Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-23 (2016) 532–539

Modeling and Course Control of Sailboats

Kristian L. Wille * **Vahid Hassani** *,**,[1] **Florian Sprenger** **

* *Department of Marine Technology, Norwegian university of science and technology (NTNU), Trondheim, Norway*
** *The Norwegian Marine Technology Research Institute (MARINTEK), Trondheim, Norway*

Abstract: This paper proposes a method of transforming a heading controller into a course controller in sailboats by adding a small correction term. Sailboats usually have large drift angles because of considerable side forces, mainly caused by the sail. Even small deviations from the correct course angle can cause large errors, and thus course controllers in sailboats plays an essential role. The solution is based on the dynamics of the system, and the model presented builds upon previous work and adds detail in areas such as how to model wind, current and drag. The solution has roots in the fact that the purpose of the keel is to create a side force that counterbalances the unwanted drift forces. Simulation results show the effectiveness of the proposed approach.

Keywords: Course Control, Sailboat, Modeling.

1. INTRODUCTION

The history of sailboats goes back to one of the earliest civilizations of mankind in Mesopotamia where the Tigris and Euphrates rivers inspired the development of many watercrafts, including sailboats, see Carter (2012). However, autonomy in sailboats is a topic that has gained attention only in recent years, see Xiao and Jouffroy (2014).

Sailboats require little to no energy to operate, making them well suited for long operations such as oceanographic research. Moreover, with the increased focus on clean energy, sailboats could also provide a green option for transporting goods, see Michael et al. (2014). The propulsion force of the sailboat is created by the wind, and the only energy needed to operate the boat is by trimming the sail and controlling the rudder. Fitting the sailboat with solar panels or extracting energy directly from the sail itself makes them very much self sustainable in terms of energy, see Jaulin and Le Bars (2014).

For a motorized boat the heading angle is almost considered synonyms with the course angle of the ship, though due to wind and current forces small drift angles may appear. This problem is easily handled by an integrator term in the controller, as unwanted drift caused by wind and current can be seen as slowly changing errors. However, in sailboats it is more challenging to control the course and this is due to the fact that the wind provides the main propulsion force but at the same time causes large heel and drift angles. The side forces are especially large when sailing closed hauled or reaching.

Solving the course control problem by an integrator term has been done for sailboats (Saoud et al. (2015)), but it has multiple shortcomings. The drift created can be bigger than what is usually observed on a motorized boat, meaning that the integrator term would have to be large. To make the matter worse, when sailing upwind, the sailboat has to constantly do tack maneuvers. This would cause a huge error to appear after each tack, as the integrator term needs time to catch up. Furthermore, one would need to be able to estimate the drift angle, which is not an easy task in sailboats.

Xiao and Jouffroy (2014) have addressed the problem by using a nonlinear system solver to calculate the necessary drift angles to keep a desired course, then storing the results in a lookup table. The drawback of this is that the lookup table is quite large as it depends on several variables, and would need to be recomputed every time a small change is applied to the boat.

In this paper, a simple solution based on the dynamics of the system is proposed. The advantage of this solution is that the drift angle needed to sustain a course is calculated directly, which drastically reduces the need for integral action and removes any need for a lookup table. When the necessary drift angle is known it is trivial to convert an existing heading controller into a course controller. Knowing the drift angle is also useful for purposes besides control, such as state estimation.

In what follows, before the solution of the course controller is presented, a mathematical model of a sailboat is constructed in section 2. This model has many similarities to the model proposed by Xiao and Jouffroy (2014), though some modification has been made. The theory of the course controller is tightly connected to the model, and the controller will be tested on the developed model.

2. DESCRIPTION OF SYSTEM DYNAMICS

The model presented in this paper is very similar to the work done by Xiao and Jouffroy (2014). The most noticeable differences are how drag and restoration forces are handled. In addition, wind simulation has been improved,

[1] Corresponding author, (e-mail: Vahid.Hassani@ntnu.no).

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.10.490

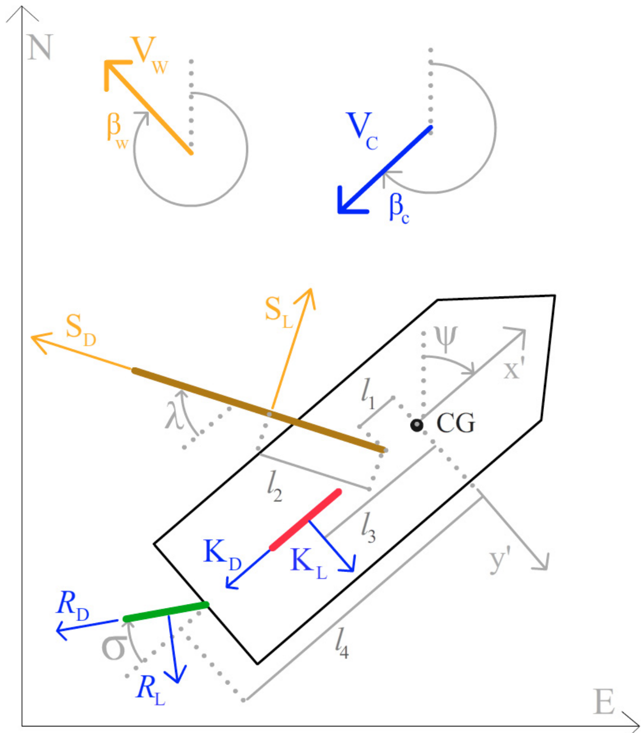

Fig. 1. Definition of positive direction of important parameters, including forces created by the sail and the keel

