

Control Properties of Underactuated Vehicles

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Abstract

This paper studies control properties of the dynamics of underactuated vehicles (e.g. underactuated surface vessels, underwater vehicles, aeroplanes or spacecraft). The unactuated dynamics implies constraints on the accelerations. Both the necessary and sufficient conditions for these constraints to be second-order nonholonomic, first-order nonholonomic or holonomic are developed. It is shown that underactuated vehicles with a gravitational field where the elements corresponding to the unactuated dynamics are zero, are not C^1 asymptotically stabilizable to a single equilibrium.

1 Introduction

In recent years there has been extensive research on motion planning for nonholonomic systems. A traditional problem in robot motion planning has been constructing an obstacle-avoiding path, i.e. finding a path subject to constraints on the position. This problem has been extended to nonholonomic motion planning, i.e. motion planning subject to nonintegrable constraints on the velocities. Li and Canny [1] proposed a path finding algorithm for an embedded manifold rolling on a flat plane, using the case where the embedded manifold is a unit ball for illustration. Based on the results of Brockett [2], Murray and Sastry [3, 4, 5] developed a nonholonomic motion planning algorithm using sinusoids at integrally related frequencies. Another approach to nonholonomic motion planning has been the use of a highly oscillatory control, [6]. These are among several important contributions to nonholonomic motion planning which all have in common that they consider the kinematics of the nonholonomic system. Consequently the models have no drift vector field.

Modeling both the kinematics and the dynamics of the system results in a model that has a drift vector

field in addition to the control vector fields. Sussmann [7] discussed the controllability of systems with drift, showing that a dynamic extension of a driftless completely controllable nonholonomic system, is completely controllable as well. Bloch and McClamroch [8], Bloch, McClamroch and Reyhanoglu [9, 10, 11] considered the kinematics and dynamics of a class of mechanical systems with nonholonomic velocity constraints. They showed that this class of systems is not C^1 asymptotically stabilizable. Instead they presented a smooth feedback law that asymptotically stabilized the system to an equilibrium manifold. For Caplygin systems, a special class of systems contained in the original class, they proposed an algorithm that asymptotically stabilizes the origin of the system.

A further extension of the motion planning problem, is motion planning subject to nonintegrable constraints on the accelerations. Inevitably, the models considered describe the dynamics of the systems, thus having a drift vector field. Nonintegrable constraints on the accelerations may appear in underactuated systems, i.e. systems where the control vector has a lower dimension than the configuration vector. Control of examples of underactuated dynamical systems have been studied by Hauser and Murray [12], Byrnes and Isidori [13], Spong [14]. Oriolo and Nakamura [15] considered a general model of an underactuated robot manipulator. They gave sufficient and necessary conditions for integrability. Also, by assuming that the gravitational term of the dynamics was zero, they showed that the manipulator was not C^1 asymptotically stabilizable. For an underactuated two-link planar manipulator, they presented a smooth feedback law that asymptotically stabilized the system to an equilibrium manifold.

In this paper we extend the results of Oriolo and Nakamura [15] by studying the dynamics of underactuated vehicles (e.g. underactuated surface vessels, underwater vehicles, aeroplanes or spacecraft). The vehicle model and the robot manipulator model have

significant points of difference from each other. Oriolo and Nakamura [15] assumed properties of the coriolis and centripetal matrix that are not satisfied for the vehicle model. Also, the vehicle model has a damping term and includes a kinematic transformation from velocity to configuration. Thus the work of Oriolo and Nakamura [15] cannot be directly applied to the study of vehicles.

Given a system having constraints on its accelerations, a question to be addressed in this paper is whether this is a second-order nonholonomic system or if it is essentially a nonholonomic system having constraints on its velocities, or if it is a holonomic system. Another question is whether the system is C^1 asymptotically stabilizable to a single equilibrium. Fossen [16] showed that a fully actuated vehicle (a vehicle where the control and configuration vector have the same dimension) can be asymptotically stabilized in position and velocity by a smooth feedback law. Byrnes and Isidori [13] showed that underactuated vehicles with zero gravitational field are not C^1 asymptotically stabilizable to a single equilibrium. In this paper we extend this result by showing that underactuated vehicles with a gravitational field $g(\eta)$ where the elements of g corresponding to the unactuated dynamics are zero, are not C^1 asymptotically stabilizable to a single equilibrium.

