Econometrics 871 – Time Series

Tutorial on non-linear models

In this tutorial we will explore the estimation of a few non-linear models on simulated data as well as pre-tests to attempt to verify the presence of the non-linearity. Simulated data corresponds to the ideal case – where data generating process corresponds exactly with our empirical model. We will show that even in this case, with many data points, the estimations are not always able to uncover the process.

Case 1: single threshold TAR(1) model

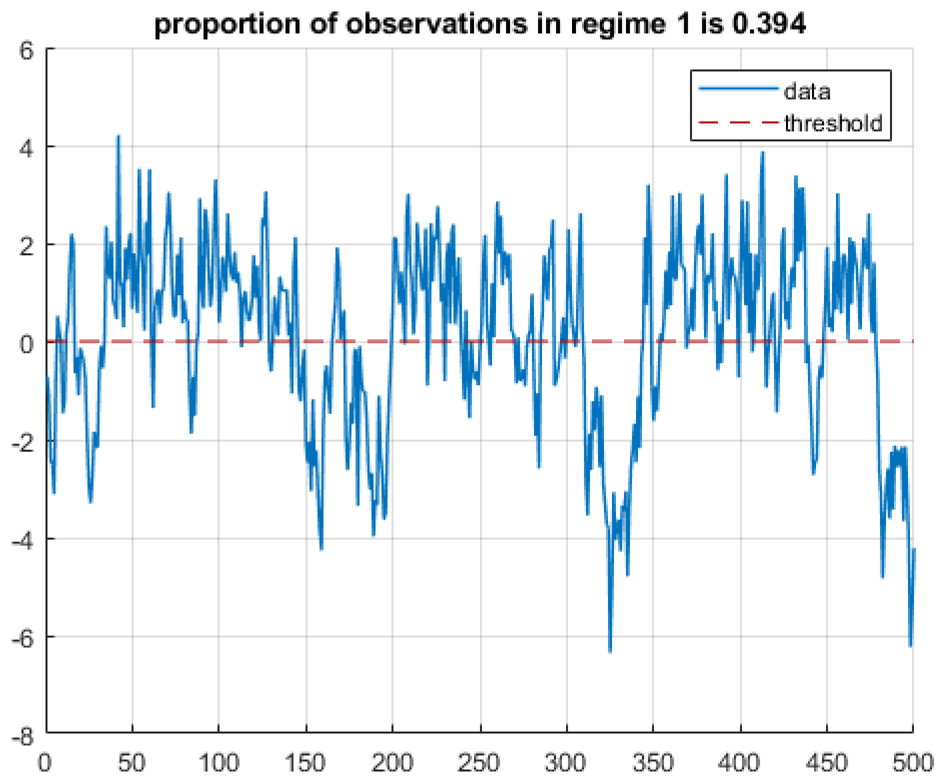
1. **The data**

The data was simulated with the Matlab script `gen_TAR1.m` and follows the data generating process:

$$y_t = \begin{cases} 0 + 0.9y_{t-1} + \varepsilon_t & \text{if } y_{t-1} < 0 \\ 1 + 0.2y_{t-1} + \varepsilon_t & \text{if } y_{t-1} \geq 0 \end{cases}$$

$$\varepsilon_t \sim N(0,1)$$

For the estimation to have any hope of identifying the threshold and coefficients closely, the process must spend enough time in each regime so that there are sufficient data points for each regime. I had to try several times (and experimented with a few sets of parameters) to find a sequence that should work well. Note that I chose to have equal error variances in both regimes, and very different processes. (A true test of this method would be to agnostically generate many samples for each parameter set and do estimations for each one to determine how often the method is likely to yield answers acceptably close to the truth). The variable `single_threshold_tar1` is plotted in the following figure:

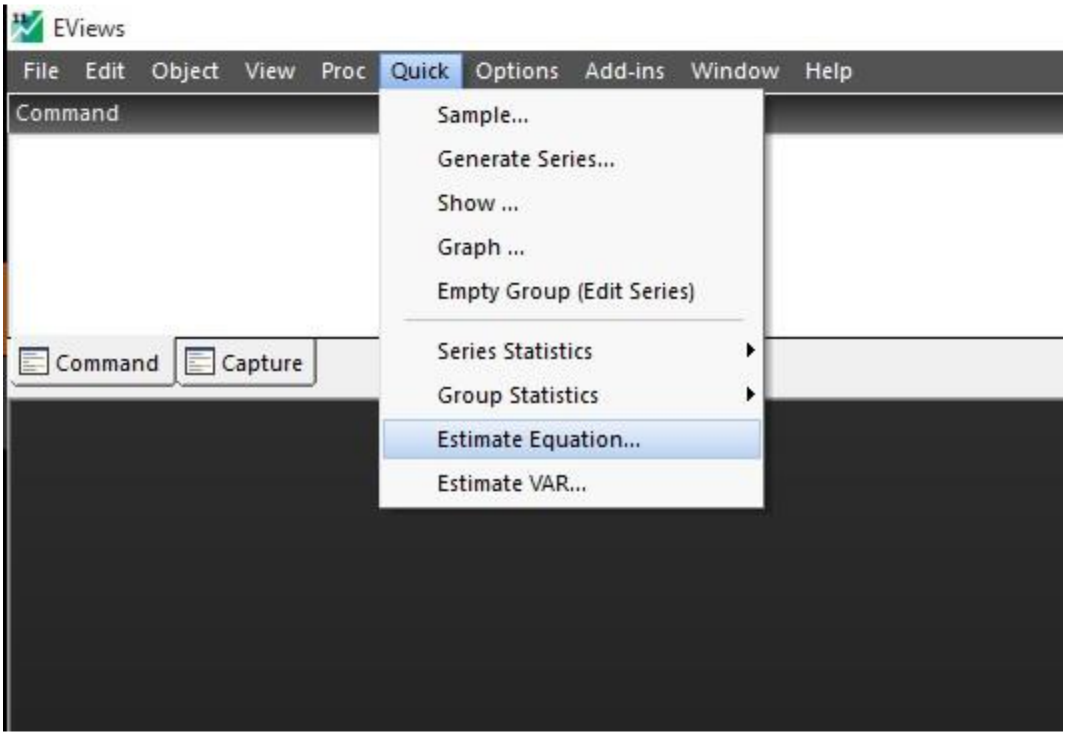


2. Preliminary analysis, linear model estimation and testing

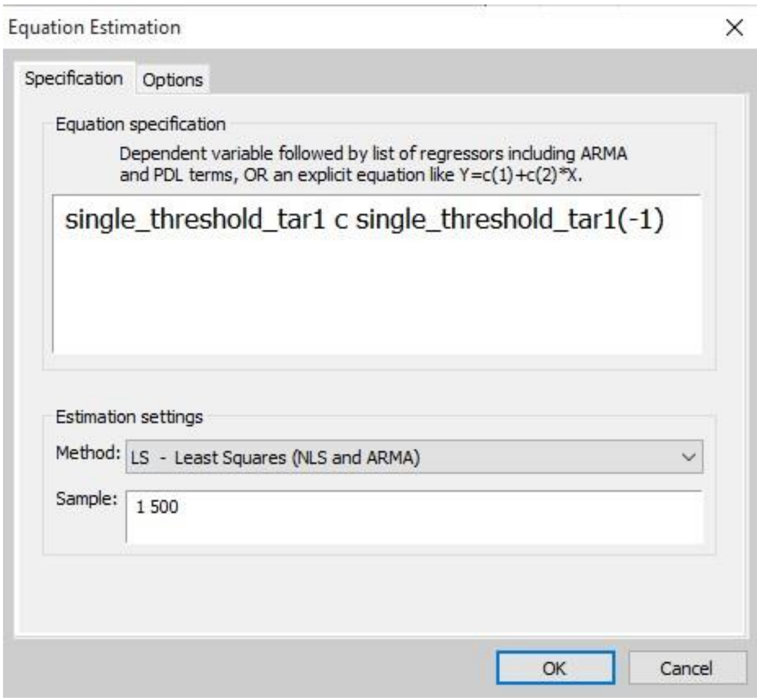
As always, the first thing to check is whether the data is stationary with the usual unit root tests. I leave this to you as exercise. The tests will show that this data indeed tests as stationary.

