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Ship Steering Design Based on NOMOTO Equation

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Abstract: This paper describes the ship's steering process and the common practice of the crew during steering operation. It analyzes and finds the mathematical theoretical basis of the crew steering control. According to its theoretical basis and considering the wind and flow conditions from the steering timing, steering rudder angle and steering time. The steering of the ship was designed and verified by the MATLAB Simulink simulation environment. The verification results show the actual operability of the design.

Keywords: ship maneuvering; steering control; NOMOTO equation

I. INTRODUCTION

When the ship sails along the planned route on the ocean, most of the time is to maintain the course or maintain the straight track. Therefore, the study of the direct track control or the course control has always been the research focus of ship motion control, but the ship Steering control is also a research topic that cannot be ignored. So far, a joint track-holding controller capable of simultaneously satisfying the ship's straight track keeping control and curve track keeping control has not been developed[1].

Most of the modern ships are only equipped with heading autopilot. Very few ships are equipped with track autopilot. Therefore, when the ship is turning at a large angle, the officer will first switch the auto to the manual rudder and complete the steering through the manual rudder. When stabilizing in the new planned line, switch the manual rudder to the autopilot to achieve the steering of the ship. This paper intends to analyze the characteristics of the officer's manual steering to find the theoretical basis of manual steering, and based on this theory, design the steering of the ship, laying the foundation for the development and design of the joint track-holding controller.

II. BASIC DESCRIPTION

A. Description of the Ship's Steering Process

When the ship turns, it is affected by many complicated and varied factors.. The direction and magnitude of environmental interference such as wind, waves, and currents will change, and the ship's speed will also change accordingly. The external manifestation is the change of wind current pressure difference angle and pressure rudder angle. See Figure 1 for details.

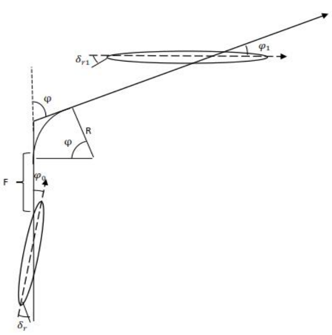

Figure 1.description of ship's turning

When the ship is sailing in a straight line, due to the influence of wind current (ignoring the influence of the asymmetry of the propeller rotation direction), there may already be a small rudder angle δ_r, which can be regarded as a constant when sailing in a straight line. When

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turning, set the steering angle relative to the forward rudder position as δ_m, then the alternating rudder angle actually acting on the ship is[2]

$$\delta = \delta_m + \delta_r \qquad (1)$$

δ is the driving rudder angle based on δ_r. ψ_0 is the yaw angle under the action of wind current and steering angle δ_r; ψ_1 is the yaw angle under the action of wind current and steering angle δ_{r1}. ψ is the steering angle and F is the lag distance.

As can be seen from the above description, the ship's nominal balance point[3] is not $[\boldsymbol{x},\boldsymbol{y},\boldsymbol{\varphi},\boldsymbol{u},\boldsymbol{v},\boldsymbol{r}]^T=[0,0,0,\boldsymbol{u},\boldsymbol{v},0]^T$ when the ship is sailing straight (whether before or after steering). If the wind & flow conditions are constant, the balance point of the system is $[\boldsymbol{x},\boldsymbol{y},\boldsymbol{\varphi},\boldsymbol{u},\boldsymbol{v},\boldsymbol{r}]^T=[0,0,\boldsymbol{\varphi_0}/\boldsymbol{\varphi_1},\boldsymbol{u},\boldsymbol{v},0]^T$. However, from the perspective of steering, the nominal balance point of the ship can be regarded as $[\boldsymbol{x},\boldsymbol{y},\boldsymbol{\varphi},\boldsymbol{u},\boldsymbol{v},\boldsymbol{r}]^T=[0,0,0,\boldsymbol{u},\boldsymbol{v},0]^T$, but corresponding the heading to be maintained has been corrected by the leeway and drift angle. During the steering process of the ship, the heading and the leeway and drift angle is constantly changing, so it is difficult to find the corresponding nominal balance point, and it is not necessary.

