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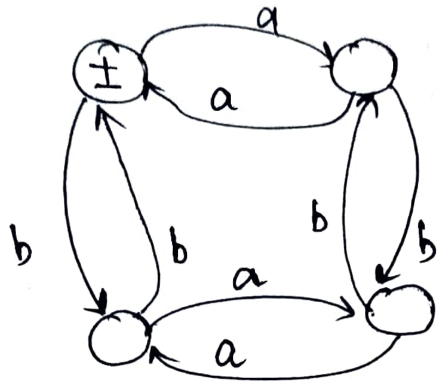
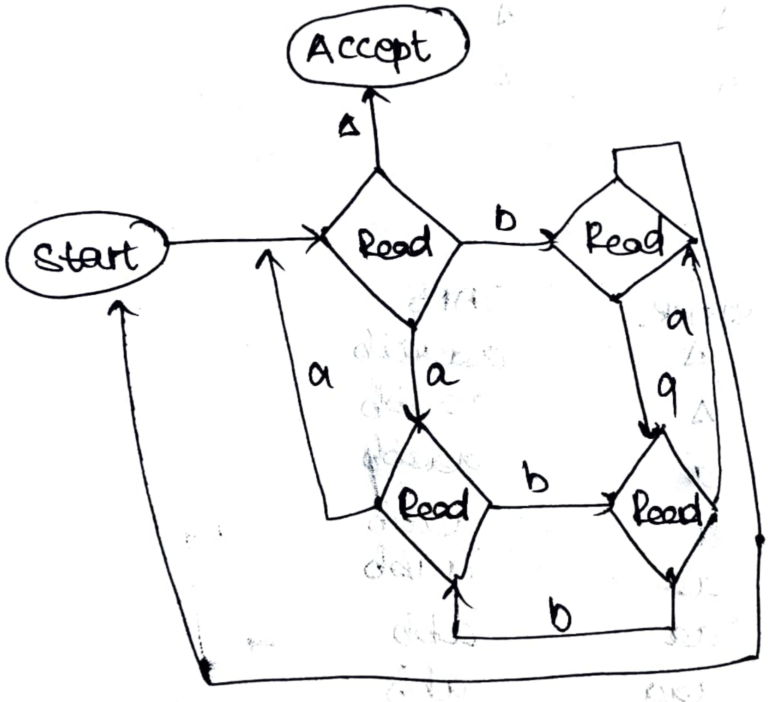
TOC Assignment 6: Pushdown Automata.

1. Convert the following into equivalent PDAs.





3 (iii) aabb

STATE	STACK	TAPE
Start	Δ	aabb
Read1	Δ	abb
Pusha	a	abb
Read2	a	bb
Pusha	aa	bb
Read2	aa	b
Read 3	aa	Δ
Pop	a	Δ
Read 4	a	Δ
Pop	Δ	Δ
Reject	[illegible]	[illegible]

(iv) aabbbb

STATE	STACK	TAPE
Start	Δ	aabbbb
Read1	Δ	abbbb
Pusha	a	abbbb
Read2	a	bbbb
Pusha	aa	bbbb
Read2	aa	bbb
Read3	aa	bb
Pop	a	bb
Read4	a	b
Read3	a	Δ
Pop	Δ	Δ
Read4	Δ	Δ
Pop	Δ	Δ
Accept		

(5.)(iv)

STATE	STACK	TAPE
Start	Δ	aaaabb
Read1	Δ	aaabb
Pusha	a	aaabb
Read2	a	aabb
Pusha	aa	aabb
Read2	aa	abb
Pusha	aaa	abb
Read2	aaa	bb
Pusha	aaaa	bb
Read2	aaaa	b
Pop	aaa	b
Read3	aaa	Δ
Pop	aa	Δ
Read3	aa	Δ
Pop	a	Δ
crash		

6. To Prove:

(i) Language accepted by machine in Prob. 5 is

$L = \{a^n S$, where S starts with b and $length(S) = n\}$

Proof:

Lets consider the machine in sections:-

1. The machine accepts Λ - $a^0 S$ where $length(S) = 0$.
2. The first letter of any other string must be a. That a is stored. As long as the machine continues to read a's they continue to be stored. No words consisting only a's can be accepted.
3. At the first b, control passes to the second loop. For each letter read (including that b) the stack is popped once. ~~for each~~ If there are fewer a's in the stack than letters from the first b to the end of the word the string crashes. When there are no more letters on the tape (read Δ) the string crashes if there are a's left on the stack. Only those words are accepted that have the same no. of letters from the first b to the end of the words as initial a's.

