

Identification of maneuverability of AUV's using the Nomoto's First Order Equation

(Identification of AUV's manoeuvrability using the Nomoto's first order equation)

William da Silva Caetano

Department of Mechatronics Engineering of
Polytechnic School of the University of São Paulo São
Paulo, Brazil.

Ettore Apolônio de Barros

Department of Mechatronics Engineering of
Polytechnic School of the University of São Paulo São
Paulo, Brazil.

Abstract — This document presents the results obtained by applying the techniques of identification of dynamical systems with the aim of describing the manoeuvrability of autonomous underwater vehicle (AUV) Pirajuba, developed on Laboratory of unmanned vehicles (LVNT), at University of São Paulo (USP). To carry out identification procedures were performed manoeuvres Zig-Zag and Turning Circle in marine environment with the AUV, it was possible to estimate Nomoto's steering quality indices of manoeuvre K and T [1] that when applied to your first order equation, describing the vehicle movement on a horizontal plane. These experiments were conducted in calm waters in the bay of Angra dos Reis, and thus it was possible to neglect the influence of waves and currents in the small parameter estimation calculations.

Keywords — *AUV, System Identification, Manoeuvrability, Nomoto's Equation, Zig-Zag Manoeuvre, Turning Circle.*

1 INTRODUCTION

Assessing the maneuverability of water vehicles is important for the design of their steering systems (rudders, thrusters, etc.) and autopilot. Mathematical models that describe the dynamics of these vehicles are based on the representation of inertial, hydrostatic and hydrodynamic efforts, which are functions of motion variables (displacements, speeds and linear and angular accelerations) of the vessel.

Tests with self-propelled scale models, or "free models", have shown a growing application potential with the increase in the quality of sensors that measure movement and the reduction of their costs, as well as the costs involved in the implementation of vehicles. Carrying out tests demands infrastructure resources and associated costs that are much lower than those of testing with captive models in test tanks. Finally, the systems identification techniques related to processing the results of these tests can be applied to full-scale sea trials for the vehicle, which is important for the

validation of parameter estimates from any other techniques applied to tests with free models, and allowing, however, the elimination of possible problems related to the scale of each vehicle.

This work describes the application of a procedure for modeling the dynamics of an autonomous underwater vehicle, AUV, based on techniques originally proposed for the study of ship maneuverability. This is the parameter estimation of a first-order transfer function, which models the vehicle's course change in response to rudder action. Section 2 reviews Nomoto's technique for estimating first-order model parameters. Section 3 applies the modeling procedure to the AUV Pirajuba, using the results of tests with pre-programmed maneuvers performed in the bay of Angra dos Reis-RJ.

Finally, section 4 presents the final considerations on the application of the technique and future developments of this investigation.

2 K E T E MANEUVERING QUALITY INDEX FIRST ORDER NOMOTO EQUATION

In his original work, based on experimental results obtained with maneuvers of several ships, Nomoto [1] proposed that the maneuver dynamics of these vehicles can be described in a practical way, through a first order linear differential equation relating their yaw angular velocity with rudder deflection (first-order Nomoto equation).

For a long time, there was a big problem to determine which measures would allow to describe the maneuverability of a marine vehicle in a reasonable way and the way to obtain these measurements.

The current estimation process of vehicle maneuverability parameters is based on the analysis of a succession of transient phases of turning maneuvers (maneuvering in

that the vehicle has its vertical rudder at a fixed angle, performing one or more 360-degree turns) with rudder angles oscillating from the same value to starboard and port according to the value reached by the vessel's heading angle (i.e., the vehicle performs a Zig Zag maneuver).

Through this type of manoeuvre, "turning manoeuvre quality indices", K and T, are estimated, which constitute a measure of the vehicle's manoeuvrability. The K parameter represents the ability or ability of the vehicle to perform curves, while the T-index represents the stability in course and the response agility of the turning maneuver. These skills are key elements of maneuverability.

2.1 FIRST ORDER MOTION EQUATION

Using the conventional nomenclature of the naval area indicated in SNAME [2], below are the linear equations of the AUV for the horizontal plane.

