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CALCULATION METHODS OF HYDRODYNAMIC COEFFICIENTS OF SHIPS IN SHALLOW WATER

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1. Introduction

1.1. Aim and intention of the project

In recent years much attention is paid to the determination of the behaviour of a ship in shallow water due to waves and manoeuvring. For this the hydrodynamic coefficients in the equation of motion have to be known. Several computational techniques are available at this moment for the determination of these coefficients for ships sailing on restricted water.
This investigation is aimed to correlate computational results from several of the available techniques with experimental results.
For that purpose forced oscillation tests have been performed by the Ship Hydromechanics Laboratory of the department of Maritime Technique of the Delft University of Technology. The hydrodynamic coefficients were established for both the vertical and horizontal direction of motion for several water depths.

These results are compared with three calculation methods:

1. a multipole-approximation method (2Dmp) developed by Professor Keil (see [1] of Appendix B) and improved and adapted by the Delft University of Technology (DUT)
2. a two-dimensional diffraction method (2Ddiffr) developed by the Delft Hydraulics Laboratory
3. a three-dimensional diffraction method (3Ddiffr) developed by the Maritime Research Institute Netherlands (MARIN).

The first two calculation methods are based on a two-dimensional computation after which the strip method is applied to determine the values for the whole model inclusive the speed influence. This speed influence has been taken into account using the method denoted in [1] and in Chapter 3.1.
For the three-dimensional diffraction method the speed influence has been introduced by an approximation method as reported in Chapter 3.2.

1.2. Background

It is well-known that the distance between the ships keel and bottom, the so-called keel-clearance, influences the behaviour of a ship. Both the manoeuvring characteristics of a ship and its motion response to waves are changed as a result of restricting water depth.
Knowledge about the behaviour of a ship in waves and its manoeuvring characteristics in such waters is important for designers, shipowners, harbour authorities and harbour designers.
On one hand the save operation of ships requires the establishment of criteria for admissible weather conditions while passing existing approach channels and harbour entrances.
On the other hand the knowledge about the behaviour of ships in shallow water is important for harbour design. For this it is necessary to be able to determine the required dimensions of approach channels and harbour entrances in relation to the resulting downtime. Costs for dredging a harbour entrance are strongly influenced by the minimum admissible keel clearance.
Moreover it is possible to incorporate certain criteria in the optimization process of ship design.

Hence accurate calculation methods for prediction of the behaviour of ships in shallow water are of great value for both the designers and operators.
All calculation methods are based on potential theory taking no viscous influence into account. For shallow water it has been proved that in this way satisfying results may be achieved for all motions except for the rolling motions, where viscous influences appear to play a prominent part in most cases. In general for shallow water a strong increase of viscous influence was also expected for the other motions.
Likewise it seems probable that two-dimensional calculations should be insufficient in view of the expected stronger flow around the fore- and aftership relative to deep water.
For this case the use of the stripmethod should not be permitted. The here presented research was intended to obtain more insight in the influence of the above mentioned fenomena. For these calculations of the hydrodynamic mass and damping coefficients were executed and the values compared with experimental results.

To obtain a better understanding longitudinal distribution of these hydrodynamic coefficients was measured using a ship model divided in seven segments. For each segment the coefficients were determined as function of water depth, oscillation-frequen-

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cy, amplitude and forward speed. These tests have been carried out in two parts. The first part was related to the higher wave frequencies while the second part was dedicated to oscillation frequencies corresponding to the lower manoeuvre-frequencies.
For the first part of the project the measured results together with the calculations according to the multipole approximation method are presented in [2] while for the second part these results are presented in [3].

From this research it appeared that contrary to expectations both above mentioned phenomena appeared to have a negligible influence on the hydrodynamic coefficients of the motions considered on shallow water. From this it was concluded that the strip method based on potential theory may deliver satisfying results not only for deep water but also for shallow water.

2. Experiments

In the following paragraph a short description of the experiments used for this study is given.
For a detailed description of the experimental set-up, the tests and the parameters investigated one is referred to the reports [2] and [3] of the Shiphydromechanics Laboratory of the department of Maritime Technique of the Delft University of Technology.

The experiments with forced oscillation have been carried out with a model of the Todd-60 series. The main-dimensions $L_{11} \times B \times T$ are 2.258 x 0.322 x 0.129 m with $C_B = 0.70$. The model is divided in seven segments, each of which is separately connected to a stiff girder by means of a strain-gauge dynamometer. These dynamometers measure vertical or horizontal forces only, depending on the direction of the forced oscillation.
For each segment the added mass and damping, or the accesory moment are determined from respectively the in-phase and quadrature component of these forces relative to motion. Vertical heave- and pitch motions as well as horizontal sway and yaw motions have been executed.
In view of comparison with the calculated results, the model has been restrained in its motions due to sinkage and trim resulting from the ships forward speed.
The experiments are carried out for the following water depth – draught ratio's:

h/T = 2.40, 1.80, 1.50, 1.20, 1.15

two forward speeds, viz.:

F_n = 0.1 and 0.2

five frequencies of oscillation:

ω = 4, 6, 8, 10, 12.

The highest frequency (ω = 12) has only been incorporated for the vertical motions, while for the horizontal motion also tests for ω = 9 have been carried out.
Tests were performed for three motion amplitudes at least in order to enable the determination of the influence of possible non-linearities.

The mean hydrodynamic cross-section coefficients are found by dividing the hydrodynamic coefficients of the segments by the length of that segment. The coefficients a_{kj} and b_{kj} are determined for each of the motions considered by integration over the model length of the cross-sectional values derived from the measurements. The seven mean cross-sectional values of the hydrodynamic coefficients are shown in the Figures 2 and 3 for h/T = 1.5 and 1.15 as a function of ω.
For the same conditions the total model values are presented in Figure 4 for F_n = 0.1 and in Figures 5 to 8 for F_n = 0.2.
It should be kept in mind that the hydrodynamic coefficients in the Figures 7 and 8 are not directly determined, but derived from the measurements. The results presented in the above mentioned figures are related to an oscillation amplitude r = 0.01 m.

3. Calculation models

For the description of the motions of a ship travelling on shallow water use is made of the following coordinate system:

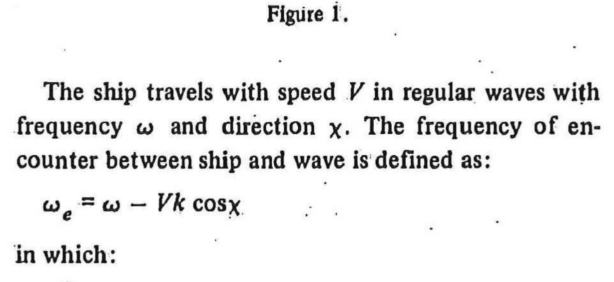

Figure 1.

The ship travels with speed V in regular waves with frequency ω and direction χ. The frequency of encounter between ship and wave is defined as:

$$\omega_e = \omega - Vk \cos\chi$$

in which:

$\omega^2 = gk \tanh(kh)$
h = the water depth.

Assuming that the fluid is non-viscous, incompressible and free of rotation the problem of a rigid ship oscillating in waves may be described by velocity potentials.
For a derivation of this see Salvesen et al. [4], Vugts [5], Inglis [6] and Semjenov-Tjan-Schanski [8].

Here only a short description of the mathematical analysis will be presented. The total potential $\tilde{\Phi}(x,y,z,t)$ should satisfy the Laplace-equation and a suitable radiation condition at infinity. For $\tilde{\Phi}(x,y,z,t)$ may be written [6]:

$$\tilde{\Phi}(x,y,z,t) = -Vx + \Phi_s(x,y,z) + \phi\, e^{i\omega_e t} \tag{1}$$

in which Φ_s represents the stationary velocity potential.