The paper is organized as follows: In Section 2 we present a model of an underactuated vehicle, describing dynamics and kinematics. In Section 3 we develop necessary and sufficient conditions for partial integrability and for total integrability of the second-order constraints. In Section 4 we show that underactuated vehicles with a gravitational field where the elements corresponding to the unactuated dynamics are zero, are not C^1 asymptotically stabilizable to a single equilibrium. In Section 5 we discuss the control properties of an underactuated underwater vehicle.

2 Model of an underactuated vehicle

We consider the class of systems described by

$$\begin{aligned} M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) &= \begin{bmatrix} \tau \\ 0 \end{bmatrix} \quad (1) \\ \dot{\eta} &= J(\eta)\nu \quad (2) \end{aligned}$$

$\eta \in \mathbb{R}^n$, $\nu \in \mathbb{R}^n$, $\tau \in \mathbb{R}^m$, $m < n$, where $\dot{M} = 0$, M and J are nonsingular matrices.

This class includes underactuated surface vessels, underwater vehicles [16], aeroplanes and spacecraft. Then ν denotes the linear and angular velocity vector with coordinates in the body-fixed frame, η denotes

the position and orientation vector with coordinates in the earth-fixed frame and τ denotes the forces and moments acting on the vehicle, with coordinates in the body-fixed frame. M is the inertia matrix, including added mass. $C(\nu)$ is the matrix of coriolis and centripetal terms, also including added mass. $D(\nu)$ is the damping matrix and $g(\eta)$ is the vector of gravitational and possibly buoyant forces and moments. Equation (2) represents the kinematics. We emphasize that the model differs from the underactuated robot manipulator of Oriolo and Nakamura [15] on several important points. In particular the coriolis and centripetal matrix $C(\nu)$ cannot be parameterized by Christoffel symbols. Moreover, the vehicle model has a damping term and includes the kinematic transformation from ν to η . Thus the work of Oriolo and Nakamura [15] cannot be directly applied to this model.

3 Integrability of second-order constraints

Let M_u denote the last $n - m$ rows of the matrix M , let $C_u(\nu)$ and $D_u(\nu)$ be defined similarly, and let $g_u(\eta)$ denote the $n - m$ last elements of the vector $g(\eta)$. Then the constraints implied by the unactuated dynamics may be written

$$M_u\dot{\nu} + C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta) = 0 \quad (3)$$

The question to be addressed in this paper is whether these are non-integrable constraints on the accelerations, i.e. second-order nonholonomic constraints, or if they can be integrated to constraints on the velocities. In the following, the constraints (3) are said to be *partially integrable* if they can be integrated to the form

$$h_P(\nu, \eta, t) = 0 \quad (4)$$

where $h_P(\cdot)$ is a function $h_P : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^{n-m}$. The following theorem gives necessary and sufficient conditions for (3) to be partially integrable:

Theorem 1 (Partially Integrable)

The constraints $M_u\dot{\nu} + C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta) = 0$ are *partially integrable* if and only if

1. g_u is constant
2. $(C_u(\nu) + D_u(\nu))$ is a constant matrix
3. The distribution Ω^\perp defined by $\Omega^\perp(\eta) = \ker((C_u + D_u)J^{-1}(\eta))$ is completely integrable.

Proof: Since $\dot{M} = 0$

$$\int_0^t (M_u \dot{\nu} + C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta)) d\tau =$$

$$M_u \nu(t) - M_u \nu(0) + \int_0^t (C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta)) d\tau$$

Thus the constraints are partially integrable if and only if $C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta) = 0$ may be integrated to the form $h(\nu, \eta, t) = 0$.

Necessity.