Next, we estimate the basic linear model and perform tests for potential non-linearity

Commands: Select "Quick>Estimate Equation" from the menus



Specify a first order AR estimation and estimate via Least Squares



This yields:

Dependent Variable: SINGLE_THRESHOLD_TAR1
Method: Least Squares
Date: 03/13/20   Time: 16:18
Sample (adjusted): 2 500
Included observations: 499 after adjustments

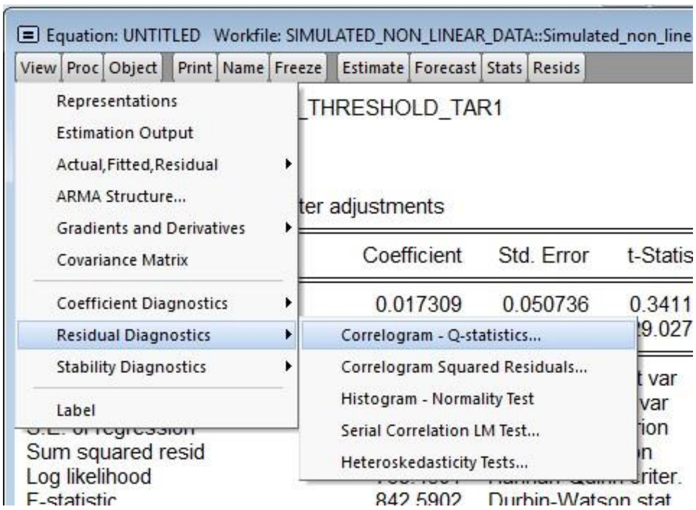
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017309	0.050736	0.341160	0.7331
SINGLE_THRESHOLD_TAR1(-1)	0.797308	0.027467	29.02740	0.0000

R-squared	0.628991	Mean dependent var	0.113001
Adjusted R-squared	0.628245	S.D. dependent var	1.854893
S.E. of regression	1.130961	Akaike info criterion	3.088012
Sum squared resid	635.6991	Schwarz criterion	3.104897
Log likelihood	-768.4591	Hannan-Quinn criter.	3.094638
F-statistic	842.5902	Durbin-Watson stat	2.254795
Prob(F-statistic)	0.000000		

Note that the fit seems quite good, but obviously, this is not the true DGP – the coefficients are mixtures of the two parts of the true DGP.

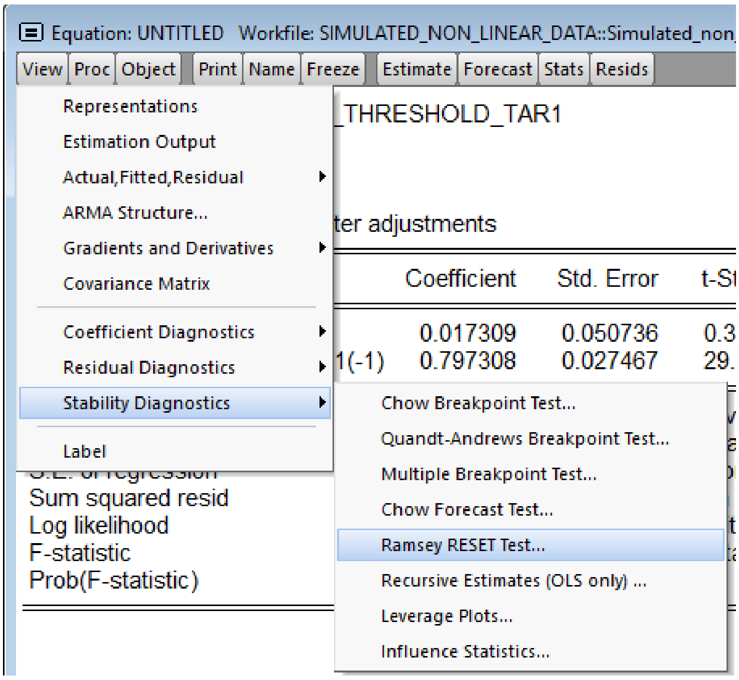
Next we test this specification.

A full battery of tests of white noise errors would be our first step.



Both the ACF and the formal LM test for autocorrelation will show problems as the 0.79 autocorrelation coefficient is not sufficient to account for the parts of the process in regime 2 with a much higher autocorrelation.

But for now, we focus on what is new: testing for non-linear effects. These are under "Stability Diagnostics"



The RESET test with just a squared fitted value added (the default) shows that there is strong evidence of some non-linear effects in this model. Adding a cubed term will not turn this conclusion around.

Ramsey RESET Test
Equation: UNTITLED
Omitted Variables: Squares of fitted values
Specification: SINGLE_THRESHOLD_TAR1 C SINGLE_THRESHOLD_TAR1(-1)

	Value	df	Probability
t-statistic	5.384155	496	0.0000
F-statistic	28.98913	(1, 496)	0.0000
Likelihood ratio	28.34401	1	0.0000

F-test summary:

	Sum of Sq.	df	Mean Squares
Test SSR	35.10237	1	35.10237
Restricted SSR	635.6991	497	1.279073
Unrestricted SSR	600.5967	496	1.210881

LR test summary:

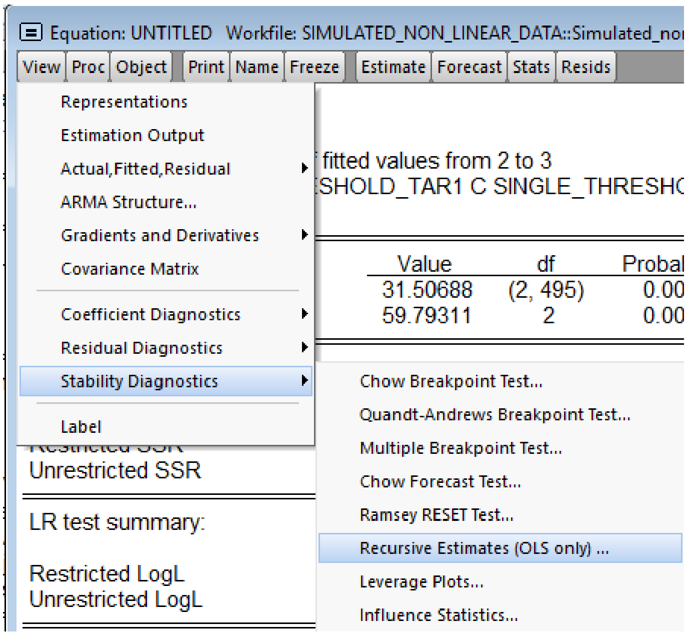
	Value
Restricted LogL	-768.4591
Unrestricted LogL	-754.2871

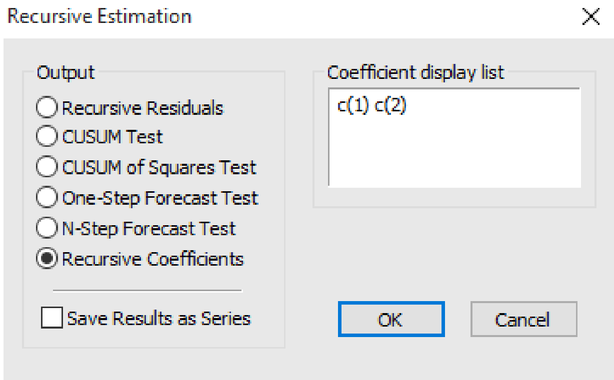
Unrestricted Test Equation:
Dependent Variable: SINGLE_THRESHOLD_TAR1
Method: Least Squares
Date: 03/13/20 Time: 16:29
Sample: 2 500
Included observations: 499

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.228541	0.063056	3.624413	0.0003
SINGLE_THRESHOLD_TAR1(-1)	0.740916	0.028704	25.81213	0.0000
FITTED^2	-0.094112	0.017479	-5.384155	0.0000

