\eta \uparrow$) or more elastic labour supply ($\gamma \uparrow$) require a higher real wage to induce the same hours.

Money Demand

Opportunity cost of holding money vs. bonds (from (2.4), (2.5) and (2.2)) :

Step 1: Combine FOC for Money and Bonds

$$\frac{\psi}{M_t} = \lambda_t - \beta\,\mathbb{E}_t[\lambda_{t+1}] \quad \text{(Equation 2.4)}$$

$$\lambda_t = \beta\,\mathbb{E}_t[\lambda_{t+1} R_t^B] \quad \text{(Equation 2.5)}$$

Step 2: Substitute λ_t into FOC of money

$$\frac{\psi}{M_t} = \beta\,\mathbb{E}_t[\lambda_{t+1} R_t^B] - \beta\,\mathbb{E}_t[\lambda_{t+1}]$$

$$\frac{\psi}{M_t} = \beta\,\mathbb{E}_t\left[\lambda_{t+1}(R_t^B - 1)\right]$$

Step 3: Substitute λ_{t+1} from FOC of Consumption

$$\lambda_{t+1} = \frac{(c_{t+1} - \eta\,c_t)^{-\theta} - \beta\,\eta\,\mathbb{E}_{t+1}[(c_{t+2} - \eta\,c_{t+1})^{-\theta}]}{P_{t+1}}$$

Step 4: Solve for M_t

$$M_t = \frac{\psi}{\beta\,\mathbb{E}_t\left[(R_t^B - 1)\cdot \dfrac{(c_{t+1} - \eta\,c_t)^{-\theta} - \beta\,\eta\,\mathbb{E}_{t+1}[(c_{t+2} - \eta\,c_{t+1})^{-\theta}]}{P_{t+1}}\right]}$$

$$\boxed{M_t \;=\; \frac{\psi}{\beta\,\mathbb{E}_t\Big[(R_t^B - 1)\;\cdot\;\dfrac{(c_{t+1} - \eta\,c_t)^{-\theta}\;-\;\beta\,\eta\,\mathbb{E}_{t+1}[(c_{t+2} - \eta\,c_{t+1})^{-\theta}]}{P_{t+1}}\Big]}} \tag{2.10}$$

Interpretation: Higher expected nominal rates $(R_t^B - 1)$ raise the opportunity cost of money, while habits and inflation expectations shape the curvature of demand. The denominator captures the liquidity premium adjusted for consumption dynamics.

Capital Euler Equation Defines Tobin's q and links required returns on capital to bond returns (from (2.7), (2.6) and (2.5)) :

Step 1: Define Tobin's q from FOC for Investment

$\lambda_t = \mu_t q_t \quad \text{where} \quad q_t \equiv 1 - \phi\left(\frac{I_t}{K_{t-1}} - \delta\right)$

Step 2: Rearrange FOC for Capital

$\mu_t = \beta\,\mathbb{E}_t\Big[\lambda_{t+1} R^k_{t+1} + \mu_{t+1}\Big(1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]\Big)\Big]$

Step 3: Substitute $\mu_t = \lambda_t/q_t$ and $\mu_{t+1} = \lambda_{t+1}/q_{t+1}$

$$\frac{\lambda_t}{q_t} = \beta\,\mathbb{E}_t\left[\lambda_{t+1} R^k_{t+1} + \frac{\lambda_{t+1}}{q_{t+1}}\left(1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]\right)\right]$$

Step 4: Factor λ_{t+1}

$$\frac{\lambda_t}{q_t} = \beta\,\mathbb{E}_t\left[\lambda_{t+1}\left(R^k_{t+1} + \frac{1}{q_{t+1}}\left(1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]\right)\right)\right]$$

