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# Control and Development of an Autonomous Sailboat

Johann Ruben van Tonder  
22569596

Thesis presented in partial fulfilment of the requirements for the degree of Master of Engineering (Electronic) in the Faculty of Engineering at Stellenbosch University

Supervisor: Prof. Jaco Versfeld & Prof. Japie Engelbrecht  
Department of Electrical and Electronic Engineering

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# **Abstract**

**English**

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# Nomenclature

## Ocean Vessel Dynamics

$X, Y, Z$	Coordinates of force vector decomposed in the body-fixed frame(surge, sway and heave forces)
$K, M, N$	Coordinates of moment vector decomposed in the body-fixed frame(roll, pitch and yaw moment)
$u, v, w$	Coordinates of linear velocity vector decomposed in the body-fixed frame(surge, sway and heave velocities)
$p, q, r$	Coordinates of angular velocity vector decomposed in the body-fixed frame(roll, pitch and yaw angular velocities)
$x, y, z$	Coordinates of position vector decomposed in the body-fixed frame(surge, sway and heave positions)
$\phi, \theta, \psi$	Coordinates of Euler angle vector decomposed in the body-fixed frame(roll, pitch and yaw Euler angles)

## Guidance and Navigation

$x_i, y_i, \psi_i$	Coordinate system for guidance control known as the <i>pose</i>
$\psi_{heading}$	Track heading
$L_{track}$	Track length
$(E_{src}, N_{src})$	Source Waypoint
$(E_{dest}, N_{dest})$	Destination Waypoint

**Acronyms and abbreviations**

SNAME	Society of Naval Architects and Engineers
CG	Centre of gravity of a vessel
CB	Centre of buoyancy of a vessel
AoA	Angle of attack
Re	Reynolds number
USV	Unmanned Surface Vehicle
AWA	Apparent Wind Angle
AWV	Apparent Wind Velocity
RC	Remote Control
GPS	Global Positioning System
MAVLink	Micro Air Vehicle Link
RTL	Return-To-Launch
GCS	Ground Control System
HITL	Hardware-In-The-Loop
SITL	Software-In-The-Loop
HUD	Heads-Up Display
HAL	Hardware Abstraction Layer

**Sailing Terminology**

Bow	Front of the sailboat
Stern	Rear of the sailboat
Luff	Leading edge of the sail
Leech	Trailing edge of the sail
Foot	Bottom edge of the sail
Boom	Attached point of the foot to the sailboat
Clew	Attachment point of the leech
Tack	Attachment point of the luff
Chord	Straight line between leading and trailing edge
Camber	Perpendicular distance from the chord line to the foil
Draft	Position of the maximum camber along the chord line
Entry	Angle of the leading edge to the chord line
Exit	Angle of the trailing edge to the chord line
AoA	Angle between oncoming flow and the chord line
Twist	Angle between the chord line and the sailboat's centre line

# **Chapter 1**

## **Introduction**

### **1.1. Background**

### **1.2. Problem Statement**

The problem is that of modeling and controlling a fixed wing unmanned surface vehicle(USV), or in layman's terms a sailboat. Control strategies and techniques are applied to find the best way of controlling a USV on a line trajectory and performing speed control.

### **1.3. Summary of Work**

The code of the modified repositories is available on my GitHub account [1].

### **1.4. Scope**

### **1.5. Format of Report**

# Chapter 2

## Literature Review

In recent years the development of robotics has enabled a new era of exploration. This exploration is made possible through USV's, more commonly known as sailboats.

Sailboats can be used to explore areas of the ocean previously unexplored. The sailboat are equipped with an array of sensors that collect data on important ocean factors and the ecosystem inside. This chapter will review the usage of sailboats to collect data and exploration usages. Also it will look into the control techniques used in sailboats and how a sailboat is controlled to stay on the designated course.

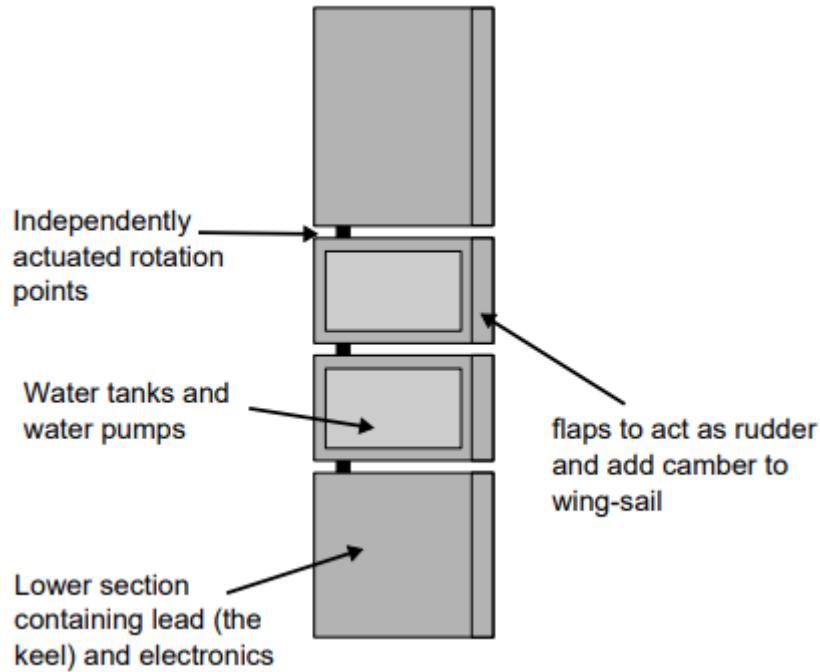
### 2.1. Related work on USV's

#### 2.1.1. South Africa Sailboat



**Figure 2.1:** UCT Sailboat

The first sailboat used for review is close to home. The sailboat was developed by the University of Cape Town(UCT) [2], the sailboat is illustrated in Figure 2.1. The study focussed on modeling and simulation of a sailboat. No sort of control techniques was



**Figure 2.2:** Variable draft sailing spar

developed. The modelling of the sailboat was done in the three and four degrees of freedom, which is the most popular degrees of freedom when considering sailboats. The four degrees of freedom are surge, sway, yaw and roll. The field tests collected valuable data which agreed with the limitations mentioned in the study. The study used a fixed wing sail but also proposed a novel sail trimming on a fixed wing sail. The platform that was developed to monitor the sailboat performed well in field test but there was one problem with the wind sensor not giving reliable data. The field tests were executed on dams near UCT.

### 2.1.2. Italy Sailboat

Now lets move over to Europe, more specifically Italy and the University of Pisa with Aeolus [3]. Aeolus is a small model sized sailing boat, provided by ETH Zurich, fitted with a wind sensor. The sailboat makes use of a fabric sail unlike the sailboat illustrated in Figure 2.1. Aeolus makes use of a main sail and a jib. The sail controller is only used to adjust the main sail angle.



**Figure 2.3:** Aeolus, sailing boat of ETH Zurich

The study focussed on developing navigation and control strategies for an autonomous sailing model boat. In the study different rudder controllers were implemented to perform the maneuvering technique called tacking and performed field test in the lake Zurich, Switzerland. The study successfully implemented a turning dynamic, real time data collecting and filtering, sailing upwind and executing tack maneuvers.

### 2.1.3. Norwegian Sailboat

Not far from Zurich lies the ancestors of the vikings who mastered the arts of sailing. The Norwegian model [4] was based on the sailboat illustrated in Figure 2.4. The thesis focused on modeling, simulation and control of a sailboat. The study first developed a mathematical model in four degrees of freedom. From this mathematical model the rudder and sail controller was designed. New strategies for a course control and a technique to reduce roll motion was also developed. A path following strategy was created and state estimation through sensors was created. All of the above mentioned controllers and strategies was tested and achieved desirable results.



**Figure 2.4:** Sailboat used for modeling

#### 2.1.4. Sweden Sailboat



**Figure 2.5:** Sailboat provided by University of Hong Kong

The study focused on developing a mathematical model with simulations of the control strategies. The control was designed for optimal trajectory approach. The tests were confined to indoor tests where fans were used to generate wind. The fans caused turbulence which impacted the tests. The modeling did not take into consideration the roll, pitch and heave and the control strategies still performed accurately. The controllers implemented were a PD controller and the force polar diagram for the sail angle.

## 2.2. Control techniques for USV's

The control of a USV is a problem of steering the vehicle in the appropriate direction and controlling the sail angle. The two common sails used are either a tradition fabric sail [5] or a fixed wing sail [6]. The steering of a vehicle is made possible through controlling the rudder. A USV is known as a underactuated vessel, which is when a vehicle has uncontrollable states. The only controllable states are the yaw rate and surge speed, which is defined in Section ??.

### 2.2.1. Rudder Control

The sailboat made use of a basic rudder control, which did not achieve desirable results. The controller used to control the rudder is a lead controller\*\*\*expand\*\*\*, although their was reported that it performed badly due to amplifying the high frequency noise. The study concluded that their is still significant amount of work required on the rudder control.

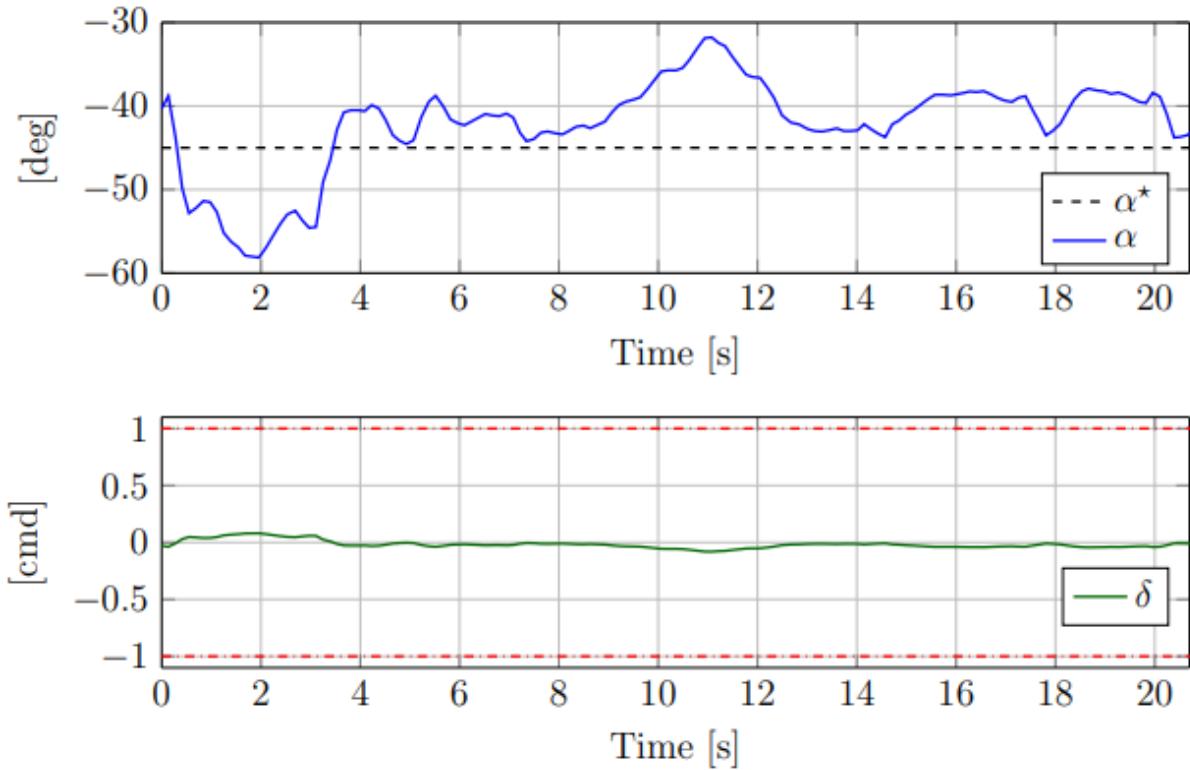
Aeolus makes use of two rudder controllers either using the one or the other depending on the circumstance. When the error signal  $e$ , which is the difference between the desired heading and the actual heading, is small the controller is defined as

$$\delta = k_p e \quad (2.1)$$

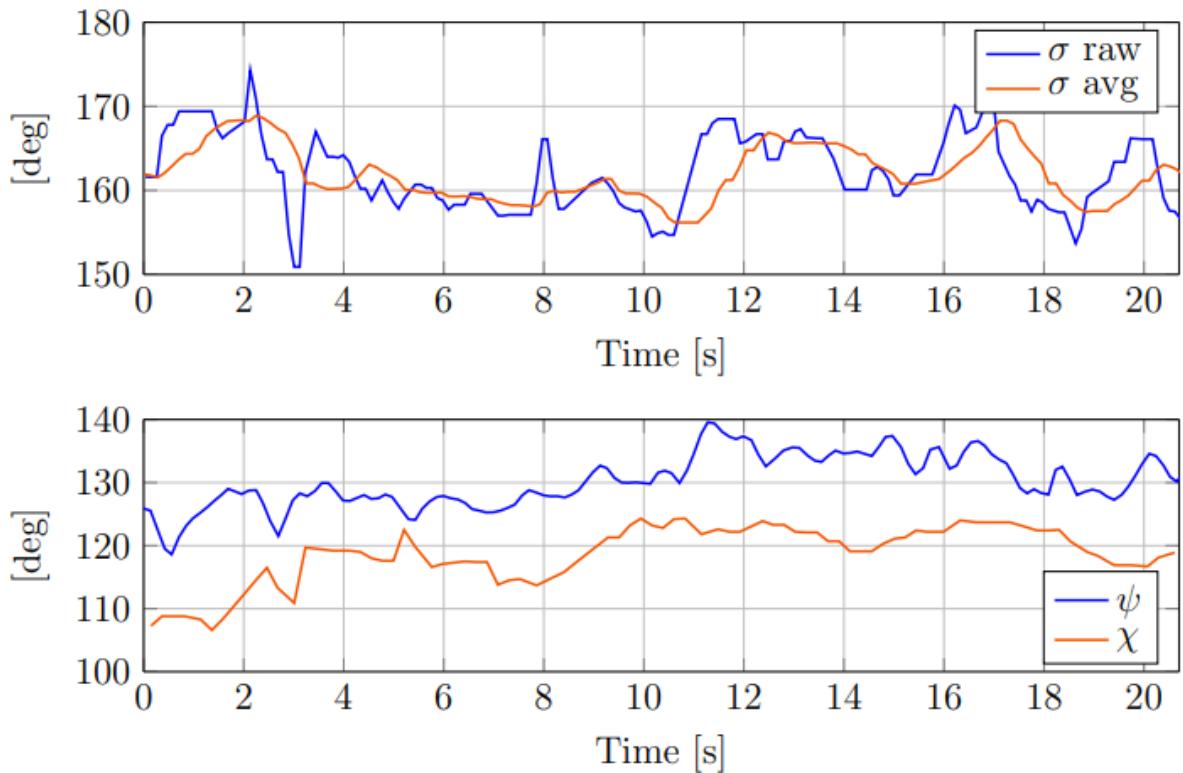
this is a simple Proportional controller for the rudder at small error signals. The second controller is used in the case of the error signal being large when implementing a non-linear controller. The non-linear controller defines a nonlinear gain  $k(e)$  as

$$k(e) = \frac{k_p}{1 + c_p |e|} \quad (2.2)$$

The constant  $c_p$  is used to tune the control action when the error is large. This enables the sailboat to perform special maneuvering called tacking. The performance of the rudder controller is illustrated in Figure 2.6 and 2.7. The rudder controller performed reasonably accurate apart from suffering from drift which causes a difference in the heading  $\psi$  and course over ground  $X$ .



**Figure 2.6:** Reference heading  $\alpha^*$ , obtained angle  $\alpha$  and rudder input  $\delta$



**Figure 2.7:** Raw and average wind direction  $\sigma$  raw and  $\sigma$  avg, heading  $\psi$  and course over ground  $X$

The rudder control of the Norwegian sailboat is by far the most complex discussed thus

far, the controller is using control Lyapunov functions and a recursive design approach. Where the new state variables are,

$$z_1 = \psi + \beta_b + X_d \quad (2.3)$$

and

$$z_2 = r - \alpha \quad (2.4)$$

where  $X_d$  is the desired heading,  $z_1$  is the error from the desired course angle,  $\psi$  is the actual heading and  $\beta_b$  is the necessary drift angle such that the desired course angle is achieved.  $\beta_b$  is a bias, the drift and course angle is defined as

$$\beta = \arctan2(v\cos(\phi), u) \quad (2.5)$$

$$X = \psi + \beta \quad (2.6)$$

The goal of the controller is to make the Lyapunov Function stable,

$$\dot{V}_1 = -K_p z_1^2 + z_1 z_2 \quad (2.7)$$

Solving the controller action  $u$  results in the state  $z_1$  to due the following procedure,

$$z_1 = \text{mod}(z_1 + \pi, 2\pi) - \pi \quad (2.8)$$

where mod is the modulo operation. This ensures that the boat always turns in the direction that has the smallest error. If one calculates  $z_1$  one will always end up tacking up-wind an jibing down-wind. The correction term  $\beta_b$  is used to compensate for the drift angle, by using a lookup-table that stores the necessary drift angles to keep a desired course. A simpler method was introduced in this study, where  $z_1$  state of a traditional heading controller is equal to

$$z_1 = \psi - \psi_d = \psi - (X_d - \beta_b) = \psi + \beta_d - X_d \quad (2.9)$$

The study found the following solution for  $\beta_b$  as follows

$$\beta_b = -\alpha_k \cos(\phi) \quad (2.10)$$

where

$$\alpha_k = -\frac{\beta}{\cos(\phi)} \quad (2.11)$$

The lift the rudder can create is limited which limits the maximum moment can create in yaw. The controller does not take this into account and therefore to smooth the rudder

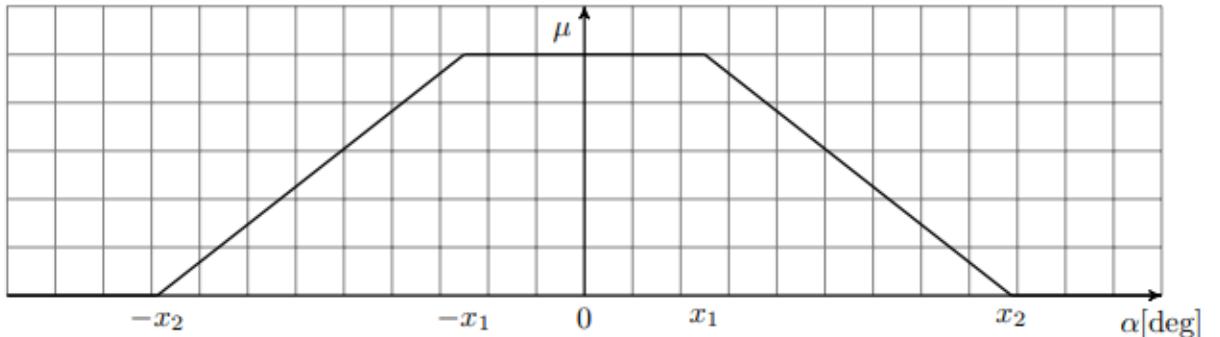
response a second order low-pass filter was added. The transfer function is shown below,

$$F(s) = \frac{\omega^2}{s^2 + 2\omega + \omega^2} \quad (2.12)$$

where  $T_\psi = \frac{1}{\omega}$  was treated as the tuning parameter.

## 2.2.2. Sail Control

The sailboat of the UCT sailboat was controlled by setting the sail angle of attack to a predetermined angle depending on the apparent wind angle. The apparent wind angle was divided into three regions  $-90 < \beta < 90$ ,  $-160 < \beta < 160$  and  $-180 < \beta < 180$ . The hardware on the ETH Zurich sailboat platform did not allow for feedback measurements of the position of the sail and therefore a simple open loop controller was implemented. The controller was based on a rule, the closer Aeolus is sailing opposite the wind direction, that is closer to  $0^\circ$ , the more the sail is closed. The rule based controller is shown in Figure 2.8. The parameters  $x_1$  and  $x_2$  can be changed at run time and the higher the sail command  $\mu$ , the more the sail is closed.



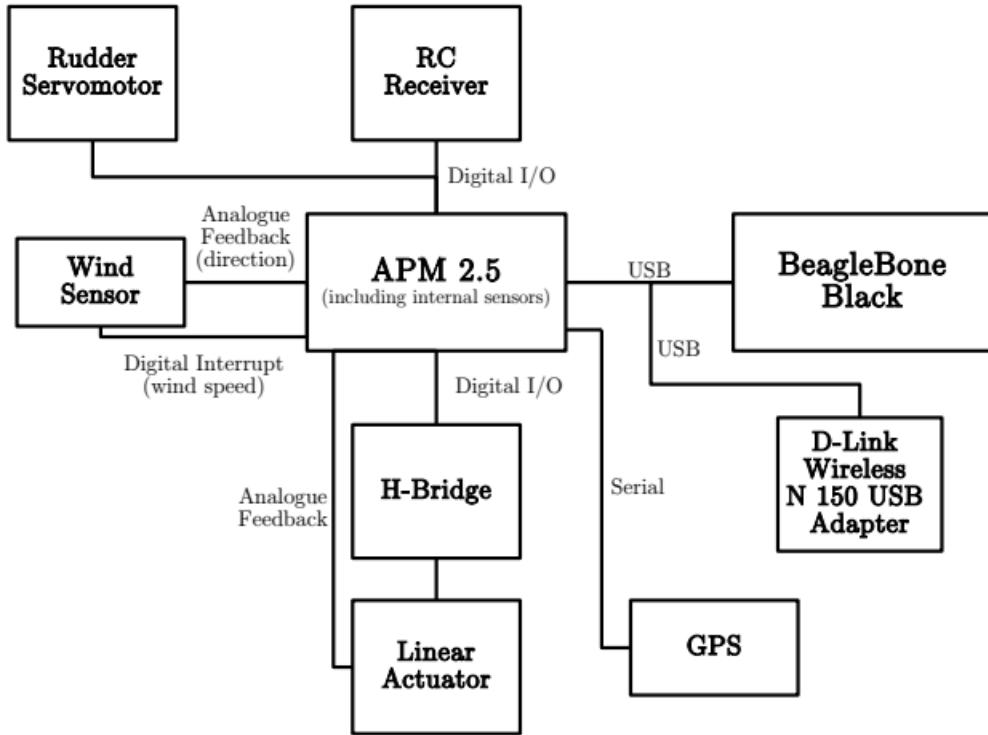
**Figure 2.8:** Rule based controller for sail

To obtain the optimal sail angle to give the highest forward acceleration for a given wind direction, making  $\lambda_d$ , which is the sail angle, a function of  $\beta_{ws}$ ,  $\lambda_d(\beta_{ws})$ . This was achieved by defining a function independent of wind speed and measuring the force it generates. The optimal sail angle was stored in a look-up table, taking into account the restriction of possible sailing angles depending on the wind direction.

## 2.3. Platform development of USV

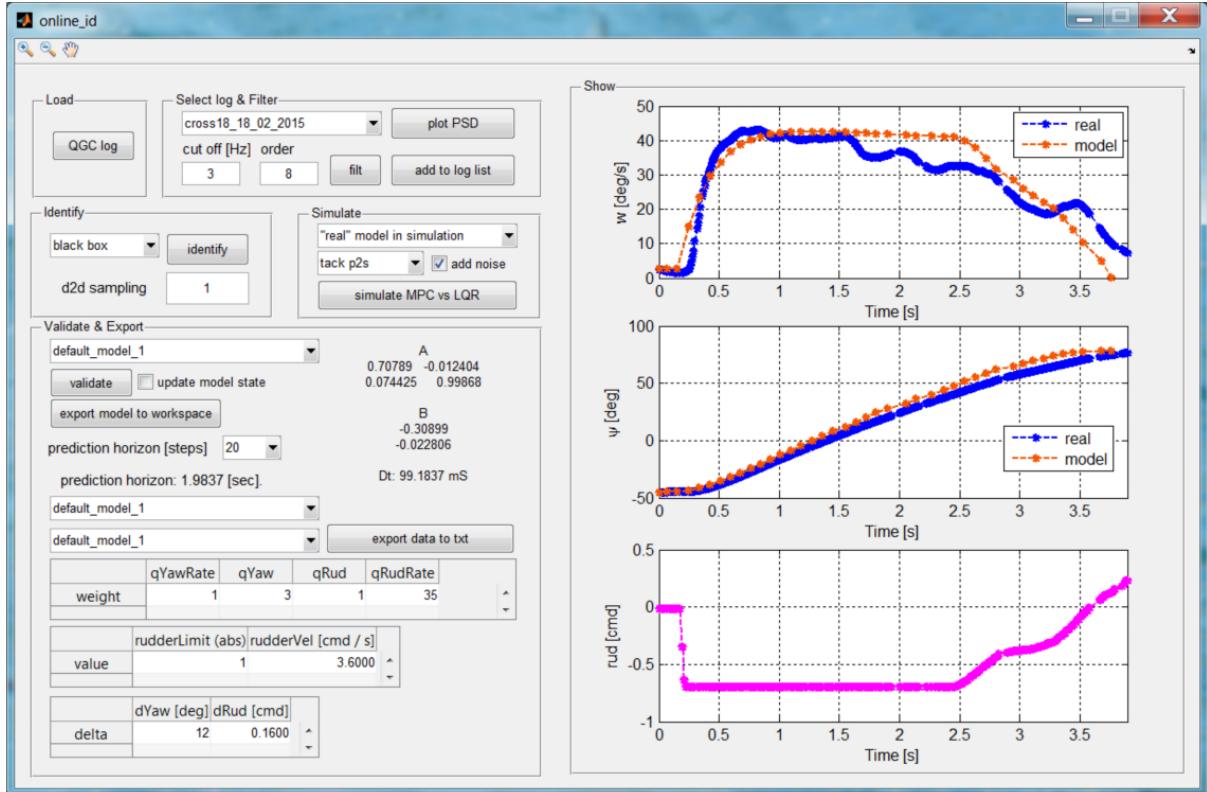
The platform required for sailboat control is quite simple. It requires a rudder angle feedback, compass or gyro, wind sensor, GPS sensor and the methods for controlling the sail and rudder. Accompanying these sensors onboard an off board station is required to monitor the sailboat's state. The difficulty in designing and implementing the platform is

the limitation of space available on the model sized sailboats. The goal of these small scale sailboats are to test control strategies for navigation and route planning. It is very seldom that you would add any other sensors apart from cameras used for obstacle avoidance.



**Figure 2.9:** Platform developed by UCT sailboat

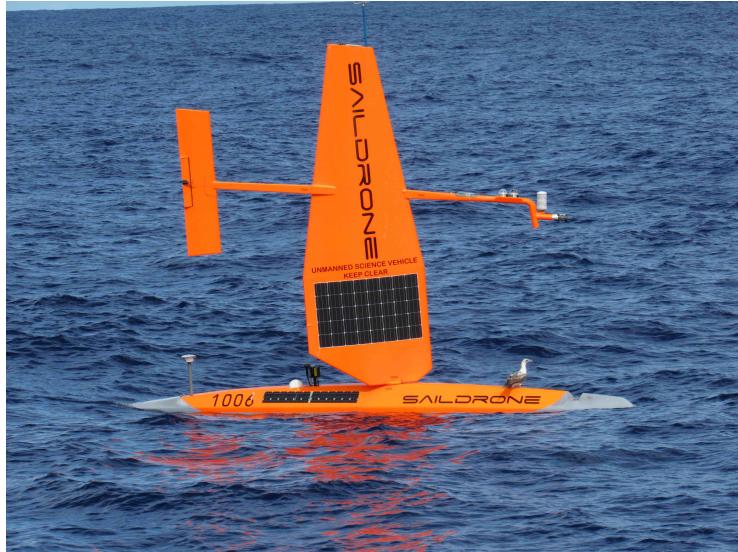
As illustrated in Figure 2.9 the rudder is controlled with a servomotor and the sail with a linear actuator. It is quite common to make use of either control methods when it comes to controlling the sail or rudder. One problem noted in [2], is the inaccurate data from the wind sensor and this will be taken into account when designing the platform. The next important part of the platform is the base station used for communicating with the sailboat. One such base station GUI developed by [3]. The base station, as previously mentioned, monitors the important states of the sailboat, as the heading angle, rudder and sail command. The base station is also used to send data and commands to the sail boat when required.



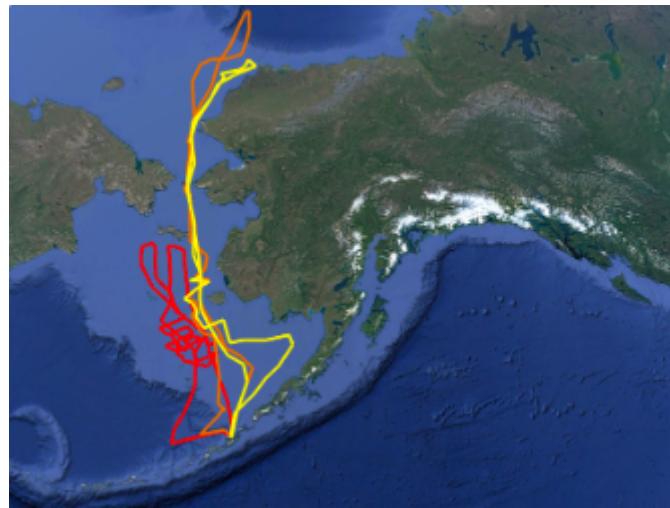
**Figure 2.10:** GUI developed for sailboat monitoring

## 2.4. Usages of USV

USV's has enabled data acquisitions that was thought to be unobtainable. Saildrone [7], founded in 2014, a company that specialize in the gathering of ocean data. The USV deployed by Saildrone is illustrated in Figure 2.11. The data that the USV gathers ranges from ocean temperature, humidity, pressure, wind speed and direction.



**Figure 2.11:** Saildrone's USV



**Figure 2.12:** Routes of the USV's in Arctic Mission

USV's also allow for exploration of unknown ocean area's, with accompanied sensor USV's can also map out the ocean floor. One of the recent missions completed by Saildrone, was the 2017 Arctic Mission. Saildrone deployed three USV's from the Dutch Harbor in Alaska. Two of the vehicles were equipped with the CO<sub>2</sub> measuring devices and camera's for capturing video and photo footage of northern fur seals. The third vehicle is equipped with an echo sounder for surveying walleye pollock, nothern fur seals and the elusive North Pacific right whale. The three routes of the vehicles are illustrated in Figure 2.12. Saildrone is one of few companies/organisations that has developed the capable technology to make use of USV. To this extent a lot of research is still needed to develop USV and also is needed to utilise the data in the oceans. This is especially true in undeveloped countries and the ocean's surrounding these countries.

## 2.5. Ardupilot

# Chapter 3

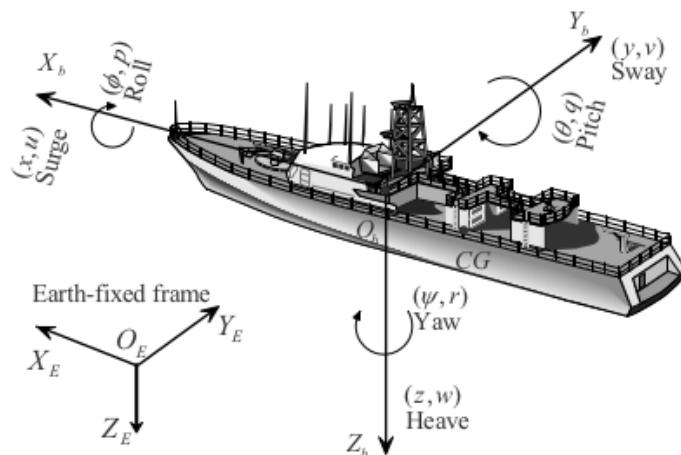
## Modelling and Linearizations

This chapter models a standard ocean vessel in six degrees of freedom, the hydrodynamic forces, the restoration forces and the external forces(wind,waves and ocean currents) acting on an ocean vessel. It then extends the modelling to a sailboat's keel, rudder and sail. The chapter ends of with a linearizations of the steering model and simplifications of the forces.

### 3.1. Modelling of Ocean Vessels

#### 3.1.1. Standard Ocean Vessel Notation

An ocean vessels are modelled in six degrees of freedom, requiring six independent coordinates to determine its position and orientation. The first three coordinates corresponding to position  $(x, y, z)$  and their first time derivatives, translation motion along the  $x$ -,  $y$ -, and  $z$ -axes. The last three coordinates  $(\phi, \theta, \psi)$  and their first time derivatives describing orientation and rotational motion [8]. Figure 3.1 illustrates the motion variables of an ocean vessel with the six independent coordinates.



**Figure 3.1:** Motion variables for an ocean vessel

The SNAME(Society of Naval Architects and Marine Engineers) established the notation for the six different motion components as *surge*, *sway*, *heave*, *roll*, *pitch* and *yaw*.

Table A.1 summarizes the SNAME notation for ocean vessels.

Two reference models are used to determine the equations of motion, namely the inertial to earth frame  $O_eX_eY_eZ_e$  that may be displaced to overlap with the vessel's fixed coordinates in some initial condition and the body-fixed frame  $O_bX_bY_bZ_b$ , illustrated in Figure 3.1. The most common used position for the body-fixed frame results in symmetry about the  $O_bX_bZ_b$ -plane and approximate symmetry about the  $O_bY_bZ_b$ . The body axes coincides with the axes of inertia and are usually defines as follows:  $O_bX_b$  is the longitudinal axis,  $O_bY_b$  is the transverse axis and  $O_bZ_b$  is the normal axis. Below are the vectors used to describe the general motion of an ocean vessel:

$$\mathbf{n} = [\mathbf{n}_1 \mathbf{n}_2]^T \quad (3.1)$$

$$\mathbf{v} = [\mathbf{v}_1 \mathbf{v}_2]^T \quad (3.2)$$

$$\boldsymbol{\tau} = [\boldsymbol{\tau}_1 \boldsymbol{\tau}_2]^T \quad (3.3)$$

$$\mathbf{n}_1 = [x \ y \ z]^T \quad (3.4) \qquad \mathbf{n}_2 = [\phi \ \theta \ \psi]^T \quad (3.5)$$

$$\mathbf{v}_1 = [u \ v \ w]^T \quad (3.6) \qquad \mathbf{v}_2 = [p \ q \ r]^T \quad (3.7)$$

$$\boldsymbol{\tau}_1 = [X \ Y \ Z]^T \quad (3.8) \qquad \boldsymbol{\tau}_2 = [K \ M \ N]^T \quad (3.9)$$

where  $\mathbf{n}$  denotes the position and orientation vector with coordinates in the earth fixed frame,  $\mathbf{v}$  denotes the linear and angular velocity vector with coordinates in the body-fixed frame and  $\boldsymbol{\tau}$  denotes the forces and moments acting on the vessel in the body-fixed frame. The vessel dynamics are divided into two parts known as *kinematics* and *kinetics*.