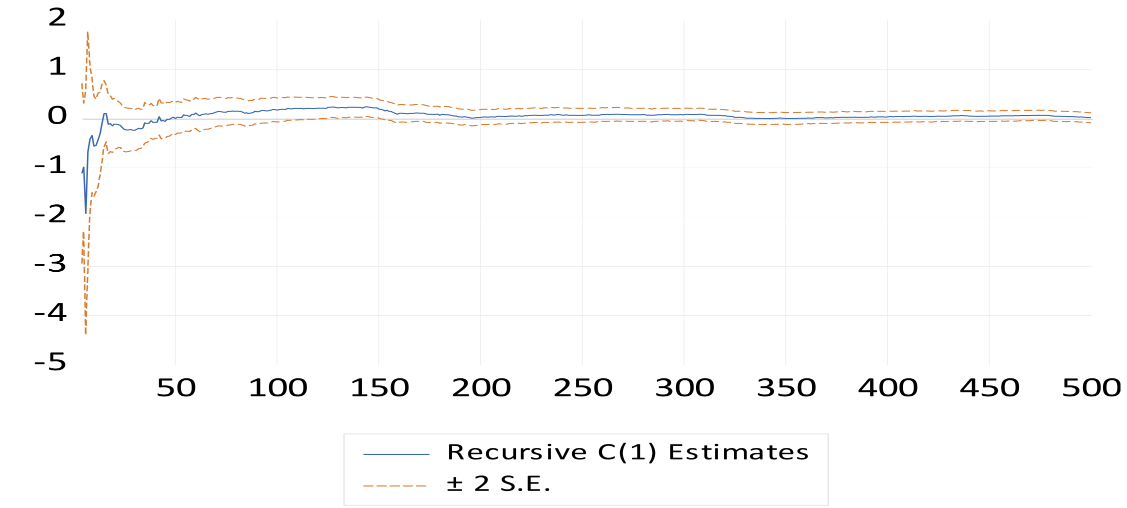
R-squared	0.649478	Mean dependent var	0.113001
Adjusted R-squared	0.648064	S.D. dependent var	1.854893
S.E. of regression	1.100400	Akaike info criterion	3.035219
Sum squared resid	600.5967	Schwarz criterion	3.060545
Log likelihood	-754.2871	Hannan-Quinn criter.	3.045158
F-statistic	459.5154	Durbin-Watson stat	2.192528
Prob(F-statistic)	0.000000		

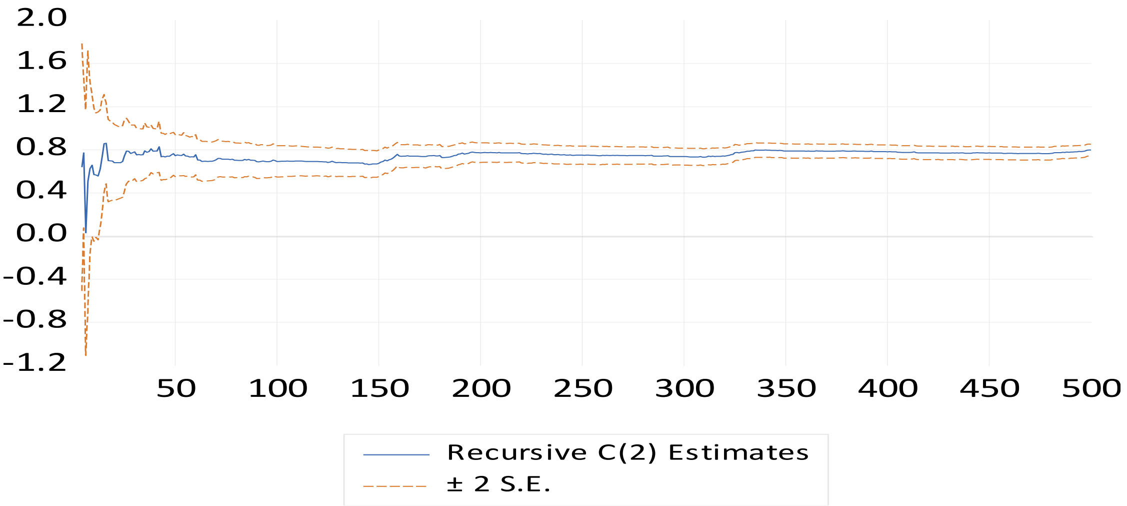
Testing the recursive stability of the estimated coefficients:





This yields the visual test:





There is some evidence of parameter instability, although not very strong. This is not surprising – since the process switches from one regime to the other and back over time, successive data-points do not push the coefficient estimates in on direction.

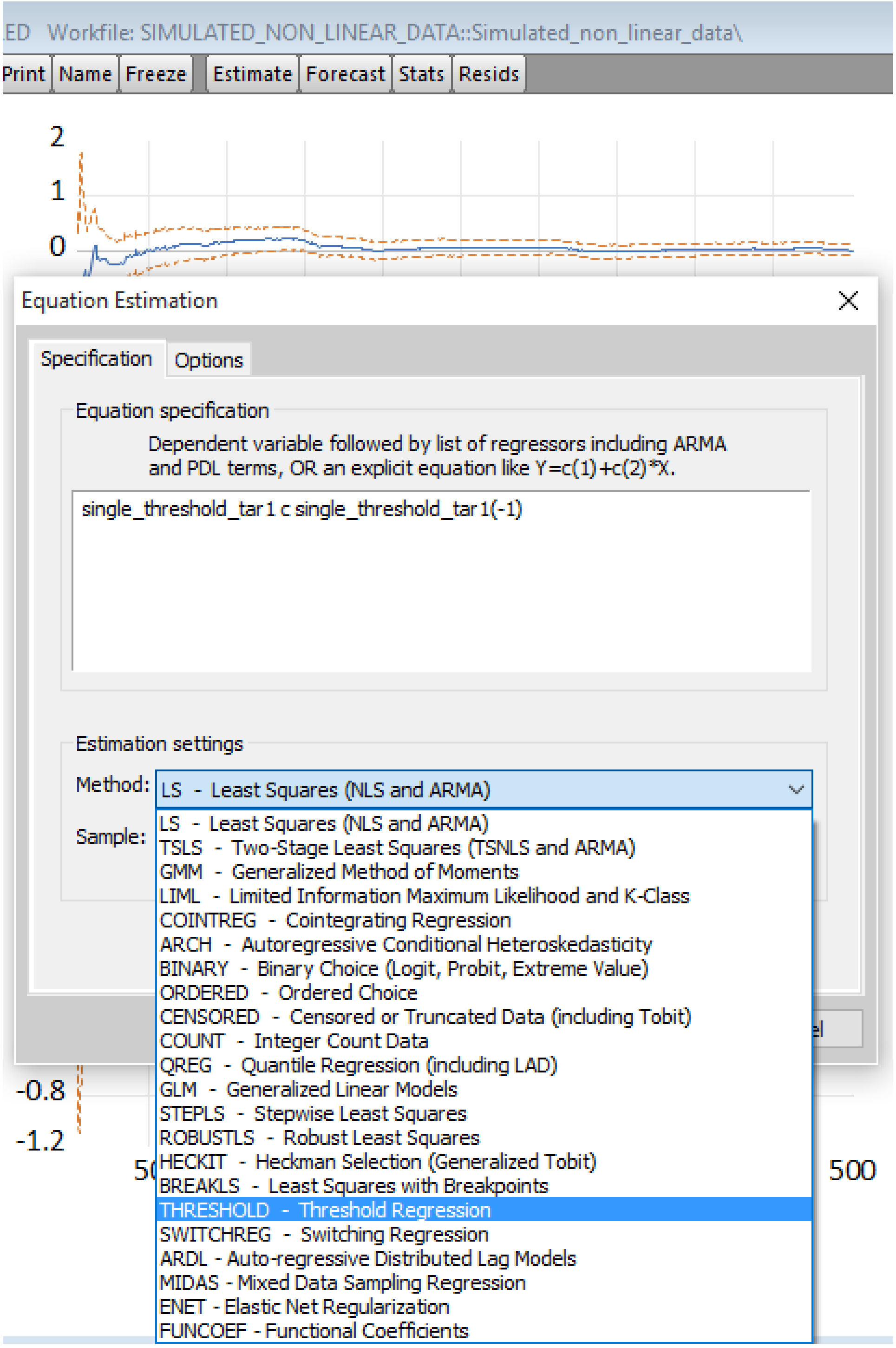
The two tests combined, however, presents the conclusion that some non-linearity may be present (as we know it is).

