

A Mathematical Model for the Tacking Maneuver of a Sailing Yacht

by

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Summary

In the present report an extension of the mathematical model for the tacking maneuver of a sailing yacht, as previously described by the same authors in Reference [1], will be presented.

There is a need for such a mathematical model because the tacking maneuver and more in particular the speed loss during such a maneuver, is of interest for handicapping purposes. If this speed loss of a large variety of sailing yachts can be calculated the differences may be incorporated in their respective handicaps. This implies however also that this mathematical model should incorporate only the use of formulations based on “generic” parameters, which describe the hull form and the sail plan of the yacht under consideration.

In the present report a more complete description of this model, as available so far, will be presented. The accent is on the hydrodynamic part of the model. As much as possible the results obtained within the Delft Systematic Yacht Hull Series (DSYHS) will be used. In a future report also the aerodynamic part will be more extensively elaborated so that a wider variety of sail plans may be dealt with.

A number of simulations with the model have been performed and checked with the results obtained during a series of full scale measurements.

1 Introduction

A maneuvering model of a sailing yacht has been the subject of research by various authors for some time now. All these research projects were carried out for a variety of reasons. Some authors were focusing on finding the optimal tacking procedure or rudder action for minimal speed loss during the tack.

These studies were in general conducted for very specific yachts such as for instance IACC yachts. The necessary coefficients for the equations of motions in the maneuvering model could in that case be determined using dedicated experiments.

Other authors were more interested in obtaining insight in the general maneuverability characteristics of sailing yachts under sail. These later studies gained importance due to the ever increasing scale of some yachts. Both the demand for proper “balance” of the hydro and aero forces and moments when sailing on a straight course as the safe operation of these yachts when sailing in confined areas where sudden maneuvers may be necessary to avoid collisions etc. Here also the coefficients were generally determined using model tests.

In the present study the emphasis has been put on formulating a set of equations containing only coefficients, which could be determined using the design data of the ship under consideration.

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This implies that the coefficients must be expressed in what we call “generic” parameters describing the hull form and sail plan.

One of the possible applications of the results of that kind of speed loss calculations is in the handicapping procedures used around the world. In those procedures it has been found that just considering the straight line up wind speed potential of the yacht alone, as is being predicted by the Velocity Prediction Programs (VPP), is actually not sufficient. Different yachts will have different “speed loss during a tack” characteristics and therefore the total time lost in an up-wind leg may differ considerably. In order to be able to calculate the differences between a large variety of yachts the development of a simulation model is necessary which yields a reliable prediction of the behavior of a sailing yacht during a tack. To develop such a model it was decided to make use of the available data obtained from extensive tests and analyses of the Delft Systematic Yacht Hull Series (DSYHS). The goal is to generate approximations for the coefficients in the equations of motions based on this well-established series.

In the first action in the present study was the selection of an appropriate mathematical model from the existing literature. It is evident that the tacking maneuver of a sailing yacht is a complex maneuver. The most important difference with “normal” maneuvering model is the incorporation of the roll motion and the large change of the aerodynamic forces during the process. After a literature survey and analysis it was decided that the model as previously presented by Y Masayuma in 1995, Ref [2], presented a good starting point. He showed in his study that with a fair determination of the coefficients a good correlation between the actual tacking maneuver at full scale and the results obtained by the simulation could be obtained.

To determine generic formulations for the coefficients some results obtained within the DSYHS and presented by Keuning and Sonnenberg in 1998 and 1999, Reference [3], could be used straight away such as the determination of the upright resistance of the hull, the resistance increase due to heel of an arbitrary hull.

Another usable result came from the report published by Keuning and Vermeulen in 2002, Reference [1]. In this report they presented a calculation method for the yaw balance of sailing yacht on a straight line and so formulated generally applicable expressions for side force and yaw moment both upright and under heel of an arbitrary hull and appendages. In the present study these expressions have been further elaborated and refined, but most important also verified by means of dedicated experiments in the towing tank of the Delft Shiphydrodynamics Laboratory. These tests showed the validity of the expressions for a variety of hulls. Only the formulations for the side force production of the appendages needed a small correction to make them more applicable for high aspect ratio appendages.

Further an extensive series of dedicated forced oscillation tests with a 6-degrees of freedom forced oscillator, the “Hexamove”, have been carried out with a number of models of the DSYHS to validate the validity and the accuracy of the expressions developed for the added mass in sway and yaw moment both upright and under heel. From these tests it became apparent that the presented expressions yield reliable results for the foreseen purpose. A summary of these results will be presented here.

Finally, to be able to check the results of the full simulations for a variety of yachts, full-scale measurements have been carried out on the tacking maneuver with three quite different yachts. The results of these tests confirmed the validity of the mathematical model for the assessment of the speed and time loss of sailing yachts during a tack and the use of these results for handicapping purposes.

2 The mathematical Model

As stated before the first action was to choose an appropriate set of equations for simulating the tacking maneuver. After some study the model as proposed and used by Y Masayuma was

selected, Reference [2]. On his turn he made use of the model and coordinate systems as presented by Hanamoto et.al. Reference [3].

In this coordinate system the origin is located on the centerline of the ship at the still waterline in the midship section at zero heel. The X-axis lies along the centerline of the ship with the positive direction forwards. The positive Y-axis points to starboard and the positive Z-axis points downwards.

Ignoring the pitch and heave motion of the yacht the mathematical model contains equations for four degrees of motion, i.e. surge, sway, yaw and roll.

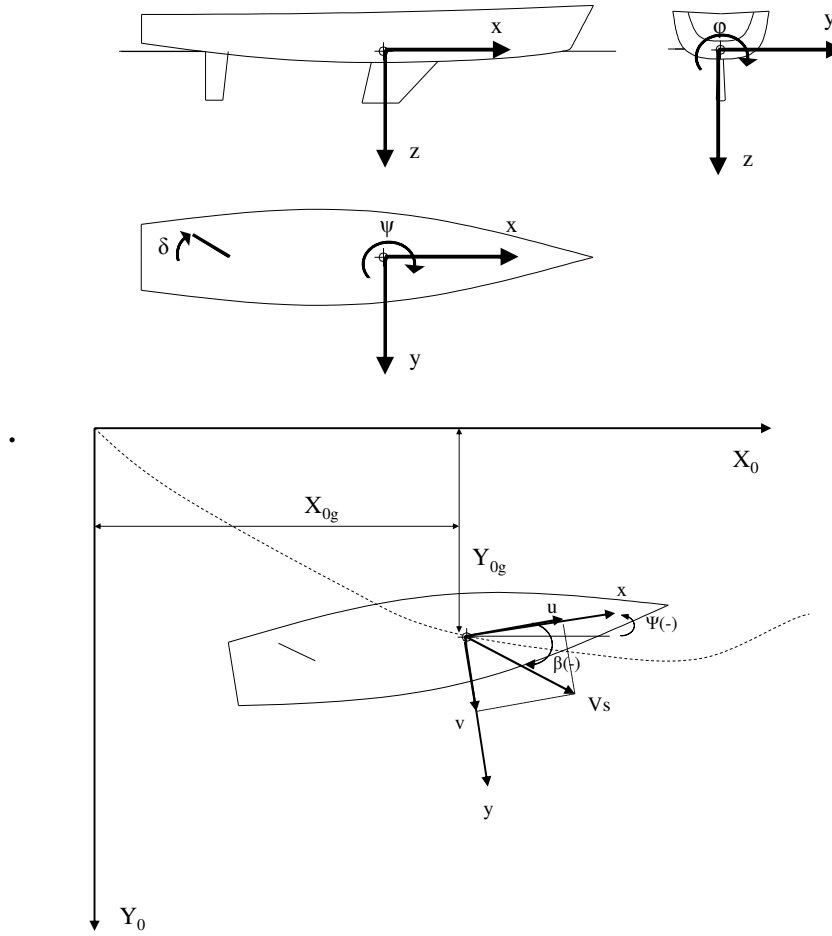


Figure 1 Coordinate System used in the mathematical model

The Eulerian equations of motions for the respective motions become:

$$m(\dot{u} - v\dot{\psi}) = X_U + X_{hull} + X_{rudder} + X_{sail}$$

$$m(\dot{v} + u\dot{\psi}) = Y_{hull} + Y_{rudder} + Y_{sail}$$

$$J_{xx}\ddot{\phi} = K_{hull} + K_{rudder} + K_{sail} + K_{stability}$$

$$J_{zz}\ddot{\psi} = N_{hull} + N_{rudder} + N_{sail}$$

in which:

u	=	velocity along the X axis
v	=	velocity along the Y axis
ϕ	=	roll angle
ψ	=	yaw angle
I_{xx}	=	(total) mass moment of inertia in roll
J_{zz}	=	(total) mass moment of inertia in yaw

Masayuma modified this set of equations in the body fixed coordinate system first through omitting the higher order terms considering that these were not significant in the present approach. He then modified the equations for taking into account the effects of the (large) heeling angle and the transformation of the expression for the motions of the ship in the earth fixed coordinate system. In addition we assume now that the centroid of the added mass is located in the Center of Gravity of the yacht and therefore no additional moments and forces originate from the possible distance between these two centers. The possible effect of a non symmetric added mass distribution over the length of the yacht may be taken into account in a later stage or further development.