### 3.1.2. Kinematics

Kinematics looks at the motion of the vessel without directly considering the forces affecting the motion. The first time derivative of the position vectors  $\mathbf{n}_1$  and  $\mathbf{n}_2$  is related to the linear velocity vector  $\mathbf{v}_1$  and  $\mathbf{v}_2$  via the following transformations,

$$\dot{\mathbf{n}}_1 = \mathbf{J}_1(\mathbf{n}_2)\mathbf{v}_1 \quad (3.10)$$

$$\dot{\mathbf{n}}_2 = \mathbf{J}_2(\mathbf{n}_2)\mathbf{v}_2 \quad (3.11)$$

where  $\mathbf{J}_1(\mathbf{n}_2)$  and  $\mathbf{J}_2(\mathbf{n}_2)$  are transformation matrices, which is related through the functions of the Euler angles: roll( $\phi$ ), pitch( $\theta$ ) and yaw( $\psi$ ). The  $\mathbf{J}_1$  transformation matrix is given by

$$\mathbf{J}_1(\mathbf{n}_2) = \begin{bmatrix} \cos(\psi)\cos(\theta) & -\sin(\psi)\cos(\theta) + \sin(\phi)\sin(\theta)\cos(\psi) & \sin(\psi)\sin(\phi) + \sin(\theta)\cos(\psi)\cos(\phi) \\ \sin(\psi)\cos(\theta) & \cos(\psi)\cos(\phi) + \sin(\phi)\sin(\theta)\sin(\psi) & -\cos(\psi)\sin(\phi) + \sin(\theta)\sin(\psi)\cos(\phi) \\ -\sin(\theta) & \sin(\phi)\cos(\theta) & \cos(\phi)\cos(\theta) \end{bmatrix} \quad (3.12)$$

and the transformation matrix  $\mathbf{J}_2$  is given by,

$$\mathbf{J}_2(\mathbf{n}_2) = \begin{bmatrix} 1 & -\sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)/\cos(\theta) & \cos(\phi)/\cos(\theta) \end{bmatrix} \quad (3.13)$$

When  $\theta = \pi/2$ , the transformation matrix  $\mathbf{J}_2(\mathbf{n}_2)$  becomes singular, however this is unlikely to happen when practically testing an ocean vessel, because of the metacentric restoring forces. Combining Equation 3.12 and Equation 3.13 results in the kinematics of an ocean vessel.

$$\begin{bmatrix} \dot{\mathbf{n}}_1 \\ \dot{\mathbf{n}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{J}_1(\mathbf{n}_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & \mathbf{J}_2(\mathbf{n}_2) \end{bmatrix} \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} = \dot{\mathbf{n}} = \mathbf{J}(\mathbf{n})\mathbf{v} \quad (3.14)$$

### 3.1.3. Kinetics

The Newton-Euler formulation [9] defines the balancing forces and moments for a rigid body with a mass of  $m$  as follows,

$$\mathbf{f}_{Ob} = \mathbf{m}[\dot{\mathbf{v}}_{Ob}^E + \dot{\mathbf{w}}_{Ob}^E \times \mathbf{r}_{Ob} + \mathbf{w}_{Ob}^E \times \mathbf{v}_{Ob} + \mathbf{w}_{Ob}^E \times (\mathbf{w}_{Ob}^E \times \mathbf{r}_{Ob})] \quad (3.15)$$

$$\mathbf{m}_{Ob} = \mathbf{I}_o \mathbf{w}_{Ob}^E + \dot{\mathbf{w}}_{Ob}^E \times \mathbf{I}_o \mathbf{w}_{Ob}^E + m \mathbf{r}_{Ob} \times (\dot{\mathbf{v}}_{Ob} + \mathbf{w}_{Ob}^E \times \mathbf{v}_{Ob}) \quad (3.16)$$

where  $\mathbf{f}_{Ob}$  is the balancing forces,  $\mathbf{m}_{Ob}$  the balancing moments and  $\mathbf{I}_o$  is the inertia matrix about  $O_b$  defined as,

$$\mathbf{I}_o = \begin{bmatrix} I_x & -I_{xy} & -I_{xz} \\ -I_{yx} & I_y & -I_{yz} \\ -I_{zx} & -I_{zy} & I_z \end{bmatrix} \quad (3.17)$$

$I_x, I_y$  and  $I_z$  are the moments of inertia about the  $O_bX_b$ ,  $O_bY_b$  and  $O_bZ_b$  axes.  $I_{xy} = I_{yx}$ ,  $I_{xz} = I_{zx}$  and  $I_{yz} = I_{zy}$  are the products of inertia. These quantities are defined as

$$I_x = \int_V (y^2 + z^2) \rho_m dV \quad (3.18)$$

$$I_{xy} = \int_V xy \rho_m dV \quad (3.19)$$

$$I_y = \int_V (x^2 + z^2) \rho_m dV \quad (3.20)$$

$$I_{xz} = \int_V xz \rho_m dV \quad (3.21)$$

$$I_z = \int_V (z^2 + y^2) \rho_m dV \quad (3.22)$$

$$I_{zy} = \int_V zy \rho_m dV \quad (3.23)$$

where  $\rho_m$  are the mass density and  $V$  the volume of the rigid body. By substituting the definitions defined in Table A.2 into Equations 3.15 and 3.16, results in the equation below,

$$\mathbf{M}_{\mathbf{RB}} \dot{\mathbf{v}} + \mathbf{C}_{\mathbf{RB}}(\mathbf{v})\mathbf{v} = \boldsymbol{\tau}_{\mathbf{RB}} \quad (3.24)$$

where  $\mathbf{v} = [u \ v \ w \ p \ q \ r]^T$  is the generalized velocity vector decomposed in the body-fixed frame and  $\boldsymbol{\tau}_{\mathbf{RB}} = [X \ Y \ Z \ K \ M \ N]^T$  is the generalized vector of external forces and moments. The rigid body system inertia matrix  $\mathbf{M}_{\mathbf{RB}}$  and the rigid body Coriolis and centripetal matrix  $\mathbf{C}_{\mathbf{RB}}$  is defined in Equation A.1 and A.2. The generalized external force and moment vector,  $\boldsymbol{\tau}_{\mathbf{RB}}$ , is a sum of the hydrodynamic force and moment vector  $\boldsymbol{\tau}_{\mathbf{H}}$ , external disturbance force and moment vector  $\boldsymbol{\tau}_{\mathbf{E}}$  and propulsion force and moment vector  $\boldsymbol{\tau}$ .

### 3.1.4. Hydrodynamic Forces and Moments

Hydrodynamic forces and moments can be defined as the forces and moments on a ocean body when the body is forced to oscillate with the wave excitation and no wave are incident on the body. As shown in [10], the hydrodynamic forces and moments acting on a rigid body can be assumed to be linearly superimposed. The forces and moments can be subdivided into three components,

1. Added mass due to the inertia of the surrounding fluid
2. Radiation-induced potential damping due to the energy carried away by the generated surface waves
3. Restoring forces due to Archimedian forces

The hydrodynamic forces and moments vector  $\boldsymbol{\tau}_{\mathbf{H}}$  is expressed in the equation below,

$$\boldsymbol{\tau}_{\mathbf{H}} = -\mathbf{M}_{\mathbf{A}} \dot{\mathbf{v}} - \mathbf{C}_{\mathbf{A}}(\mathbf{v})\mathbf{v} - \mathbf{D}(\mathbf{v})\mathbf{v} - \mathbf{g}(\mathbf{n}) \quad (3.25)$$

where  $\mathbf{M}_{\mathbf{A}}$  is the added mass matrix,  $\mathbf{C}_{\mathbf{A}}(\mathbf{v})$  is the hydrodynamic Coriolis and centripetal matrix,  $\mathbf{D}(\mathbf{v})$  is the damping matrix and  $\mathbf{g}(\mathbf{n})$  is the position and orientation depending vector of restoring forces and moments. The added mass  $\mathbf{M}_{\mathbf{A}}$  is given below,

$$\mathbf{M}_{\mathbf{A}} = \begin{bmatrix} X_{\dot{u}} & X_{\dot{v}} & X_{\dot{w}} & X_{\dot{p}} & X_{\dot{q}} & X_{\dot{r}} \\ Y_{\dot{u}} & Y_{\dot{v}} & Y_{\dot{w}} & Y_{\dot{p}} & Y_{\dot{q}} & Y_{\dot{r}} \\ Z_{\dot{u}} & Z_{\dot{v}} & Z_{\dot{w}} & Z_{\dot{p}} & Z_{\dot{q}} & Z_{\dot{r}} \\ K_{\dot{u}} & K_{\dot{v}} & K_{\dot{w}} & K_{\dot{p}} & K_{\dot{q}} & K_{\dot{r}} \\ M_{\dot{u}} & M_{\dot{v}} & M_{\dot{w}} & M_{\dot{p}} & M_{\dot{q}} & M_{\dot{r}} \\ N_{\dot{u}} & N_{\dot{v}} & N_{\dot{w}} & N_{\dot{p}} & N_{\dot{q}} & N_{\dot{r}} \end{bmatrix} \quad (3.26)$$

The hydrodynamic Coriolis and centripetal matrix is given below,

$$\mathbf{C}_A(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & 0 & -a_3 & a_2 \\ 0 & 0 & 0 & a_3 & 0 & -a_1 \\ 0 & 0 & 0 & -a_2 & a_1 & 0 \\ 0 & -a_3 & a_2 & 0 & -b_3 & b_2 \\ a_3 & 0 & -a_1 & b_3 & 0 & -b_1 \\ -a_2 & a_1 & 0 & -b_2 & b_1 & 0 \end{bmatrix} \quad (3.27)$$

where  $a_1, a_2, a_3, b_1, b_2$  and  $b_3$  are defined in Equations A.3, A.4, A.5, A.6, A.7 and A.8. The general hydrodynamic damping experienced by ocean vessels is the potential damping, skin friction, wave drift damping and damping due to vortex shedding. The hydrodynamic damping can be expressed in a general form as below,

$$\mathbf{D}(\mathbf{v}) = \mathbf{D} + \mathbf{D}_n(\mathbf{v}) \quad (3.28)$$

where the linear damping matrix  $\mathbf{D}$  is given below,

$$\mathbf{D} = - \begin{bmatrix} X_u & X_v & X_w & X_p & X_q & X_r \\ Y_u & Y_v & Y_w & Y_p & Y_q & Y_r \\ Z_u & Z_v & Z_w & Z_p & Z_q & Z_r \\ K_u & K_v & K_w & K_p & K_q & K_r \\ M_u & M_v & M_w & M_p & M_q & M_r \\ N_u & N_v & N_w & N_p & N_q & N_r \end{bmatrix} \quad (3.29)$$

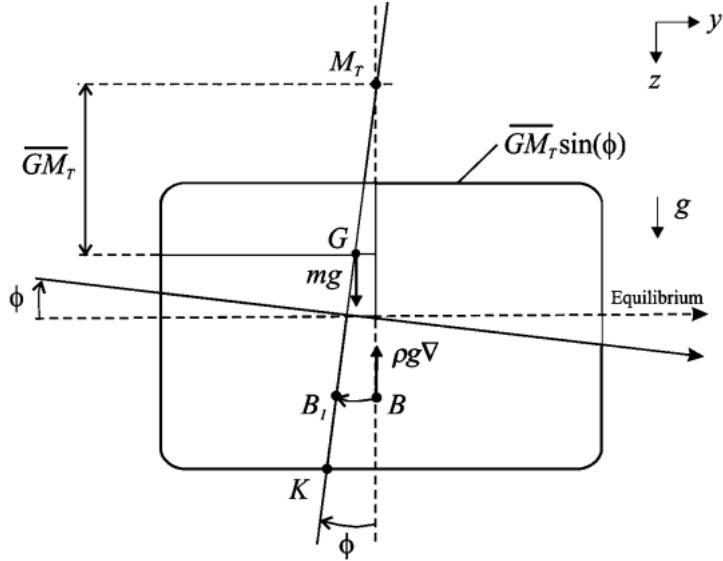
### 3.1.5. Restoring Forces and Moments

Static stability considerations due to restoring forces are usually referred to as *metacentric stability* in the hydrostatic literature. A metacentric stable vessel will resist inclinations away from its steady-state or equilibrium points in heave, roll and pitch. For surface vehicles, the restoring force will depend on the vessel's metacentric height, the location of  $CG$  and  $CB$ , as well as the shape and size of the water plane. Let  $A_{wp}$  denote the water plane area and

$$GM_T = \text{transverse metacentric height}(m) \quad (3.30)$$

$$GM_T = \text{longitudinal metacentric height}(m) \quad (3.31)$$

The metacentric height  $GM_i$ , where  $i \in T, L$ , is the distance between the metacentre  $M_i$  and the  $CG$ , as shown in Figure 3.2.



**Figure 3.2:** Transverse metacentric stability

For a floating vessel at rest, Archimedes stated that buoyancy and weight are in balance.

$$mg = \rho g \nabla \quad (3.32)$$

where  $m$  is the mass of the vessel,  $g$  gravitational force,  $\rho$  density of water and  $\nabla$  the nominal displaced water volume. Hence, the hydrostatic force in heave will be the difference between the gravitational and the buoyancy forces:

$$Z = mg - \rho g [\nabla + \delta \nabla(z)] = -\rho g \delta \nabla(z) \quad (3.33)$$

where the change in displaced water  $\delta \nabla(z)$  is due to variations in heave position  $z$ . This can be written as

$$\delta \nabla(z) = \int_0^z A_{\omega\rho}(\delta) d\delta \quad (3.34)$$

where  $A_{\omega\rho}(\delta)$  is the water plane area of the vessel as a function of the heave position. For conventional rigs and ships, however, it is common to assume that  $A_{\omega\rho}(\delta) \approx A_{\omega\rho}(0)$  is constant for small perturbations in  $z$ . Hence, the restoring force  $Z$  will be linear in  $z$ , that is

$$Z \approx -\rho g A_{\omega\rho}(0) z \quad (3.35)$$

Recall that if a floating vessel is forced downwards by an external force such that  $z \geq 0$ , the buoyancy force becomes larger than the constant gravitational force since the submerged volume  $\nabla$  increases by  $\delta \nabla$  to  $\nabla + \delta \nabla$ . This is physically equivalent to a spring with stiffness  $Z_z = -\rho g A_{\omega\rho}(0)$  and position  $z$ . The restoring force expressed in body frame  $\delta f_r^b$  can therefore be written as

$$\delta f_r^b = -\rho g \begin{bmatrix} -\sin(\theta) \\ \cos(\theta)\sin(\phi) \\ \cos(\theta)\cos(\phi) \end{bmatrix} \int_0^z A_{\omega\rho}(\delta) d\delta \quad (3.36)$$

From Figure 3.2 it is seen that the moment arms in roll and pitch can be related to the moment arms  $GM_T \sin(\phi)$  and  $GM_L \sin(\theta)$  in roll and pitch and a  $z$ -direction force pair with magnitude  $W = B = \rho g \nabla$ . Therefore,

$$r_r^b = \begin{bmatrix} -GM_L \sin(\theta) \\ GM_T \sin(\phi) \\ 0 \end{bmatrix} \quad (3.37)$$

$$f_r^b = -0\rho g \nabla \begin{bmatrix} -\sin(\theta) \\ \cos(\theta)\sin(\phi) \\ \cos(\theta)\cos(\phi) \end{bmatrix} \quad (3.38)$$

By neglecting the moment contribution due to  $\delta f_r^b$ , consider only  $f_r^b$ , implies that the restoring moment becomes

$$m_r^b = r_r^b \times f_r^b = -\rho g \nabla \begin{bmatrix} GM_T \sin(\phi) \cos(\theta) \cos(\phi) \\ GM_L \sin(\theta) \cos(\theta) \cos(\phi) \\ (GM_T - GM_L \cos(\theta)) \sin(\phi) \sin(\theta) \end{bmatrix} \quad (3.39)$$

The assumption that  $r_r^b \times \delta f_r^b = 0$  (no moments due to heave) is a good assumption since this term is small compared to  $r_r^b \times f_r^b$ . The restoring forces and moments are finally written as

$$\mathbf{g}(\mathbf{n}) = - \begin{bmatrix} \delta \mathbf{f}_r^b \\ \mathbf{m}_r^b \end{bmatrix} \quad (3.40)$$

$$\mathbf{g}(\mathbf{n}) = \begin{bmatrix} \rho g \int_0^z A_{\omega\rho}(\delta) d\delta \sin(\theta) \\ \rho g \int_0^z A_{\omega\rho}(\delta) d\delta \cos(\theta) \sin(\phi) \\ \rho g \int_0^z A_{\omega\rho}(\delta) d\delta \cos(\theta) \cos(\phi) \\ \rho g \nabla GM_T \sin(\phi) \cos(\theta) \cos(\phi) \\ \rho g \nabla GM_L \sin(\theta) \cos(\theta) \cos(\phi) \\ \rho g \nabla (GM_T + -GM_L \cos(\theta)) \sin(\phi) \sin(\theta) \end{bmatrix} \quad (3.41)$$

### 3.1.6. Environmental Disturbances

The forces and moments induced by the environmental disturbances is defined by the vector  $\tau_E$  and includes ocean currents, waves(wind generated) and wind.

$$\tau_E = \tau_E^{cu} + \tau_E^{wa} + \tau_E^{wi} \quad (3.42)$$

where  $\tau_E^{cu}$ ,  $\tau_E^{wa}$  and  $\tau_E^{wi}$  are vectors of forces and moments induced by ocean currents, waves and wind.

### Current-induced Forces and Moments

The current induced forces and moments vector  $\tau_E^{cu}$  is given by

$$\tau_E^{cu} = (\mathbf{M}_{RB} + \mathbf{M}_A)\dot{\mathbf{v}}_c + \mathbf{C}(\mathbf{v}_r)\mathbf{v}_r - \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v}_r)\mathbf{v}_r - \mathbf{D}(\mathbf{v})\mathbf{v} \quad (3.43)$$

where  $\mathbf{v}_r = \mathbf{v} - \mathbf{v}_c$  and  $\mathbf{v}_c = [u_c, v_c, w_c, 0, 0, 0]^T$  is a vector irrotational body-fixed current velocities. Take the earth-fixed velocity vector denoted by  $[u_c^E, v_c^E, w_c^E]^T$ , then the body fixed components  $[u_c, v_c, w_c]^T$  can be calculated by

$$\begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix} = \mathbf{J}_1^T(\mathbf{n}_2) \begin{bmatrix} u_C^E \\ v_C^E \\ w_C^E \end{bmatrix} \quad (3.44)$$

### Wave-induced Forces and Moments

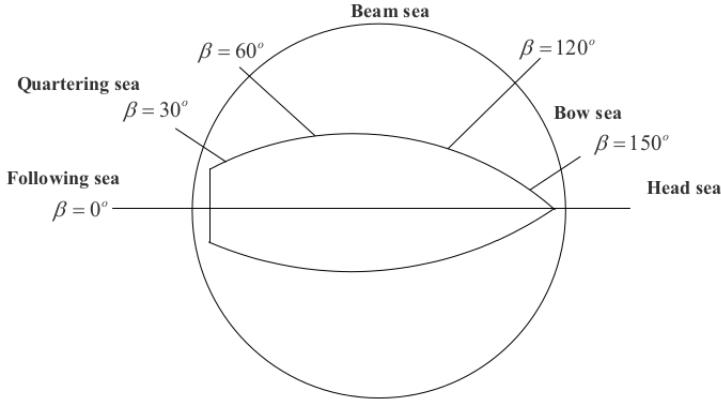
The vector  $\tau_E^{wa}$  of the wave-induced forces and moments is given by

$$\tau_E^{wa} = \begin{bmatrix} \sum_{i=1}^N \rho g B L T \cos(\beta) s_i(t) \\ \sum_{i=1}^N \rho g B L T \sin(\beta) s_i(t) \\ 0 \\ 0 \\ 0 \\ \sum_{i=1}^N \frac{1}{24} \rho g B L (L^2 - B^2) \sin(2\beta) s_i^2(t) \end{bmatrix} \quad (3.45)$$

where  $\beta$  is the vessel's heading(encounter) angle, illustrated in Figure ,  $\rho$  is the water density,  $L$  is the length of the vessel,  $B$  is the breadth of the vessel and  $T$  is the draft of the vessel. Ignoring the higher-order terms of the wave amplitude, the wave slope  $s_i(t)$  for the wave component  $i$  is defined by

$$s_i(t) = A_i \frac{2\pi}{\lambda_i} \sin(\omega_{ei} t + \phi_i) \quad (3.46)$$

where  $A_i$  is the wave amplitude,  $\lambda_i$  is the wave length,  $\omega_{ei}$  is the encounter frequency and  $\phi_i$  is a random phase uniformly distributed and constant with time  $[0, 2\pi)$  corresponding to the wave component  $i$ .



**Figure 3.3:** Ocean vessel's heading angle

### Wind-induced Forces and Moments

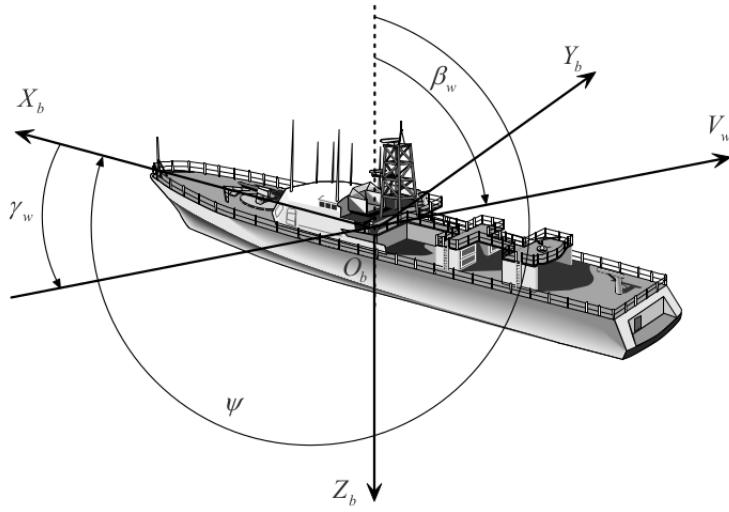
When the ocean vessel is at rest the vector  $\tau_E^{wi}$  of the wind induced forces and moments is given by

$$\tau_E^{wi} = \begin{bmatrix} C_X(\gamma_\omega)A_{F\omega} \\ C_Y(\gamma_\omega)A_{L\omega} \\ C_Z(\gamma_\omega)A_{F\omega} \\ C_K(\gamma_\omega)A_{L\omega}H_{L\omega} \\ C_M(\gamma_\omega)A_{F\omega}H_{F\omega} \\ C_N(\gamma_\omega)A_{L\omega}L_{oa} \end{bmatrix} \quad (3.47)$$

where  $V_\omega$  is the wind speed,  $\rho_a$  is the air density,  $A_{F\omega}$  is the frontal projected area,  $A_{L\omega}$  is the lateral projected area,  $H_{F=\omega}$  is the centroid of  $A_{F\omega}$  above the water line,  $H_{L\omega}$  is the centroid of  $A_{l\omega}$  above the water line,  $L_{oa}$  is the over all length of the vessel,  $\gamma_\omega$  is the angle of relative wind of the vessel bow, illustrated in Figure 3.4 and is given by

$$\gamma_\omega = \psi - \beta_\omega - \pi \quad (3.48)$$

where  $\beta_\omega$  being the wind direction. All the wind coefficients(look-up tables)  $C_X(\gamma_\omega)A_{F\omega}$ ,  $C_Y(\gamma_\omega)A_{L\omega}$ ,  $C_Z(\gamma_\omega)A_{F\omega}$ ,  $C_K(\gamma_\omega)A_{L\omega}H_{L\omega}$ ,  $C_M(\gamma_\omega)A_{F\omega}H_{F\omega}$  and  $C_N(\gamma_\omega)A_{L\omega}L_{oa}$  are computed numerically or by experiments in a wind tunnel as shown in [11].



**Figure 3.4:** Wind angle on vessel

When the vessel is moving the vector  $\tau_E^{\omega i}$  is given by

$$\tau_E^{\omega i} = \begin{bmatrix} C_X(\gamma_{r\omega})A_{F\omega} \\ C_Y(\gamma_{r\omega})A_{L\omega} \\ C_Z(\gamma_{r\omega})A_{F\omega} \\ C_K(\gamma_{r\omega})A_{L\omega}H_{L\omega} \\ C_M(\gamma_{r\omega})A_{F\omega}H_{F\omega} \\ C_N(\gamma_{r\omega})A_{L\omega}L_{oa} \end{bmatrix} \quad (3.49)$$

where

$$V_{r\omega} = \sqrt{u_{r\omega}^2 + v_{r\omega}^2} \quad (3.50)$$

$$\gamma_{r\omega} = -\arctan2(v_{r\omega}, u_{r\omega}) \quad (3.51)$$

with

$$u_{r\omega} = u - V_\omega \cos(\beta_\omega - \psi) \quad (3.52)$$

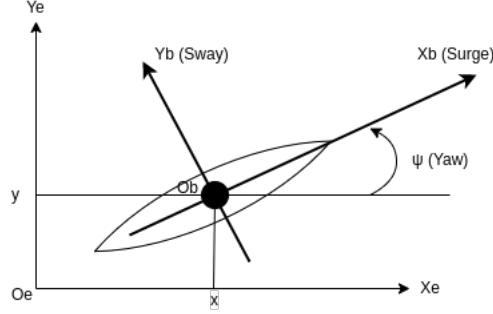
$$v_{r\omega} = v - V_\omega \cos(\beta_\omega - \psi) \quad (3.53)$$

### 3.1.7. Simplifications of 6-DOF

#### Standard 3-DOF Horizontal Model

The horizontal motion of a surface ship in a horizontal plane is often described by the motion component in surge, sway and yaw. You choose  $\mathbf{n} = [x, y, \psi]^T$  and  $\mathbf{v} = [u, v, r]^T$ .

Figure 3.5 illustrates the motion variables in this case.



**Figure 3.5:** Standard 3-DOF Horizontal Model

### Assumptions

1. The motion in roll, pitch and heave is ignored. This means that we ignore the dynamics associated with the motion in heave, roll and pitch, i.e.,  $z = 0$ ,  $w = 0$ ,  $\phi = 0$ ,  $\theta = 0$  and  $q = 0$ .
2. The vessel has homogeneous mass distribution and  $xz$ -plane of symmetry so that

$$I_{xy} = I_{yz} = 0 \quad (3.54)$$

3. The center of gravity and center of buoyancy are located vertically on the z-axis

The vessel dynamics in a horizontal plane is simplified as follows:

$$\dot{\mathbf{n}} = \mathbf{J}(\mathbf{n})\mathbf{v} \quad (3.55)$$

$$\mathbf{M}\dot{\mathbf{v}} = -\mathbf{C}(\mathbf{v})\mathbf{v} - (\mathbf{D} + \mathbf{D}_n(\mathbf{v}))\mathbf{v} + \tau + \tau_E \quad (3.56)$$

where the matrices  $\mathbf{J}(\mathbf{n})$ ,  $\mathbf{M}$ ,  $\mathbf{D}$  and  $\mathbf{D}_n(\mathbf{v})$  are given by

$$J(n) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.57)$$

$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ 0 & mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix} \quad (3.58)$$

$$C(v) = \begin{bmatrix} 0 & 0 & -m(x_g r + v) + Y_{\dot{v}} v - Y_{\dot{r}} r \\ 0 & 0 & mu - X_{\dot{u}} u \\ m(x_g r + v) - Y_{\dot{v}} v - Y_{\dot{r}} r & -mu + X_{\dot{u}} u & 0 \end{bmatrix} \quad (3.59)$$

$$D = \begin{bmatrix} X_u & 0 & 0 \\ 0 & Y_v & Y_r \\ 0 & N_v & N_r \end{bmatrix} \quad (3.60)$$

$$D_n(v) = \begin{bmatrix} X_{|u|u}|u| & 0 & 0 \\ 0 & Y_{|v|v}|v| + Y_{|r|r}|r| & Y_{|v|r}|v| \\ 0 & N_{|v|v}|v| + N_{|r|r}|r| & N_{|v|r}|v| + X_{|r|r}|r| \end{bmatrix} \quad (3.61)$$

The propulsion force and moment vector  $\tau$  is given by

$$\tau = \begin{bmatrix} \tau_u \\ 0 \\ \tau_r \end{bmatrix} \quad (3.62)$$

The above propulsion force and moment vector  $\tau$  implies that we are considering a surface vessel, which does not have an independent actuator in the sway. The environmental disturbance vector  $\tau_E$  is given by

$$\tau = \begin{bmatrix} \tau_{uE} \\ \tau_{vE} \\ \tau_{rE} \end{bmatrix} \quad (3.63)$$

### Simplified 3-DOF Horizontal Model

In some cases we can ignore the off-diagonal terms of the matrices  $\mathbf{M}$  and  $\mathbf{D}$ , all elements of the nonlinear damping matrix  $\mathbf{D}_n(\mathbf{v})$ . These assumptions hold when the vessel has three planes of symmetry, for which the axes of the body-fixed reference frame are chosen to be parallel to the principle axis of the displaced fluid, which are equal to the principle axis of the vessel. Most ships have port/starboard symmetry and moreover, bottom/top symmetry is not required fore horizontal motion. Ship fore/aft nonsymmetry implies that the off-diagonal terms of the inertia and damping matrices are nonzero. However, these terms are small compared to the main diagonal terms. Furthermore, disturbances induced by the waves, wind and ocean currents are ignored. The dynamics of the vessel in the horizontal plane is simplified as follows:

$$\dot{\mathbf{n}} = \mathbf{J}(\mathbf{n})\mathbf{v} \quad (3.64)$$

$$\mathbf{M}\dot{\mathbf{v}} = -\mathbf{C}(\mathbf{v})\mathbf{v} - \mathbf{D}\mathbf{v} + \tau \quad (3.65)$$

where the matrices  $\mathbf{J}(\mathbf{n})$ ,  $\mathbf{M}$ ,  $\mathbf{C}(\mathbf{v})$  and  $\mathbf{D}$  are given by

$$J(n) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3.66)$$

$$M = \begin{bmatrix} m_{xy} & 0 & 0 \\ 0 & m_{xy} & 0 \\ 0 & 0 & m_{33} \end{bmatrix} \quad (3.67)$$

$$C(v) = \begin{bmatrix} 0 & 0 & -m_{xy}v \\ 0 & 0 & m_{xy}u \\ m_{xy}v & -m_{xy}u & 0 \end{bmatrix} \quad (3.68)$$

$$D = \begin{bmatrix} d_{xy} & 0 & 0 \\ 0 & d_{xy} & 0 \\ 0 & 0 & d_{33} \end{bmatrix} \quad (3.69)$$

with

$$m_{xy} = m - X_{\dot{u}} = m - Y_{\dot{v}} \quad (3.70)$$

$$m_{33} = I_z - N_r \quad (3.71)$$

$$d_{xy} = -X_u = -Y_v \quad (3.72)$$

$$d_{33} = -N_r \quad (3.73)$$

### 3.1.8. Summary

The combined six degrees of freedom equations of motion is shown below:

$$\dot{n} = J(n)v \quad (3.74)$$

$$M\dot{v} = -C(v)v - D(v)v - g(n) + \tau + \tau_E \quad (3.75)$$

where

$$M = M_{RB} + M_A \quad (3.76)$$

$$C(v) = C_{RB}(v) + C_A(v) \quad (3.77)$$

**J(n)**, equation 3.12 and 3.13, is the transformation matrix which translate **v<sub>1</sub>** and **v<sub>2</sub>**

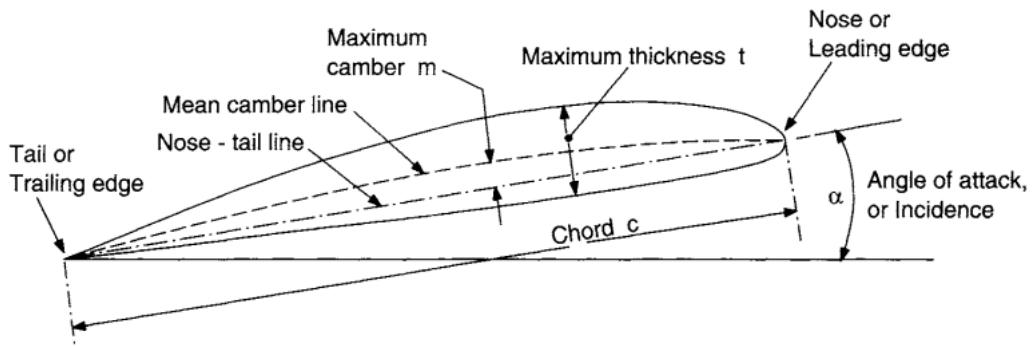
through the functions of the Euler angles to  $\dot{\mathbf{n}}_1$  and  $\dot{\mathbf{n}}_2$ .  $\mathbf{C}(\mathbf{v})$  is the linear combination of the rigid body Coriolis and centripetal matrix  $\mathbf{C}_{\text{RB}}(\mathbf{v})$ , equation A.2 and the hydrodynamic Coriolis and centripetal matrix  $\mathbf{C}_{\mathbf{A}}(\mathbf{v})$ , equation 3.27.  $\mathbf{D}(\mathbf{v})$ , equation 3.29, is the hydrodynamic damping and  $\mathbf{g}(\mathbf{n})$ , equation 3.28, is the restoring forces and moments, equation 3.41. The propulsion forces and moments is modelled by  $\tau$  and the environmental disturbances by  $\tau_E$ .

## 3.2. Modelling of a Sail, Keel and Rudder

### 3.2.1. Rudder Theory

#### Rudder Theory and Terminology

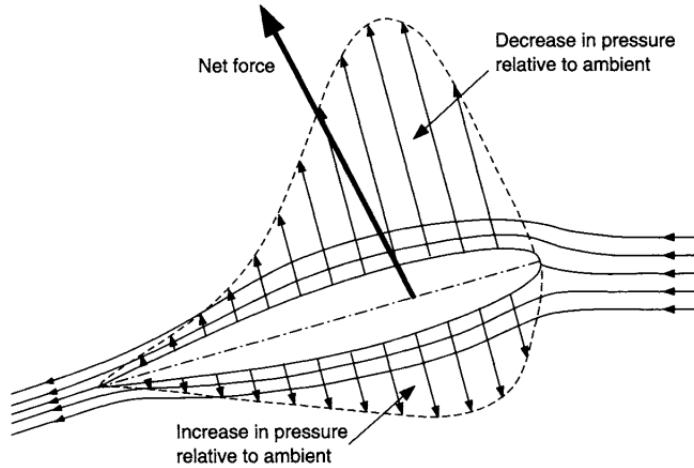
The rudder of a ocean vessel is located at the rear of the vessel and is used for steering. The rudder is usually completely submerged and therefor the forces applicable to the rudder is only hydrodynamic. A rudder changes steers a ocean vessel by generating a lifting force that causes the vessel change its heading [12]. The rudder is fundamental to the safe operation of the ocean vessel. In the case of a sailboat a rudder is very important in establishing an angle of attack and adjusting the sail direction accordingly. The terminology associated with a lifting foil is shown below, where a rudder only differs with having no maximum camber, because it is symmetrical.



**Figure 3.6:** Terminology of a lifting foil

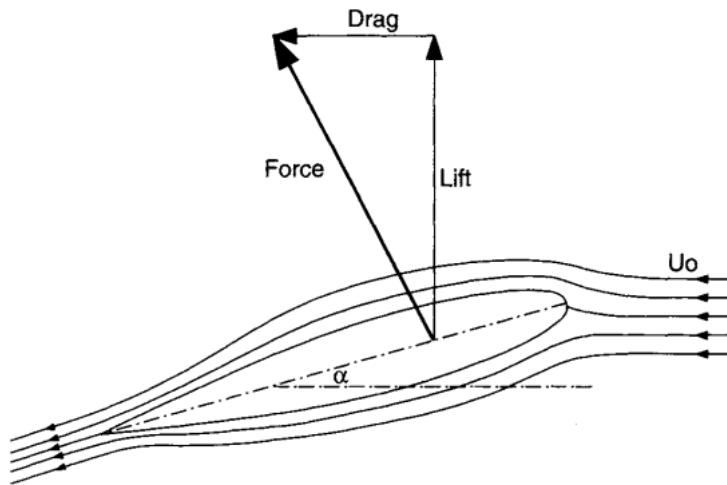
#### Hydrodynamics of a Rudder

The lifting action of the rudder arises from the difference in the average pressure of the fluid over the upper and lower surfaces of the rudder, illustrated in Figure 3.7.



**Figure 3.7:** Pressure around the rudder

The total force on the rudder consists of a *lift* component perpendicular to the fluid stream  $U_0$  and a *drag* component parallel to  $U_0$ , illustrated in Figure 3.8.



**Figure 3.8:** Lift and drag force on rudder

These components are usually presented in terms of dimensionless coefficients for a given rudder angle of attack  $\alpha$ , where  $\alpha$  is the angle between the rudder and the direction of flow.

$$C_L = \frac{\text{Lift}}{0.5\rho AU_0^2} \quad (3.78)$$

$$C_D = \frac{\text{Drag}}{0.5\rho AU_0^2} \quad (3.79)$$

where  $A$  is the rudder area,  $U_0$  the fluid free-stream speed and  $\rho$  the fluid density.  $C_L$  and  $C_D$  depend on the rudder geometry, the incidence to the incoming fluid flow and the Reynolds number( $Re$ ).

