Lect 2 Multiplying and Factoring Matrices

$$\begin{cases} A = LU & \text{elimination (Lower triangular)} \times \text{(Upper triangular)} \\ A = QR \longrightarrow & \text{Gram-Schmidt} \\ S = Q\Lambda Q^T & \\ A = X\Lambda X^{-1} & \\ A = U\Sigma V^T = (\text{orth})(\text{diag})(\text{orth}) = SVD & \end{cases}$$

singular value decomposition

$$S = \begin{bmatrix} | & & | \\ q_1 & \cdots & q_n \\ | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \begin{bmatrix} - q_1^T - \\ \vdots \\ - q_n^T - \end{bmatrix}$$

↳ orthonormal eigenvectors (n or them)

eigenvalues are all real

$$(Q\Lambda)Q^T = \boxed{\text{Sum of rank 1}} = \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \cdots + \lambda_n q_n q_n^T = S$$

(cols of $Q\Lambda$) × (rows of Q^T)

↓

$$Q\Lambda = \begin{bmatrix} q_1 \cdots q_n \end{bmatrix} \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

Look at $\boxed{Sq_1 = \lambda_1 q_1} \underbrace{(q_1^T q_1)}_{=1 = \|q_1\|} + \overbrace{\boxed{\lambda_2 q_2 \underbrace{q_2^T q_1}_{0}}}^{0}$

$$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \longrightarrow \begin{bmatrix} \underline{2} & 3 \\ 0 & \underline{1} \end{bmatrix}$$

pivots on the diagonal

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$$

multiplier

$$\begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}}_{\text{rank } 1} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}}_{\text{rank } 1}$$

$$= \begin{bmatrix} | \\ l_1 \\ | \end{bmatrix} \begin{bmatrix} - u_1^T - \end{bmatrix} + \begin{bmatrix} | \\ l_2 \\ | \end{bmatrix} \begin{bmatrix} - u_2^T - \end{bmatrix}$$

$$= (\text{col } 1)(\text{row } 1) + \begin{bmatrix} 0 & - & 0 \\ | & & \\ 0 & & A_2 \end{bmatrix}$$

4 Fundamental Subspaces $A_{m\times n}$ of rank r

Column space $C(A)$ dim $= r$

row space $C(A^T)$ dim $= r$

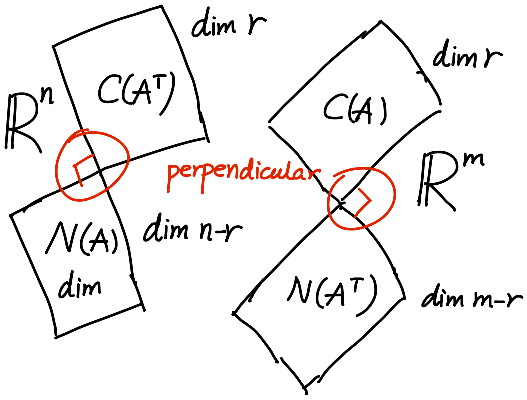
nullspace $N(A)$ dim $= n-r$

nullspace $N(A^T)$

nullspace = all solutions to $Ax = 0$

$A(x+y) = 0$ closed under addition

$A(cx) = 0$



$m \times n$

2×3

$$\begin{bmatrix} - & - & - \\ - & - & - \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} \text{—} \\ \text{—} \\ \text{—} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

dim r $n-r$ r $m-r$

row null sp

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix} \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} = 0$$

$m=2$ $n=3$ $r=1$

$n-r=2$

$$Ax = \begin{bmatrix} - \\ - \\ - \\ - \end{bmatrix} \begin{bmatrix} | & | \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$