Modeling and Autopilot Design for an Autonomous Catamaran Sailboat Based on Feedback Linearization

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***Abstract*: —This paper deals with the design of a feedback linearization and decoupling algorithm for controlling the heading and the sail opening angle of an autonomous catamaran sailboat. A nonlinear four degrees of freedom dynamic model of the sailboat was developed, it includes roll and sway motions. An autopilot was designed through the determined model in order to keep the catamaran sailboat on a predefined heading and ensure a suitable sail position. Some simulation results are presented to illustrate the behavior of the control design.**

Keywords—autonomous catamaran sailboat; autopilot; feedback linearization control.

I. Introduction

It is evident that a sailboat can not sail directly against the wind direction. However, in order to reach a target it has to follow a zigzag path as shown in Fig.1.

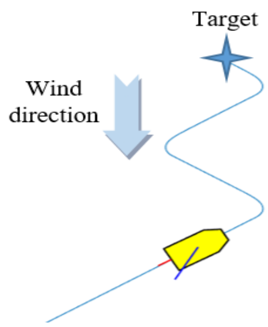

Fig. 1. Zig-zag course

Steering sailboats requires some basic sailing skills such as good adjustment of the sail opening angle and the choice of an appropriate heading; it depends on the desired heading and the wind direction.

Tack means that the sailboat heading changes through the wind direction. Therefore, in this case the wind changes from one side to the other.

By transforming the sailor's knowledge into automatic control system, sailing becomes easier and autonomous sailboats become useful in long operations such as monitoring of maritime area, oceanographic research (data acquisition for scientific use) and Microtransat challenge [1].

In sailing, autopilot is the most widespread form of marine automatic control system. In fact, it controls two actuators; rudder and sail so the sailboat becomes able to self-steering on a predefined course. By adding a high-level controller, the sailboat can autonomously reach any target position.

Autopilot can help skippers to steer their sailboats, since it controls heading and speed of the sailboat. Until now there has been many studies dedicated to autopilot design for these wind propelled vehicles. Among these works, several automatic control technics were used such as artificial intelligence [2], neural networks [3] and fuzzy control theory [4]. In other studies, [5][6][7][8][9][10], a nonlinear heading controller was developed and tested on a sailboat dynamic model with three degrees of freedom 3-DoF by excluding roll and pitch motions.

Due to external forces caused by wind and the marine current, large drift angles appear and the boat goes far from its destination. To solve this problem, a course controller for sailboat has been designed in [11] and [12] instead of heading controller [9][10]. In addition, the wind force applied on the sail causes heel angle and important roll motion. By considering this degree of freedom in sailboat dynamics, several works used a model with four degrees of freedom 4-4-DoF as it is indicated in [13] and [11]. A model with six degrees of freedom 6-DoF for a catamaran sailboat was also developed in [14] then a robust controller for the considered sailing boat named HyRaii was designed and tested.

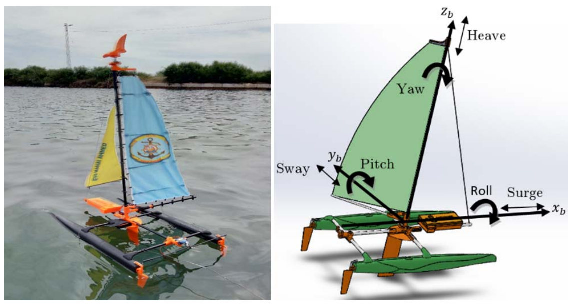

Fig. 2. 3D printing catamaran sailboat

In this paper, due to the high nonlinearity of the designed dynamic model of the catamaran sailboat, the feedback linearization method was used in order to control heading and sail opening angle. Using this method the nonlinear system transforms into a linear one. The obtained linearized system can be so controlled by standard linear techniques. The feedback linearization method was developed by the author in [15] using a monohull sailboat model with three degrees of freedom 3-DoF. One of the main contribution of this paper is the use of the feedback linearization method with a 4-DoF catamaran sailboat dynamic model.

