A Modified Model, Simulation, and Tests of a Full-Scale Sailing Yacht

Katrina Legursky
Aerospace Engineering Dept.
University of Kansas
Lawrence, KS, USA
katrina8@ku.edu

***Abstract*— Sailing yachts have great potential to act as future long-term oceanic observing platforms, yet to date there have not been complete autonomous sailing systems robust enough to handle long term operation in the harsh and continually changing ocean environment. The basis of control system design is a mathematical model capable of describing and capturing the physics based dynamics of the sailboat. The mathematical model represents the system to be controlled, however, a sailing yacht is a very difficult system to model from a controls perspective because of its heavy reliance on the uncontrolled spatial and temporal distribution of the wind. Presented in this paper is a modified aerodynamic force model which includes the sail angle as a control input to the sailing yacht system. The new model has been incorporated into a 4 degree of freedom (DOF) rigid body dynamic yacht model, and implemented in MATLAB/Simulink. The simulations shows model exhibits similar behavior to that observed in full scale sailing yacht sea trial data. Data taken aboard a Precision 23 day-sailer is analyzed, and it is found that the model is a likely candidate for including sail input to a physics based dynamic model for identification and control system design.**

I. INTRODUCTION

VPP's and race simulations represent most of the research into modeling the dynamics of a sailing yacht and have been highly motivated by competitive racing teams[1-6]. Aerodynamic data for these models is reported using the 'optimum' sail angle which produces the best "velocity made good" for the sailing yacht for either an apparent or true wind angle[7-9]. The goal of an autonomous sailing yacht will likely be very different than that of a racing boat, tenths of a knot gains in speed will not be as important as maintaining a constant course, safety, and minimizing wear on the yacht itself. It would be extremely useful for an autonomous control system for a sailing yacht to exploit the sails as real control inputs to the dynamic system. At the simplest level, that means including sail deflections in the physics based dynamic model of sailing yachts, something non-existent in current models.

Aerodynamic forces generated on a sailing yacht are very complicated to model due to the nature of flexible sails. Generally, the lift and drag forces generated by the sail plan are modeled using non-dimensional force coefficients. The sail coefficients are not only dependent upon apparent wind angle, β_A, but also highly dependent upon heel(roll) angle, □. Apparent wind velocity, V_A, is defined as the vector combination of the boat velocity, V_B, and the true wind velocity, V_T, as shown in Fig. 1. Also defined in this figure are the mainsail angle, δ_b, and leeway angle, γ.

An accepted solution to the dependence of aerodynamic coefficients on heel is to use effective angle theory[7, 10] to define an effective wind angle, β_{eff}, and velocity, V_{eff}, in the heeled plane, and use those variables rather than β_A and V_A, to define forces and corresponding coefficients. For our application of 4 DOF excluding heave and pitch, the β_{eff} and V_{eff} are defined as follows[7]:

$$\beta_{eff} = \tan^{-1}\left(\tan\beta_A \cos\phi\right) \tag{1}$$

$$V_{eff} = V_A\sqrt{1-\sin^2\beta_A \sin^2\phi} \tag{2}$$

The drawback of current aerodynamic models for sailing yachts is the assumption that sails are in optimal trim when reporting coefficients. Different methods of describing the trim of the sails away from the fully powered up state that put the sails in optimal trim for the conditions investigated have been used. The most common is the early solution that modifies the coefficients using the parameters of reef and flat[10] to represent decreases from the optimal sail coefficients that occur when sailing in real-life conditions. An alternate depowering model[7] uses a single parameter, power, which is based on the ratio between the real heeling moment coefficient and the optimum, to modify aerodynamic coefficients. Both of these models aim to capture the effect of utilizing the many secondary controls available on a sailing yacht such as reefing points, outhauls, boom vangs, etc that are used to depower and trim the sails.