## Rudder Forces and Moments

The forces and moments experienced by a sailboat due to the rudder are defined in [13]. The equations formulated for the forces and moments are illustrated below

$$X_{rud} = C_{X\delta_R} \sin(\alpha_R) \sin(\delta_R) \times \frac{1}{2} \rho_w v_B^2 L_{WL} D_K \quad (3.80)$$

$$Y_{rud} = C_{Y\delta_R} \sin(\alpha_R) \cos(\delta_R) \cos(\phi) \times \frac{1}{2} \rho_w v_B^2 L_{WL} D_K \quad (3.81)$$

$$K_{rud} = C_{K\delta_R} \sin(\alpha_R) \sin(\delta_R) \times \frac{1}{2} \rho_w v_B^2 L_{WL} D_K \quad (3.82)$$

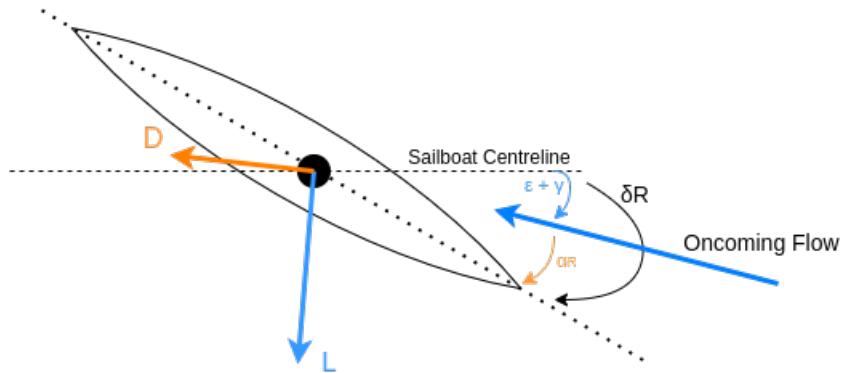
$$N_{rud} = C_{N\delta_R} \sin(\alpha_R) \cos(\delta_R) \cos(\phi) \times \frac{1}{2} \rho_w v_B^2 L_{WL} D_K \quad (3.83)$$

where  $C_{X\delta_R}$ ,  $C_{Y\delta_R}$ ,  $C_{K\delta_R}$ ,  $C_{N\delta_R}$  are non-dimensional coefficients,  $V_B$  is the boat velocity,  $\rho_w$  is the water density,  $L_{WL}$  is the length on design waterline,  $D_K$  is the design draft length,  $\delta_R$  is the physical rudder angle and  $\alpha_R$  is the effective angle of attack on the rudder as defined below

$$\alpha_R = \delta_R - \epsilon_y \gamma - \tan^{-1} \left( \frac{x_R R}{U} \right) \quad (3.84)$$

$$\epsilon = \frac{d\epsilon}{d\gamma} \times \gamma = \epsilon_\gamma \gamma \quad (3.85)$$

where  $\gamma$  is the leeway angle the sailboat is sailing and  $\epsilon$  is the angle of inflow from the downwash generated by the keel and  $x_R$  is the longitudinal distance of the quarter-chord point of the rudder to the CG of the boat. The angles  $\alpha_R$ ,  $\delta_R$ ,  $\gamma$  and  $\epsilon$  are illustrated in Figure 3.9

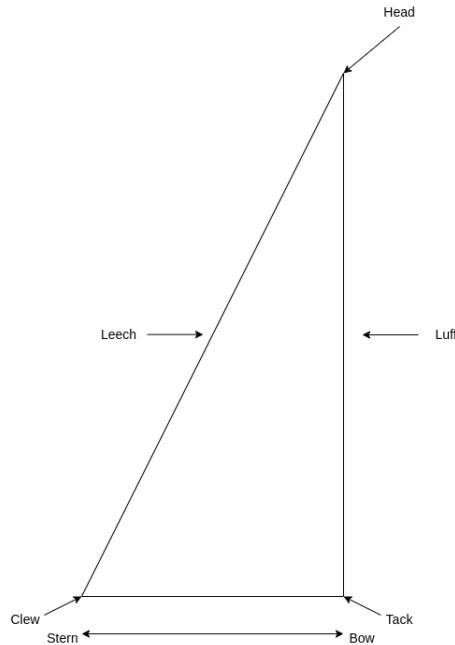


**Figure 3.9:** Definition of Rudder Angles

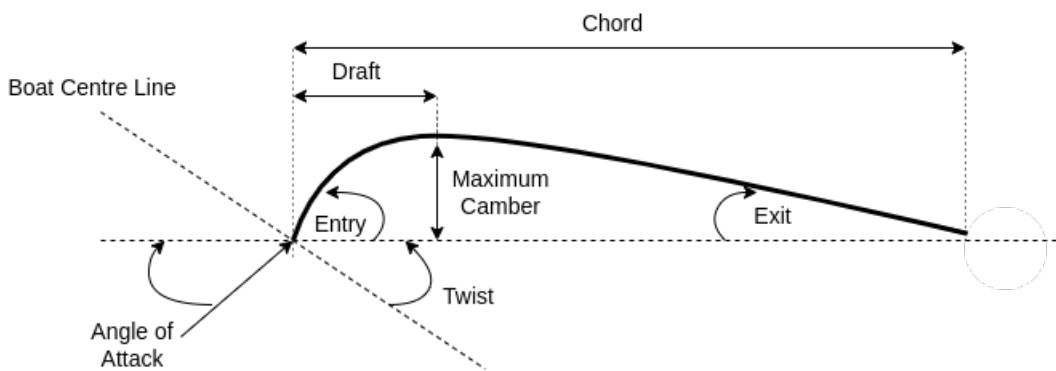
### 3.2.2. Sail Theory

#### Sail Theory and Terminology

Sail makers make use of their own language in naming for sails and sailboats [14]. It is important to know this language in order to have a conversation about sails. For instance the front of a sail is called the *bow* while the rear is called the *stern*. As shown in Figure 3.10 the *luff* is the leading edge of the sail and the *leech* is the trailing. The *foot* is the bottom edge which can be attached to the *boom* or left loose. Sails generally come to a point at the head, the attachment point to the mast at the top of the sail, but it is also common to see sails with a square top. The *tack* and the *clew* are the attachment points of the foot at the *luff* and *leech* respectively.



**Figure 3.10:** Naming convention of a sail

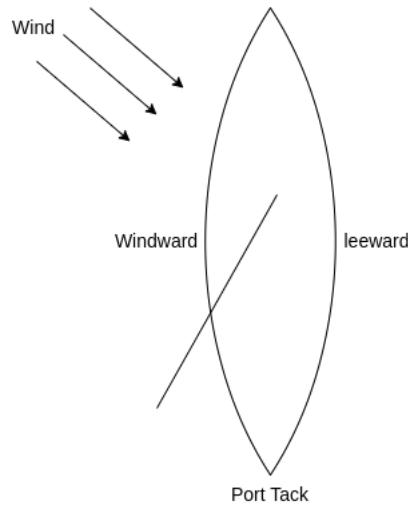


**Figure 3.11:** Terminology of a sail wing and shape

The shape of a sail can vary in its height and span and has almost no thickness, a

horizontal section can be classified using the terminology of a wing. The terms are illustrated in Figure 3.11. The *chord* is the straight line between the leading and trailing edge. *Camber* is then the perpendicular distance from the chord line to the foil. *Draft* is the position of the maximum camber along the chord line. *Entry* and *exit* angles of the foil are, respectively, the angles of the leading and trailing edge to the chord line. Angle of attack(*AoA*) is the angle between the oncoming flow and the chord line and the *twist* angle is the angle between the chord line and the sailboat's centre line.

Depending on the tack of the boat, the side from which the wind is blowing from, illustrated in Figure 3.12, is the *windward* side and the other side will be *leeward* side.



**Figure 3.12:** Windward and leeward definitions

### Aerodynamics of a Sail

The way sail function, is much alike to that of a wing with some differences. Both a sail and a wing generate a force due to a pressure difference acting over their area. The three major differences between a sail and a wing are that a sail has almost no thickness, often has a large camber and generally operate at higher angles of attack. The flow around the sail is also usually disturbed by the mast or stays leading to large areas of separation as shown in [15]. Bernoulli's equation state that the pressure is inversely related to velocity and thus it can be seen through the contraction of the streamlines over the leeward side that the flow will accelerate creating a drop in pressure.

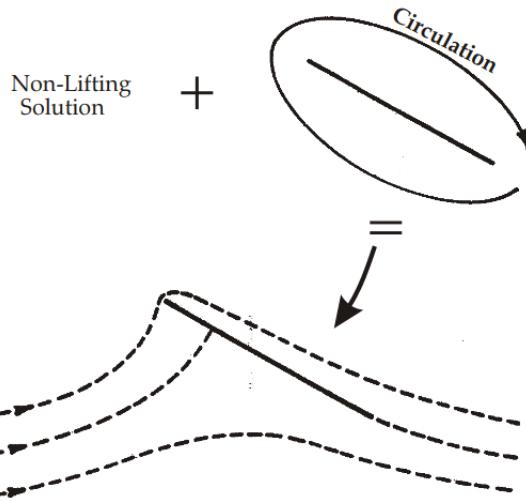
The simplified explanation of how lift is created due to the particles travelling further over the leeward side does not hold much substances for thin aerofoils such as sails. The way a airplane wing generates lift is due to the shape of the airfoil, the air flows faster over the top than it does over the bottom, because it has a further travel distance. With thin-airfoil sails the distance over the top and bottom is the same, so this the same

reasoning for lift does not hold. By looking at the numerical solution of a flat plate, found in [16], gives insight to how a sail is affected by pressure. The first results, illustrated in Figure 3.13, showed that the thin-airfoil has no lift, which is incorrect.



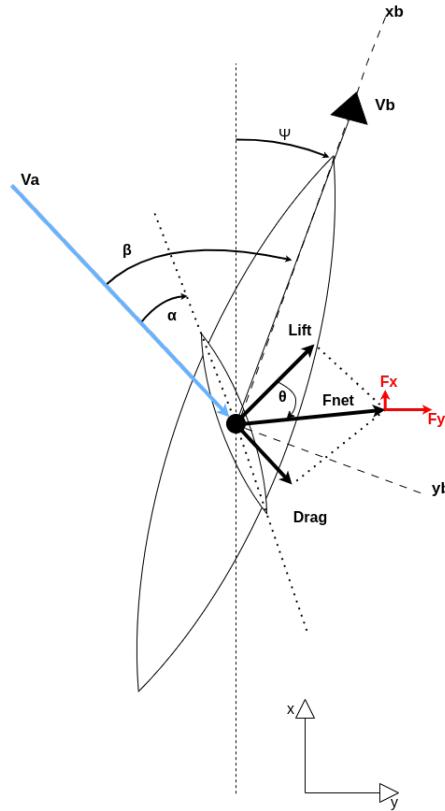
**Figure 3.13:** Flow field without circulation

Noting that these mathematical models determined streamlines that make very sharp turns in getting around the leading edge and trailing edge of the airfoil. For a thin airfoil this would mean infinite velocities, and to reduce these velocities the airfoil can be bent around the leading edge. For the flow at the trailing edge the assumption is made that the airflow will leave the airfoil smoothly in a direction determined by an imaginary slight extension of the airfoil, which is known as the Kutta condition. It has been found that the Kutta condition can be satisfied mathematically by superimposing another type of flow solution, called circulation, onto the flat plate airfoil model. The superposition is shown in Figure 3.14.



**Figure 3.14:** Superposition of circulation and non-circulation solution to give lift

Figure 3.15 illustrates the lift and drag component experienced due to wind on the sail. The drag component is always in the direction of the apparent wind and the lift is perpendicular to the apparent wind.  $\gamma_\omega$  is the apparent wind angle,  $\alpha$  is the AoA of the sail wing,  $\psi$  is the leeway angle, which is the angle between the course and the heading, and  $\theta$  is the sinusoidal angle between the net and lift force.



**Figure 3.15:** Lift and drag forces due to wind on sail wing

The lift and drag forces are defined by,

$$F_L = \frac{1}{2} C_L \rho U_A^2 A \quad (3.86)$$

$$F_D = \frac{1}{2} C_D \rho U_A^2 A \quad (3.87)$$

where  $C_L$  and  $C_D$  are the lift and drag coefficients,  $U_A$  the apparent wind speed,  $A$  area of the sail and  $\rho$  the air density. The lift and drag forces are combined into a single net aerodynamic force whose magnitude and angle relative to the apparent wind perpendicular defined as

$$|F_{net}| = F_{net} = \frac{1}{2} \rho U_A^2 A \sqrt{C_L^2 + C_D^2} \quad (3.88)$$

$$\theta = \tan^{-1} \left( \frac{C_D}{C_L} \right) \quad (3.89)$$

The component parallel to the direction of motion is the aerodynamic driving force  $F_x$ , while the force perpendicular to the direction of motion is the aerodynamic side force. Besides being a function of the angle of attack of the wing sail ( $\theta$  is a function of  $\alpha$ ),  $F_x$  and  $F_y$  are both dependent on the apparent wind angle and the wind profile in which the vessel is sailing.  $F_x$  and  $F_y$  are defined below,

$$F_x = F_{net} \sin(\gamma_\omega - \theta) \quad (3.90)$$

$$F_y = F_{net} \cos(\gamma_\omega - \theta) \quad (3.91)$$

The equations above is valid for any lifting surface, whether it is a rigid wing sail or a cloth sail. For efficient performance of a sailing vehicle, the lifting surface will maximize the aerodynamic driving force while minimizing the aerodynamic side force. At a given leeway angle, the aerodynamic force ratio ( $F_x/F_y$ ) is at a maximum when  $\theta$  is minimized. In the limit that  $C_D/C_L \rightarrow 0$ , the aerodynamic force ratio approaches the limit

$$F_x/F_y \rightarrow \tan(\gamma_\omega)$$

### Sail Forces and Moments

The sail forces and moments are expressed in their horizontal components. When the sail is in the upright position the aerodynamic coefficients  $X'_{s0}$  and  $Y'_{s0}$  are expressed using lift coefficient  $L'_{s0}$  and drag coefficient  $D'_{s0}$  as shown in [17].

$$X'_{s0} = L'_{s0} \sin(\gamma_\omega) - D'_{s0} \cos(\gamma_\omega) \quad (3.92)$$

$$Y'_{s0} = L'_{s0} \cos(\gamma_\omega) + D'_{s0} \sin(\gamma_\omega) \quad (3.93)$$

where the subscript of 0 means value at the upright condition.

In the heeled condition, when the vessel is rotated due to the sail lift and drag force, the effect of the heel on the aerodynamic forces is produced by the reduction of both the apparent wind angle and apparent wind speed. The apparent wind angle in the heeled condition  $\gamma_{\omega\phi}$  is expressed as follows using the apparent wind angle  $\gamma_\omega$  and apparent wind speed  $U_A$ :

$$\gamma_{\omega\phi} = \tan^{-1} \left( \frac{U_A \sin(\gamma_\omega) \cos(\phi)}{U_A \cos(\gamma_\omega)} \right) = \tan^{-1} (\tan(\gamma_\omega) \cos(\phi)) \quad (3.94)$$

The apparent wind speed in the heeled condition  $U_{A\phi}$  is also expressed as:

$$U_{A\phi} = \sqrt{((U_A \cos(\gamma_\omega))^2 + (U_A \sin(\gamma_\omega) \cos(\phi))^2)} = U_A \sqrt{1 - (\sin(\gamma_\omega) \sin(\phi))^2} \quad (3.95)$$

For the close-hauled condition, tacking behaviour of sailboat due to not being able to sail directly into the wind, the sail may not stall due to the small attack angle. Therefore, the lift force will decrease proportionally to the reduction of both the apparent wind angle and the dynamic pressure of flow(square of the apparent wind speed). Hence the decreasing ratio of lift force by the heel angle  $\phi$  can be described as:

$$\left(\frac{\gamma_{\omega\phi}}{\gamma_{\omega}}\right) \left(\frac{U_{A\phi}}{U_A}\right) = \left(\frac{\tan^{-1}(\tan(\gamma_{\omega})\cos(\phi))}{\gamma_{\omega}}\right) \left(1 - (\sin(\gamma_{\omega})\sin(\phi))^2\right) \quad (3.96)$$

The vector of lift force inclines with heel angle and rotates in the normal plane to the apparent wind axis. Since the angle between the apparent wind axis and the boat center line(heeling axis) is  $\gamma_{\omega}$ , the rotating angle of the lift force vector  $\phi'$  in the normal plane to the apparent wind axis is given by:

$$\phi' = \sin^{-1}(\cos(\gamma_{\omega})\sin(\phi)) \quad (3.97)$$

Therefore, the decreasing ratio of horizontal component of the lift force is expressed as:

$$\left(\frac{\gamma_{\omega\phi}}{\gamma_{\omega}}\right) \left(\frac{U_{A\phi}}{U_A}\right) \cos(\phi') = \left(\frac{\tan^{-1}(\tan(\gamma_{\omega})\cos(\phi))}{\gamma_{\omega}}\right) \left(1 - (\sin(\gamma_{\omega})\sin(\phi))^2\right) \cos(\sin^{-1}(\cos(\gamma_{\omega})\sin(\phi))) \quad (3.98)$$

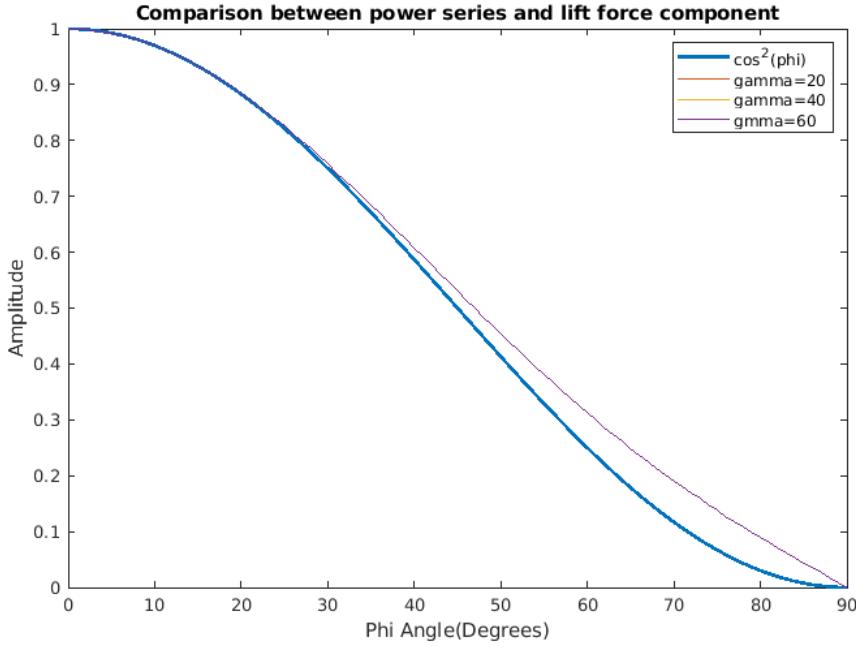
Expanding the above mention equation in a power series and assuming that  $\gamma_A$  is small, results in

$$\left(\frac{\gamma_{\omega\phi}}{\gamma_{\omega}}\right) \left(\frac{U_{A\phi}}{U_A}\right) \cos(\phi') \approx \left(\cos^2(\phi) + \frac{1}{2}\sin^2(\phi)\right) \cos(\phi) = \frac{1}{2} \left(\cos(\phi) + \cos^3(\phi)\right) \quad (3.99)$$

Equation 3.99 can be further expanded in terms of  $\phi$  and results in

$$\left(\frac{\gamma_{\omega\phi}}{\gamma_{\omega}}\right) \left(\frac{U_{A\phi}}{U_A}\right) \cos(\phi') \approx 1 - \phi^2 \quad (3.100)$$

Equation 3.100 is incidentally equal to the first two terms of the power series for the  $\cos^2(\phi)$  function. Hence the curve of the  $\cos^2(\phi)$  was compared with the calculated results in Equation 3.98 for three  $\gamma_{\omega}$  cases. The calculated results show agreement with the curve of  $\cos^2(\phi)$  in spite of the large  $\gamma_{\omega}$ . Therefore, we adopted the formula of  $\cos^2(\phi)$  to express the decreasing ratio of the horizontal component of the lift force in place of Equation 3.98.



**Figure 3.16:** Lift force component represented as a power curve

Finally, when the lift coefficient represents the variation of the lift force including the contribution of dynamic pressure of apparent wind speed, the horizontal component of lift coefficient in the heeled condition  $L'_s$  is described as:

$$L'_s = L'_{s0} \cos^2(\phi) \quad (3.101)$$

The main part of the drag is caused by the induced drag, which is in proportion to the square of the lift force. The reduction of lift force expressed by Equation 3.96 is also approximated by  $\cos(\phi)$ . The vector of the drag force is in line with the apparent wind axis and does not incline by the heel angle. Therefore the horizontal component of the drag coefficient  $D'_s$  is described as:

$$D'_s = D'_{s0} \cos^2(\phi) \quad (3.102)$$

From these results, the aerodynamic coefficients in the horizontal components  $X'_s$  and  $Y'_s$  are then expressed as follows using the coefficients at the upright condition  $L'_{s0}$  and  $D'_{s0}$ :

$$X'_s = L'_s \sin(\gamma_\omega) - D'_s \cos(\gamma_\omega) = L'_{s0} \cos^2(\phi) \sin(\gamma_\omega) - D'_{s0} \cos^2(\phi) \cos(\gamma_\omega) = X'_{s0} \cos^2(\phi) \quad (3.103)$$

$$Y'_s = L'_s \cos(\gamma_\omega) - D'_s \sin(\gamma_\omega) = L'_{s0} \cos^2(\phi) \cos(\gamma_\omega) - D'_{s0} \cos^2(\phi) \sin(\gamma_\omega) = Y'_{s0} \cos^2(\phi) \quad (3.104)$$

The moment  $K_s$  is generated mainly by the  $Y_s$  force, however it is also affected by the component normal to the mast, hence

$$K'_s = -Y'_s \left( \frac{z_{GCE}^G}{\sqrt{S_A}} \right) / \cos(\phi) \quad (3.105)$$

where  $z_{GCE}^G$  is the z-coordinate of the geometric center of effort of the sail from the CG of the boat and negative upwards.

The moment  $N_s$  is also generated mainly by the  $Y_s$  force, however, it is well known that the  $N_s$  is also affected by the heel angle  $\phi$  due to the application point of the thrust force  $X_s$  moving outboard to lee side. Therefore  $N'_s$  can be written, including the effect of  $X'_{s0}$  as

$$N'_s = \left( Y'_{s0} \frac{x_{GCE}^G}{\sqrt{S_A}} + X'_{s0} \frac{z_{GCE}^G}{\sqrt{S_A}} \sin(\phi) \right) \cos^2(\phi) \quad (3.106)$$

where  $x_{GCE}^G$  is x-coordinate of the geometric center of effort of the sail from CG of the boat.

The sail force and moment components defined above are shown below:

$$X_s = X'_{s0} \cos^2(\phi) \times \frac{1}{2} \rho_a U_A^2 S_A \quad (3.107)$$

$$Y_s = Y'_{s0} \cos^2(\phi) \times \frac{1}{2} \rho_a U_A^2 S_A \quad (3.108)$$

$$K'_s = -Y'_s \left( \frac{z_{GCE}^G}{\sqrt{S_A}} \right) / \cos(\phi) \times \frac{1}{2} \rho_a U_A^2 S_A \quad (3.109)$$

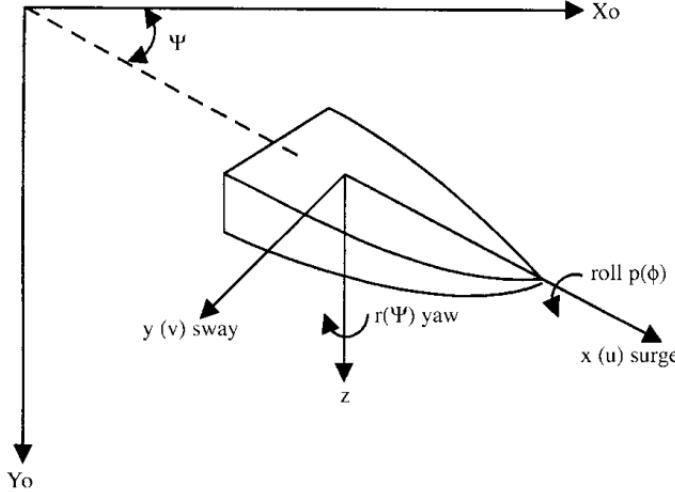
$$N'_s = \left( Y'_{s0} \frac{x_{GCE}^G}{\sqrt{S_A}} + X'_{s0} \frac{z_{GCE}^G}{\sqrt{S_A}} \sin(\phi) \right) \cos^2(\phi) \times \frac{1}{2} \rho_a U_A^2 S_A \quad (3.110)$$

### 3.3. Linearizations and Simplifications

The state space model associated with the first order Nomoto model is both observable and controllable. The state space model associated with the second order Nomoto model is also observable, however it is controllable only if the effective sway time constant is different from the effective yaw time constant. The first order model is very popular in adaptive autopilot applications. Model reduction for a fourth order transfer function ship model describing the sway-yaw-roll dynamics is conducted to reach the second order Nomoto model describing the sway-yaw dynamics and the first order Nomoto model describing the yaw dynamics itself.

### 3.3.1. Ship Steering Dynamics Model Reduction

For ship autopilots design, a simple model with average predicting capability is usually preferred. Based on the linearized surge-sway-yaw-roll equations of motion, a fourth order transfer function relating the yaw rate to the rudder angle is derived.



**Figure 3.17:** Sway-yaw-roll motion coordinate system

Upon linearization with respect to a straight line motion with a constant forward speed  $u_0$ , the surge equation is decoupled and the following linear coupled sway-yaw-roll equations follow,

$$m(\dot{v} + u_0 r) = Y_v V + Y_{\dot{v}} \dot{V} + Y_\phi \phi + Y_p P + Y_{\dot{p}} \dot{P} + Y_r r + Y_{\dot{r}} \dot{r} + Y_\delta \delta \quad (3.111)$$

$$I_x \ddot{\phi} = K_v V + K_{\dot{v}} \dot{V} + Y_\phi \phi + K_p P + K_{\dot{p}} \dot{P} + K_r r + K_{\dot{r}} \dot{r} + K_\delta \delta - mgGM\phi \quad (3.112)$$

$$I_z \ddot{\psi} = N_v V + N_{\dot{v}} \dot{V} + N_\phi \phi + N_p P + N_{\dot{p}} \dot{P} + N_r r + N_{\dot{r}} \dot{r} + N_\delta \delta \quad (3.113)$$

where  $Y_v$ ,  $Y_{\dot{v}}$ , ... indicate the hydrodynamic coefficients; for instance  $Y_v$  indicates the derivative of the sway force  $Y$  to the sway speed  $V$  evaluated at the reference condition;  $m$  is the mass of the ship;  $I_x$  is the moment of inertia about the x-axis;  $I_z$  is the moment of inertia about the z-axis;  $V$  is the sway speed;  $u$  is the surge speed;  $r$  is the yaw rate;  $\psi$  is the heading angle defined by  $\dot{\psi} = r$ ;  $p$  is the roll rate;  $\phi$  is the roll angle defined by  $\dot{\phi} = p$  and  $GM$  is the metacentric height, which indicates the restoring capability of a ship in rolling motion. Taking the Laplace transform of Equations 6.1-3 and rearranging, we have

$$a_1 V = a_2 \Phi + a_3 r + a_4 \delta \quad (3.114)$$

$$b_1\Phi = b_2V + b_3r + b_4\delta \quad (3.115)$$

$$c_1r = c_2V + c_3\Phi + c_4\delta \quad (3.116)$$

where

$$a_1 = m - Y_{\dot{v}}S - Y_v \quad (3.117)$$

$$a_2 = Y_{\dot{p}}S^2 + Y_pS + Y_\phi \quad (3.118)$$

$$a_3 = Y_{\dot{r}}S + Y_r + mu_0 \quad (3.119)$$

$$a_4 = Y_\delta \quad (3.120)$$

$$b_1 = (I_X - K_{\dot{p}})S^2 - K_pS + mgGM \quad (3.121)$$

$$b_2 = K_{\dot{v}}S + K_v \quad (3.122)$$

$$b_3 = K_{\dot{r}}S + K_r \quad (3.123)$$

$$b_4 = K_\delta \quad (3.124)$$

$$c_1 = (I_Z - N_{\dot{r}})S - N_r \quad (3.125)$$

$$c_2 = N_{\dot{v}}S + N_v \quad (3.126)$$

$$c_3 = N_{\dot{p}}S^2 + N_pS + N_\phi \quad (3.127)$$

$$c_4 = N_\delta \quad (3.128)$$

After eliminating the sway speed  $V$  and roll angle  $\phi$  from Equations 6.4-6, the following transfer function relating the yaw rate  $r$  to the rudder angle  $\delta$  can be obtained:

$$\frac{r}{\delta} = \frac{a_1(b_1c_4 + b_4c_3) + a_2(b_4c_2 - b_2c_4) + a_4(b_1c_2 + b_2c_3)}{a_1(b_1c_1 - b_3c_3) - a_2(b_2c_1 + b_3c_2) - a_3(b_1c_2 + b_2c_3)} \quad (3.129)$$

It can be easily verified that the numerator of Equation 3.129 is third order in  $S$ , while the denominator is fourth order in  $S$ . Hence, Equation 3.129 can be expressed in the following form.

$$\frac{r}{\delta} = \frac{K(1 + T_3S)(S^2 + 2n\omega_0S + w_0^2)}{(1 + T_1S)(1 + T_2S)(S^2 + 2\delta\omega_nS + w_n^2)} \quad (3.130)$$

where the quadratic factors are due to the coupling effect from the roll mode on the yaw rate. The zero  $(1 + T_3S)$  and the pole  $(1 + T_2S)$  are due to the coupling effect from the

sway mode on the yaw dynamics. If the roll mode is neglected, Equation 3.130 can be further reduced to the following form

$$\frac{r}{\delta} = \frac{K(1 + T_3 S)}{(1 + T_1 S)(1 + T_2 S)} \quad (3.131)$$

Equation 3.131 is known as the second order Nomoto model, where  $K$  is the static yaw rate gain, and  $T_1$ ,  $T_2$  and  $T_3$  are time constants. Numerical values of the parameters in Equation 3.131 for a Mariner class vessel are given by  $T_1 = 118$ ,  $T_2 = 7.8$ ,  $T_3 = 18.5$  and  $K = 0.185$ . The zero term  $(1 + T_3 S)$  and the high frequency pole term  $(1 + T_2 S)$  are due to the coupling effect from the sway mode. In practice, because the pole term  $(1 + T_2 S)$  and the zero term  $(1 + T_3 S)$  in Equation 3.131 nearly cancel each other, a further simplification can be done to give first order Nomoto model

$$\frac{r}{\delta} = \frac{K}{(1 + TS)} \quad (3.132)$$

where

$$T = T_1 + T_2 - T_3 \quad (3.133)$$

in Equation 3.133 is called the effective yaw rate time constant. It is obtained by equating the right hand side of Equation 3.131 to the right hand side of Equation 3.132 requiring the equality relationship to hold up to first order in  $S$ . If  $T_2 = T_3$ ; namely, a perfect cancellation occurs, then the equality relationship is true up to second order in  $S$ , then  $T$  is of course, equal to  $T_1$ . For the Mariner class vessel, the value of the effective constant  $T$  in Equation 3.133 is given by  $T = 107.3$ .

The first order Nomoto model defined by Equation 3.132 is widely employed in the ship steering autopilot design. The yaw dynamics is characterized by the parameters  $K$  and  $T$ , which can be easily identified from standard maneuvering tests. In practice, ship steering autopilots are designed for heading angle control. Hence, it is the transfer function relating the heading angle  $\psi$  to the rudder angle  $\delta$  being needed in the autopilot design. Since the yaw rate  $r$  is actually the time derivative of  $\psi$ , the required transfer function can be readily obtained by adding an integrator  $\frac{1}{s}$  to the transfer function models defined by Equation 3.131 and Equation 3.132.

### 3.3.2. First Order Nomoto Model

The controllability, observability and identifiability properties of the first order Nomoto model-based system will be discussed in this section. The identifiability property will be discussed with respect to the first order Nomoto transfer function model. However, the controllability and observability properties have to be discussed with respect to the

state-space model derived from the Nomoto model. This is because the transfer function model always represents the observable and controllable parts of the system dynamics. If there is any unobservable or uncontrollable parts of the dynamics, they are cancelled out before reaching the transfer function model form. Hence, it only makes sense to discuss the controllability and observability properties of the state space model, which contains the observable, controllable and uncontrollable modes. Equation 3.132 can be expressed in time domain as

$$T\dot{r} + r = K\delta \quad (3.134)$$

with the notation

$$\dot{\psi} = r \quad (3.135)$$

where  $\psi$  is the heading of the ship. Equation 3.134 can be written as

$$T\ddot{\psi} + \dot{\psi} = K\delta \quad (3.136)$$

Equation 3.135 and 3.136 can be arranged in the standard state space form

$$\dot{x} = Ax + Bu \quad (3.137)$$

$$y = Cx + du \quad (3.138)$$

where

$$x = \begin{bmatrix} \psi \\ r \end{bmatrix} \quad (3.139)$$

$$u = \delta \quad (3.140)$$

$$y = \psi \quad (3.141)$$

and

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{1}{T} \end{bmatrix} \quad (3.142)$$

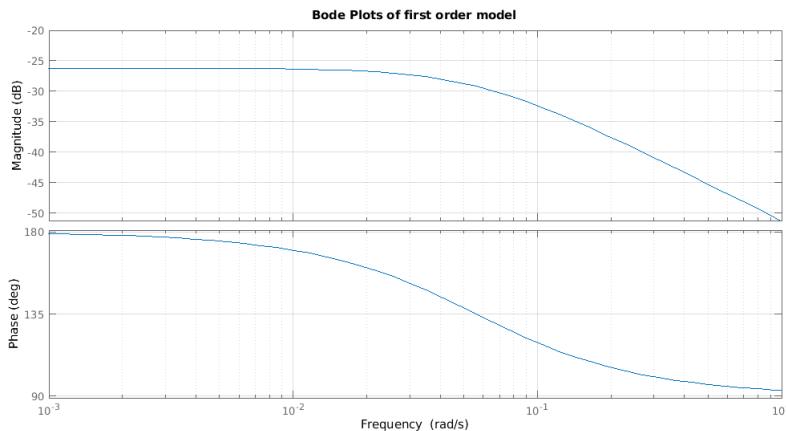
$$B = \begin{bmatrix} 0 \\ \frac{K}{T} \end{bmatrix} \quad (3.143)$$

$$C = [1 \ 0] \quad (3.144)$$

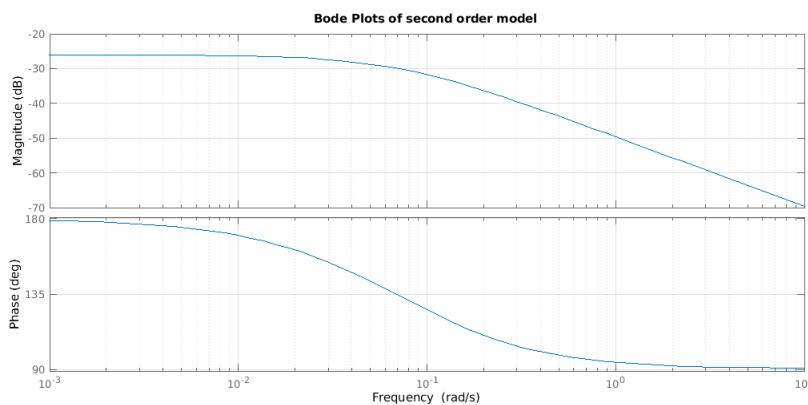
The first order system is both controllable and observable. Consequently, the first order Nomoto model satisfies the identifiability property, hence, on-line estimation of the model parameters based on the measured rudder and yaw rate information will be possible and

adaptive control strategy can be successfully implemented.

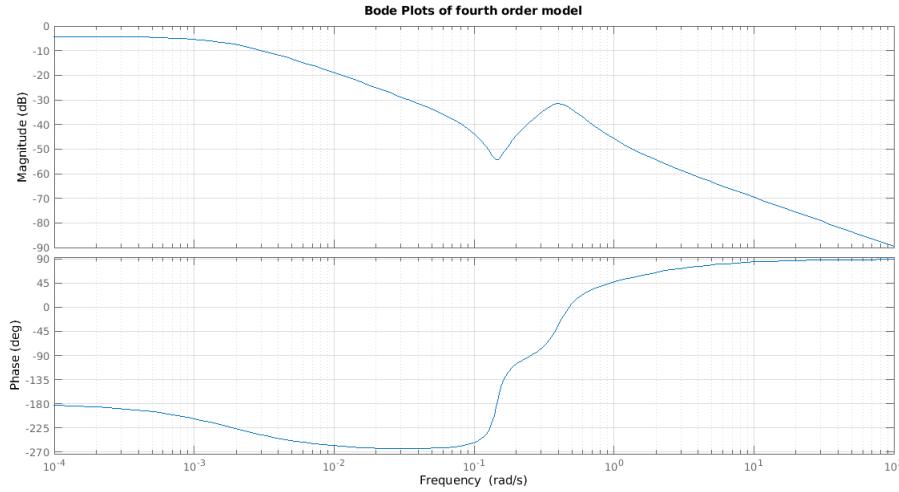
The first order Nomoto model being very popular in the design for ship steering autopilots is not without reason. Its simplicity and reasonable accuracy in describing small rudder angle yaw dynamics make it attractive. Moreover, the relative easiness in identifying the model parameters makes it suitable for adaptive autopilot application, where on-line estimation of the model parameters is important. The second order Nomoto model includes the coupling effect from the sway to the yaw mode. This introduces a zero and a high frequency pole into the transfer function. The zero structure contributes to the overshoot phenomenon, which can be seen in the yaw rate for large rudder angle maneuvers. However, the ill-conditioning problem associated with the second order Nomoto model in the identification of the model parameters from input-output data seems to outweigh the improvement gained in its modeling capability. An approach that retains the zero structure while avoiding the ill-conditioning problem during on-line estimation has been proposed. The bode plots of the first, second and fourth order model is shown below,



**Figure 3.18:** First Order bode Plot



**Figure 3.19:** Second Order Bode Plot



**Figure 3.20:** Fourth Order Bode Plot

### 3.3.3. Stabilization

This Chapter starts with open-loop analysis and maneuverability followed by state-of-the-art linear PID design methods. Conventional PID control systems have their origin from SISO linear systems theory. However, it is possible to generalize this to nonlinear MIMO systems by using results from robotics. This requires that the ocean vessel equations of motion are expressed in a vectorial setting:

$$\dot{n} = J_\Theta(n)v \quad (3.145)$$

$$M\ddot{v} + C(v)\dot{v} + D(v)v + g(n) = \tau + \omega \quad (3.146)$$

For this model class, MIMO nonlinear PID control systems can be designed by exploiting the fact that the mass matrix is positive definite and constant ( $M = M^T > 0$ ,  $\dot{M} = 0$ ), the Coriolis and centripetal matrix  $C(v) = -C^T(v)$  is skew-symmetrical and the damping matrix  $D(v) > 0$  is strictly positive.

#### Open-Loop Stability and Maneuverability

When designing a motion control system a compromise between stability and maneuverability must be made,

- *Stability* of an uncontrolled ocean vessel can be defined as the ability to return to an equilibrium point after a disturbance, without any corrective action of the actuators
- *Maneuverability*, on the other hand, is defined as the capability of the craft to carry out specific maneuvers.

It is well known that a craft that is easy to maneuver, for instance fighter aircraft or a high-speed watercraft, can be marginally stable, or even unstable in open loop. On the other hand, excessive stability implies that the control effort will be excessive in a maneuvering situation whereas a marginally stable ship is easy to maneuver. Consequently, a compromise between stability and maneuverability must be made.