3. Testing for threshold AR structure

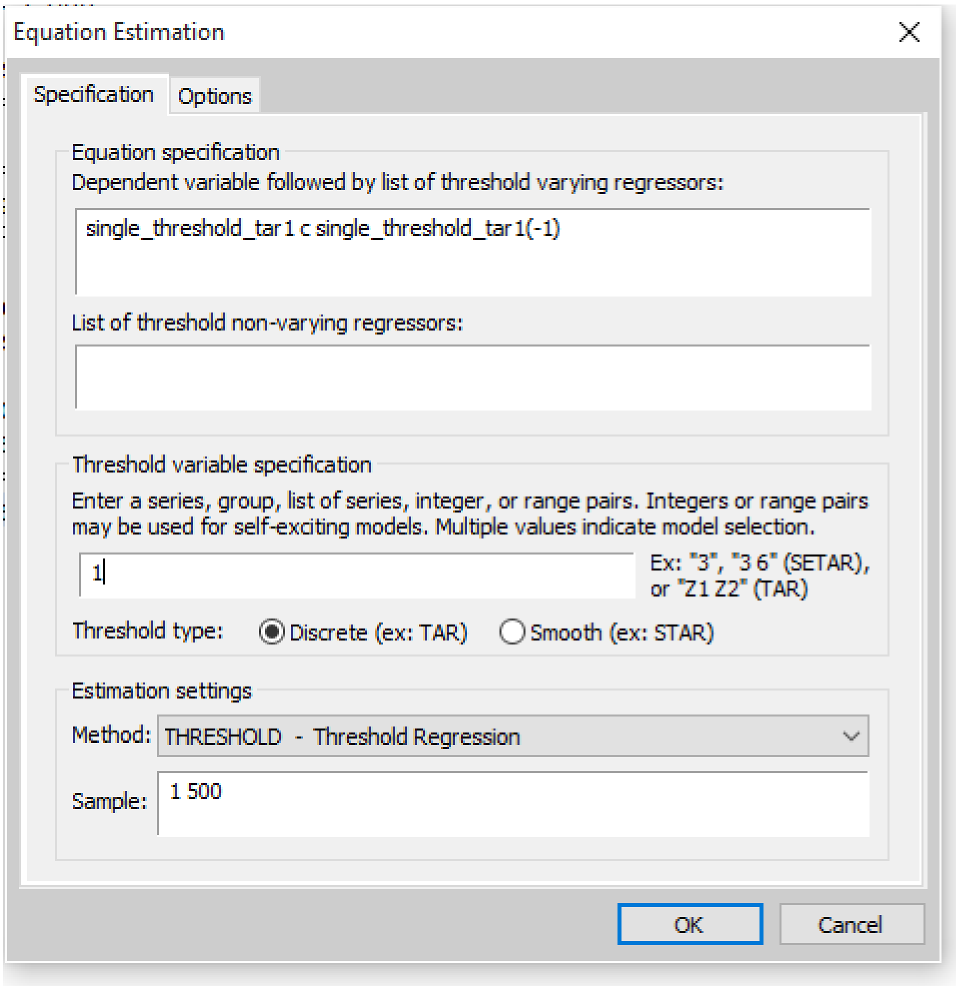
If one did not know this was a TAR process, one would also consider testing for structural breaks. I leave it to you at the end of this tutorial to see if there can be misleading conclusions from testing for the wrong type of non-linearity.

For now we take it as given and estimate a threshold model where we allow the program to select the best threshold as a value of the lagged dependent variable. It will also test for the number of thresholds, but only report the best estimation. We will turn to this in the next case.

Click on the “Estimate” button and select threshold regression from the “Method” drop-down menu.



The dialogue box opens up to allow you to specify the threshold process you are interested in:



The first box is the equation with all coefficients that can vary by regime. In this case we know both the constant and the persistence parameter are different in the two regimes. If, for instance, we knew the constant does not change across regimes, we would remove it from the first box and place it in the second. The third box "Threshold variable specification" is where we specify the delay parameter. "1" here means that the first lag of the process is what switches the regime from one to the other. So this specification is the true DGP. What Eviews will now do is consider all possible values of the lagged dependent variable (trimming 15% of the smallest and largest values of the process), select the best fitting threshold and report the estimation results.

This yields the output:

Dependent Variable: SINGLE_THRESHOLD_TAR1
Method: Discrete Threshold Regression
Date: 03/13/20 Time: 16:57
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Selection: Trimming 0.15, , Sig. level 0.05
Threshold variable: SINGLE_THRESHOLD_TAR1(-1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
SINGLE_THRESHOLD_TAR1(-1) < 0.01238415 -- 197 obs				
C	-0.110442	0.121128	-0.911783	0.3623
SINGLE_THRESHOLD_TAR1(-1)	0.842794	0.055508	15.18326	0.0000
0.01238415 <= SINGLE_THRESHOLD_TAR1(-1) -- 302 obs				
C	0.933704	0.107947	8.649689	0.0000
SINGLE_THRESHOLD_TAR1(-1)	0.206016	0.067848	3.036435	0.0025

R-squared	0.685713	Mean dependent var	0.113001
Adjusted R-squared	0.683808	S.D. dependent var	1.854893
S.E. of regression	1.043024	Akaike info criterion	2.930110
Sum squared resid	538.5105	Schwarz criterion	2.963878
Log likelihood	-727.0625	Hannan-Quinn criter.	2.943362
F-statistic	359.9971	Durbin-Watson stat	2.119005
Prob(F-statistic)	0.000000		

This output is remarkably close to the true DGP in all coefficients (all coefficient estimates are within one standard error away from the true values) as well as the estimated threshold! Of course, we had sharp differences across regimes and a lot of data, so it would have been quite damning for the method if it could not work in this setting.

You should convince yourself that the residuals are now much closer to white noise than before.

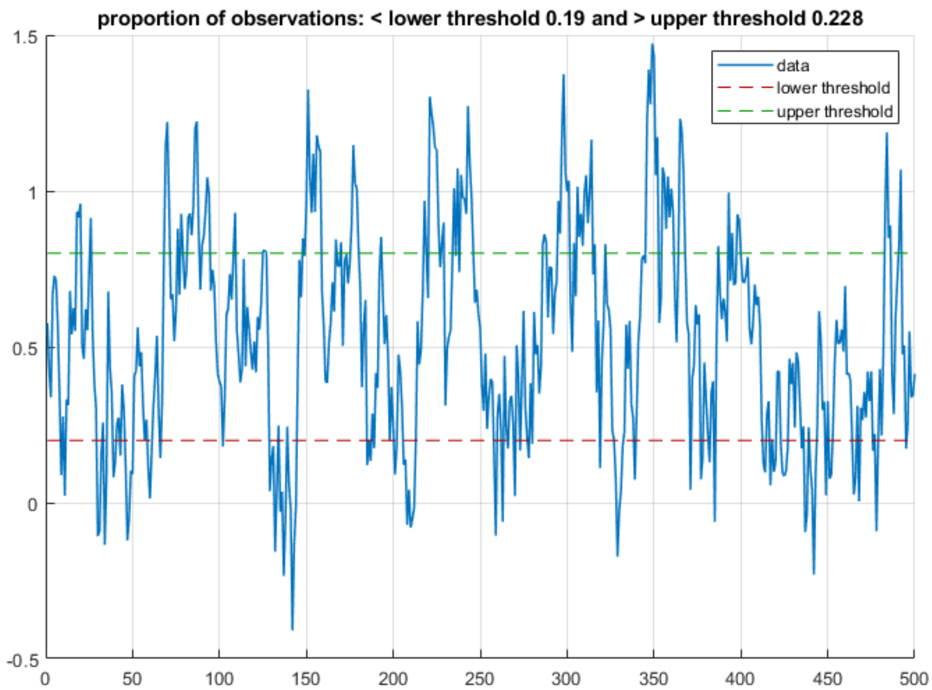
Case 2: band TAR(1) model

For this experiment, I wanted to test the band-TAR(1) model of a notional real exchange rate process we discussed in class. The data was simulated with the Matlab script `gen_bandTAR1.m` and follows the data generating process:

$$\bar{s} = 0.5$$
$$c = 0.3$$
$$s_t = \begin{cases} \bar{s} + 0.7(s_{t-1} - \bar{s}) + \varepsilon_t & \text{if } s_{t-1} < \bar{s} - c \\ \bar{s} + 1(s_{t-1} - \bar{s}) + \varepsilon_t & \text{if } \bar{s} - c \leq s_{t-1} < \bar{s} + c \\ \bar{s} + 0.8(s_{t-1} - \bar{s}) + \varepsilon_t & \text{if } s_{t-1} \geq \bar{s} + c \end{cases}$$

$$\varepsilon_t \sim N(0, 0.04)$$

Note that the middle regime implies a random walk without drift. To get the process to spend enough time within and outside of the band, I had to play around with the variance of the error relative to the width of the band, as well as the persistence outside of the band (and draw several samples) until I found the data plotted in the following figure (the variable is labelled `bandtar1`)