First, a dynamic model of the catamaran sailing vessel is developed based on Newton's second law of motion and kinematic relations. After, a feedback linearization method is applied for controlling the sailing vessel heading and sail opening angle. Finally, some simulation results were carried out to illustrate and to evaluate the studied approach.

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II. System Dynamics

The modelled sailboat has two actuators; a rudder for controlling its heading and a sail to generate the propeller force, it is described in Fig.2 and Fig.3 and All variables are presented in the next table.

TABLE 1. VARIABLE DESCRIPTION

Notation	*Description*
V_{tw}, V_{aw}	-true /apparent vector of the wind.
v_{tw}, ψ_{tw}	-true wind speed and direction .
v_{aw}, ψ_{aw}	-apparent wind speed and direction.
G	-center of gravity.
CoE	-center of effort of the sail/rudder.
(x_s, y_s, z_s)	-coordinate of the CoE of the sail in (b-frame).
(x_r, y_r, z_r)	-coordinate of the CoE of the rudder in (b-frame).
(x, y, z)	-coordinates of the sailboat's center of gravity in the (n-frame).
ψ	-heading in the (n-frame) $\in [-\pi, \pi]$.
ϕ	-roll angle in the (n-frame) $\in]-\pi/2, \pi/2[$.
δ_s	-sail opening angle in the (b-frame).
δ_r	-rudder angle in the (b-frame).
u, v	-surge and sway velocity in the (b-frame).
r, p	-yaw and roll velocity in the (b-frame).
f_s	-aerodynamic force of the wind applied on the sail in the (b-frame).
f_r	-hydrodynamic forces of the water applied on the rudder in the (b-frame).
g	-gravity constant.
p_1, p_2	-water friction coefficient.
p_3	-water angular friction coefficient.
p_4, p_5	-lift coefficient of the sail/rudder.
p_6	-distance between the mast and the CoE of the sail.
p_7	-distance between the boat's center of gravity and the mast.
p_8	-distance between G and the rudder.
m	-total mass of the boat.
I_z, I_x	-moment of inertia around Z-axis/ X-axis.
p_{12}	-roll friction coefficient.
p_{13}	-length of the equivalent pendulum in roll motion.
$X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}}, K_{\dot{p}}$	-added masses.

First, a nonlinear 4-DoF dynamic model for the catamaran sailing vessel is presented. This model is inspired from [14] and [6]. It is based on Newton's second law.

The mathematical dynamic model of the catamaran sailboat was designed under the following assumptions:

- The boat is assumed to be rigid with 4-DoF: surge, sway, roll and yaw motions.
- The mainsail and the foresail (Fig.1) are combined into one effective sail.
- The environmental disturbances are ignored and waves and marine current are not modeled.
- Added mass coefficients $X_{\dot{u}}, Y_{\dot{v}}, N_{\dot{r}}, K_{\dot{p}}$ are modeled as constants.
- The sail and the rudder are modeled as rigid foils.
- Frictional forces are seen as linear damping elements in the dynamic model of the catamaran sailboat.
- Coriolis force caused by the rotation of earth has not effects on the boat motion.
- Relative water velocity is parallel to the boat heading near to the rudder actuator.

The catamaran sailing boat model is presented in Fig. 3 and Fig. 4.

Let the coordinate system (X, Y, Z) be the inertial reference frame $(n - frame)$ and let the (x_b, y_b, z_b) be the body fixed frame $(b - frame)$.
The body fixed frame which is attached to the boat turns with angular velocity $\Omega = (p\ q\ r)^T$ relative to the $(n - frame)$.
The origin of the $(b - frame)$ is supposed to coincide with the catamaran's center of gravity G.
The sailboat linear velocity in $(b - frame)$ is $V = (u\ v\ w)^T$.

The boat is assumed to be rigid and four degrees of freedom are considered, we exclude both heave and pitch motions; $q = w = 0$ (see Fig. 2).
Vector $\vartheta = (u\ v\ r\ p)^T$ is the velocity vector in the $(b - frame)$ and $\eta = (x\ y\ \psi\ \phi)^T$ is a vector describing respectively, the position of the sailboat (x, y) in the $(n - frame)$ and its attitude (heading ψ and roll angle ϕ).

