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ATCS: Numerical Methods Project Exploring Derivatives

Simple Slope Derivative:

In this module, the derivative of each point is calculated by taking the simple slope at each point. The slope is taken at the first point, the second point, or the in-between point of each pair throughout the entire time range of the function, which in this case is from -10 to 10 of the function e^(-t^2). The accuracy of this method is calculated through the *RMS* method:

$$RMS = \sqrt{\frac{\sum |x_f^2 - x_n^2|}{N}}.$$

This compares the calculated derivative method to the true value of the derivative, defined by the function -2te^(-t^2), at each point. At first glance, the derivative taken with the in-between point has a significantly smaller RMS compared to the ones taken at the left or right points. A good result for this portion of the lab will be defined as having an RMS of under 0.01. To achieve this "good" result for the in-between derivative, around 300 points had to be sampled. To achieve this good result for the left and right derivative calculation, nearly 15000 had to be taken.

3-Point Derivative:

In this module, the derivative of each point is calculated from the slope on either side of a point. This took the points on either side. Again, the RMS is used to calculate the accuracy of this 3-Point Derivative method, and goodness is defined to be an RMS under 0.01. Around 5700 points needed to be taken to achieve this result.

Functional Fit Derivative:

Here, the derivative of each point is taken by fitting a 3-point parabolic fit to a point and then taking the derivative of it. Using the formula $y = at^2 + bt + c$, these are fitted to three points of (y_1, t_1), (y_2, t_2), and (y_3, t_3). Then, these values are used to find *a* and *b* and fit into the derivative formula:

$$\frac{dy}{dt} = 2at + b.$$

However, this method is very inaccurate in RMS terms. The goodness of the Functional Fit Derivative is never managed to be reached. At a certain point, the RMS starts to increase after a large number of points has been reached.

Five-Point Stencil:

To calculate the derivative of each point, the two points to the left and two points to the right are used. They are put into the derivative formula to calculate the derivative:

$$f'(x) = -f(x + 2h) + 8f(x + h) - 8f(x - h) + f(x - 2h) / 12h.$$

This method is the recommended way for calculating an accurate derivative, due to its high accuracy. The RMS goes below the goodness threshold at 430 points.