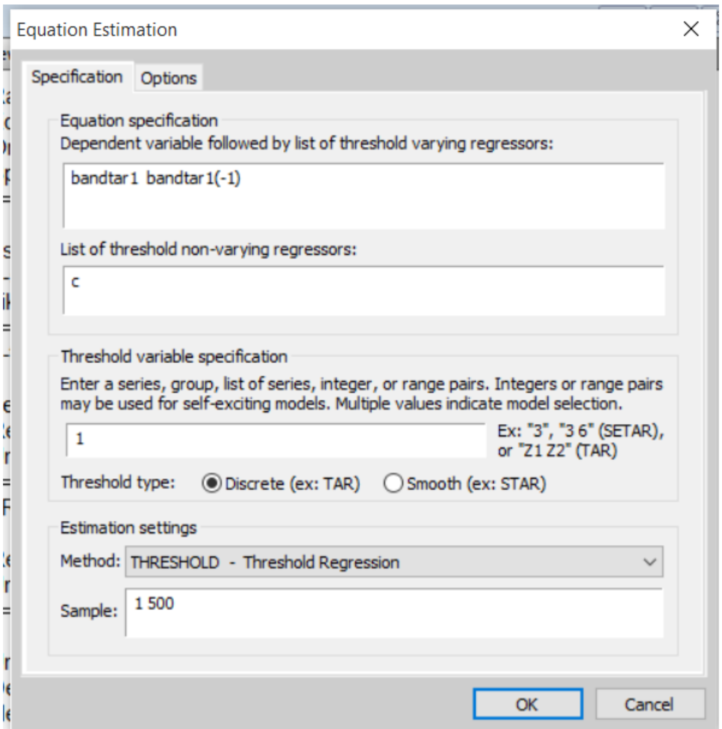
The simple linear AR1 model yields what looks like a reasonably strong fit and the residuals show no obvious autocorrelation or heteroscedasticity. The RESET test and recursive coefficients also show no obvious signs of non-linearity (see above for how to run these tests). The long run mean value is actually quite accurate: $\frac{0.1}{1-0.81} = 0.526$.

Dependent Variable: BANDTAR1
Method: Least Squares
Date: 03/14/20   Time: 10:39
Sample (adjusted): 2 500
Included observations: 499 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.100138	0.016603	6.031270	0.0000
BANDTAR1(-1)	0.809887	0.026319	30.77198	0.0000

R-squared	0.655797	Mean dependent var	0.528116
Adjusted R-squared	0.655104	S.D. dependent var	0.344928
S.E. of regression	0.202569	Akaike info criterion	-0.351476
Sum squared resid	20.39392	Schwarz criterion	-0.334592
Log likelihood	89.69336	Hannan-Quinn criter.	-0.344850
F-statistic	946.9149	Durbin-Watson stat	2.111534
Prob(F-statistic)	0.000000		

However, we know the data was generated with a band-TAR DGP, so let's see if the estimation can uncover it. We explicitly specify that the constant should not vary by regime:



The default option uses the version of Bai and Perron (2003) test where thresholds are tested for sequentially (0 vs 1, 1 vs 2 etc.). The results are:

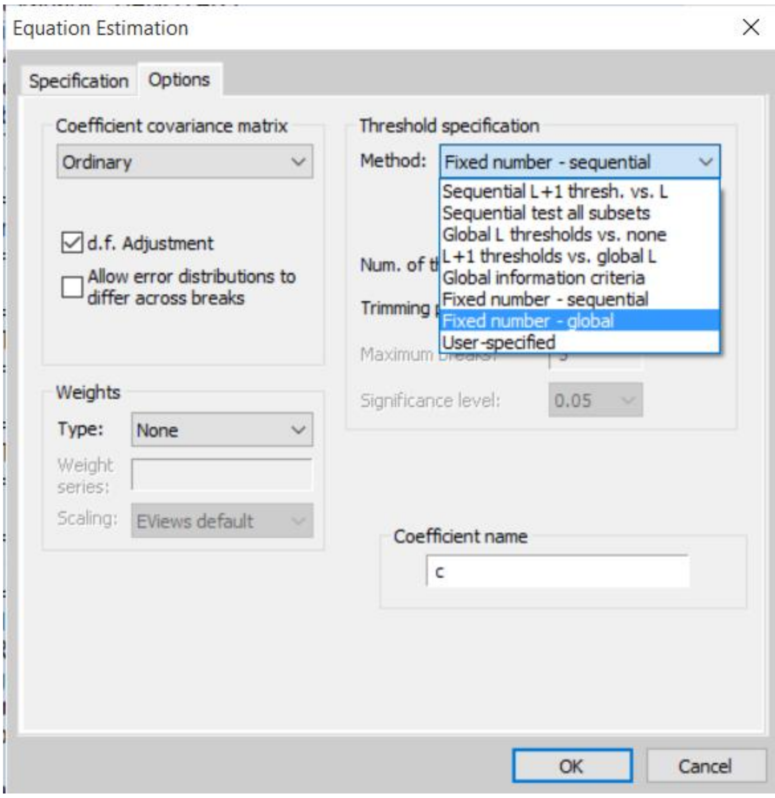
Dependent Variable: BANDTAR1
Method: Discrete Threshold Regression
Date: 03/14/20 Time: 10:46
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Selection: Trimming 0.15, , Sig. level 0.05
Threshold variable: BANDTAR1(-1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
BANDTAR1(-1) < 0.4294947 -- 207 obs				
BANDTAR1(-1)	0.536562	0.079245	6.770953	0.0000
0.4294947 <= BANDTAR1(-1) -- 292 obs				
BANDTAR1(-1)	0.777141	0.027502	28.25778	0.0000
Non-Threshold Variables				
C	0.137324	0.019306	7.113095	0.0000

R-squared	0.664806	Mean dependent var	0.528116
Adjusted R-squared	0.663455	S.D. dependent var	0.344928
S.E. of regression	0.200101	Akaike info criterion	-0.373992
Sum squared resid	19.86011	Schwarz criterion	-0.348665
Log likelihood	96.31089	Hannan-Quinn criter.	-0.364053
F-statistic	491.8706	Durbin-Watson stat	2.093537
Prob(F-statistic)	0.000000		

The best fitting model has only one threshold, not two.

We can force the model to estimate two thresholds by changing the setting in the "Options" tab of the estimation dialogue box. You will also note that under options you can allow different variances across regimes and control the trimming percentage along with other options. In the "Threshold specification" box, select "Fixed number – global" from the "Method" drop-down menu and set the number of thresholds to 2.



This yields:

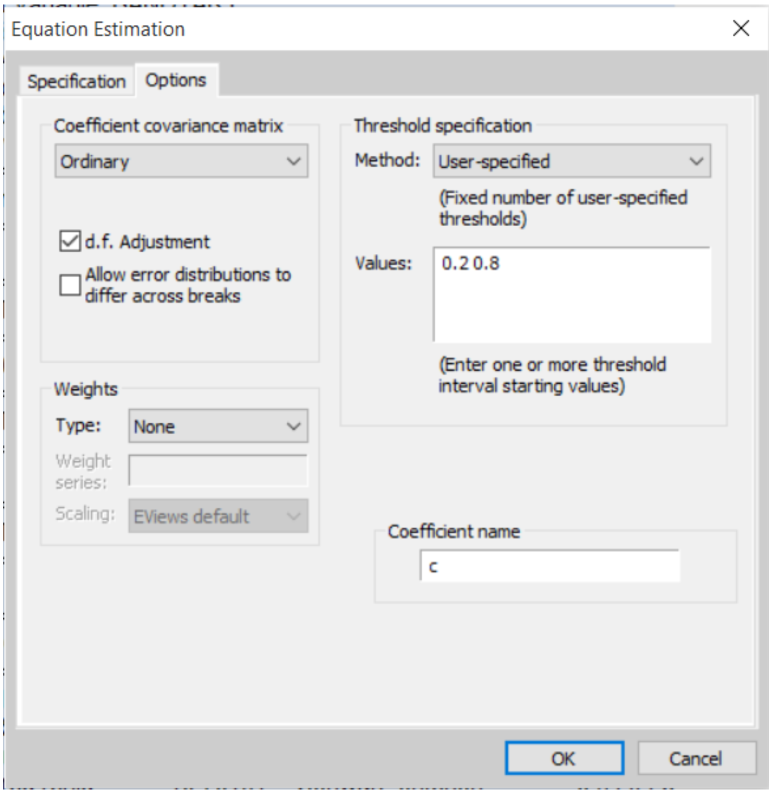
Dependent Variable: BANDTAR1
Method: Discrete Threshold Regression
Date: 03/14/20 Time: 11:03
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Threshold variable: BANDTAR1(-1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
BANDTAR1(-1) < 0.6186913 -- 305 obs				
BANDTAR1(-1)	0.685972	0.054708	12.53872	0.0000
0.6186913 <= BANDTAR1(-1) < 0.7931153 -- 77 obs				
BANDTAR1(-1)	0.882051	0.042925	20.54883	0.0000
0.7931153 <= BANDTAR1(-1) -- 117 obs				
BANDTAR1(-1)	0.780885	0.026838	29.09646	0.0000
Non-Threshold Variables				
C	0.122176	0.020041	6.096205	0.0000