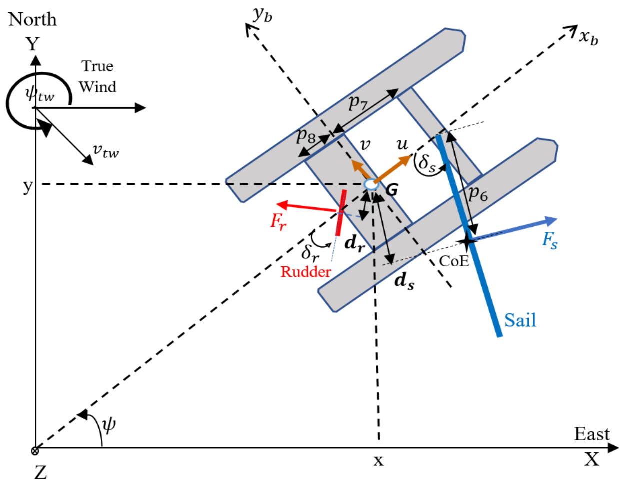

Fig. 3. Top view of the modelled catamaran sailboat

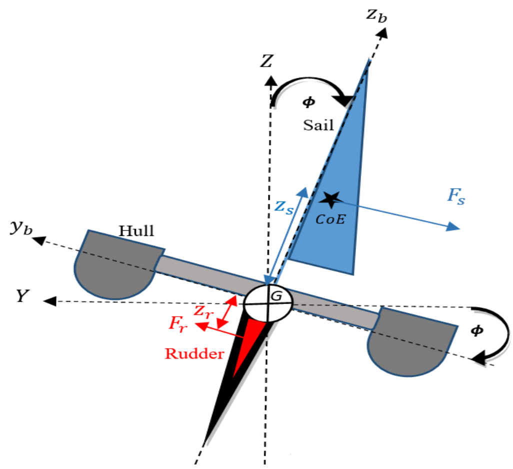

Fig. 4. Rear view of the modelled catamaran sailboat

Vector η is then derived through a coordinate transformation [16], giving the following differential equations:

$$\dot{x} = u\cos\psi - v\sin\psi\cos\phi \quad (1)$$
$$\dot{y} = u\sin\psi + v\cos\psi\cos\phi \quad (2)$$
$$\dot{\psi} = r\cos\phi \quad (3)$$
$$\dot{\phi} = p \quad (4)$$

All variables are described in TABLE 1.

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In sailing, due to the apparent wind force the sailboat advances.
The true wind vector in $(n-frame)$ is given by:

$$V_{tw}^{n-frame}=\begin{pmatrix} v_{tw}\cos\psi_{tw} \\ v_{tw}\sin\psi_{tw} \\ 0 \end{pmatrix} \quad (5)$$

It becomes in $(b-frame)$ equal to:

$$V_{tw}^{b-frame}=R_2 R_1 V_{tw}^{n-frame} \quad (6)$$

With

$$R_1=\begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (7)$$

$$R_2=\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{pmatrix} \quad (8)$$

The apparent wind vector expressed in $(b-frame)$ is given by:

$$\begin{aligned} V_{aw}^{b-frame} &= V_{tw}^{b-frame}-V-\Omega\times(x_s\ y_s\ z_s)^T \quad (9) \\ &= \begin{pmatrix} v_{tw}\cos(\psi_{tw}-\psi)-u+r\,y_s \\ v_{tw}\sin(\psi_{tw}-\psi)\cos\phi-v-r\,x_s+z_s p \\ v_{tw}\sin(\psi-\psi_{tw})\sin\phi-p\,y_s \end{pmatrix} \\ &= \begin{pmatrix} V_{aw}^{x_b} \\ V_{aw}^{y_b} \\ V_{aw}^{z_b} \end{pmatrix} \end{aligned}$$