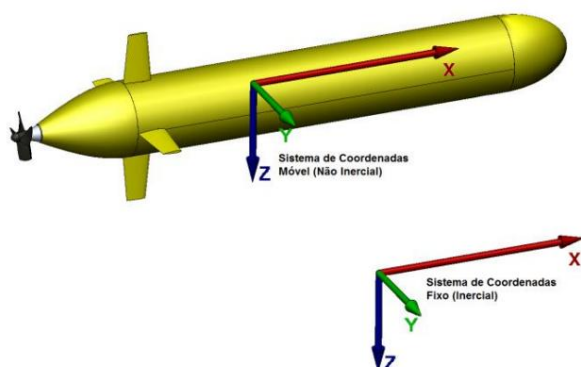


Fig. 1. Fixed and mobile coordinate system used in AUV Pirajuba.

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} U \\ V \\ W \end{pmatrix} + \begin{pmatrix} Y_r \\ X_r \\ Z_r \end{pmatrix} \quad (1)$$

$$(I_{zz} \dot{r}) + (N_v m x_G) v + N_{vv} (N_r m x_G U) r = N \quad (2)$$

Where:

- m is the mass of the vehicle [kg];
- x_G is the position of the center of gravity of the AUV [m];
- r is the rudder deflection angle [rad];
- Y_r, Y_r, N_r, \dots , represent the hydrodynamic derivatives of the system;
- r is the angular speed of course;
- U is the linear velocity of the body;
- v is the linear velocity on the y axis.

From equations (1) and (2) applying the Laplace transform to them and Cramer's rule to the system of equations, one can calculate the transfer functions that

relate drift and yaw speeds to rudder movement:

$$\frac{\dot{Y}}{r} = \frac{K_T s}{(1 + T_1 s)(1 + T_2 s)} \quad (3)$$

$$\frac{r}{\delta} = \frac{K_T}{(1 + T_1 s)(1 + T_2 s)} \quad (4)$$

The indices used in equations (3) and (4) are shown in table 1.

Table 1. Definition of transfer function terms for motion in the horizontal plane.

Termo	Expressão
$T_1 T_2$	$\frac{(m - Y_{\dot{\delta}})(I_z - N_{\dot{r}}) - (Y_r - m)(N_{\dot{\delta}} - mX_G)}{Y_v(N_r - mX_G) - N_v(Y_r - m)}$
$T_1 + T_2$	$-\frac{(m - Y_{\dot{\delta}})(N_r - mX_G) + (I_z - N_{\dot{r}})Y_v - (Y_r - mX_G)N_v + (N_{\dot{\delta}} - mX_G)(Y_r - m)}{Y_v(N_r - mX_G) - N_v(Y_r - m)}$
T_r	$\frac{(N_{\dot{\delta}} - mX_G)Y_{\dot{\delta}} + (m - Y_{\dot{\delta}})N_{\dot{\delta}}}{Y_{\dot{\delta}}N_v - Y_vN_{\dot{\delta}}}$
T_v	$\frac{(I_z - N_{\dot{r}})Y_{\dot{\delta}} - (Y_r - mX_G)N_{\dot{\delta}}}{Y_{\dot{\delta}}(N_r - mX_G) - N_{\dot{\delta}}(Y_r - m)}$
K_v	$-\frac{Y_{\dot{\delta}}(N_r - mX_G) - N_{\dot{\delta}}(Y_r - m)}{Y_v(N_r - mX_G) - N_v(Y_r - m)}$

However, it was perceived in practice that describing the maneuverability of a marine vehicle through its drift and yaw angles was a task with a high degree of difficulty and inaccuracy.

And considering that the turning motion equations are presented in the form of differential equations relating the drift motion, coupled with the angular motion of the turning vehicle in a turning maneuver, the drift angle is quite small, relatively, so that the turning motion of the vehicle can be described substantially just by defining the heading angle as a function of time.