In addition, ships have the characteristics of large inertia and time lag, and steering needs to be advanced. The lag F can be used to represent (1).

$$F = U \cdot \left(T + \frac{t}{2}\right) \qquad (2)$$

In the formula, U is the ship speed, T is the follow ability index, and t is the steering time.

B. Crew Usually Practice

Arriving near the turning point, the driver makes a quantitative prediction of the leeway and drift to obtain the heading to be maintained after turning. You can write δ as a form of a segment function:

$$\delta = \begin{cases} \delta_1(t) & (0 < t < t_1) \\ \delta_2(t) & (t_1 < t < t_2) \\ \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \\ \ldots\ldots & \ldots\ldots \\ \delta_{n-1}(t) & (t_{n-2} < t < t_{n-1}) \\ \delta_n(t) & (t_{n-1} < t < t_f) \end{cases} \qquad (3)$$

Can also be written in the form of a vector :

$$\delta = [\delta_1(t) \quad \cdots\cdots \quad \delta_n(\mathrm{t})]^{\mathrm{T}} \qquad (4)$$

For experienced AB, you only need to steer two or three times to maintain the new course.

C. *Mathematical Theoretical Basis of Steering Maneuver*

To understand the mathematical theoretical basis of steering, that is, to find the relationship between the rudder angle and the alteration angle. Naturally, you can associate with the NOMOTO equation [4]:

$$T\ddot{\varphi} + \dot{\varphi} = K\delta \qquad (5)$$

Considering the servo delay, this paper adopts the general simplified model of the steering gear (first-order inertia link) [5]:

$$\mathrm{T_E}\dot{\delta} + \delta = \mathrm{K_E}\delta_{\mathrm{E}} \qquad (6)$$

Where $\mathrm{T_E}$ is the servo time constant, usually 2.5∽3 seconds; δ is the actual rudder angle, $|\delta| \leq 35°$; $\mathrm{K_E}$ is the steering control gain, generally taken as 1; δ_{E} is the command rudder angle, that is the rudder angle command issued by the controller.

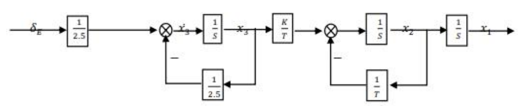
Joint equations (5) and (6) can take into account the system differential equations under servo delay:

$$\frac{T_E * T}{K}\dddot{\varphi} + \frac{T_E + T}{K}\ddot{\varphi} + \frac{1}{K}\dot{\varphi} = \delta_E \qquad (7)$$

System transfer function:

$$\mathrm{G(s)} = \frac{K}{s(Ts+1)(2.5s+1)} \qquad (8)$$

According to the system differential equation, the system state space expression can be written and the system simulation structure diagram can be drawn as:



Its state space expression is:

$$\dot{x_1} = x_2$$

$$\dot{x_2} = -\frac{1}{T}x_2 + \frac{K}{T}x_3$$

$$\dot{x_3} = -\frac{1}{2.5}x_3 + \frac{1}{2.5}\delta_E$$

$$\mathrm{y} = x_1$$

The matrix form is:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & -\frac{1}{T} & \frac{K}{T}\\ 0 & 0 & -\frac{1}{2.5}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ \frac{1}{2.5}\end{bmatrix}\delta_E \quad (9)$$

$$\mathrm{y} = [1 \quad 0 \quad 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \qquad (10)$$

The necessary and sufficient condition for the stability of the system output is its transfer function [6]:

$$\mathbf{W}(\mathrm{s}) = \mathrm{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}b \qquad (11)$$

The poles are all located in the left half plane of s.

$$\mathbf{W}(\mathrm{s}) = \mathrm{c}(s\boldsymbol{I} - \boldsymbol{A})^{-1}b$$

$$= [1 \quad 0]\left\{s\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 1\\ 0 & -\frac{1}{T}\end{bmatrix}\right\}^{-1}\begin{bmatrix}0\\ \frac{K}{T}\end{bmatrix}$$

$$= \begin{bmatrix}\frac{1}{S} & \frac{1}{S(S+\frac{1}{T})}\end{bmatrix}\begin{bmatrix}0\\ \frac{K}{T}\end{bmatrix}$$

$$= \frac{K/T}{S(S+\frac{1}{T})}$$

he system pole is $s_1 = 0$, $s_2 = -\frac{1}{T}$, indicating that the system has critical output stability, and the output stability of the system is determined by the control input.

