Robust adaptive path following for a nonlinear third order Nomoto's ship model

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Abstract— **In this paper, an adaptive backstepping method is used for a ship course tracking control. This has been done by applying a reference signal derived from a LOS algorithm. The ship model is described by a third order nonlinear model whose parameters are unknown. The control design uses estimate values of the unknown parameters of the system. Then, adaptive laws of the estimation of these values have been proposed. The stability of the controlled system has been ensured by the use of a Lyapunov function.**

Simulation results show the effectiveness of the proposed approach and the designed controller can be used to the ship course tracking with good performances.

Key-words— **Adaptive backstepping control, nonlinear control, adaptative control, Line Of Sight.**

I. Introduction

Scientific development in various areas of science and engineering techniques pushed automatic research to study more and more non-stationary systems including variable parameters.

Because of using a command with fixed parameters can lead in some works to a failure and cannot maintain or achieve "good" performance indices that are increasingly required in several industrial applications. In this paper, we focus on this paper on- the control law with parameters that can be adjusted during on line(i.e. use an Adaptive command structure), and this using the a well known technique of adaptive backstepping [1]. The adaptive version of the backstepping technique includes the ability to summarize the controllers for a large class of nonlinear systems with known structure and uncertain parameters [8]. Although the robustness of the control law has been intensively studied in the case of linear systems [3], many other robust controllers were involved in the case of nonlinear systems [13].

In this paper, we are studying the control problem of a third order Nomoto model. The study unfolds in two steps: first, an adaptive backstepping procedure is applied to an uncertain strict feedback form which presents the model of the ship, to track a given reference asymptotically and estimate the uncertain parameters which characterize the ship's model. Second, the reference signal is supplied along with the new position to be attained by the ship. To ensure this reference, a LOS algorithm is therefore presented.

The main idea of the LOS algorithm is to give an intuitive understanding of the behavior of the ship: This consists on steering the actual ship's heading (i.e. $\psi(t)$) to a desired angle called $\psi_{los}(t)$. The guidance by the LOS algorithm has been also treated by Healey, Lienard and Fossen [4], [7] and calculations have been developed in [11]. This shows that it is an effective method for marine vehicle navigation and this is thanks to its simplicity, its low cost and flexibility to sudden changes in the desired specifications for the followed path.

The reminder of this work is organized as follows. In section 2, the formulation of the problem is presented. In section 3, we will present the LOS algorithm for the path following problem. In section 4, the adaptive control based on backstepping method is described. Simulation results illustrating the effectiveness of the proposed method and the convergence to the desired path are given in section 5.

II. Problem formulation

A ship model described by the third order Nomoto model is given by [7]:

$$\psi^{(3)}(t)+\left(\frac{1}{T_1}+\frac{1}{T_2}\right)\ddot{\psi}(t) + \frac{1}{T_1T_2}H\left(\dot{\psi}(t)\right) = \frac{K}{T_1T_2}\left(T_3\dot{\delta}(t)+\delta(t)\right) \quad (1)$$

where $\psi(t)$ is the ship course and $\delta(t)$ is the rudder angle. Based on [13], we propose the following change of variables:

$$\begin{cases} x_1(t)=\psi(t) \\ x_2(t)=\dot{\psi}(t) \\ x_3(t)=\ddot{\psi}(t)-c\delta(t) \\ x_4(t)=\delta(t) \end{cases} \quad (2)$$

with the initial conditions $x_1(0)=x_2(0)=0$ and $x_3(0)=-c\delta(0)$.

As a result, the system can be described by the following state equations:

$$\begin{cases} \dot{x}_1(t)=x_2(t) \\ \dot{x}_2(t)=x_3(t) \\ \dot{x}_3(t)=-ax_3(t)-bH\left[x_2(t)\right]+x_4(t) \\ \dot{x}_4(t)=-\frac{d}{e}x_4(t)+\frac{1}{e}u(t) \end{cases} \quad (3)$$

Assumptions a, b, d and e are unknown variable parameters in the design taken as:

$$\begin{cases} a_{min}\le a\le a_{max} \\ b_{min}\le b\le b_{max} \\ d_{min}\le d\le d_{max} \\ e_{min}\le e\le e_{max} \end{cases} \quad (4)$$

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and $H[x_2(t)] = x_2^3(t) + x_2(t)$ is a nonlinear function and $u(t)$ is the controlling input.

