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Roll Stabilization Control of Sailboats

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Abstract: This paper reports a way of dynamically controlling the sail of a sailboat to reduce the heel angle and roll motion caused by the wind. This is mainly to increase robustness and safety for autonomous sailboats but could also be used to increase comfort for crew. The solution consists of a linear quadratic regulator (LQR) controlling the moment created by the sail. A lookup table will then choose the optimal angle of the sail, optimized for maximum forward acceleration, given the relative wind direction and desired moment. Simulation results are presented to show the effectiveness of the approach.

Keywords: Roll Control, Modeling, Sailboat.

1. INTRODUCTION

This paper is a continuation of Wille et al. (2016). The model and the course controller presented in Wille et al. (2016) is the foundation of the theory developed in this paper. It is recommended to read the other paper first as theory and equations from the previous paper will be referenced directly (Equations 1 - 59) and notation reused.

The large surface area of the sail provides the sailboat with forward thrust. However, it also makes the boat vulnerable against strong winds. This can cause large heeling angles, a lot of roll motion due to wind gusts, and in worst case cause the boat to capsize. Autonomous Sailboats will never be a viable option if they cannot handle a variety of different weather conditions. Robustness of a system is an important aspect of autonomy.

The solution presented in this paper controls the roll motion by utilizing the sail. It will reduce the heeling angle and the roll motion created by the wind. The controller design is simple but effective, and the states used in the feedback loop are easy to estimate.

2. SAIL CONTROLLER

2.1 Optimal angle of the sail for forward thrust

The sail is used to create forward propulsion for the sailboat. There is an optimal sail angle that gives the highest forward acceleration for a given relative wind direction. The objective is thus to create a map from relative wind direction to the optimal angle of the sail. We begin by defining a way of measuring the force created by the sail in the forward direction independent of the wind speed:

$$S_{x'_r}(\beta_{ws}, \lambda) = \frac{S_{x'}(\beta_{ws}, \lambda, V_{ws})}{V_{ws}^2}, \tag{60}$$

where $S_{x'}$ is the force created by the sail in positive surge direction (see (29)) and $S_{x'_r}$ is the force divided by the wind speed (relative to the boat) squared. $S_{x'_r}$ is a function of λ and β_{ws}, and there will be an optimal λ given a β_{ws}.

However, λ cannot be chosen freely; it is restricted to a certain range depending on the wind direction and λ_{sat}. This is because we only have control of a rope limiting $|\lambda|$, and the rope only works through tension. When the wind is hitting from starboard, it follows that $\lambda > 0$ and that the torque created by the sail around the mast has to be positive (meaning a positive angle of attack). When the wind is hitting from port side, $\lambda > 0$ and the torque has to be negative (negative angle of attack).

The following equations describe the upper and lower limits of a stable angle of the sail, $\lambda \in [\lambda_l, \lambda_u]$, given a relative wind direction:

$$\lambda_u = \begin{cases} \beta_{ws} + \pi & \text{if } \beta_{ws} < 0 \\ 0 & \text{if } \beta_{ws} > 0 \end{cases} \tag{61}$$

$$\lambda_l = \begin{cases} 0 & \text{if } \beta_{ws} < 0 \\ \beta_{ws} - \pi & \text{if } \beta_{ws} > 0 \end{cases}. \tag{62}$$

In addition to (61) and (62), λ_{sat} still applies. That is, $\lambda_u \in [0, \lambda_{sat}]$ and $\lambda_l \in [-\lambda_{sat}, 0]$.

The optimal angle of the sail can then be found by traversal of all viable λ, trying to maximize $S_{x'_r}$ for a given β_{ws}. This can be computed off-line, and the results stored in a lookup table. In this paper, an optimal λ was found for a step size of $1deg$ in β_{ws}. In order to avoid unnecessary discontinuities in the control action, when there is only a small difference in the optimal λs between a step change in the lookup table, a new optimal λ is found based on a linearized solution between the two data points (this is computed on-line).

