Macroeconomics Assignment

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Abstract

Almost an abstract

I have used an R package from Mati (2019) and should give credit. This allowed me to code everything in R while still using Dynare.

$$r^B_{t-1} = R^B_{t-1}/\pi_t$$

1. Model Specification

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{1.1}$$

$$c_t = \frac{C_t}{P_t}, \quad i_t = \frac{I_t}{P_t}, \quad g_t = \frac{G_t}{P_t}, \quad b_t = \frac{B_t}{P_t} \tag{1.2}$$

Above could note be a heading on itself. Just a paragraph that it is a core rbc foundations and what that means.

As with (1.2), it is a general rule that capital letters denote nominal amounts and lowercase denote real values, however to follow convention K_t denotes real capital.

Mainly adding capital to Sims (2024a), while also introducing adjustment costs from Sims (2024b) . Rotemberg prices was used while gleeming insights from European Central Bank (2022) .

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1.1. Households

c_t, h_t, m_t, b_t, K_t, i_t are all real values for consumption, hours worked, money, bonds, capital, and investment.

$$\max_{\{c_t, h_t, m_t, b_t, K_t, i_t\}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\frac{\Big(c_t - \eta\, c_{t-1}\Big)^{1-\theta}}{1-\theta} \;-\; \chi\, \frac{h_t^{1+\gamma}}{1+\gamma} \;+\; \psi\, \ln\!\Big(m_t\Big) \right] \tag{1.3}$$

$$c_t + i_t + b_t + m_t \leq r^B_{t-1} b_{t-1} + \frac{m_{t-1}}{\pi_t} + w_t h_t + r^k_t K_{t-1} + \Pi^r_t - \tau_t \tag{1.4}$$

$$K_t \;=\; (1-\delta)\, K_{t-1} \;+\; i_t \;-\; \frac{\phi_k}{2} \left(\frac{i_t}{K_{t-1}} - \delta \right)^2 K_{t-1} \tag{1.5}$$

Households are the owners of all intermediate-goods firms and receive the aggregate profits as lump-sum income transfers each period. Denoting aggregate real profits by Π^r_t, we can write

$$\Pi^r_t = Y_t - r^k_t K_{t-1} - w_t h_t - \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t \tag{1.6}$$

where $P_t Y_t$ is nominal revenue, $R^k_t K_{t-1}$ and $W_t h_t$ are total nominal factor payments, and the last term captures Rotemberg-style price adjustment costs in nominal terms.

Households maximize expected lifetime utility over consumption and labour supply, taking all after-tax income—wages, rental income, bond returns, and profits—as given. In particular, we treat firm profits Π_t as a lump-sum transfer from firms that enters the budget constraint passively; households do not choose or time their utility over firm profits, nor do they form expectations about future payouts.

From which we can derive the following first order conditions, found in full form in Appendix 8.1.1:

$$\lambda_t = (c_t - \eta c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t\big[(c_{t+1} - \eta c_t)^{-\theta}\big] \tag{1.7}$$

$$\lambda_t w_t = \chi h_t^{\gamma} \tag{1.8}$$

$$\lambda_t = \beta \, \mathbb{E}_t \left[\lambda_{t+1} r_t^B \right] \tag{1.9}$$

$$\frac{\psi}{m_t} = \lambda_t - \beta \, \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \right] \tag{1.10}$$

$$\lambda_t = \mu_t \left(1 - \phi_k \left(\frac{i_t}{K_{t-1}} - \delta \right) \right) \tag{1.11}$$

$$\mu_t = \beta \, \mathbb{E}_t \left[\lambda_{t+1} r_{t+1}^k + \mu_{t+1} \left((1 - \delta) - \frac{\phi_k}{2} \left(\delta^2 - \left(\frac{i_{t+1}}{K_t} \right)^2 \right) \right) \right] \tag{1.12}$$

1.2. Production

This model departs from standard RBC frameworks by introducing monopolistic competition and nominal rigidities. This is achieved through two layers of firms. The first being a perfectly competitive final goods producers, who aggregate intermediate goods, and the second being a monopolistically competitive intermediate goods producers with price-setting power. This two-tiered structure captures core New Keynesian dynamics, such as price stickiness and strategic pricing, while retaining analytical tractability.

1.2.1. Final Goods Producer

The final goods firm combines differentiated inputs $Y_t(j)$ into final output Y_t using a CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1 \tag{1.13}$$

Where ϵ captures the elasticity of substitution between varieties: the higher it is, the more easily final goods producers can substitute across inputs. Conversely, a lower ϵ implies that intermediate producers face less competition and enjoy greater market power.

Given the CES aggregator in Equation (1.13), the final goods producer chooses input varieties to minimize the cost of delivering one unit of output. The resulting demand and pricing relationships, derived in Appendix 8.2.1, are:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t \tag{1.14}$$

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{1.15}$$

Equation (1.14) shows that more expensive varieties are purchased in smaller quantities, while Equation (1.15) reflects the minimum cost of assembling one unit of final output given prevailing input prices.

1.2.2. Intermediate Goods Producers

Each intermediate firm j produces with identical Cobb-Douglas technology:

$$Y_t(j) = A_t K_t(j)^\alpha h_t(j)^{1-\alpha} \tag{1.16}$$

Cost minimization yields capital and labour demand conditions as well as the marginal cost:

$$r_t^k = mc_t \alpha A_t K_t(j)^{\alpha-1} h_t(j)^{1-\alpha} \tag{1.17}$$

$$w_t = mc_t(1-\alpha) A_t K_t(j)^\alpha h_t(j)^{-\alpha} \tag{1.18}$$

$$\mathrm{mc}_t = \frac{1}{A_t}\left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{1.19}$$

Equation (1.19), which is derived in Appendix 8.2.2, captures the firm's cost of producing one unit of output, given factor prices and technology. A rise in wages or rental rates increases marginal cost, while higher productivity A_t reduces it.