### Straight-Line, Directional and Positional Motion Stability

For ocean vessels it is common to distinguish between three type of stability, namely:

- Straight-line stability
- Directional or course stability
- Positional motion stability

This can be explained using open-loop and closed-loop stability analyzes. In order to understand the different types of stability one can consider the following test system:

$$\dot{x} = u \cos(\psi) - v \sin(\psi) \approx u_0 \cos(\psi) \quad (3.147)$$

$$\dot{y} = u \sin(\psi) + v \cos(\psi) \approx u_0 \sin(\psi) \quad (3.148)$$

$$\dot{\psi} = r \quad (3.149)$$

$$T\dot{r} + r = K\delta + \omega \quad (3.150)$$

where  $\omega$  is the external disturbances and  $u_0 = \text{constant}$  is the cruise speed. The first two equations represent the  $(x, y)$  position of the vessel while the last two equations describe the yaw dynamics modeled by Nomoto's first-order model. For simplicity, it is assumed that the yaw motion of the vessel is stabilized by a PD-controlled rudder servo:

$$\delta = -K_p(\psi - \psi_d) - K_d r \quad (3.151)$$

where  $\psi_d = \text{constant}$  denotes the desired heading angle and  $K_p$  and  $K_d$  are two positive regulator gains. Substituting the control law, in Equation 3.151, into Nomoto's first-order model, in Equation 3.149, yields the closed-loop system

$$T\ddot{\psi} + (1 + KK_d)\dot{\psi} + KK_p\psi = KK_p\psi_d + \omega \quad (3.152)$$

The closed-loop system represents a second-order mass-damper-spring system

$$m\ddot{\psi} + d\dot{\psi} + k\psi = f(t) \quad (3.153)$$

with driving input

$$f(t) = k\psi_d + \omega \quad (3.154)$$

The eigenvalues  $\lambda_{1,2}$ , the natural frequency  $\omega_n$  and the relative damping ratio  $\xi$  for the mass-damper-spring system are

$$\lambda_{1,2} = \frac{-d \pm \sqrt{d^2 - 4km}}{2m} \quad (3.155)$$

$$\omega_n = \sqrt{\frac{k}{m}} \quad (3.156)$$

$$\xi = \frac{d}{2} \frac{1}{\sqrt{km}} \quad (3.157)$$

**Instability:** For uncontrolled ocean vessel ( $K_p = K_d = 0$ ) instability occurs when

$$\lambda_1 = -\frac{d}{m} = -\frac{1}{T} > 0 \quad (3.158)$$

$$\lambda_2 = 0 \quad (3.159)$$

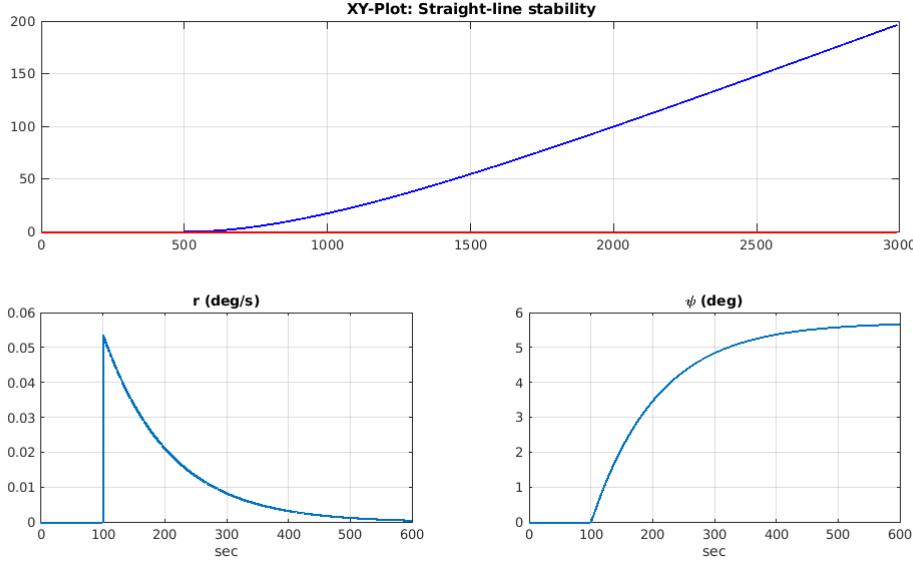
which simply states that  $T < 0$ .

**Straight-Line Stability:** Consider an uncontrolled ocean vessel ( $K_p = K_d = 0$ ) moving in a straight path. If the new path is straight after a disturbance  $\omega$  in yaw the craft is said to have straight-line stability. The direction of the new path will usually differ from the initial path because no restoring forces are present( $k=0$ ). This corresponds to

$$\lambda_1 = -\frac{d}{m} = -\frac{1}{T} < 0 \quad (3.160)$$

$$\lambda_2 = 0 \quad (3.161)$$

Consequently, the requirement  $T>0$  implies straight-line stability for the uncontrolled craft ( $\xi=0$ ).

**Figure 3.21:** Straight Line stability

**Directional Stability(Stability on Course):** Directional stability is a much stronger requirement than straight-line stability. Directional stability requires the final path to be parallel to the initial path that is obtained  $K_p > 0 \Rightarrow k > 0$ . Additional damping is added through  $K_d > 0$ . This corresponds to PD control. A ocean vessel is said to be directional stable if both eigenvalues have negative real parts, that is

$$\operatorname{Re}\{\lambda_{1,2}\} < 0 \quad (3.162)$$

The following two types of directional stability are observed: *No oscillations* ( $d^2 - 4km \leq 0$ ): This implies that both eigenvalues are negative and real, that is  $\xi \leq 1$  such that

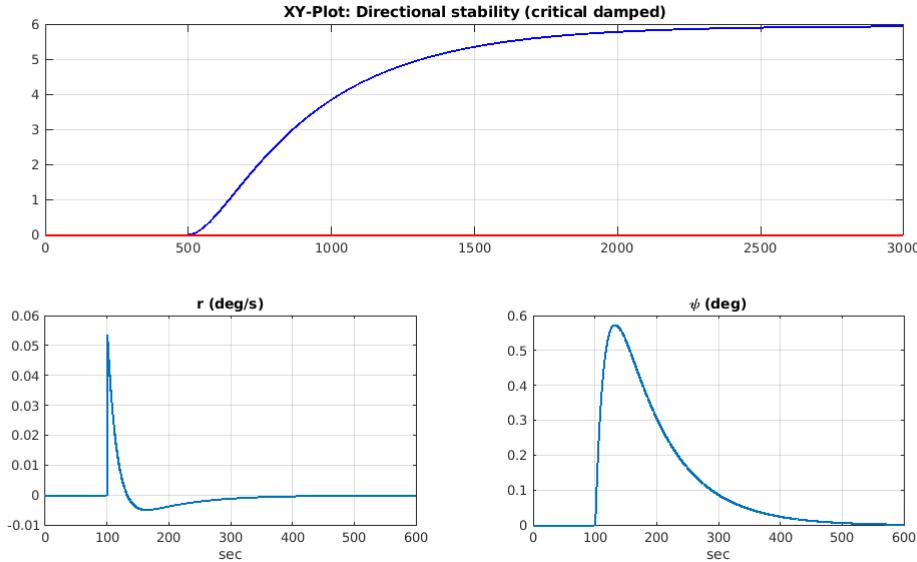
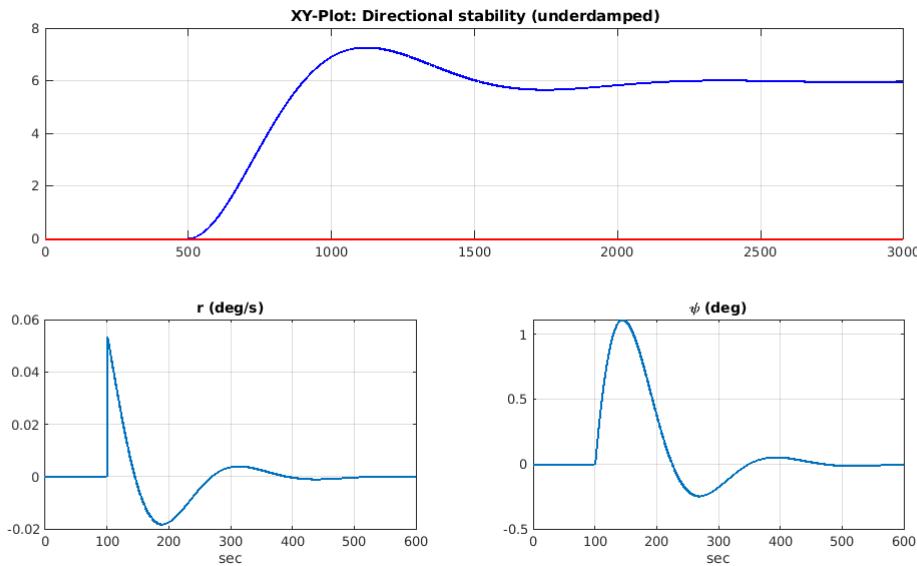
$$\lambda_{1,2} = \frac{-d \pm \sqrt{d^2 - 4km}}{2m} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right) \omega_n < 0 \quad (3.163)$$

For a critically damped system  $\xi = 1$ , such that  $\lambda_{1,2} = -1/2(d/m) = -\omega_n$ .

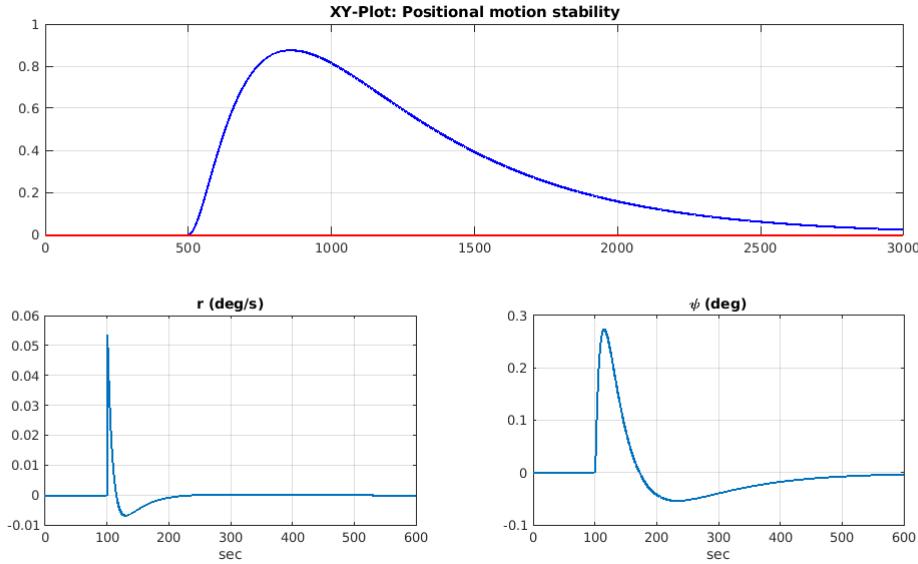
*Damped Oscillator* ( $d^2 - 4km < 0$ ): This corresponds to two imaginary eigenvalues  $\lambda_{1,2}$ , with negative real parts ( $\xi < 1$ ), that is

$$\lambda_{1,2} = \frac{-d \pm j\sqrt{d^2 - 4km}}{2m} = \left(-\xi \pm j\sqrt{\xi^2 - 1}\right) \omega_n \quad (3.164)$$

Directional stability for a critically damped and under damped ocean vessel is illustrated in Figure 3.22 and 3.23. Notice the oscillations in both positions and yaw angle in underdamped ocean vessel. Directional stability requires feedback control since there are no restoring forces in yaw. However in heave, roll and pitch where metacentric restoring forces are present ( $k > 0$ ) no feedback is required to damp out the oscillations.

**Figure 3.22:** Directional Stability(critically damped)**Figure 3.23:** Directional Stability(under damped)

**Positional Motion Stability:** Positional motion stability implies that the ship should return to its original path after a disturbance, illustrated in Figure 3.24. This can be achieved by including the integral action in the controller. Hence, a PID controller can be designed to compensate for the unknown disturbance term  $\omega$  while a PD controller will generally result in a steady-state offset.



**Figure 3.24:** Positional Motion Stability

### Criteria for Straight-Line Stability

Recall that a ship is said to be dynamically straight-line stable if it returns to a straight-line motion after a disturbance in yaw without any corrective action from the rudder. Consequently, instability refers to the case when the ship goes into a starboard or port turn without any rudder deflections. For Nomoto's first-order model straight-line motion was guaranteed for a positive time constant  $T$ . Similarly, it is possible to derive a criterion for straight-line stability for the state-space model:

$$\mathbf{M}\dot{\mathbf{v}} + \mathbf{N}(u_0)\mathbf{v} = \mathbf{b}\delta \quad (3.165)$$

where  $\mathbf{v} = [v, r]^T$ . Applications of *Laplace's transformation* to be linear model, in Equation 3.165, with  $v(0) = \mathbf{0}$  yields

$$[\mathbf{M}s + \mathbf{N}(u_0)]\mathbf{v}(s) = \mathbf{b}\delta(s) \quad (3.166)$$

consequently

$$\mathbf{v}(s) = [\mathbf{M}s + \mathbf{N}(u_0)]^{-1}\mathbf{b}\delta(s) = \frac{adj(\mathbf{M}s + \mathbf{N}(u_0))}{det(\mathbf{M}s + \mathbf{N}(u_0))}\mathbf{b}\delta(s) \quad (3.167)$$

The characteristic equation is

$$det(\mathbf{M}\sigma + \mathbf{N}(n_0)) = A\sigma^2 + B\sigma + C = 0 \quad (3.168)$$

where

$$A = \det(\mathbf{M}) \quad (3.169)$$

$$B = n_{11}m_{22} + n_{22}m_{11} - n_{12}m_{21} - n_{21}m_{12} \quad (3.170)$$

$$C = \det(N(u_0)) \quad (3.171)$$

The two roots  $\sigma_1, 2$ , both of which must have negative real parts for open-loop stability, are

$$\operatorname{Re}\{\sigma_{1,2}\} = \operatorname{Re}\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right) < 0 \quad (3.172)$$

The quantities  $\sigma_{1,2}$  are often referred to as the control-fixed stability indices for straight-line stability. Alternatively, the Routh stability criterion can be applied.

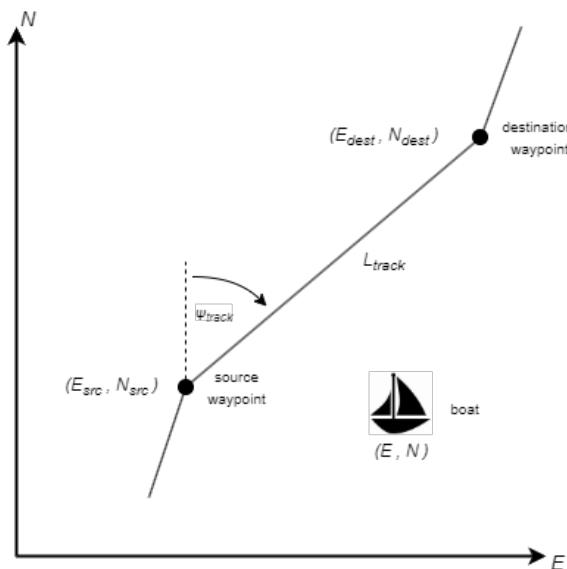
# Chapter 4

## Control Systems

### 4.1. Guidance and Waypoint Navigation

#### 4.1.1. Guidance System

The guidance system is similar to the aircraft guidance system, discussed in [18]. The defined guidance consists of a series of straight-line path segments between waypoints. Each waypoint is a set of North and East coordinates on a map. The straight line between two consecutive waypoints is defined as the ground track. The purpose of the guidance controller is to control the ocean vessel onto the ground track by controlling the cross-track position error to zero. Given that you have the source waypoint and the destination waypoint, the heading angle and the length of the ground track can be calculated as illustrated in Figure

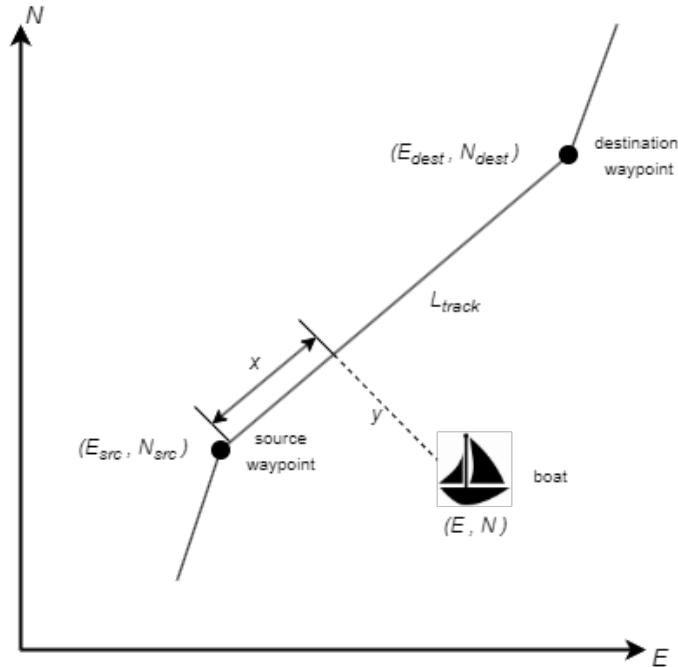


**Figure 4.1:** Ground track between source waypoint and destination waypoint

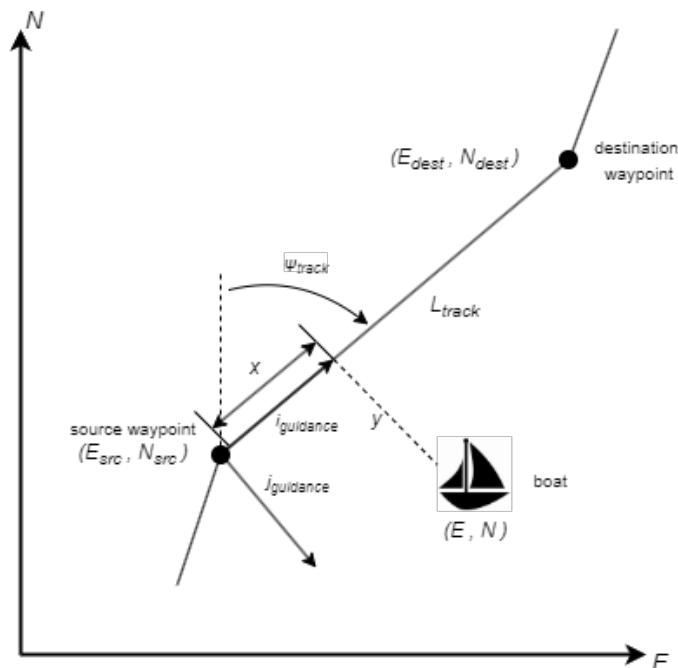
The track heading  $\psi_{track}$  and the track length  $L_{track}$  is calculated as

$$\psi_{track} = \tan^{-1} \left( \frac{E_{dest} - E_{src}}{N_{dest} - N_{src}} \right) \quad (4.1)$$

$$l_{track} = \sqrt{(N_{dest} - N_{src})^2 + (E_{dest} - E_{src})^2} \quad (4.2)$$



**Figure 4.2:** Cross-track error and in-track distance along track



**Figure 4.3:** Guidance Axis System

The origin of the guidance axis system is at the location of the source waypoint, the x-axis is parallel to the ground track and pointing in the direction of the destination waypoint, its y-axis is perpendicular to the ground track, and its z-axis coincides with the down axis of the NED axis system. The guidance axis system is obtained by rotating the NED axis system through the track heading  $\psi_{track}$ , and by moving its origin to the location of the source waypoint. To obtain the cross-track error and the in-track distance, the boat position is first transformed from the NED axes to the guidance axes. The cross-track error is then simply the y-component in the guidance axis system, and the in-track distance is simply the x-component in the guidance axis system. The boat's position is transformed from the NED axis system to the guidance axis system with the following equation

$$\begin{bmatrix} s \\ e \end{bmatrix} = \begin{bmatrix} \cos(\psi_{track}) & \sin(\psi_{track}) \\ -\sin(\psi_{track}) & \cos(\psi_{track}) \end{bmatrix} \begin{bmatrix} N - N_{src} \\ E - E_{src} \end{bmatrix} \quad (4.3)$$

The boat is following the ground track when the boat heading equals the ground track heading and the cross-track error is equal to zero. The boat will have reached the destination waypoint when its in-track distance equals the length of the ground track.

#### 4.1.2. Straight path Generation based on Waypoints

For surface craft vessels, discussed in [8], only two coordinates  $(x_k, y_k)$  for  $k = 1, \dots, n$ . The waypoint database therefore consists of

$$wpt.pos = (x_0, y_0), (x_1, y_1), \dots, (x_n, y_n) \quad (4.4)$$

Additionally other waypoint properties such as speed and heading is defined as,

$$wpt.speed = U_0, U_1, \dots, U_n \quad (4.5)$$

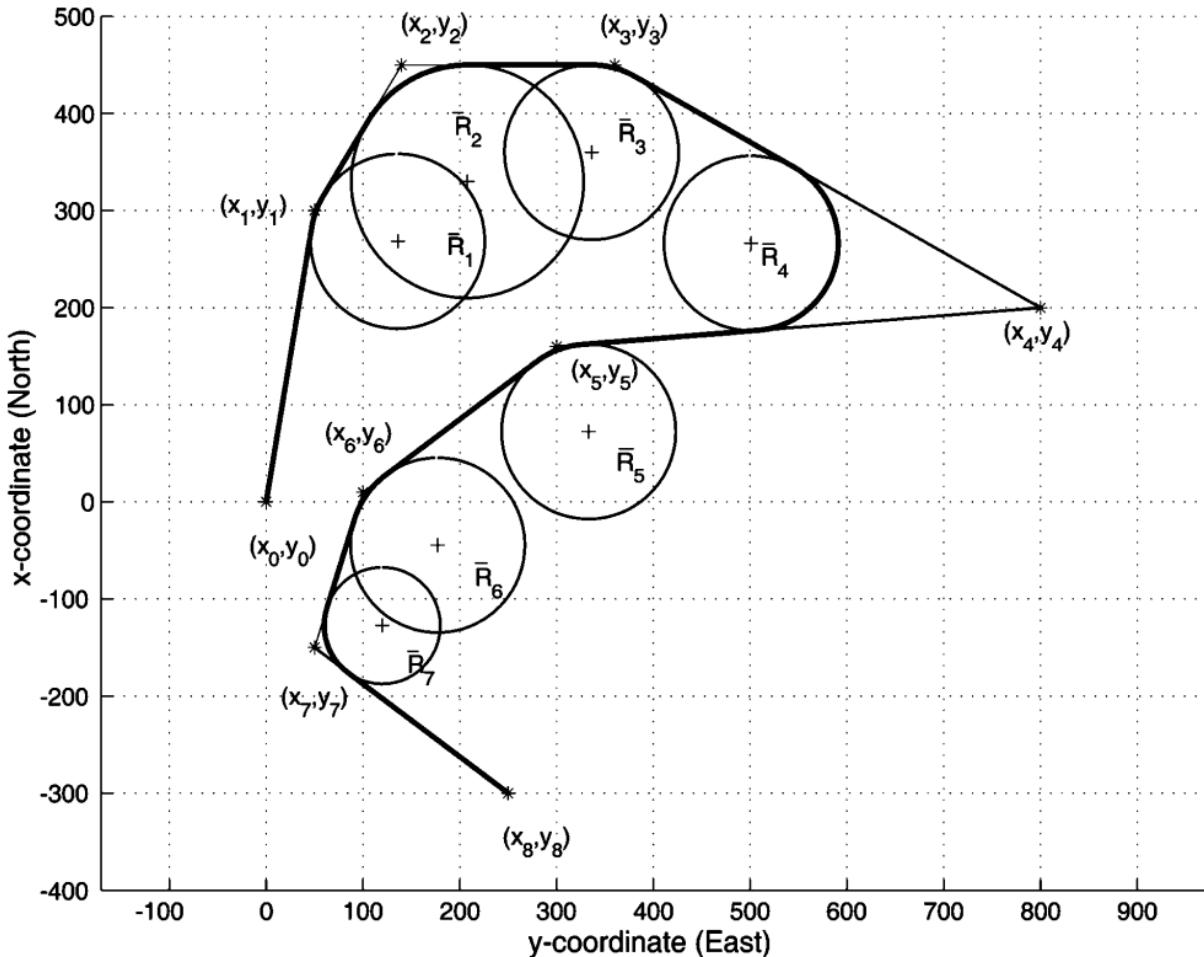
$$wpt.heading = \psi_0, \psi_1, \dots, \psi_n \quad (4.6)$$

In the case of a sailboat the speed is dependent on the wind and therefor is not considered in the waypoint navigation. The states that are thus important for sailboats during navigation are  $(x_i, y_i, \psi_i)$ , which are called the *pose*. The heading angle is usually not important but in the case of a sailboat it is important due to the constraint on sailing angles especially when performing the tacking maneuvering, which is discussed in Chapter ... .

In 1957 Dubins [19] found the shortest path for path following and can be summarised as, "The shortest path (minimum time) between two configurations  $(x, y, \psi)$  of a craft moving

at constant speed  $U$  is a path formed by a straight lines and circular arc segments.” Since a craft is considered the start and end configurations are expressed in terms of positions  $(x, y)$ , heading angle  $\psi$  and in addition it is assumed there is bounds on the turning rate  $r$ . Although this method has drawbacks with a jump in turn rate, the use of arcs and straight lines is very simplistic. Figure 4.4 illustrates a path generated by the Dubins method. Also that needs to be considered is that the operator can specify a circle with radius  $R_i$  about each waypoint and can be stored in the database as

$$wpt.radius = R_0, R_1, \dots, R_n \quad (4.7)$$



**Figure 4.4:** Straight lines and circular arc segments for waypoint guidance

### 4.1.3. Line-of-Sight (LOS) Steering Law

In order to steer the boat on a path a method known as the *lookahead – based steering* [20]. For path-following purposes, only the cross-track error is relevant since  $e(t) = 0$  means that the boat has converged to the straight line. Thus the associated control objective for straight-line path following becomes

$$\lim_{t \rightarrow \infty} e(t) = 0 \quad (4.8)$$

For lookahead-based steering, the course angle assignment is separated into two parts:

$$x_d(e) = x_p + x_r(e) \quad (4.9)$$

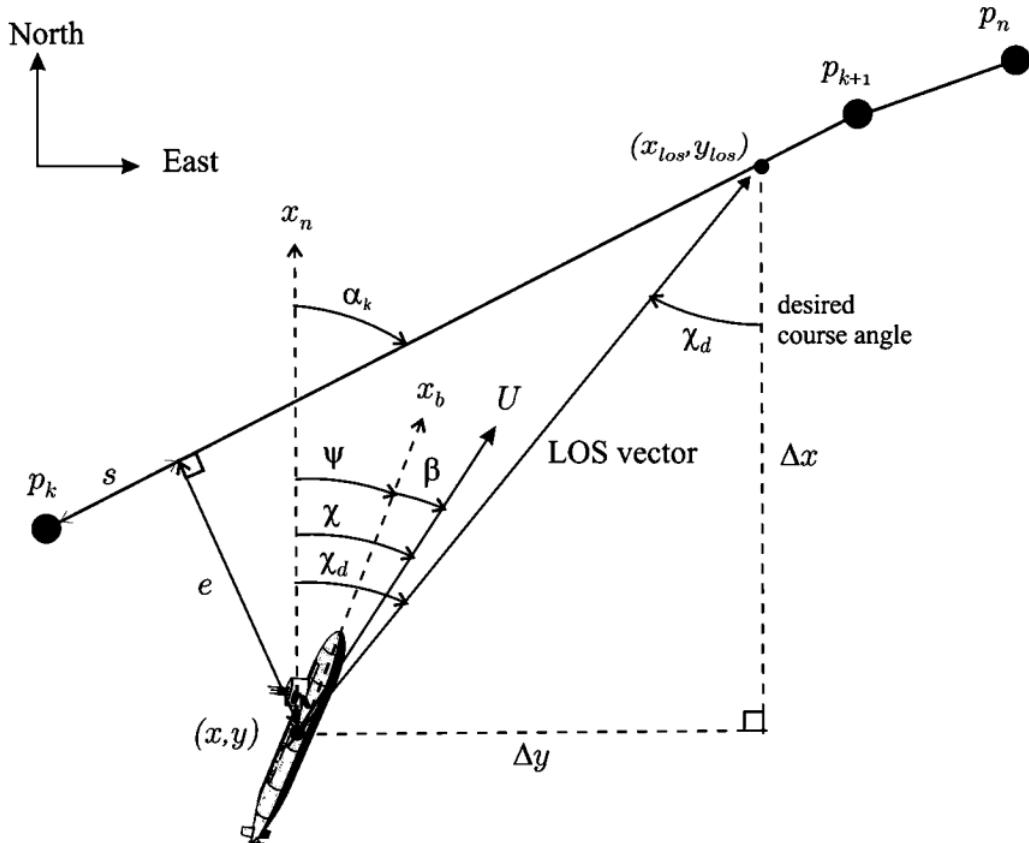
where

$$x_p = \alpha_k \quad (4.10)$$

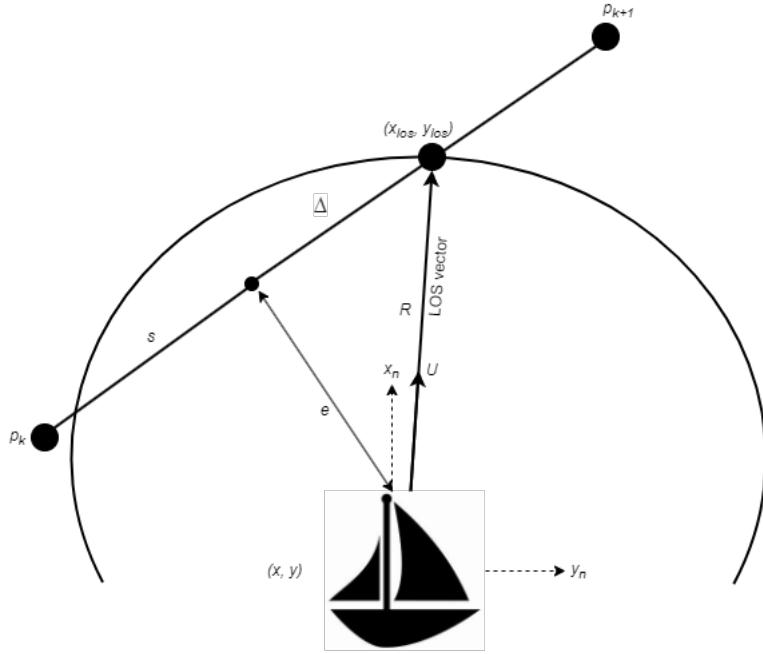
is the *path – tangential angle* illustrated in Figure 4.5, while

$$x_r(e) = \arctan\left(\frac{-e}{\Delta}\right) \quad (4.11)$$

is a *velocity-path relative angle*, which ensures that the velocity is directed toward a point on the path that is located a *lookahead distance*  $\Delta(t) > 0$  ahead of the direct projection of  $p^n(t)$  on to the path.



**Figure 4.5:** LOS guidance where the desired course angle  $x_d$  is chosen to point toward the LOS intersection point  $(x_{los}, y_{los})$



**Figure 4.6:** Circle of acceptance with constant radius  $R$

Figure 4.6 illustrates that the circle of acceptance radius  $R$  is equal to

$$e(t)^2 + \Delta(t)^2 = R^2 \quad (4.12)$$

with

$$\Delta(t) = \sqrt{R^2 - e(t)^2} \quad (4.13)$$

varying between 0 and  $R$  for  $|e(t)| = R$  and  $|e(t)| = 0$ , respectively. The steering law, in Equation 4.11, can also be interpreted as a saturating control law,

$$x_r(e) = \arctan(-K_p e) \quad (4.14)$$

where  $K_p(t) = 1/\Delta(t) > 0$ . Notice that the lookahead-based steering law is equivalent to a saturated proportional control law, effectively mapping  $e \in R$  into  $x_r(e) \in [-\pi/2, \pi/2]$ . As shown in the geometry of Figure 4.6, a small lookahead distance implies aggressive steering, which intuitively is confirmed by a correspondingly large proportional gain in the saturated control interpretation. However because a sailboat is an underactuated vessel that can only steer by attitude information and is subject to influence like ocean currents and nonzero slip angles  $B$ . This suggest that a integral controller will be a needed for the sailboat to follow straight-line. The integral controller is shown below,

$$x_r(e) = \arctan \left( -K_p e - K_i \int_0^t e(\tau) d\tau \right) \quad (4.15)$$

with  $K_i > 0$ . Considering horizontal path following along straight lines, the desired yaw angle can be computed by,

$$x_d(e) = \alpha_k + x_r(e) \quad (4.16)$$

with  $x_r(e)$  as in Equation 4.14. In practice, to avoid overshoot and windup affects, care must be taken when using integral action in the steering law. Specifically, the integral term should only be used when a steady-state off-track condition is detected.

#### 4.1.4. Path-Following Controllers

The path-following controller depends on having access to velocity measurements. This methods aim is to align the velocity and LOS vector. The desired course angle  $x_d$  is computed such that the velocity vector is along the path(LOS vector) using the *lookahead-based steering* law,

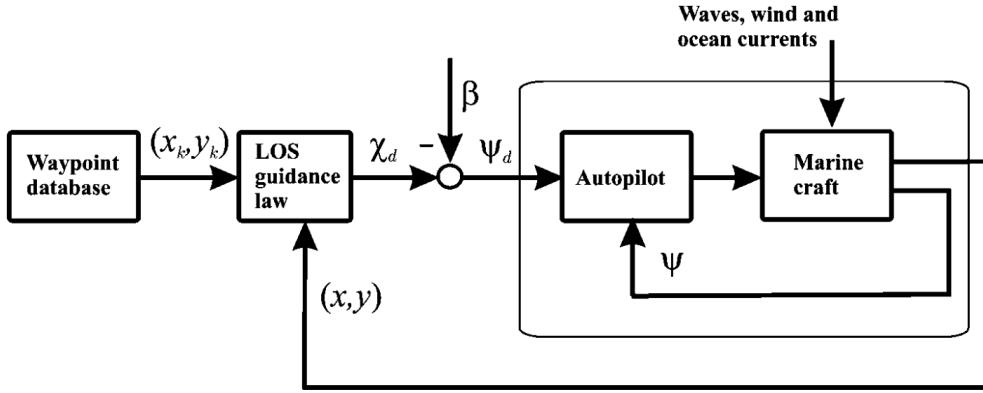
$$x_d(e) = x_p + x_r(e) = \alpha_k + \arctan(-K_p e) \quad (4.17)$$

The control objective  $x \rightarrow x_d$  is satisfied by transforming the course angle command  $x_d$  to a heading angle command  $\psi_d$ . This requires the knowledge of the  $\beta$  since, illustrated in Figure 4.7,

$$\psi_d = x_d - \beta \quad (4.18)$$

The velocity and LOS vectors can be aligned using a heading controller with the following error signal,

$$\psi = \psi - \psi_d = \psi - x_d + \beta \quad (4.19)$$



**Figure 4.7:** LOS guidance principle where the side slip angle  $B$  can be applied and compensated for by using integral action

If the velocities of the vessel are measured, the sideslip angle can be computed by

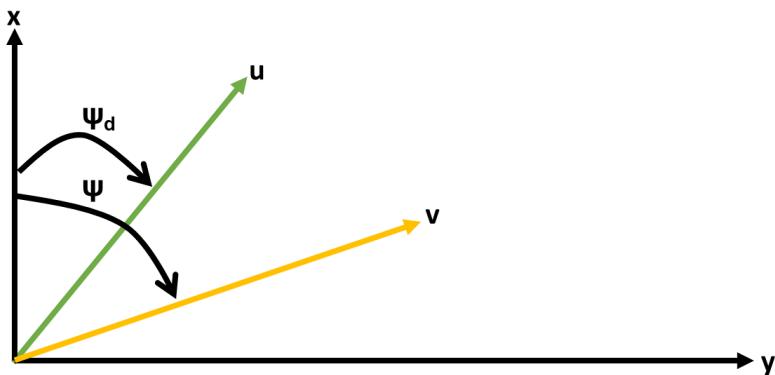
$$\beta = \arcsin\left(\frac{v}{U}\right) \quad (4.20)$$

Guidance laws of PI type avoid velocity measurements by treating  $\beta$  as an unknown slowly varying disturbance satisfying  $\beta \approx 0$ .

To ensure that the reference angle  $\psi_d$ , that is fed into the autopilot, is always the shortest angle between the waypoint and vessel position, the waypoint and vessel position are transformed into vectors and the shortest angle between the vectors are calculate. To find the angle between two vectors the following equation is used, which is derived using the Pythagorean Theorem,

$$\theta = \cos^{-1}\left(\frac{\bar{u} \cdot \bar{v}}{\|\bar{u}\| \|\bar{v}\|}\right) \quad (4.21)$$

The vectors  $\bar{u}$  and  $\bar{v}$  is illustrated in Figure 4.8. The vector  $\bar{u}$  is defined using the reference angle  $\psi_d$  and the vector  $\bar{v}$  is defined using the heading angle  $\psi$ .