The set of equations now becomes:

Surge :

$$(m + m_x)\dot{u} - (m + m_y \cos^2 \phi + m_z \sin^2 \phi)v\dot{\psi} \\ = X_U + X_{hull} + X_{v\psi}v\dot{\psi} + X_{rudder} + X_{sail}$$

Sway :

$$(m + m_y \cos^2 \phi + m_z \sin^2 \phi)\dot{v} + (m + m_x)u\dot{\psi} + 2(m_z - m_y) \sin \phi \cos \phi \cdot v\dot{\phi} \\ = Y_{hull} + Y_{\phi}\dot{\phi} + Y_{\psi}\dot{\psi} + Y_{rudder} + Y_{sail}$$

Roll :

$$(I_{xx} + J_{xx})\ddot{\phi} - \{(I_{yy} + J_{yy}) - (I_{zz} + J_{zz})\} \sin \phi \cos \phi \cdot \dot{\psi}^2 \\ = K_{hull} + K_{\phi}\dot{\phi} + K_{rudder} + K_{sail} + K_{stability}$$

Yaw :

$$\{(I_{yy} + J_{yy}) \sin^2 \phi + (I_{zz} + J_{zz}) \cos^2 \phi\} \ddot{\psi} + 2\{(I_{yy} + J_{yy}) - (I_{zz} + J_{zz})\} \sin \phi \cos \phi \cdot \dot{\psi}\dot{\phi} \\ = N_{hull} + N_{\psi}\dot{\psi} + N_{rudder} + N_{sail}$$

In the present approach the transformation of the added mass terms from the body fixed to the earth fixed coordinate system is no longer necessary because we now approximate the added mass of the asymmetric sections of the heeled yacht in the horizontal plane directly. This approach will be further described and verified later in this report but is based on the approach as suggested by Keuning and Vermeulen in Reference [1].

The terms with X, Y, K and N and their suffixes stand for the forces in X and Y direction and the moments around the X and Y axes of the body fixed coordinate system respectively as these are generated by the specific parts of the yacht mentioned in the suffix.

Masayuma showed in his report, Reference [2], that the results obtained from the simulations using this set of equations and the coefficients such as he obtained them from towing tank tests (and calculations) showed close resemblance with results obtained from full scale measurements. He concluded that no further extension of the equations is necessary to obtain valuable and reliable results for the purpose of track simulation and speed loss assessment. This is in particular

so considering the aim of the present study, in which the ability of comparing a large variety of yachts in a qualitative sense is more important than a very high accuracy.

So in the present study Masayuma's model was adopted also considering the fact that an approximation seems feasible for most of the coefficients in the model. The goal of the present report is to be able to express all these coefficients now by existing formulations or expressions derived from the data and results as obtained within the Delft Systematic Yacht Hull Series.

3 The added mass in surge, sway, yaw and roll

The equations contain on the left hand side terms with the added mass and added mass moments of inertia in the four degrees of freedom. So an assessment of the sectional added mass in sway and roll is called for. As will be seen later in particular the sway added mass is necessary for determining hull yaw moments.

Keuning and Vermeulen presented in Reference [1] a suitable approximation method of the sway added mass of an arbitrary 2-D section. These expressions are based on the simplifying assumptions that the 2-D added mass may be approximated by a half circle element as formulated by Nomoto. Using this in conjunction with a correction for the cross sectional coefficient of the section as presented by Keuning and Vermeulen proved valid for both the upright and the heeled section. So the sway added mass is obtained by integration of this 2-D value over the length of the yacht, i.e.:

$$m_{yy\phi} = \frac{\pi}{2} \rho \int_{\frac{Lwl}{2}}^{\frac{Lwl}{2}} h_{\phi}(x)^2 (3.33c_{yz}^2(x) - 3.05c_{yz}(x) + 1.39) dx$$

in which:

$h_{\phi}(x)$	is the largest draft of the section under heel.	m
$C_{yz}(x)$	sectional area coefficient of the section	-
Lwl	length of the static water line	m
ρ	density of water	kg/m ³

The added mass in sway of the keel and the rudder may be expressed as for simple oscillating plates, i.e.:

$$m_{yyk} = \frac{2\rho\pi b_k s_k}{\sqrt{ae_k^2 + 1}} \quad \text{for the keel and} \quad m_{yyr} = \frac{2\rho\pi b_r s_r}{\sqrt{ae_r^2 + 1}} \quad \text{for the rudder.}$$

$$a_{ek} = \frac{2(bk + Tc)}{\left[\frac{cre_k + ct_k}{2} \right]} \quad a_{er} = \frac{2(br + Tc)}{\left[\frac{cre_r + ct_r}{2} \right]}$$

in which:

T_c	draft of canoe body	m
b_k	span of keel	m
s_k	wetted area of keel	m ²
cre_k	root chord length of extended keel	m
ct_k	tip chord length of keel	m
ae_k	effective aspect ratio of keel	-
b_r	span of rudder	m
s_r	wetted area of rudder	m ²
cre_r	root chord length of extended rudder	m
ct_r	tip chord length of rudder	m
ae_r	effective aspect ratio of rudder	-

Multiplying the 2-D sectional added mass in sway with it's distance from the center of gravity of the yacht yields the added mass moment of inertia in yaw, i.e.:

$$J_{zz\phi} = \frac{\pi}{2} \rho \int_{\frac{L_{wl}}{2}}^{\frac{L_{wl}}{2}} x^2 h_{\phi}(x)^2 (3.33c_{yz}^2(x) - 3.05c_{yz}(x) + 1.39) dx$$

in which:

x	x-distance of midpoint section with respect to CoG	m
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in which once again the influence of the heel angle may be incorporated by using $h_{\phi}(x)$, i.e. the draft of the section when heeled.

The influence of the keel and rudder on the yaw added mass moment of inertia may be found by multiplying their respective added masses with the distance to the center of gravity squared, i.e.:

$$J_{zzkr} = m_{yk} * l_k^2 + m_{yr} * l_r^2$$

in which:

l_k	x-distance of keel(*) with respect to CoG	m
l_r	x-distance of rudder(*) with respect to CoG	m

*) *the point on the keel from which the distance is measured equals the 0.25 chord length at 43% of the span of the appendage*

Finally, the added mass in surge is approximated by a simple formulation as presented by Jacobs for commercial ships,

$$m_{xx} = 2 \frac{Tc}{Loa} m$$

in which:

Loa	length over all of the vessel	kg
m	solid mass of the vessel	kg

4 The hydrodynamic forces in the model.

Masayuma described the forces and moments acting on the hull and the appendages due to leeway and heel by means of the following expressions:

$$\begin{aligned} X_H &= X_{vv}v^2 + X_{v\phi}v\phi + X_{\phi\phi}\phi^2 + X_{vvv}v^3 \\ Y_H &= Y_vv + Y_{\phi}\phi + Y_{vvv}v^3 + Y_{vv\phi}v^2\phi + Y_{v\phi\phi}v\phi^2 + Y_{\phi\phi\phi}\phi^3 \\ K_H &= K_vv + K_{\phi}\phi + K_{vvv}v^3 + K_{vv\phi}v^2\phi + K_{v\phi\phi}v\phi^2 + K_{\phi\phi\phi}\phi^3 \\ N_H &= N_vv + N_{\phi}\phi + N_{vvv}v^3 + N_{vv\phi}v^2\phi + N_{v\phi\phi}v\phi^2 + N_{\phi\phi\phi}\phi^3 \end{aligned}$$

With his results he found that these expressions yielded good results. It should be noted however that all (or most) of these coefficients describing the hull forces were determined by applying regression on dedicated experiments in the towing tank for the one particular yacht, that was the subject of his study. These tests comprised such tests as forced oscillation tests in sway and yaw or stationary tank tests under heel and yaw. The sail forces and rudder forces were derived separately and were mostly based on thin airfoil theories and wind tunnel data.

In the present study approximations of these forces will be derived from the results of the DSYHS.

4.1 The determination of the hydrodynamic coefficients

The forces in the X direction, i.e. the resistance forces due to the forward speed of the yacht, the heel angle, the leeway angle and the side force produced, are approximated by making use of the results of the DSYHS. The expressions for the forces in the present report are based on those previously formulated in Reference [4] by Keuning and Sonnenberg. It should be noted however that there is one significant change in the formulations. This has to do with the difference in coordinate systems used between the DSYHS report and the present study. The expressions presented so far in the framework of the Delft Systematic Yacht Hull Series are derived for the

use in Velocity Prediction Programs, which implies that the X-axis is parallel to the velocity vector of the ship through the water and the lift or side force is perpendicular to that direction. In the present report however the forces are described in a body fixed coordinate system with the X-axis pointing forward along the centerline of the ship. This is common practice in maneuvering models. So the difference between the two lies in the leeway angle that the yacht makes. In general with sailing yachts however, even during a tacking maneuver, the leeway angle will remain small (at least when compared to commercial ships in a maneuver, where it may reach 30 – 40 degrees).

4.2 The forces acting up on the yacht along the X-axis due to forward velocity

For the frictional resistance of the hull use is being made of the well-known ITTC-57 formulations for the extrapolation coefficient. In correspondence with the procedure used in the DSYHS the Reynolds number of the hull is based on 70% of the waterline length. No form factor for the bare hull is being used, because such an expression for an arbitrary hull is not available. In the expressions for the viscous resistance of the appendages the generally expression based on relative thickness of the section is used.

