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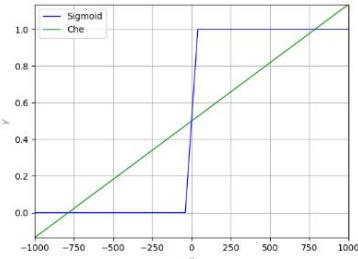
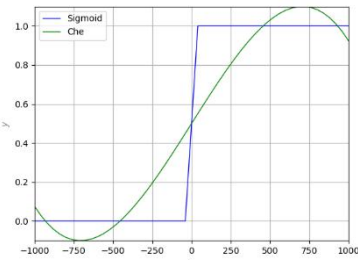
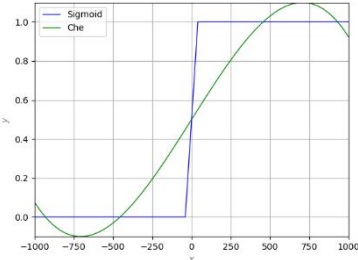
被逼近函数为 sigmoid, $f(x) = \frac{1}{1+e^{-x}} = \frac{1+\tanh\frac{x}{2}}{2}$

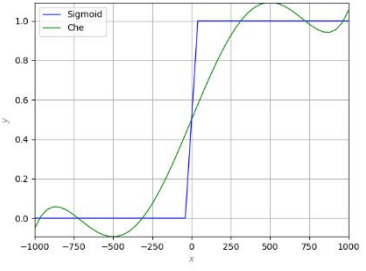
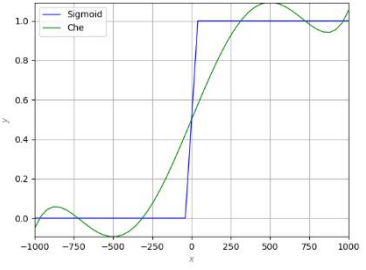
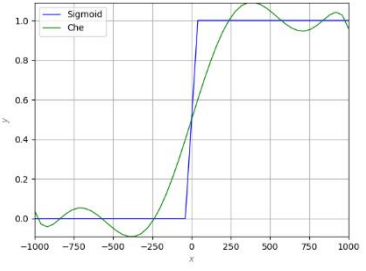
12 为切比雪夫逼近, 权函数为 $\rho(x) = \frac{1}{l\sqrt{1-(x/l)^2}}$

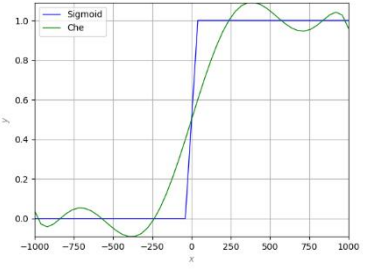
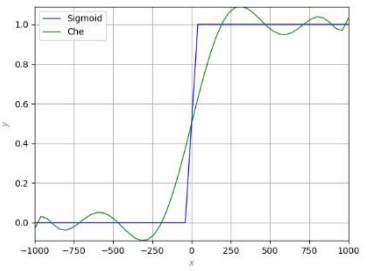
34 为切比雪夫逼近, 权函数为 $\rho(x) = e^{-(l/x)^2}$

56 为勒让德逼近, 权函数为 $\rho(x) = 1$

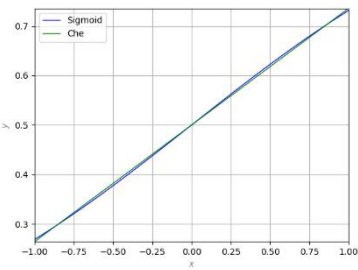
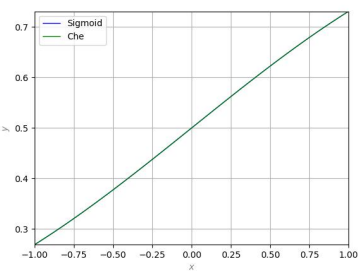
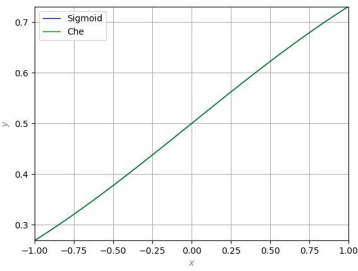
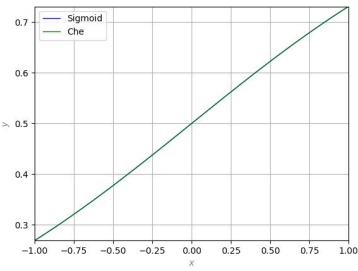
1. 基底: $\frac{1}{10^3 \sqrt{1 - (\frac{x}{10^3})^2}}$ 区间: $[-10^3, 10^3]$