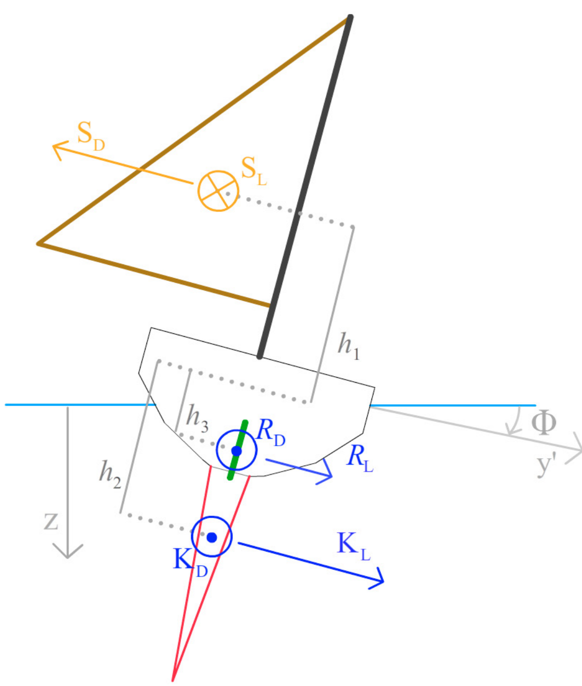

Fig. 2. Definition of positive direction of forces created by the rudder

effects due to current has been added, and a delay has been added to the actuators (rudder and sail) to make it more realistic.

Before presenting the system dynamics, description of the essential parameters in modeling the system, the direction of forces, axis and angles are presented in Figure 1 and 2 and Table 1. All lift and drag forces are shown for an angle of attack equal to zero. Throughout the paper, the notation of Society of Naval Architects & Marine Engineers (SNAME) has been adopted.

In what follows we will use two different reference frames: the north-east-down (NED) reference frame (n-frame) and the body reference frame (b-frame). The NED coordinate system will be treated as inertial, and the b-frame is connected to the body of the ship. The origin of the b-frame, CO, is set to be at the center of gravity (CG) midship and at the waterline.

2.1 System equations

Assumptions The following assumptions are used throughout the paper and are borrowed from Xiao and Jouffroy (2014), see source for a more detailed description of each assumption.

- The yacht is rigid, and movement in heave and pitch are neglected.
- Waves, and the effect caused by them (first order and second order), are not modeled.
- Added mass coefficients are modeled as constants.

Table 1. Variable description

variable	description
CG	Center of gravity
CO	Center of body frame
ρ	fluid density $[\frac{kg}{m^3}]$
g	gravity $[\frac{m}{s^2}]$
R_n	Reynolds number
β	drift angle $[rad]$
χ	course angle $[rad]$
β_w, β_c	wind/current angle $[rad]$
V_w, V_c	wind/current vector $[\frac{m}{s}]$
$U(z)$	absolute wind speed $[\frac{m}{s}]$
U_{10}	absolute wind speed at $z = -10$ $[\frac{m}{s}]$
U_{10_m}	mean absolute wind speed at $z = -10$ $[\frac{m}{s}]$
λ, σ	angle of sail/rudder $[rad]$
S_L, K_L, R_L	Sail/keel/rudder lift $[N]$
S_D, K_D, R_D	Sail/keel/rudder drag $[N]$
x, y	north/east position $[m]$
x', y'	body reference frame coordinates$[m]$
u, v	linear velocity $[\frac{m}{s}]$
u_r, v_r	linear velocity relative to current $[\frac{m}{s}]$
p, r	angular velocity in roll/yaw $[\frac{rad}{s}]$
ψ, ϕ	roll/yaw angle $[rad]$
M_{RB}, M_A	rigid/added mass matrix
$C_{RB}(\nu), C_A(\nu_r)$	rigid/added mass Coriolis-centripetal matrix
$D(\nu_r)$	drag matrix
$g(\eta)$	restoration forces
S, K, R	Forces/moments by sail/keel/rudder $[N/Nm]$
$J(\theta, \psi)$	transformation matrix from b- to n-frame
V_{ws}, V_{ck}, V_{cr}	motion of fluid relative to sail/keel/rudder $[\frac{m}{s}]$
$\beta_{ws}, \beta_{ck}, \beta_{cr}$	angle of fluid relative to sail/keel/rudder $[rad]$
$\alpha_s, \alpha_k, \alpha_r$	angle of attack of sail/keel/rudder $[rad]$
$C_L(\alpha)$,$C_D(\alpha)$	lift and drag coefficient of foil $[-]$
A_s, A_k, A_r	area of sail/keel/rudder $[m^2]$
Asp_k, Asp_r	aspect ratio of keel/rudder $[-]$
m	mass of sailboat $[kg]$
I_{xx}, I_{zz}	Inertia of sailboat about x/z axis $[kgm^2]$
$X_{\dot{u}}$, $Y_{\dot{v}}$, $K_{\dot{p}}$, $N_{\dot{r}}$	added mass in $x'/y'/\phi/\psi$ $[kg\ /\ kgm^2]$
L_{WL}	waterline length $[m]$
B_{F0}, B_L	drag coefficients in roll (friction/lift) $[-/\frac{kg}{rad}]$
S_H	wetted surface of hull $[m^2]$
Δ	displacement $[m^3]$

- The effects from the sail, keel, rudder and hull are computed independently of each other. As a result, effects caused by interactions between two (or more) of these are not modeled.
- The sail, keel and rudder are modeled as (rigid) foils. The lift and drag forces of a foil are due to the integration of pressure around the foil, though for simplification, the resultant force will be applied to a single point on the foil known as the aerodynamic center of effort.