Integrate on both sides of equation (7) on $[0 \quad t_f]$:

$$\frac{T_E * T}{K}\ddot{\varphi}\Big|_0^{t_f} + \frac{T_E + T}{K}\dot{\varphi}\Big|_0^{t_f} + \frac{1}{K}\varphi\Big|_0^{t_f} = \int_0^{t_f}\delta_E\,dt \; (12)$$

Let the initial condition be 0, that is, $\ddot{\varphi_0} = 0$, $\dot{\varphi_0} = 0$, $\varphi_0 = 0$, $\delta_0 = 0$; when the output is stable $\varphi_{t_f} = \varphi$, $\dot{\varphi}_{t_f} = 0$, $\ddot{\varphi}_{t_f} = 0$.

Then the above formula can be turned into:

$$\varphi = K\int_0^{t_f}\delta_E\,dt \qquad (13)$$

Equation (13) is the relationship between the alteration angle and the rudder angle, where δ_E is the command rudder angle and is based on the centerline rudder; the rudder angle δ_r in equation (1) is used to resist the environmental interference, δ_E is equivalent to δ_m in equation (1); it can be considered that the driving rudder angle δ of equation (1) is divided into two parts, one for steering and one for resistance to wind. The difficulty in calculating according to this understanding lies in the uncertainty of the rudder angle δ_r during the steering process.

There is another understanding of equation (13). It is considered that δ_E is the driving rudder angle based on δ_r. The steering process is from steering a nominal balance state $[\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\varphi}, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{r}]^{\mathrm{T}} = [0, 0, \varphi_0, \boldsymbol{u}, \boldsymbol{v}, 0]^{\mathrm{T}}$ shifts to another nominal balance state $[\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\varphi}, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{r}]^{\mathrm{T}} = [0, 0, \boldsymbol{\varphi_1}, \boldsymbol{u}, \boldsymbol{v}, 0]^{\mathrm{T}}$. In this process, the rudder angle is also changed from δ_r to δ_{r1}. The difficulty in calculating according to this understanding lies in the prediction of the new rudder angle δ_{r1}.

III.SHIP STEERING DESIGN

The steering design of this paper is to reduce the track deviation after steering as much as possible, so that the subsequent straight track-holding controller can make the ship better and faster to stabilize on the new planned route, thus allowing a certain amount of error. The following steering of the ship is designed according to the second understanding of the formula (13). The rudder angle of the new course can be calculated according to the leeway and drift angle of the new course, that is $\delta_{r1} = \mathrm{f}(\mathrm{t})\,\varphi_1$, (this article does not discuss the specific relationship between the two, and considers their relationship to be known).

Considering that the steering rudder angle is generally not large (within 10°) during ocean navigation, the steering design can be designed according to the linear response model, and the change of the longitudinal sip speed of the ship can be ignored, which is regarded as a constant. And the limit of steering speed can also be ignored, which is considered to be step steering [7].

Regulation: Right rudder is positive and left rudder is negative.

Assuming steering twice, the steering angle is $5°(0 < t < t_1) \cup -5°(t_1 < t < t_f)$.

Then

$$\varphi = K\int_0^{t_1} 5\,dt + K\int_{t_1}^{t_f} (-5)\,dt$$

$$\varphi = K\int_0^{2t_1 - t_f} 5\,dt$$

From the above example, it can be seen that the design of the steering of the ship requires the determination of the steering rudder angle and the corresponding steering time and steering timing.

