

# Identification of Nomoto models with integral sample structure for identification

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**Abstract:** This documentation presents the parametric identification of Nomoto models by using the proposed integral sample structure for identification (ISSI). The dataset used for validations are obtained from the free-running model tests. By analyzing the experimental data including yaw rate and rudder angle, the maneuvering indices in the 1st-order linear and nonlinear Nomoto models are approximated based on least square support vector machines (LS-SVM), where ISSI and the conventional Euler sample structure for identification (ESSI) are employed for the construction of the in-out sample pairs, respectively. The comparison between ISSI and ESSI is carried out for the validation of the proposed sample structure for identification.

**Key Words:** Nomoto models; system identification; ship maneuvering

## 1. Introduction

On the research of ship steering control, Nomoto models, i.e., ship response models, are widely employed as the basis for controller design. The fundamental advantages of the Nomoto models are their simple mathematical description, and consequently the determination of the maneuvering indices in these models are very convenient. The theoretical calculation of the indexes is based on the linear hydrodynamic derivatives, which can be easily acquired from the captive model tests, empirical regressions, computational fluid dynamics (CFD) technique, or system identification in addition with the experimental data.

In recent several decades, system identification is known as an efficient approach on the approximation of unknown parameters in the mathematical model of ship maneuvering motion. With the given sampling dataset obtained from the free-running model tests or full-scale trials, the unknown parameters can be easily approximated with the identification algorithms. Of the existing algorithms, support vector machines (SVM) are considered as an advanced and feasible approach by large numbers of validations. In this aspect, Luo and Zou, Zhang and Zou, as well as Xu et al. have made a lot of efforts<sup>[1, 2, 3]</sup>.

With respect to the parametric identification of ship maneuvering, the in-out sample pairs are usually obtained via the discretization of dynamic model based on 1st-order Euler differential method<sup>[1, 2]</sup>. The advantages of this approach are the simple structure and easy

realization. But the published work demonstrates that this approach will not be as accurate as possible when the sample interval is big, and the proposed integral sample structure for identification (ISSI) can commendably solve this problem<sup>[4]</sup>. ISSI is proofed to be more accurate and stable than ESSI<sup>[5, 6]</sup>.

This paper makes an effort on the parametric identification modeling of the Nomoto models. Firstly, Euler and integral sample structure for identification are deduced. After that the in-out sample pairs are constructed by using ESSI and ISSI, respectively. With the 15°/5° free-running model test data, the maneuvering indices in the 1st-order Nomoto models are identified based on least square SVM (LS-SVM). At last, 25°/5° zigzag tests are carried out for the validation of the proposed identification approach.

## 2. Sample structure for identification

### 2.1 Euler sample structure for identification

The core concept of ESSI is to discretize the dynamic model. For the dynamic system,

$$\begin{cases} \dot{x} = g(x, \theta, \mu) \\ y = x + \varepsilon \end{cases} \quad (1)$$

Where,  $x$  denotes motion parameter,  $\theta$  denotes the model parameters,  $\mu$  is system input,  $y$  is the observed system output,  $\varepsilon$  is measuring error.

Based on the 1st-order Euler differential method,  $\dot{x} = (x_{k+1} - x_k)/h$ . Then the Eq.(1) can be discretized as,

$$\begin{cases} x_{k+1} = x_k + hg(x_k, \theta, \mu_k) \\ y_{k+1} = x_{k+1} + \varepsilon_k \end{cases} \quad (2)$$

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Where,  $h$  is sample interval.

Generally speaking, the hydrodynamic derivatives in the mathematical model of ship maneuvering motion are considered as constant values, and the mathematical model is linear function about the hydrodynamic derivatives. Consequently,  $g(x, \theta, \mu)$  can be rewritten as,

$$g(x, \theta, \mu) = P(x, \mu) [\theta_1, \theta_2, \dots, \theta_m]^T \quad (3)$$

$$\begin{bmatrix} x_1 & P_1(x_1, \mu_1) & P_2(x_1, \mu_1) & \dots & P_m(x_1, \mu_1) \\ x_2 & P_1(x_2, \mu_2) & P_2(x_2, \mu_2) & \dots & P_m(x_2, \mu_2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & P_1(x_n, \mu_n) & P_2(x_n, \mu_n) & \dots & P_m(x_n, \mu_n) \end{bmatrix} \begin{bmatrix} 1 \\ \theta_1 \\ \vdots \\ \theta_m \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ \vdots \\ x_{n+1} \end{bmatrix} \quad (4)$$

Rewriting the above equations with uniform and compact matrix form,

$$Y = X\theta + \varepsilon \quad (5)$$

Where,  $X$  are the input samples, and  $Y$  are the output samples. It is easy to find that the Eqs.(5) is the commonly-used standard form for system identification. The parameters  $\theta$  can be obtained by using various identification algorithm.