Vectorial representation The equations of the system are based on the vectorial representation proposed by Fossen (2011). A 4DOF [2] model has been chosen because of the big roll motions observed on sailboats. Thus the states of the system become the following:

$$\eta = \begin{bmatrix} x \\ y \\ \phi \\ \psi \end{bmatrix}, \quad (1) \qquad \nu = \begin{bmatrix} u \\ v \\ p \\ r \end{bmatrix}. \quad (2)$$

The vectorial representation of the system is

$$\begin{gathered} \dot{\eta} = J(\phi,\psi)\nu \\ M_{RB}\dot{\nu} + C_{RB}(\nu)\nu + M_A\dot{\nu_r} + C_A(\nu_r)\nu_r + D(\nu_r) + g(\eta) = \\ S(\eta,\nu,\lambda,V_w) + K(\eta,\nu,V_c) + R(\eta,\nu,V_c,\sigma), \end{gathered} \tag{3}$$

where $J(\phi,\psi)$ is the transformation matrix

$$J(\phi,\psi) = \begin{bmatrix} cos(\psi) & -sin(\psi)cos(\phi) & 0 & 0 \\ sin(\psi) & cos(\psi)cos(\phi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & cos(\phi) \end{bmatrix} \tag{4}$$

and $\nu_r = [u_r \ v_r \ p \ r]^T$ is the speed of the ship relative to the water

$$\begin{bmatrix} u_r \\ v_r \end{bmatrix} = \begin{bmatrix} u \\ v \end{bmatrix} - J_{2D}^T(\psi,\phi)V_c, \tag{5}$$

where $J_{2D}(\psi,\phi)$ is

$$J_{2D}(\psi,\phi) = \begin{bmatrix} cos(\psi) & -sin(\psi)cos(\phi) \\ sin(\psi) & cos(\psi)cos(\phi) \end{bmatrix}. \tag{6}$$

Assuming $\dot{V}_c = 0$ it follows that $\dot{\nu_r} = \dot{\nu}$. M_{RB}, $C_{RB}(\nu)$, M_A, $C_A(\nu_r)$, $D(\nu_r)$, $g(\nu)$, $S(\eta,\nu,\lambda,V_w)$, $K(\eta,\nu,V_c)$ and $R(\eta,\nu,V_c,\sigma)$ are described in Table 1. $S(\eta,\nu,\lambda,V_w)$, $K(\eta,\nu,V_c)$ and $R(\eta,\nu,V_c,\sigma)$ are dependent on several different states of the system, and will from now on be written as S, K and R for readability.

Mass and Coriolis Matrices M_{RB} can be expressed as

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & 0 \\ 0 & 0 & I_{xx} & -I_{xz} \\ 0 & 0 & -I_{xz} & I_{zz} \end{bmatrix}. \tag{7}$$

[2] Degrees Of Freedom

Using the maneuvering theory presented in Fossen (2011) and assuming the sailboat to be symmetrical about the $x'z'$-plane and that $M_{A_{ij}} = M_{A_{ji}}$ gives

$$M_A = -\begin{bmatrix} X_{\dot{u}} & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & Y_{\dot{p}} & Y_{\dot{r}} \\ 0 & Y_{\dot{p}} & K_{\dot{p}} & K_{\dot{r}} \\ 0 & Y_{\dot{r}} & K_{\dot{r}} & N_{\dot{r}} \end{bmatrix}. \tag{8}$$

The rigid body Coriolis matrix can be expressed as

$$C_{RB}(\nu) = \begin{bmatrix} 0 & -mr & 0 & 0 \\ mr & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{9}$$

and the added mass Coriolis matrix, which causes the destabilizing Munk moment $(Y_{\dot{v}} - X_{\dot{u}})u_r v_r$, as

$$C_A(\nu_r) = \begin{bmatrix} 0 & 0 & 0 & Y_{\dot{v}}v_r + Y_{\dot{p}}p + Y_{\dot{r}}r \\ 0 & 0 & 0 & -X_{\dot{u}}u_r \\ 0 & 0 & 0 & 0 \\ -Y_{\dot{v}}v_r - Y_{\dot{p}}p - Y_{\dot{r}}r & X_{\dot{u}}u_r & 0 & 0 \end{bmatrix}. \tag{10}$$

For controller design purposes the smaller off-diagonal terms are neglected in M_{RB} and M_A. This gives $M_{RB} = diag\{m,\ m,\ I_{xx},\ I_{zz}\}$, $M_A = -diag\{X_{\dot{u}},\ Y_{\dot{v}},\ K_{\dot{p}},\ N_{\dot{r}}\}$ and

$$C_A(\nu_r) = \begin{bmatrix} 0 & 0 & 0 & Y_{\dot{v}}v_r \\ 0 & 0 & 0 & -X_{\dot{u}}u_r \\ 0 & 0 & 0 & 0 \\ -Y_{\dot{v}}v_r & X_{\dot{u}}u_r & 0 & 0 \end{bmatrix}. \tag{11}$$

Damping The damping matrix, $D(\nu_r)$, represents the damping caused by the hull of the ship (not the rudder and keel, which will be handled separately later). The damping matrix consists of a linear and a non-linear part:

$$D(\nu_r) = D_l\nu_r + D_q(\nu_r)\cdot \tag{12}$$

The linear damping is caused by potential damping and skin friction due to a laminar boundary layer, though only the latter will be considered. The potential damping is frequency dependant, and the effect is negligible at low frequencies. For estimation of these coefficients, see Fossen (2011) p. 125.

$$D_l = \begin{bmatrix} d_{l_{11}} & 0 & 0 & 0 \\ 0 & d_{l_{22}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{l_{66}} \end{bmatrix} \tag{13}$$

