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Identification of Ship Maneuverability Indexes by Fourier Method

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Abstract: *K* and *T* indexes are important indicators to measure the maneuverability of ship. Usually Z-type test is used to calculate the ship maneuverability indexes. This paper presents a method to identify the ship maneuverability indexes *K* and *T* by applying Fourier series approximation method. And it is verified based on the Z-type test data of training vessel YUKUN that this method is convenient and efficient.

Keywords: Fourier series, Ship Maneuverability Indexes, Z-type test, Parameter identification

1 Introduction

Up to now, there are three kinds of ship mathematical models were proposed in analyzing ship maneuverability and controllability. There are whole ship model，MMG model proposed by the Japanese Maneuvering Modeling Group and NOMOTO model. The NOMOTO model includes only simple parameters such as K, T indices which can be expressed by the linear hydrodynamic derivatives. And the indexes *K* and *T* are called ship maneuverability indexes.

Ship maneuverability indexes *K* and *T* are important indexes to express the quality of ship maneuverability and became common method to measure the ship maneuverability as its clear physical meaning. The International Maritime Organization requires vessels engaged in operations must be equipped with ship maneuverability information [1].

Although in recent years a variety of identification methods are used in the calculation of the index of ship maneuverability. But the Z-type test method is still the most basic and the most accurate method of this calculation. Therefore, a new ship is often carried out Z-type test, and use the data obtained in the test to calculate the *K* and *T* indexes. In practice, staffs draw curves and calculate the indexes by manual methods using experimental data, but this method is cumbersome and human error is larger. In recent years, the computer technology has been greatly developed. It becomes very easy to fit the curve and get function derivation. For these reasons, this paper presents a simple and easy identification method, and the specific implementation steps are given. This method allows people obtained the index conveniently based on the experimental data.

The paper is organized as follows. A review of NOMOTO ship model and ship maneuverability indexes are given in section 2. And in section 3, the specific steps of Z-type test have been introduced. Identification method of Fourier series approximation is given in details in Section 4.

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2 NOMOTO Model of Ship

2.1 Derivation of NOMOTO Model

It is commonly assumed that the influence of heave, roll, and pitch motions into the motion in the horizontal plane can be neglected. This assumption is approximately valid when the ship is in the situation of maintain the course or track keeping.

To describe the equations of motion a coordinate system fixed to the ship with the x-and y-axes as shown in Fig.1. The projections of the total ship speed V on the x-and y-axes are called the surge velocity u and the sway velocity v. The turning rate is denoted r and the heading and rudder angles are denoted ψ and δ.

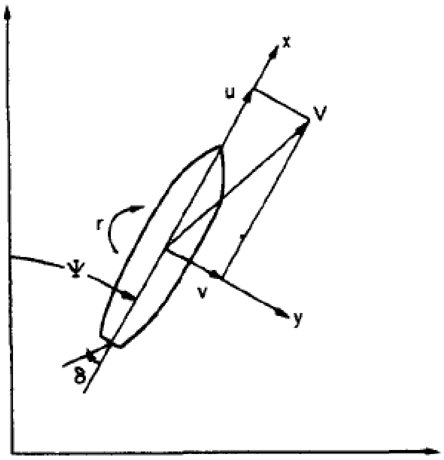

Fig. 1: Ship with coordinate system for equations of motion

The equations for the horizontal motion are obtained from Newton's laws expressing conservation of linear and angular momentum:

$$\begin{aligned} & m(\dot{u} - vr - x_G r^2) = X \\ & m(\dot{v} + ur + x_G \dot{r}) = Y \\ & I_z \dot{r} + m x_G (\dot{v} + ur) = N \end{aligned} \quad (1)$$

where X and Y are the components of the hydrodynamic forces on the x-axis and y-axis, N is the z-component of the hydrodynamic moments, m is the mass of the ship, I_z is the moment of inertia about the z-axis, and x_G is the x coordinate of the center of gravity[2].

To simplify the equations of motion, the ship center of gravity is assumed as the origin of the body-fixed coordinate system. It is viable to ignore the amount of the second order and some higher when we simplify the equations of motion. Add the linear expressions of hydrodynamic force to the equation (1). Then we derive the linear equation of ship motion as follow:

$$\begin{aligned} & (m + m_y) - Y_v v + (m + m_x) u_0 r - Y_r r = Y_\delta \delta \\ & (I_z + J_{zz}) \dot{r} - N_v v - N_r r = N_\delta \delta \end{aligned} \quad (2),$$

where m_y and m_x is the added mass of ship on the y-axis and x-axis. J_{zz} is the added moment of inertia about the z-axis.