(ii) Required CFG:

$S \rightarrow aSb \mid aSa \mid ab$

(iii) Prove

Doubt

8.(i) Show that the language $a^n b^m a^m b^n$ is context-free.

We can show this by designing a CFG for the given language.

The following CFG exist:-

$S \rightarrow aSb \mid aXb$

$X \rightarrow bXa \mid ba$.

Hence, it is context free language.

9 (ii) To show: string bbba can also be accepted by giving the trace that shows when to take the branch

We can show this by tracking the states, stack and the tape.

STATE	STACK	TAPE
START	Δ	bbba
READ1	Δ	bba
PUSH X	X	bba
READ1	X	ba
PUSH X	XX	ba
READ1	XX	a
POP1	X	a
READ2	X	Δ
POP_1	Δ	Δ
$READ_2$	Δ	Δ
POP_2	Δ	Δ
ACCEPT		

Clearly, bbba is accepted.

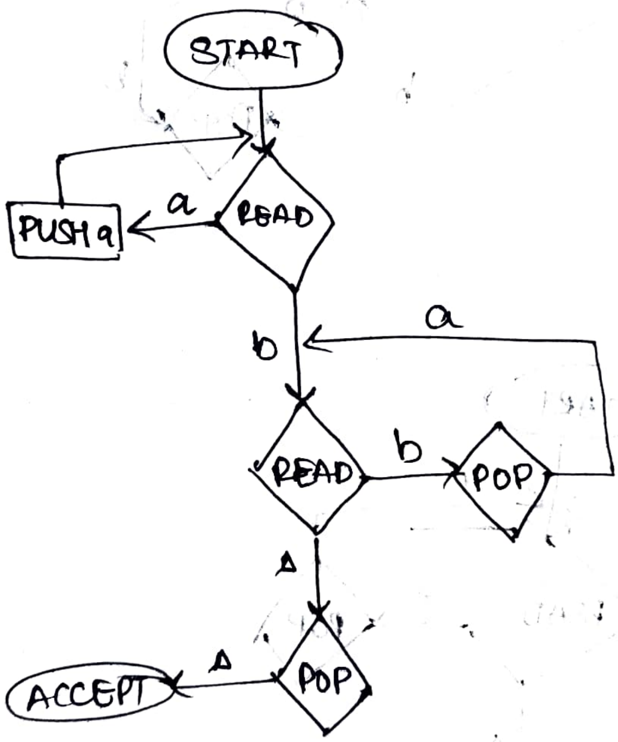
12(iii). To show that babaaab cannot be accepted.

From the PDA, we can say that, for a word to be accepted by the given machine, it must have at least one a that is not followed by b's.

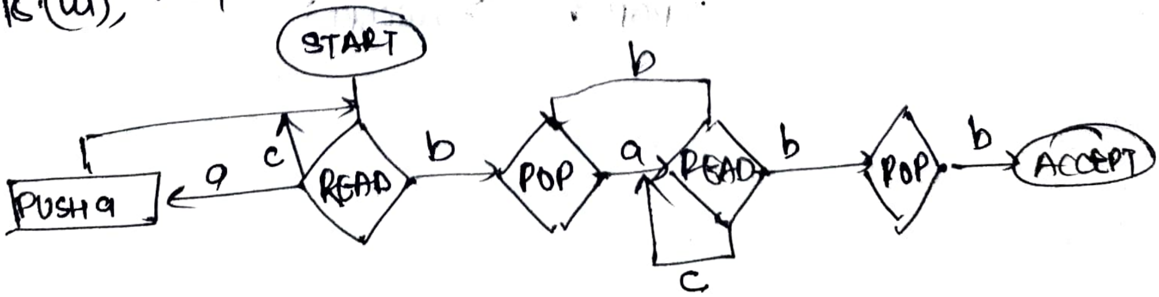
The string babaaab must crash either by reading Δ in $READ_1$, by popping Δ in POP_1 or by reading b in $READ_2$.