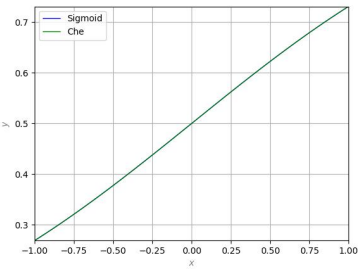
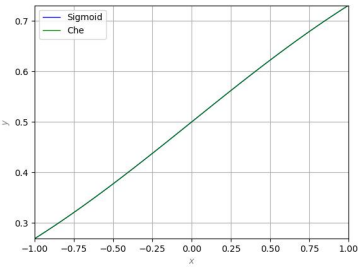
Degree	Polynomial Approximations	Sigmoid function, p(x)
2	<p>$p(x) = -2.88009325106496e-20x^2 + 0.000636618725167192x + 0.5000000000000003$</p> <p>$\ f(x) - p(x)\ _2 = 0.384422272749$</p>	
3	<p>$p(x) = -8.48813796849639e-10x^3 + 2.88009325106496e-20x^2 + 0.00127322907280519x + 0.5000000000000003$</p> <p>$\ f(x) - p(x)\ _2 = 0.277573499978$</p>	
4	<p>$p(x) = -2.82668469092201e-23x^4 + 8.48813796849639e-10x^3 + 2.82380459767495e-17x^2 + 0.00127322907280519x + 0.499999999999647$</p> <p>$\ f(x) - p(x)\ _2 = 0.277573499978$</p>	

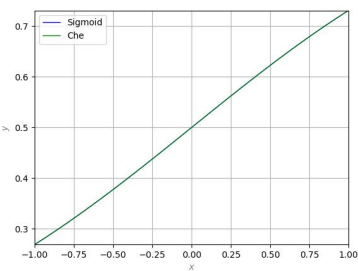
5	<p> $p(x) = 2.03709949776852e-15x^5 - 2.82668469092201e-23x^4 - 3.39518816904661e-9x^3 + 2.82380459767495e-17x^2 + 0.00190982266584751x + 0.49999999999647$ </p> <p> $\ f(x) - p(x)\ _2 = 0.227121886433$ </p>	 <p>The plot shows a blue step function (Sigmoid) and a green curve (Chebyshev polynomial) on a grid from -1000 to 1000 on the x-axis and 0.0 to 1.0 on the y-axis. The green curve approximates the step function with some oscillations.</p>
6	<p> $p(x) = 1.26509167228714e-28x^6 + 2.03709949776852e-15x^5 - 2.18030597747836e-22x^4 - 3.39518816904661e-9x^3 + 9.93994525385387e-17x^2 + 0.00190982266584751x + 0.499999999992517$ </p> <p> $\ f(x) - p(x)\ _2 = 0.227121886433$ </p>	 <p>The plot shows a blue step function (Sigmoid) and a green curve (Chebyshev polynomial) on a grid from -1000 to 1000 on the x-axis and 0.0 to 1.0 on the y-axis. The green curve approximates the step function with some oscillations.</p>
7	<p> $p(x) = -5.82005451699162e-21x^7 + 1.26509167228714e-28x^6 + 1.22221949035234e-14x^5 - 2.18030597747836e-22x^4 - 8.48773587265306e-9x^3 + 9.93994525385387e-17x^2 + 0.0025463911289349x + 0.499999999992517$ </p> <p> $\ f(x) - p(x)\ _2 = 0.196454079231$ </p>	 <p>The plot shows a blue step function (Sigmoid) and a green curve (Chebyshev polynomial) on a grid from -1000 to 1000 on the x-axis and 0.0 to 1.0 on the y-axis. The green curve approximates the step function with some oscillations.</p>

8	<p> $p(x) = -7.98386789320987e-35x^{**8} - 5.82005451699162e-21x^{**7} + 2.86186525178869e-28x^{**6} + 1.22221949035234e-14x^{**5} - 3.17828946538786e-22x^{**4} - 8.48773587265306e-9x^{**3} + 1.19359122316838e-16x^{**2} + 0.0025463911289349x + 0.499999999991893$ </p> <p> $\ f(x)-p(x)\ _2=0.196454079231$ </p>	 <p>The plot shows a blue step function (Sigmoid) and a green curve (Chebyshev polynomial) on a grid from -1000 to 1000 on the x-axis and 0.0 to 1.0 on the y-axis. The green curve approximates the step function with oscillations.</p>
9	<p> $p(x) = 1.81058825288616e-26x^{**9} - 7.98386789320987e-35x^{**8} - 4.65582900573764e-20x^{**7} + 2.86186525178869e-28x^{**6} + 4.27758714124555e-14x^{**5} - 3.17828946538786e-22x^{**4} - 1.69748681811315e-8x^{**3} + 1.19359122316838e-16x^{**2} + 0.00318292604618453x + 0.499999999991893$ </p> <p> $\ f(x)-p(x)\ _2=0.175319126035$ </p>	 <p>The plot shows a blue step function (Sigmoid) and a green curve (Chebyshev polynomial) on a grid from -1000 to 1000 on the x-axis and 0.0 to 1.0 on the y-axis. The green curve approximates the step function with oscillations, showing slightly better fit than the previous one.</p>

