

Range: Ring \mathbb{Z}^{2L} (L is a integer) or Finite Field \mathbb{F}_q (q is a prime)

Note: x to denote a scalar; \mathbf{x} to denote a vector; \mathbf{X} to denote a matrix

#case1 : scalar

ADD: $z=x+y$

INPUT

$S_0: \langle x \rangle_0, \langle y \rangle_0; \quad S_1: \langle x \rangle_1, \langle y \rangle_1$

COMPUTE

S_0 : Compute $\langle z \rangle_0 = \langle x \rangle_0 + \langle y \rangle_0$, locally

S_1 : Compute $\langle z \rangle_1 = \langle x \rangle_1 + \langle y \rangle_1$, locally

OUTPUT

$S_0: \langle z \rangle_0; \quad S_1: \langle z \rangle_1$

MUL: $z=x.y$

INPUT

$S_0: \langle x \rangle_0, \langle y \rangle_0, (\langle a \rangle_0, \langle b \rangle_0, \langle c \rangle_0)$

$S_1: \langle x \rangle_1, \langle y \rangle_1, (\langle a \rangle_1, \langle b \rangle_1, \langle c \rangle_1)$

COMPUTE

S_i : Compute $\langle e \rangle_i = \langle x \rangle_i - \langle a \rangle_i; \langle f \rangle_i = \langle y \rangle_i - \langle b \rangle_i$; , send $\langle e \rangle_i, \langle f \rangle_i$ to S_{1-i} ;

and set $e = \langle e \rangle_0 + \langle e \rangle_1, f = \langle f \rangle_0 + \langle f \rangle_1$;

Compute $\langle z \rangle_i = i.e.f + f.\langle a \rangle_i + e.\langle b \rangle_i + \langle c \rangle_i$

i.e.

S_0 : compute $\langle e \rangle_0 = \langle x \rangle_0 - \langle a \rangle_0; \langle f \rangle_0 = \langle y \rangle_0 - \langle b \rangle_0$; , send $\langle e \rangle_0, \langle f \rangle_0$ to S_1 ;

S_1 : compute $\langle e \rangle_1 = \langle x \rangle_1 - \langle a \rangle_1; \langle f \rangle_1 = \langle y \rangle_1 - \langle b \rangle_1$; , send $\langle e \rangle_1, \langle f \rangle_1$ to S_0 ;

S_0 : reconstruct $e = \langle e \rangle_0 + \langle e \rangle_1, f = \langle f \rangle_0 + \langle f \rangle_1$;

S_1 : reconstruct $e = \langle e \rangle_0 + \langle e \rangle_1, f = \langle f \rangle_0 + \langle f \rangle_1$;

S_0 : compute $\langle z \rangle_0 = f.\langle a \rangle_0 + e.\langle b \rangle_0 + \langle c \rangle_0$

S_1 : compute $\langle z \rangle_1 = e.f + f.\langle a \rangle_1 + e.\langle b \rangle_1 + \langle c \rangle_1$

OUTPUT

$S_0: \langle z \rangle_0; \quad S_1: \langle z \rangle_1$

CORRECTNESS

$z = \langle z \rangle_0 + \langle z \rangle_1 = e.f + f.a + e.b + c = (x-a)(y-b) + (y-b)a + (x-a)b + ab = x.y$

#case2 : matrix

ADD: $Z=X+Y$

INPUT

S0: $\langle X \rangle_0, \langle Y \rangle_0$; S1: $\langle X \rangle_1, \langle Y \rangle_1$

COMPUTE

S0: Compute $\langle Z \rangle_0 = \langle X \rangle_0 + \langle Y \rangle_0$, locally

S1: Compute $\langle Z \rangle_1 = \langle X \rangle_1 + \langle Y \rangle_1$, locally

OUTPUT

S0: $\langle Z \rangle_0$; S1: $\langle Z \rangle_1$

MUL: $Z=X.Y$ (For example: $Z_{m \times 1} = X_{m \times d}.Y_{d \times 1}$)

INPUT

S0: $\langle X \rangle_0$ $m \times d$, $\langle Y \rangle_0$ $d \times n$, ($\langle A \rangle_0$ $m \times d$, $\langle B \rangle_0$ $d \times n$, $\langle C \rangle_0$ $m \times n$)

S1: $\langle X \rangle_1$ $m \times d$, $\langle Y \rangle_1$ $d \times n$, ($\langle A \rangle_1$ $m \times d$, $\langle B \rangle_1$ $d \times n$, $\langle C \rangle_1$ $m \times n$)

COMPUTE

S_i: Compute $\langle E \rangle_i = \langle X \rangle_i - \langle A \rangle_i$; $\langle F \rangle_i = \langle Y \rangle_i - \langle B \rangle_i$; , send $\langle E \rangle_i$, $\langle F \rangle_i$ to S_{1-i};
and set $E = \langle E \rangle_0 + \langle E \rangle_1$, $F = \langle F \rangle_0 + \langle F \rangle_1$;

Compute $\langle Z \rangle_i = i.E.F + \langle A \rangle_i.F + E.\langle B \rangle_i + \langle C \rangle_i$

i.e.

S0: compute $\langle E \rangle_0$ $m \times d = \langle X \rangle_0$ $m \times d - \langle A \rangle_0$ $m \times d$; $\langle F \rangle_0$ $d \times n = \langle Y \rangle_0$ $d \times n - \langle B \rangle_0$ $d \times n$;
send $\langle E \rangle_0$ $m \times d$, $\langle F \rangle_0$ $d \times n$ to S1;

S1: compute $\langle E \rangle_1$ $m \times d = \langle X \rangle_1$ $m \times d - \langle A \rangle_1$ $m \times d$; $\langle F \rangle_1$ $d \times n = \langle Y \rangle_1$ $d \times n - \langle B \rangle_1$ $d \times n$;
send $\langle E \rangle_1$ $m \times d$, $\langle F \rangle_1$ $d \times n$ to S0;

S0: reconstruct E $m \times d = \langle E \rangle_0$ $m \times d + \langle E \rangle_1$ $m \times d$, F $d \times n = \langle F \rangle_0$ $d \times n + \langle F \rangle_1$ $d \times n$;

S1: reconstruct E $m \times d = \langle E \rangle_0$ $m \times d + \langle E \rangle_1$ $m \times d$, F $d \times n = \langle F \rangle_0$ $d \times n + \langle F \rangle_1$ $d \times n$;

S0: compute $\langle Z \rangle_0$ $m \times n = \langle A \rangle_0$ $m \times d.F$ $d \times n + E$ $m \times d.\langle B \rangle_0$ $d \times n + \langle C \rangle_0$ $m \times n$

S1: compute $\langle Z \rangle_1$ $m \times n = E$ $m \times d.F$ $d \times n + \langle A \rangle_1$ $m \times d.F$ $d \times n + E$ $m \times d.\langle B \rangle_1$ $d \times n + \langle C \rangle_1$ $m \times n$

OUTPUT

S0: $\langle Z \rangle_0$ $m \times n$; S1: $\langle Z \rangle_1$ $m \times n$

CORRECTNESS

$Z = \langle Z \rangle_0 + \langle Z \rangle_1 = E.F + F.A + E.B + C = (X-A)(Y-B) + (Y-B)A + (X-A)B + AB = X.Y$