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2.2 Relative moment of the sail

In the same manner that we defined the relative force in surge direction, the relative moment in roll is defined as

$$S_{\phi_r}(\beta_{ws}, \lambda) = \frac{S_\phi(\beta_{ws}, \lambda, V_{ws})}{V_{ws}^2}, \tag{63}$$

where $S_{\phi r}$ is calculated at each optimal λ and is added to the lookup table. This solution of λ will hereafter, be referred to as the unconstrained solution, λ_{100}.

Before applying the control law to reduce roll motion and heel angle, we have to be able to control the amount of moment created in roll by the sail. This is done by finding new optimal λs, though with a restriction on the maximum relative moment created at each solution.

To produce the desired result, the same traversal algorithm as discussed earlier is performed again, but with the restriction that the solution has to produce a relative moment of 67% or less compared to that of λ_{100}. This solution will hereafter be referred to as λ_{67}. Both λ_{67} and its relative moment is added to the lookup table.

This is then repeated for a 33%- and a 0% (or as low as possible) constrained solution, referred to as λ_{33} and λ_0. However, a new constraint is added, which is that $sign(\lambda_{100} - \lambda_{67}) = sign(\lambda_{67} - \lambda_{33}) = sign(\lambda_{33} - \lambda_0)$. In more practical terms, this is to ensure that the different solutions all move the sail in the same direction compared to the previous solution. This is important when we try to find a continuous solution of the relative moment in the next step.

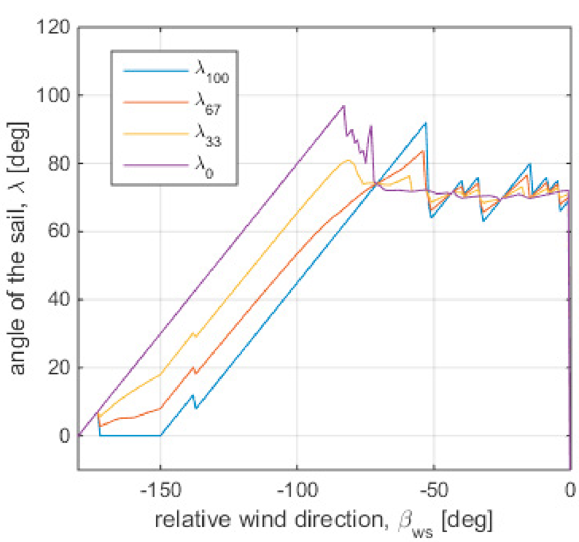

Fig. 1. Optimal position of the sail

We then have all that we need to reverse the process. By entering β_{wr} one finds the solutions of λs that corresponds to that relative wind direction. Then, by applying the restriction of the desired $S_{\phi r}$, the ideal λ is found based on a linearization (computed on-line) between the different $S_{\phi r}$ created by λ_{100}, λ_{67}, λ_{33} and λ_0.