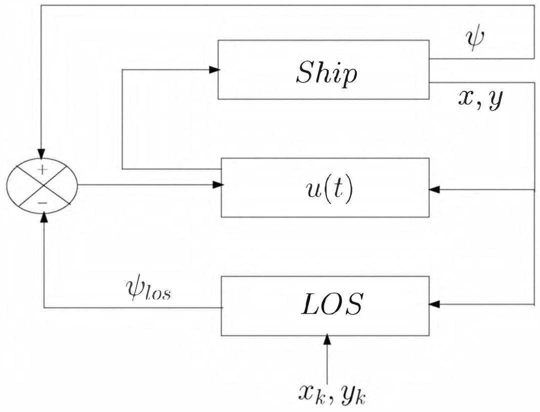

Fig. 1. Block diagram of the controlled system

Control objective: The task consists now on designing the control law $u(t)$ such that for a given $\psi_{los}(t)$, we obtain

$$\lim_{t\to\infty} |\psi(t) - \psi_{los}(t)| = 0 \tag{5}$$

where $\hat{a}$, $\hat{b}$, $\hat{d}$ and $\hat{e}$, the estimate values of a, b, d, and e, are bounded.

III. LOS algorithm

The principle of the LOS algorithm is an intuitive understanding of the behavior of a ship [6]. The principle is that, if a vessel may maintain its ship course $\psi(t)$ aligned with the angle named $\psi_{los}(t)$, then the convergence to the desired position is achieved.

In short, the geometric exercise to manoeuvre the vessel can be re-formulated as follows: force the position p of the ship to converge to a desired path by forcing its angle $\psi(t)$ to converge to $\psi_{los}(t)$ (5).

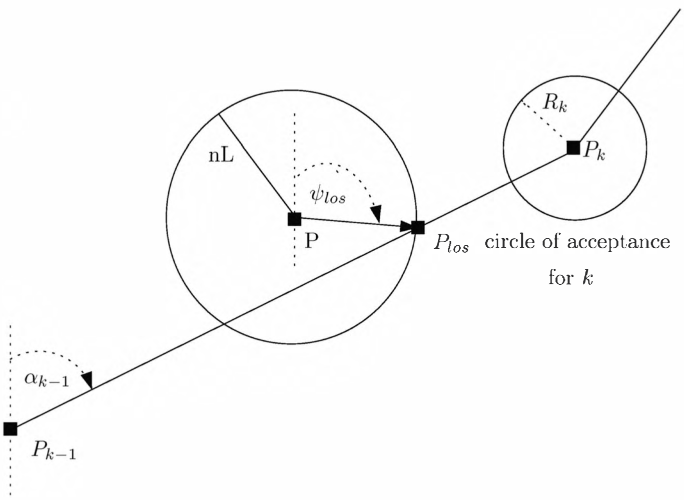

Fig. 2. Principle of the manoeuvre by the LOS algorithm [11]

The desired path consists on the collection of *way-points* in a table of points, but the LOS position is now located on the segment linking the previous and the current point, given respectively by P_{k-1} and P_k. Referring to [12], two equations are used to calculate the LOS position denoted P_{los} which are:

$$\frac{y_{los} - y_{k-1}}{x_{los} - x_{k-1}} = \frac{y_k - y_{k-1}}{x_k - x_{k-1}} = \tan(\alpha_{k-1}) \tag{6}$$

$$(x_{los} - x)^2 + (y_{los} - y)^2 = (nL)^2 \tag{7}$$

where the subscript k is relative to the current way-point, while $k-1$ is relative to the previous way-point.

To compute the LOS angle $\psi_{los}(t)$, we use the following expression:

$$\begin{aligned}\psi_{los}(t) &= \arg[(x_{los} - x) + j(y_{los} - y)] \\ &= \arctan 2\,(y_{los} - y, x_{los} - x) \end{aligned} \tag{8}$$

where x and y define the current position of the ship. The $atan2$ function ensures that $\psi_{los}(t) \in [-\pi, \pi]$.

IV. Control design

Step 1: Using (2), in this step new state variables are introduced which are defined by the following equations

$$z_1 = x_1(t) - \psi_d(t) \tag{9}$$
$$z_2 = x_2(t) - \alpha_1(z_1) \tag{10}$$
$$z_3 = x_3(t) - \alpha_2(z_1, z_2) \tag{11}$$
$$z_4 = x_4(t) - \alpha_3(z_1, z_2, z_3) \tag{12}$$

where $\psi_d(t)$ is the desired ship course, and $\alpha_1(z_1)$, $\alpha_2(z_1, z_2)$, and $\alpha_3(z_1, z_2, z_3)$ are stabilizing functions.

