Assignment 2

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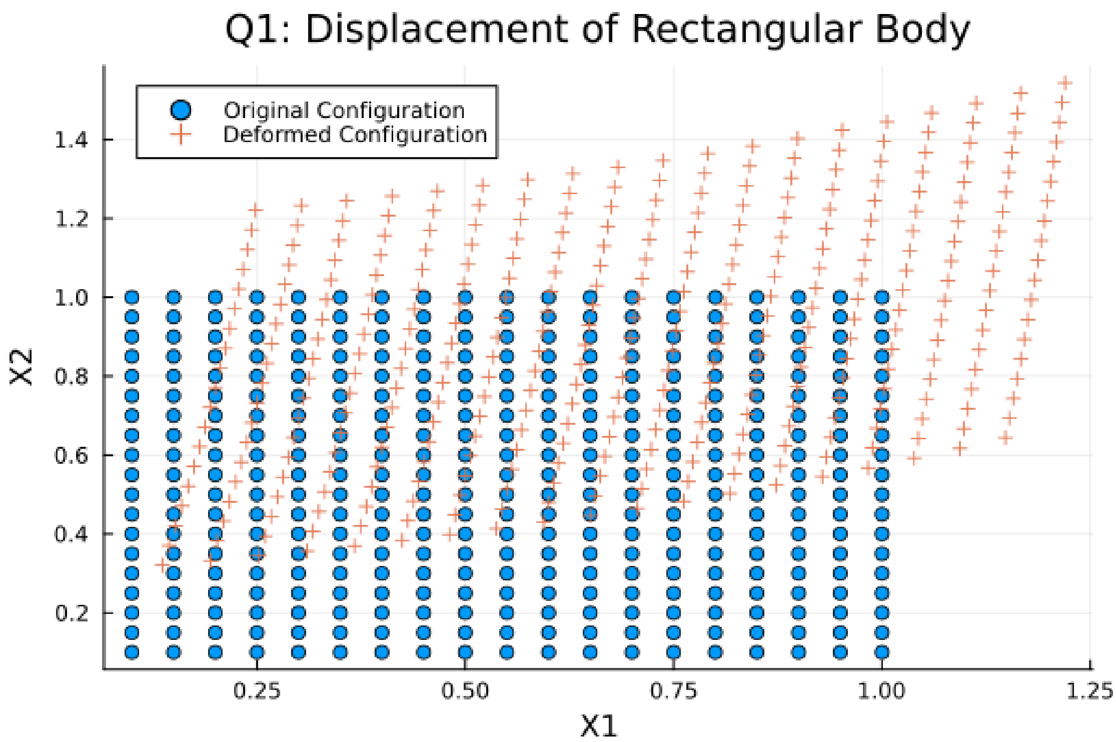
Question 1

In [1]:

```
using Plots

# 1. Define the original domain (undeformed grid)
x1_range = 0.1:0.05:1.0
x2_range = 0.1:0.05:1.0

# Lists to store coordinates
X1_orig, X2_orig = Float64[], Float64[]
x1_new, x2_new = Float64[], Float64[]

# 2. Compute displacement and new positions
for x1 in x1_range
    for x2 in x2_range
        # Original Coordinates
        push!(X1_orig, x1)
        push!(X2_orig, x2)

        # Displacement field u1, u2 [cite: 16]
        u1 = 0.2 * log(1 + x1 + x2)
        u2 = 0.2 * exp(x1)

        # Deformed Coordinates xi = Xi + ui
        push!(x1_new, x1 + u1)
        push!(x2_new, x2 + u2)
    end
end

# 3. Plotting
scatter(X1_orig, X2_orig, label="Original Configuration", marker=:circle, legend
scatter!(x1_new, x2_new, label="Deformed Configuration", marker=:cross)
title!("Q1: Displacement of Rectangular Body")
xlabel!("X1")
ylabel!("X2")
```

Out[1]:



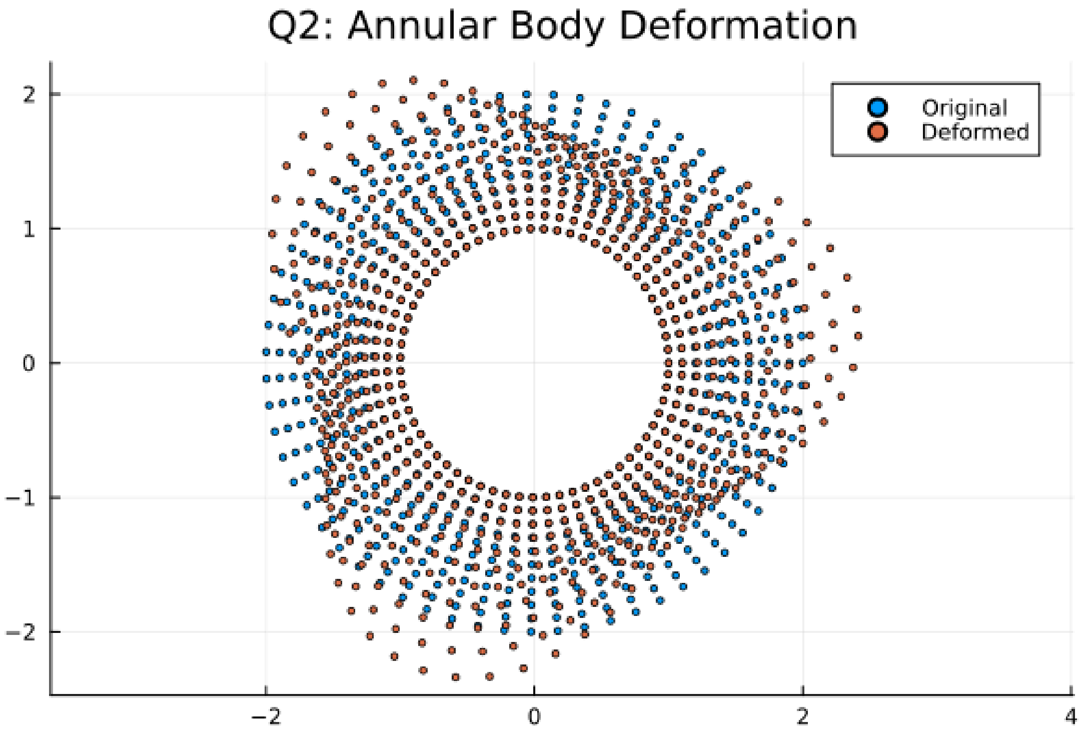
Question 2

In [2]:

```
using Plots

# 1. Define the annular domain using Polar Coordinates
R_range = 1.0:0.1:2.0
Theta_range = 0:0.1:2*pi

X1_orig, X2_orig = Float64[], Float64[]
x1_new, x2_new = Float64[], Float64[]

for r in R_range
    for theta in Theta_range
        # Convert Polar to Cartesian for Original Points [cite: 33]
        X1 = r * cos(theta)
        X2 = r * sin(theta)

        push!(X1_orig, X1)
        push!(X2_orig, X2)

        # Calculate Displacements in Polar Basis [cite: 29]
        u_r = 0.4 * (r - 1)^2 * cos(3 * theta)
        u_theta = 0.4 * (r - 1)^3

        # Transformation to Cartesian Basis
        # u1 = u_r * cos(theta) - u_theta * sin(theta)
        # u2 = u_r * sin(theta) + u_theta * cos(theta)
        # Derived from basis vector relations [cite: 25, 34]

        u1 = u_r * cos(theta) - u_theta * sin(theta)
        u2 = u_r * sin(theta) + u_theta * cos(theta)

        # Add displacement to original Cartesian coords
        push!(x1_new, X1 + u1)
        push!(x2_new, X2 + u2)
```

```
    end
end

# 3. Plotting
scatter(X1_orig, X2_orig, label="Original", aspect_ratio=:equal, markersize=2)
scatter!(x1_new, x2_new, label="Deformed", markersize=2)
title!("Q2: Annular Body Deformation")
```

Out[2]:



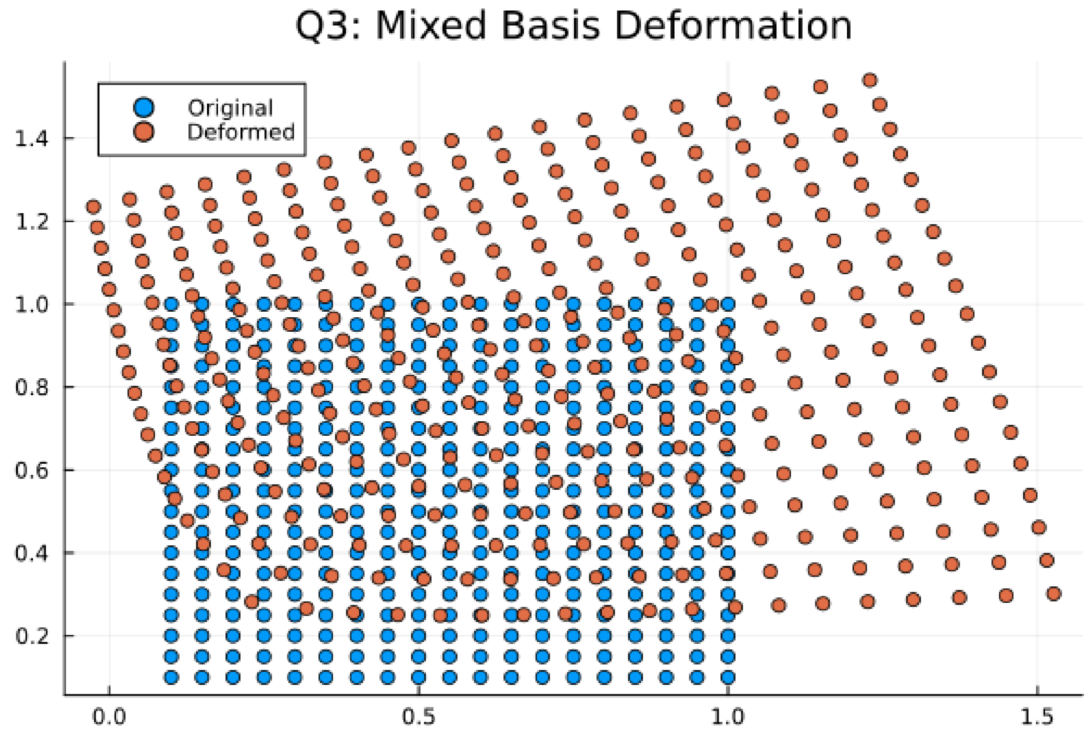
Question 3

In [3]:

```
using Plots

x1_range = 0.1:0.05:1.0
x2_range = 0.1:0.05:1.0

X1_orig, X2_orig = Float64[], Float64[]
x1_new, x2_new = Float64[], Float64[]

for X1 in x1_range
    for X2 in x2_range
        push!(X1_orig, X1)
        push!(X2_orig, X2)

        # Calculate local polar parameters for basis transformation [cite: 23]
        R = sqrt(X1^2 + X2^2)
        theta = atan(X2, X1)

        # Calculate displacements (given values) [cite: 38, 40]
        u_r_val = 0.2 * exp(X1)
        u_theta_val = 0.2 * log(1 + X1 + X2)

        # Convert to Cartesian components u1, u2 using local theta
        # u = u_r * e_r + u_theta * e_theta
        u1 = u_r_val * cos(theta) - u_theta_val * sin(theta)
        u2 = u_r_val * sin(theta) + u_theta_val * cos(theta)
```

```julia
        push!(x1_new, X1 + u1)
        push!(x2_new, X2 + u2)
    end
end

scatter(X1_orig, X2_orig, label="Original", legend=:topleft)
scatter!(x1_new, x2_new, label="Deformed")
title!("Q3: Mixed Basis Deformation")
```

Out[3]:



Question 4

In [6]:

```julia
import Gmsh: gmsh

# Initializing Gmsh
gmsh.initialize()

gmsh.model.add("PlateHole")

# --- Parameters (Converted to Meters to match your L=1 scale) ---
L = 1.0       # 1000 mm
H = 0.4       # 400 mm
R = 0.075     # Radius 75 mm
Xc = 0.5      # Center X (500 mm)
Yc = 0.2      # Center Y (200 mm)

# Mesh sizes
h_coarse = 0.05   # Coarse mesh for far boundaries
h_fine   = 0.005 # Fine mesh for the hole (CRITICAL for stress concentration)

# --- 1. Define the Outer Rectangle ---
p1 = gmsh.model.geo.addPoint(0, 0, 0, h_coarse)
p2 = gmsh.model.geo.addPoint(L, 0, 0, h_coarse)
p3 = gmsh.model.geo.addPoint(L, H, 0, h_coarse)
p4 = gmsh.model.geo.addPoint(0, H, 0, h_coarse)

l1 = gmsh.model.geo.addLine(p1, p2)
```

```python
l2 = gmsh.model.geo.addLine(p2, p3)
l3 = gmsh.model.geo.addLine(p3, p4)
l4 = gmsh.model.geo.addLine(p4, p1)

# Outer Loop
cl_rect = gmsh.model.geo.addCurveLoop([l1, l2, l3, l4])

# --- 2. Define the Inner Circle ---
# We create 5 points: Center + 4 points on the circumference (Top, Right, Bottom
# This method (Circle Arcs) is more robust for meshing than a single full circle

pc = gmsh.model.geo.addPoint(Xc, Yc, 0, h_fine) # Center
p_r = gmsh.model.geo.addPoint(Xc + R, Yc, 0, h_fine)
p_t = gmsh.model.geo.addPoint(Xc, Yc + R, 0, h_fine)
p_l = gmsh.model.geo.addPoint(Xc - R, Yc, 0, h_fine)
p_b = gmsh.model.geo.addPoint(Xc, Yc - R, 0, h_fine)

# Create 4 arcs to form the circle
c1 = gmsh.model.geo.addCircleArc(p_r, pc, p_t)
c2 = gmsh.model.geo.addCircleArc(p_t, pc, p_l)
c3 = gmsh.model.geo.addCircleArc(p_l, pc, p_b)
c4 = gmsh.model.geo.addCircleArc(p_b, pc, p_r)

# Inner Loop
cl_hole = gmsh.model.geo.addCurveLoop([c1, c2, c3, c4])

# --- 3. Create Surface with Hole ---
# The first loop is the boundary, subsequent loops are holes
s = gmsh.model.geo.addPlaneSurface([cl_rect, cl_hole])

# --- 4. Physical Groups (Boundary Conditions) ---
# Domain
gmsh.model.addPhysicalGroup(2, [s], 1, "Plate")

# Boundaries
gmsh.model.addPhysicalGroup(1, [l4], 2, "FixedLeft")  # Left Edge
gmsh.model.addPhysicalGroup(1, [l2], 3, "LoadRight")  # Right Edge
gmsh.model.addPhysicalGroup(1, [c1, c2, c3, c4], 4, "HoleEdge") # Hole boundary

# Synchronize and Mesh
gmsh.model.geo.synchronize()
gmsh.model.mesh.generate(2)

# Write mesh to file
gmsh.write("PlateHole.msh")

# Launch GUI to see the result
gmsh.fltk.run()
gmsh.finalize()
```

```
Info    : Meshing 1D...
Info    : [  0%] Meshing curve 1 (Line)
Info    : [ 20%] Meshing curve 2 (Line)
Info    : [ 30%] Meshing curve 3 (Line)
Info    : [ 40%] Meshing curve 4 (Line)
Info    : [ 60%] Meshing curve 5 (Circle)
Info    : [ 70%] Meshing curve 6 (Circle)
Info    : [ 80%] Meshing curve 7 (Circle)
Info    : [ 90%] Meshing curve 8 (Circle)
Info    : Done meshing 1D (Wall 0.00638914s, CPU 0.015625s)
Info    : Meshing 2D...
Info    : Meshing surface 1 (Plane, Frontal-Delaunay)
Info    : Done meshing 2D (Wall 0.0616488s, CPU 0.046875s)
Info    : 766 nodes 1539 elements
Info    : Writing 'PlateHole.msh'...
Info    : Done writing 'PlateHole.msh'
-------------------------------------------------------
Version       : 4.13.1
License       : GNU General Public License
Build OS      : Windows64-sdk
Build date    : 19700101
Build host    : amdci7.julia.csail.mit.edu
Build options : 64Bit ALGLIB[contrib] ANN[contrib] Bamg Blossom Cairo DIntegratio
n DomHex Eigen[contrib] Fltk GMP Gmm[contrib] Hxt Jpeg Kbipack MathEx[contrib] Me
sh Metis[contrib] Mmg Mpeg Netgen Nii2mesh NoSocklenT ONELAB ONELABMetamodel Open
CASCADE OpenCASCADE-CAF OpenGL OpenMP OptHom Parser Plugins Png Post QuadMeshingT
ools QuadTri Solver TetGen/BR TinyXML2[contrib] Untangle Voro++[contrib] WinslowU
ntangler Zlib
FLTK version  : 1.3.8
OCC version   : 7.7.2
Packaged by   : root
Web site      : https://gmsh.info
Issue tracker : https://gitlab.onelab.info/gmsh/gmsh/issues
-------------------------------------------------------
Warning : Unknown entity of dimension 2 and tag 1 in physical group 1
Warning : Unknown entity of dimension 1 and tag 4 in physical group 2
Warning : Unknown entity of dimension 1 and tag 2 in physical group 3
Warning : Unknown entity of dimension 1 and tag 5 in physical group 4
Warning : Unknown entity of dimension 1 and tag 6 in physical group 4
Warning : Unknown entity of dimension 1 and tag 7 in physical group 4
Warning : Unknown entity of dimension 1 and tag 8 in physical group 4
```

In [5]:

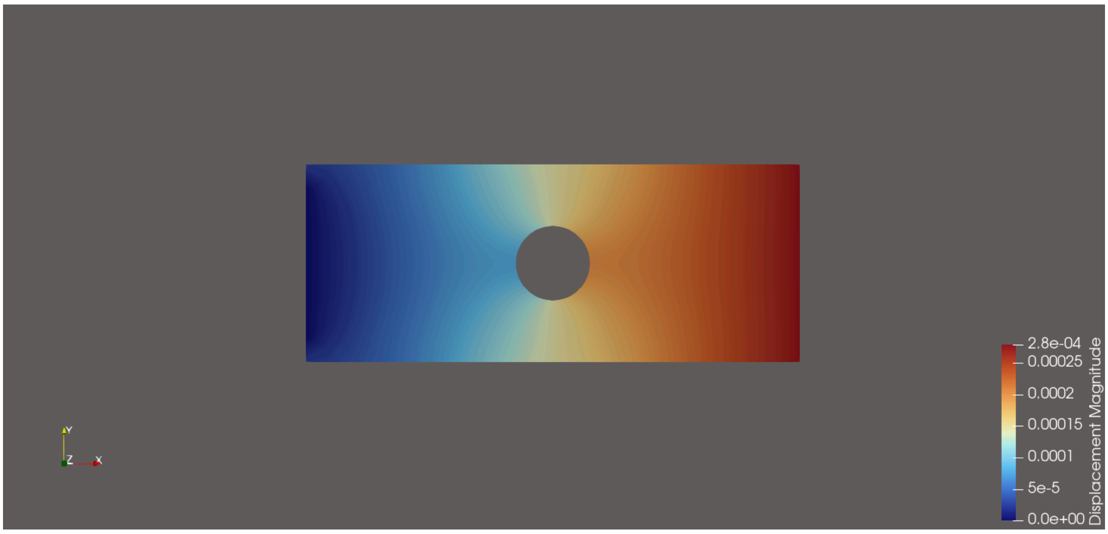
```julia
using Gridap
using GridapGmsh
using LinearAlgebra # Required for tr() and dot products

# 1. Read the Mesh
model = GmshDiscreteModel("PlateHole.msh")

# 2. Material Parameters (Steel)
const E = 210000.0  # MPa
const ν = 0.3       # Poisson's ratio
const thickness = 2.0 # mm

# Lamé parameters for Plane Stress
const μ = E / (2 * (1 + ν))
const λ = (E * ν) / ((1 + ν) * (1 - 2 * ν))
const λ_ps = (2 * λ * μ) / (λ + 2 * μ) # Plane stress correction

# Pre-define Identity Tensor for 2D
```

```julia
# We use TensorValue(xx, xy, yx, yy) instead of SymTensorValue to avoid import e
const I = TensorValue(1.0, 0.0, 0.0, 1.0)

# Constitutive Law: σ = C : ε
function σ(ε)
    return λ_ps * tr(ε) * I + 2 * μ * ε
end

# 3. Define FE Spaces
reffe = ReferenceFE(lagrangian, VectorValue{2, Float64}, 1)
V = TestFESpace(model, reffe, conformity=:H1, dirichlet_tags="FixedLeft")
U = TrialFESpace(V, VectorValue(0.0, 0.0))

# 4. Numerical Integration
degree = 2
Ω = Triangulation(model)
dΩ = Measure(Ω, degree)

Γ = BoundaryTriangulation(model, tags="LoadRight")
dΓ = Measure(Γ, degree)

# 5. Define Weak Form
# Traction: 50 MPa in X direction
t_vec = VectorValue(50.0, 0.0)

# Internal Work (Stiffness)
a(u, v) = ∫( ε(v) ⊙ (σ ∘ ε(u)) ) * dΩ

# External Work (Load)
l(v) = ∫( v ⋅ t_vec ) * dΓ

# 6. Solve
op = AffineFEOperator(a, l, U, V)
uh = solve(op)

# 7. Post-Process
writevtk(Ω, "results_PlateHole", cellfields=["Displacement" => uh, "Stress" => c

println("Analysis Complete! Open 'results_PlateHole.vtu' in Paraview.")
```

```
Info    : Reading 'PlateHole.msh'...
Info    : 18 entities
Info    : 765 nodes
Info    : 1490 elements
Info    : Done reading 'PlateHole.msh'
Analysis Complete! Open 'results_PlateHole.vtu' in Paraview.
```

In [7]:
```julia
writevtk(Ω, "results_PlateHole", cellfields=["Displacement" => uh, "Stress" => c
```

Out[7]:
```
(["results_PlateHole.vtu"],)
```



Screenshot attached from paraview

Question 5

In [8]:

```julia
using Gmsh
using Gridap
using GridapGmsh
using LinearAlgebra

# ==========================================
# PART 1: MESH GENERATION
# ==========================================
function generate_mesh()
    mesh_file = "cantilever_q5.msh"

    # 1. Force cleanup of old files
    if isfile(mesh_file)
        rm(mesh_file)
        println("Deleted old mesh file to ensure clean generation.")
    end

    # 2. Initialize Gmsh
    if gmsh.isInitialized() == 1
        gmsh.finalize()
    end
    gmsh.initialize()
    gmsh.option.setNumber("General.Terminal", 1)
    gmsh.model.add("Cantilever_Patch")

    # 3. Geometry Parameters
    L = 1000.0
    H = 200.0
    W = 250.0
    PatchRadius = 5.0 # 5mm radius patch

    # 4. Create Geometry
    println("Generating geometry...")

    # A. Beam Box
    beam_tag = gmsh.model.occ.addBox(0, 0, 0, L, H, W)
```