R-squared	0.667481	Mean dependent var	0.528116
Adjusted R-squared	0.665466	S.D. dependent var	0.344928
S.E. of regression	0.199502	Akaike info criterion	-0.377996
Sum squared resid	19.70161	Schwarz criterion	-0.344228
Log likelihood	98.31012	Hannan-Quinn criter.	-0.364745
F-statistic	331.2128	Durbin-Watson stat	2.110605
Prob(F-statistic)	0.000000		

The autocorrelation coefficients in the lower and upper regime are very close to their true values, and in the intermediate regime the estimate is almost within two standard errors of the truth. The estimated upper thresholds is accurate (true value 0.8, estimated at 0.79), but the lower threshold is not (true value 0.2, estimated at 0.62).

Finally, we can force the estimation procedure to start searching at the true values. In the “Threshold specification” box, select “User specified” from the “Method” drop-down menu and enter the true thresholds.



This gives the correct thresholds but the coefficient estimates are now a little poorer, and the statistical fit weaker.

Dependent Variable: BANDTAR1
Method: Discrete Threshold Regression
Date: 03/14/20 Time: 11:15
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Threshold variable: BANDTAR1(-1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
BANDTAR1(-1) < 0.2004052 -- 95 obs				
BANDTAR1(-1)	0.843045	0.177229	4.756820	0.0000
0.2004052 <= BANDTAR1(-1) < 0.8018334 -- 290 obs				
BANDTAR1(-1)	0.832059	0.041642	19.98143	0.0000
0.8018334 <= BANDTAR1(-1) -- 114 obs				
BANDTAR1(-1)	0.809121	0.026601	30.41714	0.0000
Non-Threshold Variables				
C	0.093558	0.019343	4.836776	0.0000

R-squared	0.656125	Mean dependent var	0.528116
Adjusted R-squared	0.654041	S.D. dependent var	0.344928
S.E. of regression	0.202880	Akaike info criterion	-0.344415
Sum squared resid	20.37445	Schwarz criterion	-0.310647
Log likelihood	89.93167	Hannan-Quinn criter.	-0.331164
F-statistic	314.8262	Durbin-Watson stat	2.112514
Prob(F-statistic)	0.000000		

From this I conclude that even in the very simplest case with a lot of data, the estimation procedure is not very likely to be accurate. This is a weak conclusion: as above, a proper test of the estimation method is to repeat this process thousands of times, varying parameters and sample sizes, and documenting the distribution of results relative to the true values of parameters.

Case 3: an AR(1) model with 1 structural break

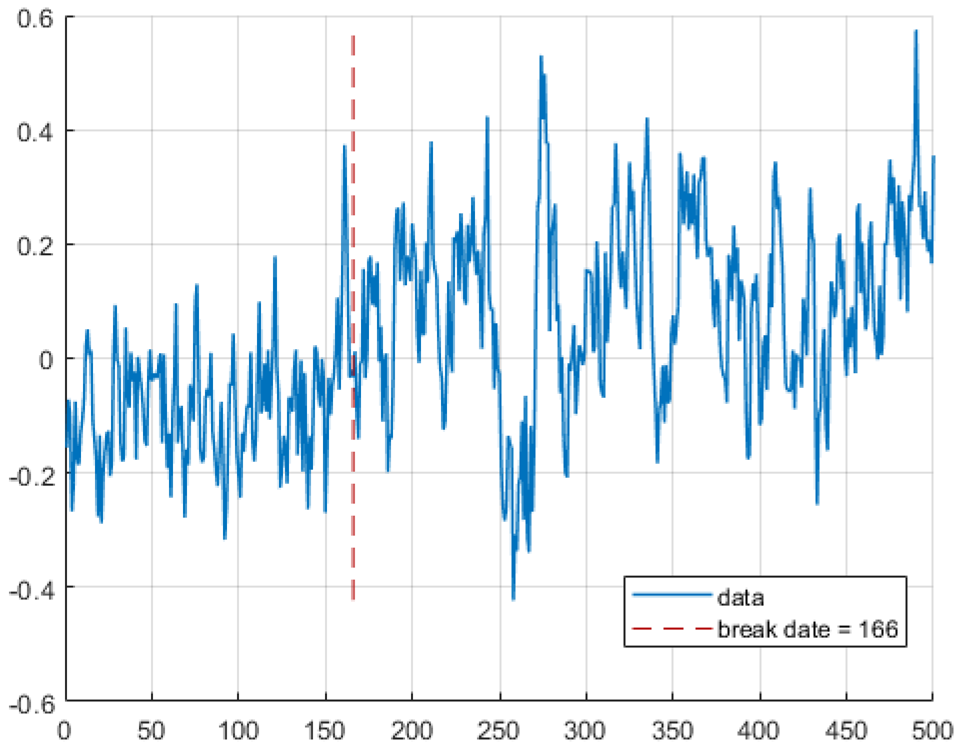
The data was simulated with the Matlab script `gen_Structural_Break_AR.m` and follows the data generating process:

$$y_t = \begin{cases} -0.05 + 0.5y_{t-1} + \varepsilon_t & \text{if } t < 166 \\ 0.02 + 0.8y_{t-1} + \varepsilon_t & \text{if } t \geq 166 \end{cases}$$

$$\varepsilon_t \sim N(0{,}0.01)$$

Note that both the constant and the persistence changes at the break date. This is likely to be an easy case to correctly estimate.

The variable `ar1_single_break` is plotted in the following figure:



The structural break in this series is not very sharp, so you will find that the process tests as stationary. Estimating the linear AR(1) model yields:

Dependent Variable: AR1_SINGLE_BREAK
Method: Least Squares
Date: 03/14/20 Time: 11:21
Sample (adjusted): 2 500
Included observations: 499 after adjustments

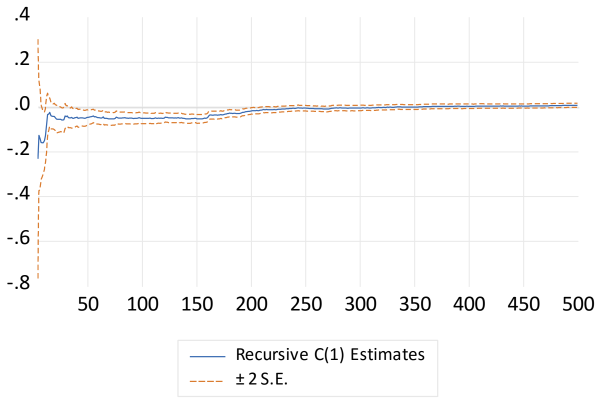
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.008081	0.004609	1.753275	0.0802
AR1_SINGLE_BREAK(-1)	0.805728	0.026738	30.13367	0.0000

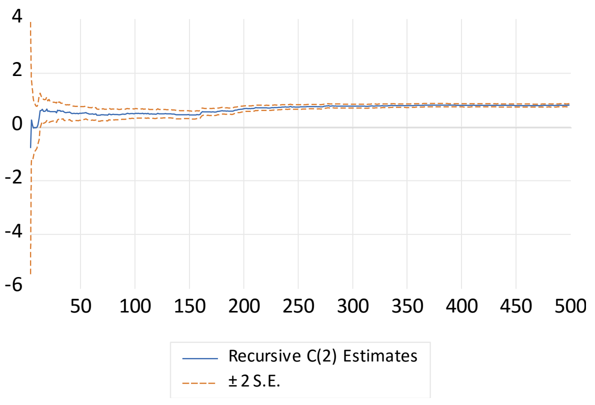
R-squared	0.646273	Mean dependent var	0.037356
Adjusted R-squared	0.645561	S.D. dependent var	0.169043
S.E. of regression	0.100639	Akaike info criterion	-1.750544
Sum squared resid	5.033768	Schwarz criterion	-1.733660
Log likelihood	438.7608	Hannan-Quinn criter.	-1.743918
F-statistic	908.0381	Durbin-Watson stat	2.131862
Prob(F-statistic)	0.000000		

Since most of the series is spent in the high persistence regime, this dominates the estimation and the AR coefficient accurately captures the true degree of persistence although the mean is imprecisely estimated.