Suppose $\exists h(\nu, \eta, t) = \int_0^t (C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta)) d\tau$. Then

$$\frac{dh}{dt} = C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta) \quad (5)$$

and

$$\frac{dh}{dt} = \frac{\partial h}{\partial \nu} \dot{\nu} + \frac{\partial h}{\partial \eta} \dot{\eta} + \frac{\partial h}{\partial t} \quad (6)$$

This implies

$$\frac{\partial h}{\partial \nu} = 0 \quad (7)$$

i.e. h is not a function of ν ,

$$h = h(\eta, t) \quad (8)$$

As t does not appear explicitly in the constraints, h must be in the following form

$$h(\eta, t) = h_\eta(\eta) + kt \quad (9)$$

for some function $h_\eta : \mathbb{R}^n \rightarrow \mathbb{R}^{n-m}$ and constant vector k . Then we have

$$\frac{dh}{dt} = \frac{\partial h_\eta(\eta)}{\partial \eta} \dot{\eta} + k \quad (10)$$

Since $\dot{\eta}$ is an independent variable, this implies using (2) and (5)

$$\frac{\partial h_\eta(\eta)}{\partial \eta} J(\eta)\nu = (C_u(\nu) + D_u(\nu))\nu \quad (11)$$

$$k = g_u \quad (12)$$

Thus g_u must be constant, i.e. 1. is a necessary condition. Eq. (11) implies that $(C_u(\nu) + D_u(\nu))$ cannot be a function of ν , and must thus be a constant matrix, proving the necessity of condition 2. We denote this matrix $(C_u + D_u)$. Eq. (11) implies

$$\frac{\partial h_\eta(\eta)}{\partial \eta} = (C_u + D_u)J^{-1}(\eta) \quad (13)$$

We define the codistribution Ω by $\Omega(\eta) = \text{span}\{\omega_1(\eta), \dots, \omega_{n-m}(\eta)\}$, where $\omega_1(\eta), \dots, \omega_{n-m}(\eta)$ denote the rows of the matrix $(C_u + D_u)J^{-1}(\eta)$. Its

annihilator is the distribution $\Omega^\perp(\eta) = \ker((C_u + D_u)J^{-1}(\eta))$. The elements of $h_\eta = [h_\eta^1, \dots, h_\eta^{n-m}]^T$ must thus satisfy

$$\frac{\partial h_\eta^k(\eta)}{\partial \eta} = \omega_k(\eta) \quad (14)$$

By the assumed existence of a function $h(\cdot)$, and thus the function $h_\eta(\cdot)$ which must satisfy (14), we can conclude that Ω^\perp is completely integrable [17]. Thus the necessity of condition 3. is proved.

Sufficiency.

Suppose conditions 1. and 2. are satisfied. Then the constraints may be written $M_u \dot{\nu} + (C_u + D_u)\nu + g_u = 0$. The time integral gives

$$\int_0^t (M_u \dot{\nu} + (C_u + D_u)J^{-1}(\eta)\dot{\eta} + g_u) d\tau = 0 \quad (15)$$

Due to the definition of complete integrability [17], condition 3. implies that there exists a function $h_\eta(\eta)$ s.t. $\frac{\partial h_\eta(\eta)}{\partial \eta} = (C_u + D_u)J^{-1}(\eta)$. Thus

$$M_u \nu(t) - M_u \nu(0) + \int_{\eta(0)}^{\eta(t)} \frac{\partial h_\eta(\eta)}{\partial \eta} d\eta + g_u t = 0 \quad (16)$$

Thus there is a constant vector C_1 s.t.

$$M_u \nu + h_\eta(\eta) + g_u t + C_1 = 0 \quad (17)$$

i.e. the constraints can be integrated to

$$h_P(\nu, \eta, t) = M_u \nu + h_\eta(\eta) + g_u t + C_1 = 0 \quad (18)$$

and are consequently partially integrable. \square

Remark 1 Note that a possible integral $h_P(\nu, \eta, t) = 0$ of the constraints (3) must be in the form (18).

In the following, the constraints (3) are said to be *totally integrable* if they may be further integrated to the form $h_T(\eta, t) = 0$, where $h_T(\cdot)$ is a function $h_T : \mathbb{R}^n \times \mathbb{R} \rightarrow \mathbb{R}^{n-m}$.

