MEng Title

Johann Ruben van Tonder
22569596

Thesis presented in partial fulfilment of the requirements for the degree of Master of Engineering (Electronic) in the Faculty of Engineering at Stellenbosch University

Supervisor:

November 2022

Acknowledgements

I would like to sincerely thank the following people for assisting me in the completion of my project:

UNIVERSITEIT·STELLENBOSCH·UNIVERSITY
jou kennisvennoot • your knowledge partner

Plagiaatverklaring / *Plagiarism Declaration*

1. Plagiaat is die oorneem en gebruik van die idees, materiaal en ander intellektuele eiendom van ander persone asof dit jou eie werk is.
   Plagiarism is the use of ideas, material and other intellectual property of another's work and to present is as my own.

2. Ek erken dat die pleeg van plagiaat 'n strafbare oortreding is aangesien dit 'n vorm van diefstal is.
   I agree that plagiarism is a punishable offence because it constitutes theft.

3. Ek verstaan ook dat direkte vertalings plagiaat is.
   I also understand that direct translations are plagiarism.

4. Dienooreenkomstig is alle aanhalings en bydraes vanuit enige bron (ingesluit die internet) volledig verwys (erken). Ek erken dat die woordelikse aanhaal van teks sonder aanhalingstekens (selfs al word die bron volledig erken) plagiaat is.
   Accordingly all quotations and contributions from any source whatsoever (including the internet) have been cited fully. I understand that the reproduction of text without quotation marks (even when the source is cited) is plagiarism

5. Ek verklaar dat die werk in hierdie skryfstuk vervat, behalwe waar anders aangedui, my eie oorspronklike werk is en dat ek dit nie vantevore in die geheel of gedeeltelik ingehandig het vir bepunting in hierdie module/werkstuk of 'n ander module/werkstuk nie.
   I declare that the work contained in this assignment, except where otherwise stated, is my original work and that I have not previously (in its entirety or in part) submitted it for grading in this module/assignment or another module/assignment.

22569596 Studentenommer / *Student number*	Handtekening / *Signature*
J.R. van Tonder Voorletters en van / *Initials and surname*	February 18, 2023 Datum / *Date*

Abstract

English

Afrikaans

Contents

List of Figures

List of Tables

Nomenclature

Ocean Vessel Dynamics

$X,\ Y,\ Z$	Coordinates of force vector decomposed in the body-fixed frame(surge, sway and heave forces)
$K,\ M,\ N$	Coordinates of moment vector decomposed in the body-fixed frame(roll, pitch and yaw moment)
$u,\ v,\ w$	Coordinates of linear velocity vector decomposed in the body-fixed frame(surge, sway and heave velocities)
$p,\ q,\ r$	Coordinates of angular velocity vector decomposed in the body-fixed frame(roll, pitch and yaw angular velocities)
$x,\ y,\ z$	Coordinates of position vector decomposed in the body-fixed frame(surge, sway and heave positions)
$\phi,\ \theta,\ \psi$	Coordinates of Euler angle vector decomposed in the body-fixed frame(roll, pitch and yaw Euler angles)

Acronyms and abbreviations

SNAME	Society of Naval Architects and Engineers
CG	Center of gravity of the vessel

Chapter 1

Introduction

1.1. Background

1.2. Problem Statement

1.3. Summary of Work

1.4. Scope

1.5. Format of Report

Chapter 2

Literature Review

Chapter 3

Modeling of Ocean Vessels

This chapter models a standard ocean vessel in six degrees of freedom. It also introduces the definitions associated with movement in each direction of freedom. The chapter also take into account the forces and moments generated by hydrodynamics and restoration of an ocean vessel. The chapter continues to model the environmental disturbances experience by a semi-submerged ocean vessel. The environmental disturbances are wind, waves and ocean currents.

3.1. Standard Ocean Vessel Notation

An ocean vessels are modelled in six degrees of freedom, requiring six independent coordinates to determine its position and orientation. The first three coordinates corresponding to position (x, y, z) and their first time derivatives, translation motion along the x-, y-, and z-axes. The last three coordinates (ϕ, θ, ψ) and their first time derivatives describing orientation and rotational motion [1]. Figure 3.1 illustrates the motion variables of an ocean vessel with the six independent coordinates.

