Lect 4 Eigenvalues & Eigenvectors

A $n \times n$

$Ax_i = \lambda_i x_i \quad i = 1, 2, \cdots, n$

$A^k x = \lambda^k x$

$A^{-1} x = \frac{1}{\lambda} x$

Any vector $v = c_1 x_1 + \cdots + c_n x_n$

$A^k v = c_1 \lambda_1^k x_1 + \cdots + c_n \lambda_n^k x_n$

B similar to A

$B = M^{-1} A M$

Similar matrices — same eigenvalues

$\boxed{M^{-1} A M y = \lambda y}$

$A(My) = \lambda(My)$

AB same nonzero eigenvalues as BA

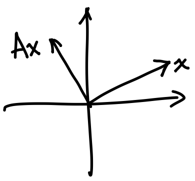
Want: $M(AB)M^{-1} = BA$

Take $M = B$

Eigenvalues of $A + B \neq$ eigenvalues of A + eigenvalues of B

$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$\theta = \frac{3}{2}\pi$



$(A - \lambda I)x = 0$

$\det(A - \lambda I) = 0$

$A - \lambda I = \begin{bmatrix} -\lambda & 1 \\ -1 & -\lambda \end{bmatrix} \qquad \det(A - \lambda I) = \lambda^2 + 1 = 0$

$\lambda = i, -i$

two checks

$\boxed{\text{add } \lambda\text{'s} = \text{add diagnal of } A}$ = trace of A

$\boxed{\text{multiply } \lambda\text{'s} = \det A}$

Symmetric

eigenvalues real eigenvectors orthognal

$S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$\lambda = 1, -1 \qquad x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\boxed{M^{-1} S M = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}}_{\Lambda}}$ similar

$SM = M \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$S \begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} Sx_1 & Sx_2 \end{bmatrix} = \begin{bmatrix} x_1 & -x_2 \end{bmatrix}$

$A \quad \lambda_1, \cdots, \lambda_n \quad x_1, \cdots, x_n$

$A \begin{bmatrix} x_1 \cdots x_n \end{bmatrix} = \begin{bmatrix} x_1, \cdots, x_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$

$AX = X\Lambda ; \quad A = X \Lambda X^{-1}$

$A^2 = X \Lambda X^{-1} X \Lambda X^{-1} = X \Lambda^2 X^{-1}$

$S = Q \Lambda Q^{-1} = Q \Lambda Q^T$ spectral theorem

$\underbrace{Q^{-1} = Q^T}_{\text{orthogonal}}$