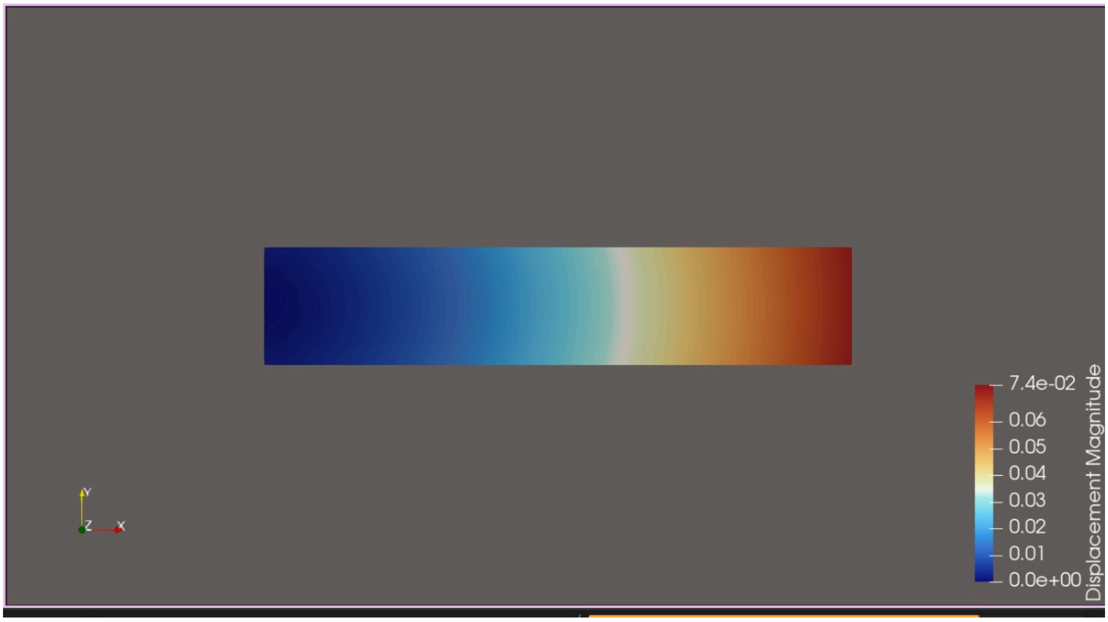
```
    # B. Load Patch Disk
    # Created at (L, H/2, W/2). By default, addDisk is in the XY plane (Normal Z
    disk_tag = gmsh.model.occ.addDisk(L, H/2.0, W/2.0, PatchRadius, PatchRadius)

    # CRITICAL FIX: Rotate the disk 90 degrees (pi/2) around the Y-axis.
    # This aligns its normal from Z (0,0,1) to X (1,0,0), matching the beam face
    # Center of rotation: (L, H/2, W/2)
    gmsh.model.occ.rotate([(2, disk_tag)], L, H/2.0, W/2.0, 0, 1, 0, pi/2)

    # C. Fragment (Imprint disk onto beam)
    # This merges the aligned disk into the beam face.
    gmsh.model.occ.fragment([(3, beam_tag)], [(2, disk_tag)])
    gmsh.model.occ.synchronize()

    # 5. Physical Groups

    # A. Fixed End (X=0)
    fixed_surfaces = gmsh.model.getEntitiesInBoundingBox(-0.1, -0.1, -0.1, 0.1,
    if isempty(fixed_surfaces)
        error("Generation Failed: Fixed end not found.")
    end
    gmsh.model.addPhysicalGroup(2, [e[2] for e in fixed_surfaces], 1, "fixedEnd"
    gmsh.model.setPhysicalName(2, 1, "fixedEnd")

    # B. Load Patch (X=L, Center)
    # Because we rotated the disk, it now lies flat on X=L.
    # We search for the surface strictly inside the patch area to distinguish it
    tol = 0.1
    load_surfaces = gmsh.model.getEntitiesInBoundingBox(
        L-0.1, H/2.0 - PatchRadius - tol, W/2.0 - PatchRadius - tol,
        L+0.1, H/2.0 + PatchRadius + tol, W/2.0 + PatchRadius + tol, 2
    )

    if isempty(load_surfaces)
        # Fallback debug if specific bound check fails
        error("Generation Failed: Load patch surface not found. The boolean frag
    else
        println("Load Patch Surface Found: ", [e[2] for e in load_surfaces])
    end

    gmsh.model.addPhysicalGroup(2, [e[2] for e in load_surfaces], 2, "loadPatch"
    gmsh.model.setPhysicalName(2, 2, "loadPatch")

    # C. Beam Volume
    vol_tags = [e[2] for e in gmsh.model.getEntities(3)]
    gmsh.model.addPhysicalGroup(3, vol_tags, 3, "beam")
    gmsh.model.setPhysicalName(3, 3, "beam")

    # 6. Mesh Settings
    # Refine on patch
    gmsh.model.mesh.setSize(gmsh.model.getBoundary(load_surfaces, false, false,
    gmsh.option.setNumber("Mesh.CharacteristicLengthMin", 25)
    gmsh.option.setNumber("Mesh.CharacteristicLengthMax", 50)

    # 7. Generate and Write
    gmsh.model.mesh.generate(3)
    gmsh.write(mesh_file)
    gmsh.finalize()
    println("SUCCESS: New mesh generated at $mesh_file")
end
```

```julia
# ==========================================
# PART 2: FEM SOLVER
# ==========================================
function solve_fem()
    mesh_file = "cantilever_q5.msh"
    # mesh_file = "CantileverBeam.msh"
    if !isfile(mesh_file)
        error("Mesh file missing despite generation attempt.")
    end

    println("\nLoading mesh into Gridap...")
    model = GmshDiscreteModel(mesh_file)

    # Verify Tags
    labels = get_face_labeling(model)
    tags = labels.tag_to_name
    println("Tags detected in mesh: ", tags)
    if !("loadPatch" in tags)
        error("CRITICAL: 'loadPatch' tag still missing.")
    end

    # Material
    E = 25000.0; ν = 0.2
    λ = (E * ν) / ((1 + ν) * (1 - 2 * ν))
    μ = E / (2 * (1 + ν))
    σ(ε) = λ * tr(ε) * one(ε) + 2 * μ * ε

    # FE Spaces
    reffe = ReferenceFE(lagrangian, VectorValue{3, Float64}, 1)
    V = TestFESpace(model, reffe, conformity=:H1, dirichlet_tags="fixedEnd")
    U = TrialFESpace(V, VectorValue(0.0, 0.0, 0.0))

    # Measures
    Ω = Triangulation(model)
    dΩ = Measure(Ω, 2)
    Γ_load = BoundaryTriangulation(model, tags="loadPatch")
    dΓ_load = Measure(Γ_load, 2)

    # Distributed Load Calculation
    force_total = 1000.0 # N
    patch_radius = 5.0
    patch_area = π * patch_radius^2
    traction_val = force_total / patch_area
    traction = VectorValue(0.0, -traction_val, 0.0) # Downwards

    println("Applying Traction: $traction_val N/mm^2 on patch.")

    # Solver
    a(u, v) = ∫( ε(v) ⊙ (σ ∘ ε(u)) )dΩ
    l(v) = ∫( v ⋅ traction )dΓ_load

    println("Solving system...")
    op = AffineFEOperator(a, l, U, V)
    uh = solve(op)
    println("Solution Found.")

    # Export
    σ_h = σ ∘ ε(uh)
```

```
    # Manual Von Mises function
    function vm_manual(σ_t)
        s = σ_t - (tr(σ_t)/3)*one(σ_t)
        return sqrt(1.5 * (s ⊙ s))
    end

    output_file = "cantilever_q5_solution"
    # output_file = "cantilever_solution"