In what follow, we will consider only the vectorial representation of the apparent wind velocity in the (x_b, y_b) plane.
Then we get:

$$\begin{aligned} \psi_{aw} &= atan2(V_{aw}^{y_b}, V_{aw}^{x_b}) \quad (10) \\ &= 2atan\left(\frac{V_{aw}^{y_b}}{\sqrt{{V_{aw}^{y_b}}^2+{V_{aw}^{x_b}}^2}+V_{aw}^{x_b}}\right) \end{aligned}$$

$$\begin{aligned} v_{aw} &= \|V_{aw}^{b-frame}\| \quad (11) \\ &= \sqrt{(V_{aw}^{x_b})^2+(V_{aw}^{y_b})^2} \end{aligned}$$

According to the projection of the applied forces described in (Fig.3) the vectorial representation of the aerodynamic force vector F_s into the (x_b, y_b) plane is:

$$F_s=\begin{pmatrix} f_s\sin\delta_s \\ -f_s\cos\delta_s\cos\phi \end{pmatrix} \quad (12)$$

With $f_s=\|F_s\|$.
According to Fig.3 and using trigonometric formulas, the angle of attack on the sail is determined by the direction of the apparent wind vector V_{aw} and the sail angle. It is equal to $\pi-(\delta_s-\psi_{aw})$ [17].
The aerodynamic force f_s applied on the CoE of the sail is hence equal to:

$$f_s=p_4 {v_{aw}}^2\sin(\delta_s-\psi_{aw}) \quad (13)$$

The vectorial representation of the hydrodynamic force F_r in (x_b, y_b) plane is

$$F_r=\begin{pmatrix} -f_r\sin\delta_r \\ f_r\cos\delta_r\cos\phi \end{pmatrix} \quad (14)$$

With $f_r=\|F_r\|$.
The angle of attack on the rudder is determined by the relative water velocity and the rudder angle. It is equal to δ_r [17] because it is assumed that the relative water velocity is parallel to the boat heading near to the rudder actuator. The water generates hence a hydrodynamic force on the rudder which is equal to:

$$f_r=p_5 u^2\sin\delta_r \quad (15)$$

We will assume that the friction forces applied to the catamaran sailboat in $(b-frame)$ are equal to $-p_1 u$, $-p_2 v$ and $-p_3 r$. These force frictions could also be seen as linear damping elements in the model.
According to Newton's second law of motion applied in the $(b-frame)$, we have:

$$(m-X_{\dot u})\dot u=f_s\sin\delta_s-f_r\sin\delta_r-p_1 u \quad (16)$$

$$(m-Y_{\dot v})\dot v=(-f_s\cos\delta_s+f_r\cos\delta_r)\cos\phi-p_2 v \quad (17)$$

$$(I_z-N_{\dot r})\dot r=d_s f_s-d_r f_r-p_3 r$$

With

$$d_s=p_6-p_7\cos\delta_s \quad (18)$$

$$d_r=p_8\cos\delta_r \quad (19)$$

Where
d_s (respectively d_r) denotes the distance between the center of gravity of the sailboat and the axis of the aerodynamic force (respectively hydrodynamic force) passing through CoE of the sail (respectively the rudder).
Equation (17) becomes:

$$(I_z-N_{\dot r})\dot r=(p_6-p_7\cos\delta_s)f_s-p_8\cos\delta_r\,f_r-p_3 r \quad (20)$$

For modelling roll motion in sailing, two approaches were developed in literature. The first one which is based on restoring forces was given by Fossen (see [16], page 62), and the second one where the roll motion is supposed to be pendulum is used in [18]. We used the second approach in this work as follows.

$$\dot p=\frac{z_s f_s\cos\delta_s\cos\phi-p_{13}mg\sin\phi-p_{12}p}{I_x-K_{\dot p}} \quad (21)$$

Therefore, the differential equation system which is composed of (1),(2),(3),(4),(15),(16),(20),(21) is highly nonlinear, it has the following form; $\dot Z=h(Z, W, \psi_{tw}, v_{tw})$
With:
$Z=(x\ y\ \psi\ \phi\ u\ v\ r\ p)^T$ the state vector.
$W=\begin{pmatrix} \delta_s \\ \delta_r \end{pmatrix}$ the input vector.
Now we will rewrite the differential equation system in the form:

$$\dot X=f(X, \psi_{tw}, v_{tw})+g(X)U \quad (22)$$

So, we proceed as follows;
Let's perform a change of variable by taking as new state vector:

