MEng Title

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jou kennisvennoot • your knowledge partner

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Abstract

English

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Nomenclature

Ocean Vessel Dynamics

$X,\ Y,\ Z$	Coordinates of force vector decomposed in the body-fixed frame(surge, sway and heave forces)
$K,\ M,\ N$	Coordinates of moment vector decomposed in the body-fixed frame(roll, pitch and yaw moment)
$u,\ v,\ w$	Coordinates of linear velocity vector decomposed in the body-fixed frame(surge, sway and heave velocities)
$p,\ q,\ r$	Coordinates of angular velocity vector decomposed in the body-fixed frame(roll, pitch and yaw angular velocities)
$x,\ y,\ z$	Coordinates of position vector decomposed in the body-fixed frame(surge, sway and heave positions)
$\phi,\ \theta,\ \psi$	Coordinates of Euler angle vector decomposed in the body-fixed frame(roll, pitch and yaw Euler angles)

Acronyms and abbreviations

SNAME	Society of Naval Architects and Engineers
CG	Center of gravity of the vessel

Chapter 1

Introduction

1.1. Background

1.2. Problem Statement

1.3. Summary of Work

1.4. Scope

1.5. Format of Report

Chapter 2

Literature Review

Chapter 3

Modeling of Ocean Vessels

This chapter models a standard ocean vessel in six degrees of freedom. It also introduces the definitions associated with movement in each direction of freedom. The chapter also take into account the forces and moments generated by hydrodynamics and restoration of an ocean vessel. The chapter continues to model the environmental disturbances experience by a semi-submerged ocean vessel. The environmental disturbances are wind, waves and ocean currents.

3.1. Standard Ocean Vessel Notation

An ocean vessels are modelled in six degrees of freedom, requiring six independent coordinates to determine its position and orientation. The first three coordinates corresponding to position (x, y, z) and their first time derivatives, translation motion along the x-, y-, and z-axes. The last three coordinates (ϕ, θ, ψ) and their first time derivatives describing orientation and rotational motion [1]. Figure 3.1 illustrates the motion variables of an ocean vessel with the six independent coordinates.

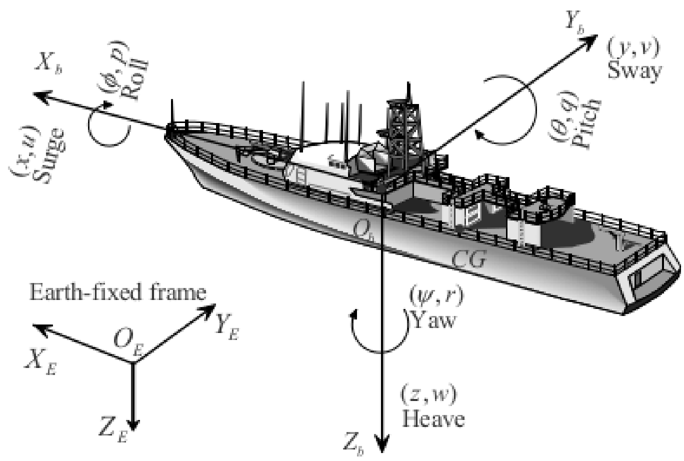

Figure 3.1: Motion variables for an ocean vessel

The SNAME(Society of Naval Architects and Marine Engineers) established the notation for the six different motion components as *surge*, *sway*, *heave*, *roll*, *pitch* and *yaw*. Table A.1 summarizes the SNAME notation for ocean vessels.

Two reference models are used to determine the equations of motion, namely the inertial to earth frame $O_eX_eY_eZ_e$ that may be displaced to overlap with the vessel's fixed coordinates in some initial condition and the body-fixed frame $O_bX_bY_bZ_b$, illustrated in Figure 3.1. The most common used position for the body-fixed frame results in symmetry about the $O_bX_bZ_b$-plane and approximate symmetry about the $O_bY_bZ_b$. The body axes coincides with the axes of inertia and are usually defines as follows: O_bX_b is the longitudinal axis, O_bY_b is the transverse axis and O_bZ_b is the normal axis. Below are the vectors used to describe the general motion of an ocean vessel:

$$\mathbf{n} = [\mathbf{n_1}\mathbf{n_2}]^T \tag{3.1}$$

$$\mathbf{v} = [\mathbf{v_1}\mathbf{v_2}]^T \tag{3.2}$$

$$\tau = [\tau_\mathbf{1}\tau_\mathbf{2}]^T \tag{3.3}$$

$$\mathbf{n_1} = [\,x \;\; y \;\; z\,]^T \tag{3.4}$$

$$\mathbf{n_2} = [\,\phi \;\; \theta \;\; \psi\,]^T \tag{3.5}$$

$$\mathbf{v_1} = [\,u \;\; v \;\; w\,]^T \tag{3.6}$$

$$\mathbf{v_2} = [\,p \;\; q \;\; r\,]^T \tag{3.7}$$

$$\tau_\mathbf{1} = [\,X \;\; Y \;\; Z\,]^T \tag{3.8}$$

$$\tau_\mathbf{2} = [\,K \;\; M \;\; N\,]^T \tag{3.9}$$

where $\mathbf{n}$ denotes the position and orientation vector with coordinates in the earth fixed frame, $\mathbf{v}$ denotes the linear and angular velocity vector with coordinates in the body-fixed frame and τ denotes the forces and moments acting on the vessel in the body-fixed frame. The vessel dynamics are divided into two parts known as *kinematics* and *kinetics*.

3.2. Kinematics

Kinematics looks at the motion of the vessel without directly considering the forces affecting the motion. The first time derivative of the position vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ is related to the linear velocity vector $\mathbf{v_1}$ and $\mathbf{v_2}$ via the following transformations,

$$\dot{\mathbf{n}}_\mathbf{1} = \mathbf{J_1}(\mathbf{n_2})\mathbf{v_1} \tag{3.10}$$

$$\dot{\mathbf{n}}_\mathbf{2} = \mathbf{J_2}(\mathbf{n_2})\mathbf{v_2} \tag{3.11}$$

where $\mathbf{J_1}(\mathbf{n_2})$ and $\mathbf{J_2}(\mathbf{n_2})$ are transformation matrices, which is related through the functions of the Euler angles: roll(ϕ), pitch(θ) and yaw(ψ). The $\mathbf{J_1}$ transformation matrix is given by

$$\mathbf{J_1(n_2)} = \begin{bmatrix} \cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\theta)+\sin(\phi)\sin(\theta)\cos(\psi) & \sin(\psi)\sin(\phi)+\sin(\theta)\cos(\psi)\cos(\phi) \\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\phi)+\sin(\phi)\sin(\theta)\sin(\psi) & -\cos(\psi)\sin(\phi)+\sin(\theta)\sin(\psi)\cos(\phi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix} \tag{3.12}$$

and the transformation matrix $\mathbf{J_2}$ is given by,

$$\mathbf{J_2(n_2)} = \begin{bmatrix} 1 & -\sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \tag{3.13}$$

When $\theta = \pi/2$, the transformation matrix $\mathbf{J_2(n_2)}$ becomes singular, however this is unlikely to happen when practically testing an ocean vessel, because of the metacentric restoring forces. Combining Equation 3.12 and Equation 3.13 results in the kinematics of an ocean vessel.

$$\begin{bmatrix} \mathbf{\dot{n}_1} \\ \mathbf{\dot{n}_2} \end{bmatrix} = \begin{bmatrix} \mathbf{J_1(n_2)} & 0_{3\times3} \\ 0_{3\times3} & \mathbf{J_2(n_2)} \end{bmatrix} \begin{bmatrix} \mathbf{v_1} \\ \mathbf{v_2} \end{bmatrix} = \mathbf{\dot{n}} = \mathbf{J(n)v} \tag{3.14}$$

