Knowledge Tracing Machines: Families of models for predicting student performance

Jill-Jênn Vie
RIKEN Center for Advanced Intelligence Project, Tokyo

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Predicting student performance

Data

A population of students answering questions

- Events: "Student i answered question j correctly/incorrectly"

Goal

- Learn the difficulty of questions automatically from data
- Measure the knowledge of students
- Potentially optimize their learning

Assumption

Good model for prediction $\rightarrow$ Good adaptive policy for teaching

Learning outcomes of this tutorial

- Logistic regression is amazing
  - Unidimensional
  - Takes IRT, PFA as special cases
- Factorization machines are even more amazing
  - Multidimensional
  - Take MIRT as special case
- It makes sense to consider deep neural networks
  - What does deep knowledge tracing model exactly?

Families of models

- Factorization Machines (Rendle 2012)
  - Multidimensional Item Response Theory
  - Logistic Regression
    - Item Response Theory
    - Performance Factor Analysis
- Recurrent Neural Networks
  - Deep Knowledge Tracing (Piech et al. 2015)

Steffen Rendle (2012). "Factorization Machines with libFM". In: *ACM Transactions on Intelligent Systems and Technology (TIST)* 3.3, 57:1–57:22. DOI: 10.1145/2168752.2168771

Chris Piech et al. (2015). "Deep knowledge tracing". In: *Advances in Neural Information Processing Systems (NIPS)*, pp. 505–513

Problems

Weak generalization

Filling the blanks: some students did not attempt all questions

Strong generalization

Cold-start: some new students are not in the train set

Dummy dataset

- User 1 answered Item 1 correct
- User 1 answered Item 2 incorrect
- User 2 answered Item 1 incorrect
- User 2 answered Item 1 correct
- User 2 answered Item 2 ???

user	item	correct
1	1	1
1	2	0
2	1	0
2	1	1
2	2	0

`dummy.csv`

Task 1: Item Response Theory

Learn abilities θ_i for each user i
Learn easiness e_j for each item j such that:

$$Pr(\text{User } i \text{ Item } j \text{ OK}) = \sigma(\theta_i + e_j)$$
$$\text{logit } Pr(\text{User } i \text{ Item } j \text{ OK}) = \theta_i + e_j$$

Logistic regression

Learn $\boldsymbol{w}$ such that $\text{logit } Pr(\boldsymbol{x}) = \langle \boldsymbol{w}, \boldsymbol{x} \rangle$
Usually with L2 regularization: $||\boldsymbol{w}||_2^2$ penalty $\leftrightarrow$ Gaussian prior

Graphically: IRT as logistic regression

Encoding of “User i answered Item j”:

$\boldsymbol{w}$	$\boxed{\theta_i}$	$\boxed{e_j}$
	U_i	I_j
$\boldsymbol{x}$	1	1
	Users	Items

$$\text{logit}\, Pr(\text{User } i \text{ Item } j \text{ OK}) = \langle \boldsymbol{w}, \boldsymbol{x} \rangle = \theta_i + e_j$$

Encoding

`python encode.py --users --items`

Users			Items		
U_0	U_1	U_2	I_0	I_1	I_2
0	1	0	0	1	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	1	0	1	0
0	0	1	0	0	1

`data/dummy/X-ui.npz`

Then logistic regression can be run on the sparse features:

`python lr.py data/dummy/X-ui.npz`

Oh, there's a problem

```
python encode.py --users --items
```

```
python lr.py data/dummy/X-ui.npz
```

	Users			Items				
	U_0	U_1	U_2	I_0	I_1	I_2	$\boldsymbol{y}_{\text{pred}}$	$\boldsymbol{y}$
User 1 Item 1 OK	0	1	0	0	1	0	0.575135	1
User 1 Item 2 NOK	0	1	0	0	0	1	0.395036	0
User 2 Item 1 NOK	0	0	1	0	1	0	0.545417	0
User 2 Item 1 OK	0	0	1	0	1	0	0.545417	1
User 2 Item 2 NOK	0	0	1	0	0	1	0.366595	0

We predict the same thing when there are several attempts.