Step 5: Substitute FOC for Bonds ($\lambda_t = \beta\,\mathbb{E}_t[\lambda_{t+1} R^B_t]$)

$$\frac{\beta\,\mathbb{E}_t[\lambda_{t+1} R^B_t]}{q_t} = \beta\,\mathbb{E}_t\left[\lambda_{t+1}\left(R^k_{t+1} + \frac{1}{q_{t+1}}\Gamma_{t+1}\right)\right]$$

where $\Gamma_{t+1} \equiv 1 - \delta + \frac{\phi}{2}\left[(I_{t+1}/K_t)^2 - \delta^2\right]$

$$\boxed{\begin{aligned} q_t \;&\equiv\; 1 - \phi\left(\tfrac{I_t}{K_{t-1}} - \delta\right) \\ \frac{\beta\,\mathbb{E}_t[\lambda_{t+1}\, R^B_t]}{q_t} \;&=\; \beta\,\mathbb{E}_t\Big[\lambda_{t+1}\Big(R^k_{t+1} + \frac{1}{q_{t+1}}\Big(1 - \delta + \tfrac{\phi}{2}\big[(I_{t+1}/K_t)^2 - \delta^2\big]\Big)\Big)\Big] \end{aligned}} \tag{2.11}$$

Interpretation: The bond-return-scaled discount factor divided by q_t equals expected return on capital plus adjustment-cost terms. If $I_t/K_{t-1} > \delta$, then $q_t > 1$ signals profitable expansion; disinvestment flips the sign. Adjustment costs (ϕ) create investment frictions.

Components breakdown:

Expenditures:

Consumption: $P_t c_t$

Investment: $P_t i_t$

Bonds: B_t

Money holdings: M_t

Income sources:

Bond returns: $(1 + i_{t-1})B_{t-1}$

Money carryover: M_{t-1}

Labor income: $W_t h_t$

Capital returns: $R_t^k K_{t-1}$ (Key addition missing in Sims)

Firm profits: Π_t

Net transfers: $-P_t \tau_t$

The utility function above (**??**) must be maximised subject to some sort of flow constraint. Note that the flow budget is undefined as of now because i am unsure if captial shows up there (which it should), bonds must too, hours worked and consumption (taxes too surely?)

$$K_t = (1 - \delta)K_{t-1} + i_t - \frac{\phi}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1} \tag{2.12}$$

Key Improvements over Sims Capital integration:

Explicit rental rate R_t^k for capital services

Physical capital stock K_t in accumulation process

Convex adjustment costs ($\phi > 0$)

Real money balances:

Maintains money-in-utility (MIU) specification

Consistent with Walsh (2010) framework

Habit persistence:

$c_t - \eta c_{t-1}$ with $\eta \in (0,1)$

Generates consumption inertia matching SA data

2.2. Firm Sector with Nominal Rigidities

Production function with capital:

$$Y_t(i) = A_t K_t(i)^\alpha H_t(i)^{1-\alpha}, \quad \alpha \in (0,1) \tag{2.13}$$

Cost minisations:

$$\min_{K_t(i),H_t(i)} R_t^k K_t(i) + W_t H_t(i) \quad \text{s.t.} \quad Y_t(i) = A_t K_t(i)^\alpha H_t(i)^{1-\alpha} \tag{2.14}$$

2.2.1. Perfect competion final goods firl

CES aggreagation:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{2.15}$$

demand function:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \tag{2.16}$$

2.2.2. Pricing setting (calvo)

$$P_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \lambda_{t+k} MC_{t+k} P_{t+k}^{\epsilon} Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta)^k \lambda_{t+k} P_{t+k}^{\epsilon-1} Y_{t+k}} \tag{2.17}$$

Price index dynamics:

$$P_t^{1-\epsilon} = \theta P_{t-1}^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon} \tag{2.18}$$

devidenet distribution:

$$\Pi_t = \int_0^1 \left[P_t(i)Y_t(i) - W_t H_t(i) - R_t^k K_t(i) \right] di \tag{2.19}$$

Aggregate equivalent:

$$\Pi_t = P_t Y_t - W_t H_t - R_t^k K_t \tag{2.20}$$

$Y_t = A_t K_t^\alpha H_t^{1-\alpha}$ Cobb-Douglas

Capital accumulation: $K_{t+1} = (1-\delta)K_t + I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t$

2.2.3. Nominal Rigidities

Probability $\theta = 0.75$ of price non-adjustment

Phillips Curve derivation: $\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t$