Intermediate firms set prices subject to Rotemberg-style adjustment costs. The firm chooses a price path to maximize the expected discounted sum of profits, which includes a quadratic penalty for deviating from past prices.

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left[\underbrace{\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j)}_{\text{real revenue}} - \underbrace{\mathrm{mc}_{t+s} \cdot Y_{t+s}(j)}_{\text{real cost}} - \underbrace{\frac{\phi_p}{2}\left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1\right)^2 Y_{t+s}}_{\text{price adjustment cost}} \right] \tag{1.20}$$

In equation (1.20), $\Lambda_{t,t+s} = \beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the stochastic discount factor, capturing how households value future profits relative to today, where λ_t is the marginal utility of consumption. The parameter $\phi_p > 0$ is the magnitude of price adjustment costs - a higher ϕ_p implies greater penalties for changing prices rapidly. Finally, $Y_{t+s}(j)$ is the firm-specific demand, which depends on the relative price $\frac{P_{t+s}(j)}{P_{t+s}}$ and aggregate output Y_{t+s}, as derived from the CES demand curve in Equation (1.14).

Each intermediate firm j chooses its price path to maximize the expected discounted sum of real profits, taking into account Rotemberg-style quadratic adjustment costs. In symmetric equilibrium—

where $P_t(j) = P_t$ for all j and $\pi_t \equiv P_t/P_{t-1}$—the first-order condition collapses into the familiar Rotemberg Phillips Curve (see Appendix 8.2.2 for the full derivation):

$$(\pi_t - 1)\pi_t = \frac{\epsilon}{\phi_p}\left(mc_t - \frac{\epsilon-1}{\epsilon}\right) + \beta\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{Y_t}(\pi_{t+1}-1)\pi_{t+1}\right] \tag{1.21}$$

$$mc_t = \frac{\mathrm{MC}_t}{P_t} \tag{1.22}$$

In this expression, $mc_t = \frac{\mathrm{MC}_t}{P_t}$ denotes the real marginal cost, and $\frac{\epsilon-1}{\epsilon}$ is its flexible-price benchmark. The left-hand side captures current inflation's departure from unity, while the first term on the right relates that departure to real marginal cost gaps scaled by the adjustment-cost parameter ϕ_p. A larger ϕ_p makes prices stickier, damping the response of inflation to cost pressures. The second term brings in expected future inflation, discounted by β, adjusted for households' intertemporal marginal-utility ratio λ_{t+1}/λ_t and relative output growth Y_{t+1}/Y_t. Together, these components link today's inflation dynamics to both real-economic conditions and expectations of tomorrow's price adjustments.

1.3. Government Sector

Government sector is from Sims (2024c)'s notes. Government chooses spending, (term), exogenously. It finances spending with lump-sum taxes and issues new debt.

Gov budget constraint (real)

$$g_t + r^B_{t-1}d_{t-1} \leq d_t + \tau_t \tag{1.23}$$

With r^Bt is the ex-post real gross return for bonds held from time t.

To ensure internal consistency, I introduce a simple government sector. The government issues one-period nominal bonds purchased by households, uses tax revenues to finance an exogenous stream of government spending, and services its debt obligations. The government budget constraint equates the sum of nominal spending and interest payments to the sum of new debt issuance and tax revenues. I assume lump-sum taxation and do not model Ricardian equivalence effects explicitly. Bonds held by households are thus assumed to be government-issued, closing the financial side of the model.

1.4. Monetary Authority

The central bank sets the nominal interest rate R_t according to:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[R_* + \kappa_\pi(\pi_t - \pi_*) + \kappa_y\left(\frac{Y_t - Y_*}{Y_*}\right)\right] + \varepsilon_t^R \tag{1.24}$$

To avoid confusion with adjustment costs, κ is used instead of the standard ϕ

2. Market Clearing and Equilibrium

2.1. Resource Constraint

Goods Market Clearing

$$Y_t = c_t + i_t + g_t + \frac{\phi_k}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + \frac{\phi_p}{2}(\pi_t - 1)^2 Y_t \tag{2.1}$$

Fisher Equation

$$R_t = \underbrace{r_t^B \cdot \mathbb{E}_t[\pi_{t+1}]}_{\text{ex-ante real rate}} + \text{inflation risk premium}$$

assume there is no without risk premium.

$$R_t = r_t^B \cdot \mathbb{E}_t[\pi_{t+1}] \tag{2.2}$$

2.2. Factor Market Clearing

Labour Market Clearing
Aggregate labour demand (from intermediate firms) equals aggregate labour supply (from households):

$$h_t = \int_0^1 h_t(j)\, dj \tag{2.3}$$

$$h_t(j) = h_t \quad \forall j \tag{2.4}$$

Capital Market Clearing Aggregate capital demand (from intermediate firms) equals aggregate capital supply (from households):

$$K_{t-1} = \int_0^1 K_t(j)\, dj \tag{2.5}$$

$$K_t(j) = K_{t-1} \quad \forall j \tag{2.6}$$

Aggregate Production Function In symmetric equilibrium, factor market clearing and the Cobb–Douglas technology imply:

$$Y_t = A_t\, K_{t-1}^{\alpha}\, h_t^{1-\alpha} \tag{2.7}$$

2.3. Definition of Equilibrium

A competitive equilibrium is a set of prices () and allocations () such that (i) household and rm optimality conditions all hold, (ii) the rm hires all the labour and capital supplied by the household, (iii) the household and rm budget constraints hold with equality, and (iv) household bond-holdings equal government debt issuance in all periods (i.e. Bt+1 = Dt+1, and we require that Bt = Dt initially), given values and stochastic processes of Gt and At, as well as initial values of government debt and household bond-holdings, which must be equal (e.g. Bt =Dt).

$$b_t = d_t \tag{2.8}$$

3. Exogenous Processes

Government spending, taxes, and technology evolve according to exogenous AR(1) processes.

