$B = QAP$

then $A \equiv B$

② $\text{rank}(A) = \text{rank}(B)$

$A \equiv B$

Equal $\quad \underset{m_1 \times n_1}{B} = \underset{m_2 \times n_2}{A}$ if shape is same & entry wise same

$m_1 = m_2$

$n_1 = n_2$

$A_{ij} = B_{ij} \ \forall i,j$

① If B is obtained by elementary row and column operation

$AB \neq BA$ matrix world

Equivalence $\quad B \equiv QAP$

if $B = QAP \quad \exists Q, P$ s.t $B = QAP$.

AP

$\to$ L.C of Columns

$$\begin{array}{cc} c_1 & c_2 \end{array}$$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} = \begin{bmatrix} 5\begin{pmatrix}1\\3\end{pmatrix} + 6\begin{pmatrix}2\\4\end{pmatrix} & 7\begin{pmatrix}1\\3\end{pmatrix} + 8\begin{pmatrix}2\\4\end{pmatrix} \end{bmatrix}$$

You know

$$\begin{bmatrix} 17 & 23 \\ 39 & 53 \end{bmatrix} = \begin{bmatrix} 17 & 23 \\ 39 & 53 \end{bmatrix}$$

Row ope

$PA \to$ L.C of Rows

$$\begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5[1\ 2] + 7[3\ 4] \\ 6[1\ 2] + 8[3\ 4] \end{bmatrix} = \begin{bmatrix} [5\ 10] + [21\ 28] \\ 27\ 38 \end{bmatrix}$$

$$\begin{bmatrix} 27 & 38 \\ 30 & 44 \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$$

if you get B, then A and B are equivalent.

$\cancel{\tfrac{1}{2}} \cancel{\tfrac{2}{3}} \cancel{\tfrac{3}{4}} \cancel{\tfrac{4}{5}}$

$T: V \to W$

$\beta_1 \quad \gamma_1$

$\beta_2 \quad \gamma_2$

$A = [T]_{\beta_1}^{\gamma_1}$

$B = [T]_{\beta_2}^{\gamma_2}$

equivalent.

$$\underset{B}{[T]_{\beta_2}^{\gamma_2}} = \left(Q_{\gamma_1}^{\gamma_2}\right)\underset{A}{[T]_{\beta_1}^{\gamma_1}}\left(P_{\beta_2}^{\beta_1}\right)$$

$P \to$ express β_2 in terms of β_1

Subscript $\quad$ Superscript

$Q \to$ express γ_1 in terms of γ_2

matrix rep of given transformation w.r.t differ basis are all equivalent. $\longrightarrow$ $B \equiv A$