Nonlinear damping is caused by wave drift and vortex shedding. Waves are not modeled, and it follows that wave drift damping is not modeled either.

$$D_q(\nu_r) = [D_{q_X}(\nu_r)\ D_{q_Y}(\nu_r)\ D_{q_K}(\nu_r)\ D_{q_N}(\nu_r)]^T \tag{14}$$

The nonlinear damping in surge will be based on the flat plate friction ITTC 1957 (Lewis, 1989)

$$D_{q_X}(\nu_r) = \frac{1}{2}\rho_w S_H(1+k)C_F(R_n)|u_r|u_r, \quad (15)$$

where

$$C_F(R_n) = \frac{0.075}{(log_{10}(R_n)-2)^2}, \quad (16)$$

ρ_w is the density of sea water, S_H is the wetted surface area of the ship, k is the form factor coefficient of the hull, $C_{DV}(R_n)$ is the friction coefficient due to nonlinear viscous effects and R_n is the Reynolds number.

The nonlinear damping in sway and yaw will be modeled using the cross-flow drag principle based on strip theory (Faltinsen, 1990):

$$D_{q_Y}(\nu_r) = \frac{1}{2}\rho_w \int_L C_D(x')T(x')|v_r + x'r|(v_r + x'r)dx' \quad (17)$$

$$D_{q_N}(\nu_r) = \frac{1}{2}\rho_w \int_L C_D(x')T(x')x'|v_r + x'r|(v_r + x'r)dx', \quad (18)$$

where $T(x')$ is the draft and $C_D(x')$ is the two-dimensional drag coefficient. $C_D(x')$ can be estimated using Hoerner's curve (Hoerner, 1965).

The roll damping will be modeled by a friction and a lift damping component

$$D_{q_K}(\nu_r) = B_{F0}(L_{WL} + 4.1\frac{u_r}{\omega_{roll}L_{WL}})p_r + B_L p_r u_r, \quad (19)$$

where B_{F0} is the linear friction coefficient at zero speed, B_L is the lift damping coefficient and L_{WL} is the water line length. ω_{roll} is the frequency of the motion in roll, but this will be simplified by using the natural frequency, ω_{roll_n}.

Eddy damping, caused by flow separation at the bottom of the ship hull due to roll motion, has not been included as it is negligible when the Froude number is larger than 0.2. For a small to medium sized sailboat, this can safely be neglected. See Himeno (1981) for more detail on how to model roll motion.

Restoration forces The restoring forces are calculated by

$$g = \begin{bmatrix} 0 \\ 0 \\ \rho_w g \nabla GM_t sin(\phi)cos(\phi) \\ 0 \end{bmatrix}, \quad (20)$$

where ∇ is the total displacement of the ship and GM_t is the transverse metacentric height. See Fossen (2011) p. 64, for more information.

Sail, Keel and Rudder The sail, keel and rudder can all be modeled using foils. The lift and drag of a foil can be calculated using the following formulas:

$$F_L = \frac{1}{2}\rho A C_L(\alpha)V^2 \quad (21)$$

$$F_D = \frac{1}{2}\rho A C_D(\alpha)V^2, \quad (22)$$

where ρ is the density of the fluid (in this context, air or water), A is the area of the foil, V is the absolute speed of the fluid relative to the foil and $C_L(\alpha)$ and $C_D(\alpha)$ are the lift and drag coefficient which are function of the angle of attack, α. The drag and lift coefficients for the rudder are based on NACA0015, while the keel is based on NACA0009 (Sheldahl and Klimas, 1981). Lift and drag coefficients for the sail were found in C.A.Marchaj (2000) p. 587.

The lift coefficients for the rudder and keel are corrected to the new aspect ratios according to (Wagner (1948))

$$C_L(\alpha) = \frac{C_{l0}(\alpha)(1+\frac{2}{Asp_0})}{(1+\frac{2}{Asp})}, \quad (23)$$

where Asp_o is the original aspect ratio of the foil used when calculating the lift coefficient, C_{l0} is the original lift coefficient, and Asp is the aspect ratio of the foil we want to scale the results to.

The drag coefficients for the rudder and keel only account for the induced drag $C_{Di}(\alpha)$ and they do not take into account the viscous drag. The viscous drag is added to the drag coefficients

$$C_D(\alpha) = C_{Di}(\alpha) + 2(1 + 2\frac{t_{max}}{c})C_F(R_n), \quad (24)$$

where t_{max} is the thickness of the foil and c is the chord length.

The velocity of the fluid relative to a foil on the sailboat is equal to

$$V_f = J_{2D}^T(\psi,\phi)V_{w/c} - \begin{bmatrix} u \\ v \end{bmatrix} - \begin{bmatrix} -ry'_f \\ rx'_f - pz'_f \end{bmatrix} - \begin{bmatrix} \dot{x}'_f \\ \dot{y}'_f \end{bmatrix}, \quad (25)$$

where x_f, y_f and z_f are the x', y' and z' coordinates of the center of effort of the foil in relation to CO. $V_{w/c}$ is the wind/current vector. This gives us the following equation for the relative wind speed to the sail:

$$\begin{aligned} V_{ws} = \begin{bmatrix} V_{ws_u} \\ V_{ws_v} \end{bmatrix} &= J_{2D}^T(\psi,\phi)V_w - \begin{bmatrix} u \\ v \end{bmatrix} \\ - \begin{bmatrix} rl_2 sin(\lambda) \\ -r(l_1 + l_2 cos(\lambda)) + ph_1 \end{bmatrix} &- \begin{bmatrix} \dot{\lambda}l_2 sin(\lambda) \\ -\dot{\lambda}l_2 cos(\lambda) \end{bmatrix}, \end{aligned} \quad (26)$$