In addition the following boundary conditions apply:

1. The free-surface condition

$$\frac{D}{Dt}P = -\rho\frac{D}{Dt}\left|\left(\frac{\partial\tilde{\Phi}}{\partial t} + \frac{1}{2}|\nabla\Phi|^2\right) + gz\right| = 0 \tag{2}$$

for $z = \zeta(x,y,t)$

in which D/Dt is the substantial derivative and $\zeta(x,y,t)$ the displacement of the wave surface.

This yields:

$$\frac{\partial^2\tilde{\Phi}}{\partial t^2} + 2\nabla\tilde{\Phi}\cdot\nabla\tilde{\Phi} + \frac{1}{2}\nabla\tilde{\phi}\cdot\nabla(\nabla\tilde{\phi}\cdot\nabla\tilde{\Phi}) + g\tilde{\Phi}_z = 0 \tag{3}$$

for $z = \zeta(x,y,t)$

This is a non-linear boundary condition.

2. Body surface

$$(\underline{V}_s - \underline{v})\cdot\underline{n} = 0 \text{ on the body surface} \tag{4}$$

There is no fluid transport through the ships hull. $\underline{V}_s$ is the forward ship speed, $\underline{v}$ is the water velocity and $\underline{n}$ the outward normal on the body surface.

3. Sea bed

$$\frac{\partial\Phi}{\partial n} = 0 \quad \text{for } z = -h \tag{5}$$

If one assumes that the motions are small and that the geometry of the ship's hull is such that the part of ϕ_s in the potential $\tilde{\Phi}$ may considered to be small too, it is than possible to linearize the boundary conditions [6].

The oscillatory potential may now be written as

$$\phi = \sum_{j=1}^{6}\eta_j\phi_j \tag{6}$$

with

j = the mode number of motion

η_j = the motion amplitude

j = 1,2,3,4,5,6 represents the motions: surge, sway, heave, roll, pitch and yaw.

On the free surface of the fluid now holds:

$$\Phi_{tt} + 2V\Phi_{xt} + V^2\Phi_{xx} + g\Phi_z = 0 \quad \text{for } z = 0 \tag{7}$$

or also

$$\left|\left(i\omega_e - V\frac{\partial}{\partial x}\right)^2 + g\frac{\partial}{\partial z}\right|\phi = 0 \quad \text{for } z = 0 \tag{8}$$

Assuming that the frequency of encounter ω_e is high with respect to $V\,\partial/\partial x$ these free surface boundary conditions may be rewritten as:

$$-\omega_e^2\phi + g\phi_z = 0 \quad \text{for } z = 0$$

This is the linearized free surface condition of the fluid for zero speed. The motion potential ϕ_j has to satisfy the relative complex boundary conditions.

It is possible, however, as shown by Salvesen et al. [4], to express the motion potentials into speed independent potentials:

$$\begin{aligned}
\phi_j &= \phi_j^o \qquad \text{for } j = 1,2,3,4\\
\phi_5 &= \phi_5^o + \frac{V}{i\omega_e}\phi_3^o\\
\phi_6 &= \phi_6^o - \frac{V}{i\omega_e}\phi_2^o
\end{aligned} \tag{9}$$

with ϕ_j^o a speed independent potential satisfying the boundary condition for the ship in its mean position.

If the motion potentials ϕ_j are known the pressure may be determined using the linearized Bernouilli equation:

$$p = -\rho\left(\frac{\partial}{\partial t} - V\frac{\partial}{\partial x}\right)\phi\ , \tag{10}$$

in which

$$\phi = \sum_{j=1}^{6}\eta_j\phi_j$$

The hydrodynamic reaction force is determined using:

$$F_k = -\iint_S p\cdot n_k\, dS \tag{11}$$

All calculation methods considered here are based on the prevailing derivations.

Differences between these methods emerge at the calculation of the motion potentials ϕ^o for zero speed and at the incorporation of the forward speed influence. The calculation of ϕ_j^o is dependent on the use of a two-or three-dimensional method for solution. For the two-dimensional approximation method an additional requirement for the ship's geometry applies:

$$L/B \sim \nu(e^{-1})$$

3.1. Two-dimensional multipole approximation (2Dmp) and two-dimensional diffraction method (2Ddiffr)

Starting from a two-dimensional motion potential ϕ_j^o the hydrodynamic reaction force F_k may be written with due regard to (10) as:

$$F_k = +\iint_S \rho\left(i\omega_e - V\frac{\partial}{\partial x}\right)\phi\, n_k\, dS$$

$$F_k = \sum_{j=1}^{6} \iint_S \rho(i\omega_e - V\frac{\partial}{\partial x})\eta_j\phi_j n_k\, dS \qquad (12)$$

Assuming

$$F_k = \sum_{j=1}^{6} T_{kj} = \sum_{j=1}^{6} -\omega_e^2 a_{kj} + i\omega_e b_{kj}$$

it follows that:

$-\omega_e^2 a_{kj}$ = real part of T_{kj}

$+i\omega_e b_{kj}$ = imaginary part of T_{kj}

If

$$\phi_j^o(x,y,z) = \phi_j^o(y,z)\cdot f(x) \qquad (13)$$

in which $f(x)$ represents the dependency of ship's geometry at section x than:

$$T_{kj} = \int_L \int_{C_o(x)} \rho(i\omega_e - V\frac{\partial}{\partial x})\eta_j\phi_j n_k\, dC\, dL \qquad (14)$$

with

$$T_{kj} = \int_L \widetilde{T}_{kj}(x)\, dL$$

in which $\widetilde{T}_{kj}$ is defined as:

$$\widetilde{T}_{kj}(x) = \rho \int_{C_o(x)} (i\omega_e - V\frac{\partial}{\partial x})\phi_j n_k\, dC$$

$$= \rho\, i\omega_e \int_{C_o(x)} \phi_j n_k\, dC - V\frac{\partial}{\partial x}\int_{C_o(x)} \phi_j n_k\, dC \qquad (15)$$

If for ϕ_j equation (9) is substituted than:

$$\widetilde{T}_{kj}(x) = \rho\, i\omega_e \int_{C_o(x)} \eta_j\phi_j^o n_k\, dC -$$

$$-\rho V\frac{\partial}{\partial x}\int_{C_o(x)} \eta_j\phi_j^o n_k\, dC \qquad (16)$$

$$j = 1,2,3,4$$

$$\widetilde{T}_{k5}(x) = \rho\, i\omega_e \int_L \eta 5(\phi_5^o + \frac{V}{i\omega_e}\phi_3^o)\, n_k\, dC -$$

$$-\rho V\frac{\partial}{\partial x}\int_L \eta 5(\phi_5^o + \frac{V}{i\omega_e}\phi_3^o)\, n_k\, dC$$

An analogous procedure is valid for $j = 6$. For $j = 1,2,3,4$ it holds that the first term in (16) represents the speed independent part of the hydrodynamic reaction force, while the second term represents the speed dependent part.

For the calculation of ϕ_j^o the two-dimensional multipole approximation follows the method of Keil; for this case use is made of Lewis-transformations. The two-dimensional diffraction method starts from a two dimensional source distribution method. Further elaborations of these methods are to find in Appendix A and Appendix B.

Two versions of the strip theory may be used:

- Version 1 leads to the ordinary strip theory method, which lacks some of the symmetry relations in the damping cross coupling coefficients.
- Version 2 includes these additional terms so that the symmetry relations are present; [1] and [2].