Real taxes follow Equation (3.1),

$$\tau_t = (1 - \rho_\tau)\bar{\tau} + \rho_\tau \tau_{t-1} + \varepsilon_t^\tau \tag{3.1}$$

where $\bar{\tau}$ is the steady-state level, ρ_τ controls persistence, and ε_t^τ is a fiscal shock.

Government spending is governed by Equation (3.2),

$$g_t = (1 - \rho_g)\bar{g} + \rho_g g_{t-1} + \varepsilon_t^g \tag{3.2}$$

with similar dynamics: mean reversion around $\bar{g}$ and shocks ε_t^g.

Technology evolves log-linearly as in Equation (3.3),

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a \tag{3.3}$$

ensuring a unit steady-state level and allowing for persistent TFP shocks.

4. Steady State

The equilibrium when prices are fully flexible. Therefore set ϕ_p to 0 and solve. This should set prices opitimally each period without adjustment costs. Inflation dynamics vanish, simplifying the philips curve. Output and the natrual real rate are purely determined by real economic forces (technology, preferences, and capital accumulation). Equations in this section are derived under appendix 8.3.

The equation firms are simplifies to:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left[\underbrace{\frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j)}_{\text{real revenue}} - \underbrace{\frac{\mathrm{MC}_{t+s}}{P_{t+s}} \cdot Y_{t+s}(j)}_{\text{real cost}} \right] \tag{4.1}$$

Thus we get

$$mc_t^* = \frac{\epsilon - 1}{\epsilon} \tag{4.2}$$

Real marginal cost mc_t is constant at $\frac{\epsilon-1}{\epsilon}$ (inverse markup). The natural level of output Y_t^* is the output level consistent with this mc_t in the flexible-price equilibrium.

$$k = \left(\frac{\frac{\epsilon-1}{\epsilon}\, \alpha\, A}{\frac{1}{\beta} - (1-\delta)} \right)^{\frac{1}{1-\alpha}} \tag{4.3}$$

$$w^* = \frac{\epsilon - 1}{\epsilon} (1-\alpha)\, A\, k^{\alpha} \tag{4.4}$$

$$D\, h^* - \bar{g} = \Omega\, (h^*)^{-\gamma/\theta} \tag{4.5}$$

$$K^* = k\, h^* \tag{4.6}$$

$$Y^* = A\,k^\alpha\,h^* \tag{4.7}$$

$$c^* = D\,h^* - \bar{g} \tag{4.8}$$

$$i^* = \delta\,k\,h^* \tag{4.9}$$

$$\Omega = \frac{1}{1-\eta}\left(\frac{(1-\beta\eta)\,w^*}{\chi}\right)^{\frac{1}{\theta}} \tag{4.10}$$

$$D = A\,k^\alpha - \delta\,k \tag{4.11}$$

Equation (4.3) shows the capital-labour ratio. It depends on how productive capital is (adjusted for market power and technology) relative to its return.

Equation (4.4) gives the real wage from labour demand. The first term reflects the markup from monopolistic competition, and the rest is just the marginal product of labour.

Equation (4.5) pins down hours worked by setting labour supply equal to demand. The left side is output minus depreciation and government spending, and the right side captures the disutility of working, adjusted for consumption habits.

Equation (4.6) just says total capital equals capital per worker times hours worked.

Equation (4.7) is the standard production function, written in terms of capital per worker and hours.

Equation (4.8) comes from the resource constraint. Output net of depreciation minus government spending gives consumption.

Equation (4.9) defines investment as depreciation times the capital stock.

Equation (4.10) defines the labour supply shifter. It's a function of wages, habits, and how much people dislike working.

Equation (4.11) is net output per worker—output minus depreciation, both on a per-worker basis.

5. Full set of Conditions

$$c_t + i_t + b_t + m_t \leq r^B_{t-1} b_{t-1} + \frac{m_{t-1}}{\pi_t} + w_t h_t + r^k_t K_{t-1} + \Pi^r_t - \tau_t \tag{1.4}$$

$$K_t \ = \ (1-\delta)\, K_{t-1} \ + \ i_t \ - \ \frac{\phi_k}{2} \left(\frac{i_t}{K_{t-1}} - \delta \right)^2 \, K_{t-1} \tag{1.5}$$

$$\Pi^r_t = Y_t - r^k_t K_{t-1} - w_t h_t - \frac{\phi_p}{2} (\pi_t - 1)^2 Y_t \tag{1.6}$$

$$\lambda_t = (c_t - \eta c_{t-1})^{-\theta} - \beta, \eta, \mathbb{E}t\big[(ct + 1 - \eta c_t)^{-\theta}\big] \tag{1.7}$$

$$h^\gamma_t = \frac{\lambda_t w_t}{\chi} \tag{1.8}$$

$$1 = \beta, \mathbb{E}t \left[\frac{\lambda t + 1}{\lambda_t} r^B_t \right] \tag{1.9}$$

$$\lambda_t = \frac{\psi}{m_t} + \beta, \mathbb{E}t \left[\frac{\lambda t + 1}{\pi_{t+1}} \right] \tag{1.10}$$