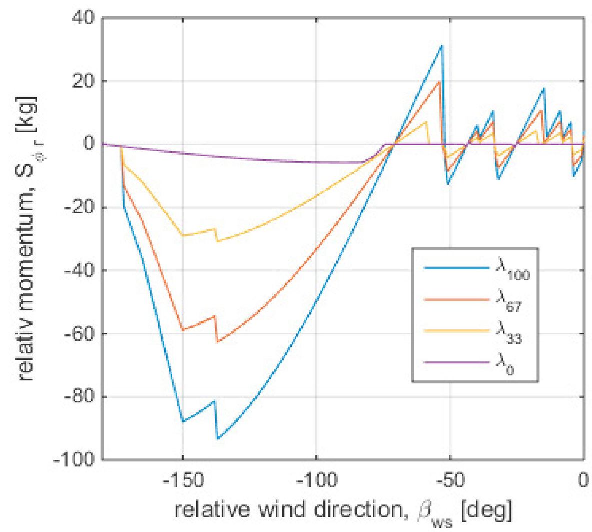

Fig. 2. Relative moment in roll caused by optimal sail position

3. MOMENT ANALYSIS WHEN BEAM REACHING

Beam reaching describes the maneuver of sailing perpendicular to the wind. Figure 3 shows the result of the moment in roll when beam reaching at $U_{m_{10}} = 5\frac{m}{s}$. It is easy to see that the sail and the restoring moment are dominating. Together they generate roughly 91% of the moment in roll. The keel produces another 8%. From (9) and (10) one can see that the two Coriolis terms provides no moment in roll, and it follows that the remaining 1% is caused by the rudder and the drag.

4. ROLL CONTROLLER

The roll controller will be split into two mirrored controllers which will step in when the heeling angle gets too large. For now, we will look into the controller which will govern for roll (ϕ) bigger than zero. The heeling angle

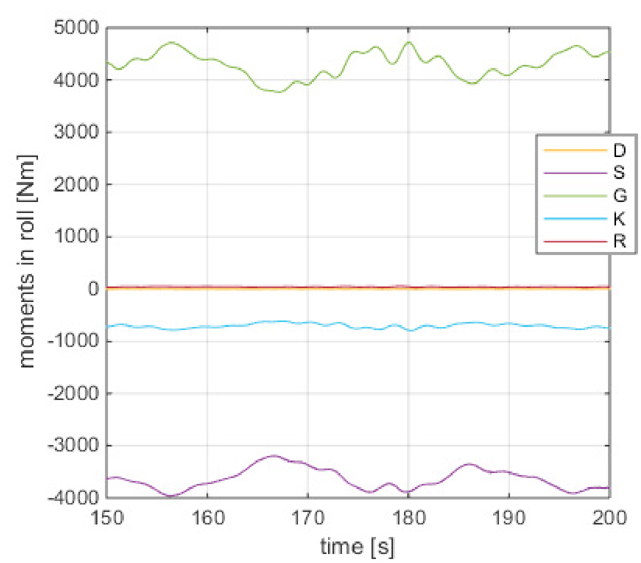

Fig. 3. Moments while beam reaching

should not be larger than 10 degrees for a comfortable ride, and the desired heel angle, ϕ_d, is thus set to 10 degrees.

A linear quadratic regulator (LQR) will be used to control the roll motion. The first step is thus to make a linearized system of the roll motion about an equilibrium point, which is selected to be the desired heel angle. The linearized system can be written as

$$x = \begin{bmatrix} \phi \\ p \end{bmatrix} \quad (64) \qquad \dot{x} = Ax + Bu\cdot \quad (65)$$

Figure 3 and the moment analysis showed which of the moments that are important. The two biggest contributions are from the sail and the restoring moment. The keel also provides some moment, though estimating the moment created by the keel is fairly difficult as it requires measurements of the speed and side slip, see (32) and (33). However, it was argued in Wille et al. (2016) that the undesirable side force F_U ((56) and (57)) is equal to the lifting force of the keel. Replacing K_L with F_U in (33) shows that when $\beta_{ck} \approx \pi$ then the moment created by the keel in roll is equal to $-F_U h_2$. $\beta_{ck} \approx \pi$ is a good approximation when the drift angle is low and there is little current. The restoring moment (20) when linearized becomes

$$G_\phi = \rho_w g \nabla GM_t((1 - 2sin(\phi_w)^2)\phi + C_1, \qquad (66)$$

where ϕ_w is the equilibrium point and C_1 is equal to

$$C_1 = \rho_w g \nabla GM_t cos(\phi_w) sin(\phi_w)\cdot \qquad (67)$$

Making C_1 and the moment generated by the keel a part of our control input u linearizes the system, and the linearized system model do then only includes the restoring moment. Matrices A and B in our linearized system then becomes

$$A = \begin{bmatrix} 0 & 1 \\ -\dfrac{\rho_w g \nabla GM_t(1 - 2sin(\phi_w)^2)}{I_{xx} - K_{\dot{p}}} & 0 \end{bmatrix} \qquad (68)$$

$$B = \begin{bmatrix} 0 \\ \dfrac{1}{I_{xx} - K_{\dot{p}}} \end{bmatrix}, \qquad (69)$$

where u is defined as

$$u = -K \begin{bmatrix} \phi - \phi_d \\ p \end{bmatrix} - K_i \int_0^t \phi - \phi_d d\tau + C_1 + F_U h_2\cdot \quad (70)$$