As result for the sectional added mass and damping now holds:

Heave:

$$a'_{33} = a^o_{33} + \left[\frac{V}{\omega_e^2}\frac{d}{dx} b^o_{33}\right]$$

$$b'_{33} = b^o_{33} - V\frac{d}{dx} a^o_{33}$$

$$a'_{35} = -a^o_{33}x - [2]\frac{V}{\omega_e^2} b^o_{33} - \left[x\frac{V}{\omega_e^2}\frac{d}{dx} b^o_{33}\right] + \qquad (17a)$$

$$+\frac{V^2}{\omega_e^2}\frac{d}{dx} a^o_{33}$$

$$b'_{35} = -b^o_{33}x + 2V a^o_{33} + x V\frac{d}{dx} a^o_{33} + \left[\frac{V^2}{\omega_e^2}\frac{d}{dx} b^o_{33}\right]$$

Pitch:

$$a'_{55} = a^o_{33}x^2 + 2\frac{V}{\omega_e^2} b^o_{33}x - \frac{V^2}{\omega_e^2} x\frac{d}{dx} a^o_{33} +$$

$$+\left[\frac{V}{\omega_e^2} x^2 \frac{d}{dx} b^o_{33}\right] = -a'_{35}x$$

$$b'_{55} = b^o_{33}x^2 - 2Vx\, a^o_{33} - Vx^2\frac{d}{dx} a^o_{33} - \qquad (17b)$$

$$-\left[\frac{V^2}{\omega_e^2} x\frac{d}{dx} b^o_{33}\right] = b'_{35}x$$

$$a'_{53} = -x\, a^o_{33} - \left[x\frac{V}{\omega_e^2}\frac{d}{dx} b^o_{33}\right] = -x\, a'_{33}$$

$$b'_{53} = -x\, b^o_{33} + x V\frac{d}{dx} a^o_{33} = -x\, b'_{33}$$

Sway:

$$a'_{22} = a^o_{22} + \left[\frac{V}{\omega_e^2}\frac{d}{dx} b^o_{22}\right]$$

$$b'_{22} = b^o_{22} - V\frac{d}{dx} a^o_{22}$$

$$a'_{26} = -x\, a^o_{22} - [2]\frac{V}{\omega_e^2} b^o_{22} - \left[x\frac{V}{\omega_e^2}\frac{d}{dx} b^o_{22}\right] + \qquad (17c)$$

$$+\frac{V^2}{\omega_e^2}\frac{d}{dx} a^o_{22}$$

$$b'_{26} = -x\, b^o_{22} + 2V a^o_{22} + x V\frac{d}{dx} a^o_{22} + \left[\frac{V^2}{\omega_e^2}\frac{d}{dx} b^o_{22}\right]$$

Yaw:

$$a'_{66} = a_{22}x^2 + 2x\frac{V}{\omega_e^2}b^o_{22} + \left[x^2\frac{V}{\omega_e^2}\frac{d}{dx}b^o_{22}\right] -$$

$$-x\frac{V^2}{\omega_e^2}\frac{d}{dx}a^o_{22} = x\,a'_{26}$$

$$b'_{66} = b^o_{22}x^2 - 2x\,V a^o_{22} - x^2\,V\frac{d}{dx}a^o_{22} - \qquad (17d)$$

$$-x\frac{V^2}{\omega_e^2}\frac{d}{dx}b^o_{22} = x\,b'_{26}$$

$$a'_{62} = -a^o_{22}x - \left[\frac{V}{\omega_e^2}x\frac{d}{dx}b^o_{22}\right] = x\,a'_{22}$$

$$b'_{62} = -b^o_{22}x + V x\frac{d}{dx}a^o_{22} = x\,b'_{22}$$

Version 1 = coefficients excluding terms between brackets.
Version 2 = coefficients including terms between brackets.

The added mass and damping for the ship in total is obtained by integration of the coefficients of (17) over the length of the ship.

The motion equation for the ship now follows from Newton's law:

$$\frac{dMV}{dt} = F_{\text{excitation}} + F_{\text{reaction}}$$

which results into:

$$(M + M_a)\ddot{\underline{x}} + B\dot{\underline{x}} + C\underline{x} = \underline{F}_{\text{exc}} \qquad (18)$$

in which M is the inertia matrix and C is the hydrostatic reaction matrix.

For M_a holds:

$$M_a = \begin{bmatrix} a_{22} & 0 & 0 & a_{26} \\ 0 & a_{33} & a_{35} & 0 \\ 0 & a_{53} & a_{55} & 0 \\ a_{62} & 0 & 0 & a_{66} \end{bmatrix}$$

and

$$B = \begin{bmatrix} b_{22} & 0 & 0 & b_{26} \\ 0 & b_{33} & b_{35} & 0 \\ 0 & b_{53} & b_{55} & 0 \\ b_{62} & 0 & 0 & b_{66} \end{bmatrix}$$

The hydrodynamic coefficients are calculated and compared with the experimental results for the Todd-60 model mentioned in Chapter 2.
The sectional values of the hydrodynamic coefficients and the values of these coefficients for the whole model are presented for version 1 in the Figures 2 to 8 with respect to two water depths (h/T = 1.5 and 1.15) and five frequencies (ω_e = 4, 6, 8, 10 and 12) and the indicated speeds.
For zero speed the added mass and damping are presented in the Tables 1 and 2 for heave and sway as well as the accessory coupling coefficients for these motions.

3.2. *Three-dimensional diffraction method (3Ddiffr)*

Based upon the three-dimensional motion potential ϕ_j it follows that (see (12)):

$$T_{kj} = \rho \iint_S (i\omega_e - V\frac{\partial}{\partial x})\,\eta_j\phi_j n_k\,dS =$$

$$= \rho \iint_S i\omega_e\,\eta_j\phi_j n_k - \rho\,V \iint_S \eta_j\frac{\partial}{\partial x}\phi_j n_k\,dS$$

For $\frac{\partial}{\partial x}\phi_j$ the velocity in x-direction may be taken, which is directly determined in the related computer program so that locally the x-velocities on the surface of the ship as a consequence of unitary oscillation motions are known. After integration of $\phi_j n_k$ and $\frac{\partial}{\partial x}\phi_j n_k$ over the surface T_{kj} is known.

Determination of the in-phase and quadrature component of T_{kj} yields respectively the added mass and damping:

$$-\omega_e^2 a_{kj} = \text{real part of } T_{kj}$$

$$+i\omega_e\,b_{kj} = \text{imaginary part of } T_{kj}$$

The calculation of the motion potential ϕ_j results from a three-dimensional source distribution method; see for the description Appendix C and [17].

Also for this method the derived sectional values and the hydrodynamic coefficients for the whole model are presented in the Figures 2 to 8 with respect to two water depths (h/T = 1.5 and 1.15) and four frequencies (ω_e = 4, 6, 8 and 10) and the denoted speeds.
For zero speed the same coefficients are presented in the Tables 1 and 2 as denoted in Chapter 3.1.

4. Comparison of results

1. For the linear motions, sway and heave, it appears that the measured and calculated values of the hydrodynamic coefficients agree generally very well with only two exceptions.
   The first case concerns the added mass at the water depth draught ratio 1.15.
   It appears from the Figures 2 and 6 that for the lower frequencies the results of the three-dimensional method related to the distribution fairly deviate.
   The second exception is related to the damping