Based on the above analysis, the in-out sample pairs for ESSi can be confirmed as,

$$\begin{cases} \text{input: } \{x_k, P_1(x_k, \mu_k), P_2(x_k, \mu_k), \dots, P_m(x_k, \mu_k)\} \\ \text{output: } \{x_{k+1}\} \end{cases}$$

## 2.2 Integral sample structure for identification

Integrating the both sides of the dynamic model as shown in Eqs.(1) gives,

$$x_k = \int_0^k g(x, \theta, \mu) dt, \quad x_{k+1} = \int_0^{k+1} g(x, \theta, \mu) dt \quad (6)$$

Where,  $k$  is time step. Subtracting  $x_k$

$$\begin{bmatrix} \int_{t_1}^{t_2} P_1(x, \mu) dt & \int_{t_1}^{t_2} P_2(x, \mu) dt & \dots & \int_{t_1}^{t_2} P_m(x, \mu) dt \\ \int_{t_2}^{t_3} P_1(x, \mu) dt & \int_{t_2}^{t_3} P_2(x, \mu) dt & \dots & \int_{t_2}^{t_3} P_m(x, \mu) dt \\ \vdots & \vdots & \ddots & \vdots \\ \int_{t_n}^{t_{n+1}} P_1(x, \mu) dt & \int_{t_n}^{t_{n+1}} P_2(x, \mu) dt & \dots & \int_{t_n}^{t_{n+1}} P_m(x, \mu) dt \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_m \end{bmatrix} + \begin{bmatrix} \varepsilon_{1+\frac{1}{2}} \\ \varepsilon_{2+\frac{1}{2}} \\ \vdots \\ \varepsilon_{n+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ \vdots \\ x_{n+1} - x_n \end{bmatrix} \quad (10)$$

The above equations are the standard format for system identification, and the in-out sample pairs for ISSi can be obtained,

Where,

$P(x, \mu) = [P_1(x, \mu), P_2(x, \mu), \dots, P_m(x, \mu)]$ ,  $m$  is the number of the identified parameters.

If there are totally  $n+1$  sample data, the problem of system identification turns to the solution of  $n$ -dimension linear equations. Replacing the state variable  $x_i$  with its observation  $y_i$ , Eqs.(2) give,

from  $x_{k+1}$  gives,

$$x_{k+1} - x_k = \int_k^{k+1} g(x, \theta, \mu) dt \quad (7)$$

Since

$$y_{k+1} = x_{k+1} + \varepsilon_{k+1}, \quad y_k = x_k + \varepsilon_k \quad (8)$$

Subtracting  $y_k$  from  $y_{k+1}$  gives,

$$y_{k+1} - y_k = \int_k^{k+1} g(x, \theta, \mu) dt + (\varepsilon_{k+1} - \varepsilon_k) \quad (9)$$

Obviously, if the statistical characteristic of  $\varepsilon$  is zero-mean Gaussian white noise, the series  $[\varepsilon_2 - \varepsilon_1, \varepsilon_3 - \varepsilon_2, \dots, \varepsilon_{k+1} - \varepsilon_k, \dots]$  can also be approximately considered as zero-mean Gaussian white noise.

If the sample number is totally  $n+1$ , separating the constant variables from  $g(x, \theta, \mu)$  based on Eq.(3), and replacing the observation value  $y_i$  with its system value  $x_i$ , the problem of system identification is transferred to the solution of the following  $n$ -dimension linear equations,

$$\begin{cases} \text{input: } \left\{ \int_k^{k+1} P_1(x, \mu) dt, \int_k^{k+1} P_2(x, \mu) dt, \dots, \int_k^{k+1} P_m(x, \mu) dt \right\} \\ \text{output: } \{x_{k+1} - x_k\} \end{cases}$$

Obviously, there are many other algorithms for the solution of the integration. For simplification, herein trapezoid method is employed for calculating the integration and the formulation is shown in Eq.(11).

$$\int_{t_k}^{t_{k+1}} P_i(x, \mu) dt = \frac{(t_{k+1} - t_k) [P_i(x_{k+1}, \mu_{k+1}) + P_i(x_k, \mu_k)]}{2} \quad (11)$$

### 3. Nomoto models

Compared with the hydrodynamic models such as Abkowitz model or MMG model, Nomoto models have extraordinary simple structure because they only deal with the relationship between the rudder and the yaw rate. They intuitively describe the steering characteristics of a ship. The Nomoto models contain 1st-order and 2nd-order models. Unfortunately, the application of ISSI for 2nd-order models is still not figured out. Therefore, 1th-order models are considered in this paper.