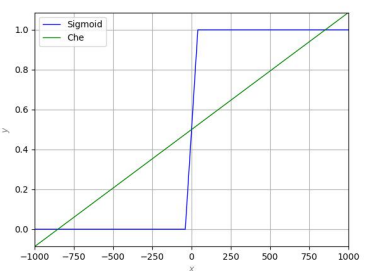
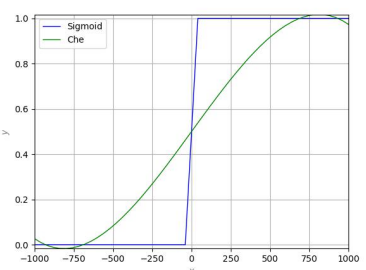
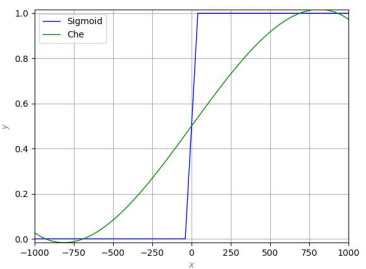
2. 基底: $\frac{1}{\sqrt{1-x^2}}$ 区间: $[-1,1]$

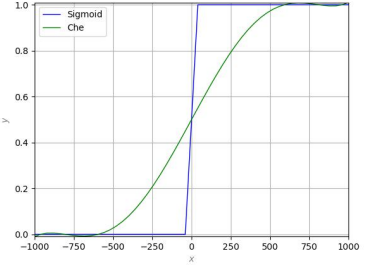
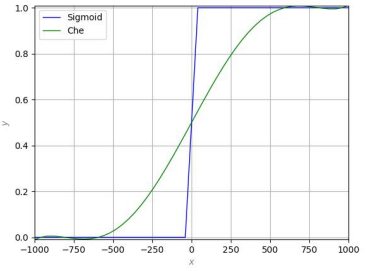
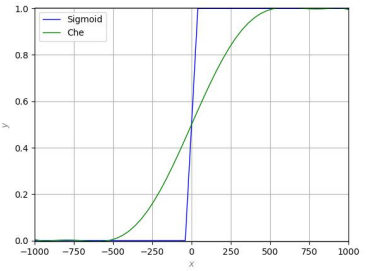
Degree	Polynomial Approximations	Sigmoid function, $p(x)$
2	$p(x) = 1.5491123157387e-14x^2 + 0.235571413924028x + 0.499999999999994$ $\ f(x)-p(x)\ _2 = 0.00579206329109$	
3	$p(x) = -0.0184803669414559x^3 + 1.5491123157387e-14x^2 + 0.249431689130122x + 0.499999999999994$ $\ f(x)-p(x)\ _2 = 0.000135757801158$	
4	$p(x) = -3.78730459342239e-12x^4 + 0.0184803669414559x^3 + 3.80279571656336e-12x^2 + 0.249431689130122x + 0.499999999999952$ $\ f(x)-p(x)\ _2 = 0.000135757801158$	
5	$p(x) = 0.00175743741756684x^5 - 3.78730459342239e-12x^4 - 0.0206771637134062x^3 + 3.80279571656336e-12x^2 + 0.249980888323105x + 0.499999999999952$	

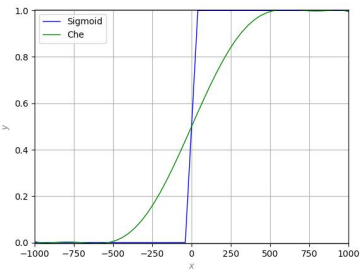
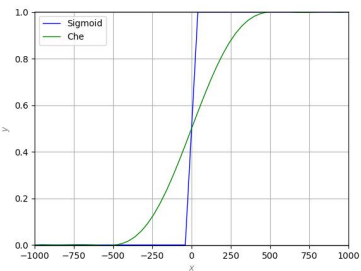
	$\ f(x)-p(x)\ _2=3.26965839264e-06$	
6	$p(x)=1.17288649126526e-11x^{**6} + 0.00175743741756684x^{**5} - 2.13806019630373e-11x^{**4} - 0.0206771637134062x^{**3} + 1.04002822305244e-11x^{**2} + 0.249980888323105x + 0.499999999999154$ $\ f(x)-p(x)\ _2=3.26965839266e-06$	
7	$p(x)=-0.000169323602892859x^{**7} + 1.17288649126526e-11x^{**6} + 0.00205375372264411x^{**5} - 2.13806019630373e-11x^{**4} - 0.0208253218659553x^{**3} + 1.04002822305244e-11x^{**2} + 0.249999408092176x + 0.499999999999154$ $\ f(x)-p(x)\ _2=7.88505194259e-08$	
8	$p(x)=-2.54810357413633e-10x^{**8} - 0.000169323602892859x^{**7} + 5.21349579143191e-10x^{**6} + 0.00205375372264411x^{**5} - 3.39893547849677e-10x^{**4} - 0.0208253218659553x^{**3} + 7.41028712624025e-11x^{**2} +$	