The derivative of the first equation in the sub-system (2) is given by:

$$\dot{z}_1 = z_2 + \alpha_1(z_1) - \dot{\psi}_d(t) \tag{13}$$

The role of $\alpha_1(z_1)$ is to stabilize equation (13). This has been done by considering the first Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \tag{14}$$

Its derivative, with respect to time, is given by:

$$\dot{V}_1(z_1) = z_1 \dot{z}_1 = z_1 \left(\alpha_1 - \dot{\psi}_d(t)\right) + z_1 z_2 \tag{15}$$

In order to ensure the convergence to 0 of the error z_1, $\alpha_1(z_1)$ takes the following form:

$$\alpha_1(z_1) = -k_1 z_1 + \dot{\psi}_d(t) \tag{16}$$

Finally, the derivative of $V_1(z_1)$ is expressed by:

$$\dot{V}_1(z_1) = -k_1 z_1^2 + z_1 z_2 \tag{17}$$

Substituting (16) into (13) gives:

$$\dot{z}_1 = -k_1 z_1 + z_2 \tag{18}$$

where $k_1 > 0$ is a tuned parameter.

Step 2: In this step, the derivative of the second equation (10) takes the form

$$\dot{z}_2 = z_3 + \alpha_2(z_1, z_2) - \dot{\alpha}_1(z_1) \tag{19}$$

where $\alpha_2(z_1, z_2)$ is the second stabilizing function
The derivative of $\alpha_1(z_1)$ is

$$\dot{\alpha}_1(z_1) = -k_1(-k_1 z_1 + z_2) + \ddot{\psi}_d(t) \tag{20}$$

The second Lyapunov function is defined by

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 \tag{21}$$

and its derivative takes the form:

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1(z_1) + z_2 \dot{z}_2 \\ &= -k_1 z_1^2 + z_2 \left(z_1 + \alpha_2 - \dot{\alpha}_1(z_1) \right) + z_2 z_3 \end{aligned} \tag{22}$$

In (22), we made the following substitution to ensure the negativity of $\dot{V}_2(z_1, z_2)$:

$$-k_2 z_2 = z_1 + \alpha_2(z_1, z_2) - \dot{\alpha}_1(z_1) \tag{23}$$

In fact, the obtained expression of $\alpha_2(z_1, z_2)$ is

$$\alpha_2(z_1, z_2) = -k_2 z_2 - z_1 + \dot{\alpha}_1(z_1) \tag{24}$$

Finally, $\dot{V}_2(z_1, z_2)$ is defined by

$$\dot{V}_2(z_1, z_2) = -k_1 z_1^2 - k_2 z_2^2 + z_2 z_3 \tag{25}$$

Substituting (24) into (19), we obtain

$$\dot{z}_2 = -k_2 z_2 - z_1 + z_3 \tag{26}$$

where $k_2 > 0$ is a tuned parameter chosen later.
Step 3: The derivative of the equation (11) is given by

$$\begin{aligned} \dot{z}_3 = &- bH\left(x_2(t)\right) - a x_3(t) + z_4 \\ &+ \alpha_3(z_1, z_2, z_3) - \dot{\alpha}_2(z_1, z_2) \end{aligned} \tag{27}$$

where the time derivative of $\alpha_2(z_1, z_2)$ takes the form

$$\begin{aligned} \dot{\alpha}_2 &= -\dot{z}_1 - k_2 \dot{z}_2 + \ddot{\alpha}_1(z_1) \\ &= (k_1^2 - 1)\dot{z}_1 - (k_1 + k_2)\dot{z}_2 + \psi_d^{(3)}(t) \end{aligned} \tag{28}$$

Consider the Lyapunov function $V_3(z_1, z_2, z_3)$

$$V_3 = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2}\gamma_1^{-1}\tilde{a}^2 + \frac{1}{2}\gamma_2^{-1}\tilde{b}^2 \tag{29}$$

where γ_1 and γ_2 are constant. The derivative of V_3 is given by:

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + z_3 \dot{z}_3 - \gamma_1^{-1}\dot{\hat{a}}\tilde{a} - \gamma_2^{-1}\dot{\hat{b}}\tilde{b} \\ &= -k_1 z_1^2 - k_2 z_2^2 - \gamma_1^{-1}\dot{\hat{a}}\tilde{a} - \gamma_2^{-1}\dot{\hat{b}}\tilde{b} + z_3 z_4 \\ &\quad + z_3 \left(z_2 - bH\left(x_2(t)\right) - a x_3(t) + \alpha_3 - \dot{\alpha}_2 \right) \end{aligned} \tag{30}$$

The third stabilizing function $\alpha_3(z_1, z_2)$ which ensures the stability of the system, has therefore the following form:

$$\begin{aligned} \alpha_3 &= \dot{\alpha}_2(z_1, z_2) + \hat{a} x_3(t) + \hat{b} H\left(x_2(t)\right) \\ &\quad - z_2 - k_3 z_3 \end{aligned} \tag{31}$$

Substituting and (31) into (30) gives:

$$\begin{aligned} \dot{V}_3 &= -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 + z_3 z_4 \\ &- \tilde{a}\left(\gamma_1^{-1}\dot{\hat{a}} + x_3(t) z_3\right) - \tilde{b}\left(\gamma_2^{-1}\dot{\hat{b}} + H\left(x_3(t)\right) z_3\right) \end{aligned} \tag{32}$$

In order to ensure the stability of the system, and the negativity of $\dot{V}_3$, expressions of the adaptive laws $\dot{\hat{a}}$ et $\dot{\hat{b}}$ take the following forms:

$$\dot{\hat{a}} = -\gamma_1 x_3(t) z_3 \tag{33}$$

$$\dot{\hat{b}} = -\gamma_2 H\left(x_2(t)\right) z_3 \tag{34}$$

Step4: In the final step, our aim is to determine expressions of adaptive laws $\dot{\hat{d}}$ et $\dot{\hat{e}}$, and then, the expression of the adaptive control $u(t)$.
In fact, the global Lyapunov function is given by

$$V_4 = V_3 + \frac{1}{2} e z_4^2 + \frac{1}{2}\gamma_3^{-1}\tilde{d}^2 + \frac{1}{2}\gamma_4^{-1}\tilde{e}^2 \tag{35}$$

The derivative of the fourth state variable is

$$\dot{z}_4 = \dot{x}_4 - \dot{\alpha}_3 = -\frac{d}{e} x_4(t) + \frac{1}{e} u(t) - \dot{\alpha}_3 \tag{36}$$

Substituting (36) into the expression of $\dot{V}_4(z_1, z_2, z_3, z_4)$, we obtain

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + e\dot{z}_4 z_4 - \gamma_3^{-1}\dot{\hat{d}}\tilde{d} - \gamma_4^{-1}\dot{\hat{e}}\tilde{e} \\ &= -\sum_{i=1}^{3} k_i z_i^2 - \gamma_3^{-1}\dot{\hat{d}}\tilde{d} - \gamma_4^{-1}\dot{\hat{e}}\tilde{e} \\ &\quad + z_4 \left[z_3 - d x_4(t) - e\dot{\alpha}_3 + u(t) \right] \end{aligned} \tag{37}$$

From (37), we pick the control law $u(t)$ as

$$u(t) = -k_4 z_4 - z_3 + \dot{\hat{d}} x_4(t) + \dot{\hat{e}}\dot{\alpha}_3 \tag{38}$$

with adaptive laws given by

$$\begin{cases} \dot{\hat{a}} = -\gamma_1 x_3(t) z_3 \\ \dot{\hat{b}} = -\gamma_2 H\left(x_2(t)\right) z_3 \\ \dot{\hat{d}} = -\gamma_3 x_4(t) z_4 \\ \dot{\hat{e}} = -\gamma_4 \dot{\alpha}_3 z_4 \end{cases} \tag{39}$$

Theorem 1: Consider the nonlinear model of the ship given by equation (1), with uncertain parameters a, b, d and e, having the bounds (4). Applying the control law $u(t)$ given by (38) and the update laws (39) of estimated parameters $\hat{a}$, $\hat{b}$, $\hat{d}$ and $\hat{e}$, states (x_1, x_2, x_3, x_4) are asymptotically stable and $(\hat{a}, \hat{b}, \hat{d}, \hat{e})$ are bounded.