The 1st-order linear Nomoto model can be expressed as follows,

$$T\dot{r} + r = K\delta \quad (12)$$

Where,  $r = \dot{\psi}$  is yaw rate,  $\psi$  is yaw angle,  $\delta$  is rudder angle,  $T$  and  $K$  are maneuvering indices. Based on ESSI, the in-out sample pairs can be gained as follows,

$$\text{input: } \{r_k, \delta_k\} \rightarrow \text{output: } \{r_{k+1}\}$$

The in-out sample pairs can be gained as follows based on ISSI,

$$\text{input: } \{r_k + r_{k+1}, \delta_k + \delta_{k+1}\} \rightarrow \text{output: } \{r_{k+1} - r_k\}$$

The 1st-order nonlinear Nomoto model can be expressed as follows,

$$T\dot{r} + r + \alpha r^3 = K\delta \quad (13)$$

Where,  $\alpha$  is a time-invariant constant. The in-out sample pairs can be gained as follows based on ESSI:

$$\text{input: } \{r_k, r_k^3, \delta_k\} \rightarrow \text{output: } \{r_{k+1}\}$$

Based on ISSI, the in-out sample pairs can be gained as,

$$\begin{aligned} \text{input: } & \{r_k + r_{k+1}, r_k^3 + r_{k+1}^3, \delta_k + \delta_{k+1}\} \\ \rightarrow \text{output: } & \{r_{k+1} - r_k\} \end{aligned}$$

### 4. Validations

To identify the maneuvering indices, least square SVM (LS-SVM) is employed as the identification algorithm. LS-SVM is a special type of SVM due to its choosing square cost function, which leads to the loss of sparse solution. However, LS-SVM converts the solution of quadratic optimization problem to a linear system of equations, which greatly simplifies the solution problem. The resolution flow of LS-SVM is given as follows.

The feature space representation of LS-SVM is given by

$$y(x) = w^T \phi(x) + b \quad (x \in R^m, y \in R) \quad (14)$$

Where,  $w$  is a vector in the so-called high dimensional feature space, is the mapping function that maps the input data to a high dimensional feature space.  $b$  is a bias for the regression model. The solution of LS-SVM can be converted to a linear system of equations via a series of transformation. The set of equations are shown as follows<sup>[7]</sup>,

$$\begin{bmatrix} 0 & \mathbf{1} \\ \mathbf{1}^T & \Omega + C^{-1}I \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} \tilde{0} \\ y \end{bmatrix} \quad (15)$$

where,  $\Omega = \phi(x_i)^T \phi(x_j) = K(x_i, x_j)$ ,

$K(x_i, x_j)$  is kernel function,  $y = [y_1, \dots, y_n]^T$ ,

$\mathbf{1} = [1, \dots, 1]^T$ ,  $\alpha = [\alpha_1, \dots, \alpha_n]^T$ ,  $\tilde{0} = [0, \dots, 0]^T$ .

Once Eqs.(15) are solved, the regression model can be gained,

$$y(x) = \sum_{i=1}^n \alpha_i K(x_i, x) + b \quad (16)$$

For the problem of parametric identification, linear kernel function is usually adopted, i.e.,

$K(x, x') = (x \cdot x')$ . The identified parameters can be regressed by using linear kernel based LS-SVM, and the regression model is,

$$\theta = \sum_{i=1}^n \alpha_i x_i \quad (17)$$

Where,  $\theta$  denotes the identified parameters.

The experimental dataset used for validations are obtained from free-running model tests which was implemented at Hamburg Ship Model Basin (HSVA)<sup>[8]</sup>. The ship model is a KVLCC, and the scale ratio is 1:45.714. Some physical parameters of this model are shown in Table.1.

Table.1 Physical parameters of KVLCC

Lpp(m)	Lwl(m)	Bwl(m)	D(m)
7	7.1204	1.2688	0.6563
T(m)	C <sub>B</sub>	C <sub>M</sub>	U <sub>0</sub> (m/s)
0.455	0.8098	0.9980	1.179

To approximate the maneuvering indices, the sampling dataset of 15°/5° zigzag maneuver is studied. The time history of the yaw angle and rudder angle are shown in Fig.1. In the identification process, the sample interval is 1s, and the sample number is 160. The rule factor for SVM is set as  $4.0 \times 10^6$ . The identified maneuvering indices are shown in Table.2.