Thus, eliminating the motion drift equation and representing the maneuverability of these vehicles through the heading angle that is easily obtained, it was generated by Nomoto an equation of motion as follows:

$$\ddot{\psi} + (\frac{1}{T_1} + \frac{1}{T_2})\dot{\psi} + \ddot{\psi} = K_T \delta \quad (5)$$

Where:

- $\dot{\psi}$ represents the angular velocity of the vehicle heading and is equal to $\dot{\psi}$ [°/s or rad/s].
- δ represents the rudder angle of the vehicle as a function of time [° or rad].

- K being the ratio between the stable angular velocity of rudder curve and angle, representing the capacity vehicle turn speed [1/s].

Since K, T1, T2 and T3 depend on the hull shape, relative dimensions of the rudder and other vehicle factors, the previous equation can be further approximated by a first-order equation, using the index $T = T1+T2-T3$, thus resulting in Nomoto's first-order equation [1]:

$$\ddot{\psi} + \ddot{\psi} = \ddot{\psi}) 6 ($$

Where:

- T represents the ongoing stability and accumulation of the angular motion of rotation [s].

However, to identify the K and T indices, the responses of a Zig-Zag maneuver are used, which is a well-known maneuver used to verify the maneuverability of marine vehicles. The techniques used to estimate the K and T indices are defined in the following item.

2.2 DETERMINATION OF THE QUALITY INDEX OF MANEUVER USING ZIG-ZAG MANEUVER

The principle of analysis of the Zig-Zag maneuver below is find the value of the K and T indices, with which the first-order equation of motion can describe an observed motion of a marine vehicle. Therefore, the procedure is indicated below:

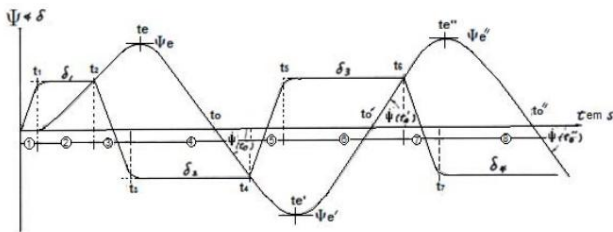


Fig. 2. Notations used for the analysis of the Zig-Zag maneuver, (NOMOTO, 1960).

As the equation of motion requires the vehicle on a natural trajectory in a straight line with the rudder amidships, it is necessary to perform a rudder angle correction noted above before using it in the Nomoto equation, so it is assumed that:

$$\ddot{\psi} + (\ddot{\psi})\ddot{\psi} = (\ddot{\psi})\ddot{\psi} \quad (7)$$

where $(\ddot{\psi})\ddot{\psi}$ (is the observed rudder angle and $\ddot{\psi}$ is the residual rudder angle, this being the difference between $(\ddot{\psi})\ddot{\psi}$ and $(\ddot{\psi})\ddot{\psi}$).

This residual angle can be considered as an unknown constant at the beginning of the analysis.

Substituting equation (6) in equation (7) we get:

$$\ddot{\psi} + \ddot{\psi} = K\ddot{\psi} + K\ddot{\psi}(t) \quad (8)$$

Integrating both sides from $t = 0$ to $t = t$:

$$\dot{\psi} = \dot{\psi} + \dot{\psi}, \ddot{\psi} \dot{\psi} \quad (9)$$

Setting the origin time at the beginning of the test, measuring $\dot{\psi}(t)$ from the baseline, and knowing that the vehicle was traveling in a straight line before the start of the tests, the initial conditions are considered as:

$$\dot{\psi} = 0 \text{ and } \ddot{\psi} = 0 \quad \text{at } t = 0$$

With this, one obtains

$$\dot{\psi} = \dot{\psi} \quad (10)$$

This being the fundamental equation for the present procedure. Then, applying the previous equation for $t = te'$ and te'' we obtain:

$$\dot{\psi} = \dot{\psi} = Ps \quad (11)$$

$$\dot{\psi} = \dot{\psi} = Ps \quad (12)$$

Since $\ddot{\psi} = 0$ at these instants. The integrals $\dot{\psi} \dot{\psi}(t)$, can be obtained through a simple calculation indicated in the sequence. Thus, the simultaneously unknown parameters in these equations are the values of K and $\dot{\psi}$. These parameters can be determined by solving the previous equations simultaneously. K calculated in this step equivalent to the second half of the test period is called K