Proof: Substituting in (37) the control law $u(t)$ given by (38) and (39), we can write:

$$\begin{aligned} \dot{V}_4 &= -\sum_{i=1}^{4} k_i z i^2 - \tilde{d}\left(\gamma_3^{-1}\dot{\hat{d}} + x_4(t) z_4\right) - \tilde{e}(\gamma_4^{-1}\dot{\hat{e}} + \dot{\alpha}_3 z_4) \\ &= -\sum_{i=1}^{4} k_i z i^2 \end{aligned} \tag{40}$$

Then, it is obvious that $\dot{V}_4 < 0$. Thus, we conclude, using Barbalat's lemma [5], that (x_1, x_2, x_3, x_4) and $(\hat{a}, \hat{b}, \hat{d}, \hat{e})$ are bounded, and therefore, z_1, z_2, z_3 and z_4 converge to 0.

V. Simulation results

In this section, we consider numerical simulations to show the effectiveness of the proposed controllers. Controller gains and adaptive gains are chosen as: $k_1 = 0.51$, $k_2 = 0.4$, $k_3 = 0.5$, $k_4 = 0.4$, $\gamma_1 = 0.4$, $\gamma_2 = 0.2$, $\gamma_3 = 0.25$, and $\gamma_4 = 0.3$. Initial conditions are $[x_1(0)\ x_2(0)\ x_3(0)\ x_4(0)]=[\frac{\pi}{4}\ 0\ 0\ 0]$.

In figure 3, we can see a good convergence of the ship course $\psi(t)$ to the angle $\psi_{los}(t)$ for which is obvious that the error $(\psi(t) - \psi_{los}(t))$ converges to 0. This allows us to say that the principle of the LOS algorithm is successfully achieved for this system. In figure 6, we observe the convergence of the error variables to 0, and figures 5 and 7 show that state variables and estimate parameters are bounded. In figure 8, it is clear that estimation errors are also bounded. The control input $u(t)$, in figure 4, shows a good convergence to the origin yielding $\psi(t) = \psi_{los}(t)$. Figure 9 shows that the vessel, in the plan (x, y) realizes a good tracking.