The polynomial for the residuary resistance of the bare hull is the latest version from 1999. The residuary resistance of the keel appendage is from the same origin. So the complete resistance expressions used read:

The total force due to the forward velocity along the X-axis from Reference [4]:

$$Fx_u = -Rfh_u - Rrh_u - Rvk_u - Rvr_u - Rrk_u$$

$$Rfh_u = \frac{1}{2} \rho u^2 S C_f$$

$$C_f = \frac{0.075}{(\log(Rn) - 2)^2}$$

$$\frac{Rrh_u}{\nabla c \rho g} = a_0 + \left(a_1 \frac{LCB}{Lwl} + a_2 Cp + a_3 \frac{\nabla c^{2/3}}{Aw} + a_4 \frac{Bwl}{Lwl} + a_5 \frac{\nabla c^{2/3}}{Sc} + a_6 \frac{LCB}{LCF} + a_7 \left(\frac{LCB}{Lwl} \right)^2 + a_8 Cp^2 \right) \frac{\nabla c^{2/3}}{Lwl}$$

$$Rvk_u = Rfk_u (1 + k_k)$$

$$Rfk_u = \frac{1}{2} \rho u^2 S_k C_{fk}$$

$$(1 + k_k) = \left(1 + 2 \frac{t_k}{c_k} + 60 \left(\frac{t_k}{c_k} \right)^4 \right)$$

$$Rvr_u = Rfr (1 + k_r)$$

$$Rfr_u = \frac{1}{2} \rho u^2 S_r C_{fr}$$

$$(1 + k_r) = \left(1 + 2 \frac{t_r}{c_r} + 60 \left(\frac{t_r}{c_r} \right)^4 \right)$$

$$\frac{Rrk_u}{\nabla k \rho g} = A_0 + A_1 \frac{Tc + bk}{Bwl} + A_2 \frac{Tc + Zcbk}{\nabla k^{1/3}} + A_3 \frac{\nabla c}{\nabla k}$$

In which:

Rrh	residuary resistance of canoe body	N
Rfh	frictional resistance of canoe body	N
Rvk	viscous resistance of keel	N
Rvr	viscous resistance of rudder	N
Rrk	residuary resistance of keel	N
Cf	frictional resistance coefficient of canoe body	-
Rn	reynolds nuber of canoe body	-
∇_c	volume of displacement of canoe body	m ³
g	acceleration of gravity	m/s ²
Bwl	beam of waterline	m
LCB	longitudinal position center of buoyancy to fpp	m
LCF	longitudinal position center of flotation to fpp	m
Cp	prismatic coefficient	-
A_w	water plane area at zero speed	m ²
Sc	wetted surface canoe body at zero speed upright	m ²
Cf_k	frictional resistance coefficient keel	-
k_k	form factor keel	-
t_k	mean thickness keel	m
c_k	mean chord length keel	m
Cf_r	frictional resistance coefficient rudder	-
k_r	form factor rudder	-
t_r	mean thickness rudder	m
c_r	mean chord rudder	m
∇_k		
$Zcbk$	volume of displacement of keel	m ³
	vertical position of center of buoyancy of keel	m

Suffixes:

u	longitudinal velocity	m/s
v	transverse velocity	m/s
φ	heeling angle	rad
ψ	yaw angle	rad

The coefficients of the polynomials are presented in the tables below:

Table 1: Coefficients for Polynomial: Residuary resistance of canoe body

Fn	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
a_0	-0.0014	0.0004	0.0014	0.0027	0.0056	0.0032	-0.0064	-0.0171	-0.0201	0.0495	0.0808
a_1	0.0403	-0.1808	-0.1071	0.0463	-0.8005	-0.1011	2.3095	3.4017	7.1576	1.5618	-5.3233
a_2	0.0470	0.1793	0.0637	-0.1263	0.4891	-0.0813	-1.5152	-1.9862	-6.3304	-6.0661	-1.1513
a_3	-0.0227	-0.0004	0.0090	0.0150	0.0269	-0.0382	0.0751	0.3242	0.5829	0.8641	0.9663
a_4	-0.0119	0.0097	0.0153	0.0274	0.0519	0.0320	-0.0858	-0.1450	0.1630	1.1702	1.6084
a_5	0.0061	0.0118	0.0011	-0.0299	-0.0313	-0.1481	-0.5349	-0.8043	-0.3966	1.7610	2.7459
a_6	-0.0086	-0.0055	0.0012	0.0110	0.0292	0.0837	0.1715	0.2952	0.5023	0.9176	0.8491
a_7	-0.0307	0.1721	0.1021	-0.0595	0.7314	0.0223	-2.4550	-3.5284	-7.1579	-2.1191	4.7129
a_8	-0.0553	-0.1728	-0.0648	0.1220	-0.3619	0.1587	1.1865	1.3575	5.2534	5.4281	1.1089

Table 2: Coefficients for Polynomial: Residuary Resistance of Keel

F_n	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60
A_0	-0.00104	-0.00550	-0.01110	-0.00713	-0.03581	-0.00470	0.00553	0.04822	0.01021
A_1	0.00172	0.00597	0.01421	0.02632	0.08649	0.11592	0.07371	0.00660	0.14173
A_2	0.00117	0.00390	0.00069	-0.00232	0.00999	-0.00064	0.05991	0.07048	0.06409
A_3	-0.00008	-0.00009	0.00021	0.00039	0.00017	0.00035	-0.00114	-0.00035	-0.00192

The additional resistance due to the heeling of the yacht without induced resistance is presented by the expressions from Reference [4]:

$$F_{x_{u\varphi}} = -\Delta R r h \varphi_{u\varphi} - \Delta R r k \varphi_{u\varphi} - \Delta R f h \varphi_{u\varphi}$$

$$\frac{\Delta R r h_{u\varphi}}{\nabla c \rho g} = \left(u_0 + u_1 \frac{Lwl}{Bwl} + u_2 \frac{Bwl}{Tc} + u_3 \left(\frac{Bwl}{Tc} \right)^2 + u_4 LCB + u_5 LCB^2 \right) \cdot 6 \cdot \varphi^{1.7}$$

$$\frac{\Delta R r k_{u\varphi}}{\nabla k \rho g} = \left(H_1 \frac{Tc}{(Tc + bk)} + H_2 \frac{Bwl}{Tc} + H_3 \frac{Tc}{(Tc + bk)} \frac{Bwl}{Tc} + H_4 \frac{Lwl}{\nabla c^{1/3}} \right) F_n^2 \varphi$$

$$\Delta R f h_{u\varphi} = R f h_{u\varphi} - R f h_u$$

$$R f h_{u\varphi} = \frac{1}{2} \rho u^2 C_f S_{c\varphi}$$

$$S_{c\varphi} = S_c \left(1 + \frac{1}{100} \left(s_0 + s_1 \frac{Bwl}{Tc} + s_2 \left(\frac{Bwl}{Tc} \right)^2 + s_3 C_m \right) \right)$$

$$F_n = \frac{u}{\sqrt{gLwl}}$$

in which:

$\Delta R r h \varphi$	change in residuary resistance of canoe body due to heel	N
$\Delta R f h \varphi$	change in frictional resistance of canoe body due to heel	N
$\Delta R r k \varphi$	change in residuary resistance of keel due to heel	N
$S_{c\varphi}$	Wetted surface of canoe body under heel	m ²
F_n	Froude number	-
C_m	midship section coefficient	-

The coefficients of the polynomials for the additional resistance due to heel for the bare hull, the appendages and the change in wetted area due to heel are presented in the tables below:

Table 3: Coefficients for Polynomial: Delta Resistance Hull due to 20° Heel

Coefficients are multiplied by 1000							
Fn	0.25	0.30	0.35	0.40	0.45	0.50	0.55
u ₀	-0.0268	0.6628	1.6433	-0.8659	-3.2715	-0.1976	1.5873
u ₁	-0.0014	-0.0632	-0.2144	-0.0354	0.1372	-0.1480	-0.3749
u ₂	-0.0057	-0.0699	-0.1640	0.2226	0.5547	-0.6593	-0.7105
u ₃	0.0016	0.0069	0.0199	0.0188	0.0268	0.1862	0.2146
u ₄	-0.0070	0.0459	-0.0540	-0.5800	-1.0064	-0.7489	-0.4818
u ⁵	-0.0017	-0.0004	-0.0268	-0.1133	-0.2026	-0.1648	-0.1174

Table 4: Coefficients for Polynomial: Delta Resistance of the Keel due to Heel

H ₁	-3.5837
H ₂	-0.0518
H ₃	0.5958
H ₄	0.2055

Table 5: Coefficients for Polynomial: Wetted Surface under Heel

φ	5	10	15	20	25	30	35
s ₀	-4.112	-4.522	-3.291	1.850	6.510	12.334	14.648
s ₁	0.054	-0.132	-0.389	-1.200	-2.305	-3.911	-5.182
s ₂	-0.027	-0.077	-0.118	-0.109	-0.066	0.024	0.102
s ₃	6.329	8.738	8.949	5.364	3.443	1.767	3.497

4.3 Forces along the X-axis due to the sway velocity

The additional force in the ship bound X-direction is due to the side force production of the hull and appendages and is composed now of two components: one (small) component due to the lift and the second (larger) component which is due to the induced resistance. So:

$$F_{x_{uv\varphi}} = Fh \cos(\varphi)_{uv\varphi} \sin\left(\frac{-v}{u}\right) - Ri_{uv\varphi} \cos\left(\frac{-v}{u}\right)$$

$$Fh \cos(\varphi)_{uv\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right) \\ * \left(\frac{-v}{u} + \beta_{Fh=0} \right) \frac{1}{2} \rho V s^2 Sc$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{Tc} \frac{Tc}{(Tc + bk)} \varphi^2 Fn$$

$$Drag_{uv\varphi} = Ri_{uv\varphi}$$

$$Ri_{uv\varphi} = \frac{Fh_{uv\varphi}^2}{\pi T e^2 \frac{I}{2} \rho V_s^2}$$

$$\frac{Te}{(Tc + bk)} = \left(A_1 \frac{Tc}{(Tc + bk)} + A_2 \left(\frac{Tc}{(Tc + bk)} \right)^2 + A_3 \frac{Bwl}{Tc} + A_4 TR \right) (B_0 + B_1 Fn)$$