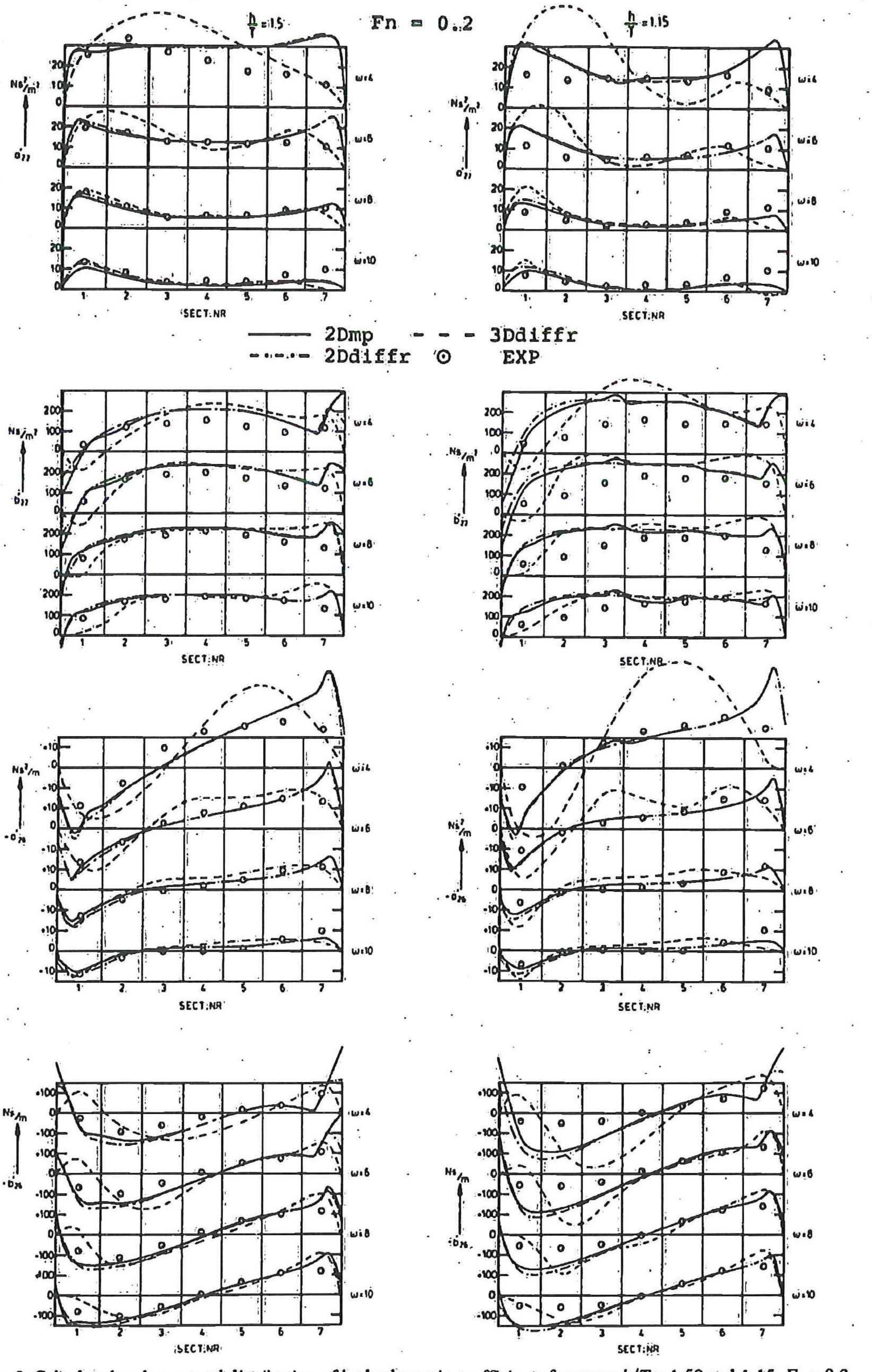

Figure 2. Calculated and measured distribution of hydrodynamic coefficients for sway; $h/T = 1.50$ and 1.15, $F_n = 0.2$.

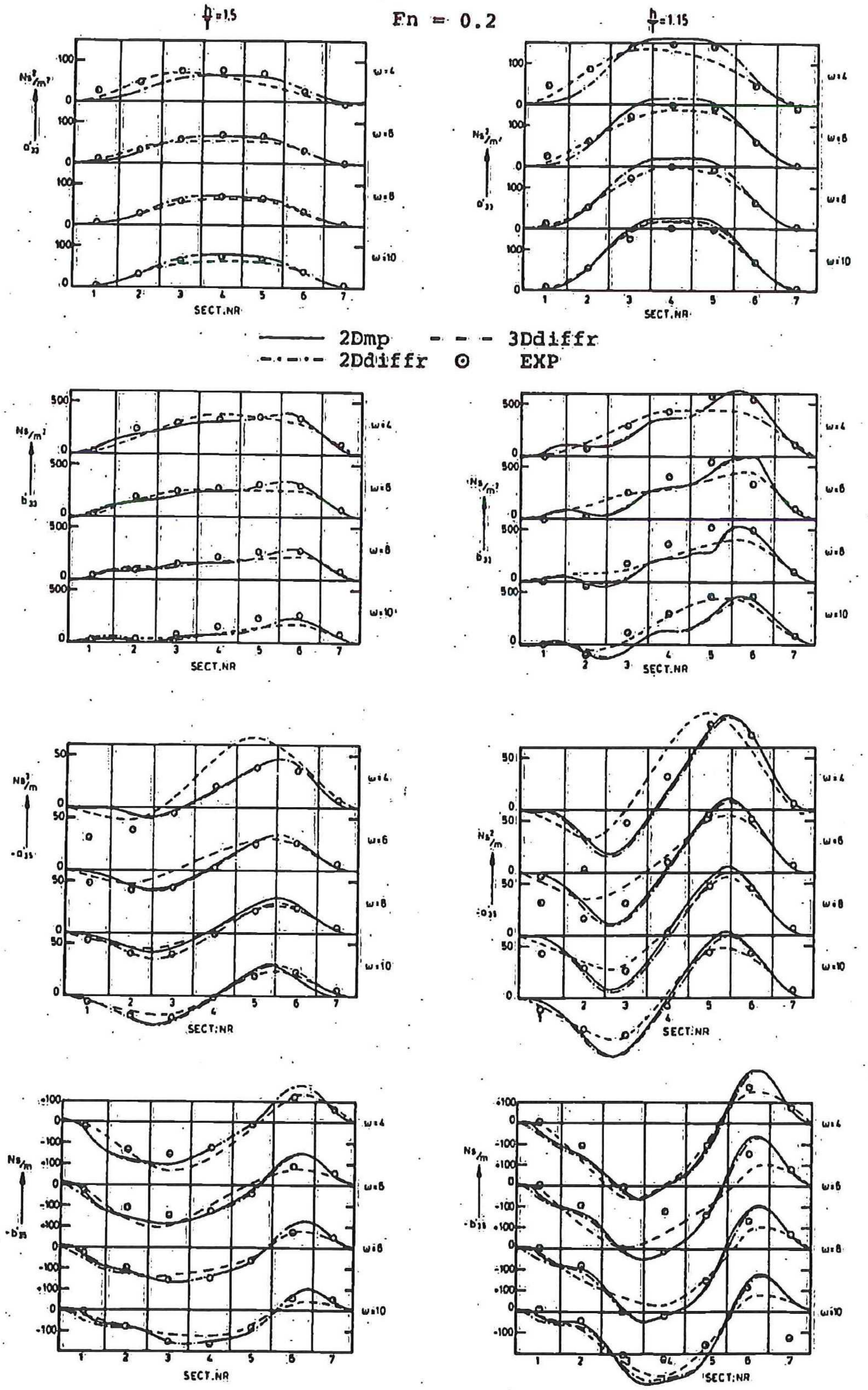

Figure 3. Calculated and measured distribution of hydrodynamic coefficients for heave; $h/T = 1.50$ and 1.15, $F_n = 0.2$.