    writevtk(Ω, output_file, cellfields=[
        "Displacement" => uh,
        "Strain" => ε(uh),
        "Stress" => σ_h,
        "VonMises" => vm_manual ∘ σ_h
    ])
    println("Results saved to $output_file.vtu")
end

# ==========================================
# EXECUTION
# ==========================================
generate_mesh()
solve_fem()
```

```
Generating geometry...
Load Patch Surface Found: Int32[7]ting solids
Info    : Meshing 1D...
Info    : [  0%] Meshing curve 13 (Ellipse)
Info    : [ 10%] Meshing curve 14 (Line)
Info    : [ 20%] Meshing curve 15 (Line)
Info    : [ 30%] Meshing curve 16 (Line)
Info    : [ 40%] Meshing curve 17 (Line)
Info    : [ 40%] Meshing curve 18 (Line)
Info    : [ 50%] Meshing curve 19 (Line)
Info    : [ 60%] Meshing curve 20 (Line)
Info    : [ 70%] Meshing curve 21 (Line)
Info    : [ 70%] Meshing curve 22 (Line)
Info    : [ 80%] Meshing curve 23 (Line)
Info    : [ 90%] Meshing curve 24 (Line)
Info    : [100%] Meshing curve 25 (Line)
Info    : Done meshing 1D (Wall 0.00244498s, CPU 0s)
Info    : Meshing 2D...
Info    : [  0%] Meshing surface 7 (Plane, Frontal-Delaunay)
Info    : [ 20%] Meshing surface 8 (Plane, Frontal-Delaunay)
Info    : [ 30%] Meshing surface 9 (Plane, Frontal-Delaunay)
Info    : [ 50%] Meshing surface 10 (Plane, Frontal-Delaunay)
Info    : [ 60%] Meshing surface 11 (Plane, Frontal-Delaunay)
Info    : [ 80%] Meshing surface 12 (Plane, Frontal-Delaunay)
Info    : [ 90%] Meshing surface 13 (Plane, Frontal-Delaunay)
Info    : Done meshing 2D (Wall 0.0601931s, CPU 0.0625s)
Info    : Meshing 3D...
Info    : 3D Meshing 1 volume with 1 connected component
Info    : Tetrahedrizing 566 nodes...
Info    : Done tetrahedrizing 574 nodes (Wall 0.015733s, CPU 0s)
Info    : Reconstructing mesh...
Info    :  - Creating surface mesh
Info    :  - Identifying boundary edges
Info    :  - Recovering boundary
Info    : Done reconstructing mesh (Wall 0.031929s, CPU 0.015625s)
Info    : Found volume 1
Info    : It. 0 - 0 nodes created - worst tet radius 6.0106 (nodes removed 0 0)
Info    : 3D refinement terminated (726 nodes total):
Info    :  - 0 Delaunay cavities modified for star shapeness
Info    :  - 0 nodes could not be inserted
Info    :  - 2586 tetrahedra created in 0.01948 sec. (132751 tets/s)
Info    : 0 node relocations
Info    : Done meshing 3D (Wall 0.0993979s, CPU 0.0625s)
Info    : Optimizing mesh...
Info    : Optimizing volume 1
Info    : Optimization starts (volume = 5e+07) with worst = 0.0249179 / average =
0.741669:
Info    : 0.00 < quality < 0.10 :        12 elements
Info    : 0.10 < quality < 0.20 :        30 elements
Info    : 0.20 < quality < 0.30 :        31 elements
Info    : 0.30 < quality < 0.40 :        61 elements
Info    : 0.40 < quality < 0.50 :        80 elements
Info    : 0.50 < quality < 0.60 :       165 elements
Info    : 0.60 < quality < 0.70 :       459 elements
Info    : 0.70 < quality < 0.80 :       646 elements
Info    : 0.80 < quality < 0.90 :       749 elements
Info    : 0.90 < quality < 1.00 :       350 elements
Info    : 72 edge swaps, 1 node relocations (volume = 5e+07): worst = 0.300048 /
average = 0.758596 (Wall 0.00387096s, CPU 0s)
Info    : No ill-shaped tets in the mesh :-)
```

```
Info    : 0.00 < quality < 0.10 :         0 elements
Info    : 0.10 < quality < 0.20 :         0 elements
Info    : 0.20 < quality < 0.30 :         0 elements
Info    : 0.30 < quality < 0.40 :        65 elements
Info    : 0.40 < quality < 0.50 :        81 elements
Info    : 0.50 < quality < 0.60 :       156 elements
Info    : 0.60 < quality < 0.70 :       458 elements
Info    : 0.70 < quality < 0.80 :       652 elements
Info    : 0.80 < quality < 0.90 :       758 elements
Info    : 0.90 < quality < 1.00 :       348 elements
Info    : Done optimizing mesh (Wall 0.008605s, CPU 0s)
Info    : 726 nodes 3781 elements
Info    : Writing 'cantilever_q5.msh'...
Info    : Done writing 'cantilever_q5.msh'
SUCCESS: New mesh generated at cantilever_q5.msh

Loading mesh into Gridap...
Info    : Reading 'cantilever_q5.msh'...
Info    : 30 entities
Info    : 726 nodes
Info    : 2580 elements
Info    : Done reading 'cantilever_q5.msh'
Tags detected in mesh: ["fixedEnd", "loadPatch", "beam"]
Applying Traction: 12.732395447351626 N/mm^2 on patch.
Solving system...
Solution Found.
Results saved to cantilever_q5_solution.vtu
```