Integral control action is added to remove errors caused by the linearization of the system (and of linearization used when calculating optimal λ for a given β_{wr} and $S_{\phi r}$), and other system uncertainties. As a note, the moment generated by the keel is fairly steady while keeping a set course (Figure 3), and the error by not including it in our model could be handled by the use of the integral control action if a simpler controller design is desired, or if F_U is difficult to compute.

The LQR controller gain, K, were tuned by setting the matrices Q and R, where Q is the weighting matrix in the quadratic cost caused by an error in x, and R weighs the quadratic cost of using the actuator. R was tuned to be $R = 1e\text{-}10$ and Q was tuned to

$$Q = \begin{bmatrix} 0.75 & 0 \\ 0 & 1 \end{bmatrix}. \qquad (71)$$

The LQR gain was then calculated in matlab by using the function lqr(A,B,Q,R,0). The result was $K = [9.0e4 \;\; 1.0e5]$. K_i was manually tuned to be equal to 2500.

The controller is very similar when the heeling angle is negative. The only difference is that ϕ_w and ϕ_d is changed to $-10deg$, and C_1 is recalculated for the new equilibrium point (changes sign). In practice, only one controller is used, and the signs in front of ϕ_w, ϕ_d and C_1 switches depending on the sign of the heel angle.

The controller should only be turned on when the heel angle is above 10 degrees. Hysteresis has also been implemented to avoid the controller turning on and off too fast, causing discontinues in the control action. It is important to note that the controller would never actually be able to create more moment than the optimal unconstrained solution (λ_{100}) gives, and it follows that the roll controller would not be able to increase the heeling angle further than what was previously possible. It follows that steps has to be taken to avoid integral build up if the wind speed is to low and the sail is not able to provide enough moment to keep the heeling angle at 10 degrees. To avoid integral build up the integral action is not allowed to further grow when $\lambda_d = \lambda_{100}$, i.e. when no more moment can be provided.

It might seem counter-intuitive to keep the controller on if the heeling angle is lower than 10 degrees. Though this might be because the integral action has not yet reached a steady value, and the sail is then restricting the moment too much. Restricting the allowed moment in roll comes at a cost of reducing the force in forward direction, see Figure 4 and will thus in general decrease the speed of the sailboat.

