Lec1 The Column Space of A Contains All Vectors Ax

$$\underbrace{\begin{bmatrix}2&1&3\\3&1&4\\5&7&12\end{bmatrix}}_{A}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=x_1\begin{bmatrix}2\\3\\5\end{bmatrix}+x_2\begin{bmatrix}1\\1\\7\end{bmatrix}+x_3\begin{bmatrix}3\\4\\12\end{bmatrix}$$

① dot products (row) x

all Ax = column space = $C(A)$ all combinations of the cols

$$A=\begin{bmatrix}1&3&8\\1&3&8\\1&3&8\end{bmatrix}=\underbrace{\begin{bmatrix}1\\1\\1\end{bmatrix}}_{3\times1}\underbrace{\begin{bmatrix}1&3&8\end{bmatrix}}_{1\times3}=\underbrace{uv^T}_{3\times3}$$

$C(A)$ = line

rank$(A)=1$ = # independent columns

$$\underbrace{\begin{bmatrix}2&1&3\\3&1&4\\5&7&12\end{bmatrix}}_{A}=\underbrace{\begin{bmatrix}2&1\\3&1\\5&7\end{bmatrix}}_{C}\underbrace{\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}}_{R_{2\times3}}$$

← basis for the row space

$\boxed{\text{column rank } r=2 \quad \text{row rank}}$

row space of A

$\equiv$ column space of A^T: $C(A^T)$

Two ways to see matrix multiplication

$$\begin{bmatrix}2&1&3\\3&1&4\\5&7&12\end{bmatrix}=\begin{bmatrix}2&1\\3&1\\5&7\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}$$

Combinations of C to give cols.

$$\begin{bmatrix}2&1&3\\3&1&4\\5&7&12\end{bmatrix}=\begin{bmatrix}2&1\\3&1\\5&7\end{bmatrix}\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}$$

Combinations of R to give rows.

key idea: $A=CR$ factorization

C → real cols in A

R → row reduced echelon form

Note $ABCx$ is also in $C(A)$

because $A(BCx)$

AB

$$\underset{A}{\begin{bmatrix}\text{——} \\ \text{row}\end{bmatrix}}\underset{B}{\begin{bmatrix}| \\ \text{col}\end{bmatrix}}=\text{dot product } r\cdot c$$

Combination of columns × rows

$$AB=\underbrace{\begin{bmatrix}\text{col } k\end{bmatrix}}_{m\times n}\underbrace{\begin{bmatrix}\text{row } k\end{bmatrix}}_{n\times p}=\text{Sum of } \underset{A}{(\text{col } k)}\underset{B}{(\text{row } k)}$$

(outer product)

$k=1,2,\cdots,n$

Old way:

$(m\times n)(n\times p)=\underbrace{(m\times p)}_{\#\text{ inner products}}\cdot n$ # muls per dot product

New way: $\underbrace{n}_{\#\text{ outer products}}\cdot(m\times p)$ # muls per inner product