Count successes and failures

Keep track of what the student has done before:

user	item	skill	correct	wins	fails
1	1	1	1	0	0
1	2	2	0	0	0
2	1	1	0	0	0
2	1	1	1	0	1
2	2	2	0	0	0

`data/dummy/data.csv`

Task 2: Performance Factor Analysis

W_{ik}: how many successes of user i over skill k (F_{ik}: #failures)

Learn β_k, γ_k, δ_k for each skill k such that:

$$\text{logit } Pr(\text{User } i \text{ Item } j \text{ OK}) = \sum_{\text{Skill } k \text{ of Item } j} \beta_k + W_{ik}\gamma_k + F_{ik}\delta_k$$

```
python encode.py --skills --wins --fails
```

Skills			Wins			Fails		
S_0	S_1	S_2	S_0	S_1	S_2	S_0	S_1	S_2
0	1	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0

`data/dummy/X-swf.npz`

Better!

```
python encode.py --skills --wins --fails
```

```
python lr.py data/dummy/X-swf.npz
```

	Skills			Wins			Fails				
	S_0	S_1	S_2	S_0	S_1	S_2	S_0	S_1	S_2	y_{pred}	y
User 1 Item 1 OK	0	1	0	0	0	0	0	0	0	0.544	1
User 1 Item 2 NOK	0	0	1	0	0	0	0	0	0	0.381	0
User 2 Item 1 NOK	0	1	0	0	0	0	0	0	0	0.544	0
User 2 Item 1 OK	0	1	0	0	0	0	0	1	0	0.633	1
User 2 Item 2 NOK	0	0	1	0	0	0	0	0	0	0.381	0

Task 3: a new model (but still logistic regression)

```
python encode.py --items --skills --wins --fails

python lr.py data/dummy/X-iswf.npz
```

Here comes a new challenger

How to model side information in, say, recommender systems?

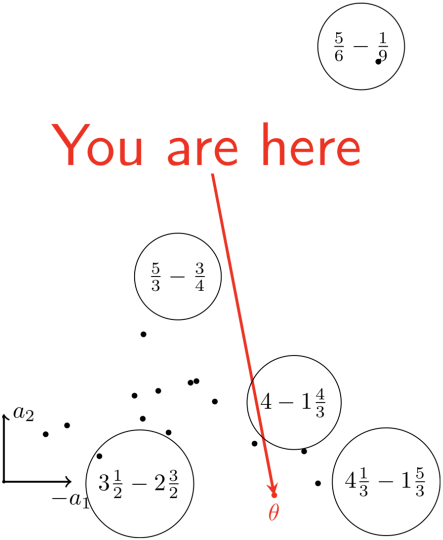
Logistic Regression

Learn a bias for each feature (each user, item, etc.)

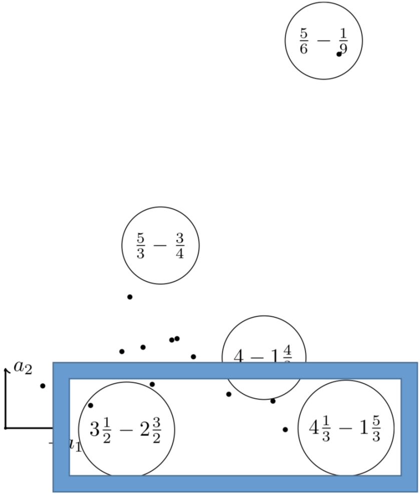
Factorization Machines

Learn a bias and an embedding for each feature

What can be done with embeddings?



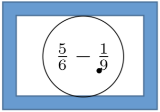
Interpreting the components



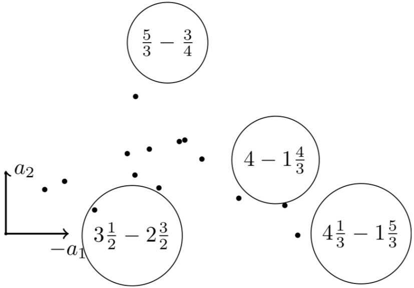
Items that discriminate only over one dimension

$3\frac{1}{2} - 2\frac{3}{2}$	$4\frac{1}{3} - 2\frac{4}{3}$	$4\frac{1}{3} - 1\frac{5}{3}$
$b = 0.13$	$b = -0.46$	$b = -1.99$
$-a_1 = 2.01$	$-a_1 = 4.65$	$-a_1 = 5.66$
$a_2 = -0.03$	$a_2 = -0.02$	$a_2 = 0.00$