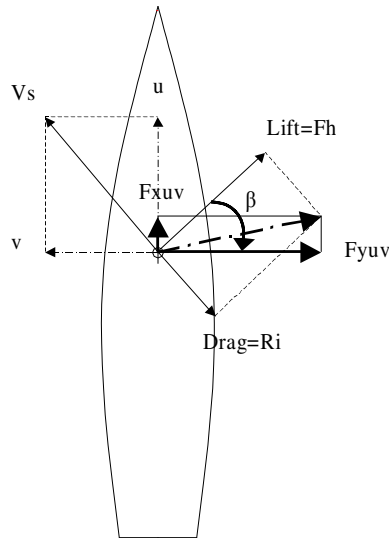


Figure 2: side force due to transverse velocity

It should be noted that the leeway angle β is defined as the fracture of the transverse velocity v and the forward velocity u .

$$\beta = \frac{-v}{u}$$

In which:

Fh	Heeling force	N
Ri	Induced resistance	N
TR	Taper ratio of keel	-
Fn	Froude number	-
Te	effective span	m

Table 6: Coefficients for the Polynomial: Effective Span

φ	0	10	20	30
A_1	3.7455	4.4892	3.9592	3.4891
A_2	-3.6246	-4.8454	-3.9804	-2.9577
A_3	0.0589	0.0294	0.0283	0.0250
A_4	-0.0296	-0.0176	-0.0075	-0.0272
B_0	1.2306	1.4231	1.5450	1.4744
B_1	-0.7256	-1.2971	-1.5622	-1.3499

Table 7: Coefficients for the Polynomial: Lift Curve Slope

φ	0	10	20	30
b_1	2.025	1.989	1.980	1.762
b_2	9.551	6.729	0.633	-4.957
b_3	0.631	0.494	0.194	-0.087
b_4	-6.575	-4.745	-0.792	2.766

4.4 The forces along the Y-axis due to the sway and roll velocity

The forces on the yacht along the Y-axis are components associated with the lift and drag also. Lift is generated due to the sway velocity (or leeway angle) of the yacht and due to the additional angle of attack that originates from the combination of the induced velocities caused by the roll velocity and the forward velocity. Similar to the situation with the forces along the X-axis here also a component of the resistance enters the equations.

So the Y forces due to the sway velocity become:

$$Fy_{uv\varphi} = \left(Fh \cos(\varphi)_{uv\varphi} \cos\left(\frac{-v}{u}\right) + Ri_{uv\varphi} \sin\left(\frac{-v}{u}\right) \right)$$

$$Fh \cos(\varphi)_{uv\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right)$$

$$* \left(\frac{-v}{u} + \beta_{Fh=0} \right) \frac{1}{2} \rho V s^2 Sc$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{Tc} \frac{Tc}{(Tc + bk)} \varphi^2 Fn$$

$$Ri_{uv\varphi} = \frac{Fh_{uv\varphi}^2}{\pi T e^2 \frac{1}{2} \rho V s^2}$$

$$\frac{Te}{(Tc+bk)} = \left(A_1 \frac{Tc}{(Tc+bk)} + A_2 \left(\frac{Tc}{(Tc+bk)} \right)^2 + A_3 \frac{Bwl}{Tc} + A_4 TR \right) (B_0 + B_1 Fn)$$

and the additional lift forces due to the roll velocity:

$$Fh \cos(\varphi)_{uv\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right) \\ * \left(\left(\frac{h\dot{\varphi}}{u} \right) + \beta_{Fh=0} \right) \frac{1}{2} \rho V s^2 Sc$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{T_c} \frac{T_c}{(T_c + bk)} \varphi^2 Fn$$

$$\beta_{Fh=0} = B_3 \varphi^2 Fn$$

$$h = 0.43(bk + Tc)$$

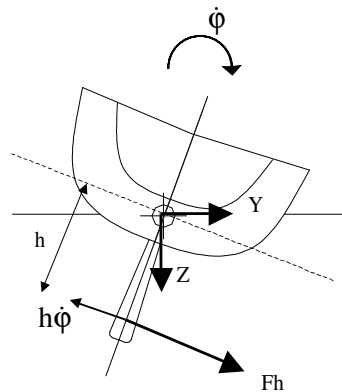


Figure 3: side force due to Roll velocity

4.5 Forces along the X and Y-axis due to the yaw velocity

The assumption is now made that the sailing yachts under consideration have a more or less traditional appendage layout. This implies that the keel is positioned close to the longitudinal position of the center of gravity of the yacht. This makes the assumption justifiable that the influence of the yaw velocity on the forces on the keel is neglectable. So the forces due to the yaw velocity are restricted to the forces on the rudder only.

For the force in X direction we find:

$$Fxr_{u\dot{\psi}} = Fhr_{u\dot{\psi}} \cos(\varphi) \sin\left(\frac{-l_r \dot{\psi}}{u}\right) - Dr_{u\dot{\psi}} \cos\left(\frac{-l_r \dot{\psi}}{u}\right)$$

and for the force in Y direction:

$$Fyr_{u\dot{\psi}} = Fhr_{u\dot{\psi}} \cos(\varphi) \cos\left(\frac{-l_r \dot{\psi}}{u}\right) + Dr_{u\dot{\psi}} \sin\left(\frac{-l_r \dot{\psi}}{u}\right)$$

$$Fhr_{u\dot{\psi}} = \frac{1}{2} \rho V s^2 Al_{at_r} \frac{\partial Cl_r}{\partial \beta} \left(\frac{-l_r \dot{\psi}}{u}\right)$$

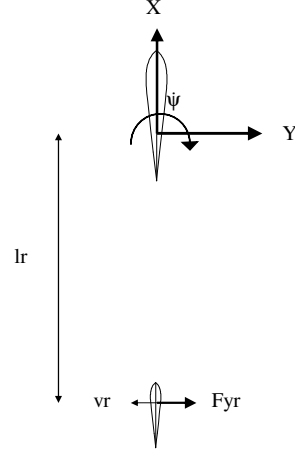


Figure 4: side force due to Yaw velocity

in which the lift curve slope is calculated using the well known expression from Faulkner:

$$\frac{\partial Cl_r}{\partial \beta} = \frac{5.7 a_{er}}{1.8 + \cos \Lambda \sqrt{\frac{a_{er}^2}{\cos^4 \Lambda r} + 4}}$$

$$a_{er} = \frac{2(br + Tc)}{\left[\frac{cre_r + ct_r}{2} \right]}$$

and for the additional induced resistance airfoil theory is used to express:

$$Dr_{u\dot{\psi}} = \frac{1}{2} \rho V s^2 Al_{at_r} Cdi$$

$$Cdi = \frac{Cl_r^2}{\pi a_{er}}$$

$$Dr_{u\dot{\psi}} = \frac{Fh^2}{\frac{1}{2} \rho V s^2 Al_{at_r} \pi a_{er}}$$

In which:

Fhr	heeling force rudder	N
Ri	induced resistance rudder	N
$Alat_r$	lateral area rudder	m ²
Cdi	induced drag coefficient rudder	-
Clr	lift coefficient rudder	-
Ar	sweep back angle	deg

4.6 Moments around the X axis due to heel angle and the sway- and roll velocity

The moment around the X-axis due to the heel angle of the yacht is the obvious and well-known stability moment of the yacht. Since detailed information of the yacht is not assumed necessary yet a feasible simplification using the GM value only is being used. This simplification has been proven quite accurate for a large variety of yachts for heeling angles up to 30 or 40 degrees. So:

$$K_{\varphi\varphi} = -GM \sin(\varphi) \Delta$$

For the approximation of the heeling moment due to the sway velocity use is being made of the approximated vertical position of the center of effort of the total side force on the hull, rudder and keel as derived from the results obtained within the Delft Systematic Yacht Hull Series:

$$K_{uv\varphi} = Fh_{uv\varphi} * 0.43(bk + Tc)$$

$$Fh_{uv\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right) * \left(\frac{-v}{u} + \beta_{Fh=0} \right) \frac{l}{2} \rho V_s^2 Sc \frac{l}{\cos(\varphi)}$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{Tc} \frac{Tc}{(Tc + bk)} \varphi^2 Fn$$

The same approximation for the center of effort of the side force is used when approximating the additional heel moment due to the induced roll velocities, i.e.:

$$K_{u\dot{\varphi}\varphi} = -Fh_{u\dot{\varphi}\varphi} * 0.43(bk + Tc)$$

$$Fh_{u\dot{\varphi}\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right) * \left(\frac{\dot{\varphi} * 0.43(bk + Tc)}{u} + \beta_{Fh=0} \right) \frac{1}{2} \rho V_s^2 Sc \frac{l}{\cos(\varphi)}$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{Tc} \frac{Tc}{(Tc + bk)} \varphi^2 Fn$$

4.7 The yaw moment due to the sway velocity

For the approximation of this Yaw moment extensive use is being made of the results reported by Keuning and Vermeulen in Reference [1].

They assumed the total yawing moment of the appended hull to be composed of three separate contributions: the hull, the keel and the rudder.