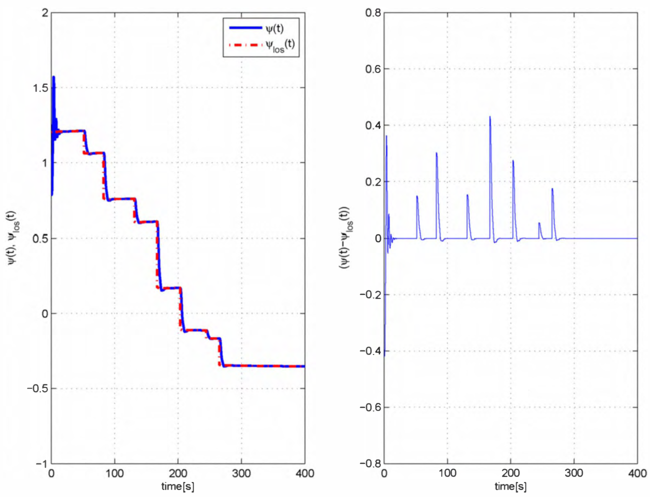

Fig. 3. (a)$\psi(t)$ and $\psi_{los}(t)$, (b) Error $(\psi(t) - \psi_{los}(t))$

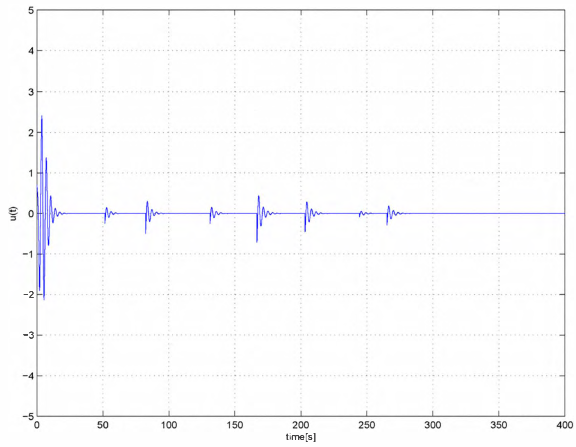

Fig. 4. Control input $u(t)$

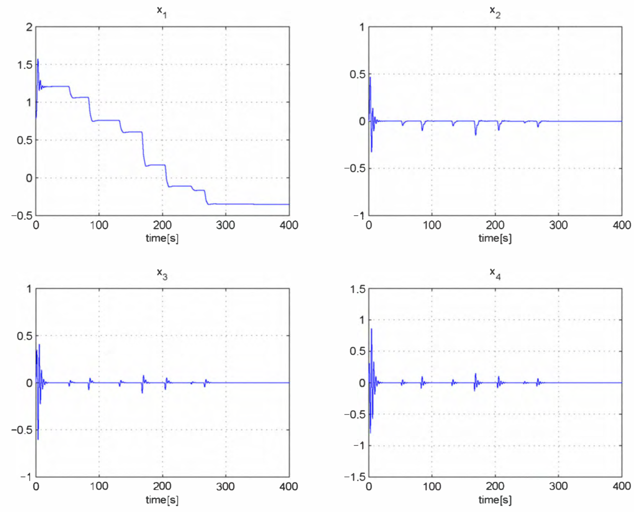

Fig. 5. States $x_1, x_2, x_3$and x_4

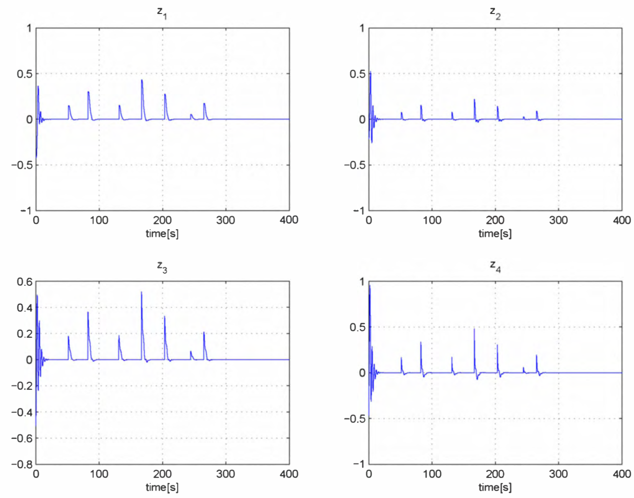

Fig. 6. Error variables z_1, z_2, z_3 and z_4

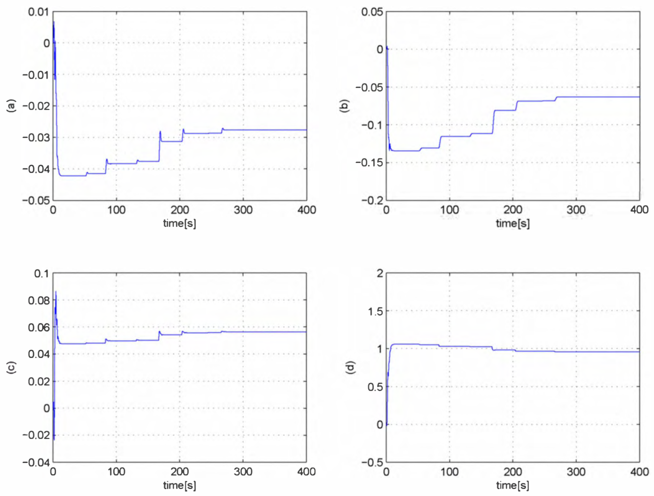

Fig. 7. Estimation parameters $\hat{a}(t)$, $\hat{b}(t)$, $\hat{d}(t)$ and $\hat{e}(t)$

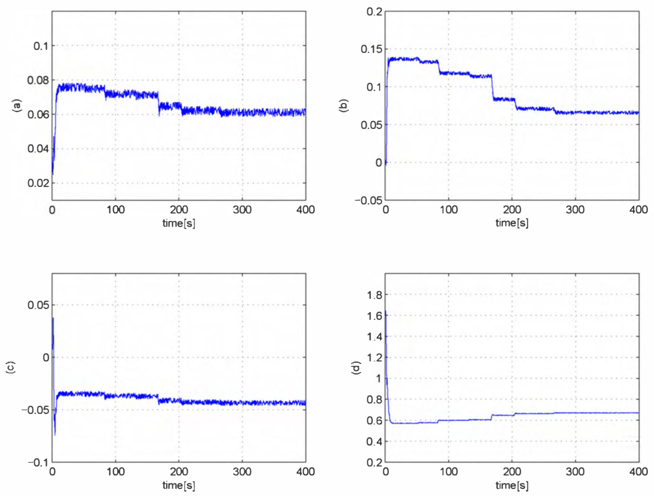

Fig. 8. Parameter errors $\tilde{a}(t)$, $\tilde{b}(t)$, $\tilde{d}(t)$ and $\tilde{e}(t)$