One case of equivalence, Sq matrices $\quad B = QAP$

$Q = P^{-1}$

$B = P^{-1}AP$

$\text{sq}\, A \equiv \text{sq}\, B$

① A and B must equivalent

② $\text{rank}(A) = \text{rank}(B)$

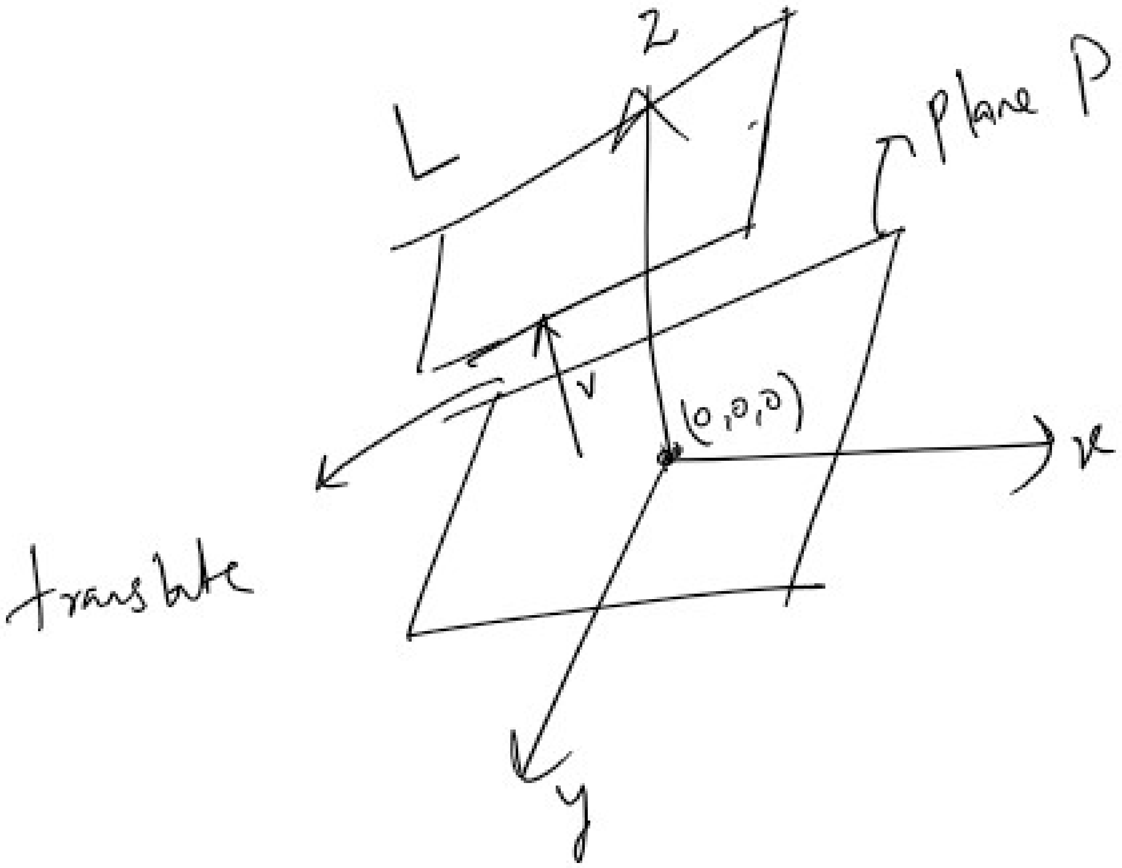
③ $|B| = |A|$

$|B| = |P^{-1}|\,|A|\,|P|$

$= \frac{1}{|P|}|A|\,|P|$

Affine Subspace

It is Affine Space



is $\mathbb{R}^3$ V.S? ✓

is P V.S? ✓

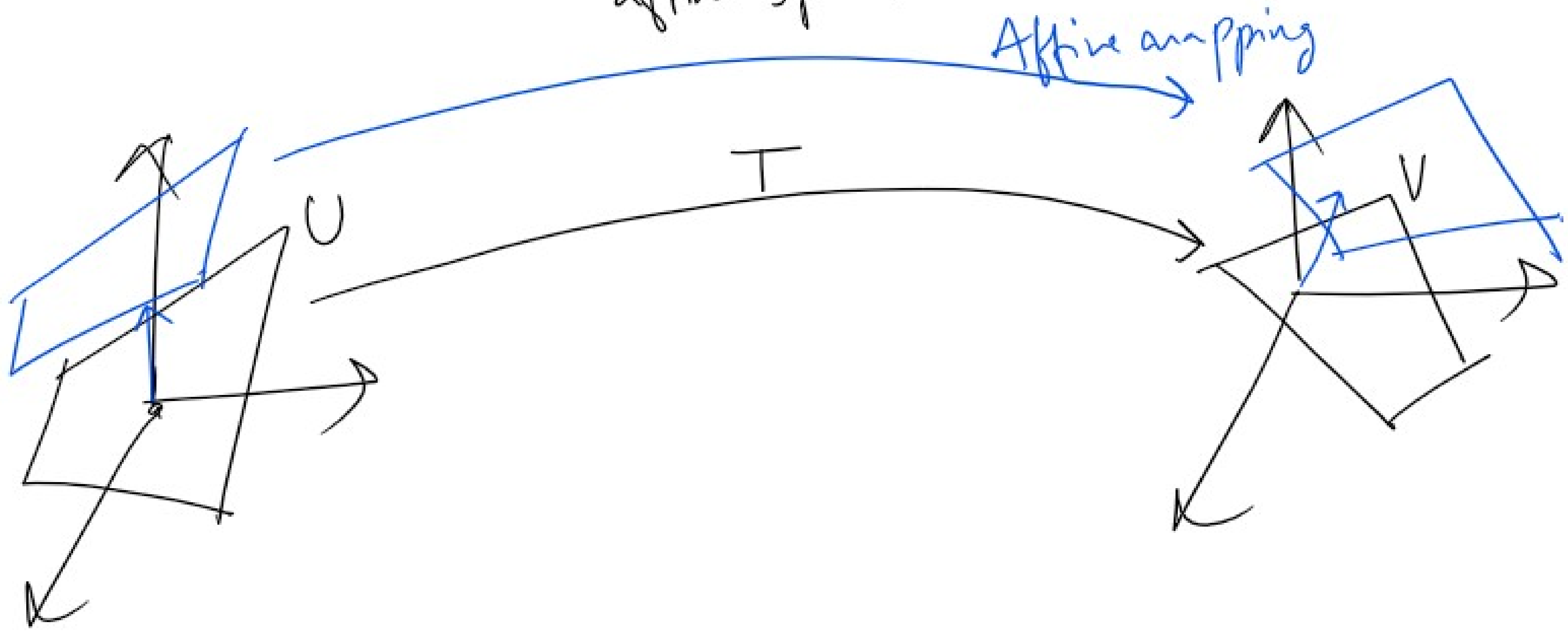
is P Subspace of $\mathbb{R}^3$? ✓

+, * must be close in P
and
'0' of parent space ∈ Subspace

$$\underset{=P}{U} \subseteq \underset{=\mathbb{R}^3}{V}$$

$$L = v + U = \{ v+u \mid u \in U \}$$

↳ affine space.



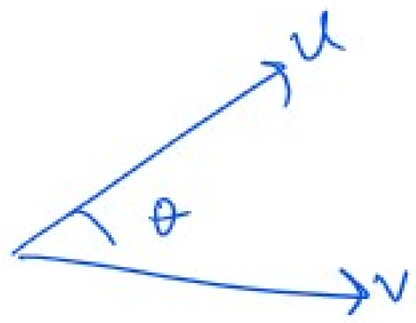
length

$\sqrt{v.v} = \|v\|$

norm

$\perp$

perpendicular



$$\cos\theta = \frac{u.v}{\|u\|\|v\|}$$

"On Earth" (dot product)

"orthogonal"

Inner product

4 rules

DIY

positivity
Bilinearity
Symmetry
Scalar multiple.

On Earth dot product

Norm

3 rules

DIY

$\triangle^{le}$ inequality
Scaling
positivity of length

On Earth length

Using inner product and Norm

Find angle b/w 2 vectors