So the principal contributions to the yaw moment read:

$$N_{uv\varphi} = N_{uv\varphi}^{Keel} + N_{uv\varphi}^{Rudder} + N_{uv\varphi}^{Hull}$$

$$N_{uv\varphi}^{keel} = Fy_{k_{uv\varphi}} * lk$$

$$N_{uv\varphi}^{Rudder} = Fy_{r_{uv\varphi}} * lr$$

Due to the fact that the DSYHS expression given in Reference [4] is used to calculate the total side force, the separate side force contributions of keel and rudder are not known. So, in order to account for the separate contribution of keel and rudder, a distribution between the side force on the (extended) keel and the rudder is assumed, similar to that used in the upright case. Using this distribution, the side force contribution from hull and keel can be separated from the contribution of the rudder. There are two reasons for using the DSYHS expression instead of the Extended Keel Method to determine the hull, keel and rudder contributions to the total side force:

1. The Extended Keel Method does not take in account the effect of heel on the lift generating capabilities
2. The ‘downwash effect’ from the keel on the lift generated by the rudder is implicitly incorporated in the DSYHS expressions,

The downwash of the keel diminishes the effective angle of attack of the rudder. The effect will be dependent on the keel loading, the aspect ratio and the distance between the two foils. In particular when the tip vortex “hits” the rudder a strong reduction may be expected. For the usual layout with moderate to high aspect foils and a reasonably large distance between the two, the following formulation has been proven to be valid:

$$\phi = \frac{1.6Cl_k}{\pi AR_k}$$

The procedure to determine the side force distribution in the upright case is given in the 4 steps below:

- 1 Calculation of the induced drag and side force for the known velocities in the upright case, using the DSYHS formulation Reference [4]:

$$Fh_{uv} = \left(2.025 \frac{(Tc+bk)^2}{Sc} + 9.551 \left(\frac{(Tc+bk)^2}{Sc} \right)^2 + 0.631 \frac{Tc}{(Tc+bk)} - 6.575 \frac{Tc}{(Tc+bk)} \frac{(Tc+bk)^2}{Sc} \right) \\ * \left(\frac{-v}{u} \right) \frac{1}{2} \rho V_s^2 Sc \\ Ri_{uv} = \frac{Fh_{uv}^2}{\pi T e^2 \frac{1}{2} \rho V_s^2} \\ \frac{Te(\varphi=0)}{(Tc+bk)} = \left(3.7455 \frac{Tc}{(Tc+bk)} - 3.6246 \left(\frac{Tc}{(Tc+bk)} \right)^2 + 0.0589 \frac{Bwl}{Tc} - 0.0296TR \right) (1.2306 - 0.7256Fn)$$

- 2 Calculation of the side force and related induced drag hull and keel, using the Extended Keel Method, note that for the upright case, the Extended Keel Method is valid:

$$\frac{\partial Cl_k}{\partial \beta} = \frac{5.7a_{ek}}{1.8 + \cos \Lambda \sqrt{\frac{a_{ek}^2}{\cos^4 \Lambda} + 4}} \\ a_{ek} = \left[\frac{2(bk + Tc)}{cre_k + ct_k} \right] \\ Fhk_{uv} = \frac{1}{2} \rho V_s^2 Al_{at_k} \frac{\partial Cl_k}{\partial \beta} \left(\frac{-v}{u} \right) \\ Ri_{uv} = \frac{Fhk_{uv}^2}{\frac{1}{2} \rho V_s^2 Al_{at_k} \pi a_{ek}}$$

- 3 Calculation of the fraction of keel+hull side force and induced resistance with respect to the total side force:

$$f_{hk} = \frac{F h k_{uv}}{F h_{uv}}$$

$$f_{rk} = \frac{R i k_{uv}}{R i_{uv}}$$

- 4 The contribution of the rudder now becomes::

$$f_{hr} = 1 - f_{hk}$$

$$f_{rr} = 1 - f_{rk}$$

This distribution of the side force and induced resistance for the upright situation in terms of f_{hk} , f_{rk} , f_{hr} and f_{rr} is now used to calculate the separate contributions of keel and rudder under heel, using the DSYHS formulation Reference [4] for the total side force under heel:

Keel:

$$F y k_{uv\varphi} = \left(F h \cos(\varphi)_{uv\varphi} (f_{hk}) \cos\left(\frac{-v}{u}\right) + R i_{uv\varphi} (f_{rk}) \sin\left(\frac{-v}{u}\right) \right)$$

$$F h \cos(\varphi)_{uv\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right)$$

$$* \left(\frac{-v}{u} + \beta_{Fh=0} \right) \frac{1}{2} \rho V s^2 Sc$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{Tc} \frac{Tc}{(Tc + bk)} \varphi^2 Fn$$

$$Drag_{uv\varphi} = R i_{uv\varphi}$$

$$R i_{uv\varphi} = \frac{F h_{uv\varphi}^2}{\pi T e^2 \frac{1}{2} \rho V s^2}$$

$$\frac{T e}{(Tc + bk)} = \left(A_1 \frac{Tc}{(Tc + bk)} + A_2 \left(\frac{Tc}{(Tc + bk)} \right)^2 + A_3 \frac{Bwl}{Tc} + A_4 TR \right) (B_0 + B_1 Fn)$$

Rudder:

$$Fy_{r_{uv\varphi}} = \left(Fh \cos(\varphi)_{uv\varphi} (fhr) \cos\left(\frac{-v}{u}\right) + Ri_{uv\varphi} (frr) \sin\left(\frac{-v}{u}\right) \right)$$

$$Fh \cos(\varphi)_{uv\varphi} = \left(b_1 \frac{(Tc + bk)^2}{Sc} + b_2 \left(\frac{(Tc + bk)^2}{Sc} \right)^2 + b_3 \frac{Tc}{(Tc + bk)} + b_4 \frac{Tc}{(Tc + bk)} \frac{(Tc + bk)^2}{Sc} \right) \\ * \left(\frac{-v}{u} + \beta_{Fh=0} \right) \frac{I}{2} \rho V s^2 Sc$$

$$\beta_{Fh=0} = 0.0092 \frac{Bwl}{Tc} \frac{Tc}{(Tc + bk)} \varphi^2 Fn$$

$$Drag_{uv\varphi} = Ri_{uv\varphi}$$

$$Ri_{uv\varphi} = \frac{Fh_{uv\varphi}^2}{\pi T e^2 \frac{I}{2} \rho V s^2}$$

$$\frac{Te}{(Tc + bk)} = \left(A_1 \frac{Tc}{(Tc + bk)} + A_2 \left(\frac{Tc}{(Tc + bk)} \right)^2 + A_3 \frac{Bwl}{Tc} + A_4 TR \right) (B_0 + B_1 Fn)$$

in which:

fhk	fraction of the total side force due to hull and keel	-
frk	fraction of the total induced resistance due to hull and keel	-
fhr	fraction of the total side force due to rudder	-
frr	fraction of the total induced resistance due rudder	-

The side force on the appendages is located on the quarter cord length of each foil and their respective contribution to the yawing moment is calculated using these position with respect to the center of gravity of the yacht.

The yaw moment on the bare hull was formulated by an improved method for the assessment of the Munk moment and based on the theory formulated by Nomoto in Reference [6] for the yaw moment of an arbitrary hull. For the improved formulation of the Munk moment use is being made of the integration of the change in sway added mass over the entire length of the hull instead of over just half the length, as is common practice with commercial vessels. The sway added mass is calculated using the approximation method of Nomoto with a correction for different Cm values of the sections. Under hell the sway added mass is approximated using the actual maximum depth of the section when heeled. Finally an “additional” leeway angle is introduced in the Nomoto expression to take care of the “additional” yaw moment of a yacht hull caused by the asymmetry of the hull when heeled over.

For a more detailed description of the method developed to asses the yaw moment, reference is made to the report by Keuning and Vermeulen on this subject in Reference [1].

The Munk moment is a fully inviscid flow phenomenon and calculated using the change of momentum of the oncoming fluid. In a real fluid however it is assumed that this type of side force generation reduced by viscous effects, such as vortex shedding and flow separation. This reduces the yaw moment when compared to the full potential flow. This effect increases with increasing leeway angle. In the literature this effect is associated with athwart forces related to the so called cross flow drag, i.e. drag forces arising from a cross flow over the sections due to the sway velocity of the ship. For commercial ships this effect will be different from a yacht hull. The more V shaped sections at the bow will have a higher drag than the flat bottom or rounded sections in the stern of the yacht. This will tend to increase the yaw moment. Also the effect of the bow wave due to the higher Froude numbers will be more significant.

Finally the yaw moment on the bare hull is approximated using the expressions as derived in Reference [3], presenting the corrected Munk moment and the additional moment due to the asymmetry of the hull when heeled, i.e.:

$$N_{uv\varphi hull} = \frac{\pi}{2} \rho u^2 \left(\frac{-v}{u} \right)^2 \int_{-\frac{Lwl}{2}}^{\frac{Lwl}{2}} h_{\varphi}(x)^2 (3.33c_{yz}(x)^2 - 3.05c_{yz}(x) + 1.39) dx + M_{z0}$$

$$M_{z0} = CM_{z0} \frac{1}{2} \rho V_s^2 Lwl A_{lat}$$

$$CM_{z0} = \frac{0.01 B w l^2}{Lwl Tc}$$

4.8 Yaw moment due to yaw velocity

Once again assuming the “normal” layout of the appendages the side force generated on the keel due to a rotational velocity in yaw is considered neglectible, because it is positioned close to the centre of gravity of the yacht. Also in a turn there is supposed to be no down wash effect from the keel on the rudder. Under these assumptions the yaw moment due to the yaw velocity reduces to:

$$N_{u\dot{\psi}} = -F_y r_{u\dot{\psi}} l_r$$

in which the force in the Y direction is composed of both a lift and a drag component according to:

$$F_{x r_{u\dot{\psi}}} = F_{h r_{u\dot{\psi}}} \cos(\varphi) \sin\left(\frac{-l_r \dot{\psi}}{u}\right) - D_{r_{u\dot{\psi}}} \cos\left(\frac{-l_r \dot{\psi}}{u}\right)$$

and for the force in Y direction:

$$Fy_{r_{u\psi}} = Fhr_{u\psi} \cos(\varphi) \cos\left(\frac{-l_r \dot{\psi}}{u}\right) + Dr_{u\psi} \sin\left(\frac{-l_r \dot{\psi}}{u}\right)$$

$$Fhr_{u\psi} = \frac{1}{2} \rho V_s^2 Al_{at_r} \frac{\partial Cl_r}{\partial \beta} \left(\frac{-l_r \dot{\psi}}{u}\right)$$

in which the lift curve slope is calculated using the well known expression from Faulkner and for the additional induced resistance airfoil theory is used to express:

$$Dr_{u\psi} = \frac{1}{2} \rho V_s^2 Al_{at_r} Cdi$$

$$Cdi = \frac{Cl_r^2}{\pi a_{er}}$$

$$Dr_{u\psi} = \frac{Fh^2}{\frac{1}{2} \rho V_s^2 Al_{at_r} \pi a_{er}}$$

4.9 Additional forces on the rudder.

For the lift forces on the rudder the lift curve slope approximation of Faulkner is being used, quite similar to the procedure used with the keel. As explained in Reference [3] the Extended Keel Method as presented by Gerritsma in Reference [5] is used to take into account the end plate effect of the hull and the increased velocity over the keel being below the hull