The angle of the wind can then be calculated as [3]

$$\beta_{ws} = arctan2(V_{ws_v}, V_{ws_u}) \quad (27)$$

and the angle of attack as

$$\alpha_s = \beta_{ws} - \lambda + \pi\cdot \quad (28)$$

S_L (lift of sail) and S_D (drag of sail) can then be calculated using formula (21), (22) and (28). Using trigonometry and the definitions of figure 1, S can be formulated by

[3] $arctan2(y,x) \in [-\pi, \pi]$ is the four-quadrant inverse tangent

$$S = \begin{bmatrix} -S_L sin(\beta_{ws}) + S_D cos(\beta_{ws}) \\ S_L cos(\beta_{ws}) + S_D sin(\beta_{ws}) \\ h_1(S_L cos(\beta_{ws}) + S_D sin(\beta_{ws})) \\ S_{x'} l_2 sin(\lambda) - S_{y'}(l_1 + l_2 cos(\lambda)) \end{bmatrix}, \tag{29}$$

where $S_{x'}$ and $S_{y'}$ are the forces from S in x' and y' direction respectively.

For the keel we follow a similar approach. As with the sail, one first have to calculate the relative speed of the water to the ship:

$$V_{ck} = \begin{bmatrix} V_{ck_u} \\ V_{ck_v} \end{bmatrix} = -\begin{bmatrix} u_r \\ v_r \end{bmatrix} - \begin{bmatrix} 0 \\ -rl_3 - ph_2 \end{bmatrix} \cdot \tag{30}$$

The angle of the relative speed of the water passing by can be calculated as

$$\beta_{ck} = arctan2(V_{ck_v}, V_{ck_u}) \cdot \tag{31}$$

Then, we find the angle of attack using

$$\alpha_k = -\beta_{ck} + \pi \cdot \tag{32}$$

Notice that the angle of attack is negative of β_{cr}. This is due to the definition of direction of forces (see figure 1).

We can now calculate K_L and K_D using (21), (22) and (32). This gives K as:

$$K = \begin{bmatrix} K_L sin(\beta_{ck}) + K_D cos(\beta_{ck}) \\ -K_L cos(\beta_{ck}) + K_D sin(\beta_{ck}) \\ h_2(K_L cos(\beta_{ck}) - K_D sin(\beta_{ck})) \\ l_3(K_L cos(\beta_{ck}) - K_D sin(\beta_{ck})) \end{bmatrix} \cdot \tag{33}$$

The equation of the water speed relative to the rudder is

$$V_{cr} = \begin{bmatrix} V_{cr_u} \\ V_{cr_v} \end{bmatrix} = -\begin{bmatrix} u_r \\ v_r \end{bmatrix} - \begin{bmatrix} 0 \\ -rl_4 - ph_3 \end{bmatrix} \cdot \tag{34}$$

Calculating the angle of the relative speed of the water in relation to the rudder, β_{cr}, is accomplished by using (31) and inserting V_{cr} for V_{ck}. The angle of attack of the rudder can then be found by

$$\alpha_r = -\beta_{cr} + \sigma + \pi \cdot \tag{35}$$

R is then found in the same manner as S and K:

$$R = \begin{bmatrix} R_L sin(\beta_{cr}) + R_D cos(\beta_{cr}) \\ -R_L cos(\beta_{cr}) + R_D sin(\beta_{cr}) \\ h_3(R_L cos(\beta_{cr}) - R_D sin(\beta_{cr})) \\ l_4(R_L cos(\beta_{cr}) - R_D sin(\beta_{cr})) \end{bmatrix} \cdot \tag{36}$$

Actuators The two actuators, the sail and rudder, have physical limitations. The rudder can not move or accelerate infinitely fast and the maximum angle is restricted. In the model this is accomplished by enforcing $|\sigma| \leq \sigma_{sat}$ and by implementing a first order low-pass filter with the time constant T_σ, effectively causing a delay when moving the rudder.

The sail follows similar limitations. $|\lambda| \leq \lambda_{sat}$ and the movement is modeled by a first order low-pass filter with the time constant T_λ. It should be noted that in reality the sail is moved by the wind, and the only means of controlling it is by restricting the maximum angle of the sail by controlling the length of a rope that is connected to the boom. While the model presented in this paper do not capture the full dynamics of the boom/sail it should give a decent approximation for small changes in λ_d (the desired angle of the sail). The value of T_λ is difficult to set due to no such actuator existing, and the value used in this paper is chosen as something the authors thought would be reasonable.

Wind The wind is simulated by the sum of a mean component and a gust component. The mean wind speed at 10 meter above the sea level will be referred to as $U_{m_{10}}$. The wind gust, $U_g(z)$, is then estimated using the following wave spectra (NORSOK standard, 2007):

$$S(f,z) = 320 \frac{(\frac{U_{m_{10}}}{10})^2 (\frac{-z}{10})^{0.45}}{(1 + x(z)^n)^{\frac{5}{3n}}} \tag{37}$$

$$x(z) = 172 f \left(\frac{-z}{10}\right)^{\frac{2}{3}} \left(\frac{U_{m_{10}}}{10}\right)^{-0.75} \tag{38}$$

$$\delta = \sqrt[2]{2S(f)\Delta f} cos(2\pi f t + \Upsilon), \tag{39}$$

where f is the frequency of the gust, δ is the amplitude of the gust, t is the time, z is the height above sea level (n-frame), n is a constant equal to 0.468 and Υ is an evenly distributed phase for each f. Frequencies between 0 and 0.4 Hz will be simulated as this covers most of the energy in the spectrum. Υ is not changed between simulations, this is to ensure easier comparisons between different simulation results. The mean wind speed has to be corrected for being lower when closer to the sea level:

$$\begin{gathered} U_m(z) = U_{m_{10}} \frac{5}{2} \sqrt{k} \log \frac{z}{z_0} \\ z_0 = 10 e^{-\frac{2}{5\sqrt{k}}}, \end{gathered} \tag{40}$$

where $U_m(z)$ is the mean wind speed at height z and $k = 0.0026$. The total wind speed is then given by

$$U(z) = U_m(z) + U_g(z) \cdot \tag{41}$$

The center of effort of the sail is located at $z = -h_1 cos(\phi)$, which gives V_w as

$$V_w = U\big(-h_1 cos(\phi)\big) \begin{bmatrix} cos(\beta_w) \\ sin(\beta_w) \end{bmatrix} \cdot \tag{42}$$

In addition to the gust, the wind direction will fluctuate, modeled by

$$\begin{gathered} \beta_w = \beta_{w_m} + \beta_{w_{flux}} \\ \dot{\beta}_{w_{flux}} = w, \end{gathered} \tag{43}$$

where β_{w_m} is the average wind direction and w is white noise. $\beta_{w_{flux}}$ is saturated at 5 degrees.

2.2 Parameter estimation

The parameters are based on a sailboat that has a water line length of 8.8 meters and a displacement of 1600kg. See appendix A for the values used in the simulator.

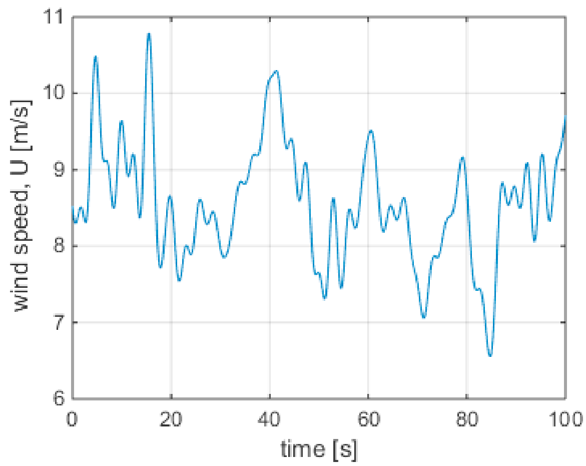

Fig. 3. Wind speed simulated over 100 seconds at height h_1 ($5.2m$) and $U_{m_{10}} = 10\frac{m}{s}$

3. COURSE CONTROLLER

3.1 Heading controller

The course angle, χ, is defined by

$$\chi = \psi + \beta, \tag{44}$$

where β is the drift angle

$$\beta = arctan2(v\ cos(\phi), u)\cdot \tag{45}$$

The heading controller will try to minimize the following error terms:

$$e_1 = \psi - \psi_d, \quad (46) \qquad e_2 = r, \quad (47)$$

where ψ_d is the desired heading angle. When a heading controller is used it is assumed that $\beta \approx 0$ which makes $\chi \approx \psi$.

The controller design method chosen is state feedback linearization (Freund, 1973; Isidori, 1989). The heading controller presented is not meant to be an applicable controller in a real system but rather as a facilitator to show how the course correction term will work. This is because the controller relies on difficult to estimate states such as ν_r and require a very accurate model of the system to function properly. However, it is easy to verify that by using the following control law all the nonlinearities in the yaw (heading) sub-dynamics are cancelled and that the linear system can easily be controlled by proper selection of the control parameters [4]:

$$R_{\psi_d} = -C_\psi(S + K - C_A(\nu_r)\nu_r) - K_p e_1 - K_d e_2, \tag{48}$$

where R_{ψ_d} is the desired rudder moment and C_ψ is a matrix defined as $C_\psi = [0\ 0\ 0\ 1]^T$. K_p and K_d are our control parameters. Using (36) it is possible to calculate the optimal σ to create the moment desired by the controller. This is done by linearizing the lift equation (21) around zero degrees and neglecting drag, which is a good approximation for small angles of attack (less than 10-15 degrees).

The linearized lift can be written as

$$F_{L0} = \frac{1}{2}pAC_{L0}\alpha V^2\cdot \tag{49}$$

C_{L0_r} and C_{L0_k} are the linearized lift coefficients for the rudder and keel respectively. By neglecting drag in (36) and linearizing the lift we have

$$\begin{aligned} R_{\psi_d} &= l_4 R_L cos(\beta_{B_c r}) \\ &= l_4\frac{1}{2}\rho_w A_r C_{L0_r}\alpha_r V_{cr}^2 cos(\beta_{cr}) \\ &= l_4\frac{1}{2}\rho_w A_r C_{L0_r}(-\beta_{cr} + \sigma + \pi)V_{cr}^2 cos(\beta_{cr}) \end{aligned} \tag{50}$$

from which it follows

$$\sigma = \beta_{cr} - \pi + \frac{2R_{\psi_d}}{l_4\rho_w A_r C_{L0_r} V_{cr}^2 cos(\beta_{cr})}\cdot \tag{51}$$

3.2 Updating a heading controller to a course controller

The heading controller can be turned into a course controller by adding a small correction term in (46):

$$\begin{aligned} e_1 &= \psi - \psi_d \\ &= \psi - (\chi_d - \beta_b) \\ &= \psi + \beta_b - \chi_d, \end{aligned} \tag{52}$$

where χ_d is the desired course angle and β_b is the necessary drift angle which is needed to achieve the desired course angle, also referred to as the correction term in this paper.