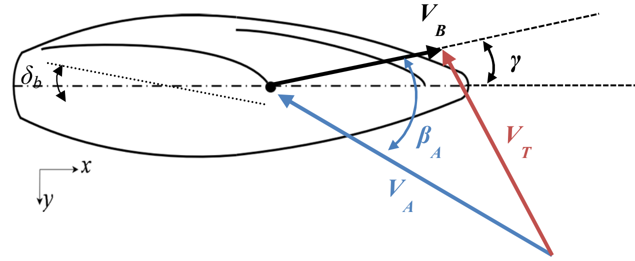

Figure 1: The Wind Triangle

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The goal of control is to define the simplest model that effectively captures the vehicle dynamics, and neither of the above models relate the sail coefficients to a single, simple control. The most straightforward way to depower a sail plan is to let the main sail out (decrease its deflection angle, δ_b) to decrease heeling force. When this is insufficient, the entire sail area should be reduced (reefing). For control applications, it is the effect of the whole sail plan that is important, meaning the combined effect of the main sail and jib, genoa, or spinnaker. The model explored in this work is inspired by those used in aerospace control applications where total airplane aerodynamic coefficients are modified by linear curves. For example, the typical breakdown for the total lift coefficient of an aircraft is[11]:

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{i_h}} i_h + C_{L_{\delta e}}\delta_e \tag{3}$$

Where, α is angle of attack, i_h is the incidence angle of the horizontal tail, and δ_e is the elevator deflection angle. C_{L0} is the lift coefficient when $\alpha = i_h = \delta_e = 0$, and the rest are linear slopes of the change in total lift with changes in α, i_h and δ_e. This project presents a new model similar to that in Eq. (3), and describes the effect of the entire sail plan, then investigates its ability to model sailing yacht dynamics through simulation and analysis of data collected aboard a full scale yacht.

II. THE NEW MODEL

Aerodynamic sail forces and moments in the new model are described in 4DOF by a vector of driving force, X_S, side force, Y_S, rolling moment, K_S, and yawing moment, N_S:

$$f_{aero} = \begin{bmatrix} X_S & Y_S & K_S & N_S \end{bmatrix}^T \tag{4}$$

The new model is a modification of a previous model[12] that describes the aerodynamic forces in the body coordinate system, with origin at the center of gravity (c.g.), rather than in terms of lift and drag coefficients:

$$\begin{aligned} f_{aero} &= \tfrac{1}{2}\rho_a V_{eff}^{\,2} S_A * \ldots \\ &\begin{bmatrix} X'_S & Y'_S & \sqrt{S_A}K'_S & \sqrt{S_A}N'_S \end{bmatrix}^T \end{aligned} \tag{5}$$

The non-dimensional coefficients, $X'_S, Y'_S, K'_S,$ and N'_S, represent the total aerodynamic force and moment coefficients. S_A is the reference sail area, and ρ_a is the air density. Typical curves for these are shown in Fig. 2 as functions of β_{eff} . The curves are the coefficients optimally trimmed sails. The new model assumes this angle is known for each effective wind angle. The total sail angle δ_b, measured w.r.t. the centerline of the yacht (see Fig. 1) is then the optimal sail angle plus the deviation of the angle from the optimum, $\Delta\delta_{b0}$:

$$\delta_b = \delta_{b_0} + \Delta\delta_{b_0} \tag{6}$$

The new aerodynamic model proposed includes an additional coefficient representing the change in force or moment coefficient w.r.t. $\Delta\delta_{b0}$:

$$X'_{S_{\delta b}} = \frac{dX'_S}{d\delta_b} \tag{7}$$

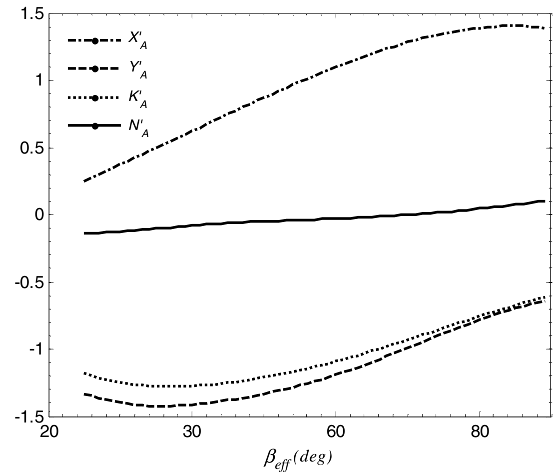

Figure 2: Typical Total Sail Coefficient Curves

Therefore, the new expression for the total sail force coefficient is written as:

$$X'_S = \left(X'_{S_0} + X'_{S_{\delta b}} \Delta\delta_{b_0} \right) \tag{8}$$

The term X'_{S_0} represents the sail coefficient at the optimal sail deflection angle, and thus also represents the curves in Fig. 2. This new model has the advantage of directly relating a control variable to the resulting forces and moments, which has not been common in past models. It would be ideal if $X'_{S\delta b}$was constant for all β_{eff}, but this is likely not the case, and it too will be dependent, as X'_{S_0} is, on β_{eff}. This is investigated using the full scale data described later in this paper.

III. SIMULATION

As a first test to the viability of this model, it is implemented in a Simulink model based on Fossen's [13] equations of motion, and hydrodynamic and rudder forces based on yacht parameters given for a 10m IMS racer [12, 14]. The 4DOF equations of motion used in this simulation are given by:

$$\begin{aligned} \dot{U} &= X/m + VR \\ \dot{V} &= Y/m - UR \\ \dot{P} &= \left(I_{zz}K + I_{xz}N\right)\left(I_{xx}I_{zz} - I_{xz}^2\right)^{-1} \\ \dot{R} &= \left(I_{xx}N + I_{xz}K\right)\left(I_{xx}I_{zz} - I_{xz}^2\right)^{-1} \end{aligned} \tag{9}$$

Where U and V are the velocities along the surge and sway axes, and P and R are the angular velocities about the roll and yaw axes, all in the body fixed coordinate system. X and Y represent the total external forces acting along the surge and sway axes; K, and N represent the total external moments applied around the roll and yaw axes. For a sailing yacht, the forces and moments are generated by the sails, hull, keel, and rudder. The force and moment model is broken down as follows with sail, hull/keel, and added mass force and moment coefficients[14]:

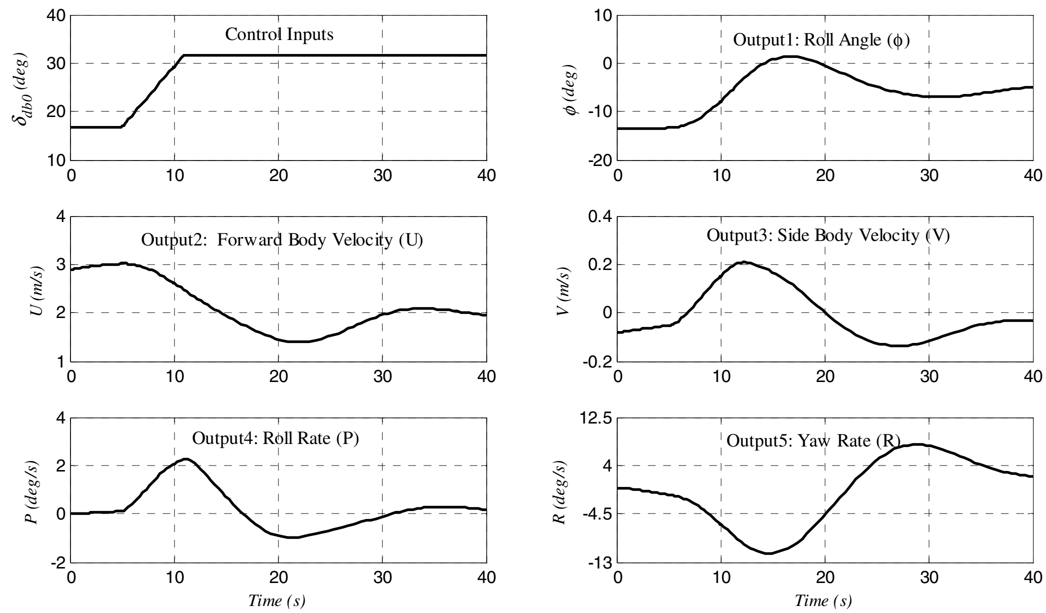

Figure 3: Simulation Results of a Sail Input with New Model

$$
\begin{aligned}
X &= X_0 + X_H + X_{VR} VR + X_{Rd} + X_S \\
Y &= Y_H + Y_P P + Y_R R + Y_{Rd} + Y_S \\
K &= K_H + K_P P + K_{Rd} + K_S - mg\overline{GM}\sin\phi \\
N &= N_H + N_R R + N_{Rd} + N_S
\end{aligned}
\quad (10)
$$

where X_0 represents total upright hull drag, the subscript H denotes hydrodynamic forces and moments, Rd denotes forces and moments generated by the rudder, and S denotes aerodynamic forces and moments generated by the sail plan. g is acceleration due to gravity, $\overline{GM}$ is the yacht's metacentric height, and the rest of the terms X_{VR}, Y_P, Y_R, K_P, and N_R are the hydrodynamic derivatives for added mass.

The optimal sail coefficients for the simulated yacht are given in [12], but the optimal sail angles are not. Thus, an assumption is made that for points of sail ranging from upwind through a beam reach, that the sail deflection angle varies linearly with β_A. In other words, for every degree further from the wind the boat is sailing, the sail angle must increase a fixed amount. For the simulation, δ_{b0} is calculated from the following equation:

$$\delta_{b_0} = 0.630\beta_A \quad (11)$$

For close-hauled apparent wind of 25°, the optimal angle will be ~16° from centerline. For the simulation it is arbitrary what the angle is, what is important is the deviation away from δ_{b0}. When looking at data from the full size yacht, a better estimate of these optimal angles will be necessary. The values chosen for the change in coefficient with sail deflection are shown in Table 1. A total of six simulations are run for the balanced yacht sailing in three true wind directions from 45° to 90° representing sailing conditions from upwind through a beam reach, and two wind speeds. The goal of the simulations are to show that when the sail angle is increased by 15°, i.e. moved further away from centerline by 15°, that the boat will decrease in heel and slow down, the response expected with a decrease in force acting upon the sails. This kind of response has been observed several times in the full scale data.

Table 1: Chosen Sail Derivatives for Simulation

$X'_{S_{\delta b}}$	$Y'_{S_{\delta b}}$	$K'_{S_{\delta b}}$	$N'_{S_{\delta b}}$
-2.84	3.07	-3.00	2.81

The results of one of the six simulations are shown in Fig. 3. The true wind for this simulation is a 5 m/s breeze +45° from the bow, corresponding to a starboard tack. As shown in the upper left plot of Fig. 3, the sail angle input changes starting at t = 5s, and increases by 15° over a period of 6 seconds. This input was chosen based upon similar maneuvers in the full scale data. As the simulation results in the rest of Fig. 3 show, there is a rapid decrease in roll angle accompanied by a slow decrease in forward velocity. The yacht also experiences a turn to leeward (away from the wind), which is also expected, especially without adjustment of the rudder to maintain force and moment equilibrium. Since the preliminary simulation shows the behavior expected and observed on the water, further evaluation and investigation of the proposed model will be done with full scale sailing data.

IV. FULL SCALE TEST PLATFORM AND DATA

The test vehicle is a Precision 23ft day-sailer, which has been equipped to perform system identification. All tests were performed on Clinton Lake in Lawrence, KS. Data is collected from a variety of sensors [15] at 5 Hz by a LabView Virtual Instrument. The data includes GPS position, heading, and velocity, hull speed, Euler angles, rate of change of Euler angles, linear accelerations, apparent wind speed and velocity, rudder deflection angle, and boom (mainsail) deflection angle.