	$0.249999408092176 \cdot x + 0.499999999997163$ $\ f(x) - p(x)\ _2 = 7.88505194818 \times 10^{-8}$	
9	$p(x) = 1.62992910768299 \times 10^{-5} x^9 - 2.54810357413633 \times 10^{-10} x^8 - 0.000205997007799754 x^7 + 5.21349579143191 \times 10^{-10} x^6 + 0.00208125877631007 x^5 - 3.39893547849677 \times 10^{-10} x^4 - 0.0208329621586357 x^3 + 7.41028712624025 \times 10^{-11} x^2 + 0.249999981114126 x + 0.499999999997163$ $\ f(x) - p(x)\ _2 = 1.91455342714 \times 10^{-9}$	

3.基底： $e^{-(1000/x)^2}$ ，当 $x=0$ 时，取 $x=0.00001$ 区间： $[-1000,1000]$

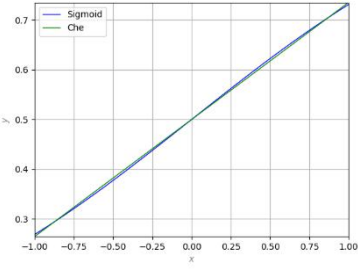
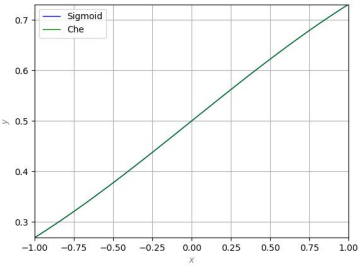
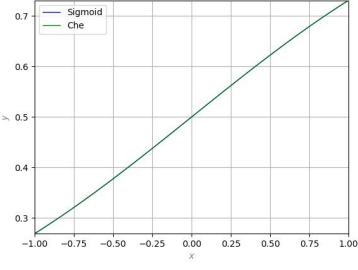
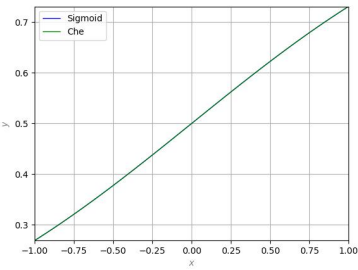
Degree	Polynomial Approximations	Sigmoid function, $p(x)$
2	$p(x)=$ $4.73923210183516e-16x^{**2}$ $+$ $0.000586995282474419x + 0.499999999663507$ $ f(x)-p(x) _2=0.976650896962$	
3	$p(x)=$ $-4.80591420542768e-10x^{**3}$ $+$ $4.73923210183516e-16x^{**2} + 0.000953862994164936x +$ 0.499999999663507 $ f(x)-p(x) _2=0.270652475585$	
4	$p(x)=$ $-1.50228252955222e-20x^{**4}$ $-$ $4.80591420542768e-10x^{**3} + 2.04294971199605e-14x^{**2} +$ $0.000953862994164936x + 0.4999999993637141$ $ f(x)-p(x) _2=0.270652475585$	

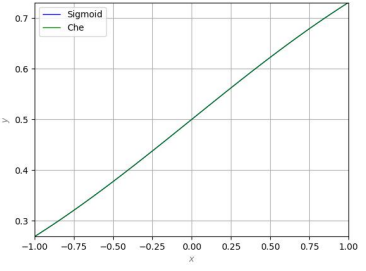
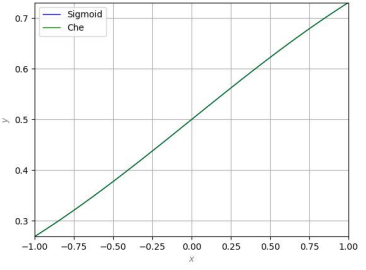
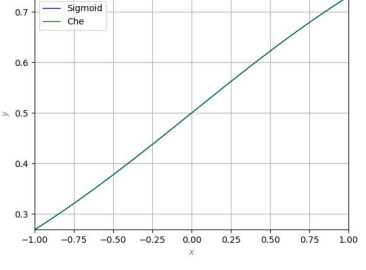
5	$p(x) = 6.65326014392383e-16x^5 - 1.50228252955222e-20x^4 - 1.41373000865643e-9x^3 + 2.04294971199605e-14x^2 + 0.00125843090769336x + 0.499999993637141$ $\ f(x) - p(x)\ _2 = 0.0941592146779$	
6	$p(x) = 2.85753271019652e-25x^6 + 6.65326014392383e-16x^5 - 5.62766836563708e-19x^4 - 1.41373000865643e-9x^3 + 3.46463057805243e-13x^2 + 0.00125843090769336x + 0.499999993468174$ $\ f(x) - p(x)\ _2 = 0.0941592146779$	
7	$p(x) = -1.09888498097995e-21x^7 + 2.85753271019652e-25x^6 + 2.8681970128863e-15x^5 - 5.62766836563708e-19x^4 - 2.80283391311553e-9x^3 + 3.46463057805243e-13x^2 + 0.00152931479143287x + 0.499999993468174$ $\ f(x) - p(x)\ _2 = 0.0372119260932$	