Thus, applying the time $t = te$ in equations (11) and (12), we get:

$$\dot{\psi} = \dot{\psi} \quad (13)$$

Since this K refers to the first half of the test period, then called K. The values of K and K are

more or less different from each other, as a result of the speed reduction in the test and the non-linear effects for the turning maneuver quality indices. Usually the average between the values of K and K' can be considered as the value to be used for the K index. After finding the value of K , it is applied in equation (13) for $t = t_0$ to $t = t_{end}$, then obtaining:

$$= \frac{\quad}{0} + \quad \quad (14)$$

$$= \frac{1}{2} + \frac{1}{2} \quad (15)$$

$$= \frac{1}{2} + \frac{1}{2} \quad (16)$$

The times to, to' and to'' being recorded when $\ddot{y} = 0$. When carrying out the calculations, it is reasonable to use K for the first equation and to use K for the other two. The values for $\ddot{y}(t)$, $\ddot{y}'(t')$ and $\ddot{y}''(t'')$ can be determined from the graph of $\dot{y}(t)$.

The integral $\int_0^T \ddot{y} \ddot{y} \ddot{y}(\ddot{y})$ can be calculated easily, and the procedure for carrying out this calculation is indicated below. The value of T obtained for equation (14), which represents the first half of the period analysis, is called T_1 and the T calculated for the second half of the analysis being the period are called T_2 average between these constants the final value of T . Below are the equations used to solve the integral

$$\ddot{y} \ddot{y} (4) + \ddot{y} + (\ddot{y} - \ddot{y})$$

$$\ddot{y} \ddot{y} (6) \quad + \quad - \quad - \quad -$$

$$\bullet) + (\bullet + \bullet) + \bullet - (\bullet + \bullet) + \bullet$$

$$\begin{aligned} & \ddot{y} \ddot{y} (8) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) \\ & \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) + \ddot{y} \ddot{y} (\ddot{y} + \ddot{y}) \end{aligned}$$

3 CHARACTERIZATION OF MANEUVERABILITY AND PIRAJIURA AUV STABILITY

In 2007, the Laboratory of Unmanned Vehicles (LVNT) at the University of São Paulo (USP) started the project and later developed an autonomous underwater vehicle (AUV) that was named Pirajuba. The Pirajuba AUV was created due to the need for an experimental validation platform, which aimed to support some research carried out in the laboratory, including the investigation of derivatives and hydrodynamic functions of

underwater vehicles. More details about the AUV can be seen in m de Barros et. al. [3][4].

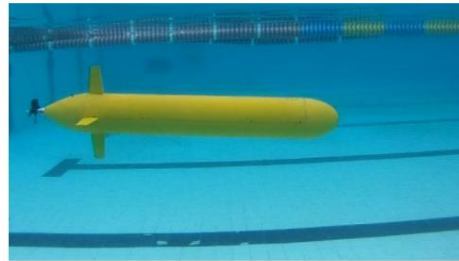


Fig. 3. AUV Pirajuba performing tests in a restricted environment.

The AUV Pirajuba during the last years has undergone some updates in its mechanical and electrical-electronic structures referring to its rudder actuators, thrusters, total mass relocation (vehicle inertia plus additional mass when submerged), hardware and software on board, where this In this way, it was necessary to verify the maneuverability and performance of the vehicle when it is carrying out missions both in restricted environments such as tanks, swimming pools, lakes and ponds, and in open environments such as large rivers, seas and oceans.

For this, Zig Zag maneuver graphs were used to estimate the maneuverability indexes of Nomoto K and T of the AUV Pirajuba, using the equations indicated in the previous items, so that it is possible to describe in a simple and qualitative way the maneuverability of the vehicle.

3.1 MANEUVER IN MARINE ENVIRONMENT: BAÍA DE ANGRA DOS REIS

To estimate the K and T indices of the Pirajuba AUV, Zig-Zag maneuvers were performed with the vehicle having a longitudinal forward speed of 1m/s in the x axis. A Zig-Zag maneuver was performed for a 5° rudder input. Due to different dynamic conditions in relation to the moment of execution of the tests, four Zig-Zag maneuvers were used and the K and T indices were calculated referring to each maneuver, and after obtaining these values, the arithmetic mean was used to estimate the final value.