3.3. Kinetics

The Newton-Euler formulation [2] defines the balancing forces and moments for a rigid body with a mass of m as follows,

$$\mathbf{f}_{Ob} = \mathbf{m}[\mathbf{\dot{v}}^E_{Ob} + \mathbf{\dot{w}}^E_{Ob} \times \mathbf{r}_{Ob} + \mathbf{w}^E_{Ob} \times \mathbf{v}_O b + \mathbf{w}^E_{Ob} \times (\mathbf{w}^E_{Ob} \times \mathbf{r}_{Ob})] \tag{3.15}$$

$$\mathbf{m}_{Ob} = \mathbf{I}_o \mathbf{w}^E_{Ob} + \mathbf{\dot{w}}^E_{Ob} \times \mathbf{I}_o \mathbf{w}^E_{Ob} + m\mathbf{r}_{Ob} \times (\mathbf{\dot{v}}_O b + \mathbf{w}^E_{Ob} \times \mathbf{v}_{Ob}) \tag{3.16}$$

where $\mathbf{f}_{Ob}$ is the balancing forces,$\mathbf{m}_{Ob}$ the balancing moments and $\mathbf{I}_o$ is the inertia matrix about O_b defined as,

$$\mathbf{I_o} = \begin{bmatrix} I_x & -I_{xy} & -Ixz \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_Z \end{bmatrix} \tag{3.17}$$

I_x,I_y and I_z are the moments of inertia about the O_bX_b, O_bY_b and O_bZ_b axes. $I_{xy} = I_{yx}$, $I_{xz} = I_{zx}$ and $I_{yz} = I_{zy}$ are the products of inertia. These quantities are defined as

$$I_x = \int_V (y^2 + z^2)\rho_m dV \tag{3.18}$$

$$I_x y = \int_V xy\rho_m dV \tag{3.19}$$

$$I_y = \int_V (x^2 + z^2)\rho_m dV \tag{3.20}$$

$$I_x z = \int_V xz\rho_m dV \tag{3.21}$$

$$I_z = \int_V (z^2 + y^2)\rho_m dV \tag{3.22}$$

$$I_z y = \int_V zy\rho_m dV \tag{3.23}$$

where ρ_m are the mass density and V the volume of the rigid body. By substituting the definitions defined in Table A.2 into Equations 3.15 and 3.16, results in the equation below,

$$\mathbf{M_{RB}\dot{v}} + \mathbf{C_{RB}(v)v} = \tau_{\mathbf{RB}} \tag{3.24}$$

where $\mathbf{v} = [u\ v\ w\ p\ q\ r]^T$ is the generalized velocity vector decomposed in the body-fixed frame and $\tau_{\mathbf{RB}} = [X\ Y\ Z\ K\ M\ N]^T$ is the generalized vector of external forces and moments. The rigid body system inertia matrix $\mathbf{M_{RB}}$ and the rigid body Coriolis and centripetal matrix $\mathbf{C_{RB}}$ is defined in Equation A.1 and A.2 The generalized external force and moment vector, $\tau_{\mathbf{RB}}$, is a sum of the hydrodynamic force and moment vector $\tau_{\mathbf{H}}$, external disturbance force and moment vector $\tau_{\mathbf{E}}$ and propulsion force and moment vector τ.

3.4. Hydrodynamic Forces and Moments

Hydrodynamic forces and moments can be defined as the forces and moments on a ocean body when the body is forced to oscillate with the wave excitation and no wave are incident on the body. As shown in [3], the hydrodynamic forces and moments acting on a rigid body can be assumed to be linearly superimposed. The forces and moments can be subdivided into three components,

1. Added mass due to the inertia of the surrounding fluid
2. Radiation-induced potential damping due to the energy carried away by the generated surface waves
3. Restoring forces due to Archimedian forces

The hydrodynamic forces and moments vector $\tau_H isexpressedintheequationbelow$,

$$\tau_{\mathbf{H}} = -\mathbf{M_A}\dot{\mathbf{v}} - \mathbf{C_A}(\mathbf{v})\mathbf{v} - \mathbf{D}(\mathbf{v})\mathbf{v} - \mathbf{g}(\mathbf{n}) \tag{3.25}$$

where $\mathbf{M_A}$ is the added mass matrix, $\mathbf{C_A}(\mathbf{v})$ is the hydrodynamic Coriolis and centripetal matrix, $\mathbf{D}(\mathbf{v})$ is the damping matrix and $\mathbf{g}(\mathbf{n})$ is the position and orientation depending vector of restoring forces and moments. The added mass $\mathbf{M_A}$ is given below,

$$\mathbf{M_a} = \begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix} \tag{3.26}$$

