Analyzing Retirement Portfolios with an ESG

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Abstract

Effective retirement and financial planning requires tools to model and simulate future financial markets. We build an Economic Scenario Generator (ESG) to simulate these potential future markets. Components of the ESG include but are not limited to inflation, interest rates, equity returns, and bond returns. This ESG provides the tools needed to analyze longevity risk for specific financial situations, and in turn helps answer questions like "How much should be annuitized?" and "What level of spending is sustainable?"

1 Introduction

Financial and retirement plans are constructed based on a model of the future economy. The effectiveness of a specific plan can be evaluated by how well it performs given the model of the future economy. It follows then that the quality of these financial plan evaluations depends mostly on the quality of the model being used to represent states of the future economy. A poor model of the future will not yield reliable results regarding financial plan evaluations.

Currently, the financial planning sector widely uses deterministic techniques in their future economy modeling. Economic factors such as inflation, equity returns, and interest rates are given a fixed value. A popular approach for determining these values is to take averages from the past. These fixed values are then applied to a financial plan and the success of the plan can be evaluated.

A more thorough technique for future modeling includes directly using economic variable paths from the past. In this technique the model for the future is fully determined by real values from the past economy. The quality of a financial plan is determined by then "applying" the plan to a specific year from the past and evaluating how well it would have performed given the past economic factors. This process can then be iterated using different starting points, representing different cohorts. This technique tells us exactly how a financial plan would have performed in the past and it may be reasonable to posit that this gives planners a good understanding of how the plan will perform in the future.

However, both of these approaches are deterministic and, by nature, ignore the uncertainty and volatility of future financial markets. To more thoroughly understand the performance of a financial plan, we propose a stochastic process through the use of an Economic Scenario Generator (ESG). ESG's simulate future economic scenarios by modeling, jointly, economic variables and financial market values. ESG's are widely used in the financial and actuarial world to price financial products and quantify the risk associated with financial markets. We extend the application of ESG's to the personal financial planning sector and build an ESG specifically for the purpose

of evaluating personal financial plans. With an ESG that properly models underlying economic variables and their uncertainty, we can better quantify the risk associated with a given financial plan.

The rest of the paper is organized as follows: Section 2 introduces the overall structure of the proposed ESG, and explains each piece of the ensemble model of the economy.

2 Economic Scenario Generator

Many different ESG structures have been proposed in the financial literature. Our approach borrows heavily from existing models for the individual components, and while it does not introduce novel modeling techniques, it is uniquely structured to address the specific financial planning questions we aim to answer. In our ESG, we model inflation, interest rates, equity returns, fixed-income returns, and mortality rates. To account for dependencies between these variables, we adopt a cascade-style structure. This structure is illustrated in Figure 1 below.
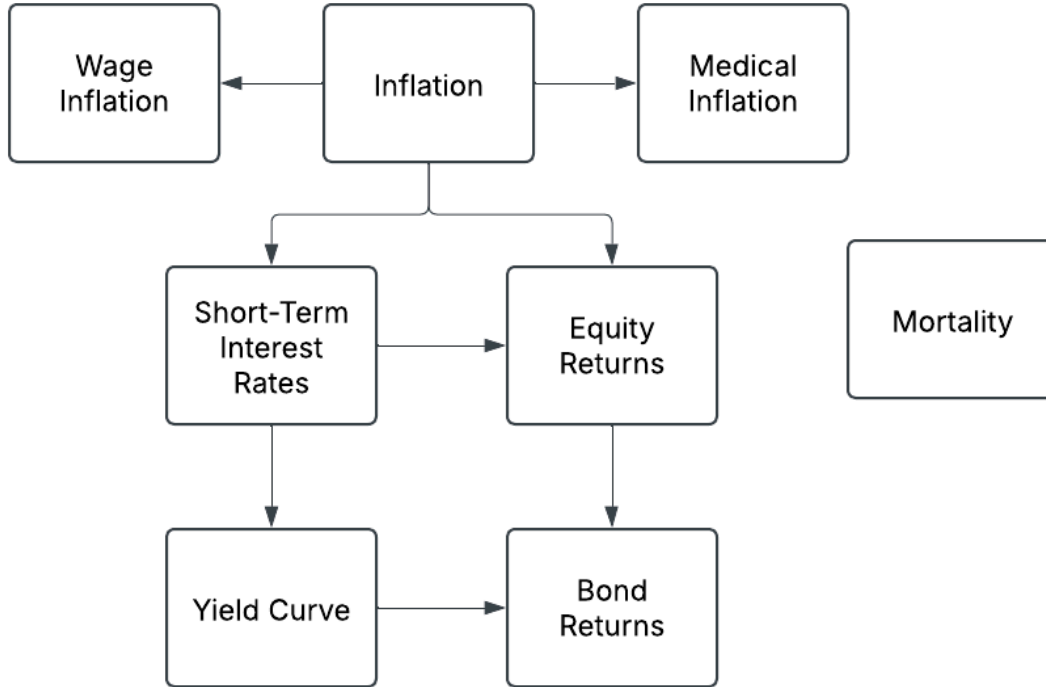