$$\mu_t = \lambda_t \left(1 - \phi_k \left(\frac{i_t}{K_{t-1}} - \delta \right) \right)^{-1} \tag{1.11}$$

$$1 = \beta, \mathbb{E}t \left[\frac{\lambda t + 1}{\mu_t} r^k_{t+1} + \frac{\mu_{t+1}}{\mu_t} \left((1-\delta) - \frac{\phi_k}{2} \left(\delta^2 - \left(\frac{i_{t+1}}{K_t} \right)^2 \right) \right) \right] \tag{1.12}$$

$$Y_t = A_t K^\alpha_{t-1} h^{1-\alpha}_t \tag{2.7}$$

$$(\pi_t - 1)\pi_t = \frac{\epsilon}{\phi_p} \left(mc_t - \frac{\epsilon - 1}{\epsilon} \right) + \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\lambda_t} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1)\pi_{t+1} \right] \tag{1.21}$$

$$mc_t = \frac{1}{A_t}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{1.19}$$

$$g_t + r_{t-1}^B d_{t-1} \leq d_t + \tau_t \tag{1.23}$$

$$Y_t = c_t + i_t + g_t + \frac{\phi_k}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + \frac{\phi_p}{2}(\pi_t - 1)^2 Y_t \tag{2.1}$$

$$b_t = d_t \tag{2.8}$$

$$R_t = r_t^B \cdot \mathbb{E}_t[\pi_{t+1}] \tag{5.1}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[R_* + \kappa_\pi(\pi_t - \pi_*) + \kappa_y\left(\frac{Y_t - Y_*}{Y_*}\right)\right] + \varepsilon_t^R \tag{1.24}$$

$$\tau_t = (1-\rho_\tau)\bar{\tau} + \rho_\tau \tau_{t-1} + \varepsilon_t^\tau \tag{3.1}$$

$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \varepsilon_t^g \tag{3.2}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a \tag{3.3}$$

6. Full set of Conditions older

these are the set of conditions i thought i needed when starting to code in dynare

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{1.1}$$

$$c_t + i_t + b_t + m_t \leq r_{t-1}^B b_{t-1} + \frac{m_{t-1}}{\pi_t} + w_t h_t + r_t^k K_{t-1} + \Pi_t^r - \tau_t \tag{1.4}$$

$$K_t \;=\; (1-\delta)\,K_{t-1} \;+\; i_t \;-\; \frac{\phi_k}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1} \tag{1.5}$$

$$\Pi_t^r = Y_t - r_t^k K_{t-1} - w_t h_t - \frac{\phi_p}{2}(\pi_t - 1)^2 Y_t \tag{1.6}$$

$$\lambda_t = (c_t - \eta c_{t-1})^{-\theta} - \beta, \eta, \mathbb{E}t\big[(ct + 1 - \eta c_t)^{-\theta}\big] \tag{1.7}$$

$$h_t^\gamma = \frac{\lambda_t w_t}{\chi} \tag{1.8}$$

$$1 = \beta, \mathbb{E}t\left[\frac{\lambda t + 1}{\lambda_t} r_t^B\right] \tag{1.9}$$

$$\lambda_t = \frac{\psi}{m_t} + \beta, \mathbb{E}t\left[\frac{\lambda t + 1}{\pi_{t+1}}\right] \tag{1.10}$$

$$\mu_t = \lambda_t \left(1 - \phi_k \left(\frac{i_t}{K_{t-1}} - \delta\right)\right)^{-1} \tag{1.11}$$

$$1 = \beta, \mathbb{E}t\left[\frac{\lambda t + 1}{\mu_t} r_{t+1}^k + \frac{\mu_{t+1}}{\mu_t}\left((1-\delta) - \frac{\phi_k}{2}\left(\delta^2 - \left(\frac{i_{t+1}}{K_t}\right)^2\right)\right)\right] \tag{1.12}$$

$$Y_t = A_t K_{t-1}^\alpha h_t^{1-\alpha} \tag{2.7}$$

$$r_t^k = \alpha m c_t \frac{Y_t}{K_{t-1}} \tag*{(*Derived from (1.17))}$$

$$w_t = (1-\alpha) m c_t \frac{Y_t}{h_t} \tag*{(*Derived from (1.18))}$$

$$(\pi_t - 1)\pi_t = \frac{\epsilon}{\phi_p}\left(m c_t - \frac{\epsilon - 1}{\epsilon}\right) + \beta \mathbb{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{Y_{t+1}}{Y_t}(\pi_{t+1} - 1)\pi_{t+1}\right] \tag{1.21}$$

$$mc_t = \frac{1}{A_t}\left(\frac{r_t^k}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{1.19}$$

$$g_t + r_{t-1}^B d_{t-1} \leq d_t + \tau_t \tag{1.23}$$

$$Y_t = c_t + i_t + g_t + \frac{\phi_k}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1} + \frac{\phi_p}{2}(\pi_t - 1)^2 Y_t \tag{2.1}$$

$$b_t = d_t \tag{2.8}$$

$$R_t = r_t^B \cdot \mathbb{E}_t[\pi_{t+1}] \tag{6.1}$$

$$R_t = \rho_R R_{t-1} + (1-\rho_R)\left[R_* + \kappa_\pi(\pi_t - \pi_*) + \kappa_y\left(\frac{Y_t - Y_*}{Y_*}\right)\right] + \varepsilon_t^R \tag{1.24}$$

$$k = \left(\frac{\frac{\epsilon-1}{\epsilon}\,\alpha\,A}{\frac{1}{\beta} - (1-\delta)}\right)^{\frac{1}{1-\alpha}} \tag{4.3}$$