The rudder however is situated in the steady state condition (i.e. on a steady close hauled course) in the downwash of the keel. This effect is felt by a reduction of the effective angle of attack on the rudder when compared with the keel and in a reduction of the “free flow” velocity over the rudder, i.e. the wake of the keel and hull. A typical velocity reduction over a rudder is presented in the Table below based on results obtained by Gerritsma in Reference [5]

Table 8: velocity measurements

U [m/s]	0.90	1.20	1.50	1.80
U_2 [m/s]	0.83	1.02	1.30	1.63
ΔU [-]	0.92	0.85	0.87	0.91

Based on these results a wake factor of minus 10% of the free flow velocity is implied on the rudder velocity.

Besides the effective rudder angle due to the leeway of the yacht in the present model account has to be taken of active rudder manipulation by the helmsman or course control. This implies additional lift and drag forces on the rudder as function of the rudder angle applied. These can be divided in force components along the X-axis and the Y-axis respectively.

The forces along the X-axis are:

$$Fxr_{u\delta} = Fhr_{u\delta} \cos(\varphi) \sin\left(\frac{-v}{u}\right) - Dr_{u\delta} \cos\left(\frac{-v}{u}\right)$$

$$Fyr_{u\delta} = Fhr_{u\delta} \cos(\varphi) \cos\left(\frac{-v}{u}\right) + Dr_{u\delta} \sin\left(\frac{-v}{u}\right)$$

$$Fyr_{u\delta} = \frac{1}{2} \rho (0.9Vs)^2 Alar_r \frac{\partial Clr}{\partial \delta} \delta$$

in which the lift curve slope is calculated using the well known expression from Faulkner and for the additional induced resistance airfoil theory is used to express:

$$Dr_{u\delta} = \frac{1}{2} \rho Vs^2 Alar_r Cdi$$

$$Cdi = \frac{Clr^2}{\pi a_{er}}$$

$$Dr_{u\delta} = \frac{Fyr_{u\delta}^2}{\frac{1}{2} \rho Vs^2 Alar_r \pi a_{er}}$$

In which:

δ rudder angle rad

5 The sail forces.

In the mathematical model presented in this present paper only a very limited aerodynamic model for determining the forces on the sails and superstructure is presented. For the present study, in which the applicability of the derived mathematical model will only be demonstrated using full-scale data of more or less “usual” sailing yachts, no extensive model is implemented to take care of a large variety of sail plans and sails. Since the tacking model is going to be used for the comparison of a much larger variety of sailing yachts this certainly is a draw back. In the near future this will be overcome by implementing a far more versatile and complete sail force model like those presently used in the Velocity Prediction Programs.

The forces on the sails in the present model are calculated using a slightly adopted and simplified procedure.

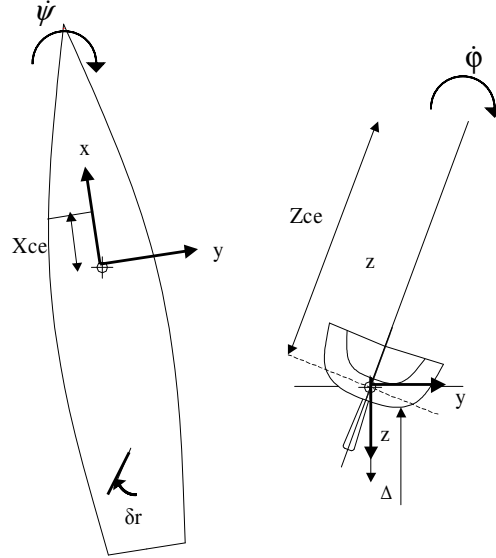
The basics of this aero model are in a self-defined lift and drag coefficient curve of the sails over a range of angles of attack between 0 and 90 degrees. This range is different from the usual procedure in a VPP because now these coefficients also need to be defined for very small angles of attack such as these occur during a tacking procedure. The uniform sail plan in the present approach consists of the full main sail and 100% fore triangle and a masthead rig. The values of the lift and the drag coefficient in the “usual” range of angles of attack for functional sails are taken from publications about the IMS sail force model as presented a.o. by Claughton e.a. in Reference [7].

The apparent wind speed and the apparent angle of attack on the sails is affected by the forward velocity of the yacht and the induced velocities at the assumed center of effort of the sail by the roll- and the yaw motions. The center of effort of the sails is assumed to be in the geometrical

centroid of the sail plan and Z_{ce} is the vertical distance and X_{ce} the horizontal distance with respect to the center of gravity of the yacht.

Figure 5: Definition of the sail forces on the yacht

The expressions for the apparent wind speed and direction read:



$$V_{aw} = \sqrt{(\dot{\phi} Z_{ce} + \dot{\psi} X_{ce} + V_{aw} \sin(\beta_{aw}))^2 + (V_{aw} \cos(\beta_{aw}))^2}$$

$$\beta_{aw} = \tan^{-1}\left(\frac{\dot{\phi} Z_{ce} + \dot{\psi} X_{ce} + V_{aw} \sin(\beta_{aw})}{V_{aw} \cos(\beta_{aw})}\right)$$

The plot of the lift and drag coefficients values used in the calculations is presented below. It should be noted that there is a significant resistance force due to the sails at very small angles of attack to account for the resistance of the floppy sails.

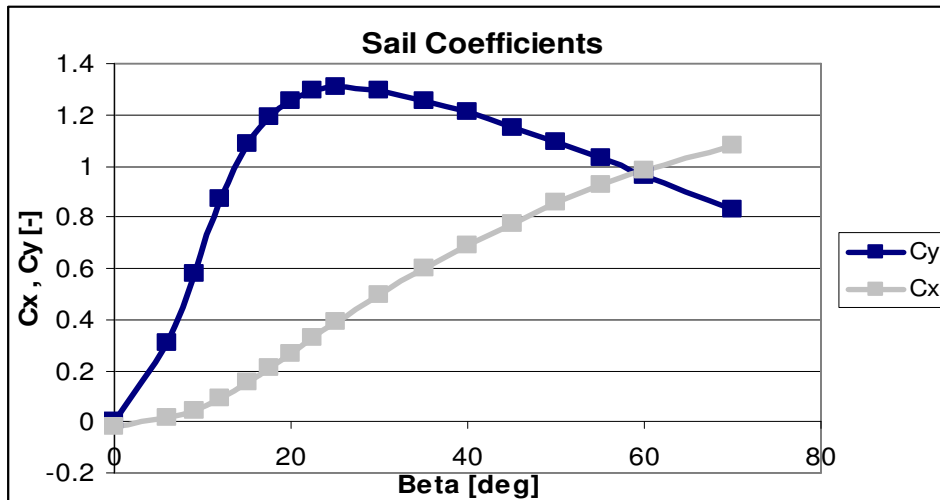


Figure 6: The lift and drag coefficient of the sails.

In the aero model also account is been taken of the windage of the hull and rig in a similar fashion. The wind speed over the hull is reduced with respect to the wind speed at 10 meters height, but no change in apparent wind angle is assumed.

In the present maneuvering model, when a tack is being simulated, the aerodynamic coefficients decrease from values in the close-hauled region to values even smaller than zero until the apparent wind shifts “to the other side”. From there on it will take a few seconds to trim the sails to the new condition and to regain stationary flow over the sails. During this “time-lag” the sail forces are assumed to increase linear with time. This parameter can be modified to suite the circumstances present at particular boats, which might be of use to asses the capabilities from boats difficult to handle in that respect. Further more this time lag will depend on all kind of parameters and procedures, such as the steering procedure followed i.e. the rudder angle as function of time. In the simulations presented here for the same half ton yacht as used by Masayuma for his validations this time lag is set at 5 seconds.

The X and Y forces of the sails as well as the K and N moments induced by the sails are expressed by the following equations:

$$X_{sail} = \frac{1}{2} \cdot \rho_A \cdot V_{aw} \cdot Sa \cdot C_x$$

$$Y_{sail} = \frac{1}{2} \cdot \rho_A \cdot V_{aw} \cdot Sa \cdot C_y$$

$$K_{sail} = Y_{sail} \cdot Zce \cdot \cos \varphi$$

$$N_{sail} = X_{sail} \cdot Zce \cdot \sin \varphi + Y_{sail} \cdot Xce$$

In which:

$$C_x = Cl \cdot \sin(\beta_{aw}) - Cd \cdot \cos(\beta_{aw})$$

$$C_y = Cl \cdot \cos(\beta_{aw}) + Cd \cdot \sin(\beta_{aw})$$

Apart from the heeling moment due to the sail forces also a yawing moment is introduced when the boat is heeled over, a rather significant component in the equations.

6 Results from the forced oscillation experiments.

In the framework of the present study forced oscillation tests have been carried out in the #1 towing tank of the Shiphydromechanics Department of the Delft University of Technology.

