CS251 Assignment Report

Aryan Mathe & Yash Ruhatiya

22 August

Contents

1 Question-1

1.0.1 Laplace PDF

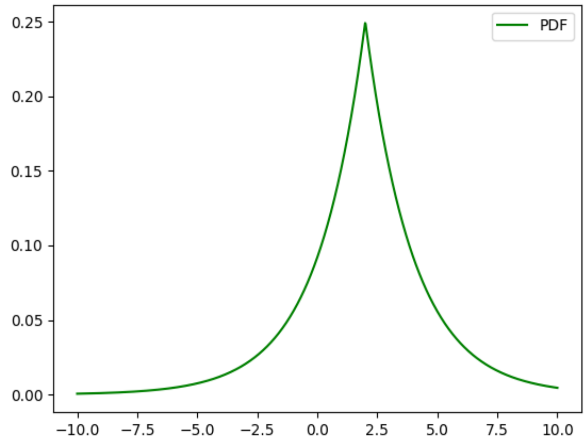

Figure 1: Plot

1.0.2 Laplace CDF

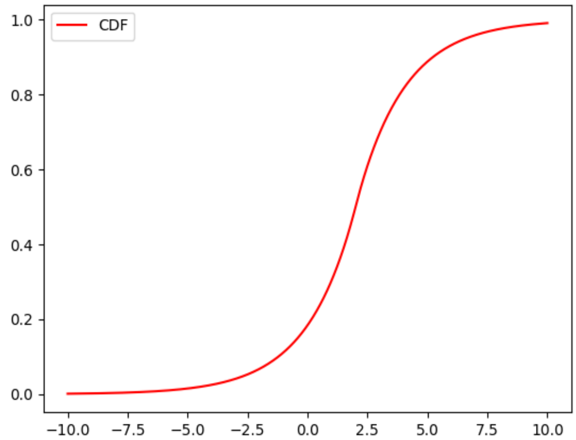

Figure 2: Plot

1.0.3 Gumbel PDF

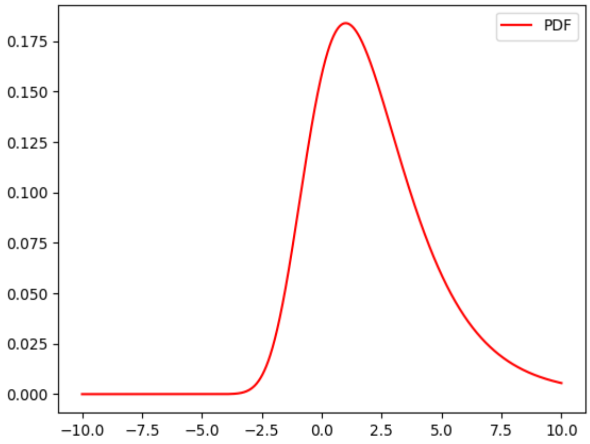

Figure 3: Plot

1.0.4 Gumbel CDF

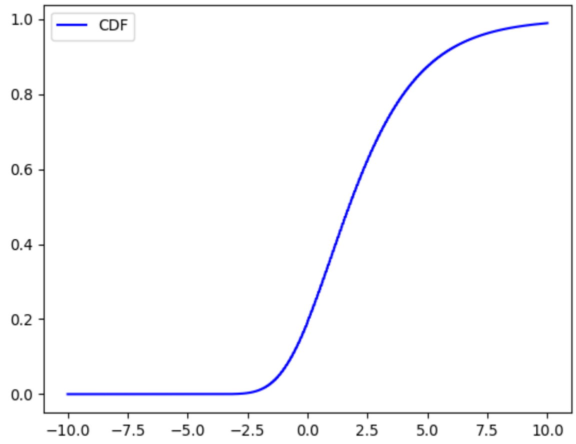

Figure 4: Plot

1.0.5 Cauchy PDF

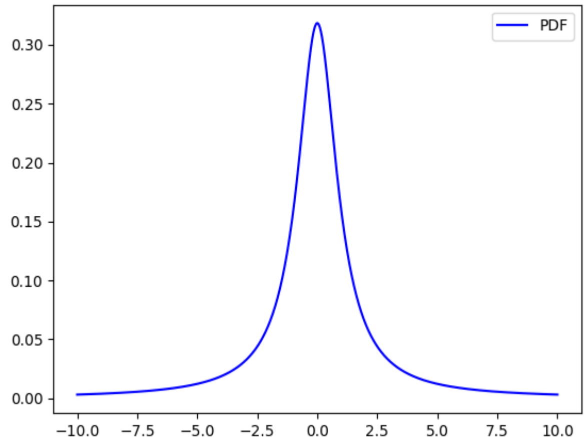

Figure 5: Plot

1.0.6 Cauchy CDF

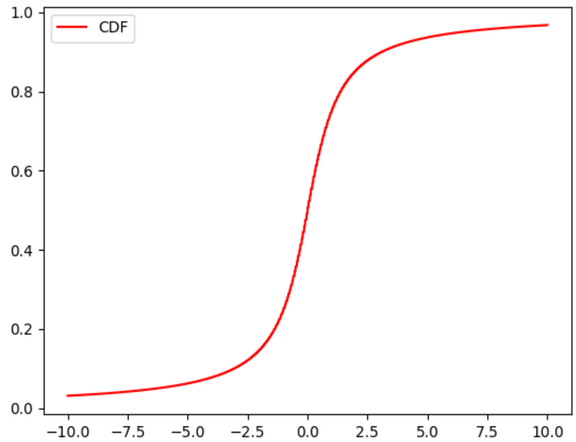

Figure 6: Plot

1.0.7 Variances

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Variance for Laplace Distribution:  7.999616605174508