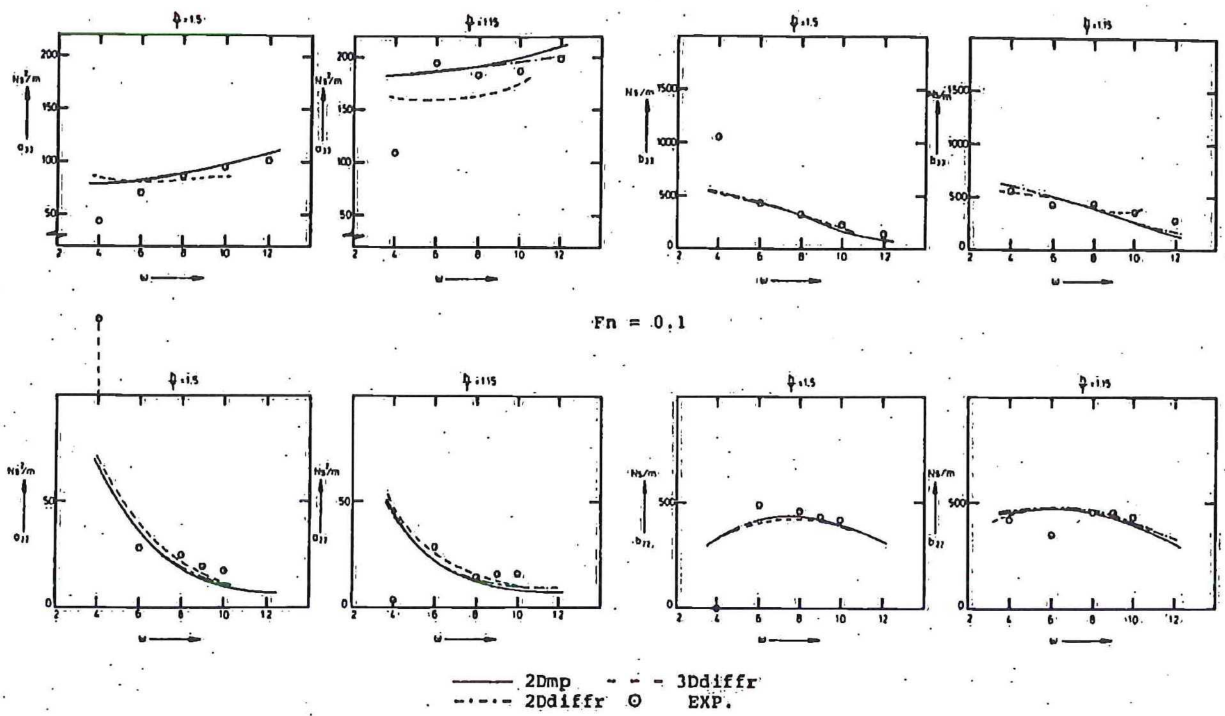

Figure 4. Calculated and measured hydrodynamic mass and damping for sway and heave, $h/T = 1.50$ and 1.15, $F_n = 0.1$.

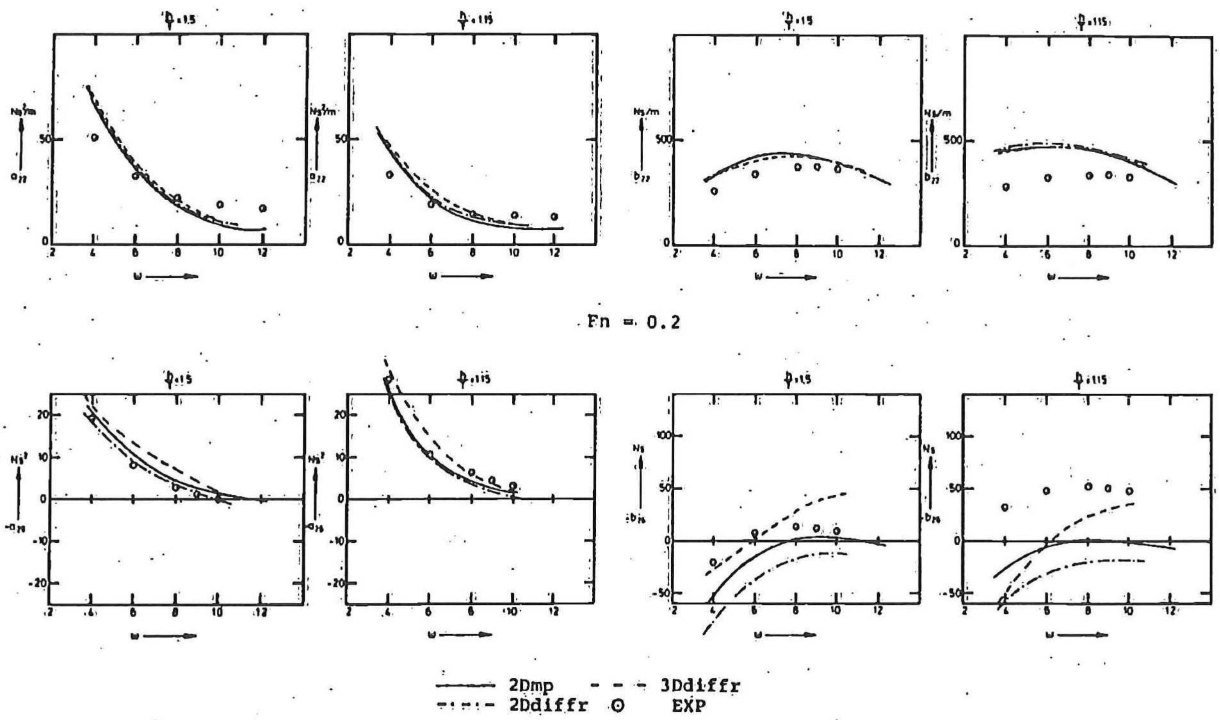

Figure 5. Calculated and measured hydrodynamic coefficients for sway, $h/T = 1.50$ and 1.15, $F_n = 0.2$.

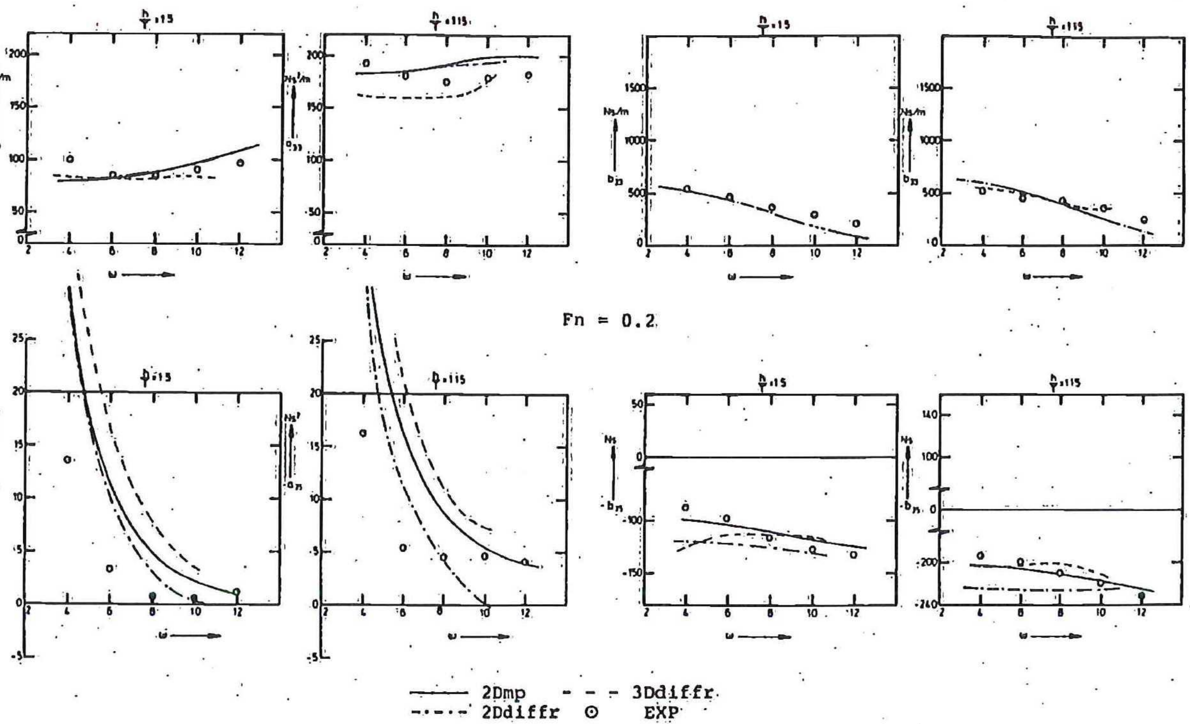

Figure 6: Calculated and measured hydrodynamic coefficients for heave, $h/T = 1.50$ and 1.15, $F_n = 0.2$.