3.3 Calculation of correction term

On a sailboat, the keel causes hydrodynamic forces that balance out undesirable forces. The undesirable forces are all the forces that cause the ship to move perpendicular to the course angle. In the body frame these are all the forces created perpendicular to the drift angle.

By assuming $\nu_r \approx \nu$ and $p = r \approx 0$ one can simplify (30) to

$$V_{ck} = -\begin{bmatrix} u \\ v \end{bmatrix}\cdot \tag{53}$$

Using (45), the assumption $v \ll u$ (low drift angle), and the approximation $tan(x) = x$ for small x, it follows that β_{ck} (31) can be computed by

$$\beta_{ck} = arctan2(V_{ck_v}, V_{ck_u}) = \frac{\beta}{cos(\phi)} + \pi \tag{54}$$

and

[4] All nonlinearities have been canceled except for damping, which is considered a "good" nonlinearity (Fossen (2011), p. 457).

$$\alpha_k = -\beta_{ck} + \pi = -\frac{\beta}{cos(\phi)}. \tag{55}$$

Assuming $cos(\phi) \approx 1$ and by applying the result from (54) into (33), one finds that the lift of the keel is approximately perpendicular to the drift angle. It then follows that the lift force from the keel is aligned with the undesirable forces because they are also perpendicular to the drift angle.

Defining the undesirable force, hereafter referred to as F_U, as -90 degrees to the drift angle causes the lift force generated by the keel and the unwanted forces to be aligned, but in opposite direction. Neither the restoring force or the Coriolis terms (assuming $r \approx 0$) contribute to the undesirable force. Putting it together we find

$$\begin{aligned} F_U =&(S_{x'} + R_{x'} - D_{x'}(\nu_r)cos(\beta - \frac{\pi}{2}) \\ &+(S_{y'} + R_{y'} - D_{y'}(\nu_r))sin(\beta - \frac{\pi}{2}), \end{aligned} \tag{56}$$

where subscripts x' and y' means that the force in x' or y' direction is being evaluated. Assuming a low drift angle this can be simplified to just the forces in negative y'-direction as $cos(-\frac{\pi}{2}) \approx 0$ and $sin(-\frac{\pi}{2}) \approx -1$. Furthermore, when the drift angle is low it follows that v is small, which implies $D_{y'}(\nu_r) \approx 0$. This gives the following simplified solution for F_U:

$$F_U \approx -S_{y'} - R_{y'}. \tag{57}$$

The lift caused by the keel should be equal to the undesirable force. Linearizing the lift formula (as was done for the rudder) the necessary angle of attack is

$$\begin{aligned} F_U &= K_L \\ &= \frac{1}{2}\rho_w A_k C_{L0_k}\alpha_k \,[u\ \ v]\,[u\ \ v]^T \\ \Rightarrow \alpha_k &= \frac{2F_U}{\rho_w A_k C_{L0_k}(u^2+v^2)}. \end{aligned} \tag{58}$$

From (55) it follows that

$$\beta_b = -\alpha_k cos(\phi). \tag{59}$$

However, it is necessary to constrain β_b due do practical reasons. If the speed of the ship is low then $u^2 + v^2$ will go towards zero and β_b will go towards infinity. By constraining the solution to be within ± 10 degrees, which has to be done anyway due to the limited range of which (49) is valid, the problem is solved. An exception should also be put in place to avoid any numerical errors when $u^2 + v^2 = 0$.

4. SIMULATION AND RESULTS

Two sets of simulations were executed. One with the heading controller ($\beta_b = 0$) and one with the course controller. In both simulations the sailboat tries to keep a steady course angle of 0 degrees (north) when the wind speed is $U_{m_{10}} = 5\frac{m}{s}$ and the direction of the wind is equal to $\beta_{w_m} = 130deg$. This is known as sailing close hauled. There is no current. The simulations lasted 200 seconds. The sail was controlled using the controller developed in Wille et al. (2016).

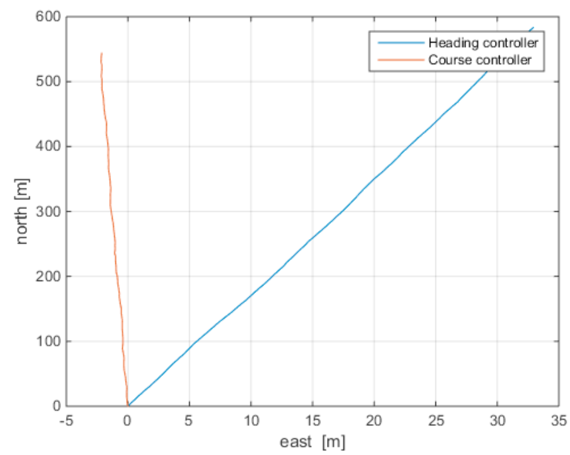

Fig. 4. position plot, heading and course controller

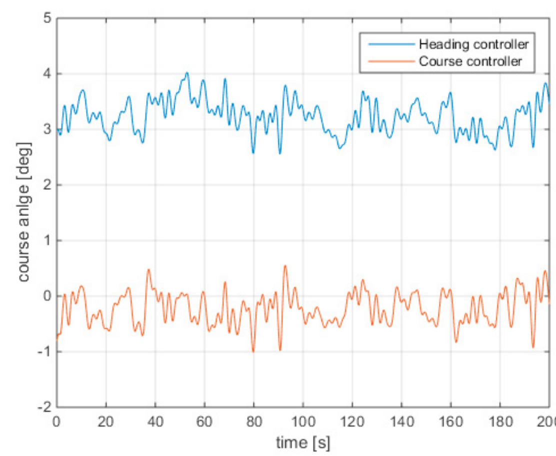

Fig. 5. Course plot, heading and course controller

The overall drift from the desired course can be estimated by figure 4. To compute an estimate of the error, $|atan(\frac{x}{y})|$ is used, where x is the distance traveled in north direction and y is the distance traveled in east direction. See Table 2 for results.

