DATE. NO.

오픈소스SW Polynomial Regression Assignment 20221396 이정민.

1-(a)

Size of vector $w = d+1$.

Size of vector $y = n$.

1-(b)

Size of matrix $A = n \times (d+1)$

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{d-1} & x_1^d \\ 1 & x_2 & x_2^2 & \cdots & x_2^{d-1} & x_2^d \\ & \vdots & & & \vdots & \\ 1 & x_n & x_n^2 & \cdots & x_n^{d-1} & x_n^d \end{bmatrix}$$

1-(c) $(d=n-1)$

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} & x_2^{n-1} \\ & \vdots & & & \vdots & \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-2} & x_n^{n-1} \end{bmatrix}$$ 라고 하고, $$V_n = \begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-2} & x_2^{n-1} \\ & \vdots & & & \vdots & \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-2} & x_n^{n-1} \end{vmatrix}$$ 라고하자.

행 연산을 통해 $$V_n = \begin{vmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-2} & x_1^{n-1} \\ 0 & x_2-x_1 & x_2^2-x_1^2 & \cdots & x_2^{n-2}-x_1^{n-2} & x_2^{n-1}-x_1^{n-1} \\ 0 & x_3-x_1 & x_3^2-x_1^2 & \cdots & x_3^{n-2}-x_1^{n-2} & x_3^{n-1}-x_1^{n-1} \\ & \vdots & & & \vdots & \\ 0 & x_{n-1}-x_1 & x_{n-1}^2-x_1^2 & \cdots & x_{n-1}^{n-2}-x_1^{n-2} & x_{n-1}^{n-1}-x_1^{n-1} \\ 0 & x_n-x_1 & x_n^2-x_1^2 & \cdots & x_n^{n-2}-x_1^{n-2} & x_n^{n-1}-x_1^{n-1} \end{vmatrix}$$ 임을 구할 수 있다.

여기서 다시 행연산을 하면 $$V_n = \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & x_2-x_1 & (x_2-x_1)x_2 & \cdots & (x_2-x_1)x_2^{n-3} & (x_2-x_1)x_2^{n-2} \\ 0 & x_3-x_1 & (x_3-x_1)x_3 & \cdots & (x_3-x_1)x_3^{n-3} & (x_3-x_1)x_3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & x_{n-1}-x_1 & (x_{n-1}-x_1)x_{n-1} & \cdots & (x_{n-1}-x_1)x_{n-1}^{n-3} & (x_{n-1}-x_1)x_{n-1}^{n-2} \\ 0 & x_n-x_1 & (x_n-x_1)x_n & \cdots & (x_n-x_1)x_n^{n-3} & (x_n-x_1)x_n^{n-2} \end{vmatrix}$$ 이다.

첫번째 row를 제외한 k번째 row에서 (x_k-x_1)를 추출할 수 있다.

그러면 $$V_n = \prod_{k=2}^{n}(x_k-x_1) \begin{vmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & x_2 & \cdots & x_2^{n-3} & x_2^{n-2} \\ 0 & 1 & x_3 & \cdots & x_3^{n-3} & x_3^{n-2} \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & x_{n-1} & \cdots & x_{n-1}^{n-3} & x_{n-1}^{n-2} \\ 0 & 1 & x_n & \cdots & x_n^{n-3} & x_n^{n-2} \end{vmatrix} = \prod_{k=2}^{n}(x_k-x_1) \cdot V_{n-1}$$ 이다.

$V_2 = \begin{vmatrix} 1 & x_1 \\ 1 & x_2 \end{vmatrix} = (x_2-x_1)$ 이므로, $V_n = \prod_{k=2}^{n}(x_k-x_1) \cdot \prod_{k=2}^{n-1}(x_k-x_1) \cdot \prod_{k=2}^{n-2}(x_k-x_1) \cdots \prod_{k=2}^{2}(x_k-x_1)$

$= \prod_{1 \le i < j \le n}(x_j - x_i)$ 이다.