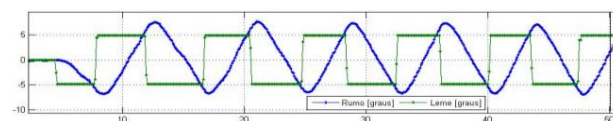


Fig. 4. 1st Zig-Zag Maneuver of the AUV Pirajuba in marine environment, with rudder at 5 degrees.

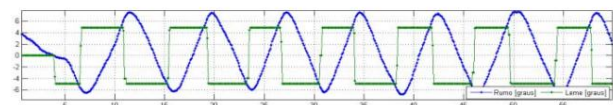


Fig. 5. 2nd Zig-Zag Maneuver of the AUV Pirajuba in marine environment, with rudder at 5 degrees.

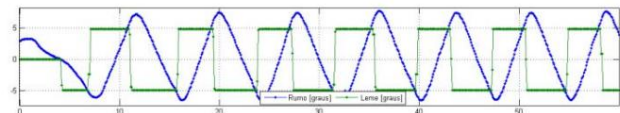
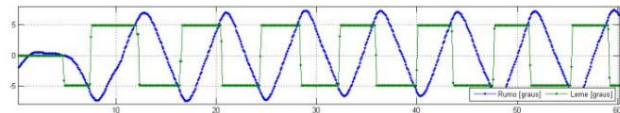


Fig. 6. 3rd Zig-Zag Maneuver of the AUV Pirajuba in marine environment, with rudder at 5 degrees.



4th Zig-Zag Maneuver of the AUV Pirajuba in marine environment, with rudder at 5 degrees.

Table 2. Estimated results for K, T indices in marine and for the residual rudder angle.

Índices de Qualidade	Gráfico 1	Gráfico 2	Gráfico 3	Gráfico 4	Média
K [s ⁻¹]	0,8997	0,8166	0,7958	0,8798	0,848
T [s]	0,4964	0,5095	0,2934	0,505	0,4511
δr [°]	1,32	0,8971	1,0905	0,7596	1,0168

It is possible to verify through the results obtained for the K and T maneuver quality indices, as indicated in table 1, that the results obtained through the Zig-Zag maneuvers returned values that are distant from each other, which can be explained by the rudder trimmings performed by the vehicle before the start of each maneuver, which can be visualized by the values of $\dot{\gamma}_r$ in the table.

3.2 USE AND VALIDATION OF K AND T – TURNING MANEUVER

To validate the values found for the K and T indices, Turning maneuvers were performed with the vehicle and their response compared with the prediction made using Nomoto's first-order equation. For this, a simulator was designed in the commercial software environment Matlab/ Simulink where, through graphs and comparative tables, it was possible to verify the quality of the prediction of the maneuverability of the AUV Pirajuba. These results are presented below:

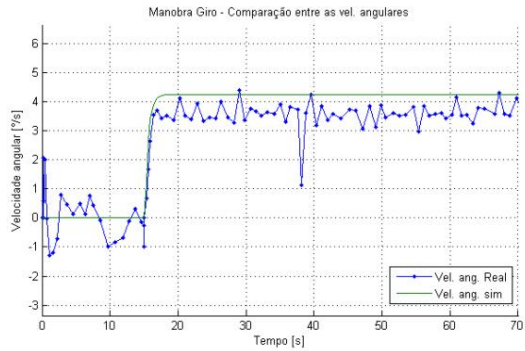


Fig. 7. Comparison between the angular speeds of the turn maneuver in a marine environment, for 5 degrees of rudder.

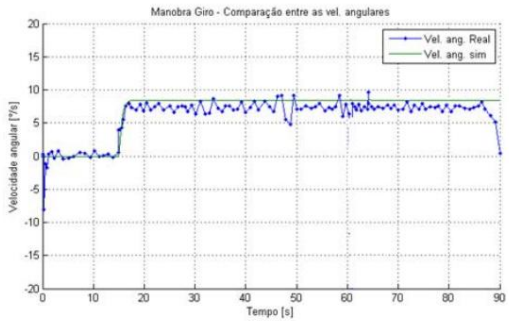


Fig. 8. Comparison between the angular speeds of the turn maneuver in a marine environment, for 10 degrees of rudder.