Question 6

In [10]:
```
using Gmsh

# Initialize Gmsh
gmsh.initialize()

# --- 1. Define Geometry Parameters ---
# Based on Question 6:
# Length = 1000 mm
```

```
# Cross-section = 250 mm (Width) x 200 mm (Height)
# Load is on the "Top Surface"
L = 1000.0
H = 200.0
W = 250.0

# --- 2. Create Geometry (OpenCASCADE Kernel) ---
# We create a box with corner at (0,0,0)
# Dimensions: dx=L, dy=H, dz=W
# Alignment:
# X-axis: Length (0 to 1000)
# Y-axis: Height (0 to 200)
# Z-axis: Width  (0 to 250)

gmsh.model.occ.addBox(0, 0, 0, L, H, W, 1)

# Synchronize CAD with Gmsh model
gmsh.model.occ.synchronize()

# --- 3. Define Physical Groups ---
# This is crucial for Gridap to apply Boundary Conditions.
# We need to group the geometric entities (Surfaces/Volumes) and give them names

# Helper function to find surfaces based on position
function get_surfaces_in_box(xmin, ymin, zmin, xmax, ymax, zmax)
    surfaces = gmsh.model.getEntities(2) # Get all surfaces (dim=2)
    selected_tags = Int[]

    for (dim, tag) in surfaces
        # Get bounding box of the surface
        bb = gmsh.model.getBoundingBox(dim, tag)
        # bb returns (xmin, ymin, zmin, xmax, ymax, zmax)

        # Check if the surface is approximately inside our target box (using a s
        tol = 0.1
        if (bb[1] >= xmin - tol) && (bb[4] <= xmax + tol) &&
           (bb[2] >= ymin - tol) && (bb[5] <= ymax + tol) &&
           (bb[3] >= zmin - tol) && (bb[6] <= zmax + tol)
            push!(selected_tags, tag)
        end
    end
    return selected_tags
end

# A. The Beam Volume (for material properties)
# Get all volume entities (dim=3)
vol_tags = [t[2] for t in gmsh.model.getEntities(3)]
gmsh.model.addPhysicalGroup(3, vol_tags, 1, "beam")

# B. The Fixed End (Support)
# Located at X = 0. It is the face in the plane X=0.
# Bounding box for check: X:[0,0], Y:[0,H], Z:[0,W]
fixed_end_tags = get_surfaces_in_box(0, 0, 0, 0.1, H, W)
gmsh.model.addPhysicalGroup(2, fixed_end_tags, 2, "fixedEnd")

# C. The Top Surface (Load Application)
# Located at Y = H (200). It is the face in the plane Y=200.
# Bounding box for check: X:[0,L], Y:[H,H], Z:[0,W]
top_surface_tags = get_surfaces_in_box(0, H-0.1, 0, L, H+0.1, W)
gmsh.model.addPhysicalGroup(2, top_surface_tags, 3, "topSurface")
```

```
# --- 4. Mesh Settings ---
# Set element size (Characteristic Length)
# Adjust Min/Max to control mesh fineness.
# A value of ~30-50mm is coarse but fast; 10-20mm is finer.
gmsh.option.setNumber("Mesh.CharacteristicLengthMin", 25)
gmsh.option.setNumber("Mesh.CharacteristicLengthMax", 50)

# --- 5. Generate and Save Mesh ---
gmsh.model.mesh.generate(3) # Generate 3D mesh

output_filename = "cantilever_q6.msh"
gmsh.write(output_filename)

println("Mesh generated successfully: $output_filename")

# Finalize Gmsh
gmsh.finalize()

# import Gmsh: gmsh

# # Initializing Gmsh
# gmsh.initialize()

# gmsh.model.add("CantileverBeam")

# # --- Parameters (Converted to Meters) ---
# L = 1.0      # Length 1000 mm
# W = 0.25     # Width 250 mm
# H = 0.20     # Height 200 mm

# # Mesh Size
# lc = 0.05    # Characteristic length

# # --- 1. Define the Cross-Section (Fixed End at X=0) ---
# p1 = gmsh.model.geo.addPoint(0, 0, 0, lc)
# p2 = gmsh.model.geo.addPoint(0, W, 0, lc)
# p3 = gmsh.model.geo.addPoint(0, W, H, lc)
# p4 = gmsh.model.geo.addPoint(0, 0, H, lc)

# l1 = gmsh.model.geo.addLine(p1, p2)
# l2 = gmsh.model.geo.addLine(p2, p3)
# l3 = gmsh.model.geo.addLine(p3, p4)
# l4 = gmsh.model.geo.addLine(p4, p1)

# cl_base = gmsh.model.geo.addCurveLoop([l1, l2, l3, l4])
# s_fixed = gmsh.model.geo.addPlaneSurface([cl_base])

# # --- 2. Extrude to Create Volume ---
# # Extrude surface along X-axis by length L
# # In Julia, the input is a vector of tuples [(dim, tag)]
# extrusion = gmsh.model.geo.extrude([(2, s_fixed)], L, 0, 0)

# # extrusion vector mapping (1-based indexing in Julia):
# # [1] = Volume
# # [2] = End Cap Surface (X=L)
# # [3+] = Side Surfaces

# vol_tag = extrusion[1][2]   # Tag of the Volume
# s_free  = extrusion[2][2]   # Tag of the End Cap Surface (X=L)
```

```
# # --- 3. Physical Groups ---
# # 1. The Beam Volume
# gmsh.model.addPhysicalGroup(3, [vol_tag], 1, "Beam")

# # 2. The Fixed Boundary (Left face at X=0)
# gmsh.model.addPhysicalGroup(2, [s_fixed], 2, "FixedBase")

# # 3. The Free End (Right face at X=L)
# gmsh.model.addPhysicalGroup(2, [s_free], 3, "FreeEnd")

# # --- Synchronize and Mesh ---
# gmsh.model.geo.synchronize()
# gmsh.model.mesh.generate(3)

# # Write mesh to file
# gmsh.write("CantileverBeam.msh")

# # Launch GUI (Julia checks ARGS for command line flags)
# if !("-nopopup" in ARGS)
#     gmsh.fltk.run()
# end

# gmsh.finalize()
```

```
Info    : Meshing 1D...
Info    : [  0%] Meshing curve 1 (Line)
Info    : [ 10%] Meshing curve 2 (Line)
Info    : [ 20%] Meshing curve 3 (Line)
Info    : [ 30%] Meshing curve 4 (Line)
Info    : [ 40%] Meshing curve 5 (Line)
Info    : [ 50%] Meshing curve 6 (Line)
Info    : [ 60%] Meshing curve 7 (Line)
Info    : [ 60%] Meshing curve 8 (Line)
Info    : [ 70%] Meshing curve 9 (Line)
Info    : [ 80%] Meshing curve 10 (Line)
Info    : [ 90%] Meshing curve 11 (Line)
Info    : [100%] Meshing curve 12 (Line)
Info    : Done meshing 1D (Wall 0.00120401s, CPU 0s)
Info    : Meshing 2D...
Info    : [  0%] Meshing surface 1 (Plane, Frontal-Delaunay)
Info    : [ 20%] Meshing surface 2 (Plane, Frontal-Delaunay)
Info    : [ 40%] Meshing surface 3 (Plane, Frontal-Delaunay)
Info    : [ 60%] Meshing surface 4 (Plane, Frontal-Delaunay)
Info    : [ 70%] Meshing surface 5 (Plane, Frontal-Delaunay)
Info    : [ 90%] Meshing surface 6 (Plane, Frontal-Delaunay)
Info    : Done meshing 2D (Wall 0.0457392s, CPU 0.046875s)
Info    : Meshing 3D...
Info    : 3D Meshing 1 volume with 1 connected component
Info    : Tetrahedrizing 510 nodes...
Info    : Done tetrahedrizing 518 nodes (Wall 0.0132098s, CPU 0s)
Info    : Reconstructing mesh...
Info    :  - Creating surface mesh
Info    :  - Identifying boundary edges
Info    :  - Recovering boundary
Info    : Done reconstructing mesh (Wall 0.026196s, CPU 0s)
Info    : Found volume 1
Info    : It. 0 - 0 nodes created - worst tet radius 2.18371 (nodes removed 0 0)
Info    : 3D refinement terminated (644 nodes total):
Info    :  - 0 Delaunay cavities modified for star shapeness
Info    :  - 0 nodes could not be inserted
Info    :  - 2242 tetrahedra created in 0.018923 sec. (118479 tets/s)
Info    : 0 node relocations
Info    : Done meshing 3D (Wall 0.0826671s, CPU 0.0625s)
Info    : Optimizing mesh...
Info    : Optimizing volume 1
Info    : Optimization starts (volume = 5e+07) with worst = 0.0256711 / average = 
0.760657:
Info    : 0.00 < quality < 0.10 :         9 elements
Info    : 0.10 < quality < 0.20 :        28 elements
Info    : 0.20 < quality < 0.30 :        19 elements
Info    : 0.30 < quality < 0.40 :        29 elements
Info    : 0.40 < quality < 0.50 :        43 elements
Info    : 0.50 < quality < 0.60 :        99 elements
Info    : 0.60 < quality < 0.70 :       395 elements
Info    : 0.70 < quality < 0.80 :       586 elements
Info    : 0.80 < quality < 0.90 :       694 elements
Info    : 0.90 < quality < 1.00 :       335 elements
Info    : 56 edge swaps, 0 node relocations (volume = 5e+07): worst = 0.303751 / 
average = 0.776857 (Wall 0.00220275s, CPU 0s)
Info    : No ill-shaped tets in the mesh :-)
Info    : 0.00 < quality < 0.10 :         0 elements
Info    : 0.10 < quality < 0.20 :         0 elements
Info    : 0.20 < quality < 0.30 :         0 elements
Info    : 0.30 < quality < 0.40 :        29 elements
```