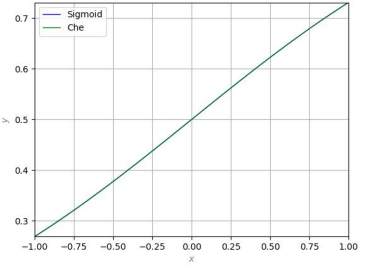
8	<p> $p(x) = -4.21283559444646e-30x^{**8} - 1.09888498097995e-21x^{**7} + 1.07747217556232e-23x^{**6} + 2.8681970128863e-15x^{**5} - 9.80575539490412e-18x^{**4} - 2.80283391311553e-9x^{**3} + 3.71679113321182e-12x^{**2} + 0.00152931479143287x + 0.499999513025745$ </p> <p> $f(x)-p(x) _2=0.0372119260934$ </p>	
9	<p> $p(x) = 2.00207839846178e-27x^{**9} - 4.21283559444646e-30x^{**8} - 6.27909899367563e-21x^{**7} + 1.07747217556232e-23x^{**6} + 7.65269878100198e-15x^{**5} - 9.80575539490412e-18x^{**4} - 4.65167102353083e-9x^{**3} + 3.71679113321182e-12x^{**2} + 0.00177789475527988x + 0.499999513025745$ </p> <p> $f(x)-p(x) _2=0.0160062427207$ </p>	

4. 基底: $e^{-(1/x)^2}$, 当 $x=0$ 时, 取 $x=0.00001$ 区间: $[-1,1]$

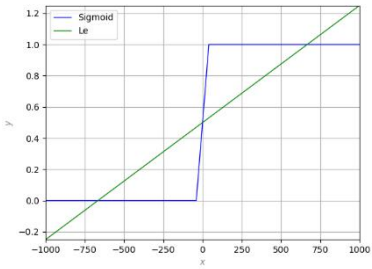
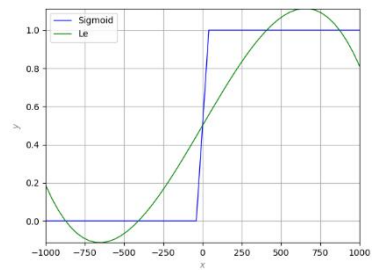
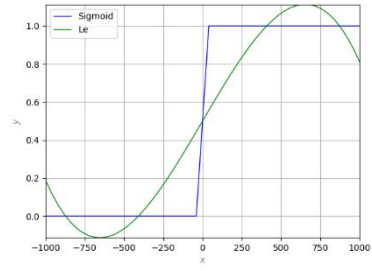
Degree	Polynomial Approximations	Sigmoid function, $p(x)$
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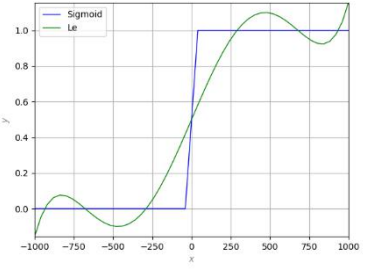
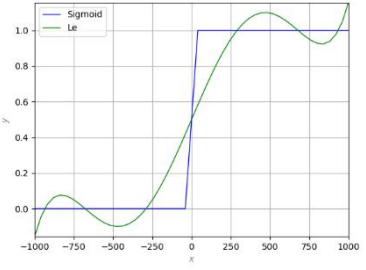
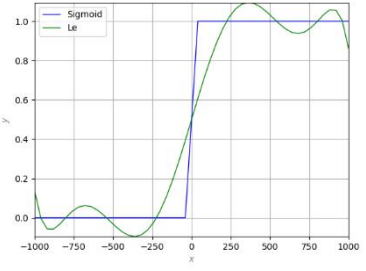
2	$p(x) = 9.50385053384505e-10x^2 + 0.235274350264956x + 0.499999999325213$ $\ f(x) - p(x)\ _2 = 0.00112440919402$	
3	$p(x) = -0.0182070283763714x^3 + 9.50385053384505e-10x^2 + 0.249172997955295x + 0.499999999325213$ $\ f(x) - p(x)\ _2 = 2.06746497109e-05$	
4	$p(x) = -3.00836326899558e-8x^4 + 0.0182070283763714x^3 + 4.09119864194152e-8x^2 + 0.249172997955295x + 0.499999987257244$ $\ f(x) - p(x)\ _2 = 2.06746497171e-05$	
5	$p(x) = 0.0017139206042993x^5 - 3.00836326899558e-8x^4 - 0.0206108505902007x^3 + 4.09119864194152e-8x^2 + 0.249957583573638x + 0.499999987257244$ $\ f(x) - p(x)\ _2 = 4.07772394306e-07$	