Theorem 2 (Totally Integrable)

The constraints $M_u \dot{\nu} + C_u(\nu)\nu + D_u(\nu)\nu + g_u(\eta) = 0$ are totally integrable if and only if

1. The constraints are partially integrable
2. $(C_u + D_u) = 0$
3. The distribution Δ defined by $\Delta(\eta) = \ker(M_u J^{-1}(\eta))$ is completely integrable

Proof: By condition 1. the constraints can be integrated to the form $h_P(\nu, \eta, t) = 0$. We need to show that $h_P(\nu, \eta, t) = 0$ can be integrated to the form $h_T(\eta, t) = 0$ if and only if the conditions 2. and 3. are satisfied. If and only if condition 1. is satisfied Remark 1 and (18) give

$$\int_0^t h_P(\nu, \eta, t) d\tau = \int_0^t (M_u \nu + h_\eta(\eta) + g_u t + C_1) d\tau = 0 \quad (19)$$

Necessity.

Suppose $\exists h_T(\eta, t) = \int_0^t (M_u \nu + h_\eta(\eta) + g_u t + C_1) d\tau$. Then

$$\frac{dh_T}{dt} = M_u \nu + h_\eta(\eta) + g_u t + C_1 \quad (20)$$

and

$$\frac{dh_T}{dt} = \frac{\partial h_T}{\partial \eta} \dot{\eta} + \frac{\partial h_T}{\partial t} \quad (21)$$

Since $\dot{\eta}$ is an independent variable, this implies using (2)

$$\frac{\partial h_T}{\partial \eta} = M_u J^{-1}(\eta) \quad (22)$$

$$\frac{\partial h_T}{\partial t} = h_\eta(\eta) + g_u t + C_1 \quad (23)$$

As t does not appear explicitly in $M_u J^{-1}(\eta)$, h_T must be in the form

$$h_T(\eta, t) = h_{1T}(\eta) + h_{2T}(t) \quad (24)$$

and thus

$$\frac{\partial h_{1T}(\eta)}{\partial \eta} = M_u J^{-1}(\eta) \quad (25)$$

$$\frac{\partial h_{2T}(t)}{\partial t} = h_\eta(\eta) + g_u t + C_1 \quad (26)$$

By the assumed existence of a function $h_T(\cdot)$, and thus the function $h_{1T}(\cdot)$ which must satisfy (25), we can conclude that Δ is completely integrable [17], condition 3.

As $h_{2T}(\cdot)$ is a function of t only, (26) implies that $h_\eta(\eta)$ is a constant vector. As $h_\eta(\eta)$ satisfies equation (13) and $J(\eta)$ is nonsingular, this implies condition 2.

Sufficiency.

Suppose condition 2. is satisfied. Then by equation (13) $h_\eta(\eta)$ is a constant vector b . Thus from (19) and (2)

$$\int_0^t h_P(\nu, \eta, t) d\tau = \int_0^t (M_u J^{-1}(\eta) \dot{\eta} + b + g_u t + C_1) d\tau = 0 \quad (27)$$

Due to the definition of complete integrability [17], condition 3. implies the existence of a solution $m(\eta)$, $m : \mathbb{R}^n \rightarrow \mathbb{R}^{n-m}$, to the equation

$$\frac{\partial m(\eta)}{\partial \eta} = M_u J^{-1}(\eta) \quad (28)$$

Thus there is a constant vector C_2 such that the constraints can be integrated to

$$h_T(\eta, t) = m(\eta) + \frac{1}{2} g_u t^2 + (b + C_1)t + C_2 = 0 \quad (29)$$

The constraints are consequently totally integrable. \square

Remark 2 Note that condition 2. in Theorem 2 implies conditions 2. and 3. in Theorem 1. Thus the condition 1. in Theorem 2 can be replaced by g_u is constant.

The integrability theorems thus allow us to determine what is the essential nature of the system. If the constraints are not partially integrable, we have a second-order nonholonomic system. If the constraints are partially but not totally integrable, it is a first-order nonholonomic system, i.e. a system with constraints on its velocities. When the constraints are totally integrable, the problem is holonomic. A vehicle will generally not be partially integrable.

4 Stabilizability

Theorem 3 Consider the system (1)-(2). Suppose that the elements of the gravitational field $g(\eta)$ corresponding to the unactuated dynamics are zero,

$$\text{i.e. } g(\eta) = \begin{bmatrix} g^1(\eta) \\ 0_{(n-m)} \end{bmatrix}.$$

Let $(\eta, \nu) = (\eta^e, 0)$ denote an equilibrium. There is no C^1 state feedback law, $\alpha(\eta, \nu) : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^m$, that makes $(\eta^e, 0)$ an asymptotically stable equilibrium.