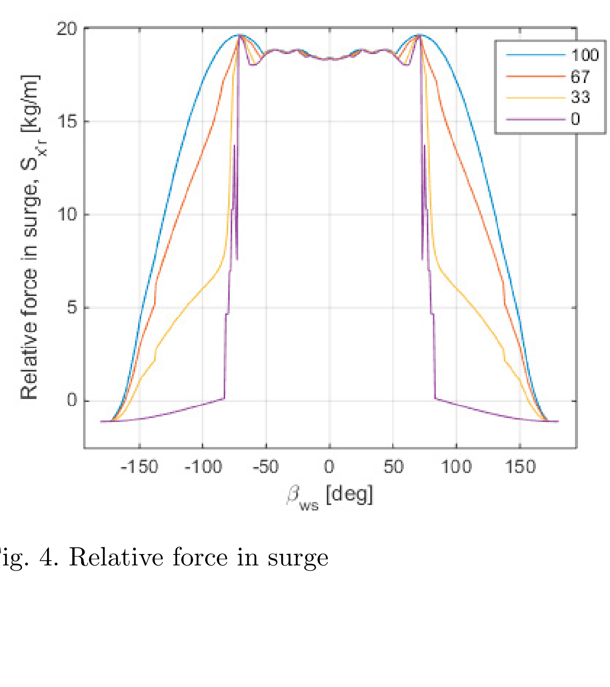

Fig. 4. Relative force in surge

5. SIMULATION AND RESULTS

Three sets of simulations have been studied in what follows. In the first simulation, the controller in roll has been turned off, meaning λ_{100} is always chosen as the desired angle of the sail. In the next simulation, the controller is turned on, but only the average wind speed is known. In the last simulation, the controller is on, and perfect wind estimation is used.

The results will show the last 50 seconds of a 300 second simulation, ensuring that the top speed in surge has been reached and that the integral term in the controller has had time to reach a steady value. The boat is set to go on a course direction perpendicular to a mean wind of $U_{m_{10}} = 10\frac{m}{s}$ (beam reaching). Notice that $\beta_{ws} \neq 90deg$, see equation (26) and (27).

Figure 5 demonstrates the effectiveness of the controller keeping the heel angle low. Furthermore, not only does the controller reduce the maximum heeling angle, it also heavily reduces the amplitude of the roll motion. In Figure 6 one can see that the rotational velocity in roll is reduced considerably, and Figure 7 shows that having perfect wind estimation improves the results even further. When perfect wind estimation is used we have an almost perfect result, and this justifies the simplifications made to the linearized version of the system as it do not effect the performance of the controller. The remaining error is likely caused by the delay in the sail actuator.

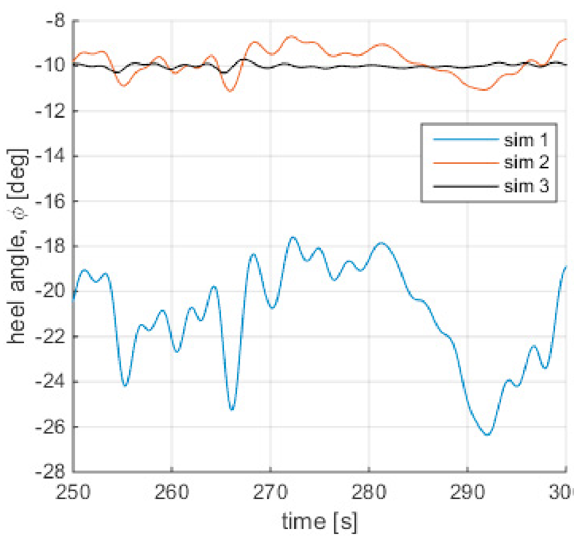

Fig. 5. Time evolution of ϕ, comparison of all simulations

Table 1. Results of roll controller test

	sim 1	sim 2	sim 3
amplitude of roll motion [deg]	4.38	1.18	0.29
maximum rotational velocity p [$\frac{deg}{s}$]	5.15	1.42	0.47
average speed in surge, u [$\frac{m}{s}$]	7.75	6.59	6.59

The results of the test is given in Table 1. The amplitude of the roll motion is reduced by 73% and the maximum rotational velocity by 72% when the average wind speed is known. However, reducing the heeling angle comes at a cost in speed. Restricting the moment created in roll comes at a cost in forward thrust from the sail, which directly effects the maximum speed. When the roll controller is enabled there is a 15% reduction in speed.

While it in general is true that restricting the moment causes a lower speed it is not correct for extreme heeling angles. Figure 8 shows the top speed in surge when ϕ_d is set at a range of different values while beam reaching at very high wind speeds. It shows that the maximum speed is reached at roughly $34deg$ at $U_{m_{10}} = 13\frac{m}{s}$ and $35deg$ at $U_{m_{10}} = 16\frac{m}{s}$. This can be explained by equation (26) and (27). When the heeling angle increases then β_{ws} approaches π (assuming positive speed in surge), i.e. as if the sailboat were sailing up-wind. The optimal heeling angle in terms of forward speed is however quite high, and keeping the heeling angle this large at such high wind speeds would be very dangerous.