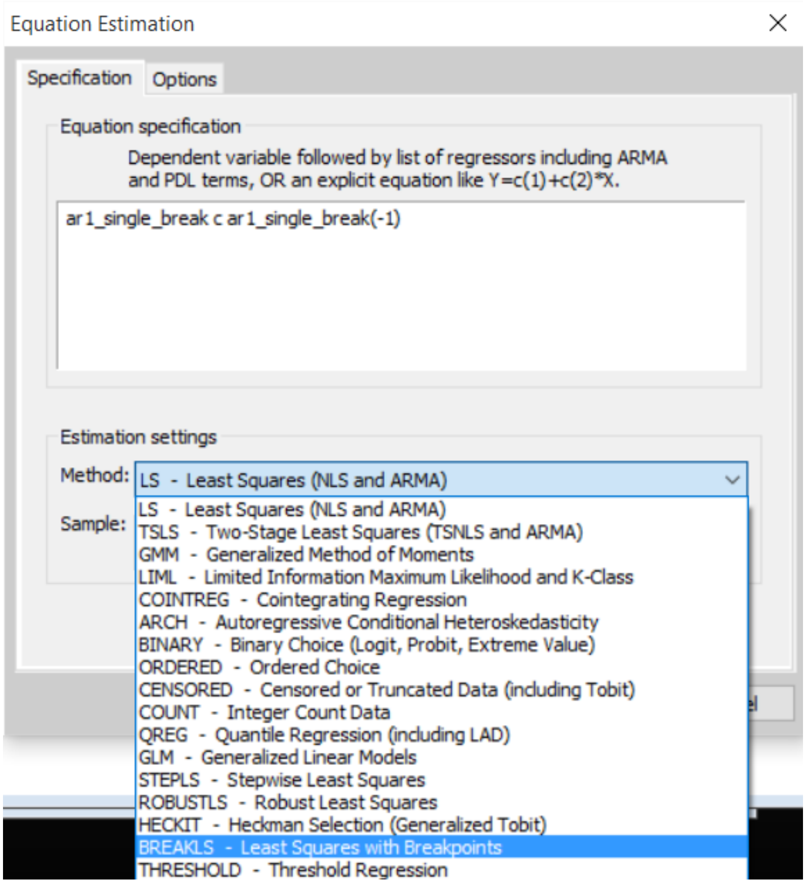
Performing the tests above, the residuals show some weak evidence of autocorrelation (at the 10% level). The RESET test with squared does not indicate remaining non-linearity, but with squared and cubed terms it does.

The recursively estimated coefficients graph shows what we would expect: at the beginning of the estimation sample, the estimated coefficients are lower than for the full sample, indicating evidence of a structural break:





To estimate a structural break model, select the breakpoint estimation:



The default specification is correct: both coefficients are subject to a break

Again the default Bai and Perron (2003) test is the sequential one: 0 breaks against 1, 1 against 2 etc. This yields

Dependent Variable: AR1_SINGLE_BREAK
Method: Least Squares with Breaks
Date: 03/14/20 Time: 11:29
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Break type: Bai-Perron tests of L+1 vs. L sequentially determined breaks
Break: 154
Selection: Trimming 0.15, , Sig. level 0.05

Variable	Coefficient	Std. Error	t-Statistic	Prob.
	2 - 153 -- 152 obs			
C	-0.053157	0.011358	-4.679989	0.0000
AR1_SINGLE_BREAK(-1)	0.438229	0.085598	5.119632	0.0000
	154 - 500 -- 347 obs			
C	0.022441	0.006050	3.709396	0.0002
AR1_SINGLE_BREAK(-1)	0.775118	0.032332	23.97380	0.0000

R-squared	0.669725	Mean dependent var	0.037356
Adjusted R-squared	0.667723	S.D. dependent var	0.169043
S.E. of regression	0.097442	Akaike info criterion	-1.811129
Sum squared resid	4.700028	Schwarz criterion	-1.777360
Log likelihood	455.8766	Hannan-Quinn criter.	-1.797877
F-statistic	334.5838	Durbin-Watson stat	2.038977
Prob(F-statistic)	0.000000		

The procedure finds a very close representation to the truth: one structural break at data point 153 (where the true break is at 166), and the coefficients of the two regimes are very accurately estimated, most within one standard error of the true values. The diagnostic tests are now unproblematic.

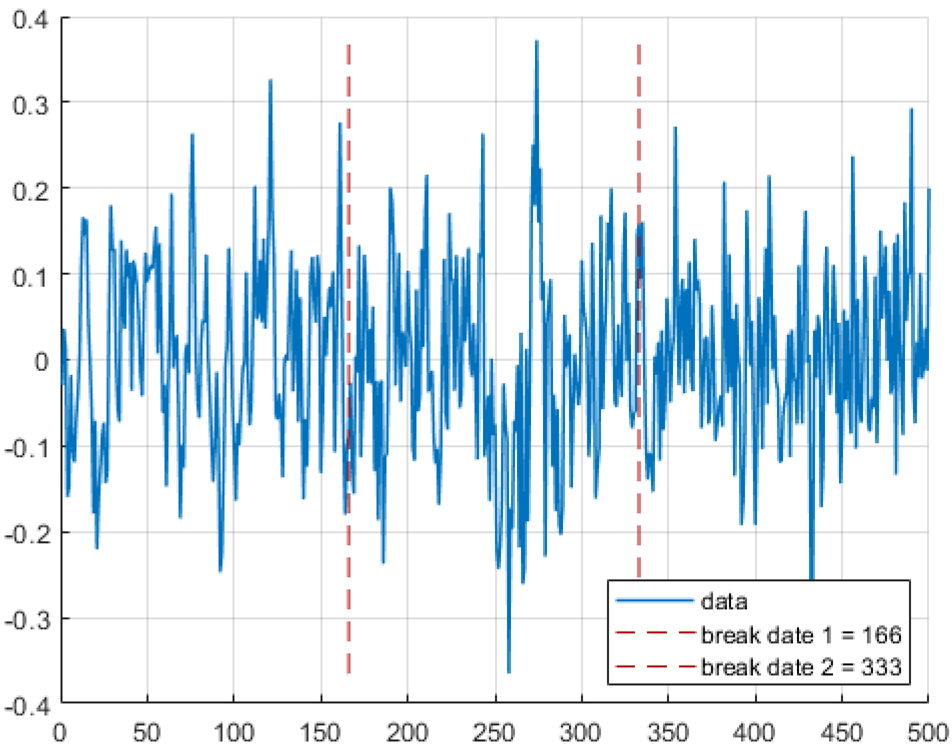
Case 4: an AR(1) model with 2 structural breaks

For the final experiment, we attempt a difficult problem, where the process has a constant mean, but the persistence decreases at two points in the sample. The data was simulated with the Matlab script `gen_Structural_Break_AR.m` and follows the data generating process:

$$y_t = \begin{cases} 0 + 0.7y_{t-1} + \varepsilon_t & \text{if } t < 166 \\ 0 + 0.5y_{t-1} + \varepsilon_t & \text{if } 166 \leq t < 333 \\ 0 + 0.2y_{t-1} + \varepsilon_t & \text{if } t > 333 \end{cases}$$

$$\varepsilon_t \sim N(0,0.01)$$

The variable `ar1_two_breaks` is plotted in the following figure:



A linear AR(1) estimation yields the expected result: a zero mean process with a AR coefficient that is some average of the true values:

Dependent Variable: AR1_TWO_BREAKS
Method: Least Squares
Date: 03/14/20 Time: 11:35
Sample (adjusted): 2 500
Included observations: 499 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.001090	0.004426	0.246293	0.8056
AR1_TWO_BREAKS(-1)	0.421957	0.040829	10.33470	0.0000