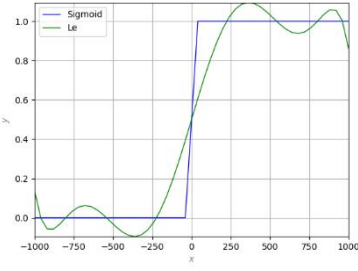
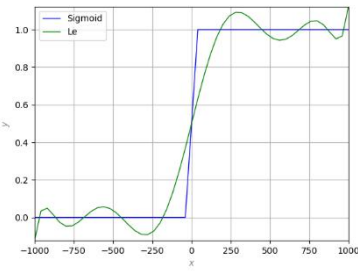
6	<p> $p(x) = 5.56153313641097e-7x^{**6} + 0.0017139206042993x^{**5} - 1.09614191783775e-6x^{**4} - 0.0206108505902007x^{**3} + 6.75461638925146e-7x^{**2} + 0.249957583573638x + 0.499999872514062$ </p> <p> $f(x)-p(x) _2 = 4.07777237783e-07$ </p>	 <p>The plot shows a Sigmoid function (blue line) and its Chebyshev polynomial approximation (green line) over the interval [-1, 1]. The x-axis ranges from -1.00 to 1.00 with increments of 0.25. The y-axis ranges from 0.3 to 0.7 with increments of 0.1. The green line closely follows the blue line, indicating a high-quality approximation.</p>
7	<p> $p(x) = -0.00016381184455958x^{**7} + 5.56153313641097e-7x^{**6} + 0.00204230470963145x^{**5} - 1.09614191783775e-6x^{**4} - 0.0208179256494335x^{**3} + 6.75461638925146e-7x^{**2} + 0.249997964495491x + 0.499999872514062$ </p> <p> $f(x)-p(x) _2 = 8.57766125944e-09$ </p>	 <p>The plot shows a Sigmoid function (blue line) and its Chebyshev polynomial approximation (green line) over the interval [-1, 1]. The x-axis ranges from -1.00 to 1.00 with increments of 0.25. The y-axis ranges from 0.3 to 0.7 with increments of 0.1. The green line closely follows the blue line, indicating a high-quality approximation.</p>
8	<p> $p(x) = -7.61968477690841e-6x^{**8} + 0.00016381184455958x^{**7} + 1.95273917968505e-5x^{**6} + 0.00204230470963145x^{**5} - 1.78138135136454e-5x^{**4} - 0.0208179256494335x^{**3} + 6.77133716349788e-6x^{**2} + 0.249997964495491x + 0.499999109868424$ </p> <p> $f(x)-p(x) _2 = 1.04261674754e-08$ </p>	 <p>The plot shows a Sigmoid function (blue line) and its Chebyshev polynomial approximation (green line) over the interval [-1, 1]. The x-axis ranges from -1.00 to 1.00 with increments of 0.25. The y-axis ranges from 0.3 to 0.7 with increments of 0.1. The green line closely follows the blue line, indicating a high-quality approximation.</p>

9	<p> $p(x) = 1.53550405081618e-5x^9 - 7.61968477690841e-6x^8 - 0.000203541757531201x^7 + 1.95273917968505e-5x^6 + 0.00207899968996794x^5 - 1.78138135136454e-5x^4 - 0.0208321054010187x^3 + 6.77133716349788e-6x^2 + 0.249999870992509x + 0.499999109868424$ </p> <p> $\ f(x) - p(x)\ _2 = 6.27758704531e-09$ </p>	
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5. 基底:1 区间:[-1000,1000]