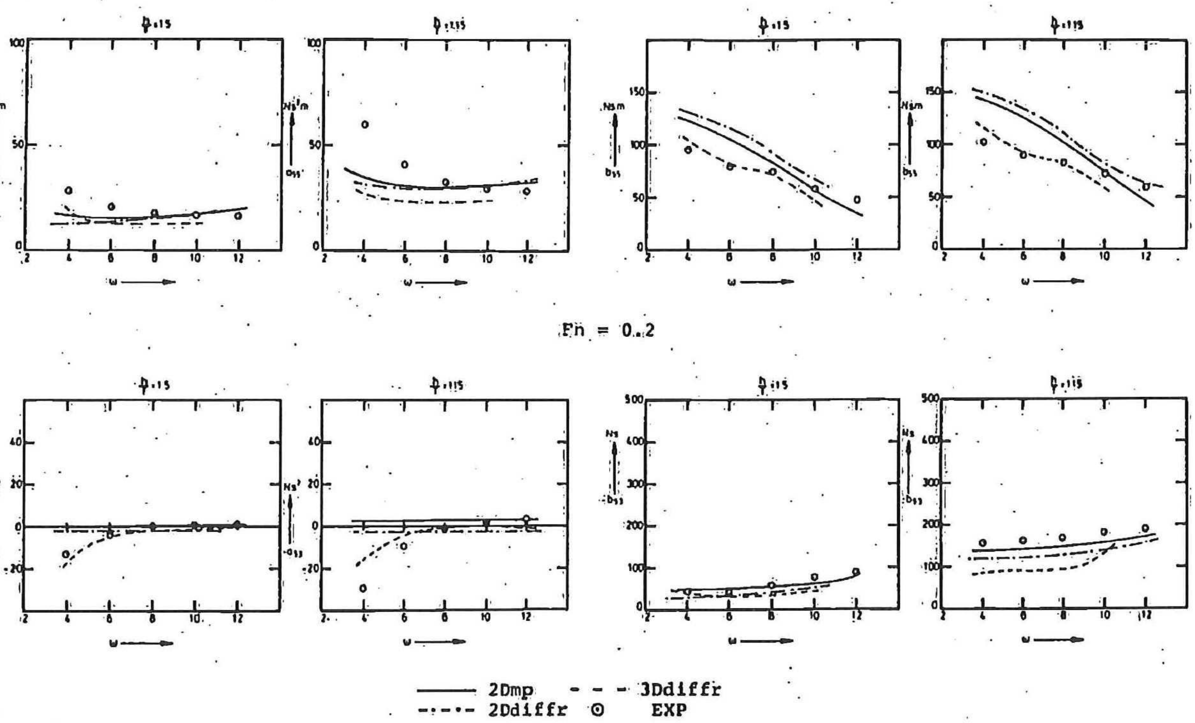

Figure 7. Calculated and measured hydrodynamic coefficients for pitch, $h/T = 1.50$ and 1.15, $F_n = 0.2$.

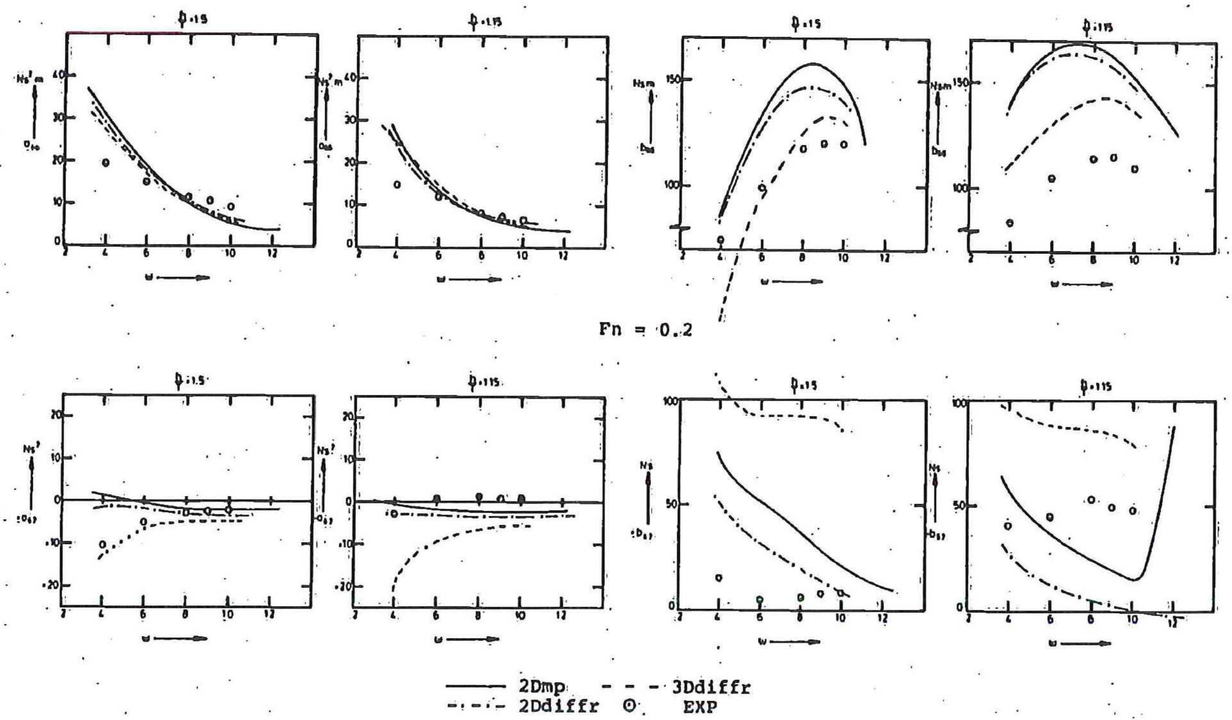

Figure 8. Calculated and measured hydrodynamic coefficients for yaw, h/T = 1.50 and 1.15, F_n = 0.2.

coefficient for sway. From Figure 5 the measured values appear to be 20 to 30% lower than all calculated results in case of F_n = 0.2.
From this it follows that also for yaw the measured damping coefficients are lower than the calculated values (see Figure 8).
It is remarkable that for this case the results calculated according to the three-dimensional diffraction theory fairly deviate from both other calculations in such a way that they follow closer the measured results.
To a less extent the same trend may be ascertained for the damping coefficient of the pitching motion, see Figure 7.

2. For the mass coupling coefficients there appears to be good agreement between the measured and calculated values especially for the horizontal motions.

3. The coupling coefficients for damping show good agreement between measured and calculated values relative to the vertical motion.
However, for the horizontal motions the deviations between the calculated values mutually and the measured values appear to be significant. This phenomenon is also clearly shown with respect to the distribution of these coefficients over the model length, see Figure 2.
From this figure it is also apparent that the measured values for e.g. b_{26} in the aft ship are a good deal lower than the calculated ones, especially for h/T = 1.15.

4. The differences between the calculated results for the coupling coefficients b_{35}, a_{35} and b_{26} with respect to h/T = 1.15 and 1.50 are mainly connected to the same differences, found for the calculations in the case of zero speed, see Tables 1 and 2.

5. Since both diffraction methods considered are based on a distribution of sources and sinks, the frequency range of these methods is restricted to the maximum of ω = 10 rad/s for this ship due to numerical instabilities, which are inherent to these type of solution-methods.

5. Conclusions and recommendations

The following conclusions and recommendations may be derived from this study.

1. In general the mutual differences between the results of the presented calculation methods are small. Clear differences between the three-dimensional and both two-dimensional methods occur only for:
   a. the added mass-distribution at low frequencies in the case of horizontal motions
   b. the values of added mass for the ship in the heave mode at h/T = 1.15.

Rather large mutual differences for the damping coefficients of the yawing motion and the accessory coupling coefficients occur, which may be caused by the differences found for zero speed.