The hydrodynamic Coriolis and centripetal matrix is given below,

$$\mathbf{C_A}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -a_3 & a_2 \\ 0 & 0 & 0 & a_3 & 0 & -a_1 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ 0 & -a_3 & a_2 & 0 & -b_3 & b_2 \\ a_3 & 0 & -a_1 & b_3 & 0 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & b_1 & 0 \end{bmatrix} \tag{3.27}$$

where a_1, a_2, a_3, b_1, b_2 and b_3 are defined in Equations A.3, A.4, A.5, A.6, A.7 and A.8.

The general hydrodynamic damping experienced by ocean vessels is the potential damping, skin friction, wave drift damping and damping due to vortex shedding. The hydrodynamic damping can be expressed in a general form as below,

$$\mathbf{D}(\mathbf{v}) = \mathbf{D} + \mathbf{D_n}(\mathbf{v}) \tag{3.28}$$

where the linear damping matrix $\mathbf{D}$ is given below,

$$\mathbf{D} = -\begin{bmatrix} X_u & X_v & X_w & X_p & X_q & X_r \\ Y_u & Y_v & Y_w & Y_p & Y_q & Y_r \\ Z_u & Z_v & Z_w & Z_p & Z_q & Z_r \\ K_u & K_v & K_w & K_p & K_q & K_r \\ M_u & M_v & M_w & M_p & M_q & M_r \\ N_u & N_v & N_w & N_p & N_q & N_r \end{bmatrix} \tag{3.29}$$

3.5. Restoring Forces and Moments

The restoring forces and moments are described by the symbol $\mathbf{g}(\mathbf{n})$. If ∇ is the volume of fluid displaced by the ocean vessel. The acceleration of gravity, g and the water density ρ. The submerged weight of the body and buoyancy forces are defined by

$$W = mg \tag{3.30}$$

$$B = \rho g \nabla \tag{3.31}$$

With the above definition for body and buoyancy forces, the restoring force and moment vector $\mathbf{g}(\mathbf{n})$ is due to gravity and buoyancy forces and is given by

$$\mathbf{g}(\mathbf{n}) = \begin{bmatrix} (W-B)sin(\theta) \\ -(W-B)cos(\theta)sin(\phi) \\ -(W-B)cos(\theta)cos(\phi) \\ -(y_gW - y_bB)cos(\theta)cos(\phi) + (z_gW - z_bB)cos(\theta)sin(\phi) \\ (z_gW - z_bB)sin(\theta) + (x_gW - x_bB)cos(\theta)cos(\phi) \\ -(x_gW - x_bB)cos(\theta)sin(\phi) - (y_gW - y_bB)sin(\theta) \end{bmatrix} \tag{3.32}$$

where (x_b, y_b, z_b) denote coordinates of the center of buoyancy.

3.6. Environmental Disturbances

The forces and moments induced by the environmental disturbances is defined by the vector $\tau_{\mathbf{E}}$ and includes ocean currents, waves(wind generated) and wind.

$$\tau_{\mathbf{E}} = \tau_{\mathbf{E}}^{\mathbf{cu}} + \tau_{\mathbf{E}}^{\mathbf{wa}} + \tau_{\mathbf{E}}^{\mathbf{wi}} \tag{3.33}$$

where $\tau_{\mathbf{E}}^{\mathbf{cu}}$, $\tau_{\mathbf{E}}^{\mathbf{wa}}$ and $\tau_{\mathbf{E}}^{\mathbf{wi}}$ are vectors of forces and moments induced by ocean currents, waves and wind.

3.6.1. Current-induced Forces and Moments

3.6.2. Wave-induced Forces and Moments

3.6.3. Wind-induced Forces and Moments

3.7. Propulsion Forces and moments

Propulsion with a fixed wing sail leading to next chapter.

