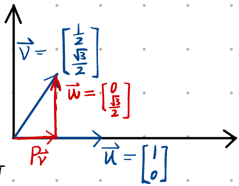
I.5 Q2

$\vec{w} = \vec{v} - \vec{u}(\vec{u}^T\vec{v})$

$= \vec{v} - (\vec{u}\vec{u}^T)v$



Let $P = u\vec{u}$, then we have $P^2 = u\vec{u}u\vec{u} = u\vec{u} = P = P^T$

Thus, Pv is the orthogonal projection of $\vec{v}$ onto the column space of P.

$\vec{u}^T\vec{w} = \vec{u}^T\vec{v} - \vec{u}^T\vec{u}\vec{u}^T\vec{v} = 0$

I.5 Q4

$(Qx)^T(Qy) = x^T(Q^TQ)y = x^Ty = \|x\|\|y\|\cos\theta$ (same θ)

Angles are preserved when all vectors are multiplied by Q.

I.5 Q6

$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$ has orthonormal columns

so $P^TP = P^{-1}P = I$ and $P^{-1} = P^T = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Every permutation matrix has unit vectors in its columns.

Those columns are orthogonal because their 1's are in different positions.

I.5 Q7

$PF = \begin{bmatrix} 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \\ 1 & 1 & 1 & 1 \end{bmatrix} = F \begin{bmatrix} 1 & & & \\ & i & & \\ & & i^2 & \\ & & & i^3 \end{bmatrix}$

$\bar{Q}^TQ = I$ For example $\bar{q}_2^T q_3 = \frac{1}{4}[1 \ -i, \ -1 \ i] \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = 0$

$\|q_2\| = \|q_3\| = 1$