Variance for Gumbel Distribution:  6.579542404785874

Variance for Cauchy Distribution:  Not defined
```

2 Question-2

2.1 Poisson Sum

2.1.1 Values for $\hat{P}(Z)$

Values of $\hat{P}(Z)$ for $k = 0, 1..., 25$ are:

k	P(Z=k)
0	0.000863
1	0.006335
2	0.022296
3	0.051755
4	0.091102
5	0.127609
6	0.149113
7	0.149113
8	0.13088
9	0.101438
10	0.071086
11	0.045162
12	0.026235
13	0.014287
14	0.006951
15	0.00336
16	0.001458
17	0.00062
18	0.000216
19	6.6e-05
20	3.8e-05
21	1.4e-05
22	3e-06
23	0
24	0
25	0

2.1.2 Sum of two Poisson Random Variables

$P(Z)$ where, $Z = X + Y$ (X and Y are two poisson random variables with average rate λ, μ) can be calculated by using the following formula

$$P(Z = k; \lambda; \mu) = \frac{e^{-(\lambda+\mu)}(\lambda + \mu)^k}{k!}$$

2.1.3 Comparision between Actual and Experimental

The figure below shows the comparision between $\hat{P}(Z)$ and $P(Z)$ graphically

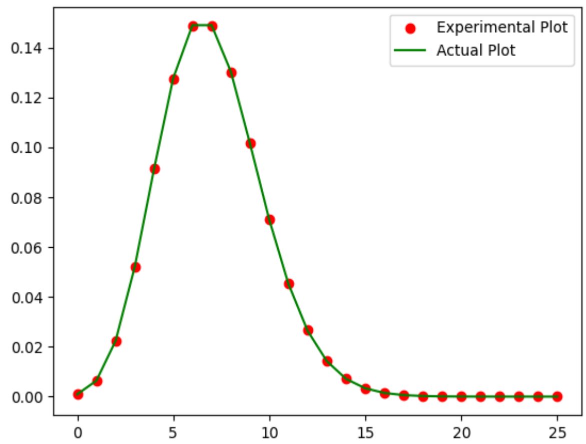

Figure 7: Comparision Plot

2.2 Poisson Thinning

2.2.1 Values for $\hat{P}(Z)$

Values of $\hat{P}(Z)$ for $k = 0, 1..., 25$ are:

k	P(Z=k)
0	0.04066
1	0.13045
2	0.20988
3	0.22203
4	0.1778
5	0.11436
6	0.05967
7	0.02821
8	0.01108
9	0.00408
10	0.00137
11	0.00032
12	6e-05
13	3e-05
14	0
15	0
16	0
17	0
18	0
19	0

20	0	
21	0	
22	0	
23	0	
24	0	
25	0	

2.2.2 Thinned Poisson Variable

$P(Z)$ where, Z is the thinned random variable out of Y (Y is a poisson random variable with average rate λ and p is the probability parameter) can be calculated by using the following formula

$$P(Z = k; \lambda; p) = \frac{e^{-\lambda p}(\lambda p)^k}{k!}$$

2.2.3 Comparision between Actual and Experimental

The figure below shows the comparision between $\hat{P}(Z)$ and $P(Z)$ graphically

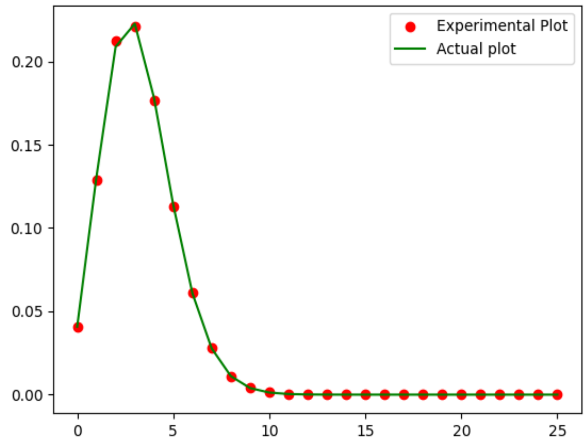

Figure 8: Comparision Plot

3 Question-3

3.0.1 Random Walk Position Plot

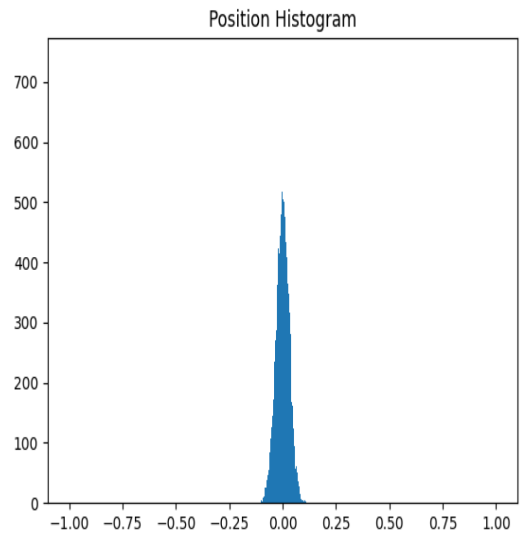

Figure 9: Position Histogram

3.0.2 Random Walk Space -Time Plot

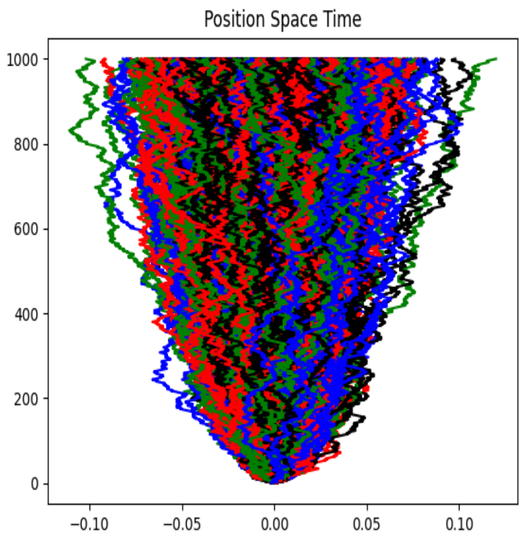

Figure 10: Space Time Plot

3.0.3 Law of Large Numbers

Mean