**Figure 4.8:** Definition of vector  $\bar{u}$  and  $\bar{v}$

Vector  $\bar{u}$  and  $\bar{v}$  is decomposed into its x- and y-components by the following equation,

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\psi) \\ \sin(\psi) \end{bmatrix} \quad (4.22)$$

The vector components are used to calculate the dot product and the magnitude for each vector as shown below,

$$\bar{u} \cdot \bar{v} = x_{\bar{u}} x_{\bar{v}} + y_{\bar{u}} y_{\bar{v}} \quad (4.23)$$

$$\|\bar{u}\| = \sqrt{x_{\bar{u}}^2 + y_{\bar{u}}^2} \quad (4.24)$$

$$\|\bar{v}\| = \sqrt{x_{\bar{v}}^2 + y_{\bar{v}}^2} \quad (4.25)$$

The calculated angle  $\theta$ , which is the shortest angle between the two vectors, is now fed into the autopilot with the following transformation

$$\psi_d = -\theta \quad (4.26)$$

#### 4.1.5. Circle of Acceptance for Surface Vessel

When a vessel is traveling along a straight line piece wise path, made up of  $n$  straight-line segments connected by  $n + 1$  waypoints. A mechanism is required for selecting the next waypoint. The next waypoint  $(x_{k+1}, y_{k+1})$  can be selected on the basis of whether or not the vessel lies within a *circle of acceptance* with radius  $R_{k+1}$  around  $(x_{k+1}, y_{k+1})$ . This means that the vessel position must at time  $t$  satisfy,

$$[x_{k+1} - x(t)]^2 + [y_{k+1} - y(t)]^2 \leq R_{k+1}^2 \quad (4.27)$$

then the next waypoint  $(x_{k+1}, y_{k+1})$  should be selected. A suggested method is to select the radius  $R$  depending on the distance between the two waypoints.

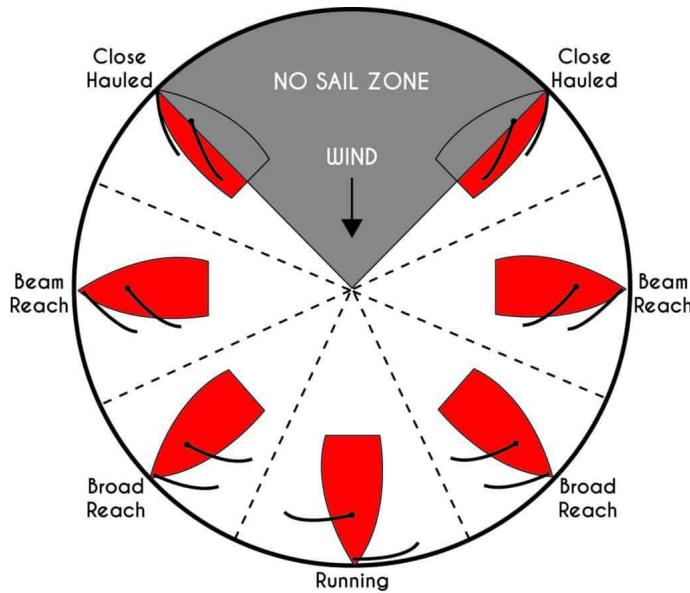
$$R = b \frac{y_{k+1} - y_k}{x_{k+1} - x_k} \quad (4.28)$$

where the value  $b$  is a constant value.

## 4.2. Sailboat Maneuvering

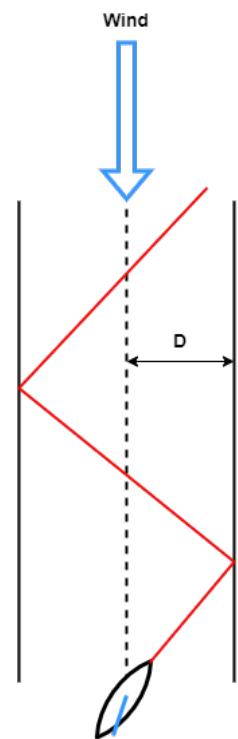
### 4.2.1. The Three forms of Sailboat Maneuvering

The maneuvering of a sailboat comes down to three different types of maneuvering [21]. The maneuverings are called tacking, jibbing and normal sailing. The tacking maneuver is used when the sailboat is required to sail into the wind, where the sailboat switches between sailing close hauled left and right, which is illustrated in Figure 4.9. The tacking maneuver is illustrated in Figure 4.10.

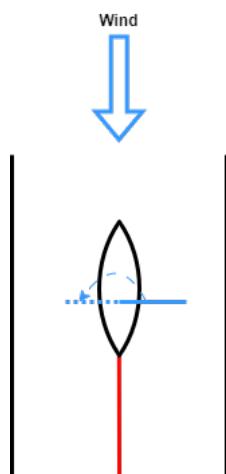


**Figure 4.9:** Sail Zone

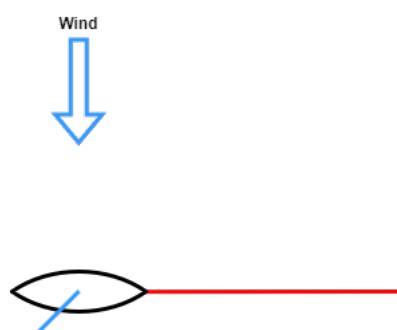
The jibbing maneuver is when the sailboat sails downwind and is known as running, illustrated in Figure 4.9. The sail angle will can be switch between  $90^\circ$  and  $-90^\circ$ , with a small rudder angle change. The act of switching the sail angle is what is known as jibbing. The last maneuver, which is normal sailing, is merely the control of the rudder and sail according to the direction of the wind and the direction in which it desires to sail in. The decision tree for the three maneuvers can be presented in the form of a finite state machine.



**Figure 4.10:** Tacking



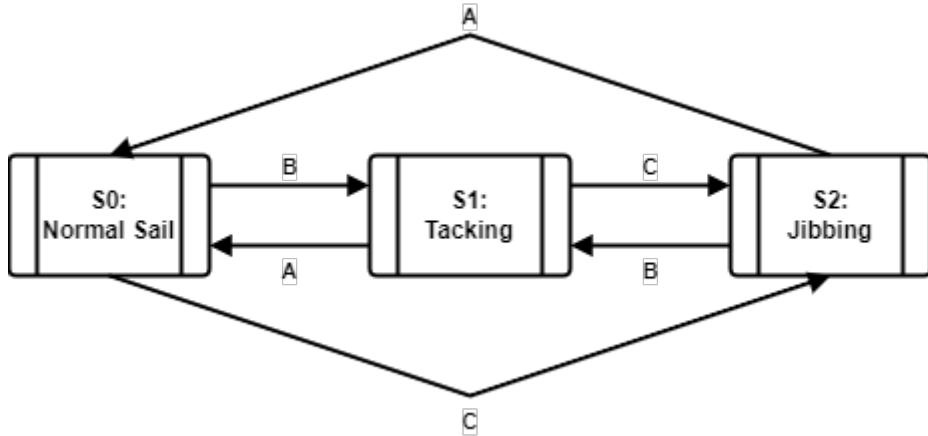
**Figure 4.11:** Jibbing



**Figure 4.12:** Normal Sailing

### 4.2.2. Finite State Machine of Sailboat Maneuvering

The three maneuver abilities of a sailboat can be implemented via a state machine, more specifically a Moore state machine. Where the states are only dependent on the input actions. The input action in this case are the apparent wind angle, defined in Equation 3.48.



**Figure 4.13:** Finite-State Machine for Sailboat Maneuvering

The state machine is illustrated in Figure 4.13. The actions are stated as **A**, **B** and **C** and is defined in Table 4.1. The angle are only expressed in the case of a positive heading angle.

**Table 4.1:** Definitions of Actions in State Machine 4.13

Action	Definition
<b>A</b>	$45 < \text{Apparent Wind Angle} < 315$ $!(150 < \text{Apparent Wind Angle} < 210)$
<b>B</b>	$\text{Apparent Wind Angle} < 45$ $\text{Apparent Wind Angle} > 315$
<b>C</b>	$150 < \text{Apparent Wind Angle} < 210$

### 4.2.3. Heading Controller

The heading controller, discussed in Section 4.2.2, is responsible for determining which state of sailing the boat is in and also adjusting the the tacking angle with an offset of  $-45^\circ$  and  $45^\circ$  from the apparent wind angle. The pseudo code of the controller is shown in Algorithm 4.1 and the implementation in Matlab is illustrated in the Appendix Subsection A.5.2.

For state 1, tacking, another finite-state machine will define the inner workings that dictates the actions of the desired heading angle. The heading angle will then be sent to the low level control which dictates the rudder angle required to reached the desired

**Algorithm 4.1:** Heading Controller

---

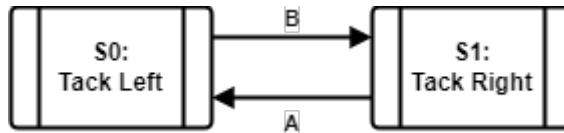
```

 $\gamma$  = Wind angle
if  $\gamma > 180$  then
     $\gamma = \gamma + 360$ 
else if  $\gamma < -180$  then
     $\gamma = \gamma - 360$ 
end if
if (absolute value of  $\gamma$ ) < 45 then                                ▷ State 1
    if tack heading < 15 then
        State = Initialized right tack
        Set tack distance
        Set tack right and tack left angle
        Reference Heading = tack right
    else
        State = Tacking
        Set tack distance
        Set tack right and tack left angle
        Reference Heading = Tack left/Tack Right
    end if
else                                                               ▷ State 0
    State = Normal Sailing
    Reference Heading = Waypoint Heading
end if

```

---

heading. For states 0, normal sail, and 2, jibbing, the heading angle is merely passed onto the rudder controller without the need for another state machine.



**Figure 4.14:** Finite-State Machine for Sailboat Maneuvering

The actions definitions for the tacking state machine is illustrated in Figure 4.14. The  $e$  is the  $y$ -component of the guidance axis, defined in Section 4.1.1, and the  $Y$  is the value that corresponds the the width,  $D$ , illustrated in Figure 4.10. The optimal heading angle for tack left is  $-45^\circ$  and tack right is  $45^\circ$ .

**Table 4.2:** Definitions of Actions in State Machine 4.14

Action	Definition
A	$e > D$
B	$e < -D$

The sailboat actions during a tack is summarized in below,

1. Tack Right

- (a) Set reference heading to  $45^\circ$
  - (b) While the sailboat turns winch in the sail accordingly
  - (c) Once the sailboat is sailing in the correct direction release the sail to the correct angle
2. Tack Left
- (a) Set reference heading to  $-45^\circ$
  - (b) While the sailboat turns winch in the sail accordingly
  - (c) Once the sailboat is sailing in the correct direction release the sail to the correct angle

There will be two rudder controllers, one controller will be used to adjust the rudder angle depending on the reference heading and then a second controller will be used only in the state of tacking. The controller is fuzzy logic controller that switches the reference heading depending on the cross-track error.

#### 4.2.4. Tacking Controller

The tacking controller is a fuzzy logic controller that has the inputs tack direction, tack distance, tack left angle and tack right angle. The pseudo code of the fuzzy logic controller is shown below,

---

**Algorithm 4.2:** Fuzzy Logic Tacking Controller
 

---

**Require:** State 1: Tacking

$\gamma$  = Apparent wind angle

**if**  $e > D$  **then**

$\psi_d = 45 + \gamma$  ▷ Tack Right Angle

**else if**  $e < -D$  **then**

$\psi_d = -45 + \gamma$  ▷ Tack Left Angle

**end if**

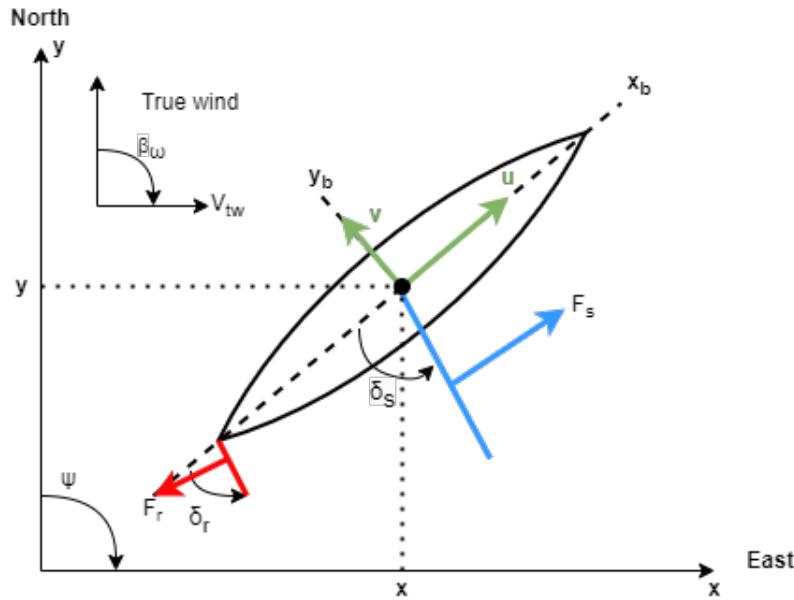
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The value of D can be adjusted to change the amount of tacks being performed to reach a waypoint. The adjustment happens inside the heading controller, discussed in Section A.5.2.

### 4.3. Rudder and Sail Control

This section takes a look at models for ocean vessels to control the heading, maneuvering and dynamic positioning. The chapter first looks at independently controlling the steering through the use of a rudder. The chapter considers the use of a linear and non-linear

heading controller. The non-linear controller has advantages in the form of better heading control with small heading errors compared to the linear controller. The references angles for the sail and rudder on the sailboat is illustrated in the figure below,



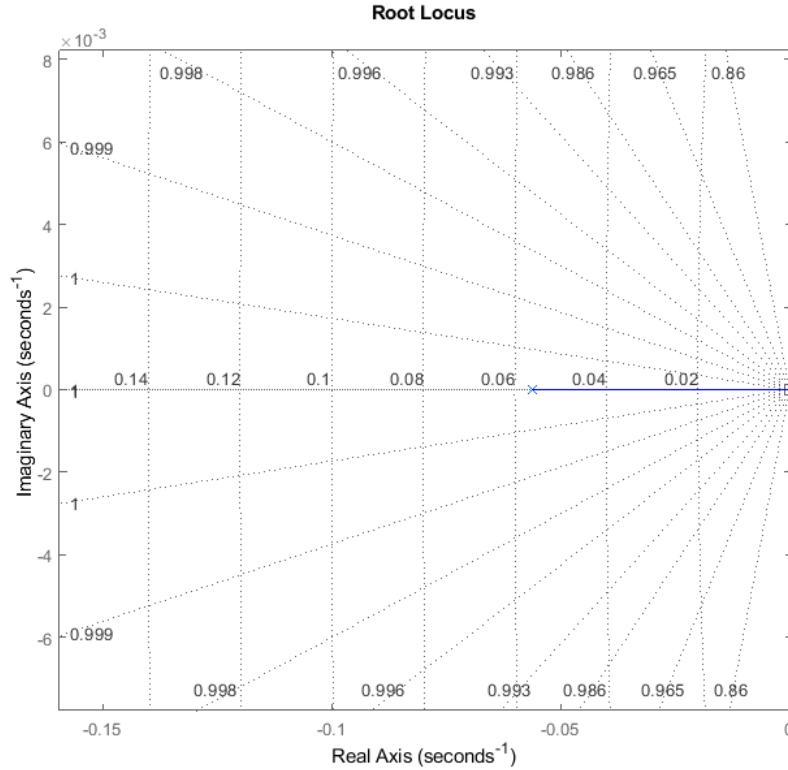
**Figure 4.15:** Reference angles of sailboat

### 4.3.1. Rudder Control

When designing the steering controller the first order Nomoto transfer function of yaw rate to rudder angle is used. The ocean vessel used in this case is a marine class vessel. The first order model is shown below,

$$\frac{r}{\delta} = \frac{K}{(1 + TS)} \quad (4.29)$$

The transfer function mentioned above is used for inner-loop control. The inner loop controller is used to stabilise the boat. The root locus of this transfer function is illustrated in Figure 4.16. The pole will always lie to the left of the imaginary axis due to uncontrolled vessels being unstable

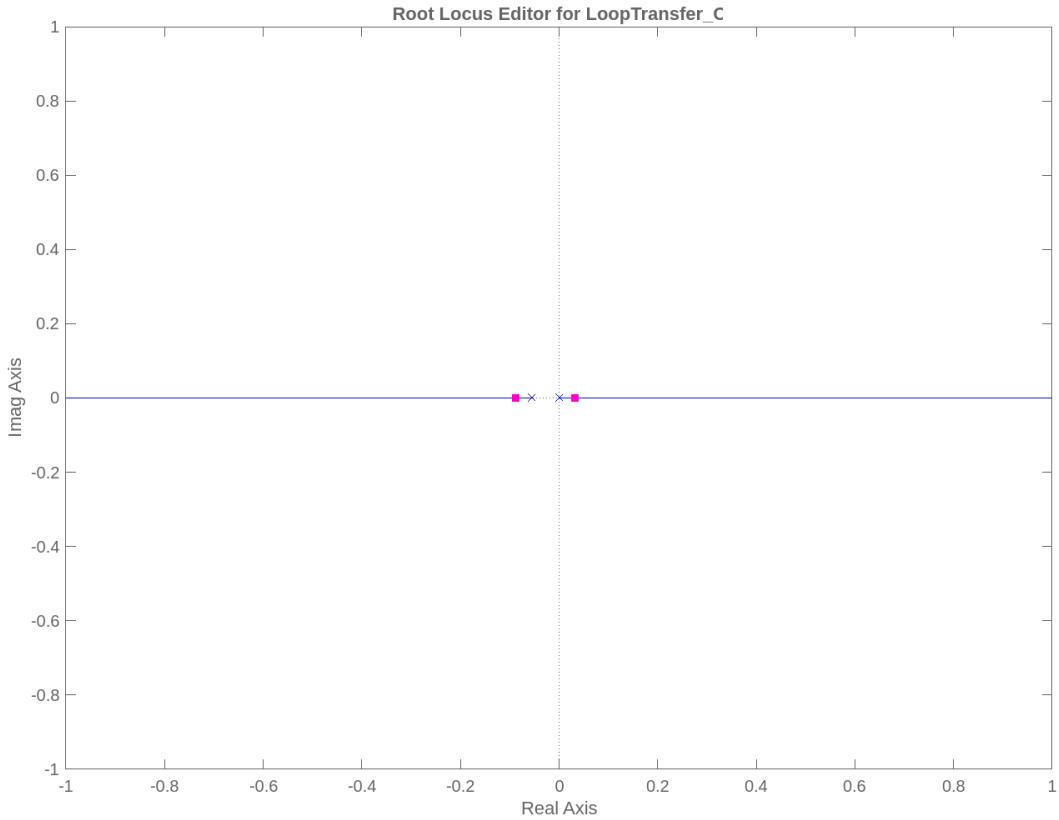


**Figure 4.16:** Caption

It makes more sense to compare the rudder angle  $\delta$  with the heading angle  $\psi$ . To obtain this transfer function multiply the transfer function mentioned above with an integrator  $\frac{1}{S}$  and also substituting in the values for  $K$  and  $T_S$  results in,

$$\frac{\psi}{\delta} = \frac{-0.049}{(1 + 17.78S)S} \quad (4.30)$$

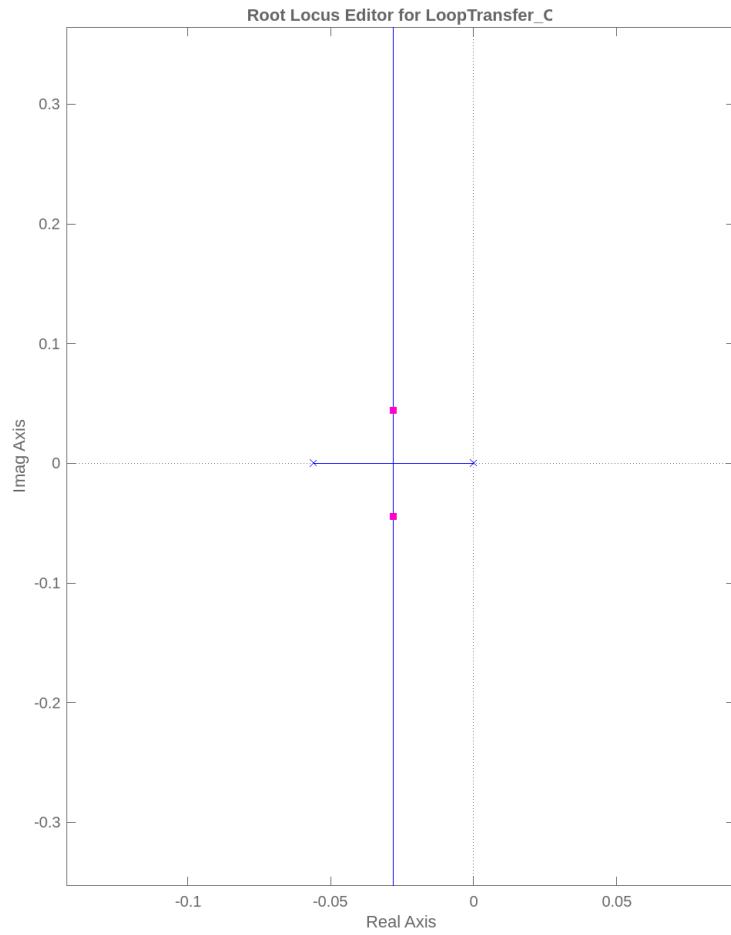
By analyzing the root locus plot of Equation 4.30. The root locus has two poles, one at  $S = -0.0566$  and one at  $S = 0$ . From the root locus plot in Figure 4.17 it clear as the gain  $k$  increases the system will become unstable.



**Figure 4.17:** Root locus of first order system

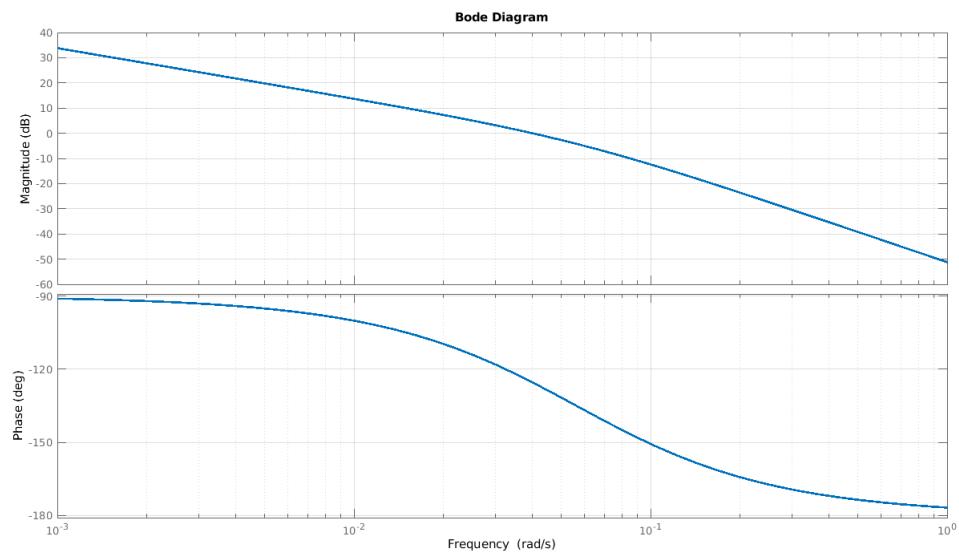
### Linear Rudder Control

In order to first stabilize the system the transfer function should be multiplied with a negative to invert the root locus, meaning only a negative gain should be used. The new root locus plot is illustrated in Figure 4.18. From the newly constructed root locus plot the system will be stable for any value of  $k$ . For  $k = -1$ , two complex poles at  $s = -0.02811 \pm j * 0.0443$ . The damping  $\delta_a = 0.563$  and the natural frequency is  $\omega_n = 0.0525$ .



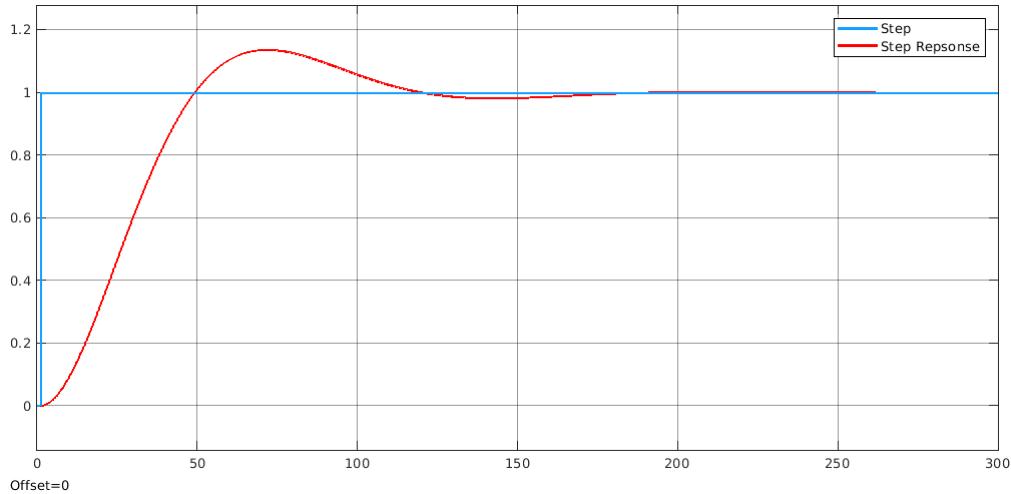
**Figure 4.18:** Corrected root locus of first order system

The bode plot is illustrated in Figure 4.19, the Phase Margin(PM) is  $PM = 55$  and the Gain Margin(GM)  $GM = 91.2dB$ .



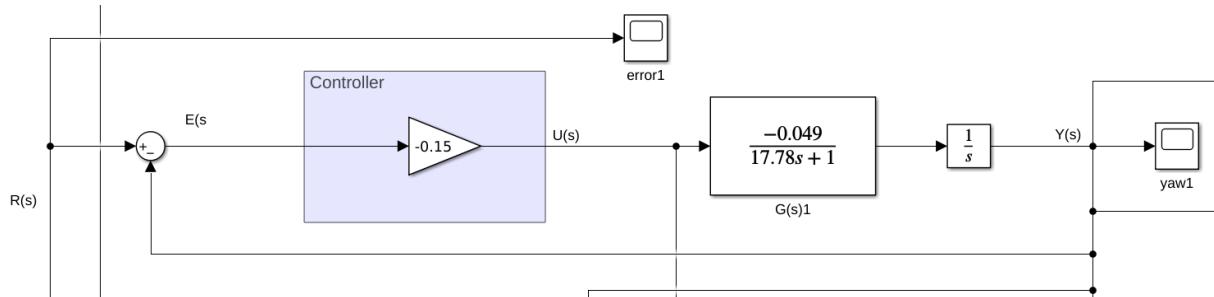
**Figure 4.19:** Bode plot

The step response of the system, illustrated in Figure 4.20 with a  $k = 1$  shows that the system is underdamped. The time specifications, namely the rise time is  $t_r = 34.29s$ , the settling time is  $t_s = 135.33s$  and maximum overshoot is  $M_p = 0.118$ .

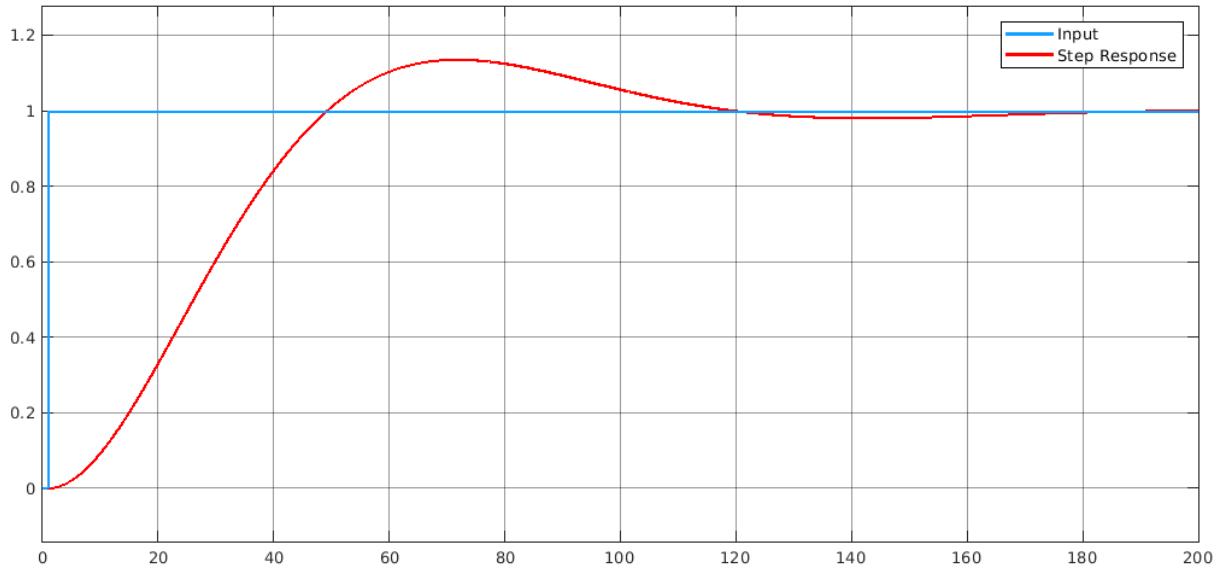


**Figure 4.20:** Step response of first order system

The system was implemented in Simulink and is illustrated in Figure 4.21. The closed-loop step response of the system, illustrated in Figure 4.22.



**Figure 4.21:** Simulink model of system



**Figure 4.22:** Closed loop step response

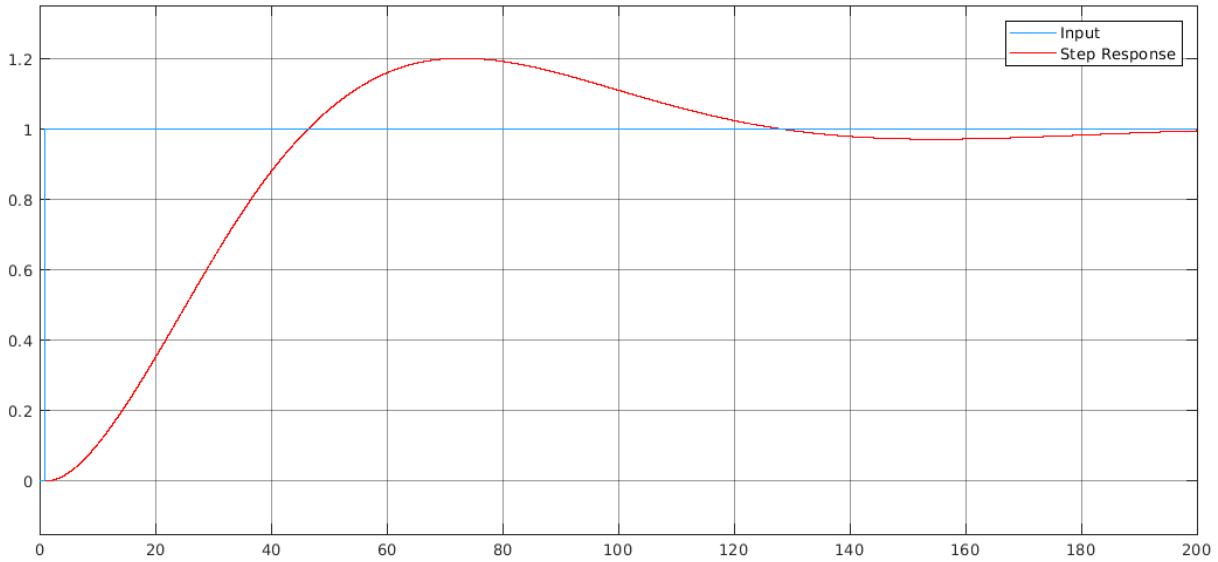
The controller used in this system is a Proportional(P) controller. The continuous controller is shown below,

$$D(s) = -K \quad (4.31)$$

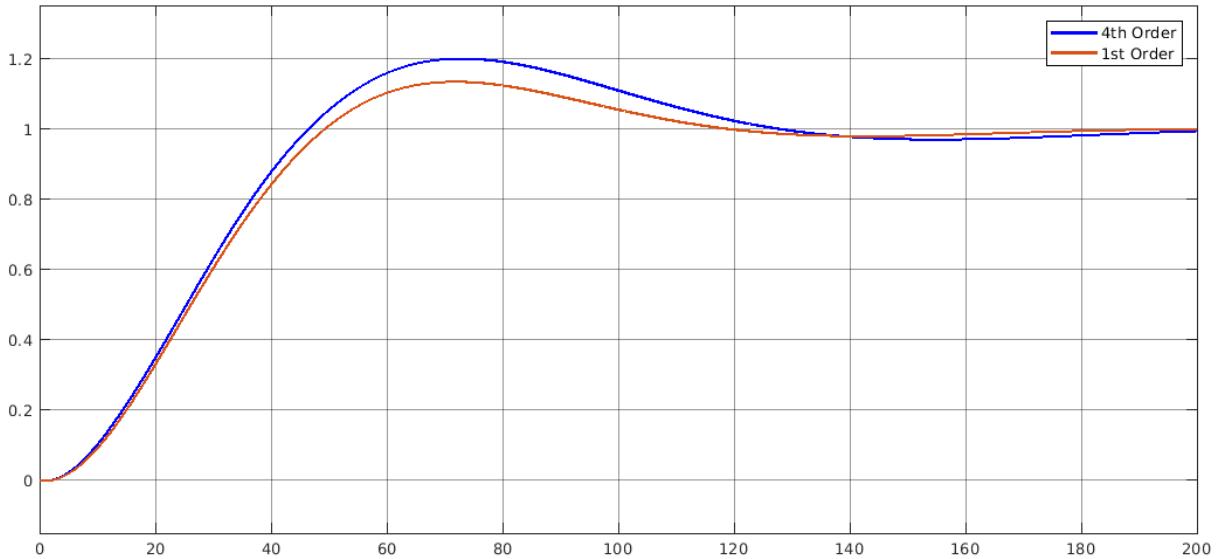
To implement a continuous controller it has to be converted to a discrete controller. The discrete controller is simply the negative gain

$$D(z) = -K \quad (4.32)$$

Implementing the controller in the fourth order transfer function results in the following response,



**Figure 4.23:** Implementation of controller in fourth order transfer function



**Figure 4.24:** Comparison between 1st order and 4th order step responses

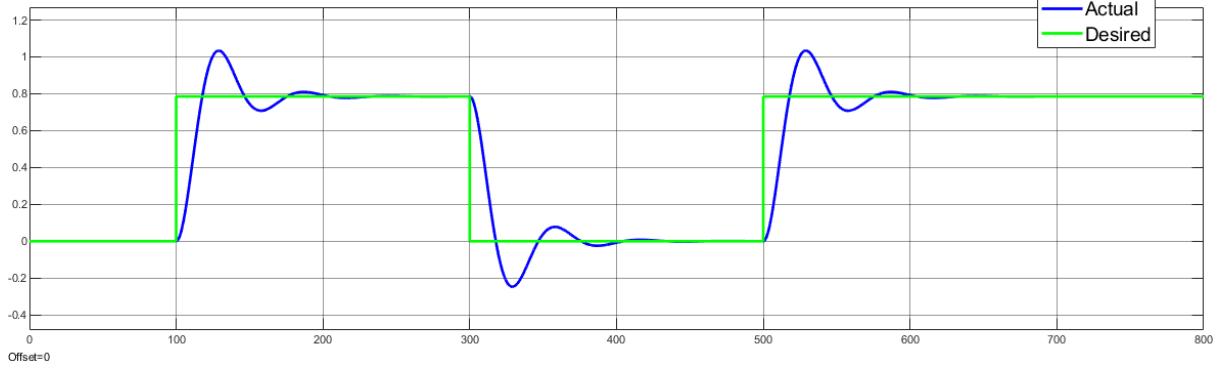
### Non-Linear Rudder Control

The linear rudder controller implemented in Section ??, known as a Proportional Controller, tends to overshoot a lot in large changes in heading angle. The actual and desired heading of a linear controller is illustrated in Figure 4.25 and the non-linear controller illustrated in Figure 4.26. The non-linear controller is shown below,

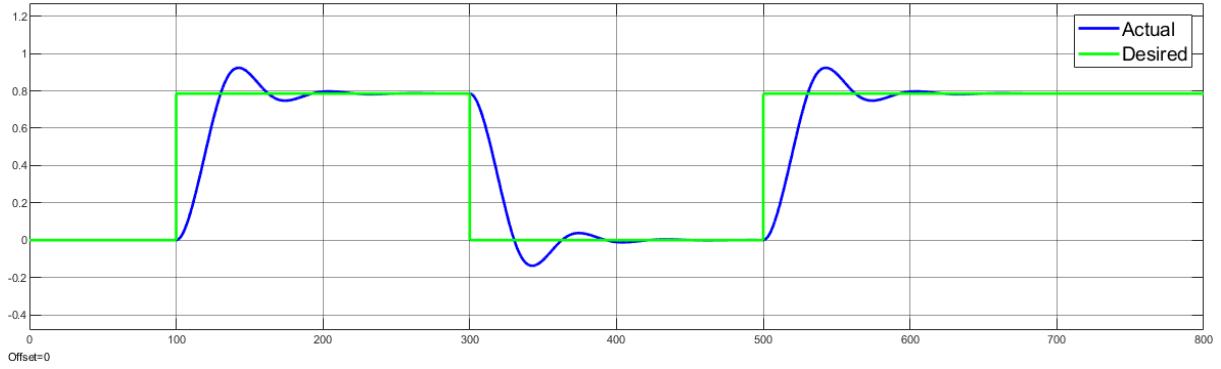
$$k(e) = \frac{k_p}{1 + c_p|e|} \quad (4.33)$$

for some  $k_p, c_p > 0$  and hence sets the rudder command to

$$\delta = \delta_{NL}(e) = k(e)e \quad (4.34)$$



**Figure 4.25:** Linear control of heading



**Figure 4.26:** Non-Linear control of heading

The benefit to using the non-linear controller compared to the linear controller is due to a more damped system when it comes to large heading changes. The smaller the heading error the more the non-linear controller will perform the same as the linear controller, as illustrated above. The linear controller tends to overshoot illustrated in Figure 4.25 and the non-linear controller has less of a overshoot Figure 4.26. The overshoots observed in Figure 4.27 and in Figure 4.28 is almost the same, having a error of around 3% of the step size compared with the overshoot observed in the case of a large heading change with is closer to a 17.5% overshoot.