Chapter 4

Modeling of a Fixed-Wing Sail

Chapter 5

Stability Analysis

Chapter 6

Platform Development

Chapter 7

Control Techniques for a Sail and Rudder

Chapter 8

Model Simulation

Chapter 9

Results

Chapter 10

Conclusion

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Appendix A

Additional Modelling Information

A.1. Notation and Vector Definitions

Table A.1: SNAME Notation for ocean vessels

Degree of freedom		Force and moment	Linear and angular velocity	Position and Euler angles
1	Surge	X	u	x
2	Sway	Y	v	y
3	Heave	Z	w	z
4	Roll	K	p	ϕ
5	Pitch	M	q	θ
6	Yaw	N	r	ψ

Table A.2: Rigid body motion vectors

Vector	Components	Definition
$\mathbf{f}_{Ob}$	$[X\ Y\ Z]^T$	force decomposed in the body-fixed frame
$\mathbf{m}_{Ob}$	$[K\ M\ N]^T$	moment decomposed in the body-fixed frame
$\mathbf{v}_{Ob}$	$[u\ v\ w]^T$	linear velocity decomposed in the body-fixed frame
$\mathbf{w}_{Ob}^E$	$[p\ q\ r]^T$	angular velocity of the body-fixed relative to the earth-fixed frame
$\mathbf{r}_{Ob}$	$[x_g\ y_g\ z_g]^T$	vector from O_b to CG decomposed in the body-fixed frame

A.2. Modeling Equations

$$\mathbf{M_{RB}} = \begin{bmatrix} m & 0 & 0 & mz_g & mz_g & -my_g \\ 0 & m & 0 & 0 & 0 & mx_g \\ 0 & 0 & m & -mx_g & -mx_g & 0 \\ 0 & -mz_g & -my_g & I_x & -Ixy & -I_xz \\ mz_g & 0 & -mx_g & -I_{xy} & I_y & -I_yz \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{A.1}$$

$$\mathbf{C_{RB}(v)} = \begin{bmatrix} 0 & 0 & 0 & m(y_gq+z_gr) & -m(x_gq-w) & -m(x_gr+v) \\ 0 & 0 & 0 & -m(y_gp+w) & m(z_gr+x_gp) & -m(y_gr-u) \\ 0 & 0 & 0 & -m(z_gp-v) & -m(z_gq+u) & m(x_gp+y_gq) \\ -m(y_gq+z_gr) & m(y_gp+w) & m(y_gp-v) & 0 & -I_{yz}q-I_{xz}q+I_zr & I_{yz}r+I_{xy}p-I_yq \\ m(x_gp-w) & -m(z_gr-x_gp) & m(z_gq+u) & I_{yz}q+I_{xz}p-I_zr & 0 & -I_{xz}r-I_{xy}q+I_Xp \\ m(x_gr+v) & m(y_gr-u) & -m(x_gp+y_gq) & -I_{yz}r-I_{xy}p+I_yq & I_{xz}r+I_{xy}q-I_xp & 0 \end{bmatrix} \tag{A.2}$$

$$a_1 = X_{\dot{u}}u + X_{\dot{v}}v + X_{\dot{w}}w + X_{\dot{p}}p + X_{\dot{q}}q + X_{\dot{r}}r \tag{A.3}$$

$$a_2 = Y_{\dot{u}}u + Y_{\dot{v}}v + Y_{\dot{w}}w + Y_{\dot{p}}p + Y_{\dot{q}}q + Y_{\dot{r}}r \tag{A.4}$$

$$a_3 = Z_{\dot{u}}u + Z_{\dot{v}}v + Z_{\dot{w}}w + Z_{\dot{p}}p + Z_{\dot{q}}q + Z_{\dot{r}}r \tag{A.5}$$

$$b_1 = K_{\dot{u}}u + K_{\dot{v}}v + K_{\dot{w}}w + K_{\dot{p}}p + K_{\dot{q}}q + K_{\dot{r}}r \tag{A.6}$$

$$b_2 = M_{\dot{u}}u + M_{\dot{v}}v + M_{\dot{w}}w + M_{\dot{p}}p + M_{\dot{q}}q + M_{\dot{r}}r \tag{A.7}$$

$$b_3 = N_{\dot{u}}u + N_{\dot{v}}v + N_{\dot{w}}w + N_{\dot{p}}p + N_{\dot{q}}q + N_{\dot{r}}r \tag{A.8}$$