Proof: A necessary condition for the existence of a C^1 state feedback law that asymptotically stabilizes this system is that the image of the mapping $f : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^{2n}$ defined by

$$f(\eta, \nu, \tau) =$$

$$\begin{bmatrix} -M^{-1}(C(\nu)\nu + D(\nu)\nu) - M^{-1}g(\eta) + M^{-1} \begin{bmatrix} \tau \\ 0 \end{bmatrix} \\ J(\eta)\nu \end{bmatrix}$$

contains some neighbourhood of zero [18].

Consider points of the form

$$\varepsilon = \begin{bmatrix} M^{-1} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \\ 0 \end{bmatrix} \quad (30)$$

where $\alpha \in \mathbb{R}^m$ is an arbitrary vector, $\beta \in \mathbb{R}^{n-m}$ an arbitrary non-zero vector. The equation $f(\eta, \nu, \tau) = \varepsilon$ implies $\nu = 0$ as $J(\eta)$ is nonsingular, and thus

$$M^{-1} \left(\begin{bmatrix} \tau \\ 0 \end{bmatrix} - \begin{bmatrix} g^1(\eta) \\ 0 \end{bmatrix} \right) = M^{-1} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (31)$$

or

$$\begin{bmatrix} \tau - g^1(\eta) \\ 0 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad (32)$$

which has no solution (η, ν, τ) as β is non-zero. Consequently ε is not in the image of f . As M is constant and α, β arbitrary vectors, points of the form ε are contained in every neighbourhood of zero, and thus Brockett's necessary condition is not satisfied. \square

Remark 3 We may then conclude that for the systems in Theorem 3, the State Space Exact Linearization Problem [17] will not be solvable. Also, transformed to normal form [17], the systems cannot have asymptotically stable zero dynamics.

5 Case Study

A general model of an underwater vehicle is [16]

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \quad (33)$$

$$\dot{\eta} = J(\eta)\nu \quad (34)$$

where $\eta = [x, y, z, \phi, \theta, \psi]^T$ is the position and orientation vector with coordinates in earth-fixed frame. The orientation is parametrized by the Euler angles, and using the xyz-convention, $J(\eta)$ is nonsingular for $|\theta| < \pi/2$. The vector $\nu = [u, v, w, p, q, r]^T$ denotes the linear and angular velocities with coordinates in the body-fixed frame. The matrix M is constant and nonsingular. Consequently, the model is in the form (1)-(2).

Suppose the underwater vehicle is underactuated, not having thruster force in the body-fixed z-direction, i.e. $\tau = [X, Y, 0, K, M, N]^T$. The second-order constraint implied by the unactuated dynamics is [16]

$$\begin{aligned} & [0, 0, m_{33}, 0, 0, 0] \dot{\nu} \\ & + [0, 0, 0, (m - Y_{\dot{v}})v, (X_{\dot{u}} - m)u, m x_G p] \nu \\ & + [0, 0, -Z_w - Z_{w|w}|w|, 0, 0, 0] \nu = 0 \end{aligned} \quad (35)$$

where m_{ij} , X_y , x_G are constant coefficients. Theorem 1 states that (35) is not partially integrable and thus the system is second-order nonholonomic.

When the distance between the center of gravity and the center of buoyancy is

$$\bar{BG} = [0, 0, \bar{BG}_z]^T$$

and W is the submerged weight of the vehicle equal to the buoyancy force, the vector of gravitational and buoyant forces and moments is [16]

$$g(\eta) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \bar{BG}_z W \cos \theta \sin \phi \\ \bar{BG}_z W \sin \theta \\ 0 \end{bmatrix} \quad (36)$$

Thus $(\eta, \nu) = (0, 0)$ is an equilibrium of the system. As $g_u(\eta) = 0$, Theorem 3 states that there is no C^1 state feedback law, $\alpha(\eta, \nu) : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^m$, that makes $(0, 0)$ an asymptotically stable equilibrium for the system.

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