Interpreting the components



Items that highly discriminate over both dimensions



$\frac{3}{4} - \frac{3}{8}$

$b = 1.09$

$-a_1 = 5.54$

$a_2 = 6.22$

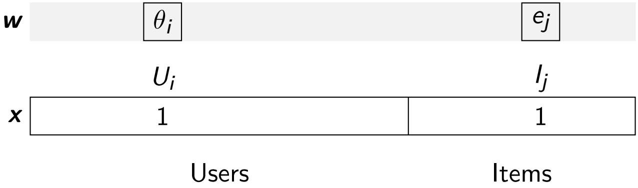
$\frac{5}{6} - \frac{1}{9}$

$b = -0.28$

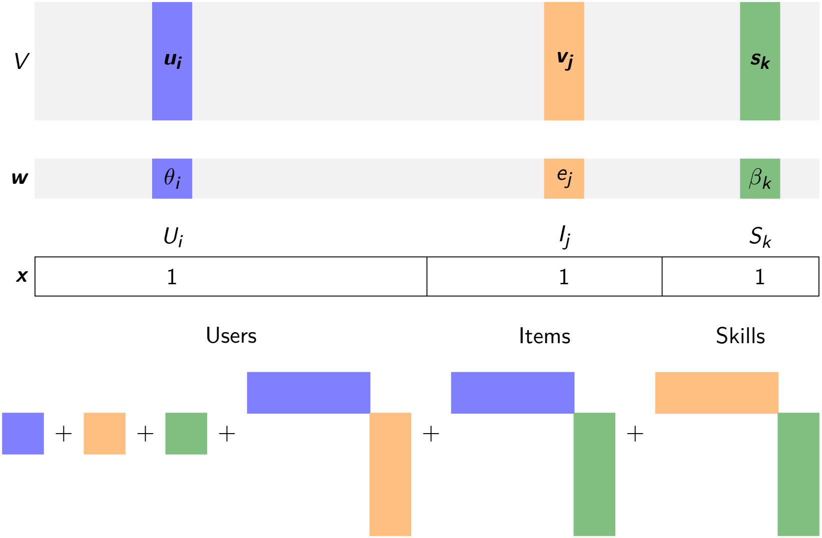
$-a_1 = 5.29$

$a_2 = 6.44$

Graphically: logistic regression



Graphically: factorization machines



Formally: factorization machines

Learn bias w_k and embedding $\boldsymbol{v_k}$ for each feature k such that:

$$\operatorname{logit} p(\boldsymbol{x}) = \mu + \underbrace{\sum_{k=1}^{N} w_k x_k}_{\text{logistic regression}} + \underbrace{\sum_{1 \leq k < l \leq N} x_k x_l \langle \boldsymbol{v_k}, \boldsymbol{v_l} \rangle}_{\text{pairwise interactions}}$$

Particular cases

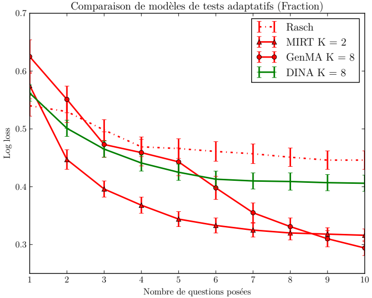
- Multidimensional item response theory: $\operatorname{logit} p = \langle \boldsymbol{u_i}, \boldsymbol{v_j} \rangle + e_j$
- SPARFA: $\boldsymbol{v_j} > \mathbf{0}$ and $\boldsymbol{v_j}$ sparse
- GenMA: $\boldsymbol{v_j}$ is constrained by the zeroes of a q-matrix $(q_{ij})_{i,j}$

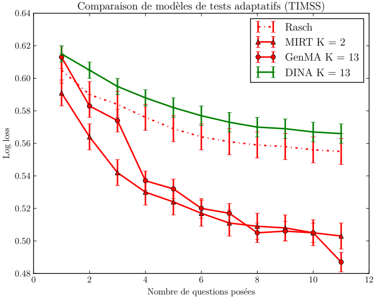
Andrew S Lan, Andrew E Waters, Christoph Studer, and Richard G Baraniuk (2014). "Sparse factor analysis for learning and content analytics". In: *The Journal of Machine Learning Research* 15.1, pp. 1959–2008

Jill-Jênn Vie, Fabrice Popineau, Yolaine Bourda, and Éric Bruillard (2016). "Adaptive Testing Using a General Diagnostic Model". In: *European Conference on Technology Enhanced Learning*. Springer, pp. 331–339

Tradeoff expressiveness/interpretability

NLL	logit p	4 q	7 q	10 q
Rasch	$\theta_i + e_j$	0.469 (79%)	0.457 (79%)	0.446 (79%)
DINA	$1 - s_j$ or g_j	0.441 (80%)	0.410 (82%)	0.406 (82%)
MIRT	$\langle \boldsymbol{u_i}, \boldsymbol{v_j} \rangle + e_j$	0.368 (83%)	0.325 (86%)	0.316 (86%)
GenMA	$\langle \boldsymbol{u_i}, \boldsymbol{\tilde{q}_j} \rangle + e_j$	0.459 (79%)	0.355 (85%)	0.294 (88%)





Assistments 2009 dataset

278608 attempts of 4163 students over 196457 items on 124 skills.

