I.8 Q6

By equation (12), we have $A^T = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix} = \frac{1}{\sqrt{10}}\begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{45} & \\ & \sqrt{5} \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$A^T = U\Sigma V^T \Rightarrow A = V\Sigma U^T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} \sqrt{45} & \\ & \sqrt{5} \end{bmatrix}\frac{1}{\sqrt{10}}\begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix}$

I.8 Q7

$\|A - \sigma_1 u_1 v_1^T\| = \sigma_2$

singular values $\sigma_2, \sigma_3, \cdots, \sigma_r$

rank $r-1$

I.8 Q8

$A = \begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$ $A^TA = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$\lambda_1 = 0$ $\lambda_2 = 4$ $\lambda_3 = 9$

$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\sigma_1 = 3$ $\sigma_2 = 2$ $\sigma_3 = 0$

$V = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$u_i = \frac{Av_i}{\sigma_i}$

$u_1 = \frac{Av_1}{\sigma_1} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$u_2 = \frac{Av_2}{\sigma_2} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$A^Tu_3 = 0 \Rightarrow u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$U = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & & \\ & 2 & \\ & & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$