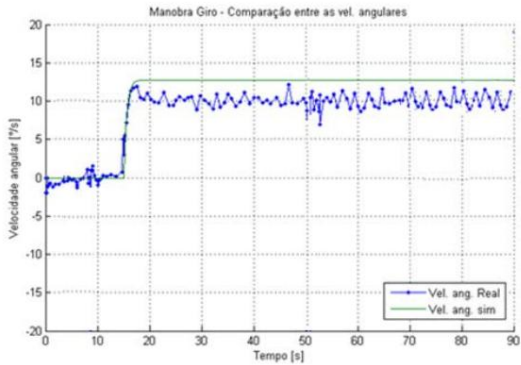


Fig. 9. Comparison between the angular speeds of the turn maneuver in a marine environment, for 15 degrees of rudder.

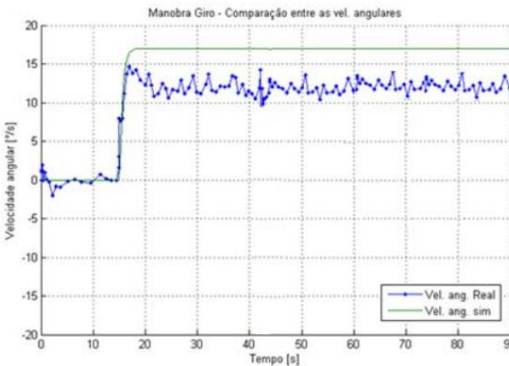


Fig. 10. Comparison between the angular speeds of the turn maneuver in a marine environment, for 20 degrees of rudder.

Table 3. Estimated time for the vehicle to complete a lap in comparison to experimental data.

Tempo para o AUV completar uma volta de 360°		
Leme [°]	Simulação [s]	Experimental [s]
5	90,36	88,93
10	47,90	48,04
15	32,78	35,25
20	25,96	29,53

Table 3. Comparison between angular velocity curves.

Velocidade angular da manobra de Giro		
Leme [°]	Simulação [°/s]	Experimental (média) [°/s]
5	4,240	3,553
10	8,480	7,146
15	12,720	10,103
20	16,960	11,989

Table 4. Comparison between gyration radii, for speed of 1 m/s.

Raio da manobra de Giro		
Leme [°]	Simulação [m]	Experimental [m]
5	13,51	16,13
10	6,76	8,02
15	4,50	5,67
20	3,38	4,78

Bearing in mind the obtained values of K and T later used in Nomoto's first order equation when compared with the real data of the behavior of the AUV in a marine environment, they were perceived through the graphs of angular velocity of r and in the tables of gyration radius that the simulated and obtained data were very close to each other, indicating excellent maneuver prediction quality. However, as expected, the prediction error increases as the rudder angles in the maneuvers reach values far from the linearization region ($\dot{\gamma} = 5^\circ$).

CONCLUSIONS

The use of Nomoto's first-order equation to describe the maneuverability of marine vehicles is normally applied to describe the behavior of ships and other surface water craft. The experiment proposal indicated in this document uses the Nomoto maneuver quality indices to predict the behavior of the AUV Pirajuba, which in turn resulted in a good approximation of reality based on standard Zig-Zag tests and were validated through the maneuvers of Cute.

It was also noticed that when the real data of the maneuvers were compared with the data generated by the simulator that uses the Nomoto equation as a basis, the predictions made for entry angles close to the linearization point had a minimum mean error in steady state, which accentuates as the angles approach the 20° limit.

The use of this maneuverability identification method when used in submarine vehicles as in the case of the AUV has the advantage of providing an estimation procedure that is simple to perform. Even with the differences between the predictions and the experimental results observed for the turning maneuver, the method effectively provides a model of the vehicle's dynamics that can be easily used for the design of its autopilot.

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