Acccording to the **Law of Large Numbers**, the average of the results obtained from a large number of trials tends to the expected value as more trials are performed.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{X_i}{n} = \overline{X} = E(X)$$

Variance
Expected Variance $\hat{V}(X)$ tends to true variance $V(X)$ when the number of elements N tends to ∞

Here $\hat{M}$ is the expectation value for the random variables $M := E[X]$

$$\hat{V}(X) = \frac{\sum_{i=1}^{N}(X_i - \hat{M})^2}{N}$$

$$\hat{V}(X) = \frac{\sum_{i=1}^{N}(X_i^2 + \hat{M}^2 - 2\hat{M}X_i)}{N}$$

$$\hat{V}(X) = \frac{\sum_{i=1}^{N} X_i^2}{N} + \hat{M}^2 - 2\hat{M}^2$$

$$\hat{V}(X) = \frac{\sum_{i=1}^{N} X_i^2}{N} - \hat{M}^2$$

Now using the **Law of large numbers** on random variable X_i^2, the expression $\frac{\sum_{i=1}^{N} X_i^2}{N}$ reduces to $E[X^2]$

$$\hat{V}(X) = E[X^2] - \hat{M}^2$$

So the expected variance $\hat{V}(X) = Var(X)$ as N tends to ∞.

3.0.4 Empirically Computed Mean and Variance

Property	Value
Mean	0.000401
Variance	0.0009984

3.0.5 True Mean and Variance

Let the random variable for position be X, Δz be the step width, p be the probability of moving left, n be the total number of steps, and x be the number of left steps taken by the walker.

Mean
Then the final position z is given as $z = \Delta z(2x - n)$

$$E(Z = \Delta z(2x - n)) = 2\Delta z E(x) - n\Delta z$$

We know that the expected value for the binomial distribution is given as $E(X) = np$, which implies

$$E(Z) = n\Delta z(2p - 1)$$

Since for a uniform distribution $p = 0.5$

$$E(Z) = 0$$

Variance
$V(Z)$ denotes the variance for the distribution of Z,

$$V(Z = \Delta z(2x - n)) = \frac{\sum_z (z - E(Z))^2}{n}$$

$$V(Z = \Delta z(2x - n)) = \frac{\sum_x (\Delta z(2x - n) - n\Delta z(2p - 1))^2}{n}$$

$$V(Z = \Delta z(2x - n)) = \frac{\sum_x (2\Delta z(x - np))^2}{n}$$

$$V(Z = \Delta z(2x - n)) = 4\Delta z^2 \frac{\sum_x (x - np)^2}{n}$$

Now we know, $V(X) = np(1 - p)$ for a binomial distribution.

$$V(Z) = 4\Delta z^2(np(1 - p))$$

3.0.6 Error in Mean and Variance