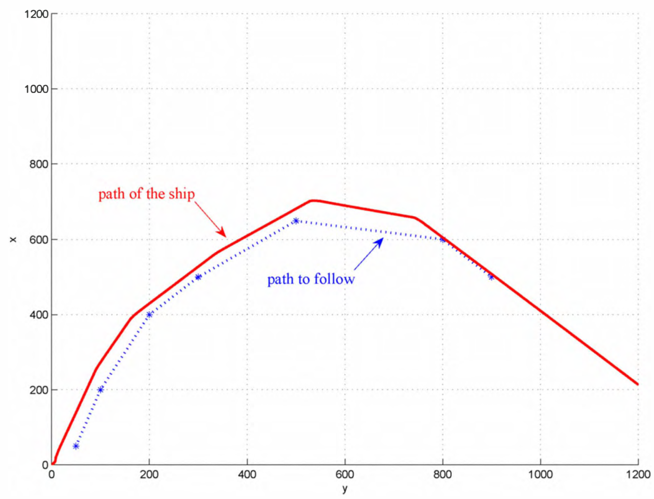

Fig. 9. The ship tracking

VI. Conclusion

In this paper, we have considered an uncertain model of a nonlinear third order Nomoto's ship, for which, we applied the adaptive backstepping technique to regulate the actual orientation of the ship, with a desired angle provided by a LOS algorithm.

This technique has been applied providing an effective tool, for nonlinear systems, allowing to built directly and systematically a control law that ensures the stability of the closed loop system.

Simulation results show the effectiveness of the proposed approach. Further, research directions will deal with NN algorithms combined with the backstepping procedure to address the trajectory tracking of such model.

References

[1] Astrom K.J and Wittenmark B., Adaptative Control. Reading, MA: AddisonWesley, 1989
[2] Brockett R. W., Asymptotic Stability and Feedback Stabilization, in Differential Geometric Control Theory . Birkhäuser, Boston, USA, 181–191, 1983
[3] Feurer A and Morse A.S., Adaptive Control of Single-Input, Single-Ouput Linear Systems. IEEE Trans. on Automatic Control. 23(4): 557–569, 1978
[4] Fossen T.I., Marine Control Systems. Guidance, Navigation, and Control of Ships, Rigs and Underwater Vehicles. Trondheim, Norway: Marine cybernetics, 2002
[5] Fernando A.C.C., Fontes and Lalo Magni. A Generalization of Barbalat's Lemma with Applications to Robust Model Predictive Control, Proc. of the MTNS'04 Mathematical Theory of Networks and Systems, Louvain, Belgium, 2004
[6] Ghommam J., Mnif F. and Derbel N., Path Following for Underactured Marine Craft using Line Of Sight, Int. Conf. on Machine Intelligent, Tozeur-Tunisia, 2005
[7] Healey A.J and Lienard D., Multivariable Sliding-Mode Control for Autonomous Diving and Steering of Unmanned Underwater Vehicles. IEEE Journal of Oceanic Engineering, 18(3): 327–339, 1993
[8] Kanellakopoulos I., Kokotovic P.V. and Morse A.S., Systernatic Design of Adaptive Controllers for Feedback Linearizable Systems. IEEE Trans on Automatic Control. Vol.36(11), 1241-1253, 1991
[9] Kokotovic P. and Arcak M. Constructive Non Linear Control: A Historical Perspective. Automatica. 37(5): 637–662, 2001
[10] Krstic M., Kanellakolulos I. and Kokotovic P.V. Non Linear and Adaptive Control Design. New York, Wiley, 1995
[11] Morten B., Nonlinear Maneuvering Control of Underactuated Ships. Master of Science Thesis, Norway University of Science and Technology, Departement of Engineering Cybernetics, Trondheim, Norway, 2003
[12] Paquin L.-N., Application du Backstepping à une Colonne de Flottation. Mémoire, Département de Mines et Métallurgie, Faculté des Sciences et De Génie, Université Laval, 2000
[13] Witkowska A., Tomera M. and Smierschalski R., A BacksteppingApproch to Ship Course Control. Dans Int. J. Appl. Math. Comput. Sciences., 17(1): 73–85, 2007