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$$X = \begin{pmatrix} Z \\ W \end{pmatrix} \tag{23}$$

In another way, an integrator has been added before of each input which means that the inputs u_1 and u_2 of the system are the differentials of the angles δ_s and δ_r.
The dynamic model of the catamaran sailboat becomes hence:

$$\begin{cases} \dot{x} = u\cos\psi - v\sin\psi\cos\phi \\ \dot{y} = u\sin\psi + v\cos\psi\cos\phi \\ \dot{\psi} = r\cos\phi \\ \dot{\phi} = p \\ \dot{u} = \frac{f_s\sin\delta_s - f_r\sin\delta_r - p_1 u}{m - X_{\dot{u}}} \\ \dot{v} = \frac{(-f_s\cos\delta_s + f_r\cos\delta_r)\cos\phi - p_2 v}{m - Y_{\dot{v}}} \\ \dot{r} = \frac{(p_6 - p_7\cos\delta_s)f_s - p_8\cos\delta_r f_r - p_3 r}{I_z - N_{\dot{r}}} \\ \dot{p} = \frac{z_s f_s\cos\delta_s\cos\phi - p_{13}p_9 g\sin\phi - p_{12}p}{I_x - K_{\dot{p}}} \\ \dot{\delta}_s = u_1 \\ \dot{\delta}_r = u_2 \\ f_r = p_5 u^2\sin\delta_r \\ f_s = p_4 v_{aw}{}^2\sin(\delta_s - \psi_{aw}) \end{cases} \tag{24}$$

The ten differential equations represent the system state dynamic model of the sailboat, it has the new following form;

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{pmatrix} Z \\ W \end{pmatrix} = \begin{pmatrix} h(Z, W, \psi_{tw}, v_{tw}) \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} U \tag{25}$$

With:
$\begin{pmatrix} Z \\ W \end{pmatrix}$ the state vector.
$U = \begin{pmatrix} \dot{\delta}_s \\ \dot{\delta}_r \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$ the input vector.
In the next section an autopilot is developed using feedback linearization for controlling the sailboat heading and sail opening angle.

III. Autopilot Design

The proposed autopilot scheme is given by the following figure.

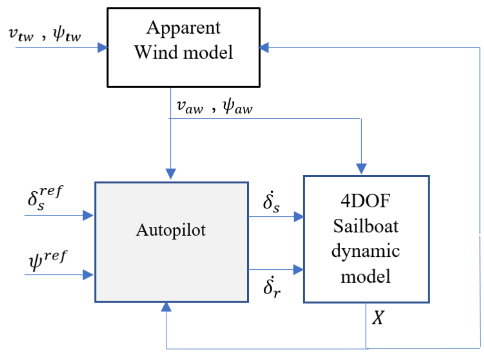

Fig. 5. Proposed control scheme

The mathematical model presented by (24) is highly nonlinear. The state vector is of dimension ten.
Using the feedback linearization method [19][20], the output vector $Y = (\delta_s, \psi)^T$ was differentiated many times as the relative degree requires it [15], in other words three times for ψ and once for δ_s.
We obtain hence:

$$\begin{pmatrix} \dot{\delta}_s \\ \dddot{\psi} \end{pmatrix} = A_1\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + A_2\begin{pmatrix} \dot{f}_s \\ \dot{f}_r \end{pmatrix} + B_1 \tag{26}$$

With:

$$A_1 = \begin{pmatrix} 1 & 0 \\ \frac{p_7 f_s\sin\delta_s\cos\phi}{I_z - N_{\dot{r}}} & \frac{p_8 f_r\sin\delta_r\cos\phi}{I_z - N_{\dot{r}}} \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 \\ \frac{(p_6 - p_7\cos\delta_s)\cos\phi}{I_z - N_{\dot{r}}} & -\frac{p_8\cos\delta_r\cos\phi}{I_z - N_{\dot{r}}} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 \\ -\cos\phi\left(\frac{p_3\dot{r}}{I_z - N_{\dot{r}}} + rp^2\right) - \sin\phi\,(2\dot{r}p - r\dot{p}) \end{pmatrix}$$