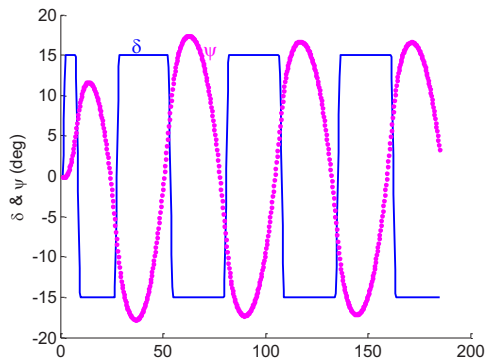


Fig.1 Time histories of rudder angle and yaw angle

Table.2 Identified maneuvering indices

1st-order linear model	$K$		$T$
ESSI	-0.249		15.615
ISSI	-0.254		15.860
1st-order nonlinear model	$K$	$\alpha$	$T$
ESSI	-0.324	225.999	20.311
ISSI	-0.309	164.666	19.321

Because there are no authoritative numerical values in the published work for the evaluation of the identified parameters, the effectiveness of the maneuvering indices can only be validated via the maneuvering prediction by using the identified Nomoto models. Based on the identified Nomoto models, 25°/5° zigzag maneuver is carried out. The prediction results of the yaw rate and yaw angle based on 1st-order linear Nomoto model is shown in Fig.2, and the corresponding prediction error is shown in Fig.3. The prediction results of the yaw rate and yaw

angle based on 1st-order nonlinear Nomoto model is shown in Fig.4, and the corresponding prediction error is shown in Fig.5. These figures demonstrate that all the identified Nomoto models are appropriate for the maneuvering prediction, which also suggests the good generalization performance of the identified Nomoto models. By comparison, the accuracy of the ISSI-based models is a little higher than the ESSI-based models, which can be easily found from Fig.3 and Fig.5.

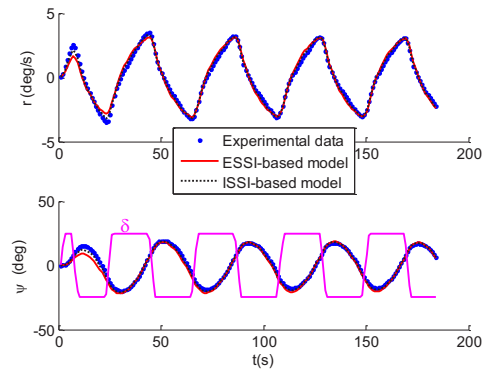


Fig.2 Prediction of yaw rate and yaw angle with identified 1st-order linear Nomoto model

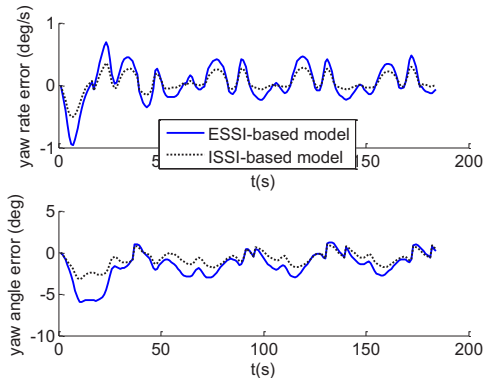


Fig.3 Prediction error of yaw rate and yaw angle with identified 1st-order linear Nomoto model

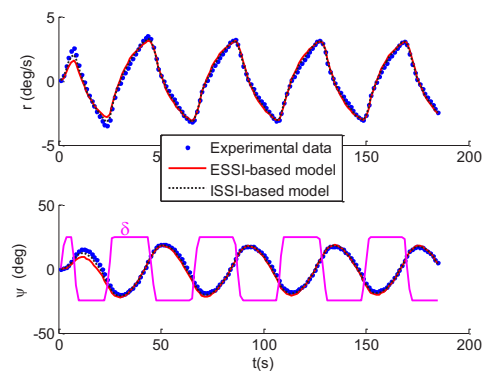


Fig.4 Prediction of yaw rate and yaw angle with identified 1st-order nonlinear Nomoto models

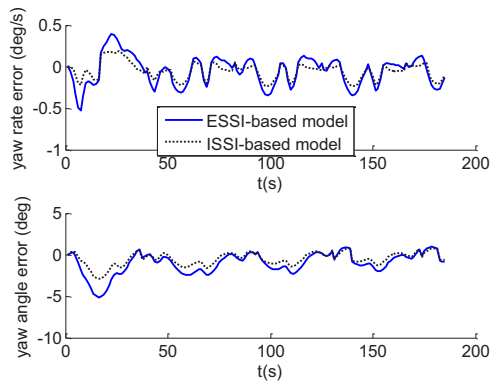


Fig.5 Prediction error of yaw rate and yaw angle with identified 1st-order nonlinear Nomoto models

## 5. Conclusions

Parametric identification of Nomoto models by using LS-SVM with ISSI is implemented in this paper. With the free-running model test data, the maneuvering indices are approximated and used for maneuvering predictions. By the comparison of the prediction results, the effectiveness of the proposed ISSI is validated. This identification approach can be used as accurate modeling for ship maneuvering. The next work will be concentrated on the application of ISSI for 2nd-order Nomoto models.

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