In order to show this design idea simply and more conveniently and intuitively, the following is based on a real container ship data.

A. Ship Information

Table 1 ship information

Type	Δ	L_{PP}	B	d_m	U	C_B	K'	T'
Con	33200	245	32.2	7.9	29.0	0.520	1.70	2.10

In the table, Δ is the displacement; L_{PP} is the length between the vertical lines; B is the type width; d_m is the average draft; Δd is the difference in draft; ***U*** is the ship speed; C_B is the square coefficient; K' 、 T' is the non-dimensional maneuverability index[8].

Dimensioning the K' 、T' dimensionless maneuverability index[9]:

$$K = \left(\frac{U}{L}\right)K', T = \left(\frac{L}{U}\right)T' \qquad (14)$$

K =0.201，T =17.741.

B. *Ship Motion Balance under Wind and Flow Conditions*

In the case of wind & flow conditions, the ship must have a corresponding rudder angle δ_r in order to maintain the planned route to resist the leeway and drift . Therefore, when sailing in a straight line, the rudder angle and the heading are not $[\delta, \varphi]^T=[0，0]^T$, but $[\delta, \varphi]^T=[\delta_r，\varphi_0]^T$, which is the dynamic balance of the ship during straight-line navigation.

The task to be completed in the steering process is to transition the ship from a dynamic balance state to another dynamic balance state, as shown in Figure 1, that is, from $[\delta, \varphi]^T=[\delta_r，\varphi_0]^T$ to $[\delta, \varphi]^T=[\delta_{r1}，\varphi_1]^T$.

C. Steering Design

According to the analysis of the dynamic balance state of the ship under wind & flow conditions , the corresponding formula (13) can be rewritten as

$$\varphi + \varphi_1 - \varphi_0 = K\int_0^{t_f} \delta_E\,dt \qquad (15)$$

The steering amount during the steering process is shown in Figure 2. The steering amount refers to the integral of the steering rudder angle with respect to the time when the heading is stabilized on the new course [10].

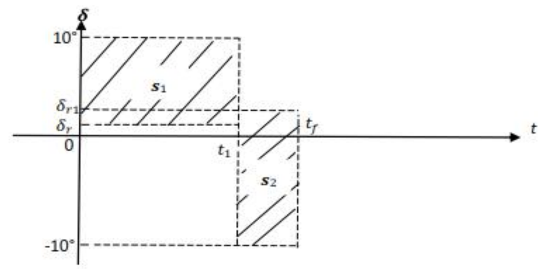


Figure 2. steering amount

In Figure 2, the steering speed limit is ignored, which is considered to be step steering, the rudder angle from δ_r step to δ_E at 0 o'clock; the rudder angle from δ_E step to $-\delta_E$ at t_1 time; the rudder angle from $-\delta_E$ step to δ_{r1} at t_f time; The right rudder is positive, the left udder is negative; δ_E is taken as 10°; the rudder angles δ_r and δ_{r1} can be positive, can be negative, can be the same number, or can be different.

$$S_1 = (\delta_E - \delta_r) \times (t_1 - 0) \qquad (16)$$

$$S_2 = |(\delta_E - \delta_{r1})| \times (t_f - t_1) \qquad (17)$$

Then $\varphi + \varphi_1 - \varphi_0 = K(S_1 - S_2)$ (18)

The steering timing (active lag distance), steering rudder angle and steering time can be determined according to equations (2) and (18).

The following table is calculated according to the data of the actual ship according to Section A, wherein the steering rudder angle (command rudder angle) δ_E is taken as 10° (other values may be taken, but not too large according to actual conditions), φ is the alteration angle.