The time derivative of equations (12) and (14) is:

$$\begin{pmatrix} \dot{f}_s \\ \dot{f}_r \end{pmatrix} = A_3\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} + B_2 \tag{27}$$

With

$$A_3 = \begin{pmatrix} p_4 v_{aw}{}^2\cos(\delta_s - \psi_{aw}) & 0 \\ 0 & p_5 u^2\cos\delta_r \end{pmatrix}$$

$$B_2 = \begin{pmatrix} p_4\left(2V_{aw}\dot{v}_{aw}\sin(\delta_s - \psi_{aw}) - v_{aw}{}^2\,\dot{\psi}_{aw}\cos(\delta_s - \psi_{aw})\right) \\ 2p_5\dot{u}u\sin\delta_r \end{pmatrix}$$

$$\dot{\psi}_{aw} = 2\frac{V_{aw}^{\dot{y}_b}(V_{aw}^{x_b} + v_{aw}) - V_{aw}^{y_b}(V_{aw}^{\dot{x}_b} + \dot{v}_{aw})}{\left(1 + \left(\frac{V_{aw}^{y_b}}{V_{aw}^{x_b} + V_{aw}}\right)^2\right)(V_{aw}^{x_b} + v_{aw})^2}$$

$$\dot{v}_{aw} = \frac{(V_{aw}^{\dot{x}_b}V_{aw}^{x_b} + V_{aw}^{\dot{y}_b}V_{aw}^{y_b})}{v_{aw}}$$

$$V_{aw}^{\dot{x}_b} = \dot{\psi}v_{tw}\sin(\psi_{tw} - \psi) - \dot{u} + \dot{r}y_s$$

$$V_{aw}^{\dot{y}_b} = -\dot{\psi}\cos\phi\, v_{tw}\cos(\psi_{tw} - \psi) - p\sin\phi\, v_{tw}\sin(\psi_{tw} - \psi) - \dot{v} + z_s\dot{p} - \dot{r}x_s$$

By replacing (27) in (26) we get:

$$\begin{pmatrix} \dot{\delta}_s \\ \dddot{\psi} \end{pmatrix} = AU + B \tag{28}$$

With
$A = A_1 + A_2A_3$
$B = A_2B_2 + B_1$
In order to set $\begin{pmatrix} \dot{\delta}_s \\ \dddot{\psi} \end{pmatrix}$ to a certain setpoint $Q = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix}$, we must take:

$$U = A^{-1}(Q - B) \tag{29}$$

It means that the system becomes linear and it can be presented with the following differential equations:

$$S_L\colon \begin{cases} \dot{\delta}_s = Q_1 \\ \dddot{\psi} = Q_2 \end{cases} \tag{30}$$

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Equations given in (30) are decoupled and linear, its order is equal to four instead of ten. We have thus lost control over six variables which happen to be x, y, ϕ, u, v, p . The loss of control over x and y was predictable (we want the sailboat to move ahead and therefore it is only natural that this corresponds to an instability for these two variables x and y). As for the loss of control over u,v, ϕ and p this is without effect since its dynamics are stable.

By computing the determinant of matrix A, we can show that we have a singularity when this quantity is equal to zero, in other words:

$$\det(A) = 0$$
$$p_8 p_5 u \cos\phi\,(2\sin^2\delta_r - 1) = 0 \qquad (31)$$

Then $u = 0$ or $\delta_r = \frac{\pi}{4} + k\frac{\pi}{2}$ Since $\cos\phi \neq 0\ \forall\ \phi \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$.

This configuration corresponds to a singularity that should be avoided. Therefore, the maximum rudder angle will be set to $|{\delta_r}^{max}| = \frac{\pi}{6}$.

The singularity corresponding to $u = 0$ is relatively simple to understand: when the sailboat is not advancing, we can no longer control its heading. Previously we proposed a feedback linearization loop for our boat. We still have to regulate correctly the obtained linear system (30).

The order of the first subsystem $\dot{\delta}_s = Q_1$ is equal to one, it can be correctly stabilized with a proportional controller.