Data has been collected on several days in differing weather conditions. The data from each day includes segments where the boat is sailed at constant apparent wind angles on both port and starboard tacks ranging from close hauled (true wind ~ 45°) to a beam reach (true wind ~ 90°). During such a segment inputs are given to the system through the rudder and main sail angle. The Precision 23 is a sloop rig, meaning it has a main sail and a jib, and is not equipped to measure the set of the jib. Thus, during all constant β_A time segments, the jib is set and not varied, and its contribution to the sail plan will be consistent. Also, the secondary sail controls are never adjusted or utilized, even in between days, in order present the most consistent data possible.

V. Data Analysis

The data analyzed and presented here all represents data collected while sailing on port tack at varying apparent wind angles. The optimal sail deflections are not known for the Precision 23, so they are estimated from the data. A total of seven data segments representing a total of about 10 minutes of near steady state trimmed sailing are individually averaged, resulting in seven pairs of (β_{eff} , δ_{b0}), shown in Fig. 4. The standard deviations of these averages are very small, so the points are an accurate representation of the measurement for that particular time segment. A linear curve is fitted to these average points, resulting in Eq. (12), which is used in all subsequent analysis to calculate δ_{b0} from measured β_{eff}.

$$\delta_{b_0} = -0.58\beta_{eff} - 20 \tag{12}$$

Twelve segments of data are selected, representing about 40s of sailing data each, with each segment containing a change in sail deflection. The collection of these segments results in 2,062 individual data points with which to carry out the analysis of the new aerodynamic model.

Currently, a hydrodynamic model of the hull, keel, and rudder has not been accurately estimated for the test vehicle. An estimate of the hydrodynamic forces and moments acting upon the hull are needed in order to directly estimate aerodynamic coefficients from the data. The results of the Delft Systematic Yacht Hull Series (DSYHS)[16, 17] are used to generate all residuary resistance/drag forces generated by the hull, both upright and due to heel. The lift and drag of the keel and rudder are estimated using Hembold's modification of lifting line theory for low-aspect-ratio wings[18]. A summary of hull, rudder, and keel characteristics necessary to generate these estimates is in Tables 2 and 3.

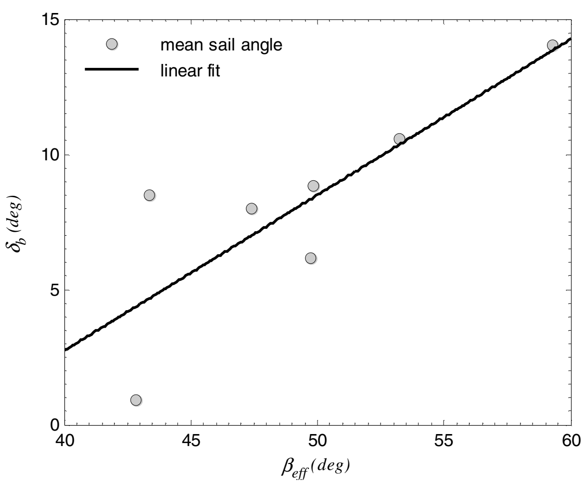

Figure 4: Data Used to Define Optimal Sail Deflection

Table 2: DSYHS Paramters for Precision 23

	Description	*Value*		*Description*	*Value*
S_W	*Wetted surface area*	9.100m²	A_w	*Waterplane aera*	7.47m²
BWL	*Max beam @ waterline*	2.042m	S_{Wk}	*Wetted surface of keel*	2.788m²
LWL	*Length @ waterline*	6.096m	t/c_k	*Thickness to chord of keel*	0.10
t_c	*Draft of canoe body*	0.180m	∇_k	*Displacement of keel*	0.044m³
∇	*Total displacement*	3.240m³	z_k	*Vertical position of center buoyancy, keel*	0.125m
∇_c	*Displacement of canoe body*	2.965m³	b_k	*Span of keel*	1.219m
lcf	*Longitudinal center of flotation*	-3.335m	λ_k	*Taper ratio of keel*	0.583
lcb	*Longitudinal center of buoyancy*	-3.231m	S_{Wr}	*Wetted surface of rudder*	0.464m²
C_P	*Prismatic coefficient*	0.475	t/c_r	*Thickness to chord of rudder*	0.07
C_M	*Midship section coefficient*	0.71			