Then we can get a linear equation of ship motion by eliminate the parameter u in equation (2).As given below, the equation is also called the second order NOMOTO model.

$$T_1 T_2 \ddot{r} + (T_1 + T_2)\dot{r} + r = K\delta + K T_3 \dot{\delta} \quad (3),$$

where T_1, T_2, T_3, K are four parameters. The NOMOTO model is also called the respond model, which reflects the relationship between the system input, the rudder angle, and the system output, the heading.

To further simplify the equation, we should give a comprehensive consideration of the parameters T_1, T_2, T_3 .Then we get the first order NOMOTO model.

$$T\dot{r} + r = K\delta \quad (4),$$

where $T = T_1 + T_2 - T_3$ in the NOMOTO model[3][4].

2.2 Maneuverability Index K and T

The NOMOTO model is widely used in the study of ship maneuverability and controllability. By analyzing (4) carefully, we find that the parameters K and T in NOMOTO model have a clear physical significance. The parameter K is called ship turning quality index, which represents the ratio of rotating moment and damping moment generated by per unit rudder angle in the rotary motion of the ship. The parameter K is a kind of static gain, which determines the value of ship turning rate. The parameter T is called ship turning lag index, which represents the ratio of moment of inertia and damping moment in the rotary motion of the ship. The parameter T determines the response time of a given rudder angle.

Maneuverability and controllability of ship can be judged through the maneuverability index K and T. It is a very important thing for seafarers to know the ship maneuverability index exactly. There are several measures can calculate the index of ship maneuverability. In these methods, Z-type test is the most convenient and accurate method to calculate the ship maneuverability index.

3 Traditional Z-type Test and Calculation of Ship Maneuverability Index

3.1 Z-type Test

(1).To be maintaining constant voyage with stated speed.
(2).Turn the rudder right for angle10° and maintained.
(3).Turn the rudder left for angle 10° and maintained when the angle of ship head turn right to maximum angle 10°.
(3).Turn the rudder right for angle 10° and maintained

when the angle of ship head turn left to maximum angle 10°.

(4). Then repeat the steps (2) and (3) above several times.

In the process of the test, the time of the rudder angle reached the designated location, the characteristics time of turning head and the angle of inertia exceed should be accurate record[8]. Drew the curve δ —t, ψ —t with these data as Fig.2.

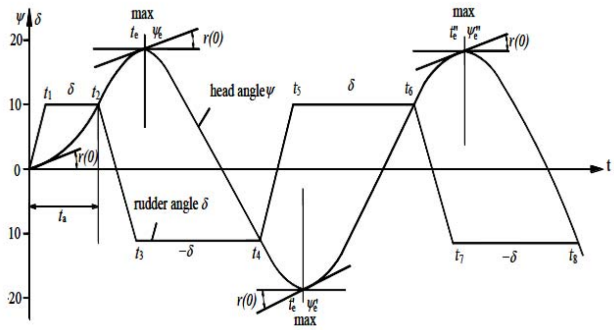

Fig.2: the test curve of ψ —t and δ —t

In the figure above, t_a is the initial rotary time of the rudder, which is the time needed for the ship turning certain angle after turning the rudder for the angle δ. The greater t_a the worse of the initial turning function, and vice versa. The angle of inertia exceeds and the lag time of turning head could be used for evaluating the ability of ship inhibitory deflection. The angle of inertia exceed refer to the difference between the instantaneous angle steering opposite rudder and the maximum heading angle. As shown in Figure 2 ψ_1, ψ_2, ψ_3. The lag time of turning head *T* refer to the time intervals between the instantaneous times of the rudder passing the position of zero angles to the maximum turning angle as shown in figure [5].

3.2 Calculation of Maneuverability Index

The traditional method of ship maneuvering index calculation is to find some characteristic points of the curve in Figure 2, and then the first-order response equation transformed into an algebraic equation, thereby calculate the *K* and *T*.

ψ is the heading of ship, and r is the turning rate. Considering the relationship between ψ and r, $\psi = \dot{r}$ .The NOMOTO model can be expressed in that form:

$$T\ddot{\psi} + \dot{\psi} = K\delta \qquad (5).$$

Considering the fixed rudder of rudder angle δ_r , the equation (5) should be

$$T\ddot{\psi} + \dot{\psi} = K(\delta + \delta_r) \qquad (6).$$

Then the integral style of equation (6):

$$\int_0^t T\frac{d\dot{\psi}}{dt}dt + \int_0^t \dot{\psi}dt = \int_0^t K(\delta + \delta_r) \qquad (7)$$

The results:

$$T[r(\mathrm{t}) + \mathrm{r}(0)] + [\psi(\mathrm{t}) - \psi(0)] = K[\delta_r(t-0) + \int_0^t \delta dt] \qquad (8)$$

Then find three characteristic points of the curve in Figure 2. In general, we will choose the vertices of the curve, such as t_e, t'_e, t''_e. Take the data corresponding to the three characteristic points into equation (8). Thus we get three equations. There are three unknowns in these equations; we can calculate three unknowns, that is, the index *K* and *T*. The specific methods and procedures of that can be obtained from the reference [2][8].