- Download `http://jiji.cat/weasel2018/data.csv`
- Put it in `data/assistments09`

`python fm.py data/assistments09/X-ui.npz`
etc. or `make big`

AUC	users + items	skills + w + f	items + skills + w + f
LR	0.734 (IRT) 2s	0.651 (PFA) 9s	0.737 23s
FM	0.730 2min9s	0.652 43s	0.739 2min30s

Results obtained with FM $d = 20$

Deep Factorization Machines

Learn layers $W^{(\ell)}$ and $b^{(\ell)}$ such that:

$$\begin{aligned}
\boldsymbol{a}^0(\boldsymbol{x}) &= (\boldsymbol{v}_{\texttt{user}}, \boldsymbol{v}_{\texttt{item}}, \boldsymbol{v}_{\texttt{skill}}, \ldots) \\
\boldsymbol{a}^{(\ell+1)}(\boldsymbol{x}) &= \mathrm{ReLU}(W^{(\ell)} \boldsymbol{a}^{(\ell)}(\boldsymbol{x}) + \boldsymbol{b}^{(\ell)}) \quad \ell = 0, \ldots, L-1 \\
y_{DNN}(\boldsymbol{x}) &= \mathrm{ReLU}(W^{(L)} \boldsymbol{a}^{(L)}(\boldsymbol{x}) + \boldsymbol{b}^{(L)})
\end{aligned}$$

$$\operatorname{logit} p(\boldsymbol{x}) = y_{FM}(\boldsymbol{x}) + y_{DNN}(\boldsymbol{x})$$

Jill-Jênn Vie (2018). "Deep Factorization Machines for Knowledge Tracing". In: *The 13th Workshop on Innovative Use of NLP for Building Educational Applications*. URL: https://arxiv.org/abs/1805.00356

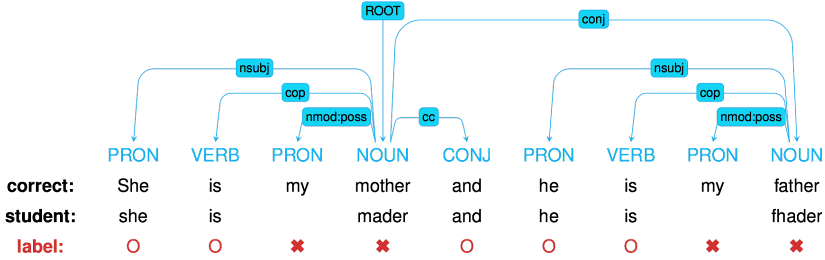
Comparison

- FM: y_{FM} factorization machine with $\lambda = 0.01$
- Deep: y_{DNN}: multilayer perceptron
- DeepFM: $y_{DNN} + y_{FM}$ with shared embedding
- Bayesian FM: $w_k, v_{kf} \sim \mathcal{N}(\mu_f, 1/\lambda_f)$
  $\mu_f \sim \mathcal{N}(0, 1)$, $\lambda_f \sim \Gamma(1, 1)$ (trained using Gibbs sampling)

Various types of side information

- first: `<discrete>` (`user`, `token`, `countries`, etc.)
- last: `<discrete>` + `<continuous>` (`time` + `days`)
- pfa: `<discrete>` + `wins` + `fails`