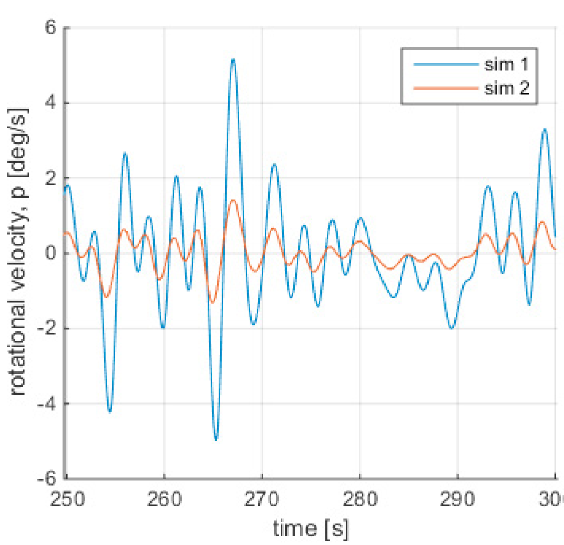

Fig. 6. Time evolution of p, comparison of first and second simulation

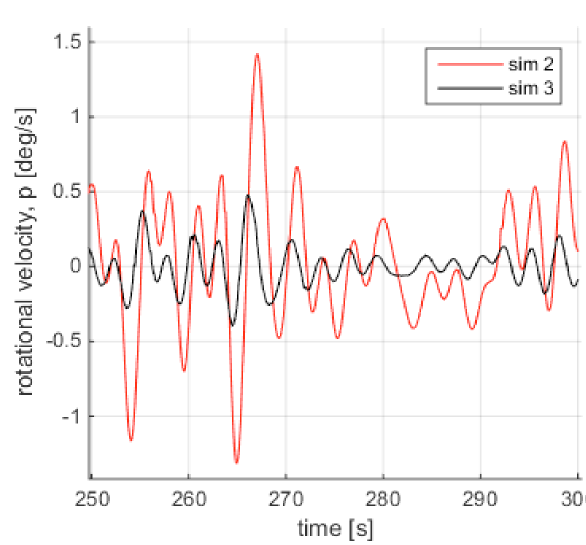

Fig. 7. Time evolution of p, comparison of second and third simulation

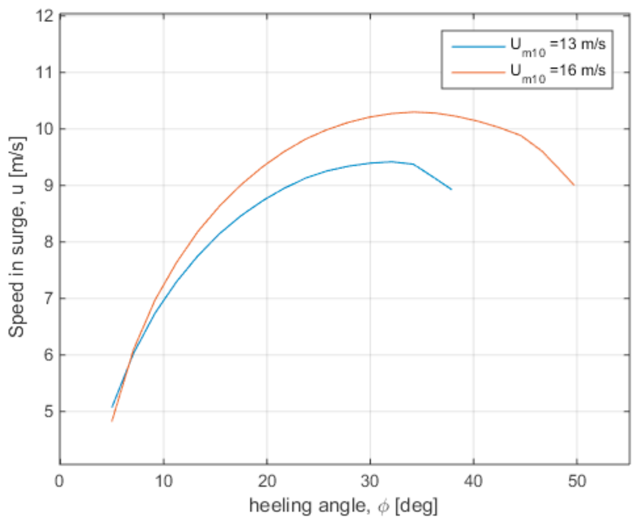

Fig. 8. Relation between roll angle and maximum speed in surge

6. CONCLUSION

This paper presented a new roll reduction technique in sailboats by trimming the sail. The roll controller works as intended, keeping the heeling angle low and reducing roll motion. Having perfect wind estimation improves the result, but the controller is effective regardless. Reducing the heeling angle come at a small cost in speed, though the increase in robustness and safety will make it worth it in many applications.

REFERENCES

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