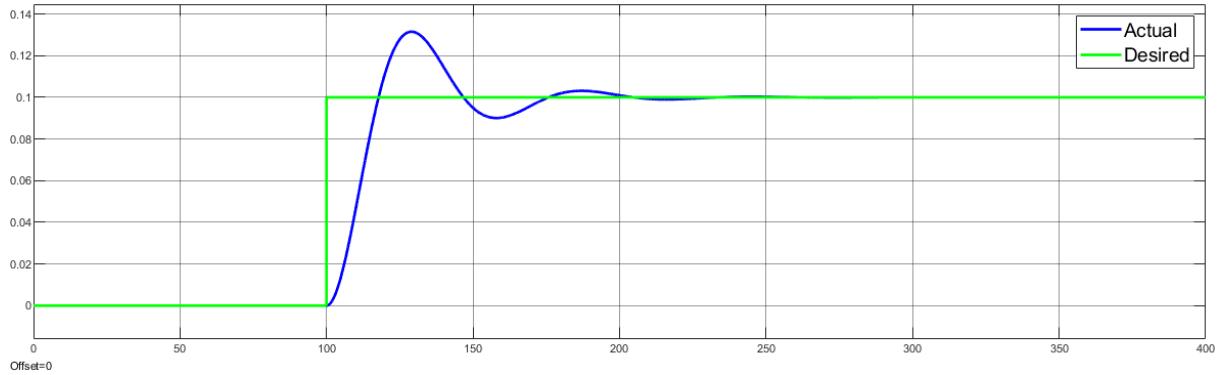


Figure 4.27: Linear control of heading

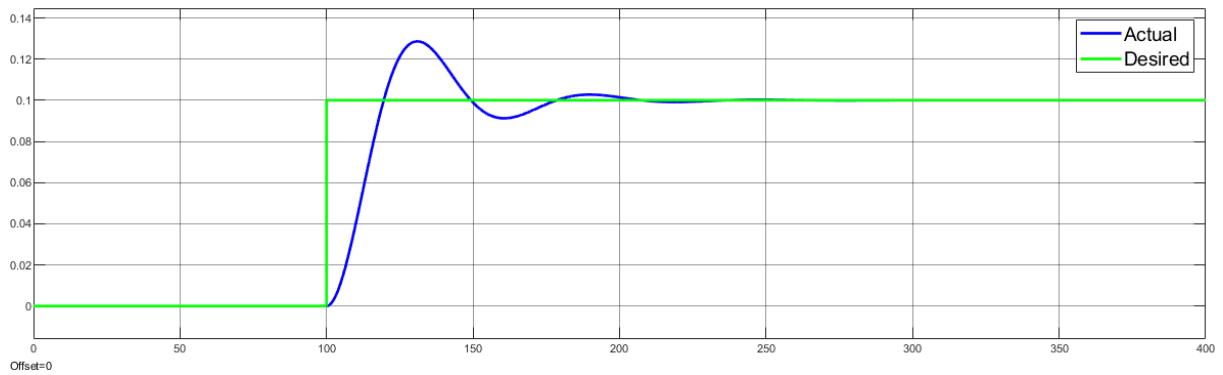


Figure 4.28: Non-Linear control of heading

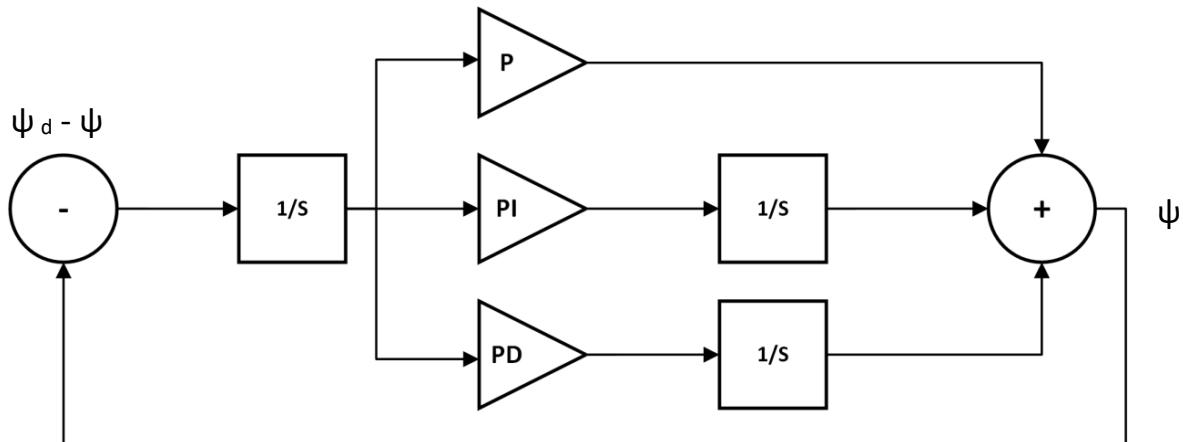


Figure 4.29: PID Controller for rudder

### 4.3.2. Sail Control

The only propulsion source for sailboats are the wind. To propel the sailboat forward the sail needs to be aligned such that it catches the wind at the right angle. Firstly the reference angles need to be assigned. An example of a sailboat's sail angle is illustrated in Figure 4.15. The sail force is defined in [22], the article assumed a four degree of freedom model for the sailboat, surge sway heave and roll. The vector  $n$  was derived as

$$\dot{n} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\psi} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} u \cos(\psi) - v \sin(\psi)\cos(\phi) \\ u \sin(\psi) - v \cos(\psi)\cos(\phi) \\ r \cos(\phi) \\ p \end{bmatrix} \quad (4.35)$$

The apparent wind velocity will only be considered in the  $(x_b, y_b)$  plane. This results in the following definitions for apparent wind and velocity, which is defined as the wind experienced by the sailboat,

$$\psi_{aw} = \text{atan}2(V_{aw}^{y_b}, V_{aw}^{x_b}) = 2\text{atan}\left(\frac{V_{aw}^{y_b}}{\sqrt{(V_{aw}^{y_b})^2 + (V_{aw}^{x_b})^2}} + V_{aw}^{x_b}\right) \quad (4.36)$$

$$v_{aw} = \|V_{aw}^{b-frame}\| = \sqrt{(V_{aw}^{x_b})^2 + (V_{aw}^{y_b})^2} \quad (4.37)$$

The described force  $F_s$  define in Figure 4.15, can be represented in the vectorial representation of the aerodynamic force vector  $F_s$  into  $(x_b, y_b)$  plane is,

$$F_s = \begin{bmatrix} f_s \sin(\delta_s) \\ -f_s \cos(\delta_s) \cos(\phi) \end{bmatrix} \quad (4.38)$$

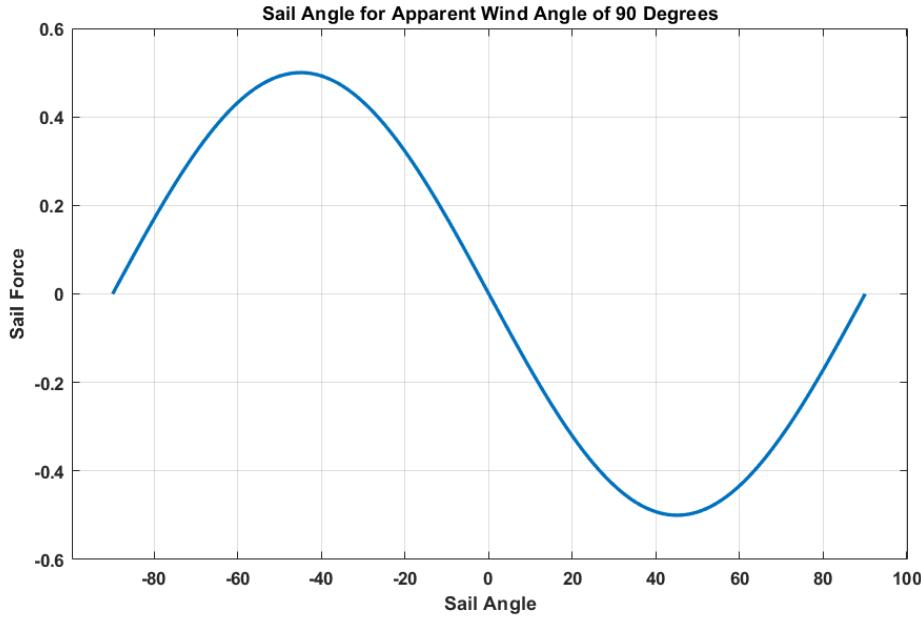
with  $f_s = \|F_s\|$ . The angle of attack on the sail is determined by the direction of the apparent wind vector  $V_{aw}$  and the sail angle, which is equal to

$$AoA = \pi - (\delta_s - \psi_{aw}) \quad (4.39)$$

The force generated by sail was discussed in Chapter 3, the force can be simplified in the surge direction as,

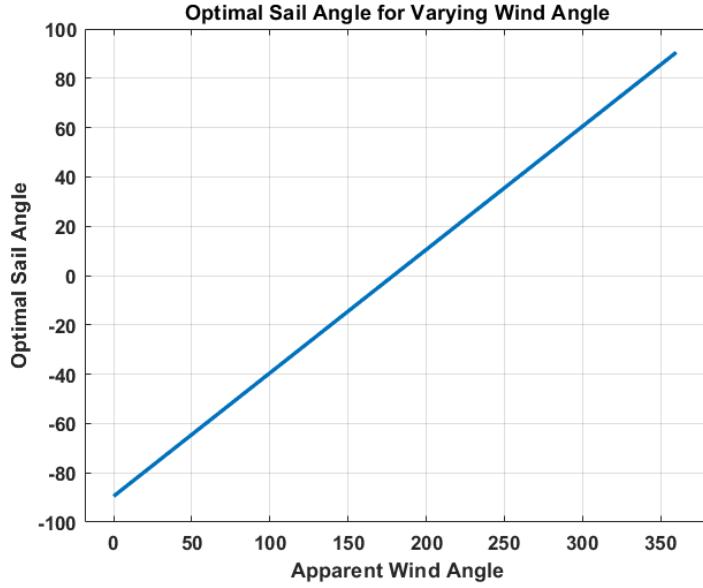
$$f_s = K v_{aw}^2 \sin(\delta_s - \psi_{aw}) \sin(\delta_s) \quad (4.40)$$

where  $K$  is a constant value dependent on the physical dimensions of the sail,  $\delta_s$  is the sail angle and  $v_{aw}$  is the apparent wind velocity.



**Figure 4.30:** Sail Angle for Apparent wind angle of  $90^\circ$

The control techniques for a sail was discussed in Chapter 2.2.2. The majority of control techniques are based on some look-up table form, where for a specific Apparent Wind Angle(AWA) a specific sail angle is used to generate the maximum sail force. Figure 4.30 illustrates the sail force in the surge direction, while varying the sail angle. The figure clearly shows that the sail will generate a positive force while it maintains a negative sail angle. The limitations imposed on the sail is that it will never open further than  $90^\circ$  in either directions and that it cannot abruptly changes its angle. The optimal sail angle was calculated for every wind angle and is shown in Table A.3. The calculated optimal sail angle when limiting the sail to  $-90 < \delta_s < 90$ , results in a straight line illustrated in Figure 4.31.

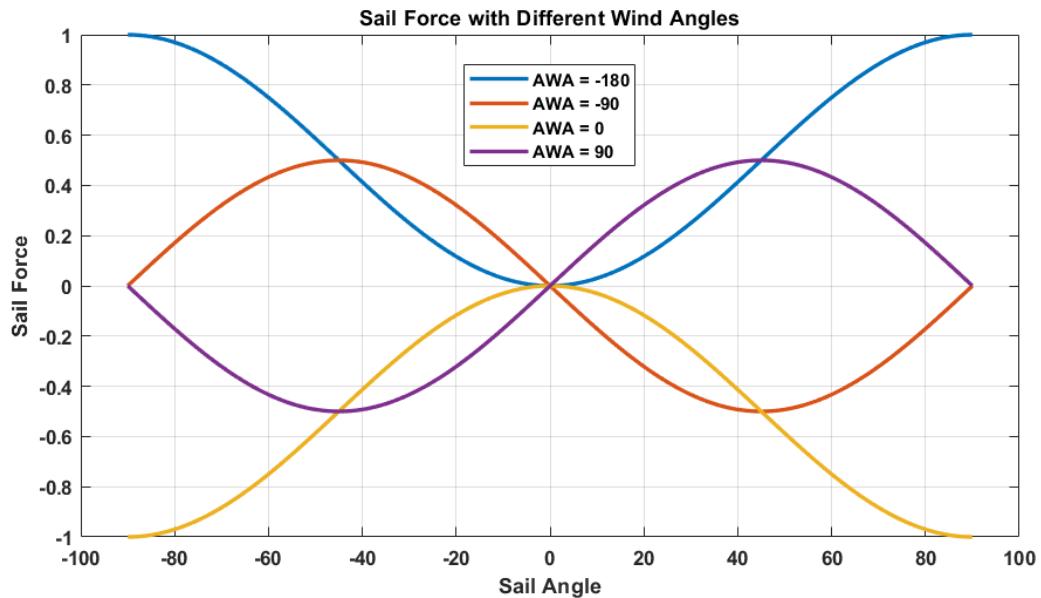


**Figure 4.31:** Optimal sail angle for varying wind angle

The straight line equation is represented by the following equation,

$$\delta_s = -0.5\lambda_\omega - 90 \quad (4.41)$$

This equation can be used to always set the sail angle to the optimal angle. There is however more limitations to consider. A sailboat cannot sail upwind. The zone wherein a sailboat cannot directly sail into the wind will be defined as the no-go zone. The sailboat is however possible to perform a maneuvering tactic called tacking that will allow it to follow a heading upwind while not sailing upwind. The sail force for AWA -180, -90, 0 and 90 is illustrated in Figure 4.32. When sailing upwind the AWA = 0 and sailing downwind the AWA = -180. The sail will generate a force,  $f_s > 0$  for downwind sailing and  $f_s < 0$  for upwind sailing for any  $\delta_s$ .



**Figure 4.32:** Sail force for specific apparent wind angles

---

**Algorithm 4.3:** Sail Controller

---

```

 $\gamma_w$  = Apparent Wind angle
y = Sail Angle
if  $\gamma_w > 180$  then
    y =  $-0.5 \times \gamma_w + 180$ 
else
    y =  $-0.5 \times \gamma_w$ 
end if
```

---

# Chapter 5

## Ardupilot

This chapter gives a overview of the Ardupilot software, more specifically the rover firmware. The sailboat is a extension of the rover firmware, although there is a sailboat option there is still limited functionality. This chapter will also introduce the Ardupilot code flow and then give detailed explanation on extensions made to improve or add functionality.

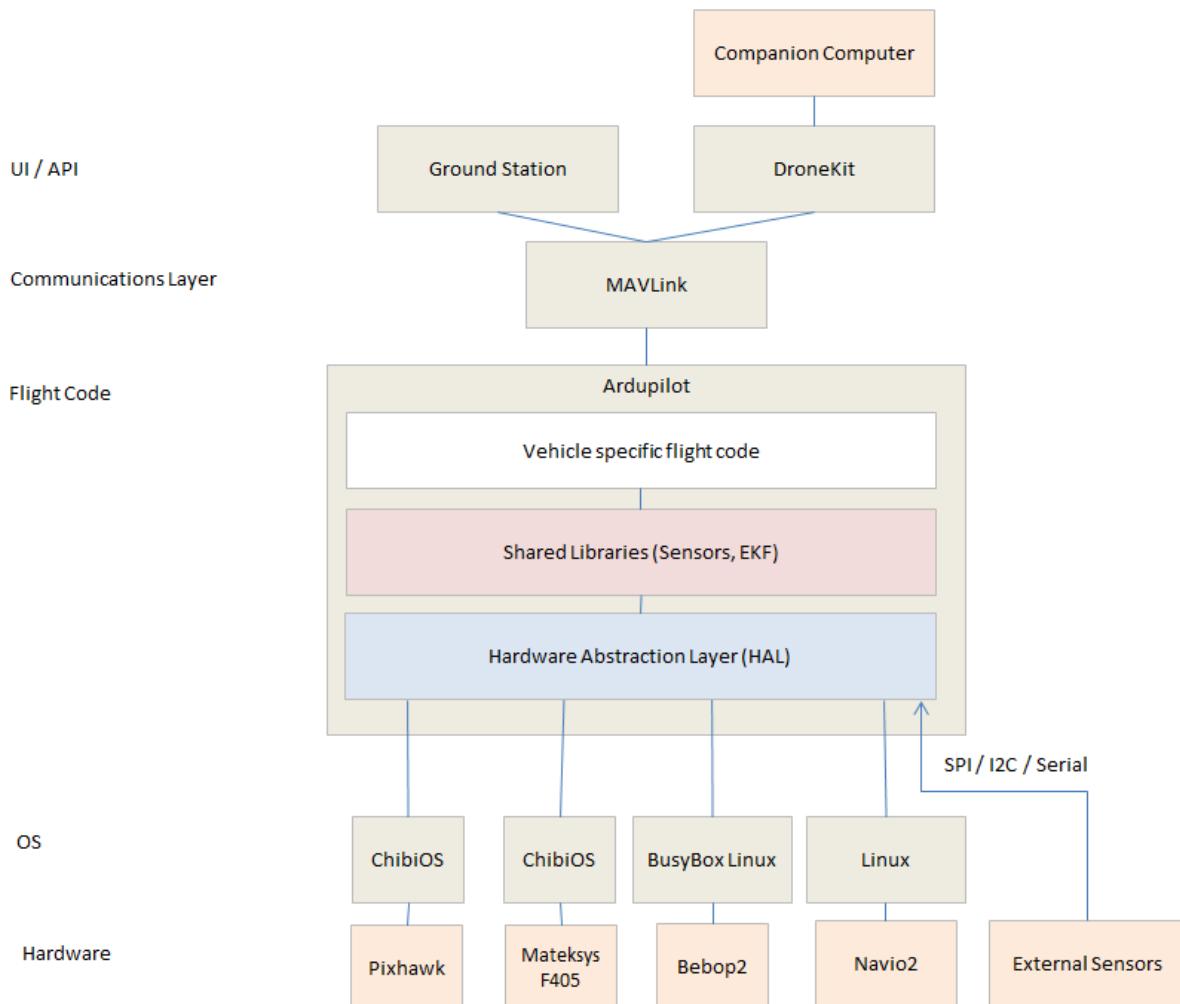
### 5.1. Architecture

Before looking at the architecture of Ardupilot you need to understand exactly what Ardupilot is [23]. Ardupilot is what is known as an autopilot, a system that automatically controls a vehicle. The control can consists of speed control, waypoint control and altitude control to name a few. Ardupilot is created for the use in multiple vehicles with a list below,

- Plane
- Copter
- Rover
- Sub
- Blimp
- AntennaTracker

The above vehicles are known as vehicle types and each type further has specific frameworks that is specific to a type of plane, copter or rover. Each framework has different actuators and control abilities. Inside the rover vehicle type the framework of interest, sailboat, resides. Although a sailboat is not a type of rover due to the its novelty it does not have its own vehicle type but rather makes use of the rover class.

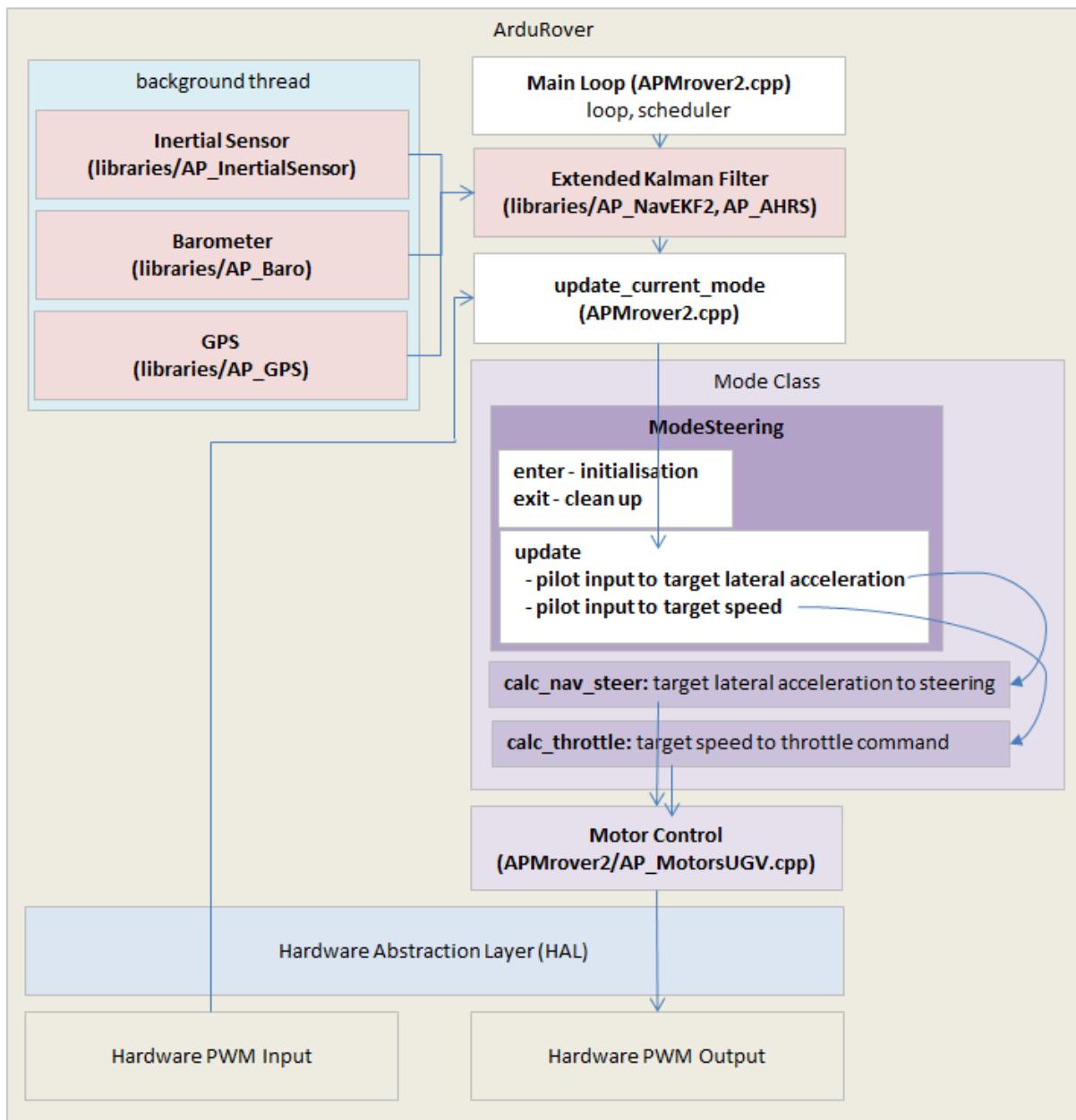
The high level architecture of the ArduPilot software is illustrated below. As seen in Figure 5.1 the Ardupilot is responsible for three things, the vehicle specific code, the sensor libraries and the hardware abstraction layer(HAL).



**Figure 5.1:** High Level Architecture of Ardupilot Software

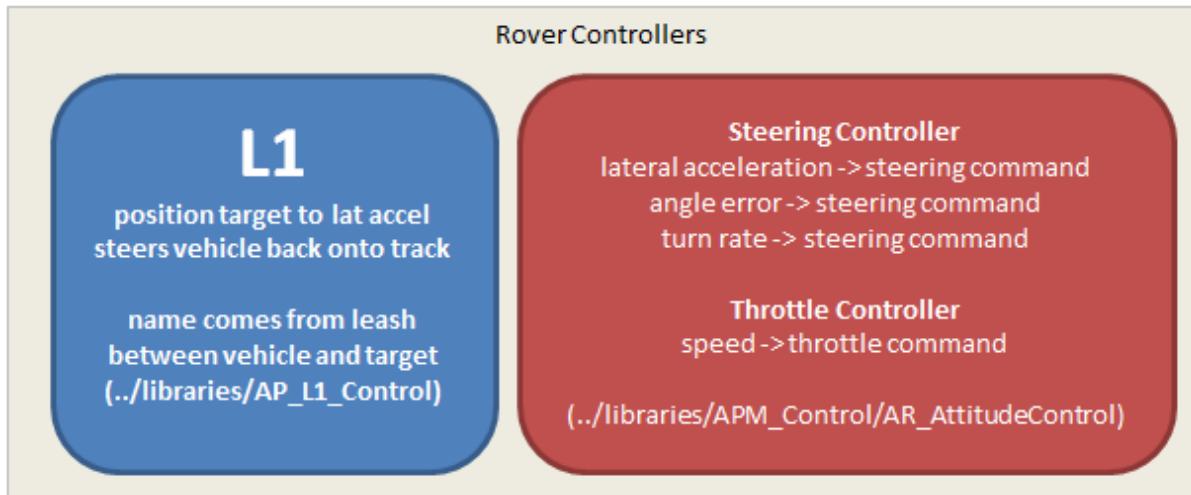
## 5.2. Rover Architecture

Now that the high level overview of where Ardupilot fits in is explained a deep dive into specifically the rover code is required. The rover code architecture is illustrated in Figure 5.2. The rover architecture shows the background threads are running the sensors defined in the libraries class. The main loop is where the rover code is initialized and also where the scheduler runs, which will be discussed in detail later in this chapter. The Mode class is important and it is inside the mode that the specific framework of the sailboat will be called from. The mode determines the how the sailboats rudder and sail will be controlled



**Figure 5.2:** Rover architecture

The rover vehicle has three high level controllers, L1 controller, Steering Controller and Throttle controller. The three controllers are illustrated in Figure 5.3. The L1 controller converts an origin point and a destination point into a lateral acceleration to make the vehicle travel along the path from the origin to the destination. This lateral acceleration is then passed to the steering controller.

**Figure 5.3:** Rover Controllers

The steering controller converts the desired lateral acceleration, the angle error or the desired turn rate into a steering output command that is fed into the motor library. The throttle controller converts the desired speed into a throttle command that is fed into the motor library. For the sailboat a new controller was designed that will calculate the optimal sail angle.

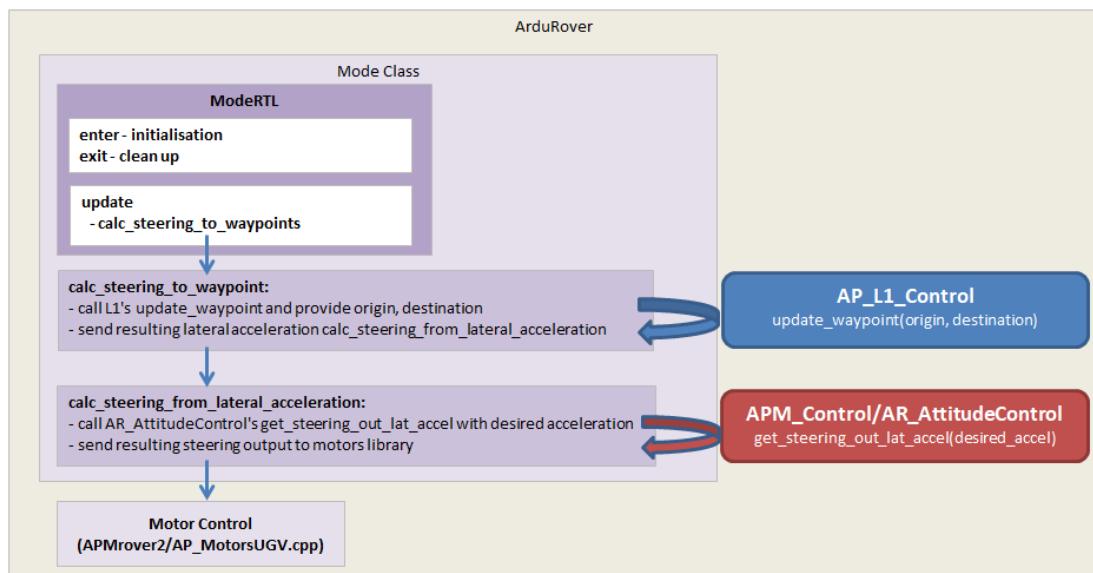
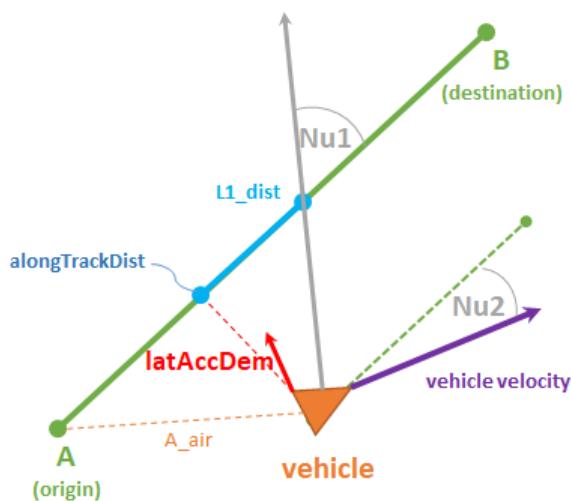
**Figure 5.4:** Rover navigation overview

Figure 5.4 gives an overview of how the rover navigation architecture works. On every iteration of the main loop (50hz) a call is made to the active mode's update method. While in Auto, Guide, RTL and SmartRTL mode, the update calls into the Mode class's **calc\_steering\_to\_waypoint** method. Mode's **alc\_steering\_to\_waypoint** then calls the **AP\_L1\_controller** library's **update\_waypoint** method providing it the location that the rover should drive towards. The **AP\_L1\_controller**'s **update\_waypoint** method

returns a desired **lateral\_acceleration** which is passed into Mode's **calc\_steering\_from\_lateral\_acceleration**. Mode's **calc\_steering\_from\_lateral\_acceleration** sends the desired acceleration to **APM\_Control/AR\_AttitudeControl**'s **get\_steering\_out\_lat\_accel** which uses a PID controller to calculate a steering output. The steering output is sent into the **AP\_MotorsUGV** library using the **set\_steering** method. The final output of the L1 controller's **update\_waypoint** method is a desired lateral acceleration which should bring the vehicle back to the line between the origin and destination. The formulas used are shown below,



**alongTrackDist**: distance from A to closest point on line from A to B

**L1\_dist**: distance from alongTractDist to target point on path

$$L1\_dist = 0.3 * damping * period * speed$$

**Nu1**: angle from vehicle to L1\_dist point (relative to line from A to B)

**Nu2**: vehicle velocity angle relative (relative to line from A to B)

**latAccDem**: desired acceleration output along red cross track line

$$LatAccDem = \frac{4 * damping^2 * speed^2 * \sin(Nu1 + Nu2)}{L1\_dist}$$

**Figure 5.5:** Formulas for L1 controllers

### 5.2.1. Sailboat Firmware

Due to ArduPilots software being open source, there is limited capability in the control of the sailboat, the most notable shortages are:

- Sail Controller
- Roll Controller
- Certain data not being logged

The firmware will be expanded to support the above mentioned shortcomings. The firmware has the option of modes to select that dictates the sailboats behaviour. The modes of interest are:

- Manual
  - The sail and rudder is directly controller via the transmitter input. The minimum throttle value sets the sail to fully in and the maximum throttle releases the sail fully. This mode is used as a fail safe when an error or problem occurs in the other modes that controls the sailboat. A GPS position cannot be given to guide the sailboat.
- Acro
  - In Acro mode the user's steering stick controls the vehicle's turn rate and the throttle stick controls the vehicle's speed. The sail is automatically trimmed to the wind direction using to the wind vane. The sailboat will only try to sail if the transmitter gives a forward speed. With a 0 throttle input the sailboat will let out the sail to attempt to stop. This mode will only be used to estimate parameters of the sailboat that will be used to configure the steering control.
- Loiter
  - Loiter mode sets the sailboat to stay in the same position. If the sailboat does not have any speed and is within the Loiter radius, the sailboat simply drifts otherwise the sailboat will perform a figure 8 type of motion to keep its position.
- RTL
  - RTL stands for Return-To-Launch. The sailboat will return to the home position that is set on the ground control software. The sailboat will tack if it is required to reach the home position. Once it reaches the home position it will loiter around the home position.

- Auto
  - In Auto mode the sailboat will follow a pre-programmed mission that is stored in the ground control software. The mission consists of waypoints which the sailboat will follow in its numerical order and the sailboat will tack if required to reach a waypoint.
- Guided
  - In guided mode the sailboat is controlled via the ground control software by giving the sailboats points to sail towards. The sailboat can tack if required.

Within the Rover firmware a *cpp* file named sailboat contains the firmware that is specific to the sailboat. Inside this file is where the added features and functionality will be added.

### 5.2.2. Configuring and Hardware Setup for Sailboats

The configuring of the rover firmware for a sailboat vehicle type requires to enable and initialise certain parameters. The parameter **SAIL\_ENABLE** needs to be set to 1 and the sailboat requires a rudder which is setup by assigning a servo output channel to function 26(Ground Steering). Although it is not absolutely required but for the best control the sailboat should be fitted with a wind vane. The wind vane is fully discussed in Section 6.3.3.

The sailboat also needs a way to control the sail angle. This is done with a servo sail winch motor to controller the sail angle. Due to the build of the sailboat the wind dictates the directing of the sail and the servo only controls the angle. It is important to define the min and max sail angles which limits the range the sail can be winched in and out. The parameters can be defined in **SAIL\_ANGLE\_MIN** and **SAIL\_ANGLE\_MAX**, then it should be tested in manual mode. The throttle will dictate the sail angle and it should operate within the limits.

# Chapter 6

## Platform Development

This chapter details the platform used for testing and developing the control strategies, the detailed hardware and software platform. The hardware controller used is a Pixhawk 6C flight controller, which is generally used for airplanes, however the software is Ardupilots rover firmware which has a basic implemented framework for a sailboat.

### 6.1. Sailboat

The sailboat used in developing and testing the control strategies is a dragonflite 95 model RC boat [24]. The full specifications of the sailboat is shown in Table A.5. Below, in Figure 6.1, is a photo of the dragonflite sailboat. The sailboat has a mainsail and a jib sail, the sails are controlled via a servo motor that controls the sail angle. The main and jib sail cannot be operated at different angles. For steering control another servo motor is controller the rudder angle.



**Figure 6.1:** Dragonflite 95 RC model sailboat

## 6.2. Flight Controller

The hardware based around the Pixhawk 6C flight controller [25]. Traditionally the controller is used for airplanes and drones(copters). But there has been a recent shift to use the controller for rovers and boats. The technical specifications of the Pixhawk 6C is shown in Chapter A.7, the dimensions of the Pixhawk 6C is illustrated in Figure A.1 and the pinout diagram of the Pixhawk 6C is illustrated in Figure A.2.



**Figure 6.2:** Pixhawk 6C

## 6.3. External Sensors

### 6.3.1. GPS and Compass Sensor

In addition to the flight controller the platform requires more sensors to allow for the autonomous control of the sailboat. The system requires a GPS and compass sensor, both these sensors are inside the M8N GPS sensor from Holybro [26]. The sensor is illustrated in Figure 6.3. The technical and feature specifications are shown in Chapter A.8 and the pinout diagram is illustrated in Figure A.3.



**Figure 6.3:** M8N GPS

### 6.3.2. Telemetry System

The telemetry system is required to communicate with the sailboat during missions and testing. The radios communicate via the MAVLink protocol [27], communication protocol and setup is explained in the Chapter ??.



**Figure 6.4:** Telemetry radio

The high level overview of the telemetry system is illustrated in Figure 6.5, one of the telemetry radios will plug into the PC, that runs the ground station software and communicate via serial protocol. The other telemetry radio will be onboard the sailboat. The two radios will communicate via the MAVLink protocol.



**Figure 6.5:** High Level Communication

### 6.3.3. Wind Vane

Due to the limited space on the sailboat and the importance of knowing what the wind direction is, a custom wind vane will be built to result in the best accuracy.