Duolingo dataset




```
# user:D2inSf5+  countries:MX  days:1.793  client:web  session:lesson  format:reverse_translate  time:16
8rgJEAPw1001  She      PRON   Case=Nom|Gender=Fem|Number=Sing|Person=3|PronType=Prs|fPOS=PRON++PRP    nsubj     4  0
8rgJEAPw1002  is       VERB   Mood=Ind|Number=Sing|Person=3|Tense=Pres|VerbForm=Fin|fPOS=VERB++VBZ    cop       4  0
8rgJEAPw1003  my       PRON   Number=Sing|Person=1|Poss=Yes|PronType=Prs|fPOS=PRON++PRP$              nmod:poss 4  1
8rgJEAPw1004  mother   NOUN   Degree=Pos|fPOS=ADJ++JJ                                                 ROOT      0  1
8rgJEAPw1005  and      CONJ   fPOS=CONJ++CC                                                           cc        4  0
8rgJEAPw1006  he       PRON   Case=Nom|Gender=Masc|Number=Sing|Person=3|PronType=Prs|fPOS=PRON++PRP   nsubj     9  0
8rgJEAPw1007  is       VERB   Mood=Ind|Number=Sing|Person=3|Tense=Pres|VerbForm=Fin|fPOS=VERB++VBZ    cop       9  0
8rgJEAPw1008  my       PRON   Number=Sing|Person=1|Poss=Yes|PronType=Prs|fPOS=PRON++PRP$              nmod:poss 9  1
8rgJEAPw1009  father   NOUN   Number=Sing|fPOS=NOUN++NN                                               conj      4  1

# user:D2inSf5+  countries:MX  days:2.689  client:web  session:practice  format:reverse_translate  time:6
oMGsnnH/0101  When     ADV    PronType=Int|fPOS=ADV++WRB                                              advmod    4  1
oMGsnnH/0102  can      AUX    VerbForm=Fin|fPOS=AUX++MD                                               aux       4  0
oMGsnnH/0103  I        PRON   Case=Nom|Number=Sing|Person=1|PronType=Prs|fPOS=PRON++PRP               nsubj     4  1
oMGsnnH/0104  help     VERB   VerbForm=Inf|fPOS=VERB++VB                                              ROOT      0  0
```

Results

Model	d	epoch	train	first	last	pfa
Bayesian FM	20	500/500	–	0.822	–	–
Bayesian FM	20	500/500	–	–	0.817	–
DeepFM	20	15/1000	0.872	0.814	–	–
Bayesian FM	20	100/100	–	–	0.813	–
FM	20	20/1000	0.874	0.811	–	–
Bayesian FM	20	500/500	–	–	–	0.806
FM	20	21/1000	0.884	–	–	0.805
FM	20	37/1000	0.885	–	0.8	–
DeepFM	20	77/1000	0.89	–	0.792	–
Deep	20	7/1000	0.826	0.791	–	–
Deep	20	321/1000	0.826	–	0.79	–
LR	0	50/50	–	–	–	0.789
LR	0	50/50	–	0.783	–	–
LR	0	50/50	–	–	0.783	–

Duolingo ranking

Rank	Team	Algo	AUC
1	SanaLabs	RNN + GBDT	.857
2	singsound	RNN	.854
2	NYU	GBDT	.854
4	CECL	LR + L1 (13M feat.)	.843
5	TMU	RNN	.839
7 (off)	JJV	Bayesian FM	.822
8 (off)	JJV	DeepFM	.814
10	JJV	DeepFM	.809
15	Duolingo	LR	.771

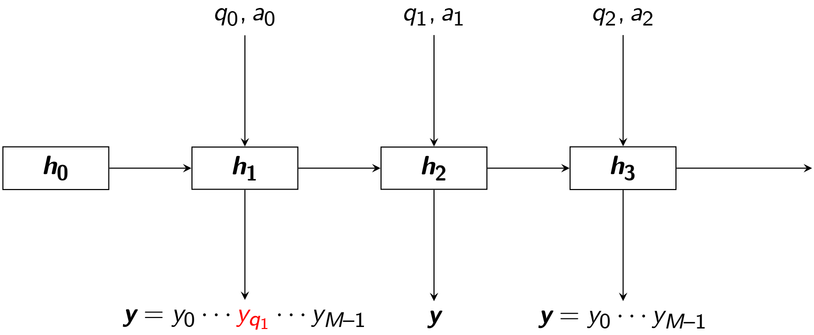
Burr Settles, Chris Brust, Erin Gustafson, Masato Hagiwara, and Nitin Madnani (2018). "Second language acquisition modeling". In: *Proceedings of the Thirteenth Workshop on Innovative Use of NLP for Building Educational Applications*, pp. 56–65. URL: http://sharedtask.duolingo.com

What 'bout recurrent neural networks?