(14) Required PDA for

$$L = \{ a^n b^{n+1} ;\ n = 1, 2, \ldots \}.$$



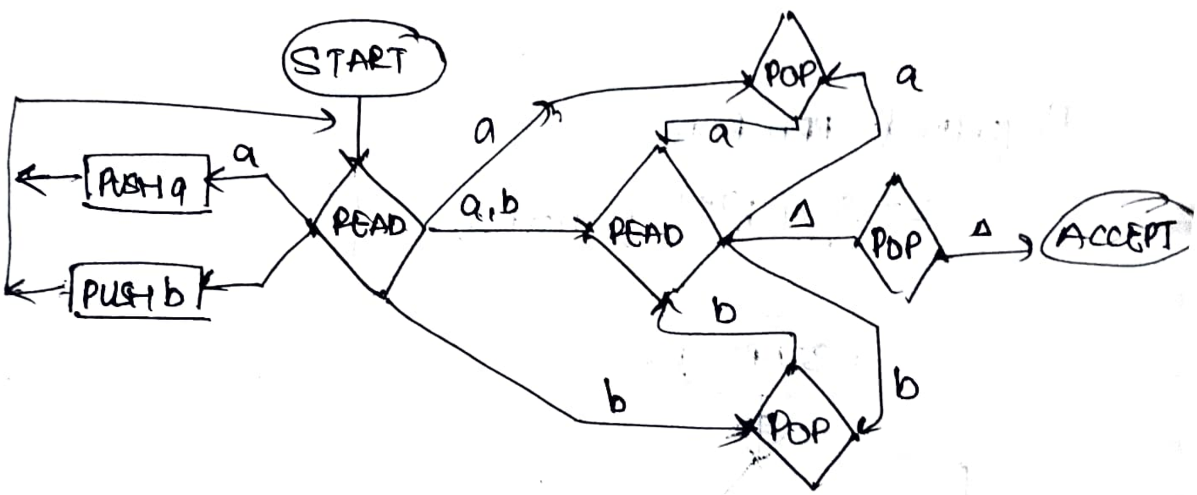
15(iii), Required PDA.



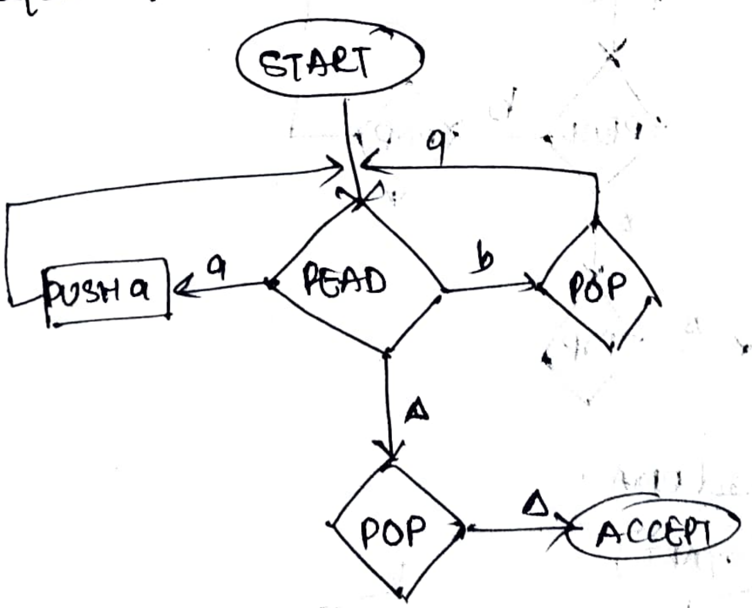
(iv) Required CFG

$S \rightarrow YaXbY \mid Y$
$X \rightarrow aXb \mid XY \mid YX$
$Y \rightarrow cY \mid L$

(16) PDA for PALINDROME (EVEN + ODD)



(20) (ii) Required PDA :-



(iii) Prove L is not regular.

Let us assume that L is regular, by pumping lemma, $\exists$ a pumping length p

consider any string $w \in L$ where $|w| \geq p$ and split it as

$w = xyz$ where,

$|xy| \leq p$

$|y| > 0$

$xy^iz \in L \ \forall i \geq 0$.

Structure of L :-

L consist of strings where

1. No. of a's = no. of b's

2. As we go from left to right, there are never more b's than a's at any point in the string.

We have following cases for y.

Case 1 : y consists only of a's

- pumping down ($i=0$) – Reduces no. of a's but no. of b remains unchanged. violates 1st ~~2nd~~ 2nd condition
- pumping up ($i=2$). Changes no. of a without changing no. of b's. Hence, condition 1 is violated.

Case 2 : y consists only of b's

pumping down or pumping up would change the no. of ~~b~~ ~~a~~ without changing no. of a's.

Hence, violates required condition.

Case 3 : y consists of both a's and b's

- causes imbalance b/w no. of a's and no. of b's.
- result is more b's than a's at some point in string.

Hence $\exists$ no y such that it can be pumped. Hence, proved

(iv) Required CFG

$S \to SS \mid aSb \mid ab$