Dividend specification: $\Pi_t = Y_t - w_t h_t - r_t^k k_t$

2.3. Government Sector

Fiscal rule: $T_t = \tau Y_t$ (lump-sum taxes)

Monetary authority: - Taylor Rule: $R_t = \rho R_{t-1} + (1-\rho)[\phi_\pi \pi_t + \phi_y \hat{Y}_t] + \varepsilon_t^r$

-Money Growth Rule: $\ln \mu_t = \rho_\mu \ln \mu_{t-1} + \varepsilon_t^m$

2.4. Exogenous Processes

- TFP shock: $\ln A_t = (1-\rho_A)\ln A_{ss} + \rho_A \ln A_{t-1} + \varepsilon_t^A$
- Monetary policy shocks $(\varepsilon_t^r, \varepsilon_t^m$

2.5. Equilibrium and Model Closure

- Output Gap: $\hat{Y}_t = Y_t - Y_t^n$ (natural rate output)
- Market clearing conditions
- Determinacy Requirements: Blanchard-Kahn conditions for policy rules

3. Solution Strategy

3.1. Steady State Derivation

3.2. Log-Linearization Techniques

3.3. Determinacy Analysis

Taylor principle verification ($\phi_\pi > 1$)

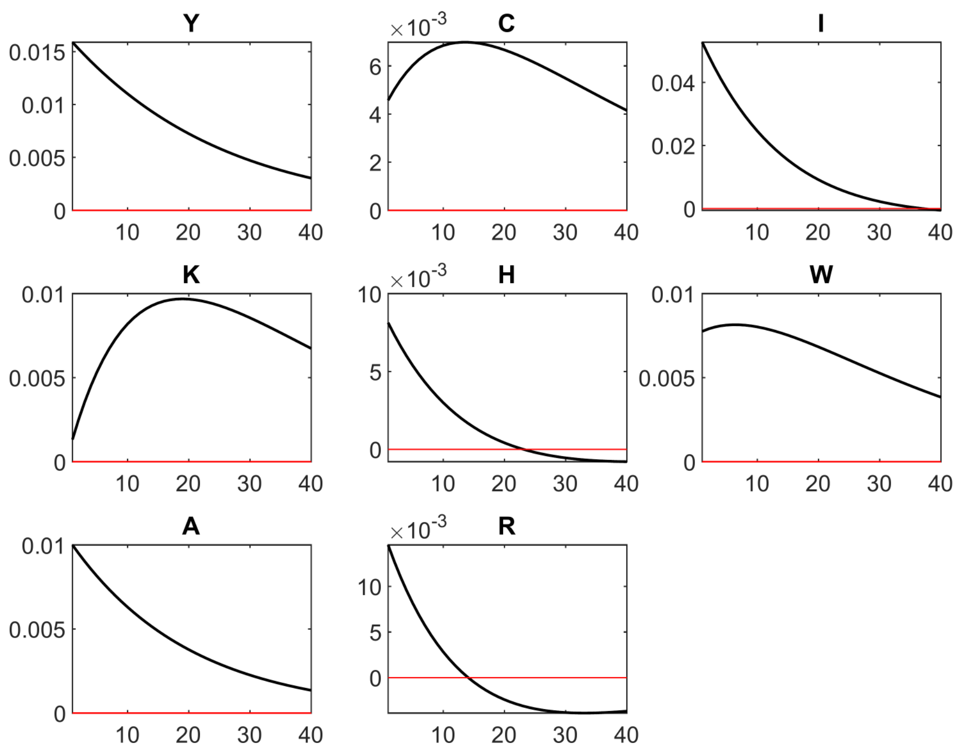

Figure 3.1: image

4. Parameterization

4.1. Calibration Table

4.2. Data Alignment

5. Quantitative Analysis

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## [1] "works"
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References

Mati, S. 2019. DynareR: Bringing the power of Dynare to R, R Markdown, and Quarto. *CRAN*. [Online], Available: https://CRAN.R-project.org/package=DynareR.