Figure 1: Proposed Cascade-Structure ESG

2.1 Model Selection Criteria

As previously alluded to, models for the ESG were primarily based on models used in existing ESG literature. For our use case, factors most important in model selection (ordered by relative importance) are

a) the behavior of simulated draws

b) model fit

c) model parsimony

As is described in Begin 2021, from an out-of-sample perspective, simpler models underlying an ESG perform better or similar to complex methods. Because of this, we try to keep our models as parsimonious as possible while still maintaining good model fit and reasonable simulated draws.

2.2 Inflation Models

The base level of our cascade-structure ESG is inflation. Unique to our use-case, we are interested in modeling general inflation, medical inflation, and wage inflation separately. Because we are modeling medical inflation separately and general inflation measures normally include cost of medical goods, we should note that we are modeling general inflation excluding medical goods.

2.2.1 General Inflation

We measure general inflation with the consumer price index (CPI) without medical goods. Let CPI_t denote the index at time t. Because CPI is non-stationary and generally increases over time, we model changes in CPI rather than its level. Specifically, we consider the log-ratio:

$$q_t = \log \frac{\text{CPI}_t}{\text{CPI}_{t-1}}$$

We assume that q_t follows an ARMA(1,1) process with constant mean, defined as:

$$q_t = \mu + \phi q_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$$

$$\epsilon_t \sim N(0, \sigma^2)$$

Here ϕ represents the auto-regressive component and θ represents the moving average component, both of order 1.

2.2.2 Medical Inflation

We measure medical inflation using a consumer price index for just medical care. Let MedCPI_t denote the index at time t. The same way we modeled general inflation, we model medical inflation as a log-ratio:

$$m_t = \log \frac{\text{MedCPI}_t}{\text{MedCPI}_{t-1}}$$

Where m_t follows an ARMA(1,1) process with a constant mean.

2.2.3 Wage Inflation

We measure wage inflation using the employee cost index (ECI). Let ECI_t denote the index at time t. Similar to the previous two inflation measures, we take the log-ratio of wage inflation:

$$w_t = \log \frac{\text{ECI}_t}{\text{ECI}_{t-1}}$$

We found, however, that the w_t series was still non-stationary per the Augmented Dickey-Fuller Test (p-value = .5703), rendering potential fitted coefficients for an ARMA process unreliable. To account for this we difference w_t and model:

$$\Delta w_t = w_t - w_{t-1}$$

Where Δw_t follows an ARMA(1,1) process with constant mean, and, per our cascading structure, general inflation (q) is included as an external regressor:

$$\Delta w_t = \mu + q_{t-1} + \phi \Delta w_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$$

$$\epsilon_t \sim N(0, \sigma^2)$$

Also notable, is that q_{t-1} is included as opposed to q_t. Regressing the lagged value of q_t onto Δw_t improved model fit. This choice reflects the economic reality that wages often adjust with a delay to changes in consumer prices. In a practical sense, adjustments to employee compensation react to consumer price changes, but this reaction would not occur in real time. It happens with some delay relative to the consumer price change, leading to lagged CPI measures better explaining variations in ECI then contemporaneous CPI values.

2.3 Interest Rates

The next piece of our ESG is interest rates. Perhaps one of the most crucial pieces of our ESG, interest rates directly affect the price and returns of certain fixed-income assets and indirectly affect the behavior of the prices and returns of other assets, most importantly equity returns. We are interested in modeling interest rates of varying duration to properly price fixed income assets such as annuities. As such, we model the yield curve by breaking it up into 3 components: level, slope, and curvature. These 3 components give us a reasonable representation of the yield curve.