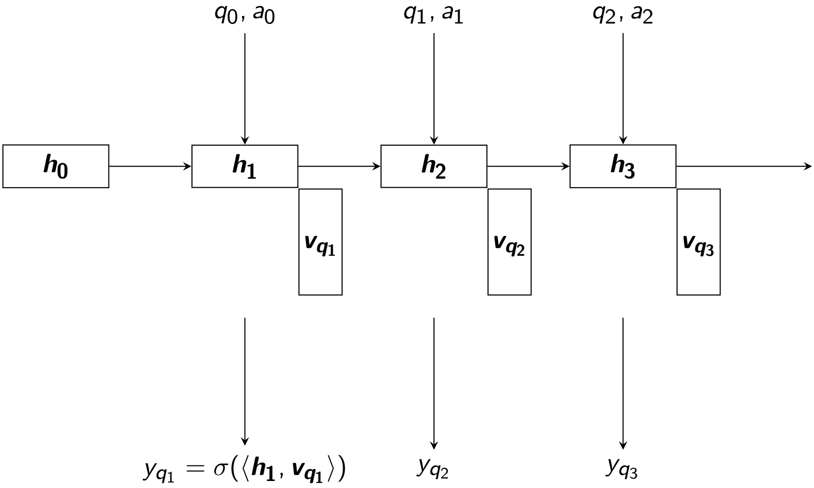
Deep Knowledge Tracing: model the problem as sequence prediction

- Each student on skill q_t has performance a_t
- How to predict outcomes $\boldsymbol{y}$ on every skill k?
- Spoiler: by measuring the evolution of a latent state $\boldsymbol{h_t}$

Graphically: deep knowledge tracing



Graphically: there is a MIRT in my DKT



Drawback of Deep Knowledge Tracing

DKT does not model individual differences.

Actually, Wilson even managed to beat DKT with (1-dim!) IRT.

By estimating on-the-fly the student's learning ability, we managed to get a better model.

AUC	BKT	IRT	PFA	DKT	DKT-DSC
Cognitive Tutor	0.61	0.81	0.76	0.79	0.81
Assistments 2009	0.67	0.75	0.70	0.73	0.92
Assistments 2012	0.61	0.74	0.67	0.72	0.80
Assistments 2014	0.64	0.67	0.69	0.72	0.86

Sein Minn, Yi Yu, Michel Desmarais, Feida Zhu, and Jill-Jênn Vie (2018). "Deep Knowledge Tracing and Dynamic Student Classification for Knowledge Tracing". In: *Proceedings of the 18th IEEE International Conference on Data Mining*, to appear. URL: https://arxiv.org/abs/1809.08713

Take home message

Factorization machines are a strong baseline that take many models as special cases

Recurrent neural networks are powerful because they track the evolution of the latent state (try simpler dynamic models?)

Deep factorization machines may require more data/tuning, but neural collaborative filtering offer promising directions

Any suggestions are welcome!

Feel free to chat:

`vie@jill-jenn.net`

All code:

`github.com/jilljenn/ktm`

Do you have any questions?

- Lan, Andrew S, Andrew E Waters, Christoph Studer, and Richard G Baraniuk (2014). "Sparse factor analysis for learning and content analytics". In: *The Journal of Machine Learning Research* 15.1, pp. 1959–2008.
- Minn, Sein, Yi Yu, Michel Desmarais, Feida Zhu, and Jill-Jênn Vie (2018). "Deep Knowledge Tracing and Dynamic Student Classification for Knowledge Tracing". In: *Proceedings of the 18th IEEE International Conference on Data Mining*, to appear. URL: https://arxiv.org/abs/1809.08713.
- Piech, Chris, Jonathan Bassen, Jonathan Huang, Surya Ganguli, Mehran Sahami, Leonidas J Guibas, and Jascha Sohl-Dickstein (2015). "Deep knowledge tracing". In: *Advances in Neural Information Processing Systems (NIPS)*, pp. 505–513.
- Rendle, Steffen (2012). "Factorization Machines with libFM". In: *ACM Transactions on Intelligent Systems and Technology (TIST)* 3.3, 57:1–57:22. DOI: 10.1145/2168752.2168771.

Settles, Burr, Chris Brust, Erin Gustafson, Masato Hagiwara, and Nitin Madnani (2018). "Second language acquisition modeling". In: *Proceedings of the Thirteenth Workshop on Innovative Use of NLP for Building Educational Applications*, pp. 56–65. URL: http://sharedtask.duolingo.com.

Vie, Jill-Jênn (2018). "Deep Factorization Machines for Knowledge Tracing". In: *The 13th Workshop on Innovative Use of NLP for Building Educational Applications*. URL: https://arxiv.org/abs/1805.00356.

Vie, Jill-Jênn, Fabrice Popineau, Yolaine Bourda, and Éric Bruillard (2016). "Adaptive Testing Using a General Diagnostic Model". In: *European Conference on Technology Enhanced Learning*. Springer, pp. 331–339.