The aim of these tests was to validate the approach as presented by Keuning and Vermeulen in Reference [1] for the approximation of the sway added mass both in the upright condition as under heel. It was decide to investigate the influence of the following parameters on the sway added mass :

- The influence of hull depth
- The influence of heel angle
- The influence of the forward velocity
- The influence of the frequency of oscillation

Hereto four very different models of the DSYHS, one from Sub-Series 1 and three of Sub-Series 2, have been tested underneath the new 6 degrees of freedom oscillator from the Department. The principal dimensions of these models are depicted in the table below. For more detailed

information about the body plans of these models reference is made to the literature dealing with the DSYHS, such as Reference [4].

Table 9 Syssers used for the validation

SYSSER	Lwl/Bwl	Bwl/Tc	Lwl/Volc ^{1/3}	LCB %	LCF %
6	3.155	2.979	4.339	-2.40	-3.42
24	3.497	10.958	6.935	-2.09	-5.84
25	4.000	5.388	6.003	-1.99	-5.54
27	4.496	2.460	5.011	-1.88	-5.24

The tests have been carried out at three different heel angles, i.e. 0, 20 and 30 degrees, at two different speeds, i.e. $F_n=0.30$ and $F_n=0.40$ and at a number of different oscillation frequencies between 0.447 rad/sec to 0.373 rad/sec.

Some of the results of these tests are presented in Figure 7. In these figures the added mass in sway is compared against the results of the calculations using the procedure as presented in Reference [1]. The results are shown for the four models and for the zero degrees of heel and the 20 degrees of heel condition. The results are depicted for a number of different forward speeds and oscillation frequencies. The thick spot at the $\omega = 0$ axis is the result of the approximation, which is independent of speed and oscillation frequency.

As is obvious from these results at zero degrees of heel the approximation is certainly in the ball park, considering the necessary extrapolation of the measured data to zero frequency. It is also obvious however that the measured data show a considerable speed influence, which is not accounted for in the calculation. In general the sway added mass decreases with increasing speed. It should be noted that during the experiments the model was not free to sink and trim and the generated wave system may significantly influence the results. In general the trend found in the influence of the heel angle on the sway added mass corresponds between measurement and calculation, except for Sysser 24 the high beam to draft ratio model where the calculation suggests an increase in added mass much higher than the measurements do.

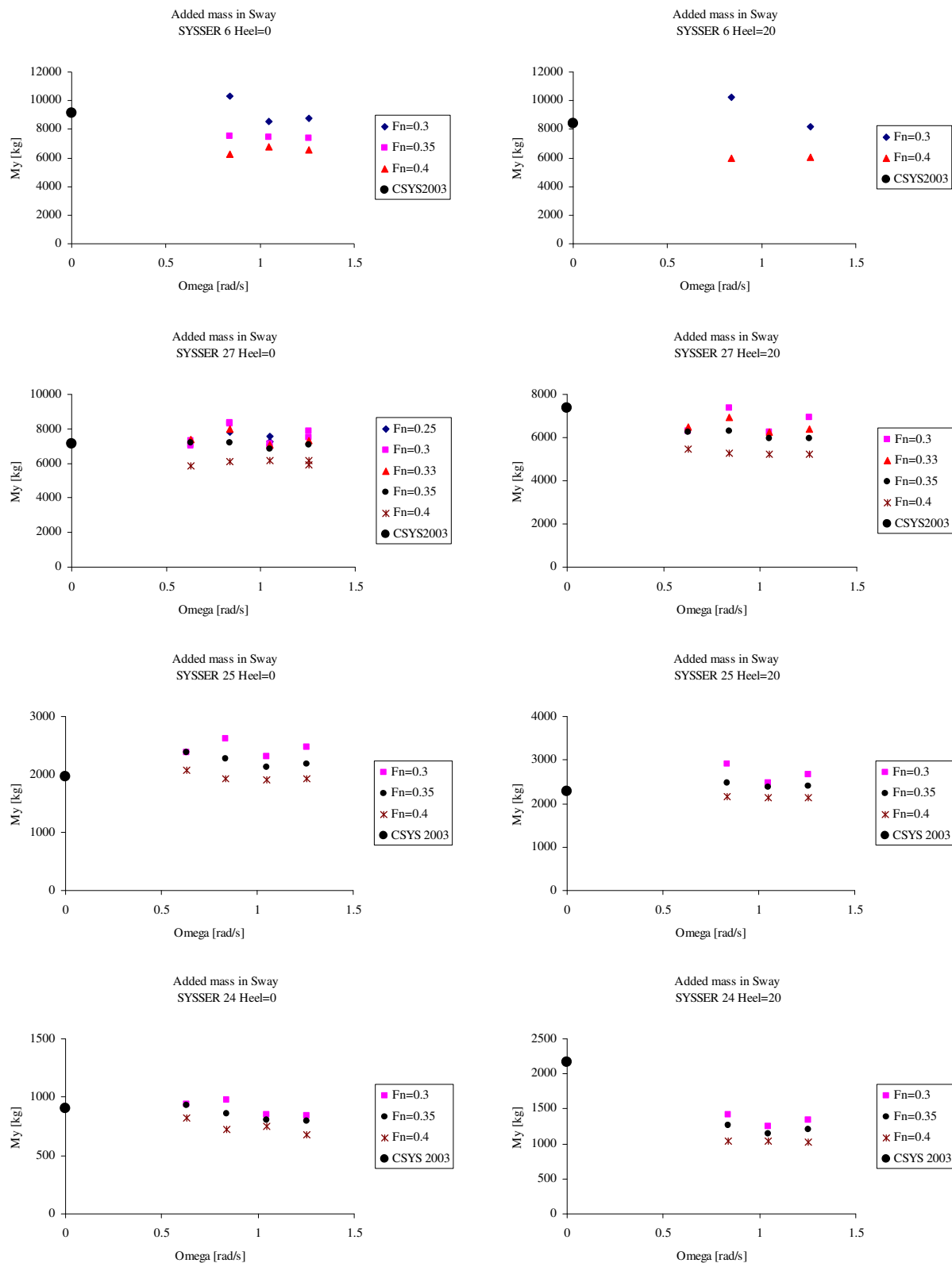


Figure 7: Added mass derived from oscillation tests compared to calculation of added mass according to CSYS2003 formulations

7. Comparison of the results of the simulations with full-scale measurements.

To check whether the present model with the calculated coefficients yields similar

results when compared with the simulated results obtained by Masayuma, who used the set of measured coefficients, the full scale measurements of Masayuma with the half-tonner were recalculated. He found a good correlation between his simulation and the full scale measurement and so does the present model. The simulated and measured track of this yacht in a tacking maneuver may be seen in Figure 8. Similar agreement was found when the results of the speed-loss, the heeling angle, the course etc. etc. were compared.

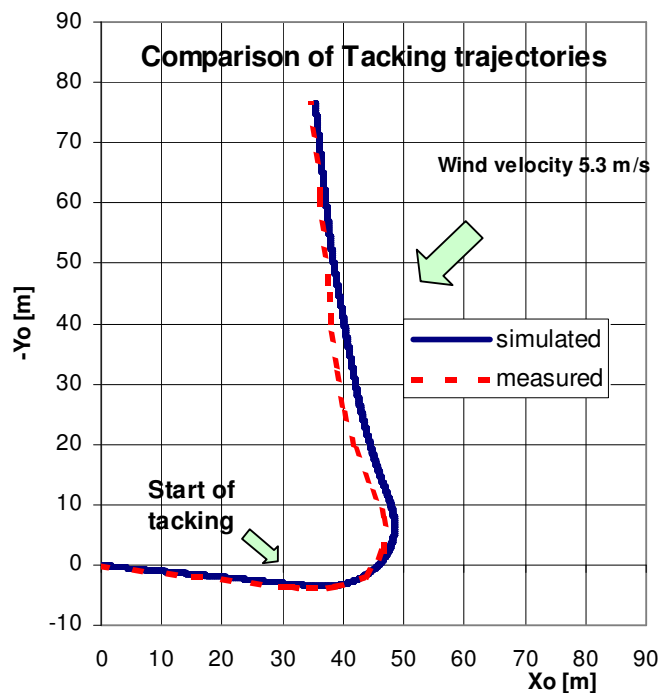


Figure 8 Comparison of simulated results with Masayuma's measurements

In the scope of the present study also some full-scale measurements on the tacking maneuver have been carried out for validation purposes with three rather different yachts. The measurements with one of these yachts, i.e. those with the Bashford 41 "Checkmate", produced the most reliable results due to the environmental conditions at the time of the measurements and the equipment onboard. On board was a very sensitive and highly accurate dedicated GPS receiver, capable of measuring displacements in the order of magnitude of 10 centimeters and with a very high sampling rate. The result of one of the tacking maneuvers is presented in Figure 9.