This hydrodynamic model is used to calculate the forces and moments generated by the hull at each of the measured data points, and the following equations are then used to solve for the total aerodynamic force and moment coefficients:

$$X'_S = \frac{(\dot{U}+VR)(m+m_x) - X_H - X_K - X_{Rd}}{\frac{1}{2}\rho_A V_{eff}^2 S_A}$$
$$Y'_S = \frac{(\dot{V}+UR)(m+m_y) - Y_H - Y_K - Y_{Rd}}{\frac{1}{2}\rho_A V_{eff}^2 S_A} \tag{13}$$

$$K'_S = \frac{(I_{xx}+J_{xx})\dot{P} - K_H - K_K - K_{Rd} - mg\overline{GM}\sin\phi}{\frac{1}{2}\rho_A V_{eff}^2 S_A z_{COE}}$$
$$K'_S = \frac{(I_{zz}+J_{zz})\dot{R} - N_H - N_K - N_{Rd}}{\frac{1}{2}\rho_A V_{eff}^2 S_A x_{COE}} \tag{14}$$

Where m_x ,m_y , J_{xx}, and J_{zz} are added mass, ρ_A is air density, S_A is the reference sail area, g is acceleration due to gravity, $\overline{GM}$ is the yacht's metacentric height, and x_{COE} and z_{COE} are the distances from the center of gravity (c.g.) to the center of effort of the sails. This data analysis makes no attempt to account for changes in the location of the center of effort of the sails at this time. A summary of the constant values used in Eq. (13) is shown in Table 4.

Table 3: Parameter to Define Keel and Rudder Model

	Keel	***Rudder***
Airfoil Section	NACA 0009	NACA 0009
Aspect Ratio	1.93	1.76
Surface Area	1.394m^2	0.232m^2
x location of center of force wrt c.g.	1.341m	-3.35m
z location of center of force wrt c.g.	0.61m	0.55

Table 4: Mass Properties of Precision 23

m	1293 kg	$I_{xx} + J_{xx}$	6074 kg-m^2
m_x	44 kg	$I_{zz} + J_{zz}$	10,740 kg-m^2
m_y	2576 kg	$\overline{GM}$	1.524 m

The total aerodynamic coefficients are thus obtained for each of the 2,062 data points, along with $\Delta\delta_{b0}$. From now on, only the surge degree of freedom will be referred to, but the process has been repeated for sway, roll, and yaw. The first step to obtaining the unknown coefficients on the right hand side of Eq. (8) is to look at the relationship between X_S' and β_{eff} present in the data, shown as the original data in Fig. 5. As may be seen, there is a substantial amount of scatter in the data. This is expected, as the data is collected on different days, at different wind speeds, and possibly with different sail shapes (due to their flexible nature). Chauvenet's criterion is applied to the data every 0.5° of β_{eff} to remove outliers in the data, and the remaining data is also shown in Fig. 5. After reduction, the remaining means and their corresponding errors are now plotted in Fig. 6. The errors are calculated by dividing the standard deviation of the sample by the square root of the sample size in each set of 0.5° β_{eff}. A quadratic least squares fit

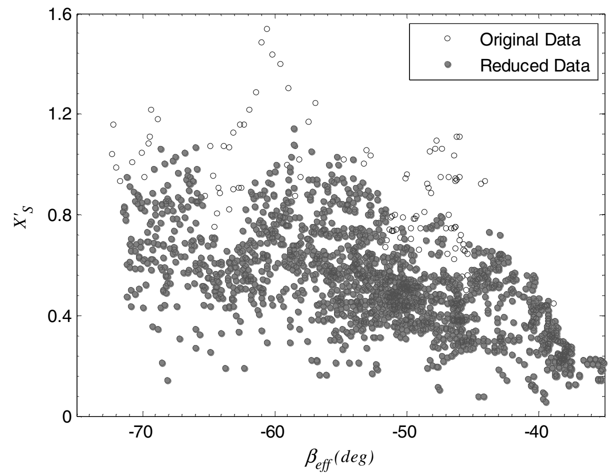

Figure 5: Original and Reduced Data for Total Sail Coefficient

is applied to this data, shown by the curves in Fig. 6.