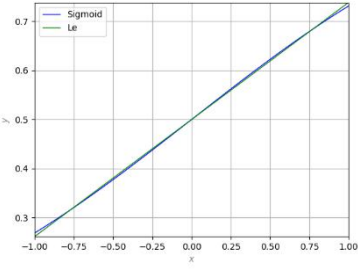
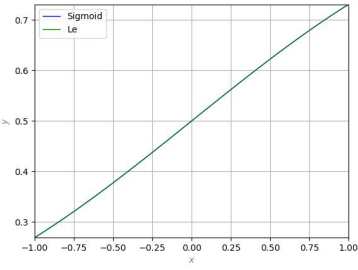
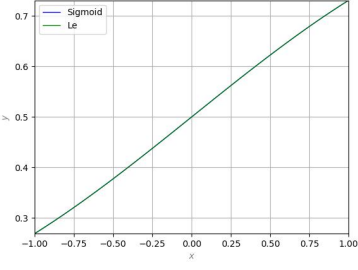
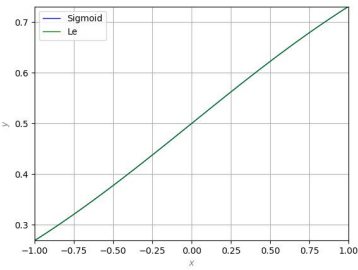
Degree	Polynomial Approximations	Sigmoid function, $p(x)$
2	$p(x) = -1.30104260698261e-22x^2 + 0.000749997532599195x + 0.499999999999998$ $\ f(x) - p(x)\ _2 = 11.1356395145$	
3	$p(x) = -1.09372841048899e-9x^3 + 1.30104260698261e-22x^2 + 0.00140623457889259x + 0.499999999999998$ $\ f(x) - p(x)\ _2 = 8.32569074172$	
4	$p(x) = -2.32283011287114e-27x^4 + 1.09372841048899e-9x^3 + 1.86089297890557e-21x^2 + 0.00140623457889259x + 0.499999999999998$ $\ f(x) - p(x)\ _2 = 8.32569074172$	

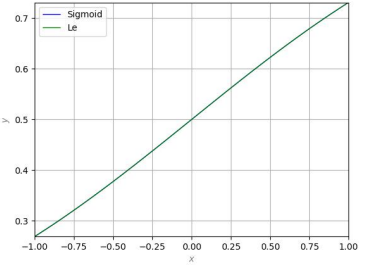
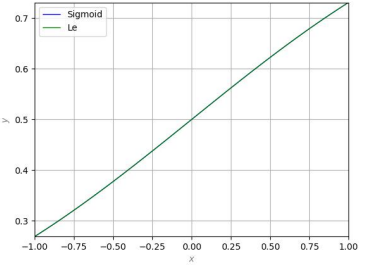
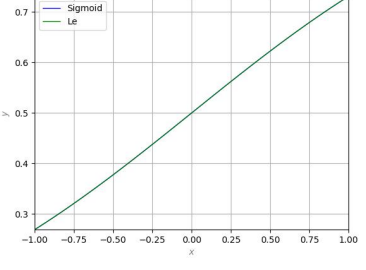
5	$p(x) = 2.70689766766883e-15x^5 - 2.32283011287114e-27x^4 - 4.10139248567655e-9x^3 + 1.86089297890557e-21x^2 + 0.00205073402357563x + 0.499999999999998$ $\ f(x) - p(x)\ _2 = 6.91627584969$	 <p>The plot shows the Sigmoid function (blue line) and the Le function (green line) over the range x from -1000 to 1000. The y-axis ranges from 0.0 to 1.0. The Sigmoid function is a smooth curve, while the Le function is a piecewise constant function that is 0 for x < 0 and 1 for x > 0.</p>
6	$p(x) = -1.08052338942169e-30x^6 + 2.70689766766883e-15x^5 + 1.47111815546216e-24x^4 - 4.10139248567655e-9x^3 - 4.89286102212774e-19x^2 + 0.00205073402357563x + 0.5000000000000021$ $\ f(x) - p(x)\ _2 = 6.91627584969$	 <p>The plot shows the Sigmoid function (blue line) and the Le function (green line) over the range x from -1000 to 1000. The y-axis ranges from 0.0 to 1.0. The Sigmoid function is a smooth curve, while the Le function is a piecewise constant function that is 0 for x < 0 and 1 for x > 0.</p>
7	$p(x) = -7.85450106007808e-21x^7 + 1.08052338942169e-30x^6 + 1.53949378416417e-14x^5 + 1.47111815546216e-24x^4 - 9.86868347384634e-9x^3 - 4.89286102212774e-19x^2 + 0.0026915441333723x + 0.5000000000000021$ $\ f(x) - p(x)\ _2 = 6.0326518198$	 <p>The plot shows the Sigmoid function (blue line) and the Le function (green line) over the range x from -1000 to 1000. The y-axis ranges from 0.0 to 1.0. The Sigmoid function is a smooth curve, while the Le function is a piecewise constant function that is 0 for x < 0 and 1 for x > 0.</p>