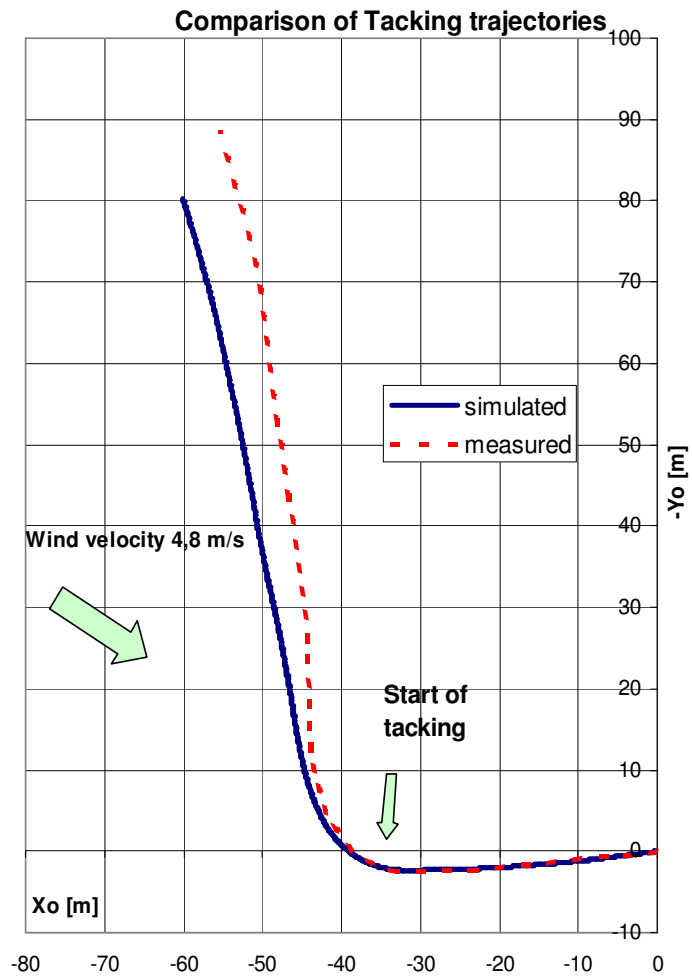


Figure 9 Comparison of tacking trajectories

Some more results as time histories of rudder angle, effective rudder angle, speed loss and course obtained from the same maneuver are presented in figure 10.

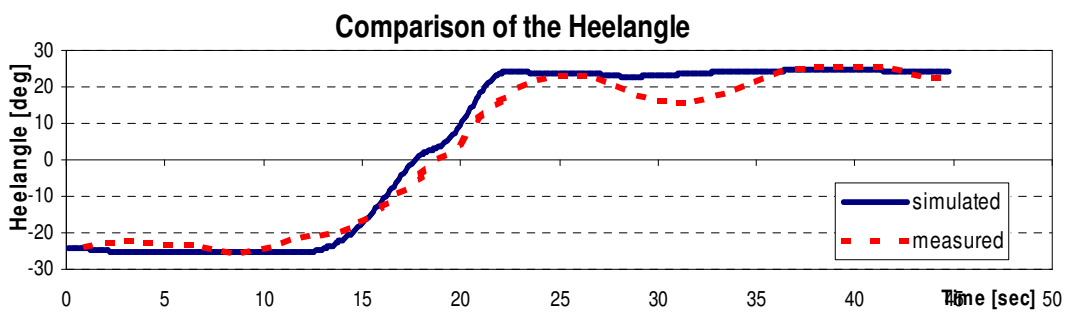
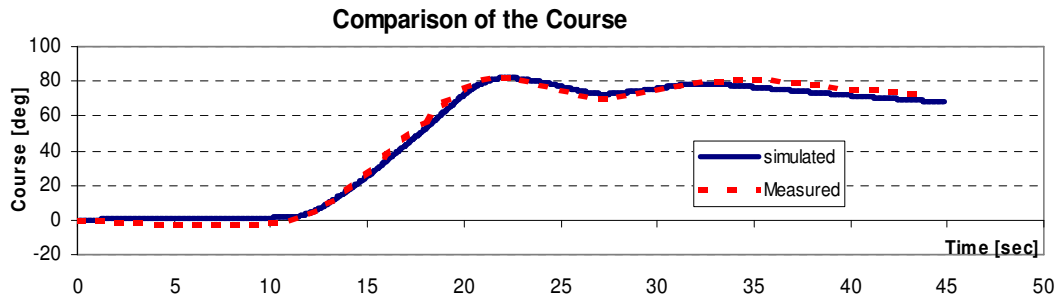
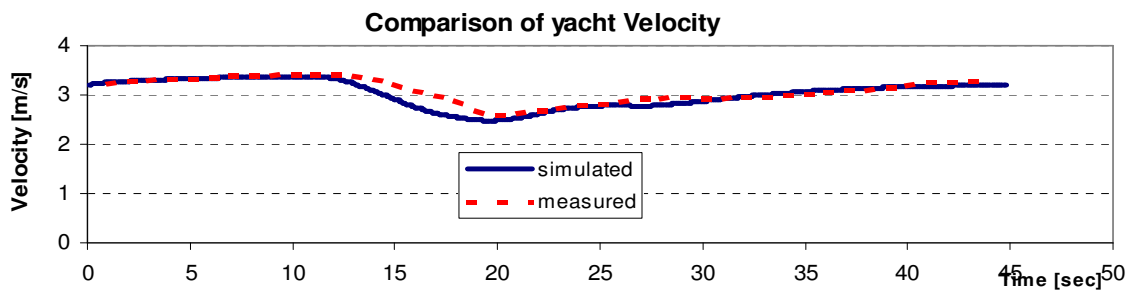
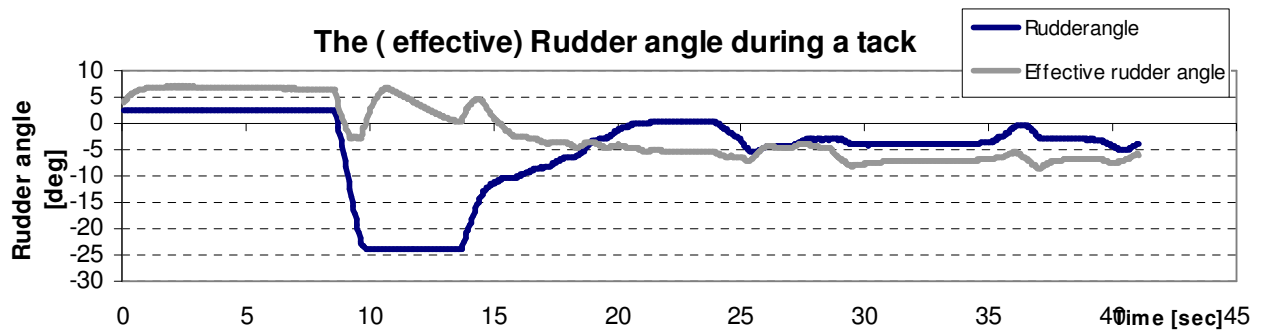


Figure 10: time histories

As may be seen from this comparison in general the comparison is quite reasonable. Note worthy however is that the simulation of the Bashford 41 track predicts a more close-hauled course than was recorded during the measurements. This will most likely be caused by a lack of sufficient windage for both the rig and the hull (superstructure) in the simulations.

8. Conclusions and Future Developments.

From the results obtained in the present study it may be concluded that a reasonably reliable tool for the prediction and simulation of the tacking maneuver and the associated speed loss of a large variety of sailing yachts is available now. The results obtained indicate that the mutual differences between various designs can be predicted with an acceptable degree of accuracy. Since only calculated coefficients and/or hydrodynamic derivatives are used the model is easily applicable and no dedicated experiments are required.

An important shortcoming of the model at this moment is the lack of a more refined aerodynamic model, which is capable of taking into account the differences between a larger variety of sail plans. This is one of the extensions of the model foreseen in the near future.

The model could be used for handicapping purposes if required. To show already one of the possible results of such a comparison simulations have been made for a number of quite different boats and their speed loss during a tacking maneuver assessed.

The main dimensions of the boats used for this comparison are presented in the Table below:

Table 10 main particulars

		Fair V 1/2 tonner	Checkmate3 Sydney 41	Swan 48 Design	Staron Huisman 42	De Ridder Design	Aldebaran J-35
Lwl	[m]	8,55	11,50	9,24	8,70	9,73	9,47
Bwl	[m]	2,42	3,14	3,17	2,90	3,10	2,85
Displ	[m ³]	3,69	7,77	9,01	7,00	6,31	5,37
SA	[m ²]	56,34	89,00	64,53	66,50	70,00	63,90
SA/Displ ^{2/3}	[-]	23,59	22,68	14,90	18,17	20,50	20,84

The comparison was made for one single tack in 10 knots of true wind. The rudder input signal was manipulated by hand to find the smallest speed loss during the procedure. This resulted among other things in a considerable wider turn for the heavy boat when compared to the light ones.

The results of the time loss calculations is presented in the table below:

Table 11 Time loss

Time loss in seconds for a tack in 10 knots of breeze							
		Fair V	Checkmate3	Swan 48	Staron	De Ridder Design	J-35
DMG loss	[m]	8,9	10,4	14,1	11,4	11,6	10,9
DMG total	[m]	113,5	152,2	106,7	112,5	131,7	153,3
DMGconst	[m/sec]	2,0	2,7	2,0	2,1	2,4	2,4
Seconds loss	[sec]	4,3	3,8	7,0	5,5	4,9	4,5

From these results it is possible to derive some more general information such as the dependency of the speed loss on design parameters, i.e.: the relation between the average speed loss and the sail area displacement ratio of the yachts under consideration.

Such an analysis becomes feasible with this tool and may result in more insight in these phenomena amongst the designers and the users. This kind of relationships could also be useful for race organisers and racing rule makers or “handicappers”.

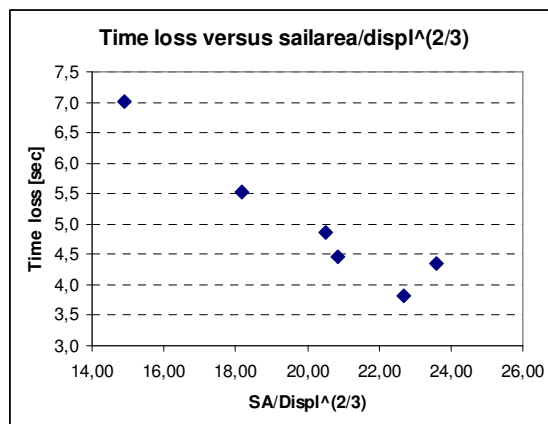


Figure 11 Time loss vs form and sail parameters

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