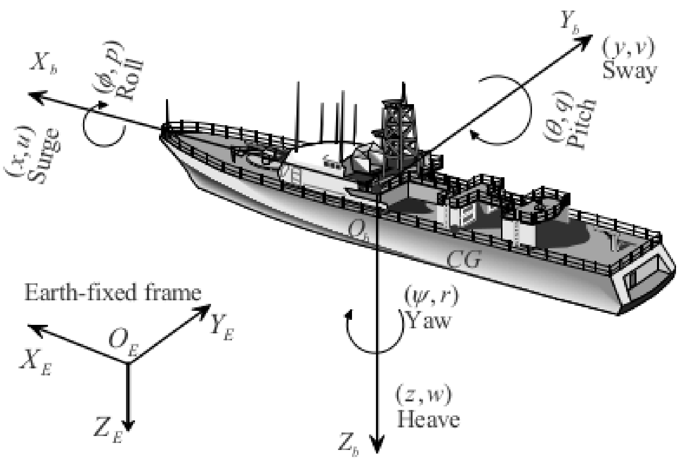

Figure 3.1: Motion variables for an ocean vessel

The SNAME(Society of Naval Architects and Marine Engineers) established the notation for the six different motion components as *surge*, *sway*, *heave*, *roll*, *pitch* and *yaw*. Table A.1 summarizes the SNAME notation for ocean vessels.

Two reference models are used to determine the equations of motion, namely the inertial to earth frame $O_eX_eY_eZ_e$ that may be displaced to overlap with the vessel's fixed coordinates in some initial condition and the body-fixed frame $O_bX_bY_bZ_b$, illustrated in Figure 3.1. The most common used position for the body-fixed frame results in symmetry about the $O_bX_bZ_b$-plane and approximate symmetry about the $O_bY_bZ_b$. The body axes coincides with the axes of inertia and are usually defines as follows: O_bX_b is the longitudinal axis, O_bY_b is the transverse axis and O_bZ_b is the normal axis. Below are the vectors used to describe the general motion of an ocean vessel:

$$\mathbf{n} = [\mathbf{n_1}\mathbf{n_2}]^T \tag{3.1}$$

$$\mathbf{v} = [\mathbf{v_1}\mathbf{v_2}]^T \tag{3.2}$$

$$\tau = [\tau_{\mathbf{1}}\tau_{\mathbf{2}}]^T \tag{3.3}$$

$$\mathbf{n_1} = [\,x \;\; y \;\; z\,]^T \tag{3.4}$$

$$\mathbf{n_2} = [\,\phi \;\; \theta \;\; \psi\,]^T \tag{3.5}$$

$$\mathbf{v_1} = [\,u \;\; v \;\; w\,]^T \tag{3.6}$$

$$\mathbf{v_2} = [\,p \;\; q \;\; r\,]^T \tag{3.7}$$

$$\tau_{\mathbf{1}} = [\,X \;\; Y \;\; Z\,]^T \tag{3.8}$$

$$\tau_{\mathbf{2}} = [\,K \;\; M \;\; N\,]^T \tag{3.9}$$

where $\mathbf{n}$ denotes the position and orientation vector with coordinates in the earth fixed frame, $\mathbf{v}$ denotes the linear and angular velocity vector with coordinates in the body-fixed frame and τ denotes the forces and moments acting on the vessel in the body-fixed frame. The vessel dynamics are divided into two parts known as *kinematics* and *kinetics*.

3.2. Kinematics

Kinematics looks at the motion of the vessel without directly considering the forces affecting the motion. The first time derivative of the position vectors $\mathbf{n_1}$ and $\mathbf{n_2}$ is related to the linear velocity vector $\mathbf{v_1}$ and $\mathbf{v_2}$ via the following transformations,

$$\dot{\mathbf{n}}_\mathbf{1} = \mathbf{J_1}(\mathbf{n_2})\mathbf{v_1} \tag{3.10}$$

$$\dot{\mathbf{n}}_\mathbf{2} = \mathbf{J_2}(\mathbf{n_2})\mathbf{v_2} \tag{3.11}$$

where $\mathbf{J_1}(\mathbf{n_2})$ and $\mathbf{J_2}(\mathbf{n_2})$ are transformation matrices, which is related through the functions of the Euler angles: roll(ϕ), pitch(θ) and yaw(ψ). The $\mathbf{J_1}$ transformation matrix is given by

