

Polynomial Regression Assignment

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1-(a)

Size of vector $W = d+1$.Size of vector $y = n$.

1-(b)

Size of matrix $A = n \times (d+1)$

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{d-1} & x_1^d \\ 1 & x_2 & x_2^2 & \dots & x_2^{d-1} & x_2^d \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{d-1} & x_n^d \end{bmatrix}$$

1-(c) ($d=n-1$)

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-2} & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-2} & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{n-2} & x_n^{n-1} \end{bmatrix} \quad \text{라고 하고, } V_n = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-2} & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \dots & x_2^{n-2} & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \dots & x_n^{n-2} & x_n^{n-1} \end{bmatrix} \quad \text{라고 하자.}$$

$$\text{연산을 통해 } V_n = \begin{bmatrix} 1 & x_1 & x_1^2 & \dots & x_1^{n-2} & x_1^{n-1} \\ 0 & x_2 - x_1 & x_2^2 - x_1^2 & \dots & x_2^{n-2} - x_1^{n-2} & x_2^{n-1} - x_1^{n-1} \\ 0 & x_3 - x_1 & x_3^2 - x_1^2 & \dots & x_3^{n-2} - x_1^{n-2} & x_3^{n-1} - x_1^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & x_{n-1} - x_1 & x_{n-1}^2 - x_1^2 & \dots & x_{n-1}^{n-2} - x_1^{n-2} & x_{n-1}^{n-1} - x_1^{n-1} \\ 0 & x_n - x_1 & x_n^2 - x_1^2 & \dots & x_n^{n-2} - x_1^{n-2} & x_n^{n-1} - x_1^{n-1} \end{bmatrix} \quad \text{임을 구할 수 있다.}$$

$$\text{여기서 다시 연산을 하면 } V_n = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & x_2 - x_1 & (x_2 - x_1)x_2 & \dots & (x_2 - x_1)x_2^{n-3} & (x_2 - x_1)x_2^{n-2} \\ 0 & x_3 - x_1 & (x_3 - x_1)x_3 & \dots & (x_3 - x_1)x_3^{n-3} & (x_3 - x_1)x_3^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & x_{n-1} - x_1 & (x_{n-1} - x_1)x_{n-1} & \dots & (x_{n-1} - x_1)x_{n-1}^{n-3} & (x_{n-1} - x_1)x_{n-1}^{n-2} \\ 0 & x_n - x_1 & (x_n - x_1)x_n & \dots & (x_n - x_1)x_n^{n-3} & (x_n - x_1)x_n^{n-2} \end{bmatrix} \quad \text{이다.}$$

첫번째 row를 제외한 k 번째 row에서 $(x_k - x_1)$ 를 추출할 수 있다.

$$\text{그러면 } V_n = \prod_{k=2}^n (x_k - x_1) \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & x_2 & \dots & x_2^{n-3} & x_2^{n-2} \\ 0 & 1 & x_3 & \dots & x_3^{n-3} & x_3^{n-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & x_{n-1} & \dots & x_{n-1}^{n-3} & x_{n-1}^{n-2} \\ 0 & 1 & x_n & \dots & x_n^{n-3} & x_n^{n-2} \end{bmatrix} = \prod_{k=2}^n (x_k - x_1) \cdot V_{n-1} \quad \text{이다.}$$

$$V_2 = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} = (x_2 - x_1) \text{ 이므로, } V_n = \prod_{k=2}^n (x_k - x_1) \cdot \prod_{k=2}^{n-1} (x_k - x_1) \cdot \prod_{k=2}^{n-2} (x_k - x_1) \dots \prod_{k=2}^2 (x_k - x_1) \\ = \prod_{1 \leq i < j \leq n} (x_j - x_i) \quad \text{이다.}$$