Table 2. Drift

simulation	x	y	error
heading controller	$32.9m$	$583.1m$	$3.23deg$
course controller	$-2.15m$	$543.7m$	$0.22deg$

Comparing with figure 5, the estimate of the errors seems reasonable. The course controller managed to follow the desired course much better than the heading controller, reducing the error of roughly 93%. Looking at the results one can conclude that the course controller behaved as intended.

Still, there is a small error using the course controller. The correction term is slightly bigger than it should be, likely caused by neglecting the drag term in (56). The drag term was neglected partly because it is difficult to estimate v. However, the absolute speed of the ship $(u^2 + v^2)$ can be estimated by GPS, and v can then be calculated using the approximation of the drift given by the correction term.

REFERENCES

C.A.Marchaj (2000). *Aero-Hydrodynamics of Sailing*. Tiller, 3rd edition.

Carter, R. (2012). *A Companion to the Archaeology of the Ancient Near East*, chapter Watercraft. John Wiley & Sons.

Faltinsen, O.M. (1990). *Sea Loads on Ships and Offshore Structures*, chapter Current and Wind Loads. Cambridge University Press.

Fossen, T.I. (2011). *Handbook of Marine Craft Hydrodynamics and Motion Control*. John Wiley & Sons.

Freund, E. (1973). *Decoupling and Pole Assignment in Nonlinear Systems*. Electronics Letter.

Himeno, Y. (1981). *Prediction of Ship Roll Damping- State of the Art*. Report of Department of Naval Architecture & Marine Engineering, University of Michigan, No. 239.

Hoerner, S.F. (1965). *Fluid Dynamic Drag*. Hartford House.

Isidori, A. (1989). *Nonlinear Control Systems*. Springer-Verlag.

Jaulin, L. and Le Bars, F. (2014). Sailboat as a windmill. In L.F. Bars and L. Jaulin (eds.), *IRSC 2013 - 6th International Robotic Sailing Conference*, pp. 81–92. Springer, Brest, France.

Lewis, E.V. (1989). *Principles of Naval Architecture*. Society of Naval Architects and Marine Engineers (SNAME), 2nd edition.

Michael, T., Gilbert, P., Walsh, C., Bows, A., Filippone, A., Stansby, P., and Wood, R. (2014). Propulsive power contribution of a kite and a flettner rotor on selected shipping routes. *Applied Energy*, vol. 113, pp. 362–372.

NORSOK standard (2007). *Actions and action effects (N-003)*. Standards Norway, 2nd edition.

Saoud, H., Hua, M.D., Plumet, F., and Amar, F.B. (2015). Routing and course control of an autonomous sailboat. In *European Conference on Mobile Robots 2015*, pp. 334–339. Lincoln, UK.

Sheldahl, R.E. and Klimas, P.C. (1981). *Aerodynamic Characteristics of Seven Symmetrical Airfoil Sections Through 180-Degree Angle of Attack for use in Aerodynamic Analysis of Vertical Axis Wind Turbines*. Sandia National Laboratories.

Wagner, H. (1948). *Planing of Watercraft*. NACA Technical Report, No. 1139. Langley Research Center.

Wille, K.L., Hassani, V., and Sprenger, F. (2016). Roll stabilization control of sailboats. 10th IFAC Conference on Control Applications in Marine Systems, Trondheim, Norway.

Xiao, L. and Jouffroy, J. (2014). Modeling and nonlinear heading control of sailing yachts. *IEEE Journal of Oceanic engineering*, Vol. 39(2), pp. 256–268.

Appendix A. BOAT COEFFICIENTS

coefficient	value
p_w	1025 $\frac{kg}{m^3}$
p_a	1.23 $\frac{kg}{m^3}$
h_1	5.2 m
h_2	0.95 m
h_3	0.7 m
l_1	-1.82 m
l_2	1.35 m
l_3	-0.66 m
l_4	3.7 m
A_s	22.8 m^2
A_k	0.93 m^2
A_r	0.30 m^2
Asp_k	1.75
Asp_r	2.97
K_p	3e5
K_d	1e6
λ_{sat}	110 deg
σ_{sat}	35 deg
T_λ	0.5
T_σ	0.2

coefficient	value
S_H	9.8 m^2
L_{WL}	8.8 m
k	0.15
$d_{l_{11}}$	10 $\frac{kg\ m}{s}$
$d_{l_{22}}$	16 $\frac{kg\ m}{s}$
$d_{l_{44}}$	40 $\frac{kg\ m^2}{rad\ s}$
B_{F0}	15 $[-]$
B_L	70 $\frac{kg}{rad}$
ω_{roll_n}	2.4 $\frac{1}{s}$
GM_t	2.4 m
m	1.6e3 kg
I_{xx}	6.8e3 $kg\ m^2$
I_{zz}	8.5e3 $kg\ m^2$
I_{xz}	-13.4 $kg\ m^2$
$X_{\dot{u}}$	-1.6e2 kg
$Y_{\dot{v}}$	-1.2e3 kg
$Y_{\dot{p}}$	0 $kg\ m$
$Y_{\dot{r}}$	-3.5e2 $kg\ m$
$K_{\dot{r}}$	0 $kg\ m^2$
$K_{\dot{p}}$	-1.0e3 $kg\ m^2$
$N_{\dot{r}}$	-2.4e3 $kg\ m^2$