The order of the second subsystem $\dddot{\psi} = Q_2$ is equal to three, it can be stabilized with a proportional derivative (PDD^2) controller. These two controllers have the following expressions

$$\begin{cases} Q_1 = k_1 e_1 \\ Q_2 = k_2(e_2 - T_{d_1}\dot{\psi} - T_{d_2}\ddot{\psi}) \end{cases} \qquad (32)$$

Where

$e_1 = w_1 - \delta_s$ represents the error of the sail opening angle.

$e_2 = w_2 - \psi$ is the error between the actual heading and the desired ons.

$k_1, k_2, T_{d_1}, T_{d_2}$ are positive design constant.

$w = (w_1\ w_2)^T = ({\delta_s}^{ref}\ \psi^{ref})^T$ is the setpoint vector.

The looped linear system is hence written as:

$$\begin{cases} \dot{\delta}_s = k_1(w_1 - \delta_s) \\ \dddot{\psi} = k_2(w_2 - \psi - T_{d_1}\dot{\psi} - T_{d_2}\ddot{\psi}) \end{cases} \qquad (33)$$

The polynomial characteristics of each equation (33) are:

$$P_1(s) = 1 + \frac{1}{k_1}s \qquad (34)$$
$$P_2(s) = s^3 + k_2 T_{d_2} s^2 + k_2 T_{d_1} s + k_2 \qquad (35)$$

The time constant $\frac{1}{k_1}$ of the equation (34) is chosen equal to 0.25 second which means that $k_1 = 4$.

The second polynomial characteristic is chosen as follow

$$P_2(s) = (1 + \tau s)(s^2 + 2\xi w_n s + w_n^2) \qquad (36)$$

Where ξ is the damping ratio. It is chosen equal to 1. $w_n = 1$ is the natural frequency of the system and $\tau = 1$ is the time constant.

Therefore, using (35) and (36) we get $k_2 = 1$, $T_{d_1} = T_{d_2} = 3$.

The controller is hence given by:

$$U = A^{-1}\left(\begin{pmatrix} 4e_1 \\ e_2 - 3\dot{\psi} - 3\ddot{\psi} \end{pmatrix} - B\right) \qquad (37)$$

With

$$\dot{\psi} = r\cos\phi$$
$$\ddot{\psi} = \dot{r}\cos\phi - rp\sin\phi$$

IV. Simulation and Results

During the simulation (Matlab and Simulink), the true wind data is set to ($\psi_{tw} = 270°, v_{tw} = 10\ m/s$). The used coefficients (given in Appendix A) was borrowed from previous works such as [20]. Based on the boat speed polar[1] shown in Fig. 6 the desired sail opening angle is given by the following function [15].

$${\delta_s}^{ref} = \pi\left[\frac{\psi}{2\pi} + \frac{1}{4}\right] + \frac{\pi}{4} - \frac{\psi}{2} \qquad (38)$$

The sailboat will try firstly to keep a steady heading angle $\psi^{ref} = 30°$ and at $t = 5s$ the desired heading angle is switched to $\psi^{ref} = 0°$. These two desired headings are not situated on the no go zone shown in the sailboat speed polar.

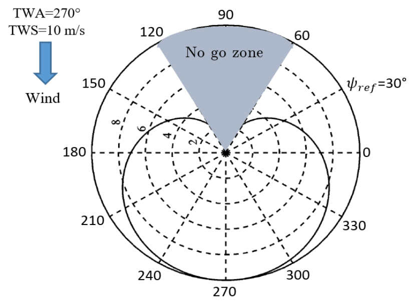

Fig. 6. Sailboat speed polar

The simulation started with initial values:

$$X(t = 0) = (0\ 0\ 45°\ 0\ 8\ 0\ 0\ 0\ \ 30°\ 0)^T \qquad (39)$$

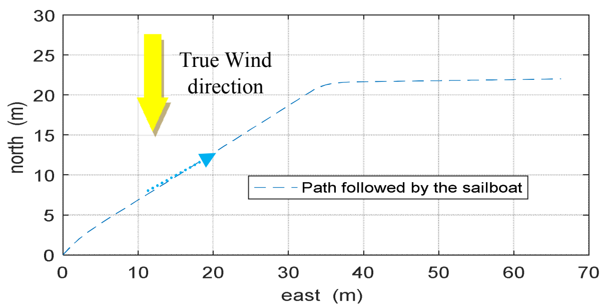

Fig. 7. Path followed by the sailboat

[1] The polar diagram of a sailboat is the set of all pairs $(\psi - \psi_{tw} - \frac{\pi}{2}, u)$ that can be reached by the sailboat when it navigates