3.3 Related Problems of the Calculation

According to methods previously described, we can calculate the ship maneuverability indexes *K* and *T*. But there are some problems with this approach.

(1) The Z-type test cure is obtained by connecting the discrete data points, which is obtained by Z-type test, using smooth curve. There are some errors in heading angle between the non-recording point and the real course angle.

(2) In the process of calculating *K* and *T*, $\ddot{\psi}$ and $\dot{\psi}$ is obtained by seeking the derivative of the curve δ —t. The traditional method of seeking the derivative is makes a tangent of the curve through hand drawn. However, such manual operations can generate errors which cannot be controlled. Different people will draw different curves by the same data, and thus may reach different conclusions [6].

In view of the above problems, this paper presents a method using Fourier approximation to get the curve. In this method, the curve and its derivative are calculated by a program which can reduce the impact of human error. This method is more simple and convenient with a better reproducibility.

4 Identification of Ship Maneuverability Index by Fourier Method

In order to calculate ship maneuverability index, this paper proposes a method, which using curve fitting and Fourier series approximation to identify the equation of $\frac{d\psi}{dt}(t)$ and $\frac{d^2\psi}{dt^2}(t)$. As a result, we have the curves and the expressions of $\psi - t$, $\dot{\psi} - t$, $\ddot{\psi} - t$ and $\delta - t$. And then, it is easy to calculate ship maneuverability index according to the method as described below.

In this section, we apply Z-type test data of training vessel YUKUN to describe how to identify the K and T indexes.

Table 1: Heading and Rudder Angle

t	0	1	3	…	268	269	270
ψ	20.4	21.6	22.6	…	-17.0	-16.4	-16.0
δ	7	-1	-3	…	10	10	10

According to the data in the table above, we can get the heading angle and rudder angle changing trends over time and the following curves of $\psi - t$ and $\delta - t$.

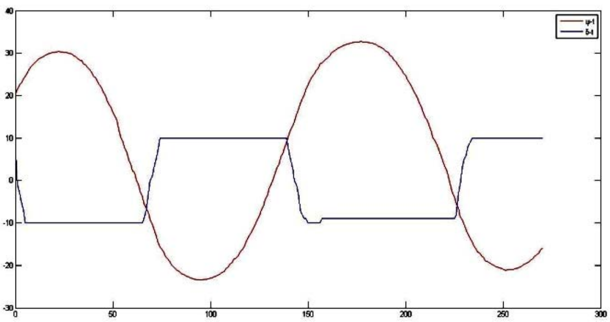

Fig.3: ψ —*t* and δ —*t*

4.1 Fourier Series Approximation

In mathematics, a Fourier series decomposes periodic functions or periodic signals into the sum of a set of simple oscillating functions, namely sins and cosines [7].For example, a functions *f(x)* can be decomposed into a Fourier series form as follows:

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

When we observe the curve of δ — t carefully, we will find that it is similar to the curve of trigonometric functions. Therefore, this paper presents Fourier series approximation method to calculate *K* and *T*.

4.2 Steps of Identification

Step 1

Using Fourier series approximation and curve fitting methods, the curve as shown below can be got. This fitted curve is in good agreement with the original data, and reflects characteristics of the original data[9][10].

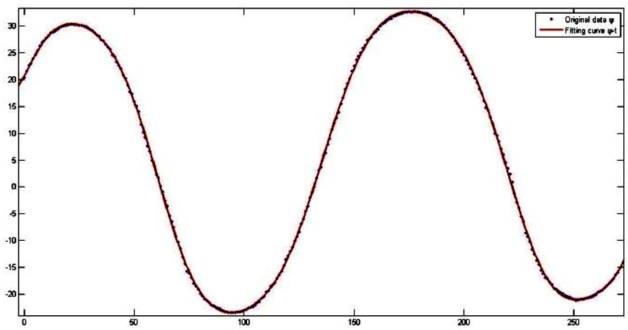

Fig.4: The curve of ψ —t fitted by Fourier series

Meanwhile, we can get the expression of fitted curve ψ —*t*. This expression in the form as follows:

$$\begin{aligned}\psi(\mathrm{t}) = \mathrm{a}_0 &+ \mathrm{a}_1\cos(\mathrm{tw}) + \mathrm{b}_1\sin(\mathrm{tw}) + \\ &\mathrm{a}_2\cos(2\mathrm{tw}) + \mathrm{b}_2\sin(2\mathrm{tw}) + \\ &...+ \mathrm{a}_8\cos(8\mathrm{tw}) + \mathrm{b}_8\sin(8\mathrm{tw})\end{aligned} \tag{9}$$

Table 2: Parameter of Fitting Results

parameter	value
a_0	31.09
a_1	31.34
b_1	-37.77
a_2	6.752
…	…
a_8	0.817
b_8	-0.907
w	0.01639

where a_0 , a_1 … a_8 , b_8 are all parameters of fitting. The R-square and Adjusted R-square of the fitting result are all

0.9998. Via observing the fitted curve and analyzing the fitting results, it can be seen that the Fourier series approximation is a feasible method.