Table 1

$F_n = 0$

ω (1/s)	h/T = 1.50						h/T = 1.15					
	a_{33} (Ns^2/m)			b_{33} (Ns/m)			a_{33} (Ns^2/m)			b_{33} (Ns/m)		
	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr
4	78.5	79.1	85.2	514.3	522.7	525.4	183.1	183.8	162.9	598.0	601.4	559.0
6	82.2	82.7	81.6	427.7	437.2	429.6	186.5	186.9	161.1	513.5	518.0	493.3
8	88.7	89.0	82.9	309.3	320.8	320.8	192.1	191.4	163.3	396.5	402.5	405.2
10	98.5	97.5	84.8	182.3	192.6	193.2	200.5	194.8	177.5	260.4	258.1	378.4
12	109.1			88.1			210.9			136.8		

ω (1/s)	h/T = 1.50						h/T = 1.15					
	a_{22} (Ns^2/m)			b_{22} (Ns/m)			a_{22} (Ns^2/m)			b_{22} (Ns/m)		
	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr
4	66.3	66.6	70.1	323.0	329.9	321.2	45.0	43.8	47.3	453.5	465.2	445.5
6	36.2	36.1	39.0	412.9	414.3	396.1	22.3	22.0	26.0	475.5	479.8	469.1
8	18.5	19.2	22.0	434.1	428.7	419.1	12.3	13.2	15.7	455.4	456.5	453.1
10	9.6	11.1	12.5	391.5	386.3	388.1	8.1	9.8	10.7	400.1	405.9	407.0
12	7.5			317.2			7.8			303.9		

Table 2

$F_n = 0$

ω (1/s)	h/T = 1.50						h/T = 1.15					
	a_{35} (Ns^2)			b_{35} (Ns)			a_{35} (Ns^2)			b_{35} (Ns)		
	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr
4	−0.1	+1.8	+0.8	+26.8	+44.3	+27.1	−2.7	+2.2	−0.7	+30.3	+50.7	+30.1
6	−0.2	+1.8	+0.5	+27.0	+43.1	+27.7	−2.8	+2.2	−1.0	+30.2	+49.1	+31.5
8	−0.3	+1.8	+0.1	+26.8	+41.1	+29.6	−2.9	+2.2	−1.4	+29.7	+46.7	+32.3
10	−0.5	+1.8	−0.2	+25.4	+37.6	+29.3	−3.0	+2.2	−1.9	+28.3	+42.2	+29.2
12	−0.6			+21.6			−3.1			+25.0		

ω (1/s)	h/T = 1.50						h/T = 1.15					
	a_{26} (Ns^2)			b_{26} (Ns)			a_{26} (Ns^2)			b_{26} (Ns)		
	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr	2Dmp	2Ddiffr	3Ddiffr
4	−1.6	+1.3	−2.0	−13.5	−1.3	− 8.4	+0.7	+2.9	−1.0	−19.0	+0.3	−12.7
6	+0.1	+1.9	−1.2	−21.6	−7.9	−17.7	+1.9	+3.2	+0.2	−18.8	+0.3	−20.4
8	+1.7	+2.9	+0.3	−22.3	−8.2	−28.3	+2.3	+3.3	+1.5	−15.1	+1.5	−23.7
10	+2.2	+3.3	+2.1	−14.4	−2.9	−27.6	+2.3	+3.4	+2.4	− 9.6	+4.6	−20.3
12	+1.9			− 6.5			+2.0			− 3.3		

2. The differences between measurements and calculations are especially significant in the damping of the horizontal motions and the accessory coupling coefficients for which the calculations indicate too high values.

3. The results of this detailed comparison of the calculated values mutually and between those and the measured values, denote that the use of potential theory in determining the added mass and damping either by two- or three-dimensional methods is certainly applicable for the calculation of a ship's response in waves in shallow water, especially for technical applications.

4. As a following to this study, determination of wave forces and responses of a ship on shallow water as verification of the calculation methods may be important to check the usefulness of the calculation methods.

5. Mutual comparison of the calculated hydrodynamic coefficients does not lead to a clear preference for one of the three calculation methods for the ship form considered.

For a good appreciation also the required computation time should be taken into account.

For the two-dimensional multipole approximation method computation time is far less than for the diffraction programs, which are based on the method of distribution of sources.

From the two diffraction programs the two-dimensional method combined with strip theory requires again considerable less computation time than the fully three-dimensional source distribution method.

Nomenclature

A_n	multipole coefficient
a_{kj}, b_{kj}	added mass and damping coefficient for the whole ship
a'_{kj}, b'_{kj}	added mass and damping coefficient for a section with forward speed V
a^o_{kj}, b^o_{kj}	added mass and damping coefficient for a section with zero forward speed
B	breadth of model, damping matrix
C	hydrostatic reaction matrix
C_B	block coefficient
d	index for diffraction-potential
F	force
F_k	hydrodynamic reaction force in the kth-mode
F_n	Froude-number
g	acceleration due to gravity
h	depth of water
j	mode of motion
k	wave number ($= 2\pi/\lambda$), index for mode
L	length of model
M	mass matrix of model
M_a	hydrodynamic mass matrix
n_k	direction normal
p	pressure
r	amplitude of oscillation, index for radiation-potential
S	surface of model
T	draught of model
T_{kj}	hydrodynamic reaction force in the kth-mode for the jth mode of motion
t	time
V	forward speed of ship
v	velocity of water, function describing the influence of the source-strength distribution
v_s	velocity of a point on the ship's surface
w	index for wave potential
x, y, z	right handed coordinate system
$\underline{x}$	position of vector
x_j	motion in the jth mode, $j = 1(1)6$
β	drift angle
ω	frequency of incoming wave
ω_e	frequency of encounter, frequency of oscillation
χ	wave direction
$\tilde{\Phi}$	total velocity potential
Φ_s	stationary motion potential
ϕ, ϕ^o	oscillating velocity potential for speed unequal to zero respectively equal to zero
ϕ_j, ϕ_j^o	oscillating velocity potential into the jth-mode for speed unequal to zero respectively equal to zero
ψ	stream function
$\underline{\zeta}$	location vector
ζ	displacement of wave surface
μ_j	complex source-strength distribution for mode j
η_j	motion amplitude for mode j
λ	wave length
ν	frequency number ($= \omega_e^2/g$)
ν_o	wave number satisfying the surface condition

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Appendix A

Two-dimensional diffraction method (2Ddiffr)

Added masses and damping coefficients are calculated from the potential ϕ_j^o which is related to the waves generated by an oscillating body.
For a situation as denoted in Figure 1 ϕ_j^o has to satify:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\phi_j^o = 0, \quad \text{everywhere in the fluid}$$

$$\frac{\partial}{\partial n}\phi_j^o = 0 \qquad \text{on the sea floor}$$

$$\frac{\partial \phi_j^o}{\partial n} = f(s) \qquad \text{on the ship}$$

$$\phi_j^o = \frac{g}{\omega^2}\frac{\partial \phi_j^o}{\partial z} \qquad \text{on the free surface of the fluid}$$

the radiation condition for ϕ_j^o at infinity.

The value $f(s)$ is found from the displacement $A(s)$ $\sin\omega t$ of a point s of the ship's surface into the direction of the normal.
The generalized boundary condition is in succession for sway and heave:

$$\frac{\partial \phi_j^o}{\partial n} = n_j \qquad j = 2,3$$

In this ϕ_j^o is the potential induced by jth mode of motion and n_j the generalized direction normal j = 2,3 (sway and heave).
For the solution of the set of equations a source distribution technique is applied. The velocity potential ϕ_j^o may be defined as:

$$\phi_j^o(x) = \int_S \mu_j(\underline{\zeta})\, \upsilon(\underline{\zeta},\underline{x})\, dS_\zeta, \quad \underline{x} \text{ in the fluid}$$

in which:

$\mu_j(\underline{\zeta})$ = complex distribution of the source strength in ζ due to the jth mode of motion of the ship
$\upsilon(\underline{\zeta},\underline{x})$ = function describing the influence of the distribution of the source strength for $\zeta \in S$
$\underline{\zeta}, \underline{x}$ = location vectors.