$$w^* = \frac{\epsilon - 1}{\epsilon}\,(1-\alpha)\,A\,k^{\alpha} \tag{4.4}$$

$$D\,h^* - \bar{g} = \Omega\,(h^*)^{-\gamma/\theta} \tag{4.5}$$

$$K^* = k\,h^* \tag{4.6}$$

$$Y^* = A\,k^{\alpha}\,h^* \tag{4.7}$$

$$c^* = D\,h^* - \bar{g} \tag{4.8}$$

$$i^* = \delta\, k\, h^* \tag{4.9}$$

$$\Omega = \frac{1}{1-\eta}\left(\frac{(1-\beta\eta)\, w^*}{\chi}\right)^{\frac{1}{\theta}} \tag{4.10}$$

$$D = A\, k^{\alpha} - \delta\, k \tag{4.11}$$

$$\tau_t = (1-\rho_\tau)\bar{\tau} + \rho_\tau \tau_{t-1} + \varepsilon_t^{\tau} \tag{3.1}$$

$$g_t = (1-\rho_g)\bar{g} + \rho_g g_{t-1} + \varepsilon_t^{g} \tag{3.2}$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^{a} \tag{3.3}$$

7. model

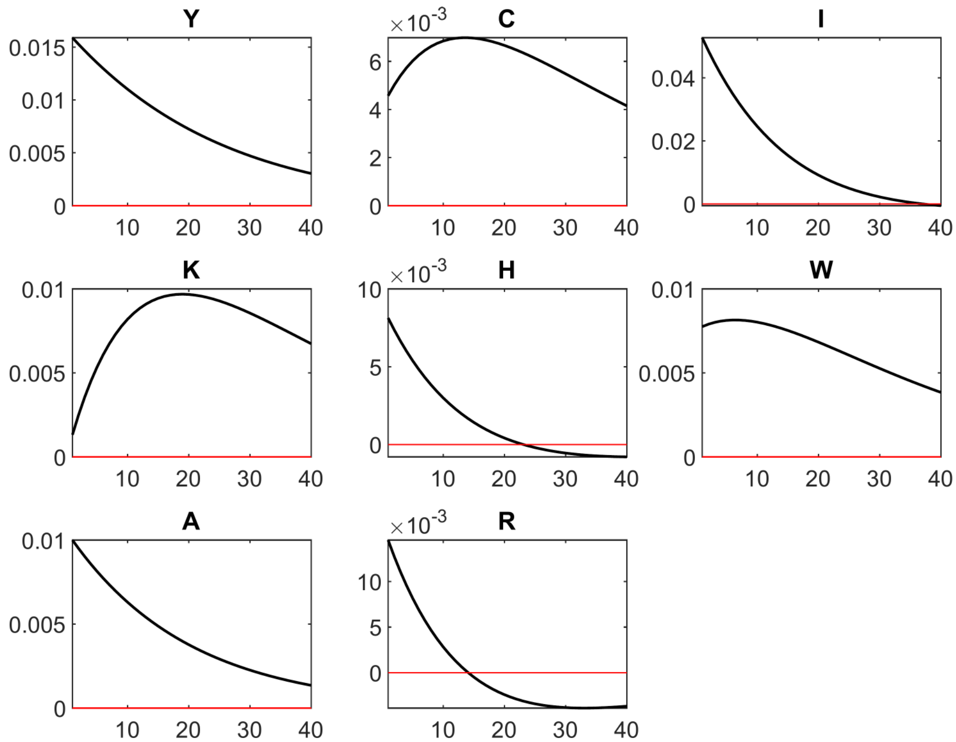

Figure 7.1: image

8. Appendix

8.1. Households

Define Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \Big\{ \underbrace{\frac{(c_t - \eta\, c_{t-1})^{1-\theta}}{1-\theta}}_{\text{Consumption utility}} - \underbrace{\chi \frac{h_t^{1+\gamma}}{1+\gamma}}_{\text{Labour disutility}} + \underbrace{\psi \ln(m_t)}_{\text{Money utility}}$$

$$+ \underbrace{\lambda_t \Big[\frac{R^B_{t-1} b_{t-1}}{\pi_t} + \frac{m_{t-1}}{\pi_t} + w_t h_t + r^k_t K_{t-1} + \Pi^r_t - \tau_t - c_t - i_t - b_t - m_t \Big]}_{\text{Real flow constraint}} + \underbrace{\mu_t \Big[(1-\delta)\, K_{t-1} \; + \; i_t \; - \; \frac{\phi_k}{2} \Big(\frac{i_t}{K_{t-1}}}_{\text{Capital accumulation}}$$

8.1.1. First Order Conditions

FOC w.r.t. Consumption

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0$$

$$\big[(c_t - \eta c_{t-1})^{-\theta} - \lambda_t \big] - \beta\, \eta\, \mathbb{E}_t \big[(c_{t+1} - \eta c_t)^{-\theta} \big] = 0$$

Rearrange

$$\lambda_t = (c_t - \eta c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t \big[(c_{t+1} - \eta c_t)^{-\theta} \big]$$

$$\boxed{\lambda_t = (c_t - \eta c_{t-1})^{-\theta} - \beta\, \eta\, \mathbb{E}_t \big[(c_{t+1} - \eta c_t)^{-\theta} \big]} \tag{8.1}$$

FOC w.r.t. Labour

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0$$

$$-\chi h_t^{\gamma} + \lambda_t w_t = 0$$

Rearrange

$$\lambda_t w_t = \chi h_t^{\gamma}$$

$$\boxed{\lambda_t w_t = \chi h_t^{\gamma}} \tag{8.2}$$

FOC w.r.t. Real Bonds:

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial b_t} = 0 \\
&-\beta^{t-1}\lambda_t + \beta^t\,\mathbb{E}_t\left[\lambda_{t+1}\cdot\frac{R_t^B}{\pi_{t+1}}\right] = 0 \\
&\text{Divide by } \beta^{t-1} \\
&-\lambda_t + \beta\,\mathbb{E}_t\left[\lambda_{t+1}\frac{R_t^B}{\pi_{t+1}}\right] = 0
\end{aligned}$$

$$\boxed{\lambda_t = \beta\,\mathbb{E}_t\left[\lambda_{t+1}\frac{R_t^B}{\pi_{t+1}}\right]} \tag{8.3}$$

FOC w.r.t. Real Money Balances:

$$\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial m_t} = 0 \\
&\beta^{t-1}\left[\frac{\psi}{m_t} - \lambda_t\right] + \beta^t\,\mathbb{E}_t\left[\lambda_{t+1}\cdot\frac{1}{\pi_{t+1}}\right] = 0 \\
&\text{Divide by } \beta^{t-1} \\
&\frac{\psi}{m_t} - \lambda_t + \beta\,\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right] = 0
\end{aligned}$$

$$\boxed{\frac{\psi}{m_t} = \lambda_t - \beta\,\mathbb{E}_t\left[\frac{\lambda_{t+1}}{\pi_{t+1}}\right]} \tag{8.4}$$

FOC w.r.t. Investment:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial i_t} = 0 \\
&\quad \beta^{t-1}\left[-\lambda_t + \mu_t \cdot \frac{\partial}{\partial i_t}\left(i_t - \frac{\phi_k}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1}\right)\right] = 0
\end{aligned}
$$

Compute derivative:

$$
\frac{\partial}{\partial i_t}\left[i_t - \frac{\phi_k}{2}\left(\frac{i_t}{K_{t-1}} - \delta\right)^2 K_{t-1}\right] = 1 - \phi_k\left(\frac{i_t}{K_{t-1}} - \delta\right)
$$

Substitute and simplify:

$$
-\lambda_t + \mu_t\left(1 - \phi_k\left(\frac{i_t}{K_{t-1}} - \delta\right)\right) = 0
$$

$$
\boxed{\lambda_t = \mu_t\left(1 - \phi_k\left(\frac{i_t}{K_{t-1}} - \delta\right)\right)} \tag{8.5}
$$

FOC w.r.t. Capital:

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial K_t} = 0 \\
&\quad \beta^{t-1}(-\mu_t) + \beta^t\,\mathbb{E}_t\left[\lambda_{t+1} r_{t+1}^k + \mu_{t+1} \cdot \frac{\partial}{\partial K_t}\left((1-\delta)K_t + i_{t+1} - \frac{\phi_k}{2}\left(\frac{i_{t+1}}{K_t} - \delta\right)^2 K_t\right)\right] = 0
\end{aligned}
$$

Compute derivative:

$$
\frac{\partial}{\partial K_t}\left[(1-\delta)K_t - \frac{\phi_k}{2}\left(\frac{i_{t+1}}{K_t} - \delta\right)^2 K_t\right] = (1-\delta) - \frac{\phi_k}{2}\left(\delta^2 - \left(\frac{i_{t+1}}{K_t}\right)^2\right)
$$

Substitute and simplify:

$$
-\mu_t + \beta\,\mathbb{E}_t\left[\lambda_{t+1} r_{t+1}^k + \mu_{t+1}\left((1-\delta) - \frac{\phi_k}{2}\left(\delta^2 - \left(\frac{i_{t+1}}{K_t}\right)^2\right)\right)\right] = 0
$$

$$
\boxed{\mu_t = \beta\,\mathbb{E}_t\left[\lambda_{t+1} r_{t+1}^k + \mu_{t+1}\left((1-\delta) - \frac{\phi_k}{2}\left(\delta^2 - \left(\frac{i_{t+1}}{K_t}\right)^2\right)\right)\right]} \tag{8.6}
$$

8.2. Production

8.2.1. Final Good Producer

Derivation of Intermediate Goods Demand and Aggregate Price Index

Final goods producer's profit:

$$\Pi_t = P_t Y_t - \int_0^1 P_t(j) Y_t(j) dj$$

$$\text{subject to } Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

Substitute production function into profit:

$$\Pi_t = P_t \left(\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 P_t(j) Y_t(j) dj$$

First-order condition for $Y_t(j)$:

$$\frac{\partial \Pi_t}{\partial Y_t(j)} = P_t \cdot \frac{\epsilon}{\epsilon - 1} \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} \cdot \frac{\epsilon - 1}{\epsilon} Y_t(j)^{-\frac{1}{\epsilon}} - P_t(j) = 0$$

$$\Rightarrow P_t \cdot Y_t^{\frac{1}{\epsilon}} Y_t(j)^{-\frac{1}{\epsilon}} = P_t(j)$$

Rearrange to obtain demand curve:

$$Y_t(j) = \left(\frac{P_t}{P_t(j)} \right)^{\epsilon} Y_t$$

Substitute demand into production function:

$$Y_t = \left(\int_0^1 \left[\left(\frac{P_t}{P_t(j)} \right)^{\epsilon} Y_t \right]^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

$$= Y_t \left(\int_0^1 \left(\frac{P_t}{P_t(j)} \right)^{\epsilon-1} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

