I.1 Q1

$$A = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

A: 4×3 x: 3×1 $\vec{0}$: 3×1

I.1 Q4

$$\vec{x} = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix} \quad \vec{y} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

There is no 3rd $\vec{z} \in \mathbb{R}^3$ with $Az=0$ and $\vec{x}, \vec{y}, \vec{z}$ are independent. If there were, then combinations of $\vec{x}, \vec{y}, \vec{z}$ would induce that $\forall \vec{w} \in \mathbb{R}^3$ solves $Aw=0$, contradiction!

I.1 Q9

If $C(A) = \mathbb{R}^3$, then $r=3$, $m \geq 3$, $n \geq 3$

I.1 Q18

If $A=CR$, then $\begin{bmatrix} 0 & A \\ 0 & A \end{bmatrix}_{2m\times(n+1)} = \begin{bmatrix} C \\ C \end{bmatrix}_{2m\times r} \begin{bmatrix} 0 & R \end{bmatrix}_{r\times(n+1)}$

$A_{m\times n}$ $C_{m\times r}$ $R_{r\times n}$