# Lect 5 Positive Definite & Semidefinite Matrices

Symmetric Positive Defintte $S$

1. All $\lambda_i > 0$
2. Energy $x^T S x > 0$ (all $x \neq 0$)
3. $S = A^T A$ (independent cols in $A$)
4. All leading determinants $> 0$
5. All pivots in elimination $> 0$

$$S = \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} \longrightarrow \begin{bmatrix} 3 & 4 \\ 0 & \underset{-\frac{1}{3}}{\underline{5 - 4 \times \frac{4}{3}}} \end{bmatrix}$$

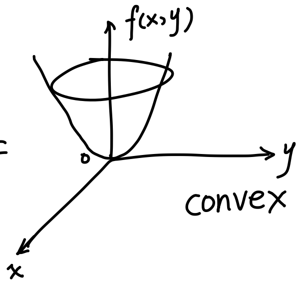
pivot $3, -\frac{1}{3}$

$$[x \ y] \begin{bmatrix} 3 & 4 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = f(x,y)$$

$$= [x \ y] \begin{bmatrix} 3x + 4y \\ 4x + 6y \end{bmatrix}$$

$$= 3x^2 + 6y^2 + \cancel{4xy + 4xy} \ 8xy$$

BOWL




convex

$S, T$ PD $\quad S + T$

$x^T (S+T) x = x^T S x + x^T T x$ ✓

$S$ PD $\quad S^{-1}$ has eigenvalues $\frac{1}{\lambda}$ ✓ YES PD

$Q^T S Q = \underline{Q^{-1} S Q}$ same eigenvalue

$$\underbrace{x^T Q^T}_{y^T} S \underbrace{Q x}_{y} = y^T S y > 0$$

# Semi-definite

$$\begin{bmatrix} 3 & 4 \\ 4 & \frac{16}{3} \end{bmatrix}$$

$\lambda_i \geq 0$

PSD

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$ PSD

eigenvalues $3, 0, 0$

$$= \lambda_1 q_1 q_1^T + \lambda_2 q_2 q_2^T + \lambda_3 q_3 q_3^T$$

$$= Q \Lambda Q^T$$

$$= 3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Big/ \sqrt{3} \ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \Big/ \sqrt{3}$$