DIS12

2 Rayleigh Quotients

Part (a)

See notes.

Part (b)

We may reformulate the optimization problem

$$\min_{w:||w||_2=1} ||Xw||_2^2 = \min_w \frac{w^\intercal X^\intercal Xw}{w^\intercal w} = \min_w R(X^\intercal X, w)$$

.
Similar for the maximum case.

Part(c)

Let $F(\lambda) = ||Ax - \lambda x||^2$. Then $\frac{dF}{d\lambda} = -2x^\intercal Ax + 2\lambda x^\intercal x$. Set the derivative to 0 gives $\lambda = R(A, x)$.
Thus the rayleigh quotient gives an interpretation as the closest equivalent to an eigenvalue for an arbitrary vector.

3 Derivation of PCA

See notes.