Simplify to obtain price index:

$$1 = \left(\int_0^1 \left(\frac{P_t}{P_t(j)} \right)^{\epsilon - 1} dj \right)^{\frac{\epsilon}{\epsilon - 1}}$$

$$\Rightarrow P_t^{1-\epsilon} = \int_0^1 P_t(j)^{1-\epsilon} dj$$

$$\Rightarrow P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}$$

$$\boxed{\begin{gathered} Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon} Y_t \\ \\ P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \end{gathered}} \tag{8.7}$$

8.2.2. Intermediate Goods Producers

Cost minimization for intermediate firm j :

$$\min_{K_t(j),h_t(j)} \left\{r_t^k K_t(j) + w_t h_t(j)\right\}$$

subject to $Y_t(j) = A_t K_t(j)^\alpha h_t(j)^{1-\alpha}$

Lagrangian:

$$\mathcal{L} = r_t^k K_t(j) + w_t h_t(j) + \lambda_t \left[A_t K_t(j)^\alpha h_t(j)^{1-\alpha} - Y_t(j)\right]$$

First-order conditions:

$$\frac{\partial \mathcal{L}}{\partial K_t(j)} = 0 : \quad r_t^k = \lambda_t \alpha A_t K_t(j)^{\alpha-1} h_t(j)^{1-\alpha}$$

$$\frac{\partial \mathcal{L}}{\partial h_t(j)} = 0 : \quad w_t = \lambda_t (1-\alpha) A_t K_t(j)^\alpha h_t(j)^{-\alpha}$$

Rearrange FOCs:

$$\lambda_t = \frac{r_t^k}{\alpha} \left(\frac{K_t(j)}{h_t(j)}\right)^{1-\alpha} \frac{1}{A_t}, \quad \lambda_t = \frac{w_t}{1-\alpha} \left(\frac{K_t(j)}{h_t(j)}\right)^{\alpha} \frac{1}{A_t}$$

Equate expressions:

$$\frac{r_t^k}{\alpha} \left(\frac{K_t(j)}{h_t(j)}\right)^{-\alpha} = \frac{w_t}{1-\alpha} \left(\frac{K_t(j)}{h_t(j)}\right)^{1-\alpha}$$

$$\Rightarrow \frac{K_t(j)}{h_t(j)} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}$$

Substitute into capital FOC:

$$\lambda_t = \frac{r_t^k}{\alpha A_t} \left(\frac{\alpha}{1-\alpha} \frac{w_t}{r_t^k}\right)^{\alpha-1}$$

$$= \frac{1}{A_t} \left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}$$

$$\boxed{mc_t = \frac{1}{A_t} \left(\frac{r_t^k}{\alpha}\right)^\alpha \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}} \tag{8.8}$$

Intermediate-goods producer's problem:

$$\max_{P_t(j)} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{t,t+s}\Big[\big(\tfrac{P_{t+s}(j)}{P_{t+s}}\big)^{1-\epsilon} Y_{t+s} - mc_{t+s}\,\big(\tfrac{P_{t+s}(j)}{P_{t+s}}\big)^{-\epsilon} Y_{t+s} - \tfrac{\phi_p}{2}\big(\tfrac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1\big)^2 Y_{t+s}\Big]$$

subject to $\quad Y_t(j) = \big(\frac{P_t(j)}{P_t}\big)^{-\epsilon} Y_t$

First-Order Condition w.r.t. $P_t(j)$:

$$\mathbb{E}_t\Big[\frac{\partial \Pi_t(j)}{\partial P_t(j)} + \beta\,\Lambda_{t,t+1}\frac{\partial \Pi_{t+1}(j)}{\partial P_t(j)}\Big] = 0$$

$$\frac{\partial \Pi_t}{\partial P_t(j)} = (1-\epsilon)\big(\tfrac{P_t(j)}{P_t}\big)^{-\epsilon}\tfrac{Y_t}{P_t} + \epsilon\, mc_t\big(\tfrac{P_t(j)}{P_t}\big)^{-\epsilon-1}\tfrac{Y_t}{P_t} - \phi_p\big(\tfrac{P_t(j)}{P_{t-1}(j)} - 1\big)\tfrac{Y_t}{P_{t-1}(j)}$$

$$\frac{\partial \Pi_{t+1}}{\partial P_t(j)} = \phi_p\big(\tfrac{P_{t+1}(j)}{P_t(j)} - 1\big)\,\tfrac{P_{t+1}(j)}{P_t(j)^2}\,Y_{t+1}$$

Impose symmetry: $P_t(j) = P_t,\ Y_t(j) = Y_t,\ \pi_t = \frac{P_t}{P_{t-1}}$.

$$(1-\epsilon) + \epsilon\, mc_t = \epsilon\big(mc_t - \tfrac{\epsilon-1}{\epsilon}\big), \quad \tfrac{P_t(j)}{P_{t-1}(j)} = \pi_t,\ \tfrac{P_{t+1}(j)}{P_t(j)} = \pi_{t+1}$$

$$0 = \epsilon\big(mc_t - \tfrac{\epsilon-1}{\epsilon}\big) - \phi_p\,(\pi_t - 1)\,\pi_t + \beta\,\mathbb{E}_t\Big[\Lambda_{t,t+1}\,\phi_p\,(\pi_{t+1}-1)\,\pi_{t+1}\,\tfrac{Y_{t+1}P_t}{Y_tP_{t+1}}\Big]$$

Noting $\Lambda_{t,t+1} = \beta\,\frac{\lambda_{t+1}}{\lambda_t}$ and $P_{t+1}/P_t = \pi_{t+1}$, the bracket simplifies to $\beta\,\frac{\lambda_{t+1}}{\lambda_t}\,\frac{Y_{t+1}}{Y_t}$.