The function $\upsilon(\underline{\zeta},\underline{x})$ is chosen in such a way that ϕ_j^o satisfies the boundary condition on the sea floor, on the free surface and the radiation condition.
The source strength should be determined in such a way that the boundary condition on the ship is also satisfied. The function $\upsilon(\underline{\zeta},\underline{x})$ may be found as presented by Wehausen and Laitone [2].
By distribution of these edges into elements (line pieces) and by assuming the source strength to be constant for each line element a set of complex linear equations arises in the unknown source strength's (see Harten and Enfrony [1]). These equations are solved by applying a LU-decomposition on the matrix of coefficients.

Literature

1. Harten, A. and Enfrony, S., 'Partition technique for the solution of potential flow problems by integral equations', J. Compt. Phys., 27, 1978, p. 71-87.
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Appendix B

Two-dimensional multipole approximation method (2Dmp)

In the following a short description is presented of the method to calculate added mass and damping as proposed by Keil [1]. See also [2].
As an example the horizontal asymmetric motions sway and yaw are taken into consideration. For symmetric motions as pitch and heave essentially a similar derivation is valid. Starting point is the potential theory, which has to satisfy a number of boundary conditions. The velocity potential describing the situation which results from the presence of a moving body in waves is divided into three parts:

$$\Phi = \Phi_w + \Phi_d + \Phi_r$$

in which:

Φ_w = the wave potential
Φ_d = the diffraction potential
Φ_r = the radiation potential.

The wave potential is the potential of the undisturbed wave while the diffraction potential presents the disturbance of the wave potential by the presence of the ship. With aid of these two potentials the excitation forces and -moments are determined for which the ship is fixed in the position considered. The radiation potential originates by the motion of the ship. For the calculation of this potential it is assumed that the ship is oscillated with a frequency equal to the frequency of encounter with the wave. The force required for these oscillations is divided into two parts. The inphase part delivers the added mass and the quadrature component delivers the damping force.

Calculation of added mass and damping

For the calculation of added mass and damping the radiation potential is determined. This potential arises from the oscillating motion of the ship. For sway the ship is oscillated into the y-direction. To describe the flow the body is replaced by pressure fluctuations in the waterline within a small strip around the middle. Since the motions are asymmetric the flow around the body is asymmetric too just as the assumed pressure fluctuations.

For the calculation of the coefficients of the strip-theory-equations a two-dimensional consideration is sufficient. This means that it is assumed that waves are emitted into the y-direction only. The potential than becomes:

$$\Phi_{or} = e^{i\omega t} A_o \int_o^\infty \frac{k \cosh\{k(z-h)\}\sin ky}{\nu \cosh(kh) - k \sinh(kh)} dk$$

in which:

A_o = strength of pressure fluctuation at x
k = wave number in y-direction ($= 2\pi/\lambda$)
h = water depth
ν = frequency-number ($= \omega^2/g$).

These potentials are constructed in such a way that the boundary conditions at the surface, the bottom and the continuity conditions are satisfied. The difference with the potentials for deep water is that these potentials satisfy the bottom condition. The potential for deep water may be separated so that the potential is composed as follows:

$$\Phi_{or} = \Phi_{or\infty} + \Phi_{orad}$$

With due regard to the radiation condition the imaginary part of the potential may be found:

$$\Phi_{oi} = e^{i\omega t} A_o \pi \nu_o \frac{\cosh(\nu_o h) \cosh\{\nu_o(z-h)\}\sin\nu_o y}{\nu_o h + \sinh(\nu_o h)\cosh(\nu_o h)}$$

Herein $\nu_o = 2\pi/\lambda$ is the wave number belonging to the waves generated by the oscillation and satisfying the boundary conditions of the surface. For ν_o holds:

$$\nu \cosh(\nu_o h) - \nu_o \sinh(\nu_o h) = 0$$

The solution of the potential found up to now satisfies all boundary conditions except that on the edge of the body.

Now multipole potentials are still added, which are based on symmetrical pressure fluctuations around the origin. These potentials also have to satisfy all boundary conditions:

The mutipole potentials become now

$$\Phi_{nr\infty} = -e^{i\omega t} A_n \int_o^\infty (k+\nu) k^{2n-1} e^{kz} \sin(ky)\, dk$$

$$\Phi_{n_{rad}} = -e^{i\omega t} A_n \int_o^\infty (k+\nu) k^{2n-1} e^{-kh}$$

$$\Phi_{ni} = e^{i\omega t} A_n \frac{\pi \nu_o^{2n-1}}{\cosh(\nu_o h)} \frac{\cosh\{\nu_o(z-h)\}\sin\nu_o y}{\nu_o h + \sinh(\nu_o h)\cosh(\nu_o h)}$$

The total potential may now be written in an abridged form as follows:

$$\Phi = A_o(\phi'_{or\infty} + \phi'_{o_{rad}} + i\phi'_{oi}) + \sum_{n=1}^{\infty} A_n(\phi'_{nr\infty} + \phi'_{n_{rad}} + i\phi'_{ni})$$

from which the coefficient A_n remains as unknown, which may be determined from the boundary conditions for the contour. For this use is made of the stream function derived from the potential with:

$$\frac{\partial\Phi}{\partial y} = \frac{\partial\psi}{\partial z}$$

The stream function on the contour may be calculated with the A_n's as unknowns. Putting this solution equal to the stream function as it follows from the boundary condition the coefficients A_n may be calculated.

As body a cylinder is chosen, which by means of a Lewis procedure is transformed to a shipform. In this way the potential function for the oscillating cylinder has been determined and from this the force required for the oscillation may be calculated.

For the calculation of the potentials series are developed as denoted in [1].

Literature

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Appendix C

Three-dimensional diffraction method (3Ddiffr.)

For the calculation of the motion potential ϕ^o one can use a three-dimensional source distribution method. This potential ϕ^o consists of:

$$\phi^o = \sum_{j=1}^{6} \eta_j \phi_j^o$$ with the jth mode of motion and η_j the amplitude of motion

The potentials ϕ_j^o have to satisfy the Laplace equation, the linearized free surface condition of the fluid, a suitable radiation condition at infinity and the boundary conditions on the sea flow and on the ship's surface.

The potential may be written as a source distribution [2] on the ship's surface:

$$\phi_j^o(\underline{x}) = \iint_{S_o} \mu_j(\underline{\zeta})\, \upsilon(\underline{\zeta},\underline{x})\, dS_\zeta \quad j = 1,2,3,4,5,6 \qquad (*)$$

with:

$\mu_j(\underline{\zeta})$= source strength for jth mode of motion

$\upsilon(\underline{\zeta},\underline{x})$= Green's function satisfying all boundary conditions except the ship's surface boundary condition

$\underline{\zeta},\underline{x}$ = location vectors.

The Green's functions $\upsilon(\underline{\zeta},\underline{x})$ are presented by Wehausen and Laitone in [1]. The unknown source strength $\mu(\underline{\zeta})$ should be determined in such a way that the boundary condition on the ship's surface is fulfilled. This yields:

$$+ i\omega n_j(\underline{x}) = -\frac{1}{2}\mu_j(\underline{x}) + \frac{1}{4\pi}\iint_S \mu_j(\underline{\zeta}) \frac{\partial}{\partial n} \upsilon(\underline{\zeta},\underline{x})\, dS_\zeta \; (**)$$

for $\underline{x}$ on the ship's surface.

n_j are the generalized direction cosines:

$n_1 = \cos(n, x_1)$

$n_2 = \cos(n, x_2)$

$n_3 = \cos(n, x_3)$

$n_4 = x_2 n_3 - x_3 n_2$

$n_5 = x_3 n_1 - x_1 n_3$

$n_6 = x_2 n_1 - x_1 n_2$

Equation (**) is a Fredholm equation of the second kind. This equation is solved by dividing the ship's surface into a number of surface elements for which constant source strength is assumed.

The integral equation (**) reduces to a set of algebraic equations for the unknown source strength $\mu(\zeta)$.

With the aid of equation (*) and the calculated source strength $\mu_j(\underline{\zeta})$ the potential $\phi_j^o(\underline{x})$ may than be calculated.

Literature

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