$$\mathbf{J_1}(\mathbf{n_2}) = \begin{bmatrix} \cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\theta)+\sin(\phi)\sin(\theta)\cos(\psi) & \sin(\psi)\sin(\phi)+\sin(\theta)\cos(\psi)\cos(\phi) \\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\phi)+\sin(\phi)\sin(\theta)\sin(\psi) & -\cos(\psi)\sin(\phi)+\sin(\theta)\sin(\psi)\cos(\phi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix} \tag{3.12}$$

and the transformation matrix $\mathbf{J_2}$ is given by,

$$\mathbf{J_2}(\mathbf{n_2}) = \begin{bmatrix} 1 & -\sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \tag{3.13}$$

When $\theta = \pi/2$, the transformation matrix $\mathbf{J_2}(\mathbf{n_2})$ becomes singular, however this is unlikely to happen when practically testing an ocean vessel, because of the metacentric restoring forces. Combining Equation 3.12 and Equation 3.13 results in the kinematics of an ocean vessel.

$$\begin{bmatrix} \dot{\mathbf{n}}_1 \\ \dot{\mathbf{n}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{J_1}(\mathbf{n_2}) & 0_{3\times 3} \\ 0_{3\times 3} & \mathbf{J_2}(\mathbf{n_2}) \end{bmatrix} \begin{bmatrix} \mathbf{v_1} \\ \mathbf{v_2} \end{bmatrix} = \dot{\mathbf{n}} = \mathbf{J}(\mathbf{n})\mathbf{v} \tag{3.14}$$

3.3. Kinetics

The Newton-Euler formulation [2] defines the balancing forces and moments for a rigid body with a mass of m as follows,

$$\mathbf{f}_{Ob} = \mathbf{m}[\dot{\mathbf{v}}_{Ob}^{E} + \dot{\mathbf{w}}_{Ob}^{E} \times \mathbf{r}_{Ob} + \mathbf{w}_{Ob}^{E} \times \mathbf{v}_{O}b + \mathbf{w}_{Ob}^{E} \times (\mathbf{w}_{Ob}^{E} \times \mathbf{r}_{Ob})] \tag{3.15}$$

$$\mathbf{m}_{Ob} = \mathbf{I}_o \mathbf{w}_{Ob}^{E} + \dot{\mathbf{w}}_{Ob}^{E} \times \mathbf{I}_o \mathbf{w}_{Ob}^{E} + m\mathbf{r}_{Ob} \times (\dot{\mathbf{v}}_{O}b + \mathbf{w}_{Ob}^{E} \times \mathbf{v}_{Ob}) \tag{3.16}$$

where $\mathbf{f}_{Ob}$ is the balancing forces,$\mathbf{m}_{Ob}$ the balancing moments and $\mathbf{I}_o$ is the inertia matrix about O_b defined as,

$$\mathbf{I_o} = \begin{bmatrix} I_x & -I_{xy} & -Ixz \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_Z \end{bmatrix} \tag{3.17}$$

I_x,I_y and I_z are the moments of inertia about the O_bX_b,O_bY_b and O_bZ_b axes. $I_{xy} = I_{yx}$, $I_{xz} = I_{zx}$ and $I_{yz} = I_{zy}$ are the products of inertia. These quantities are defined as

$$I_x = \int_V (y^2 + z^2)\rho_m dV \tag{3.18}$$

$$I_x y = \int_V xy\rho_m dV \tag{3.19}$$

$$I_y = \int_V (x^2 + z^2)\rho_m dV \tag{3.20}$$

$$I_x z = \int_V xz\rho_m dV \tag{3.21}$$

$$I_z = \int_V (z^2 + y^2)\rho_m dV \tag{3.22}$$

$$I_z y = \int_V zy\rho_m dV \tag{3.23}$$

where pm are the mass density and V the volume of the rigid body. By substituting the definitions defined in Table A.2 into Equations 3.15 and 3.16, results in the equation below,

$$\mathbf{M_{RB}}\dot{\mathbf{v}} + \mathbf{C_{RB}}(\mathbf{v})\mathbf{v} = \tau_{\mathbf{RB}} \tag{3.24}$$