Table 2.example of ship steering design

φ	φ_0	φ_1	δ_E	δ_r	δ_{r1}	F (m)	t_1 (s)	t_f (s)
20°	3°	5°	10°	2°	3°	264	20	23.8
30°	3°	5°	10°	2°	3°	264	25	28.1
30°	5°	3°	10°	3°	2°	264	25	27.9
40°	3°	5°	10°	2°	3°	264	30	32.3
50°	3°	5°	10°	2°	3°	264	35	36.5
60°	3°	5°	10°	2°	3°	264	40	41
70°	3°	5°	10°	2°	3°	264	45	45
80°	3°	5°	10°	2°	3°	264	55	57.3
90°	3°	5°	10°	2°	3°	264	60	61.5

The table lists the lag, steering rudder angle and steering time from 20° to 90° at different alteration angles. Reflected in the steering operation is the change of the steering rudder angle (δ_E) or the steering time (t_1、 t_f). In other words, the deviation of the track after the steering can be made zero by adjusting the steering rudder angle and the steering time for different wind and flow conditions.

In order to verify the practical feasibility of the above calculation data, this paper will use Simulink in MATLAB for simulation verification.

IV. SIMULATION VERIFICATION AND ANALYSIS

Considering the actual economic conditions, it is not allowed to carry out the actual ship or real ship model test verification. This paper uses the Simulink simulation environment in MATLAB to verify and analyze. The verification uses the following equation of motion [11]:

$$\begin{cases} \dot{x} = U\cos\varphi + u_C\cos\varphi_C \\ \dot{y} = U\sin\varphi + u_C\sin\varphi_C \\ \dot{\varphi} = r \end{cases} \qquad (19)$$

When the alteration angle is 30°, the simulation diagram is as follows:

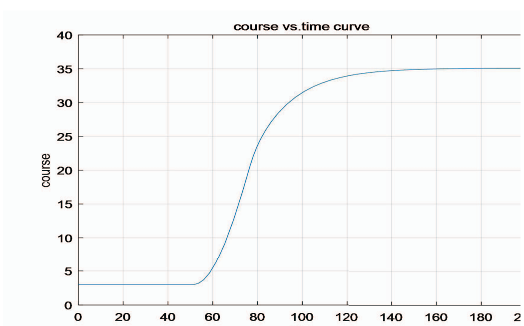


Figure 3. Course vs. time curve

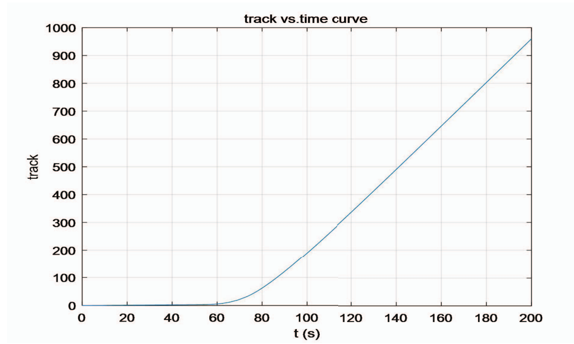


Figure 4. Track vs. time curve

Figure 3 shows the course of the course with time, and Figure 4 shows the curve of the track with time. Due to the influence of the wind and flow, there is a small yaw angle φ_0=3° in the initial course line, and a yaw angle φ_1=5° in the new course line after the steering; There is no track deviation under the rudder angle δ_r in the initial course line, and there is no track deviation due to the presence of the rudder angle δ_{r1} after steering (this conclusion is based on the ideal state, in reality due to the estimation error of the leeway & drift angle and the rudder angle is unlikely to occur when the track deviation is zero).

V. CONCLUSIOUS AND PROSPECTS

Both theoretical calculations and simulation analysis verify the practical operability of the ship steering design described above. In the ideal state, the track deviation after steering operation can be zero, which is beneficial to the following straight track keeping control; the ideal state is unlikely to exist due to the prediction error of the leeway & drift angle and the rudder angle, but can it is guaranteed that the heading is stable at a certain value, that is, the turning rate is zero, which provides a larger choice for the subsequent straight track keeping control design. The next task is to find the relationship between the rudder angle and the leeway and drift angle and the ship type (multiple regression analysis can be used to summarize the general formula if necessary), and to design and develop joint track keeping controller based on this steering design.

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