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Fig. 7 displays the path followed by the catamaran sailboat during the simulation.

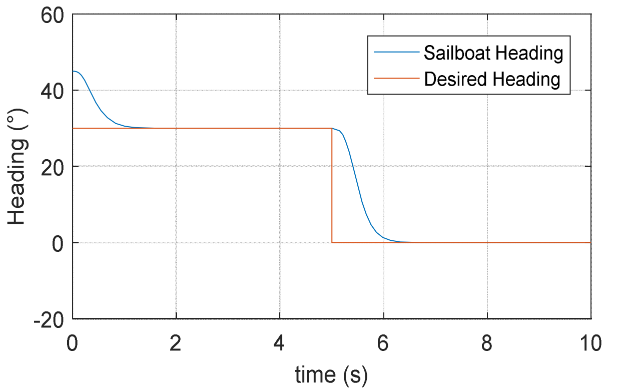

Fig. 8. Time evolution of the sailboat heading ψ and desired heading ψ^{ref}

Fig. 8 shows that $\psi(t)$ finally converges to the desired heading $\psi^{ref}(t)$ with an error $\psi^{ref}(t) - \psi(t) = 0°$.

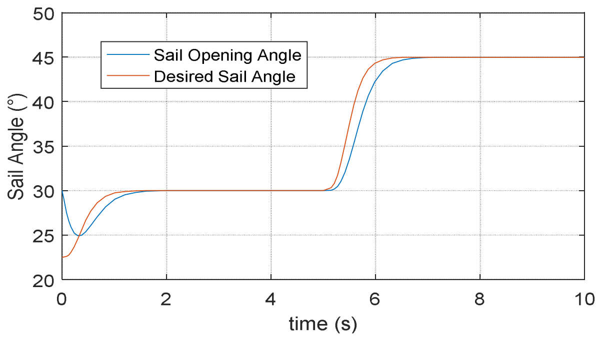

Fig. 9. Time evolution of the sail opening angle δ_s and desired sail angle $\delta_s{}^{ref}$

Fig. 9 shows also that $\delta_s(t)$ converges to the desired sail opening angle given in equation (38) with an error $\delta_s{}^{ref}(t) - \delta_s(t) = 0°$.

V. CONCLUSION

In this paper, a mathematical model describing the dynamic motion of a catamaran sailboat was derived under several approximations. The system is underactuated; it has four degrees of freedom but only two actuators[22].

A control law based on Feedback Linearization combined with PDD techniques was designed for controlling the sailboat heading and sail opening angle. The simulation results show that the used control technique gives good results in terms of regulation. Future work in this area is to validate the proposed control law through experimental test.

ACKNOWLEDGMENT

This work was supported by the "Ministry of Higher Education and Scientific Research".

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Appendix A: Boat parameters

Coefficient	Value	Coefficient	Value
p_1	10 Kg.s/m	p_{12}	40Kg.m2/rad.s
p_2	400 Kg.s/m	p_{13}	0.2 m
p_3	125 Kg.m2/rad.s	z_s	1 m
p_4	30	I_x	12.5 Kg.m2
p_5	50	g	9.8 m/s2
p_6	0.5 m	$X_{\dot{u}}$	-20 Kg
p_7	0.5 m	$Y_{\dot{v}}$	-200 Kg
p_8	1.5 m	$N_{\dot{r}}$	-50 Kg.m2
m	200 Kg	$K_{\dot{p}}$	-12 Kg.m2
I_z	50 Kg.m2	$x_s = y_s$	0

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