R-squared	0.176888	Mean dependent var	0.001552
Adjusted R-squared	0.175232	S.D. dependent var	0.108864
S.E. of regression	0.098866	Akaike info criterion	-1.786095
Sum squared resid	4.857957	Schwarz criterion	-1.769211
Log likelihood	447.6307	Hannan-Quinn criter.	-1.779469
F-statistic	106.8060	Durbin-Watson stat	2.031447
Prob(F-statistic)	0.000000		

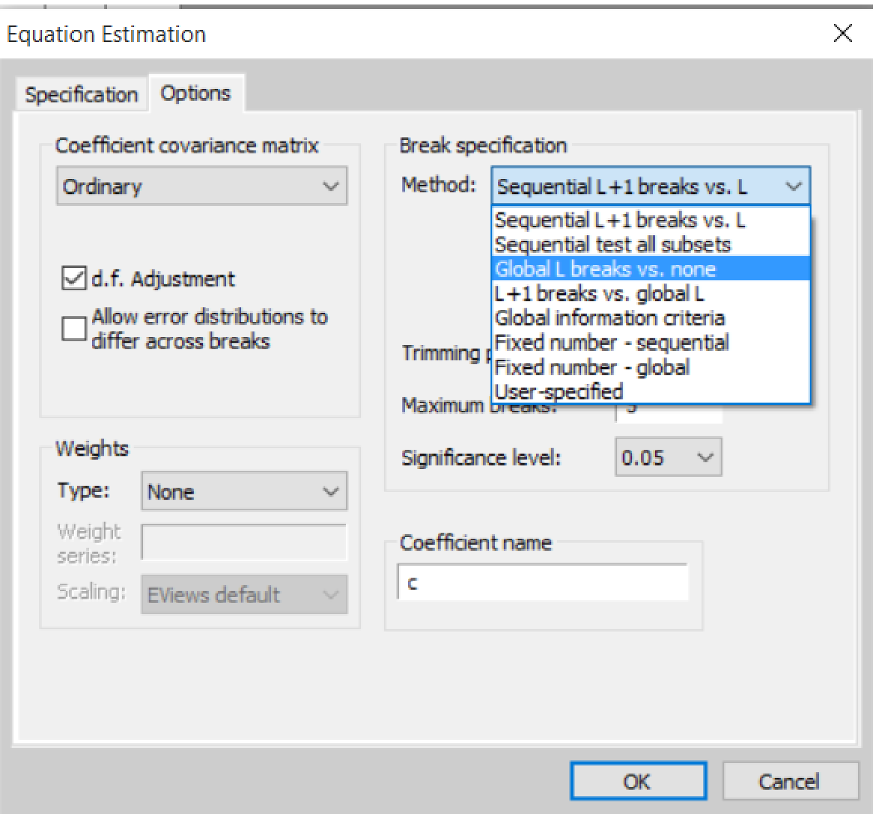
The default setting of the break-point estimation yields only one break-point, close to the true value of the second break (333, estimated as 342): The first estimated regime is close to the middle regime, and the second close to the third regime, although with an imprecise estimate of the AR coefficient:

Dependent Variable: AR1_TWO_BREAKS
Method: Least Squares with Breaks
Date: 03/14/20 Time: 11:37
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Break type: Bai-Perron tests of L+1 vs. L sequentially determined breaks
Break: 342
Selection: Trimming 0.15, , Sig. level 0.05

Variable	Coefficient	Std. Error	t-Statistic	Prob.
	2 - 341 -- 340 obs			
C	-0.000866	0.005272	-0.164218	0.8696
AR1_TWO_BREAKS(-1)	0.520106	0.046238	11.24845	0.0000
	342 - 500 -- 159 obs			
C	0.007214	0.007723	0.934165	0.3507
AR1_TWO_BREAKS(-1)	0.117251	0.081033	1.446946	0.1485

R-squared	0.207505	Mean dependent var	0.001552
Adjusted R-squared	0.202702	S.D. dependent var	0.108864
S.E. of regression	0.097206	Akaike info criterion	-1.815985
Sum squared resid	4.677259	Schwarz criterion	-1.782216
Log likelihood	457.0882	Hannan-Quinn criter.	-1.802733
F-statistic	43.20316	Durbin-Watson stat	2.024666
Prob(F-statistic)	0.000000		

Finally, if we use the global test for breakpoints allowing for up 5 breaks, we can see the results for all numbers of breaks.



The estimation output gives the maximum number of breaks (set to 5 as a default) but we can see the test results themselves under “View>Breakpoint Specification”.

Breakpoint Specification
Description of the breakpoint specification used in estimation
Equation: UNTITLED
Date: 03/14/20 Time: 11:48

Summary

Estimated number of breaks: 5
Method: Bai-Perron tests of 1 to M globally determined breaks
Maximum number of breaks: 5
Breaks: 94, 170, 245, 342, 419

Current breakpoint calculations:

Multiple breakpoint tests
Bai-Perron tests of 1 to M globally determined breaks
Date: 03/14/20 Time: 11:48
Sample: 2 500
Included observations: 499
Breaking variables: AR1_TWO_BREAKS(-1)
Non-breaking variables: C
Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05

Sequential F-statistic determined breaks:	5
Significant F-statistic largest breaks:	5
UDmax determined breaks:	1
WDmax determined breaks:	1

Breaks	F-statistic	Scaled F-statistic	Weighted F-statistic	Critical Value
1 *	18.38628	18.38628	18.38628	8.58
2 *	11.85972	11.85972	14.09369	7.22
3 *	9.299414	9.299414	13.38741	5.96
4 *	7.049069	7.049069	12.12044	4.99
5 *	5.598633	5.598633	12.28549	3.91

UDMax statistic*	18.38628	UDMax critical value**	8.88
WDMax statistic*	18.38628	WDMax critical value**	9.91

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Estimated break dates:
1: 342
2: 122, 342
3: 124, 250, 342
4: 124, 250, 342, 419
5: 94, 170, 245, 342, 419

The F statistics for all the break specifications are significant, but the largest one is for only one break, hence the result above. Note that when 2 breaks are allowed, they are close to the true break points (the last part of the table above). When we impose exactly two breaks, the estimation yields:

Dependent Variable: AR1_TWO_BREAKS
Method: Least Squares with Breaks
Date: 03/14/20 Time: 11:50
Sample (adjusted): 2 500
Included observations: 499 after adjustments
Break type: Fixed number of globally determined breaks
Breaks: 122, 342

Variable	Coefficient	Std. Error	t-Statistic	Prob.
		2 - 121 -- 120 obs		
AR1_TWO_BREAKS(-1)	0.680270	0.084025	8.096090	0.0000
		122 - 341 -- 220 obs		
AR1_TWO_BREAKS(-1)	0.451593	0.055054	8.202798	0.0000
		342 - 500 -- 159 obs		
AR1_TWO_BREAKS(-1)	0.120927	0.080577	1.500775	0.1341
		Non-Breaking Variables		
C	0.001311	0.004338	0.302121	0.7627

R-squared	0.214526	Mean dependent var	0.001552
Adjusted R-squared	0.209766	S.D. dependent var	0.108864
S.E. of regression	0.096774	Akaike info criterion	-1.824884
Sum squared resid	4.635818	Schwarz criterion	-1.791116
Log likelihood	459.3086	Hannan-Quinn criter.	-1.811633
F-statistic	45.06433	Durbin-Watson stat	2.004849
Prob(F-statistic)	0.000000		

This is reasonably close to the true DGP, except for the imprecise estimate of the final regime autocorrelation (but low autocorrelation is difficult to identify without a large sample, so this is not surprising).

To be fair, this was a case that was unlikely to be very successful – it would be difficult for any method to detect the relatively minor changes from regime to regime. A DGP with starker changes is likely to yield more definitive results. A second two-break series, `AR1_two_breaks2`, is in the work file with DGP:

$$y_t = \begin{cases} 0 + 0.7y_{t-1} + \varepsilon_t & \text{if } t < 166 \\ 0 + 0.3y_{t-1} + \varepsilon_t & \text{if } 166 \leq t < 333 \\ 0 + 0.9y_{t-1} + \varepsilon_t & \text{if } t > 333 \end{cases}$$

$$\varepsilon_t \sim N(0,0.01)$$

You will find that the default specification immediately identifies two breaks close to their true dates with coefficients also close to the truth.