Homework 1
Fundamental Algorithms, Fall 2020, Professor Yap

Due: Mon Sep 28 (Upload to GradeScope by 11:30pm)

INSTRUCTIONS:

- We have a "no late homework" policy.
  Special permission must be obtained in advance if you have a valid reason.
- The exercise numbers in this homework comes from Chapters I and II.
- If you are not familiar with Gradescope, please try to practice uploading your solutions a day in advance.

---

1. (10 Points) Exercise I.4.5, bounds on $S(1000)$. Page 20. HINT: Stirling's formula may be useful.
2. (10 Points)
   (a) Exercise I.3.9 (Drawing the comparison tree $T_{2,4}$). Page 14.
   (b) Please draw the Hasse diagram of the partial order associated with each node of the tree in Part (a).
3. (15 Points) Exercise I.3.10 (Sorting in the Tape Model). Page 14. Read the §I.11 where we described a Tape Model algorithm for merging two sorted lists. Write your algorithm using "pseudo-code" in the same style as our Tape Merge Algorithm.
4. (4 Points) Our asymptotic notations falls under two groups: O, Ω, Θ and o, ω. In the first group, we have $\Theta(f) = O(f) \cap \Omega(f)$. This suggests the "small-theta" analogue for the second group, "$\theta(f) = o(f) \cap \omega(f)$". Why was this not done?
5. (8 Points) Either prove the CLAIM or give a counter example.
   (i) We say f is **unbounded** if
   $$(\forall C > 0)(\exists x)[f(x) > C].$$
   CLAIM: f is unbounded is the same as saying "$f \succ\!\!\succ 1$".
   (ii) We say $f > g$ (i.o.) if
   $$(\forall x_0)(\exists x)[x > x_0 \wedge f(x) > g(x)].$$
   Read i.o. as "infinitely often".
   CLAIM: "$(\forall C > 0)[f > g$ (i.o.)$]$" is the same as "$f \not\preceq g$".
   Note that $\not\preceq$ is the negation of the relation $\preceq$.
6. (10 Points) Do Exercise I.7.1 (parts (A) and (B). Page 34.
7. (10 Points) Do Exercise I.7.12, Parts (a)-(h). Page 36.
8. (8 Points) Exercise II.3.4, Karatsuba recurrence. Page 16.
9. (9 Points) Exercise II.6.3, Growth types. Page 42.
10. (5 Points) Exercise II.6.5, complexity of an algorithm on binary trees. Page 42.
11. (4 Points) Exercise II.9.1, comparing T_1, T_2. Page 52.
12. (10 Points) Exercise II.9.2, Joe, Jane and John. Page 52. Show your reasoning!
13. (10 Points) Exercise II.9.4 (a)-(d), Master Theorem. Page 52. Please state whice CASE of the Master Theorem is applicable, and justify it.