where $\mathbf{v} = [u\ v\ w\ p\ q\ r]^T$ is the generalized velocity vector decomposed in the body-fixed frame and $\tau_{\mathbf{RB}} = [X\ Y\ Z\ K\ M\ N]^T$ is the generalized vector of external forces and moments. The rigid body system inertia matrix $\mathbf{M_{RB}}$ and the rigid body Coriolis and centripetal matrix $\mathbf{C_{RB}}$ is defined in Equation A.1 and A.2 The generalized external force and moment vector, $\tau_{\mathbf{RB}}$, is a sum of the hydrodynamic force and moment vector $\tau_{\mathbf{H}}$, external disturbance force and moment vector $\tau_{\mathbf{E}}$ and propulsion force and moment vector τ.

3.4. Hydrodynamic Forces and Moments

3.5. Restoring Forces and Moments

3.6. Environmental Disturbances

3.7. Propulsion Forces and moments

Propulsion with a fixed wing sail leading to next chapter.

Chapter 4

Modeling of a Fixed-Wing Sail

Chapter 5

Stability Analysis

Chapter 6

Platform Development

Chapter 7

Control Techniques for a Sail and Rudder

Chapter 8

Model Simulation

Chapter 9

Results

Chapter 10

Conclusion

Bibliography

[1] K. D. Do and J. Pan, *Control of Ships and Underwater Vehicles.* Springer Science+Business Media, LLC, 2009.

[2] R. Featherstone, *Rigid Body Dynamics Algorithms.* Springer Science+Business Media, LLC, 2008.

Appendix A

Additional Modelling Information

A.1. Notation and Vector Definitions

Table A.1: SNAME Notation for ocean vessels

Degree of freedom		Force and moment	Linear and angular velocity	Position and Euler angles
1	Surge	X	u	x
2	Sway	Y	v	y
3	Heave	Z	w	z
4	Roll	K	p	ϕ
5	Pitch	M	q	θ
6	Yaw	N	r	ψ

Table A.2: Rigid body motion vectors

Vector	Components	Definition
$\mathbf{f}_{Ob}$	$[X\ Y\ Z]^T$	force decomposed in the body-fixed frame
$\mathbf{m}_{Ob}$	$[K\ M\ N]^T$	moment decomposed in the body-fixed frame
$\mathbf{v}_{Ob}$	$[u\ v\ w]^T$	linear velocity decomposed in the body-fixed frame
$\mathbf{w}_{Ob}^E$	$[p\ q\ r]^T$	angular velocity of the body-fixed relative to the earth-fixed frame
$\mathbf{r}_{Ob}$	$[x_g\ y_g\ z_g]^T$	vector from O_b to CG decomposed in the body-fixed frame

A.2. Modeling Equations

$$
\mathbf{M_{RB}} = \begin{bmatrix} m & 0 & 0 & mz_g & mz_g & -my_g \\ 0 & m & 0 & 0 & 0 & mx_g \\ 0 & 0 & m & -mx_g & -mx_g & 0 \\ 0 & -mz_g & -my_g & I_x & -Ixy & -I_xz \\ mz_g & 0 & -mx_g & -I_{xy} & I_y & -I_yz \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{A.1}
$$

$$
\mathbf{C_{RB}}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & m(y_gq + z_gr) & -m(x_gq - w) & -m(x_gr + v) \\ 0 & 0 & 0 & -m(y_gp + w) & m(z_gr + x_gp) & -m(y_gr - u) \\ 0 & 0 & 0 & -m(z_gp - v) & -m(z_gq + u) & m(x_gp + y_gq) \\ -m(y_gq + z_gr) & m(y_gp + w) & m(y_gp - v) & 0 & -I_{yz}q - I_{xz}q + I_zr & I_{yz}r + I_{xy}p - I_yq \\ m(x_gp - w) & -m(z_gr - x_gp) & m(z_gq + u) & I_{yz}q + I_{xz}p - I_zr & 0 & -I_{xz}r - I_{xy}q + I_Xp \\ m(x_gr + v) & m(y_gr - u) & -m(x_gp + y_gq) & -I_{yz}r - I_{xy}p + I_yq & I_{xz}r + I_{xy}q - I_xp & 0 \end{bmatrix} \tag{A.2}
$$