When modeling interest rates, we must account for the fact that, in the United States, they effectively have a zero lower bound. This lower bound introduces skewness into the data, violating the assumption of normally distributed errors in standard time-series models when interest rates get close to 0. To account for this, we use a simplified version of a piece-wise transformation proposed by Begin:

$$\tilde{r}_t = \begin{cases} r_t & r_t > \overline{r} \\ \overline{r} - \overline{r}\log(\overline{r}) + \overline{r}log(r_t) & r_t \leq \overline{r} \end{cases}$$

where r_t is the 3 month rate at time t and $\overline{r}$ is the threshold at which we apply the log transformation. For our modeling we choose to set $\overline{r} = .005$, thus applying the transformation to all interest rates under .5%.

2.3.1 Short-Term

In theory, the level of a yield curve can be decided by any interest rate, but we decide to represent it by the short-term interest rate, specifically the 3-month rate. Per our cascading ESG structure, we also incorporate short term rates in our equity returns model, so a model specifically for short rates will prove beneficial for both the yield curve and the equity returns pieces of our ESG.

Short term interest rates have been found to show conditional heteroskedasticity, that is to say, the variance of these interest rates are not just non-constant, but also can depend on the variance of the values before them. For example, large shocks in the interest rates tend to be followed by periods of high variance. To capture these dynamics we choose to model the variance of the rates as a generalized auto-regressive conditional heteroskedastic process of order 1,1 (or GARCH(1,1)). The mean, like inflation, is modeled as an ARMA(1,1) process and, per our cascading structure, CPI is included as a regressor as $q_t = \log \frac{CPI_t}{CPI_{t-1}}$. More specifically:

$$\tilde{r}_t = \mu + q_t + \phi \tilde{r}_{t-1} + \theta \epsilon_{t-1} + \epsilon_t$$

$$\epsilon_t \sim \mathcal{N}(0, \sigma_t^2)$$

$$\sigma_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2$$

where α is the reaction (or ARCH) component and β is the persistence (or GARCH) component of the variance model.

2.3.2 Yield Curve

We choose to approximate the functional form of the yield curve through its level, slope, and curvature. Any point on the yield curve can then be approximated as a weighted sum of the level, slope, and curvature. We denote interest rates as $\tilde{r}_{i,t}$ where $\tilde{r}_{i,t}$ is the i year interest rate at time t (i.e. $\tilde{r}_{10,1}$ is the 10 year interest rate at time 1). We collect and use data for $i = \{3/12, 1, 2, 3, 5, 7, 10, 20, 30\}$. 3/12 denotes the 3 month interest rate, and all other interest rates are yearly integer values.

We define the level, slope, and curvature specifically in the following way:

$$\text{Level} : \tilde{r}_{3/12, t}$$

$$\text{Slope} : \tilde{r}_{30, t} - \tilde{r}_{3/12, t}$$

$$\text{Curvature} : \tilde{r}_{3/12, t} + \tilde{r}_{30, t} - 2\tilde{r}_{10, t}$$

As described above, we model the level as a GARCH process. We choose to model slope and curvature as a 2-dimensional vector auto-regressive (VAR) process of order 1 as follows:

$$\mathbf{F}_t = \boldsymbol{\mu} + \mathbf{A}\mathbf{F}_{t-1} + \mathbf{b}\tilde{r}_{3/12, t} + \boldsymbol{\epsilon}_t$$

$$\boldsymbol{\epsilon}_t \sim N_2(\mathbf{0}, \boldsymbol{\Sigma})$$

Where $\mathbf{F}_t$ is a 2×1 vector of slope and curvature at time t, $\mathbf{A}$ is a 2×2 matrix of auto-regressive components, $\mathbf{b}$ is a 2×1 vector of coefficients capturing the influence of the short-term rate at time t on each component, and $\boldsymbol{\Sigma}$ is the 2×2 variance-covariance matrix. We include the 3-month rate as a regressor to capture the dependence of slope and curvature on the overall level of the yield curve. Empirically, including this term also improves the fit of the VAR model.

We then model each interest rate as a linear function of level, slope, and curvature as follows:

$$\tilde{\mathbf{r}}_i = \mathbf{X}\boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i, \quad i = \{1, 2, 3, 5, 7, 20\}$$

$$\boldsymbol{\epsilon}_i \sim N_n(0, \sigma_i^2 \mathbf{I})$$

$\mathbf{X}$ being an $n \times 3$ matrix of level, slope, and curvature, and $\boldsymbol{\beta}_i$ being a 3×1 vector of the weightings for interest rate i. Noticeably, we exclude the 3 month, 10 year, and 30 year rates from this step because these rates are direct functions of level, slope, and curvature as described above.

In combination, the GARCH-level model, VAR for slope and curvature, and linear regressions for each interest rate provide a complete framework to approximate the yield curve across all maturities.

2.4 Equity Returns