```
Error in mean:  0.0004009999999999981
Error in variance:  -1.6004009999858598e-06
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4 Question-4

4.1 Plots

4.1.1 Histogram for M-Shaped Distribution

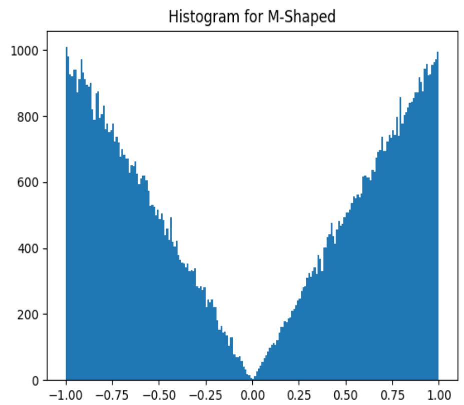

Figure 11: Histogram

4.1.2 CDF for M-Shaped Distribution

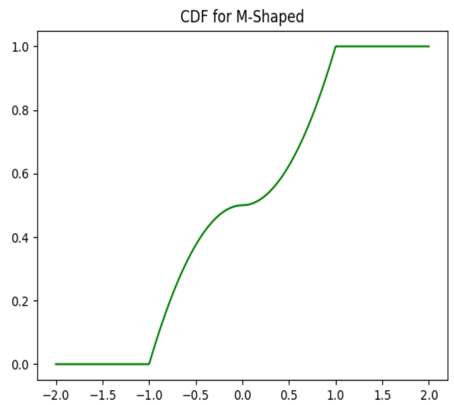

Figure 12: CDF Plot