$$\boxed{0 = \epsilon\Big(mc_t - \tfrac{\epsilon-1}{\epsilon}\Big) - \phi_p\,(\pi_t - 1)\,\pi_t + \beta\,\mathbb{E}_t\Big[\tfrac{\lambda_{t+1}}{\lambda_t}\,\phi_p\,(\pi_{t+1}-1)\,\pi_{t+1}\,\tfrac{Y_{t+1}}{Y_t}\Big]} \tag{8.9}$$

8.3. Steady State

$$\max_{P_t(j)}\ (P_t(j)/P_t)^{1-\epsilon}Y_t \ -\ mc_t\,(P_t(j)/P_t)^{-\epsilon}Y_t$$

$$\frac{\partial}{\partial P_t(j)} : (1-\epsilon)(P_t(j)/P_t)^{-\epsilon}\frac{Y_t}{P_t} \ +\ \epsilon\, mc_t\,(P_t(j)/P_t)^{-\epsilon-1}\frac{Y_t}{P_t} = 0$$

$$\Rightarrow (1-\epsilon) + \epsilon\, mc_t\,(P_t(j)/P_t)^{-1} = 0$$

$$\Rightarrow P_t(j)/P_t = \frac{\epsilon}{\epsilon-1}\,mc_t$$

In equilibrium $P_t(j) = P_t$, so

$$1 = \frac{\epsilon}{\epsilon - 1}\, mc_t$$
$$mc_t^* = \frac{\epsilon - 1}{\epsilon} \tag{8.10}$$

8.3.1. Derivation of Flexible-Price Steady-State Values

$$\begin{aligned}
&\text{Factor Prices and Technology Link}\\
&mc^* = \frac{\epsilon - 1}{\epsilon}\\
&mc^* = \frac{1}{A}\left(\frac{r^{k,*}}{\alpha}\right)^{\alpha}\left(\frac{w^*}{1-\alpha}\right)^{1-\alpha}\\
&\text{Substitute to get}\\
&\frac{\epsilon - 1}{\epsilon} = \frac{1}{A}\left(\frac{r^{k,*}}{\alpha}\right)^{\alpha}\left(\frac{w^*}{1-\alpha}\right)^{1-\alpha}
\end{aligned}$$

$$\begin{aligned}
&\text{Steady-State Real Rates}\\
&r^{B,*} = \frac{1}{\beta}\\
&r^{k,*} = \frac{1}{\beta} - (1-\delta)
\end{aligned}$$

$$\begin{aligned}
&\text{Solve for Wage } w^*\\
&\frac{K^*}{h^*} = \left(\tfrac{mc^*\,\alpha\,A}{r^{k,*}}\right)^{\frac{1}{1-\alpha}}\\
&w^* = mc^*(1-\alpha)\,A\left(\tfrac{K^*}{h^*}\right)^{\alpha}\\
&\text{Substitute to get}\\
&w^* = \frac{\epsilon - 1}{\epsilon}\,(1-\alpha)\,A\left(\tfrac{\frac{\epsilon-1}{\epsilon}\,\alpha\,A}{\frac{1}{\beta}-(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

labour Supply with Habit Formation

$\lambda^* = [c^*(1-\eta)]^{-\theta}(1-\beta\eta)$

$[c^*(1-\eta)]^{-\theta}(1-\beta\eta)\, w^* = \chi\,(h^*)^{\gamma}$

Resource Constraints

$i^* = \delta K^*$

$Y^* = c^* + i^* + \bar{g} = c^* + \delta K^* + \bar{g}$

Solve for Steady-State Values

$Y^* = A(K^*)^{\alpha}(h^*)^{1-\alpha}$

$k \equiv \frac{K^*}{h^*} = \left(\frac{\frac{\epsilon-1}{\epsilon}\,\alpha\, A}{\frac{1}{\beta}-(1-\delta)}\right)^{\frac{1}{1-\alpha}}$

$Y^* = Ak^{\alpha}h^*$

$Ak^{\alpha}h^* = \dfrac{1}{1-\eta}\left(\frac{(1-\beta\eta)\,w^*}{\chi}\right)^{\frac{1}{\theta}}(h^*)^{-\frac{\gamma}{\theta}} + \delta k h^* + \bar{g}$

$\Omega \equiv \dfrac{1}{1-\eta}\left(\frac{(1-\beta\eta)\,w^*}{\chi}\right)^{\frac{1}{\theta}}, \quad D \equiv Ak^{\alpha} - \delta k$

$D\,h^* - \bar{g} = \Omega\,(h^*)^{-\frac{\gamma}{\theta}}$

Monetary Policy Steady State

$\pi^* = \pi_*$

$R^* = r^{B,*}\,\pi^* = \dfrac{\pi_*}{\beta}$

$$\boxed{\begin{gathered} k = \left(\frac{\frac{\epsilon-1}{\epsilon}\,\alpha\, A}{\frac{1}{\beta}-(1-\delta)}\right)^{\frac{1}{1-\alpha}}, \quad w^* = \tfrac{\epsilon-1}{\epsilon}(1-\alpha)\, A\, k^{\alpha}, \\ h^* :\ D\, h^* - \bar{g} = \Omega\,(h^*)^{-\gamma/\theta}, \quad K^* = k\, h^*, \\ Y^* = A\, k^{\alpha}\, h^*, \quad c^* = D\, h^* - \bar{g}, \quad i^* = \delta\, k\, h^*, \\ \Omega = \tfrac{1}{1-\eta}\left(\tfrac{(1-\beta\eta)\, w^*}{\chi}\right)^{\frac{1}{\theta}}, \quad D = A\, k^{\alpha} - \delta\, k \end{gathered}} \tag{8.11}$$

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