Now, we have a curve to represent one of the unknowns in Eq. (8), and the only unknown left is $X'_{S\delta b}$, which is found using the data points $(\beta_{eff}, X'_S, \Delta\delta_{b0})$. Again, Chauvenet's criterion is used to reduce the data, resulting in the means and errors are shown in Fig. 7.

VI. Conclusions

The complete data presented in this study is from several different days sailing in wind speeds where the full main sail and jib were the appropriate sail choice. Thus, the wind speed, surface conditions, and trim of the boat were different among the data sets. Therefore, it is not surprising to find outliers and scatter in the data because there are so many variables that are difficult to control among experiments. Despite these difficulties, after the data is reduced according to a well-known

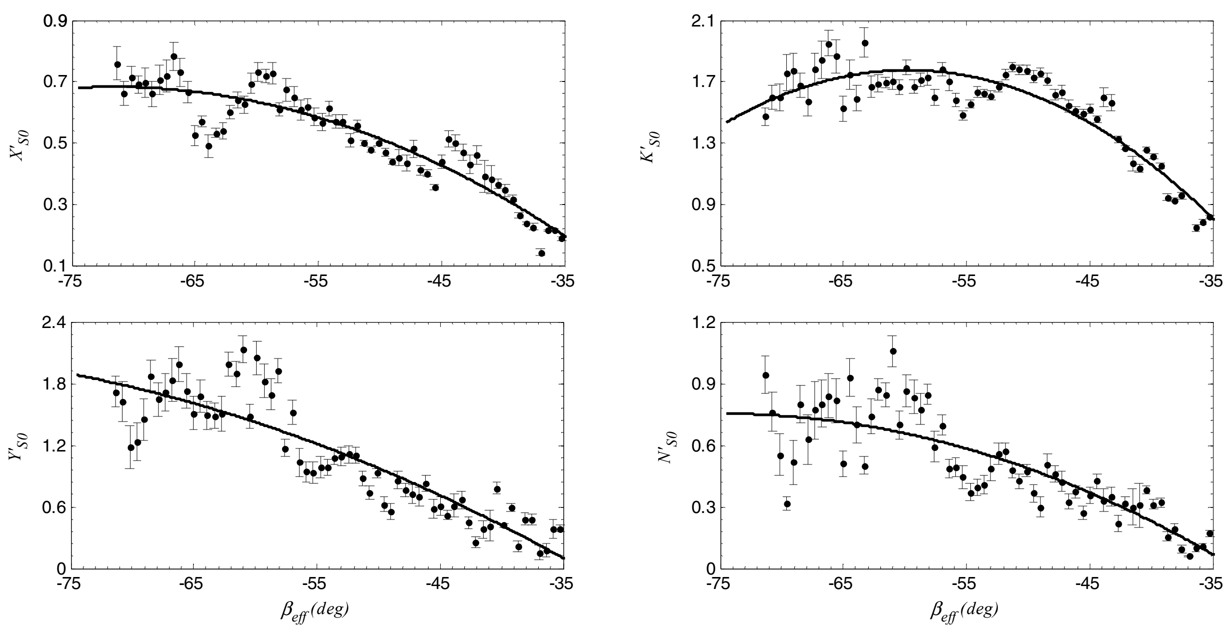

Figure 6: Results of Data Reduction to Find Optimal Sail Coefficients

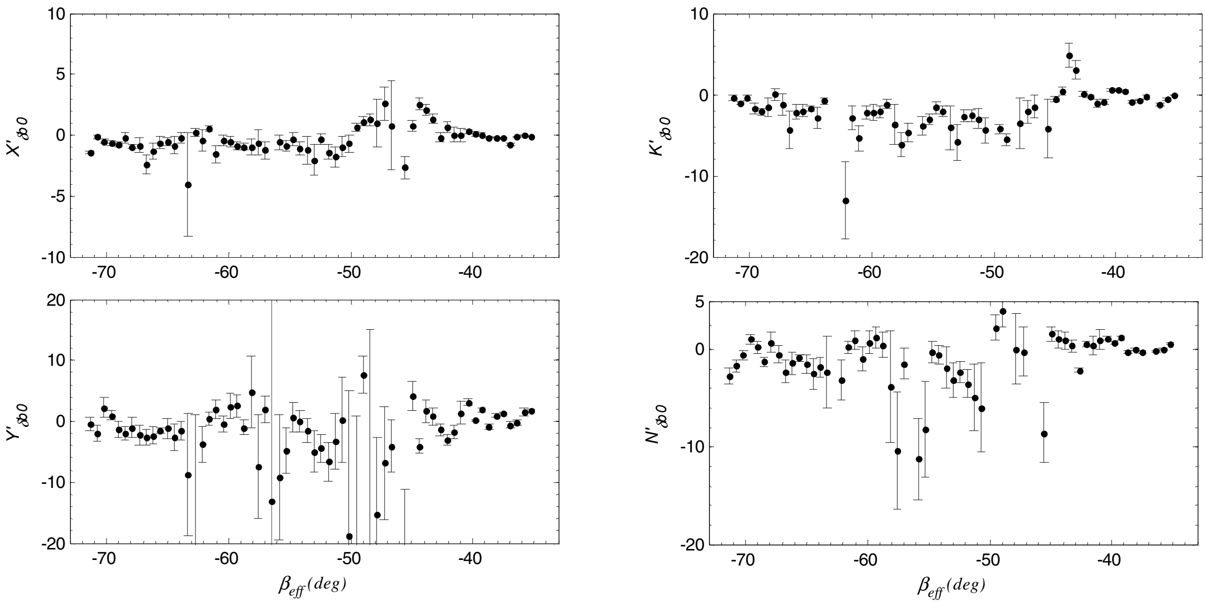

Figure 7: Results of Data Reduction to Find Aerodynamic Derivatives

and widely used statistical test, reasonable trends appear. The curves for X'_{S0} follow the trends to be found in many sources that report sail force coefficients[7, 9, 12]. The new component of the model, $X'_{S\delta b}$, looks to be relatively constant with β_{eff}. However, the values obtained for $X'_{S\delta b}$ contain larger errors than the values for X'_{S0}, particularly in sway and yaw. It is not surprising to find error in sway, as the tests are all performed at different heel angles, and it is possible the generalization of β_{eff} is not an adequate method of accounting for changes in heel angle.

This is the first time data taken with this relatively inexpensive, easy to install data acquisition system has been analyzed. The results are very encouraging, pointing to the ease of collecting similar data aboard other sloop rigged sailboats, or even yachts powered by a wing-sail.

VII. SUMMARY

A newly structured aerodynamic force model is presented for a sailing yacht which more accurately represents the control power the sail has over the dynamic behavior of a sailing yacht. The new model is tested in simulations which verify the new model exhibits good behavior and trends well. The new model is then identified from data taken aboard a Precision 23 day-sailer. The DSYHS and a standard lift and drag model were used to model the forces acting on the hull, keel, and rudder. The data represented the yacht sailing on port tack in an upwind configuration. The next steps for this work will be to include all the starboard tack data, as well as all the data taken on beam reach on both tacks. The results of this future analysis will then serve as the starting point for full non-linear system identification of the sailing yacht system from the collected data. Eventually, this line of research may lead to a better understanding of full scale sailing yacht dynamics, and will enable more robust control system design for autonomous sailing yachts.

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