**Figure 6.6:** Caption

## 6.4. Ground Control Software

The Ground Control Software(GCS) that was used is Mission Planner [28]. Mission Planner is specifically designed to work in conjunction with the ArduPilot firmware. The main reason for using a GCS can be summarized below,

- Loading firmware onto flight controller boards
- Setup and configure parameters
- Tuning control systems of vehicle
- Setup missions, that consists of waypoints and fences, that you can load onto the autopilot
- Able to download and view logs from missions.
- SITL/HITL simulation capabilities

Figure 6.7 illustrates the Mission Planner GUI. On the right hand side is the maps, the upper left the Heads-Up Display(HUD) and bottom left is the toggle screen, which can be switched to display different screens. Ranging from the Actions screen, Messages and Dataflash Logs. At the top of the GUI there is tabs that lets you switch between different views, the one shown in Figure 6.7 is the **FLIGHT DATA** view, the **FLIGHT PLAN** view is where the mission waypoints and fences can be set.



**Figure 6.7:** Mission Planner GUI

The **SETUP** view is where you load the firmware onto your board, the **CONFIGURATION** view is where parameters in the autopilot can be changed and written to the flight controller and the last important view **SIMULATION** is where either the SITL or HITL simulation can be executed from.

### 6.4.1. Setup and Connecting

### 6.4.2. Mission Planning

The first part in executing a mission is to plan accordingly. Inside the **FLIGHT PLAN** view navigate the map until you find the area where the mission should be executed. Now at the top right of the map switch from mission to fence in the drop down box. Place the fence around the area you wish the vehicle to stay within. In the case of sailboat, this would be the area of water. The fence is created by placing points at the banks that connect in the form of a polygon. Now switch back to mission in the drop down box. The waypoints are placed the same way as the fence but instead of forming a polygon the waypoints will be numbered in the order that they are placed, take this into account and how it will effect the mission. The last thing to configure before writing the mission to the autopilot is to set the home point of the mission which will be where the sailboat is launched from. An example of a mission is illustrated in Figure 6.8.



**Figure 6.8:** An example of a mission consisting of waypoints and a fence

# Chapter 7

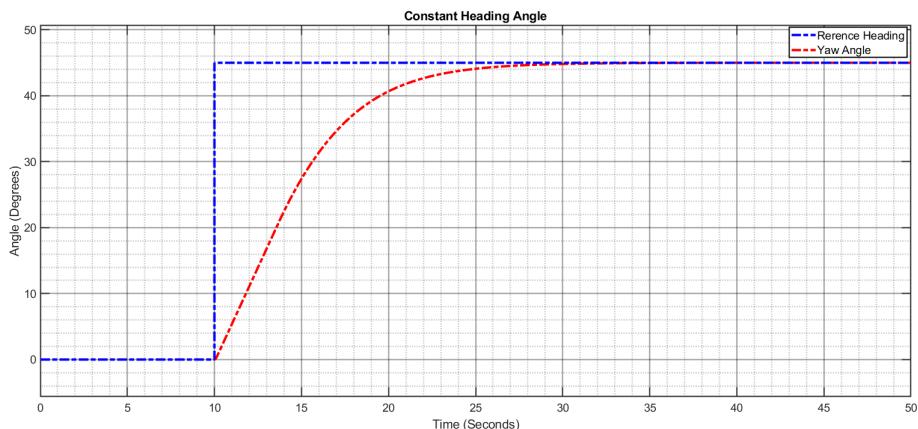
## Simulations

In this chapter the simulations and their results are discussed. The first simulation is done in Matlab with an accurate non-linear model of a boat(container vessel). The sail and rudder forces where adequately picked to steer and propel the boat model. Onto this model of the vessel the waypoint navigation, sailboat maneuvering, rudder and sail the control strategies will be implemented. The second simulations are used to test the firmware, running in the autopilot, in a SITL and HITL environment.

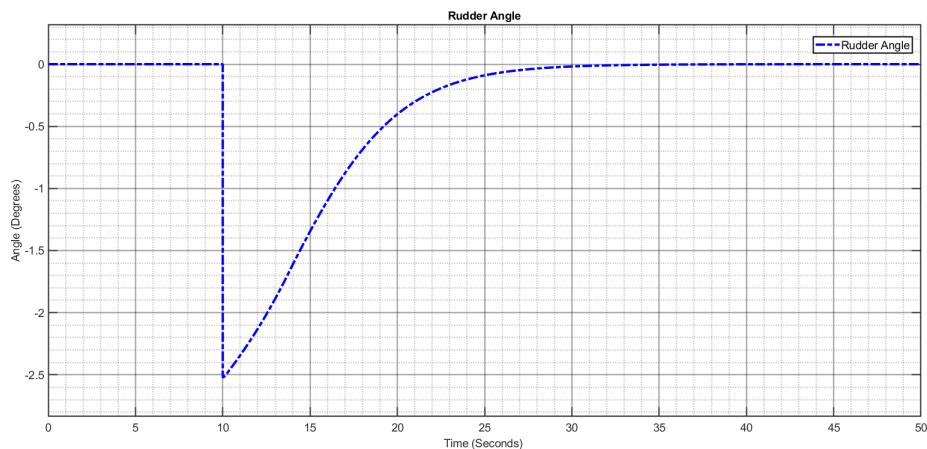
### 7.1. Matlab Simulations

The ocean vessel model is obtained from "On the Coupled motion of Steering and Rolling of a High Speed Container Ship". The model describes the vessels non-linear motion in 4-DOF(surge, sway, yaw and roll). The parameters are presented in Table A.4.

**Constant Heading Control** The first control exercised on the model is that of controlling the vessel to follow a constant heading angle. This consist of feeding the model a constant reference angle and feeding that angle to the rudder controller and measuring the vessels yaw angle. Figure 7.1 illustrates the heading and yaw angle. The heading angle is plotted as the reference heading and the yaw angle, the vessels angle, as yaw angle. As shown in the figure, the rudder controller is capable of controller the yaw angle towards a reference heading. The rudder angle is illustrated in Figure 7.2.



**Figure 7.1:** Constant Heading Angle vs Yaw Angle

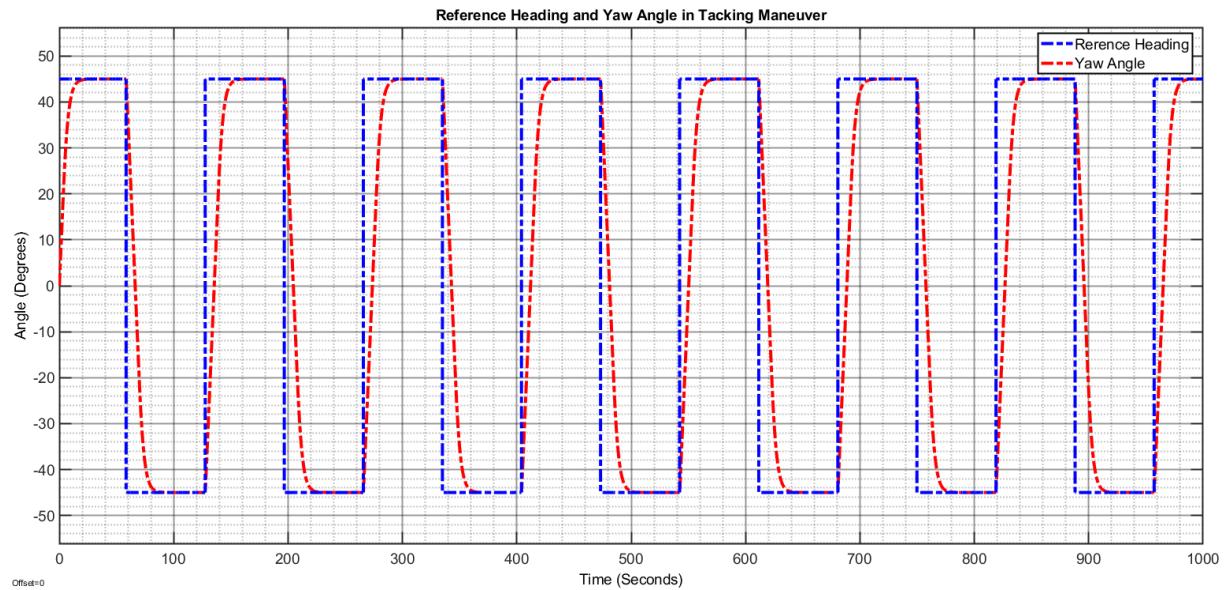


**Figure 7.2:** Rudder Angle

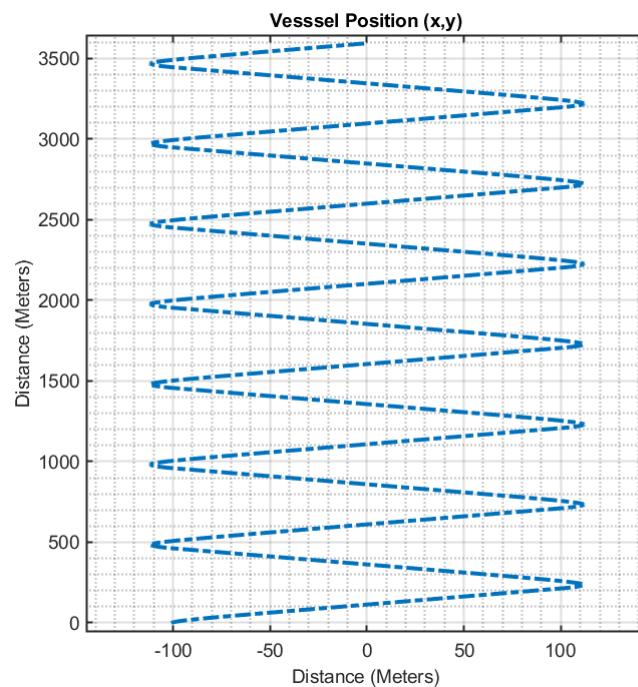
### 7.1.1. Tacking Control

The tacking state of control is when the sailboat is sailing into the wind. The vessel will perform maneuvers explained in Section 4.2.2. The desired reference headings and the distance of the tack  $D$  can be changed depending on the circumstance. A longer tack will achieve faster speeds for the ocean vessel. Whereas shorter tacks will allow for more accurate GPS positioning.

Figure 7.3 illustrates the reference heading angle and yaw angle while the vessel is performing tacking maneuvers. The reference heading is set to  $45^\circ$  and the tack distance is set to 100 meters. The wind direction is  $0^\circ$  and the sailboat is sailing directly into the wind. The vessels (x,y) global position is illustrated 7.4, noticed that the tacking angle changes when the vessels cross-track error exceeds 100m.

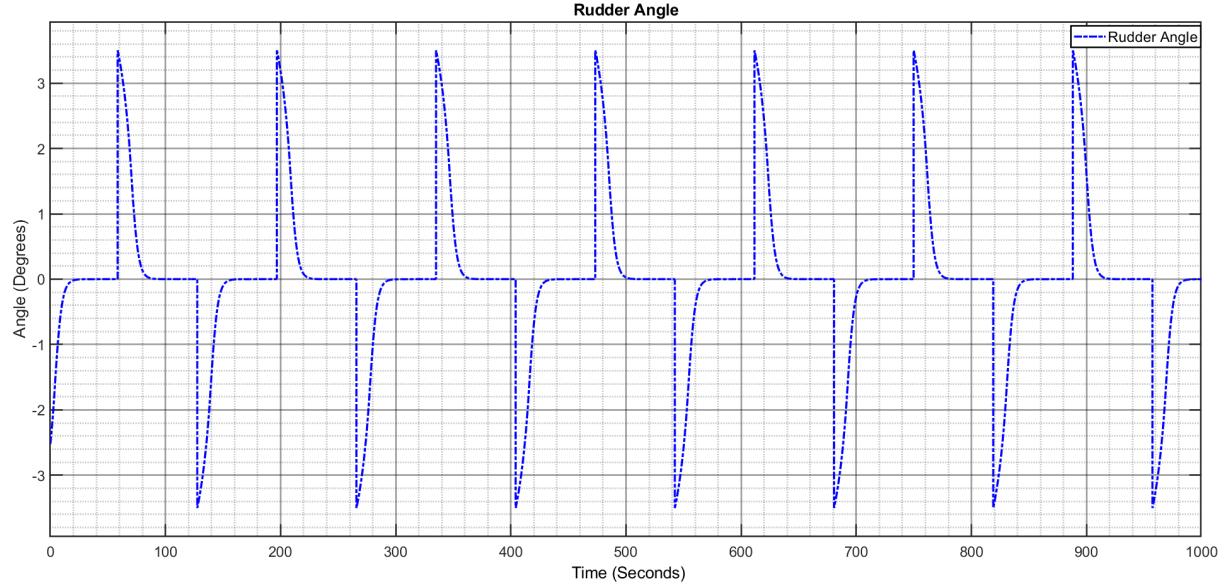


**Figure 7.3:** Tacking Maneuver



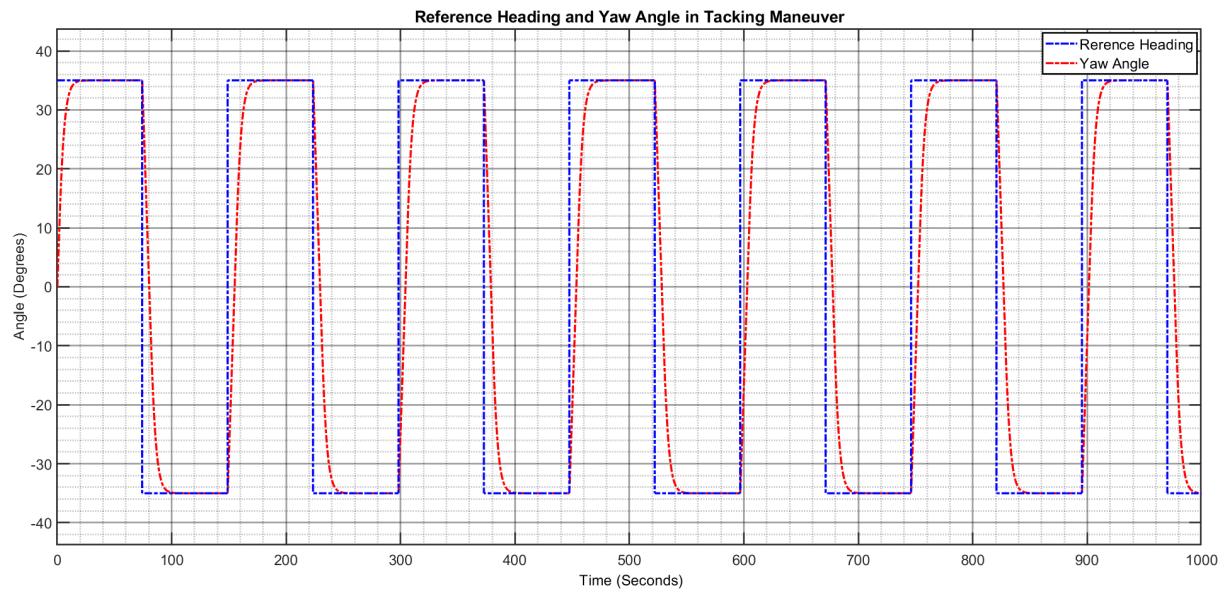
**Figure 7.4:** Vessel Position in Tacking Maneuver

The rudder angle is illustrated in Figure 7.5.

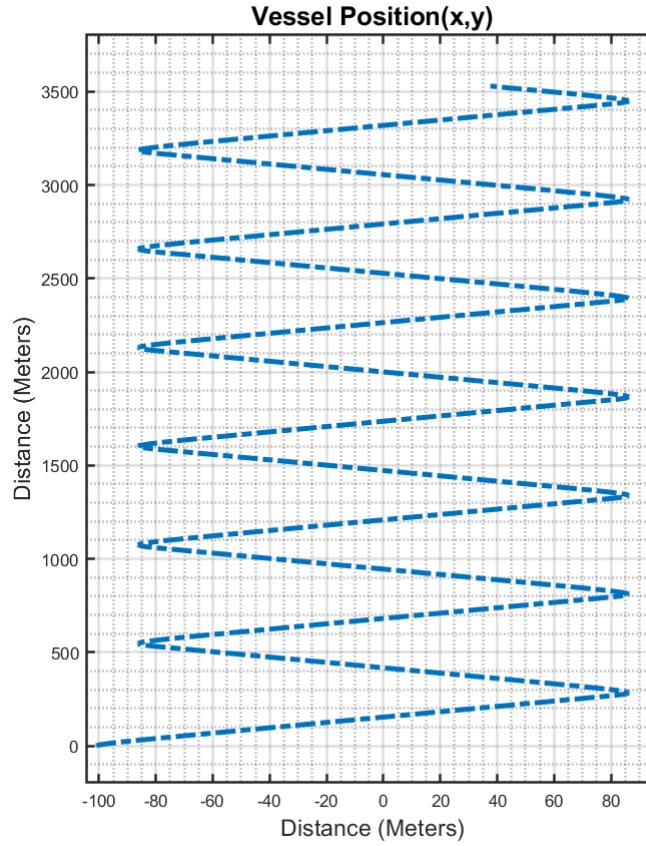


**Figure 7.5:** Rudder Angle in Tacking Maneuver

The same tacking maneuver can be achieved with different tacking angles and tacking distance,  $D$ . The reference heading and yaw angle during this tacking maneuver is illustrated in Figure 7.6, noticed that the reference heading is now  $35^\circ$ . The cross-track error is also changed to  $80m$  and is illustrated in Figure 7.7.



**Figure 7.6:** Tacking Maneuver with Reference Heading  $30^\circ$

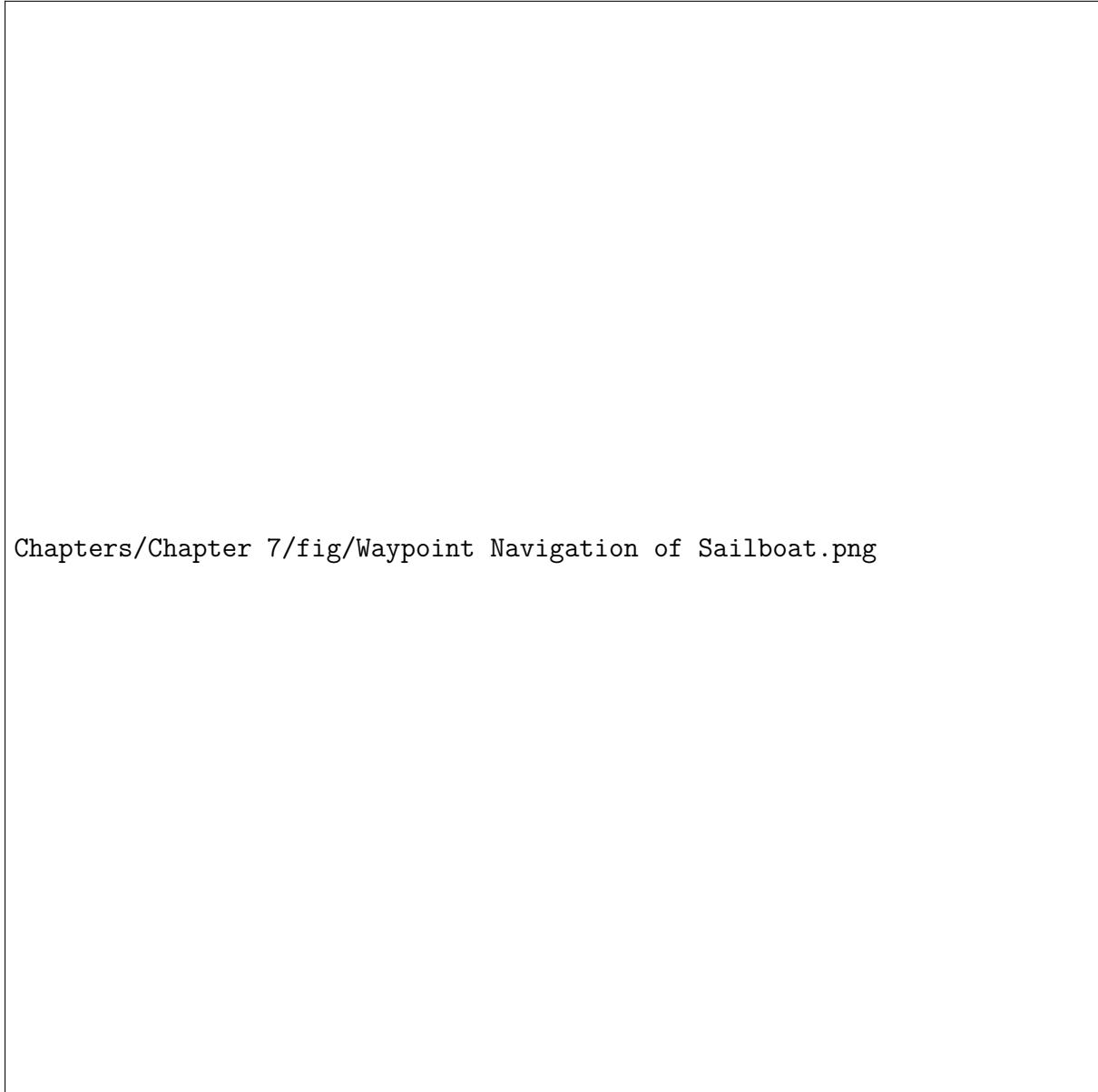


**Figure 7.7:** Vessels Position in Tacking Maneuver(D=80m)

### 7.1.2. Waypoint Navigation Simulation

The waypoint navigation will test the switching between the state from tacking and normal sailing. The waypoints are implemented with a state machine controller the current heading and then the sailboat heading controller determines which state, tacking or normal sail, it should be. The appropriate reference heading is then fed to the rudder controller as discussed in Section 4.2.2. Three waypoints are used for testing the sailboat's navigation abilities. Two waypoints chosen such that the sailboat can sail in its normal state and one waypoint chosen such that the sailboat will be required to perform a tacking maneuver.

Figure 7.8 illustrates the global position of the waypoints and the position of the sailboat. The simulation clearly shows the moment the sailboat reaches the waypoint (depending on the radius of acceptance value  $R$ ) and when the new waypoint is passed on to the sailboat heading controller state machine. The green circles illustrate the radius of acceptance around the waypoints and the waypoints are indicated with the red dots. The waypoints exact positions are show in Table 7.1.



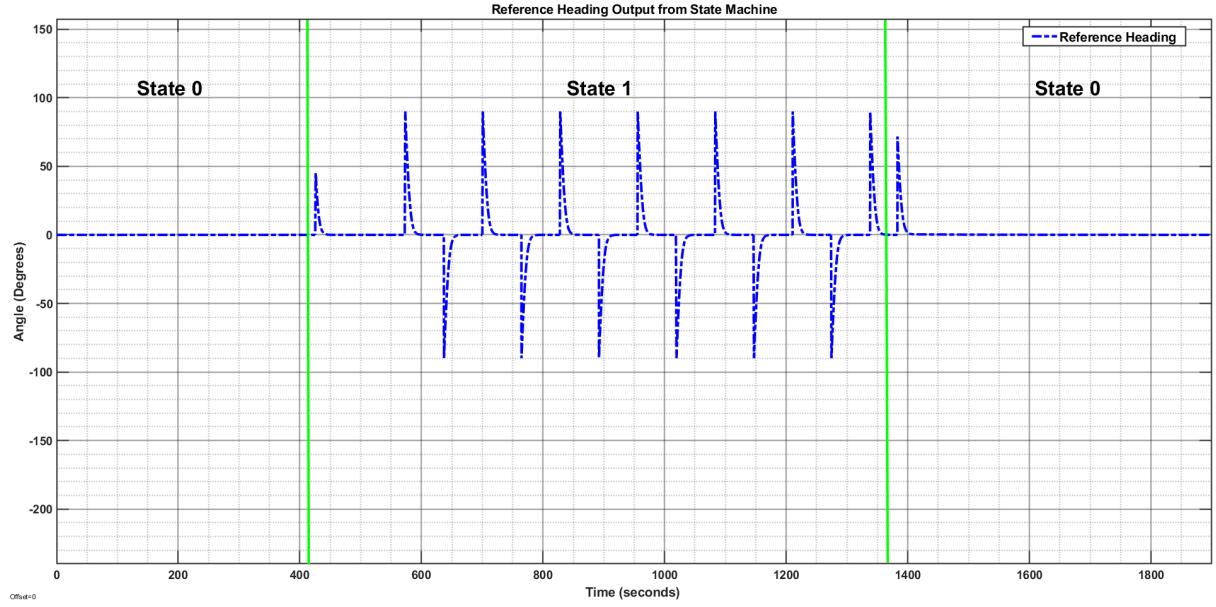
Chapters/Chapter 7/fig/Waypoint Navigation of Sailboat.png

**Figure 7.8:** Waypoint Navigation of Sailboat

**Table 7.1:** Waypoints

Waypoint	Global Position
$(x_1, y_1)$	(-1000, 0)
$(x_2, y_2)$	(-1000, 4000)
$(x_3, y_3)$	(3000, 4000)
$(x_4, y_4)$	(-1000, 0)

Lets take a deeper look into how the state machine functions. Figure 7.9 illustrates the reference heading output of the state machine. The states are separated with a green line to indicate in which state the state machine currently is in.



**Figure 7.9:** Reference Heading Output of State Machine

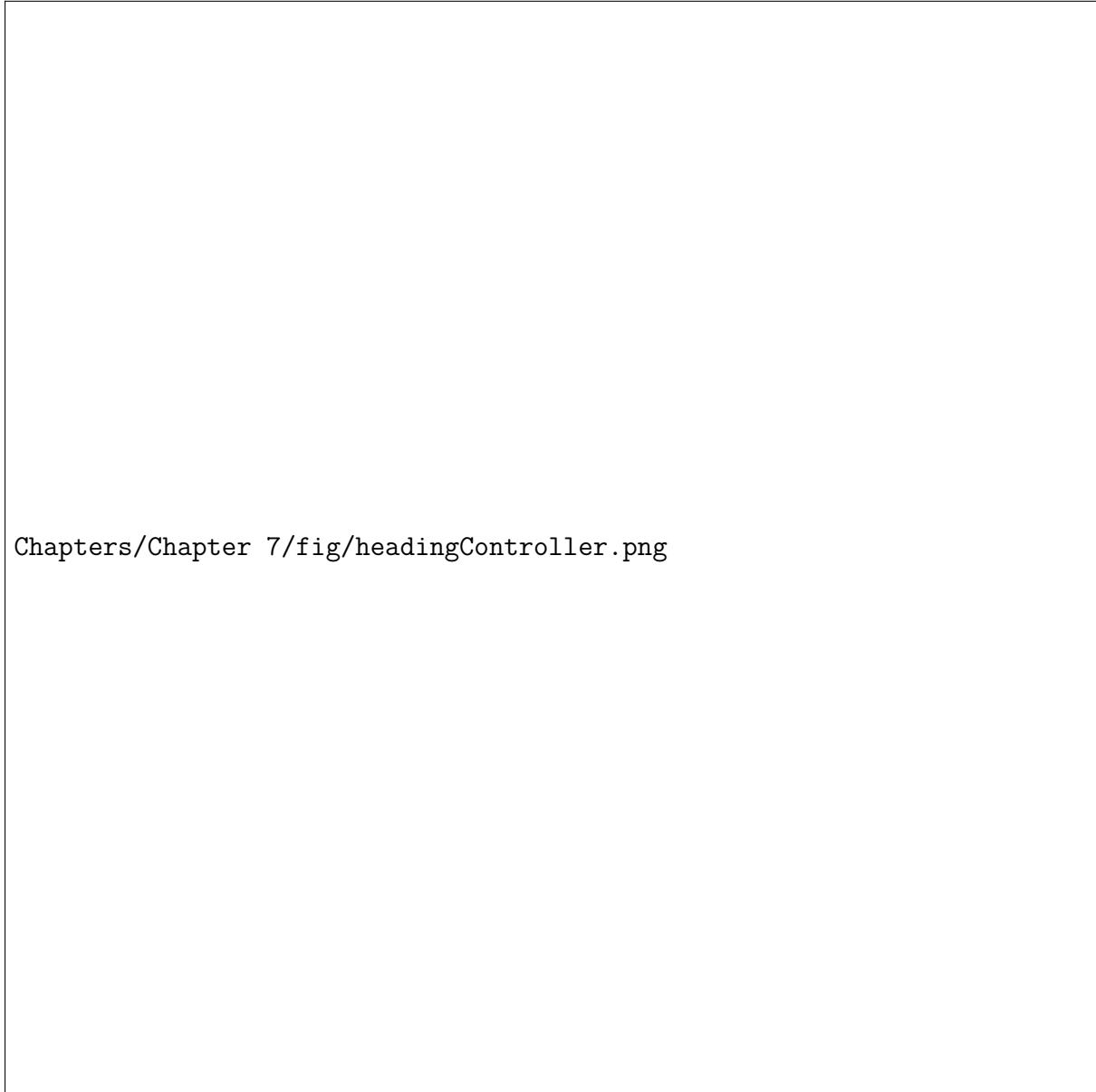
Another way to visualize the state is to look at the cross-track error. Figure 7.10 illustrates the cross-track error in each state. Take notice that the cross-track error in state 1 switches around 100 meters which is the predefined tack distance defined in the state machine.



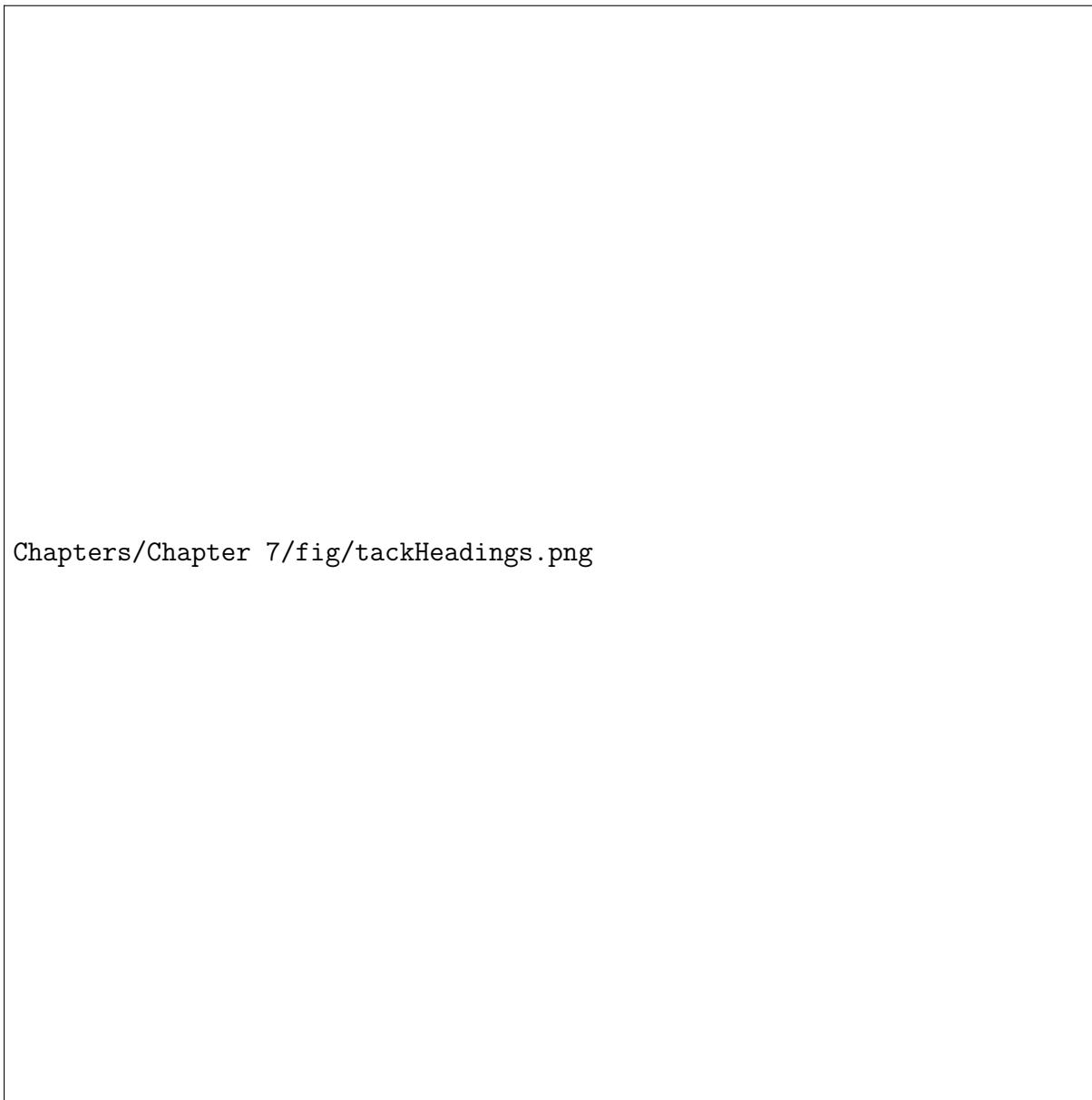
**Figure 7.10:** Cross-track error in Waypoint Navigation

Figure 7.11 illustrates the input and output of the heading controller. The output is multiplied by 100 to be visible on the same graph. The controller performs correctly

where the output is 1 if the apparent wind angle is less than  $|45^\circ|$  and 0 when the the apparent wind angle is greater than  $|45^\circ|$ . Illustrated in Figure 7.12 is the left and right tacking angles, under the condition that the apparent wind angle is  $10^\circ$ . When the sailboat is in state 1, the let tack angle is  $-35^\circ$  and the right tack angle  $55^\circ$  and then resets to  $-45^\circ$  and  $45^\circ$ . The controller is capable of switching the sailboats mode of sailing, changing the tack distance and angles of tacking.

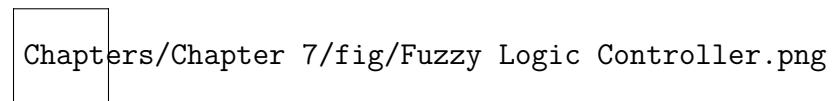


**Figure 7.11:** Input and Output of the heading controller



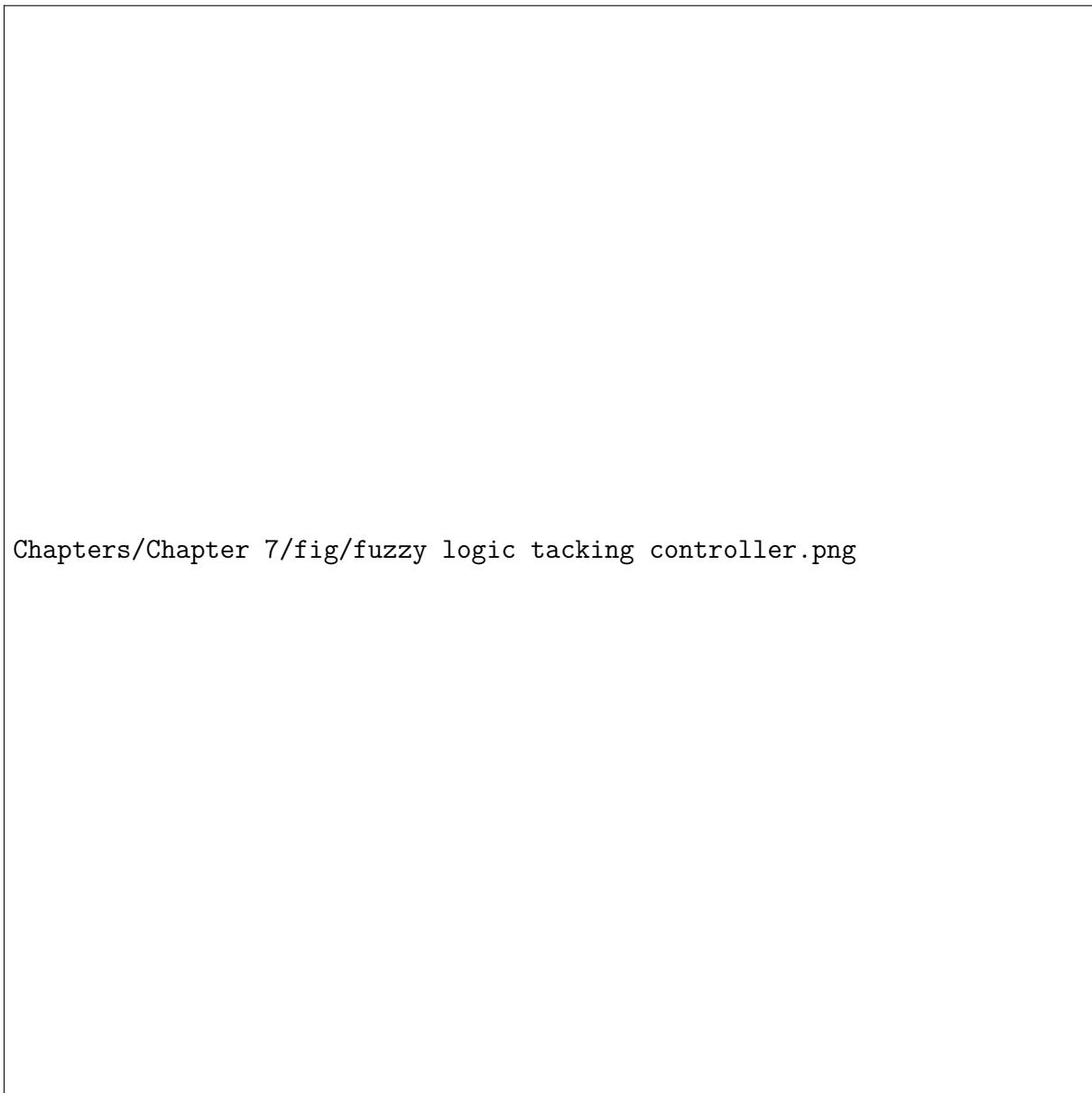
**Figure 7.12:** Left and Right Tack Headings

The Simulink implementation is illustrated in Figure 7.13.