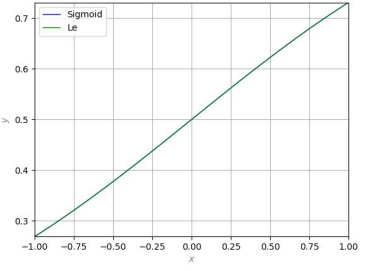
8	$p(x) = -1.07741520106648e-36x^{**8} - 7.85450106007808e-21x^{**7} + 9.30651652568123e-31x^{**6} + 1.53949378416417e-14x^{**5} + 3.10824862007252e-25x^{**4} - 9.86868347384634e-9x^{**3} - 2.78323685221168e-19x^{**2} + 0.0026915441333723x + 0.5000000000000015$ $ f(x)-p(x) _2=6.0326518198$	
9	$p(x) = 2.46639357598473e-26x^{**9} - 1.07741520106648e-36x^{**8} - 6.00840120808315e-20x^{**7} + 9.30651652568123e-31x^{**6} + 5.19555955560798e-14x^{**5} + 3.10824862007252e-25x^{**4} - 1.92432110929027e-8x^{**3} - 2.78323685221168e-19x^{**2} + 0.00333071647103233x + 0.5000000000000015$ $ f(x)-p(x) _2=5.41221067639$	

6. 基底:1 区间:[-1,1]

Degree	Polynomial Approximations	Sigmoid function, $p(x)$
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2	$p(x) = -1.54696099709161e-15x^2 + 0.238327951492686x + 0.5000000000000001$ $\ f(x) - p(x)\ _2 = 0.00400430162746$	
3	$p(x) = -0.0187229765012767x^3 - 1.54696099709161e-15x^2 + 0.249561737393452x + 0.5000000000000001$ $\ f(x) - p(x)\ _2 = 9.64135174469e-05$	
4	$p(x) = -4.92647174780368e-15x^4 - 0.0187229765012767x^3 + 2.67572907245441e-15x^2 + 0.249561737393452x + 0.5$ $\ f(x) - p(x)\ _2 = 9.64135174469e-05$	
5	$p(x) = 0.00178009152069321x^5 - 4.92647174780368e-15x^4 - 0.0207008559687136x^3 + 2.67572907245441e-15x^2 + 0.249985568707902x + 0.5$ $\ f(x) - p(x)\ _2 = 2.33591804586e-06$	

6	<p> $p(x) = 1.38943368129522e-13x^{**6} + 0.00178009152069321x^{**5} - 1.94394701015332e-13x^{**4} - 0.0207008559687136x^{**3} + 6.58318054949632e-14x^{**2} + 0.249985568707902x + 0.499999999999997$ </p> <p> $f(x)-p(x) _2 = 2.33591804587e-06$ </p>	 <p>The plot shows two curves, 'Sigmoid' (blue) and 'Le' (green), on a grid. The x-axis ranges from -1.00 to 1.00 with increments of 0.25. The y-axis ranges from 0.3 to 0.7 with increments of 0.1. Both curves are nearly identical, starting at approximately (-1.00, 0.25) and ending at (1.00, 0.75), following a smooth S-shaped path.</p>
7	<p> $p(x) = -0.000171474020962806x^{**7} + 1.38943368129522e-13x^{**6} + 0.00205708801609467x^{**5} - 1.94394701015332e-13x^{**4} - 0.0208267634666234x^{**3} + 6.58318054949632e-14x^{**2} + 0.249999558429892x + 0.499999999999997$ </p> <p> $f(x)-p(x) _2 = 5.65300989637e-08$ </p>	 <p>The plot shows two curves, 'Sigmoid' (blue) and 'Le' (green), on a grid. The x-axis ranges from -1.00 to 1.00 with increments of 0.25. The y-axis ranges from 0.3 to 0.7 with increments of 0.1. Both curves are nearly identical, starting at approximately (-1.00, 0.25) and ending at (1.00, 0.75), following a smooth S-shaped path.</p>
8	<p> $p(x) = 2.33715930638571e-12x^{**8} - 0.000171474020962806x^{**7} - 4.22375400379092e-12x^{**6} + 0.00205708801609467x^{**5} + 2.32254609047756e-12x^{**4} - 0.0208267634666234x^{**3} - 3.9179379295837e-13x^{**2} + 0.249999558429892x + 0.500000000000001$ </p> <p> $f(x)-p(x) _2 = 5.65300989513e-08$ </p>	 <p>The plot shows two curves, 'Sigmoid' (blue) and 'Le' (green), on a grid. The x-axis ranges from -1.00 to 1.00 with increments of 0.25. The y-axis ranges from 0.3 to 0.7 with increments of 0.1. Both curves are nearly identical, starting at approximately (-1.00, 0.25) and ending at (1.00, 0.75), following a smooth S-shaped path.</p>

9	<p> $p(x) = 1.65409312968551e-5x^{**9} + 2.33715930638571e-12x^{**8} - 0.000206501875473747x^{**7} - 4.22375400379092e-12x^{**6} + 0.00208160751425229x^{**5} + 2.32254609047756e-12x^{**4} - 0.020833050517433x^{**3} - 3.9179379295837e-13x^{**2} + 0.249999987092448x + 0.500000000000001$ </p> <p> $\ f(x) - p(x)\ _2 = 1.36664400078e-09$ </p>	
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