Step 2

Now the expression of the heading angle changes with time is obtained, that is (9). Then we can get the first derivative $\dot{\psi}(t)$ and the second derivative $\ddot{\psi}(t)$ of $\psi(t)$ easily [8] [9]. The fitted curve of $\psi(t)$ and the curve of $\dot{\psi}(t)$ and $\ddot{\psi}(t)$ are obtained as follows:

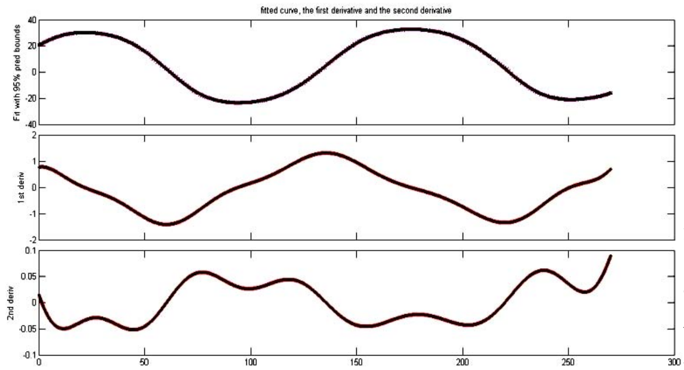

Fig.3: $\psi(t)$, $\dot{\psi}(t)$ and $\ddot{\psi}(t)$

In Figure 3, the first curve is the curve of equation (9), which is fitted by the data of table 1.The second and third curve are the first derivative $\dot{\psi}(t)$ and the second derivative $\ddot{\psi}(t)$ of $\psi(t)$.

Step 3

Ship maneuverability index *K* and *T* can be easily calculated whit figure 3.Since there are three unknowns in the equation (6), then choose three time points in Figure 3, for example, t_1, t_2 and t_3. Thereby, we will get the equations as shown below:

$$\begin{aligned} T\ddot{\psi}_{t1} + \dot{\psi}_{t1} &= K(\delta_1 + \delta_r) \\ T\ddot{\psi}_{t2} + \dot{\psi}_{t2} &= K(\delta_2 + \delta_r) \\ T\ddot{\psi}_{t3} + \dot{\psi}_{t3} &= K(\delta_3 + \delta_r) \end{aligned} \quad (10)$$

Solving the equations (10), *K*, *T* and δ_r can be calculated.

Step 4

In order to make a more accurate value of *K*, *T* and δ_r, several different time points are selected in figure 3 and then repeat step 3. Then we get $T_1, T_2 ... T_n$ and $K_1, K_2 ... K_n$.Then using the following formula to obtain the average.

$$\begin{aligned} K &= \frac{K_1 + K_2 + ... + K_n}{n} \\ T &= \frac{T_1 + T_2 + ... + T_n}{n} \end{aligned} \quad (11)$$

Through the above four steps of identification, we got ship maneuverability index of training vessel YUKUN, that is $K = 0.1, T = 16.3$. Thus we have the NOMOTO model of training vessel YUKUN. Then we carry out ship Z-type test simulation to test the feasibility of identification results. In this Z-type test simulation, we will table rudder angle values in Table1 as input. The simulation result is as follow:

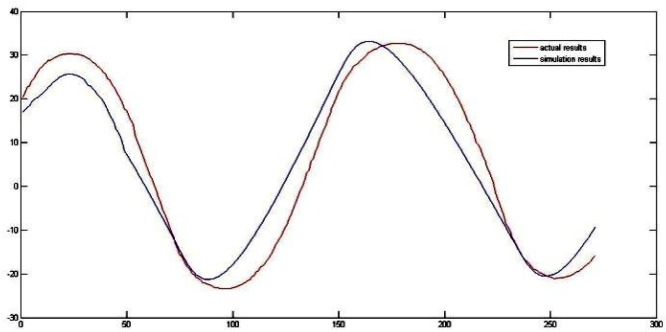

Fig.5: Z-type Test Simulation Results

From Figure 5 we can know that the accuracy of this identification method as the references of the daily operations of the ship maneuverability is sufficient.

Conclusion

Maneuverability indexes *K* and *T* directly determines the maneuverability of a ship, and it also effect responses of seafarers to different situations at sea. The purpose of this paper is to present an identification method of the ship's maneuverability index, which matches the development of modern computer technology. This identification method of Fourier series approximation is more simple and understandable, and the same time, it is more convenient and accurate than the manual method and other identification method.

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