**Figure 7.13:** Fuzzy Logic Controller

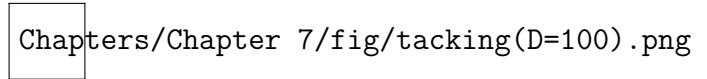
The input to the controller is the cross-track distance  $e$ . The output of the controller is the tacking direction. Once the vessel's cross-track distance,  $e$ , reaches the tack distance, the tack angle switches to the left angle tack. When the cross-track distance,  $e$ , reaches the negative tack distance the tack angle switches to the right angle tack. This is illustrated in Figure 7.14, where the blue graph is the cross-track distance and the red graph is the reference heading.



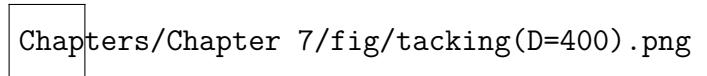
Chapters/Chapter 7/fig/fuzzy logic tacking controller.png

**Figure 7.14:** Fuzzy Logic Tacking Controller Input and Output

To illustrate that the Distance D determines the amount of tacks the sailboat perform, the tacking maneuver is simulated with  $D = 100$  and  $D = 400$ . Tacking Less will give the sailboat more speed, but has the trade off the waypoint following will be less accurate.



**Figure 7.15:** Tacking with  $D = 100$



**Figure 7.16:** Tacking with  $D = 400$

## 7.2. Firmware Simulations

The firmware of the autopilot can be tested in a SITL/HITL simulation through a python script with visualization in Mission Planner. This simulation will form as a preliminary test before testing the autopilot and controllers on the sailboat. Firstly the sail controller will be tested. The sail is controlled by a servo motor that either winches in the sail or out. The servo's output varies from 0 to 100. Where 0 means fully winched in and 100 means fully released. The implemented sail controller is illustrated below,

**Listing 7.1:** Sail Angle Controller

```

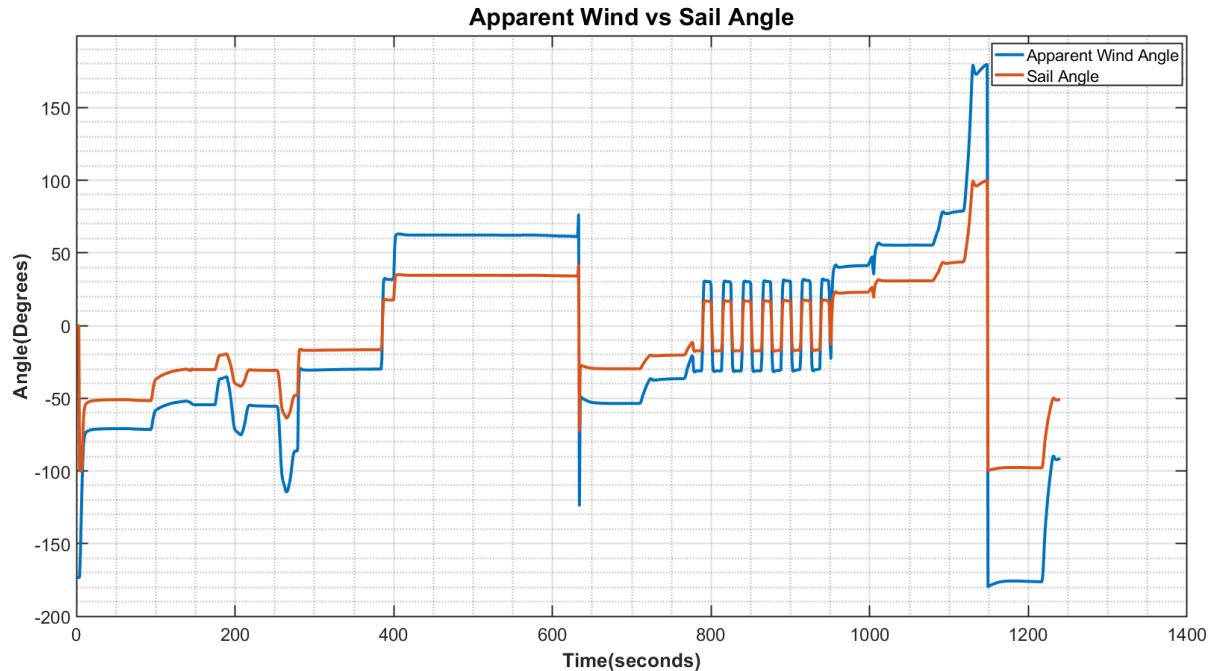
1 // 
2 // Sail Angle Control
3 //
4     if(!is_positive(desired_speed)){
5         mainsail_out = 100.0f;
6     }else{
7         // Sail angle controller
8         // Sails are sheeted the same on each side use abs wind direction
9
10        // set the main sail to the ideal angle to the wind
11        float mainsail_angle = 0.5 * wind_dir_apparent_abs;
12
13        // make sure between allowable range
14        mainsail_angle = constrain_float(mainsail_angle,sail_angle_min, ...
15                                     sail_angle_max);
16        // gcs().send_text(MAV_SEVERITY_INFO, "Sail Angle: %.1f", ...
17                         static_cast<double>(mainsail_angle*wind_dir_apparent_sign));
18
19        // linear interpolate mainsail value (0 to 100) from wind angle ...
20        mainsail_angle

```

```

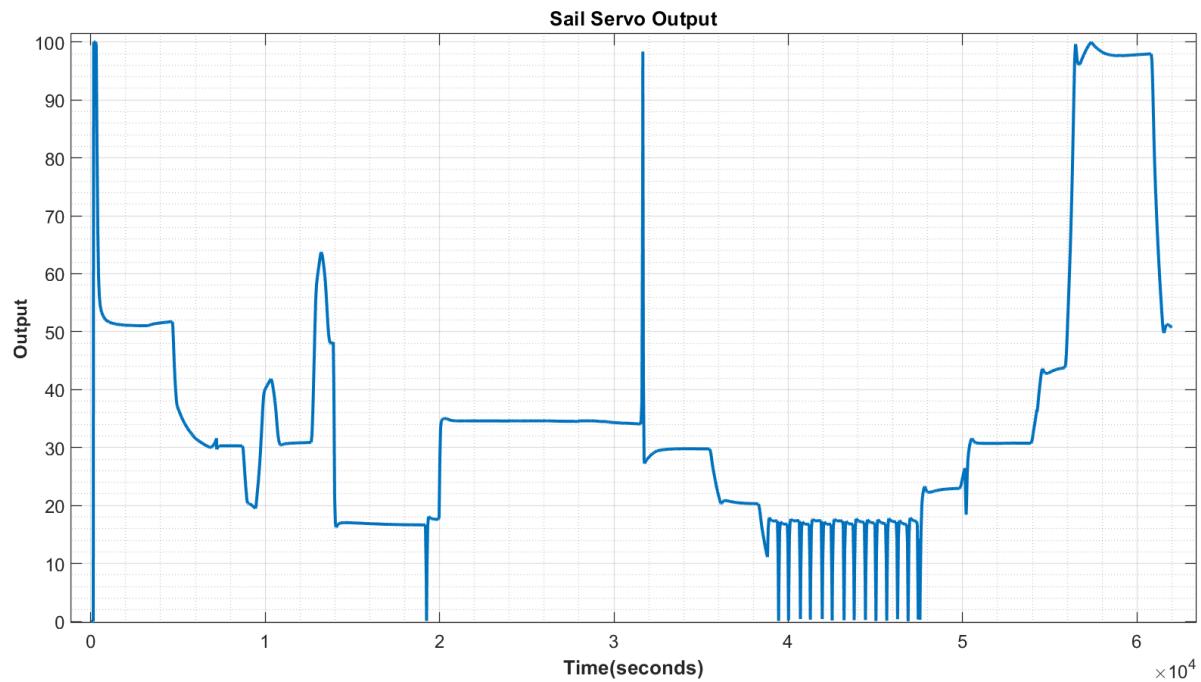
18     float mainsail_base = linear_interpolate(0.0f, 100.0f, ...
19         mainsail_angle, sail_angle_min, sail_angle_max);
20
21     mainsail_out = constrain_float((mainsail_base + pid_offset), ...
22         0.0f, 100.0f);
23     // gcs().send_text(MAV_SEVERITY_INFO, "Main Sail Out: %.1f", ...
24     static_cast<double>(mainsail_out));
25 }
```

Line 11 calculates the angle of the main sail according to the apparent wind direction. The main sail angle is then constrained to fall inside the range defined by the minimum and maximum sail angle parameters. The main sail angle is then converted to range between 0 and 100, where 0 is  $0^\circ$  and 100 is  $90^\circ/-90^\circ$ . The main sail output is then controlled via PID controller which dictates how fast the sail is winched. The results is illustrated in Figure 7.17.



**Figure 7.17:** Apparent wind compared to sail angle

The sail angle is correctly adjusted to find the optimal angle over the course of the mission. The servo output is illustrated in Figure 7.18. The sailboat speed is also positive throughout the mission and this confirms that the sail controller works correctly. The sail boat speed is illustrated in Figure 7.19. In the simulation the wind speed is 3 m/s. The sailboat maintains a positive speed throughout the mission.



**Figure 7.18:** Sailboat Speed

Chapters/Simulation Results/fig/sailboatSpeed.png

**Figure 7.19:** Sail Servo Output

# **Chapter 8**

## **Results**

# **Chapter 9**

## **Conclusion**

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# Appendix A

## Additional Modelling Information

### A.1. Notation and Vector Definitions

**Table A.1:** SNAME Notation for ocean vessels

Degree of freedom		Force and moment	Linear and angular velocity	Position and Euler angles
1	Surge	$X$	$u$	$x$
2	Sway	$Y$	$v$	$y$
3	Heave	$Z$	$w$	$z$
4	Roll	$K$	$p$	$\phi$
5	Pitch	$M$	$q$	$\theta$
6	Yaw	$N$	$r$	$\psi$

**Table A.2:** Rigid body motion vectors

Vector	Components	Definition
$\mathbf{f}_{Ob}$	$[X \ Y \ Z]^T$	force decomposed in the body-fixed frame
$\mathbf{m}_{Ob}$	$[K \ M \ N]^T$	moment decomposed in the body-fixed frame
$\mathbf{v}_{Ob}$	$[u \ v \ w]^T$	linear velocity decomposed in the body-fixed frame
$\mathbf{w}_{Ob}^E$	$[p \ q \ r]^T$	angular velocity of the body-fixed relative to the earth-fixed frame
$\mathbf{r}_{Ob}$	$[x_g \ y_g \ z_g]^T$	vector from $O_b$ to CG decomposed in the body-fixed frame

## A.2. Modeling Equations

$$\mathbf{M}_{\mathbf{RB}} = \begin{bmatrix} m & 0 & 0 & mz_g & mz_g & -my_g \\ 0 & m & 0 & 0 & 0 & mx_g \\ 0 & 0 & m & -mx_g & -mx_g & 0 \\ 0 & -mz_g & -my_g & I_x & -I_{xy} & -I_{xz} \\ mz_g & 0 & -mx_g & -I_{xy} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \quad (\text{A.1})$$

$$\mathbf{C}_{\mathbf{RB}}(\mathbf{v}) = \begin{bmatrix} 0 & 0 & 0 & m(y_g q + z_g r) & -m(x_g q - w) & -m(x_g r + v) \\ 0 & 0 & 0 & -m(y_g p + w) & m(z_g r + x_g p) & -m(y_g r - u) \\ 0 & 0 & 0 & -m(z_g p - v) & -m(z_g q + u) & m(x_g p + y_g q) \\ -m(y_g q + z_g r) & m(y_g p + w) & m(y_g p - v) & 0 & -I_{yz} q - I_{xz} q + I_z r & I_{yz} r + I_{xy} p - I_y q \\ m(x_g p - w) & -m(z_g r - x_g p) & m(z_g q + u) & I_{yz} q + I_{xz} p - I_z r & 0 & -I_{xz} r - I_{xy} q + I_X p \\ m(x_g r + v) & m(y_g r - u) & -m(x_g p + y_g q) & -I_{yz} r - I_{xy} p + I_y q & I_{xz} r + I_{xy} q - I_x p & 0 \end{bmatrix} \quad (\text{A.2})$$

$$a_1 = X_{\dot{u}} u + X_{\dot{v}} v + X_{\dot{w}} w + X_{\dot{p}} p + X_{\dot{q}} q + X_{\dot{r}} r \quad (\text{A.3})$$

$$a_2 = Y_{\dot{u}} u + Y_{\dot{v}} v + Y_{\dot{w}} w + Y_{\dot{p}} p + Y_{\dot{q}} q + Y_{\dot{r}} r \quad (\text{A.4})$$

$$a_3 = Z_{\dot{u}} u + Z_{\dot{v}} v + Z_{\dot{w}} w + Z_{\dot{p}} p + Z_{\dot{q}} q + Z_{\dot{r}} r \quad (\text{A.5})$$

$$b_1 = K_{\dot{u}} u + K_{\dot{v}} v + K_{\dot{w}} w + K_{\dot{p}} p + K_{\dot{q}} q + K_{\dot{r}} r \quad (\text{A.6})$$

$$b_2 = M_{\dot{u}} u + M_{\dot{v}} v + M_{\dot{w}} w + M_{\dot{p}} p + M_{\dot{q}} q + M_{\dot{r}} r \quad (\text{A.7})$$

$$b_3 = N_{\dot{u}} u + N_{\dot{v}} v + N_{\dot{w}} w + N_{\dot{p}} p + N_{\dot{q}} q + N_{\dot{r}} r \quad (\text{A.8})$$

**Table A.3:** Sail Angle Look-up Table

Apparent Wind Angle	Sail Angle	Apparent Wind Angle	Sail Angle
0	-90	1	-89
2	-89	3	-88
4	-89	5	-87
6	-88	7	-86
8	-87	9	-85
10	-86	11	-84
12	-85	13	-83
14	-84	15	-82
16	-83	17	-81
18	-82	19	-80
20	-81	21	-79
22	-80	23	-78
24	-79	25	-77
26	-78	27	-76
28	-77	29	-75
30	-76	31	-74
...	...	...	...
330	75	331	76
332	76	333	77
334	77	335	78
336	78	337	79
338	79	339	80
340	80	341	81
342	81	343	82
344	82	345	83
346	83	347	84
348	84	349	85
350	85	351	86
352	86	353	87
354	87	355	88
356	88	357	89
358	89	359	90

## A.3. Ocean Vessel Parameters

**Table A.4:** Parameters of Container Vessel

Parameter Definition	Parameter Value	Parameter Definition	Parameter value
$m$	0.00792	$N_v$	-0.0038545
$X_{uu}$	-0.0004226	$N_r$	-0.00222
$X_{vr}$	-0.00311	$N_p$	0.000213
$X_{rr}$	0.0002	$N_\phi$	-0.0001424
$X_{\phi\phi}$	-0.0002	$N_{vvv}$	0.001492
$X_{vv}$	-0.00386	$N_{rrr}$	-0.00229
$K_v$	0.0003026	$N_{vvr}$	-0.0424
$K_r$	-0.000063	$N_{vrr}$	0.00156
$K_p$	-0.0000075	$N_{vv\phi}$	-0.019058
$K_\phi$	-0.000021	$N_{v\phi\phi}$	-0.0053766
$K_{vvv}$	0.002843	$N_{rr\phi}$	-0.0038592
$K_{rrr}$	-0.0000462	$N_{r\phi\phi}$	0.0024195
$K_{vvr}$	-0.000588	$Y_{r\phi\phi}$	-0.001368
$K_{vrr}$	0.0010565	$Y_{rr\phi}$	0.009325
$K_{vv\phi}$	-0.0012012	$Y_{v\phi\phi}$	0.00304
$K_{v\phi\phi}$	-0.0000793	$Y_{vv\phi}$	0.04605
$K_{rr\phi}$	-0.000243	$Y_{vrr}$	-0.0405
$K_{r\phi\phi}$	0.00003569	$Y_{vvr}$	0.0214
$Y_v$	-0.0116	$Y_{rrr}$	0.00177
$Y_r$	0.00242	$Y_{vvv}$	-0.109
$Y_p$	0	$Y_\phi$	-0.000063

## A.4. Heading Controller

## A.5. Matlab Code

### A.5.1. Sail Controller

```

1 %% Sail Controller
2 function y = sailController(u)
3 % Convert apparent wind angle to the range 0 -> 360
4 if(u<360)
5     % Sail control equation implementation
6     if(u<180)
7         y = -0.5*u;
8     else
9         y=-0.5*u+180;
10    end
11 else
12     % Sail control equation implementation
13     u=u-360;
14     if(u<180)
15         y = -0.5*u;
16     else
17         y=-0.5*u+180;
18     end
19 end
20 end

```

### A.5.2. Heading Controller

```

1 %% Heading Controller
2 function [y,tack,tackDistance,tackLeft,tackRight] = ...
3     headingController(tackHeading,pathTrajectory,WA,
4 currentHeading,wayPointHeading)
5 % Change wind angle to follow same convection as waypoint ...
6     heading, ranges from -180 -> 180
5 if WA<-180
6     WA = WA + 360;
7 end

```

```
8 if WA>180
9     WA = WA - 360;
10 end
11
12 % Determine the state of sailboat
13 if      abs(wayPointHeading-WA)<45
14     % State 1: Tacking
15         % Initialize tacking right
16         if(abs(tackHeading)<15)
17             % Set tack angle to the right
18             y=WA+45;
19             % State value
20             tack=0.5;
21             % Tack Distance D
22             tackDistance=400;
23             % Tack Right Angle
24             tackRight=WA+45;
25             % Tack Left Angle
26             tackLeft=WA-45;
27     else
28         % Reference heading
29         y=tackHeading-currentHeading;
30         % State value
31         tack=1;
32         % Tack Right Angle
33         tackRight=WA+45;
34         % Tack Left Angle
35         tackLeft=WA-45;
36         % Tack Distance D
37         tackDistance=400;
38     end
39 % State 0: Normal Sail
40 else
41     % State Value
42     tack=0;
43     % Reference Heading
44     y=pathTrajectory;
45     % Reset tacking distance
46     tackDistance=100;
```

```
47 % Reset Right tack angle  
48 tackRight=45;  
49 % Reset Left tack angle  
50 tackLeft=-45;  
51 end
```

### A.5.3. Waypoint Controller

```
1 %% Heading Controller  
2 function [nextState,currentWaypoint] = ...  
    wayPointController(R,currentState)  
3 % Waypoint switching  
4 % With each added waypoint another case should be added  
5 switch currentState  
6     % First waypoint  
7     case 0  
8         % Check if waypoint lies within radius of acceptance  
9         if(R==1)  
10            % Switch current Waypoint to next waypoint  
11            nextState=1;  
12            currentWaypoint=1;  
13            % Waypoint not reached  
14        else  
15            nextState=0;  
16            currentWaypoint=0;  
17        end  
18    case 1  
19        % Check if waypoint lies within radius of acceptance  
20        if(R==1)  
21            % Switch current Waypoint to next waypoint  
22            nextState=2;  
23            currentWaypoint=2;  
24            % Waypoint not reached  
25        else  
26            nextState=currentState;  
27            currentWaypoint=1;  
28    end
```

```
29 case 2
30     % Check if waypoint lies within radius of acceptance
31     if(R==1)
32         % Switch current Waypoint to next waypoint
33         nextState=3;
34         currentWaypoint=3;
35     % Waypoint not reached
36     else
37         nextState=currentState;
38         currentWaypoint=2;
39     end
40 case 3
41     % Check if waypoint lies within radius of acceptance
42     if(R==1)
43         % Switch current Waypoint to next waypoint
44         nextState=4;
45         currentWaypoint=4;
46     % Waypoint not reached
47     else
48         nextState=currentState;
49         currentWaypoint=3;
50     end
51 case 4
52     % Check if waypoint lies within radius of acceptance
53     if(R==1)
54         % Switch current Waypoint to next waypoint
55         nextState=0;
56         currentWaypoint=0;
57     % Waypoint not reached
58     else
59         nextState=currentState;
60         currentWaypoint=4;
61     end
62 otherwise
63     nextState=0;
64     currentWaypoint=0;
65 end
```

### A.5.4. Waypoint Construction

```

1 %% Waypoints
2 Waypoints.x = [x0, x1, x2, x3];
3 Waypoints.y = [y0, y1, y2, y3];
4 Waypoints.R = [R0, R1, R2, R3];
5 waypoint = 0; % Starting Waypoint
6 Waypoints.number= [0,1,2,3];

```

## A.6. Sailboat Specifications

**Table A.5:** Specifications of Dragonflite 95 Sailboat

Parameter Definition	Parameter Value
Length	950 mm
Width	125 mm
Rig Height	105 mm
Overall Height	1470 mm
RTR Total Weight	2000 g
Sail Area (Mainsail)	2314 cm <sup>2</sup>
Sail Area (Jib)	1422 cm <sup>2</sup>
Sail Area (Overall)	3736 cm <sup>2</sup>
Hull Material	Molded plastic with painted finish and logo sticker

## A.7. Pixhawk 6C Technical Specifications

### Processors and Sensors

- FMU Processor: STM32H743
  - 32 Bit Arm® Cortex®-M7, 480MHz, 2MB memory, 1MB SRAM
- IO Processor: STM32F103
  - 32 Bit Arm® Cortex®-M3, 72MHz, 64KB SRAM
- On-board sensors
  - Accel/Gyro: ICM-42688-P
  - Accel/Gyro: BMI055
  - Mag: IST8310
  - Barometer: MS5611

### Electrical data

- Voltage Ratings:
  - Voltage Ratings:
  - USB Power Input: 4.75-5.25V
  - Servo Rail Input: 0-36V
- Servo Rail Input: 0-36V
  - TELEM1 Max output current limiter: 1.5A
  - All other port combined output current limiter: 1.5A

### Mechanical data

- Dimensions: 84.8 \* 44 \* 12.4 mm
- Weight: 59.3g

### Interfaces

- 16- PWM servo outputs (8 from IO, 8 from FMU)
  - TELEM1 - Full flow control, separate 1.5A current limit
  - TELEM2 - Full flow control

- TELEM3
- 2 GPS ports
  - GPS1 - Full GPS port (GPS plus safety switch)
  - GPS2 - Basic GPS port
- 1 I2C port
  - Supports dedicated I2C calibration EEPROM located on sensor module
- 2 CAN Buses
  - CAN Bus has individual silent controls or ESC RX-MUX control
- 2 Debug ports:
  - FMU Debug
  - I/O Debug
- Dedicated R/C input for Spektrum / DSM and S.BUS, CPPM, analog / PWM RSSIs
- 2 Power input ports (Analog)
- Other Characteristics:
  - Operating and storage temperature: -40 to 85°C

Chapters/Chapter 5/fig/Pixhawk 6C Diagram.png

**Figure A.1:** Pixhawk 6C Dimensions

Chapters/Chapter 5/fig/pixhawk 6C pinouts.png

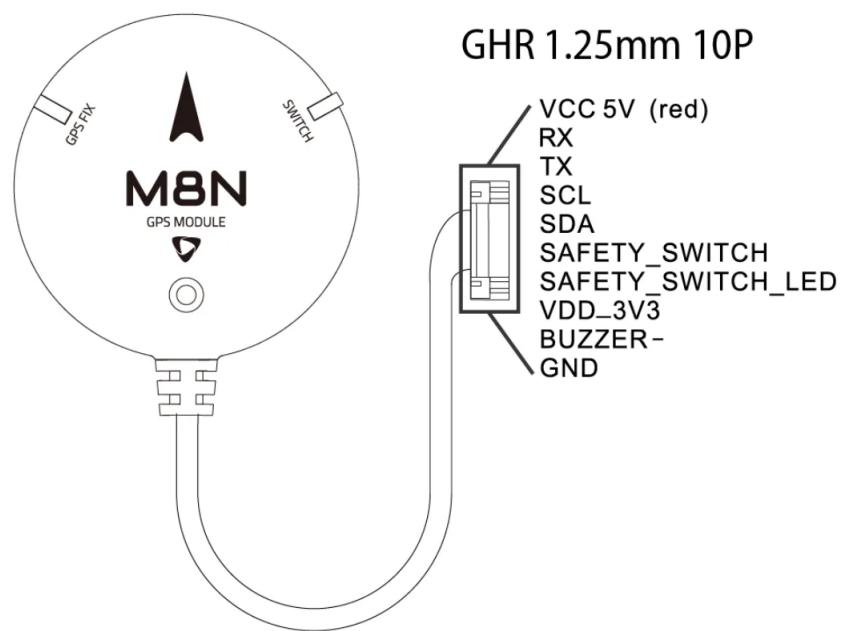
**Figure A.2:** Pixhawk 6C Pinout Diagram

## A.8. M8N GPS Technical and Features Specifications

### Features and Specifications:

- Ublox Neo-M8N module
- IST8310 compass
- Industry leading -167 dBm navigation sensitivity
- Cold starts: 26s
- LNA MAX2659ELT+

- 25 x 25 x 4 mm ceramic patch antenna
- Rechargeable Farah capacitance
- Low noise 3.3V regulator
- Current consumption: less than 150mA @ 5V
- Fix indicator LEDs
- Protective case
- Cable Length: 26cm
- Diameter 50mm total size, 32 grams with case



**Figure A.3:** M8N GPS pinout diagram

## A.9. Ardupilots Sailboat Configuration Parameters

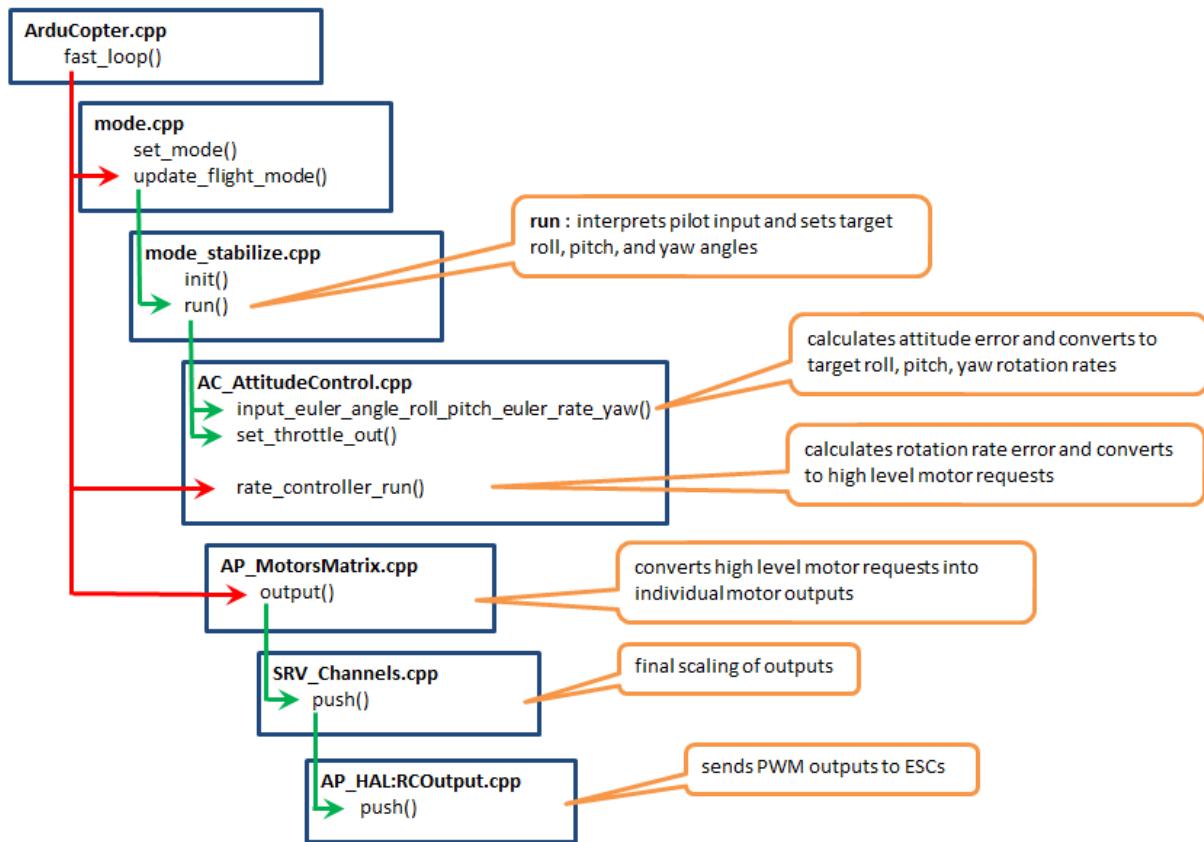
**Table A.6:** Sailboat Configuration Parameters

Parameter	Definition
SAIL_ENABLE	This enables Sailboat functionality
SAIL_ANGLE_MIN	Mainsheet tight, angle between centerline and boom
SAIL_ANGLE_MAX	Mainsheet loose, angle between centerline and boom
SAIL_ANGLE_IDEAL	Ideal angle between sail and apparent wind
SAIL_WNDSPD_MIN	Sailboat minimum wind speed to continue sail in, at lower wind speeds the sailboat will motor if one is fitted
SAIL_NO_GO_ANGLE	The typical closest angle to the wind the vehicle will sail at. The vehicle will sail at this angle when going upwind
SAIL_XTRACK_MAX	The sail boat will tack when it reaches this cross track error, defines a corridor of 2 times this value wide, 0 disables
PIVOT_TURN_RATE	Desired pivot turn rate in deg/s.
SAIL_LOIT_RADIUS	When in sailing modes the vehicle will keep moving within this loiter radius
SAIL_HELL_MAX	When in auto sail trim modes the heel will be limited to this value using PID control
ATC_SAIL_P	Sail Heel control P gain for sailboats. Converts the error between the desired heel angle (in radians) and actual heel to a main sail output (in the range -1 to +1)
ATC_SAIL_D	Sail Heel control D gain. Compensates for short-term change in desired heel angle vs actual
ATC_SAIL_I	Sail Heel control P gain for sailboats. Converts the error between the desired heel angle (in radians) and actual heel to a main sail output (in the range -1 to +1)
ATC_SAIL_IMAX	Sail Heel control I gain maximum. Constrains the maximum I term contribution to the main sail output (range -1 to +1)
SIM_SAIL_TYPE	Sailboat simulation sail type
ATC_SAIL_FF	Sail Heel control feed forward
ATC_SAIL_FILT	Sail Heel control input filter. Lower values reduce noise but add delay.

**Table A.6:** Sailboat Configuration Parameters

Parameter	Definition
SAIL_ENABLE	This enables Sailboat functionality
SAIL_ANGLE_MIN	Mainsheet tight, angle between centerline and boom
SAIL_ANGLE_MAX	Mainsheet loose, angle between centerline and boom
SAIL_ANGLE_IDEAL	Ideal angle between sail and apparent wind
SAIL_WNDSPD_MIN	Sailboat minimum wind speed to continue sail in, at lower wind speeds the sailboat will motor if one is fitted
SAIL_NO_GO_ANGLE	The typical closest angle to the wind the vehicle will sail at. The vehicle will sail at this angle when going upwind
SAIL_XTRACK_MAX	The sail boat will tack when it reaches this cross track error, defines a corridor of 2 times this value wide, 0 disables
PIVOT_TURN_RATE	Desired pivot turn rate in deg/s.
ATC_SAIL_FLTT	Target filter frequency in Hz
ATC_SAIL_FLTE	Error filter frequency in Hz
ATC_SAIL_FLTD	Derivative filter frequency in Hz
ATC_SAIL_SMAX	Sets an upper limit on the slew rate produced by the combined P and D gains

## A.10. Copter Modes Architecture



**Figure A.4:** Manual Flight Mode

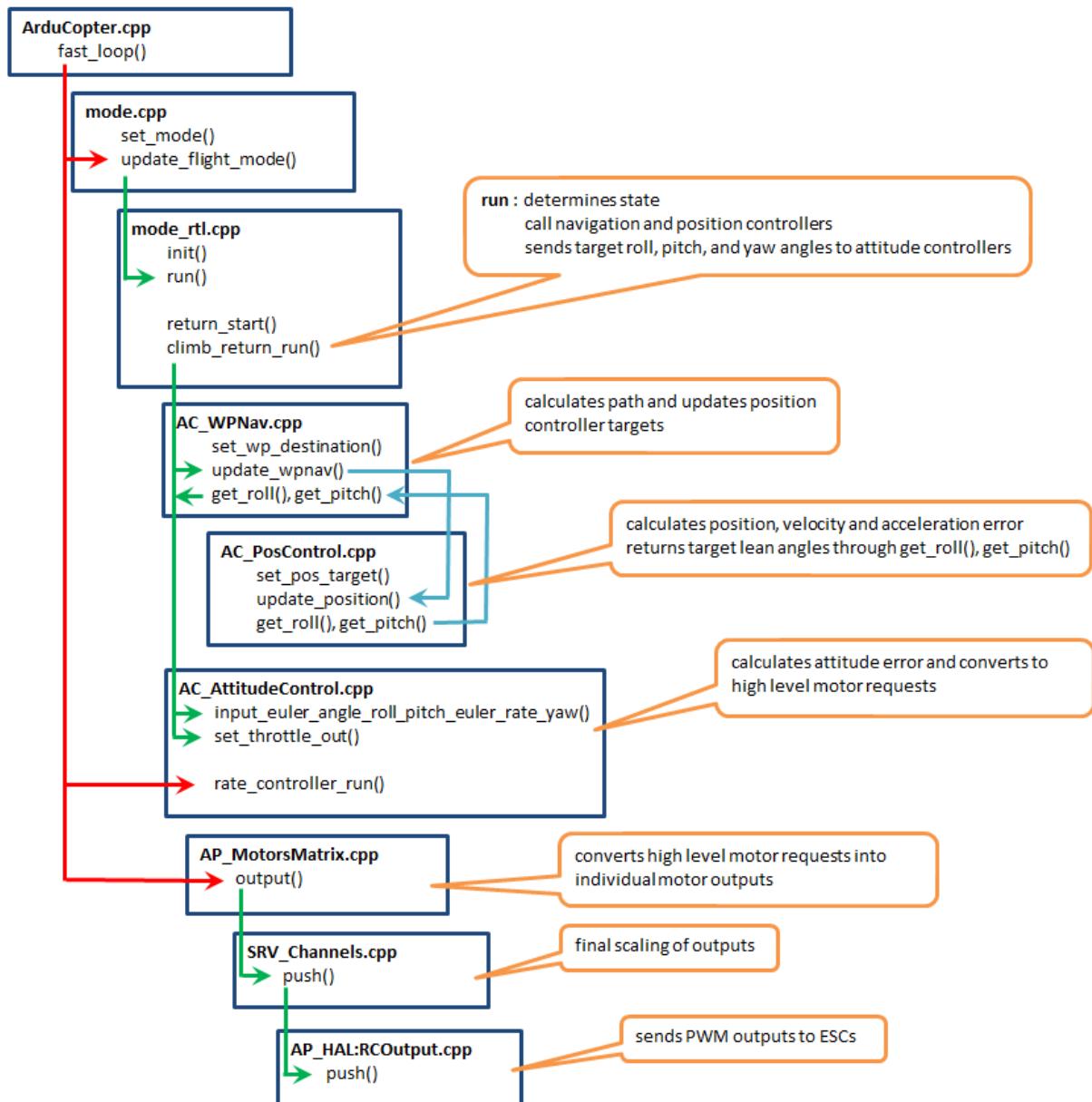


Figure A.5: Autonomous Flight Mode

## A.11. Creating Custom Log Messages

When creating a new log message first, we firstly need to create a *struct* to hold the message declarations. The name *log\_test* is given to the structure containing the information that is being sent through log messages. Every log message starts the same with *LOG\_PACKET\_HEADER* and *uint64\_t time\_us*. The rest of the structure is illustrated below

**Listing A.1:** Structure Definition

```

1 struct PACKED log_Test {
2     LOG_PACKET_HEADER;
3     uint64_t time_us;
4     float a_value;
5 }
```

The structure should be defined in the *Log.cpp* file of the **Rover** firmware. Also inside the *Log.cpp* file a function that sends the message structure defined above. The function assigns values to the message and sends it to the *logger*.

**Listing A.2:** Function Definition

```

1 void Rover::Log_Write_Test()
2 {
3     struct log_Test pkt = {
4         LOG_PACKET_HEADER_INIT(LOG_TEST_MSG),
5         time_us : AP_HAL::micros64(),
6         a_value : 1234
7     };
8     logger.WriteBlock(&pkt, sizeof(pkt));
9 }
```

The definition of the custom message should also be added to the *log\_structure*. The definition is illustrated below, the definition for the letters is defined in *LogStructure.h* file.

**Listing A.3:** Message Definition

```

1 // @LoggerMessage: TEST
2 // @Description: Custom Log Message information
3 // @Field: TimeUS: Time since system startup
4 // @Field: Test: Test Var
5 { LOG_TEST_MSG, sizeof(log_Test),
6   "TEST", "Qf", "TimeUS,Test", "sm", "F0" },
7 };
```

The log message can be called in different ways and at different rates inside the *Rover.cpp* file(remember to also define function in *Rover.h*), below are the once a second loop.

**Listing A.4:** Log Message Call

```
1      /*
2       once a second events
3   */
4 void Rover::one_second_loop(void)
5 {
6     ...
7     Log_Write_Test();
8 }
```