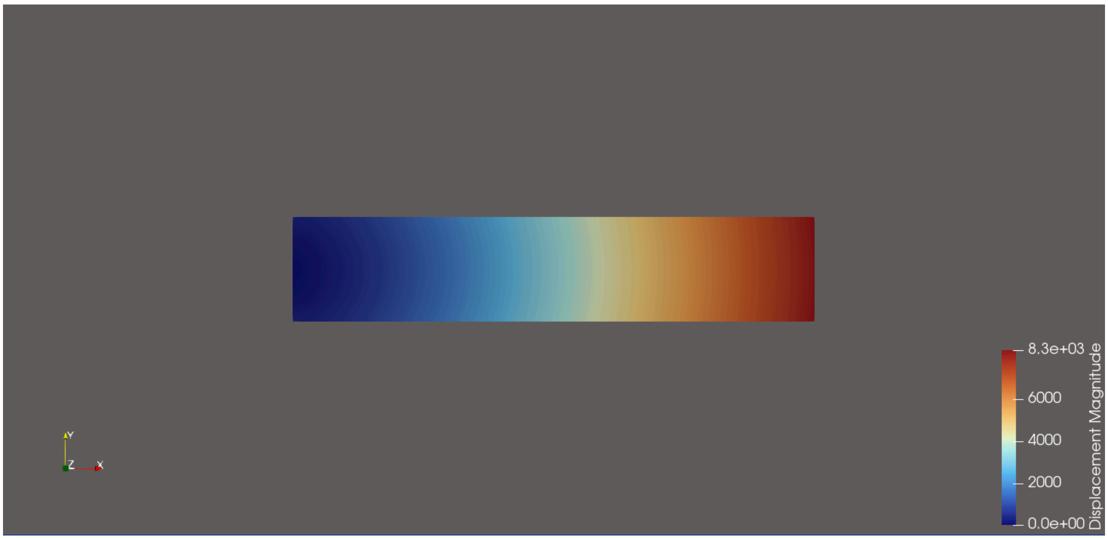
```
Info    : 0.40 < quality < 0.50 :        36 elements
Info    : 0.50 < quality < 0.60 :       101 elements
Info    : 0.60 < quality < 0.70 :       392 elements
Info    : 0.70 < quality < 0.80 :       593 elements
Info    : 0.80 < quality < 0.90 :       700 elements
Info    : 0.90 < quality < 1.00 :       334 elements
Info    : Done optimizing mesh (Wall 0.00536895s, CPU 0s)
Info    : 644 nodes 3330 elements
Info    : Writing 'cantilever_q6.msh'...
Info    : Done writing 'cantilever_q6.msh'
Mesh generated successfully: cantilever_q6.msh
```

In [11]:

```julia
using Gridap
using GridapGmsh
using LinearAlgebra

# --- 1. Load the Discrete Model ---
mesh_file = "cantilever_q6.msh"
# mesh_file = "CantileverBeam.msh"
if !isfile(mesh_file)
    error("Mesh file '$mesh_file' not found. Please run the mesh generation scri
end

model = GmshDiscreteModel(mesh_file)

# --- 2. Define Material Properties ---
E = 25000.0 # Young's Modulus (N/mm^2)
ν = 0.2     # Poisson's Ratio

# Lamé parameters
λ = (E * ν) / ((1 + ν) * (1 - 2 * ν))
μ = E / (2 * (1 + ν))

# Constitutive Law: Linear Elasticity
# FIXED: Defined as a standard function instead of using @law macro
function σ(ε)
    return λ * tr(ε) * one(ε) + 2 * μ * ε
end

# --- 3. Define FE Spaces ---
reffe = ReferenceFE(lagrangian, VectorValue{3, Float64}, 1)

# Test Space (V) - Fixed at "fixedEnd"
V = TestFESpace(model, reffe, conformity=:H1, dirichlet_tags="fixedEnd")

# Trial Space (U) - 0 displacement at boundaries
U = TrialFESpace(V, VectorValue(0.0, 0.0, 0.0))

# --- 4. Define Integration Measures ---
Ω = Triangulation(model)
dΩ = Measure(Ω, 2)

Γ_top = BoundaryTriangulation(model, tags="topSurface")
dΓ_top = Measure(Γ_top, 2)

# --- 5. Define Weak Form ---
force_density = VectorValue(0.0, -1000.0, 0.0)

# Bilinear form a(u, v)
# FIXED: Used composition (σ ∘ ε(u)) explicitly
```

```julia
a(u, v) = ∫( ε(v) ⊙ (σ ∘ ε(u)) )dΩ

# Linear form l(v)
l(v) = ∫( v ⋅ force_density )dΓ_top

# --- 6. Solve ---
println("Solving linear elasticity problem...")
op = AffineFEOperator(a, l, U, V)
uh = solve(op)
println("Solution computed.")

# --- 7. Post-Processing (Paraview Export) ---
# Compute stress field
σ_h = σ ∘ ε(uh)

# Helper for Von Mises
function von_mises(σ_tensor)
    # Calculate deviatoric stress manually: s = σ - (tr(σ)/3)*I
    # This ensures compatibility even if 'dev' is not exported
    tr_σ = tr(σ_tensor)
    s = σ_tensor - (tr_σ / 3.0) * one(σ_tensor)
    return sqrt(1.5 * (s ⊙ s))
end

output_file = "cantilever_q6_solution"
println("Writing output to $output_file.vtu ...")

writevtk(Ω, output_file, cellfields=[
    "Displacement" => uh,
    "Strain" => ε(uh),
    "Stress" => σ_h,
    "VonMises" => von_mises ∘ σ_h
])

println("Done! Open '$output_file.vtu' in Paraview.")
```

```
Info    : Reading 'cantilever_q6.msh'...
Info    : 27 entities
Info    : 644 nodes
Info    : 2490 elements
Info    : Done reading 'cantilever_q6.msh'
Solving linear elasticity problem...
WARNING: redefinition of constant Main.E. This may fail, cause incorrect answers,
or produce other errors.
WARNING: redefinition of constant Main.v. This may fail, cause incorrect answers,
or produce other errors.
WARNING: redefinition of constant Main.λ. This may fail, cause incorrect answers,
or produce other errors.
WARNING: redefinition of constant Main.μ. This may fail, cause incorrect answers,
or produce other errors.
Solution computed.
Writing output to cantilever_q6_solution.vtu ...
Done! Open 'cantilever_q6_solution.vtu' in Paraview.
```



8.3e+03
6000
4000
2000
0.0e+00
Displacement Magnitude
Y
Z
X