4.1.3 Histograms for Average M-Shaped Distribution

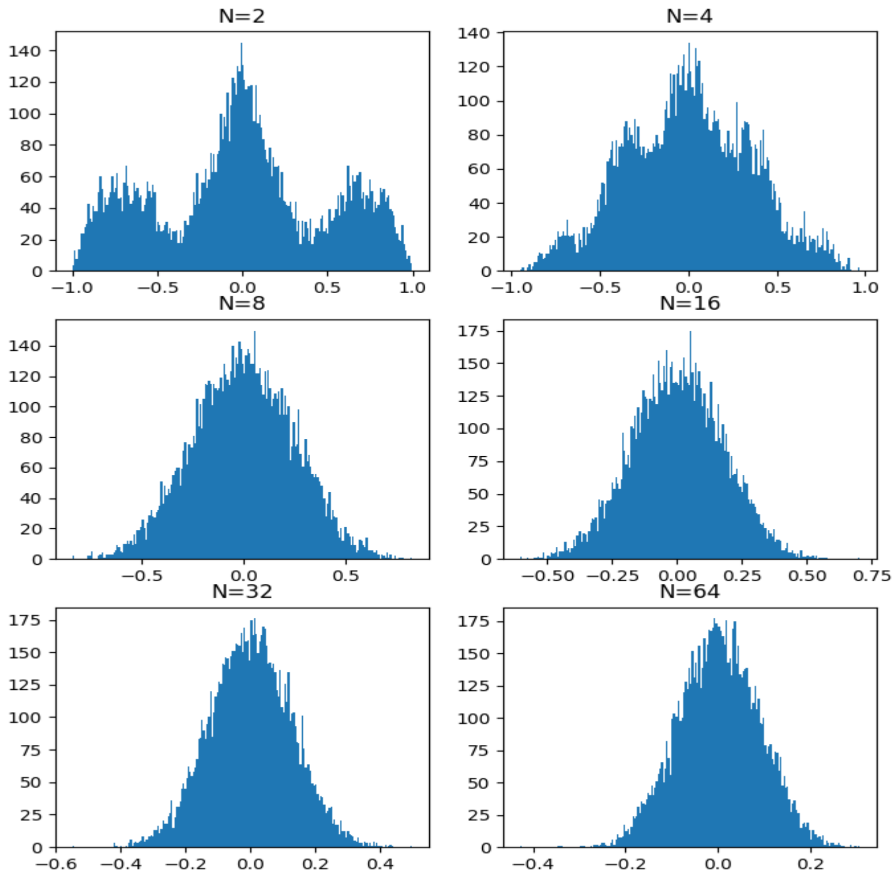

Figure 13: Histograms

4.1.4 CDFs for Average M-Shaped Distribution

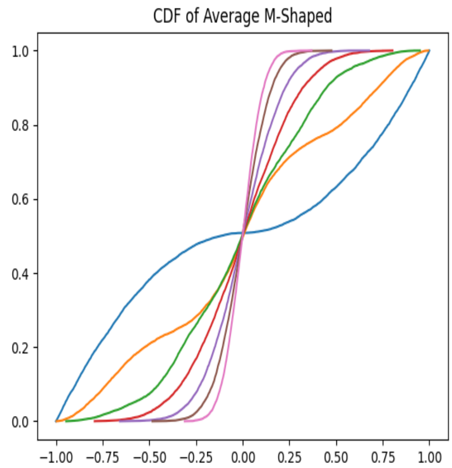

Figure 14: CDF Plots

5 Question-5

5.0.1 Unifrorm Distribution Error - Box Plot

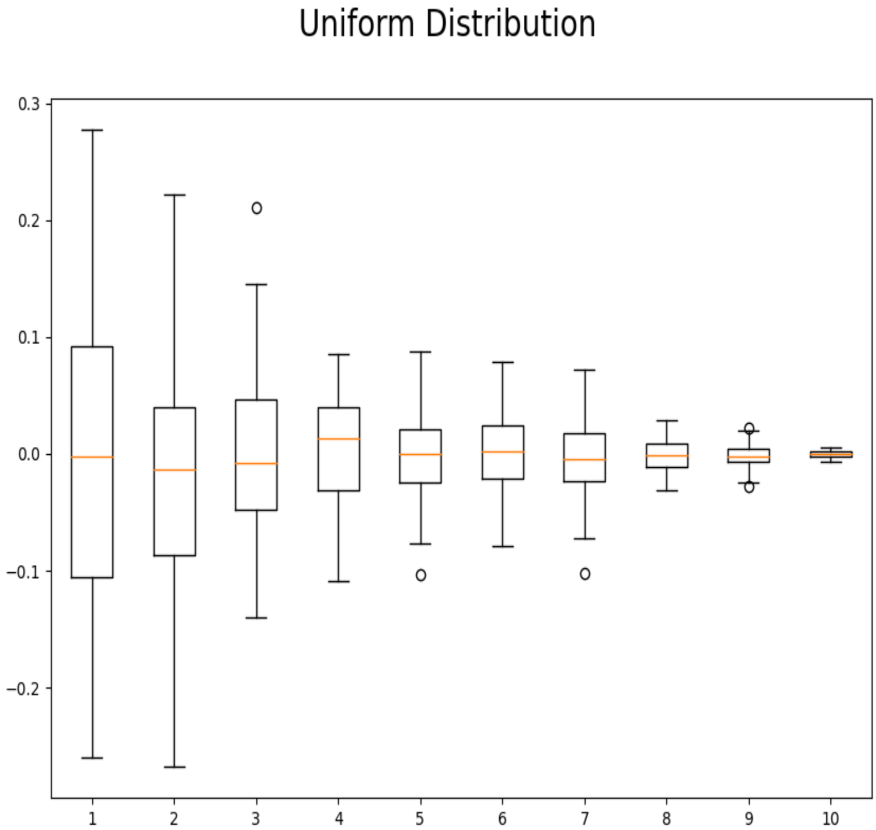

Figure 15: Error Boxplot

5.0.2 Gaussian Distribution Error - Box Plot

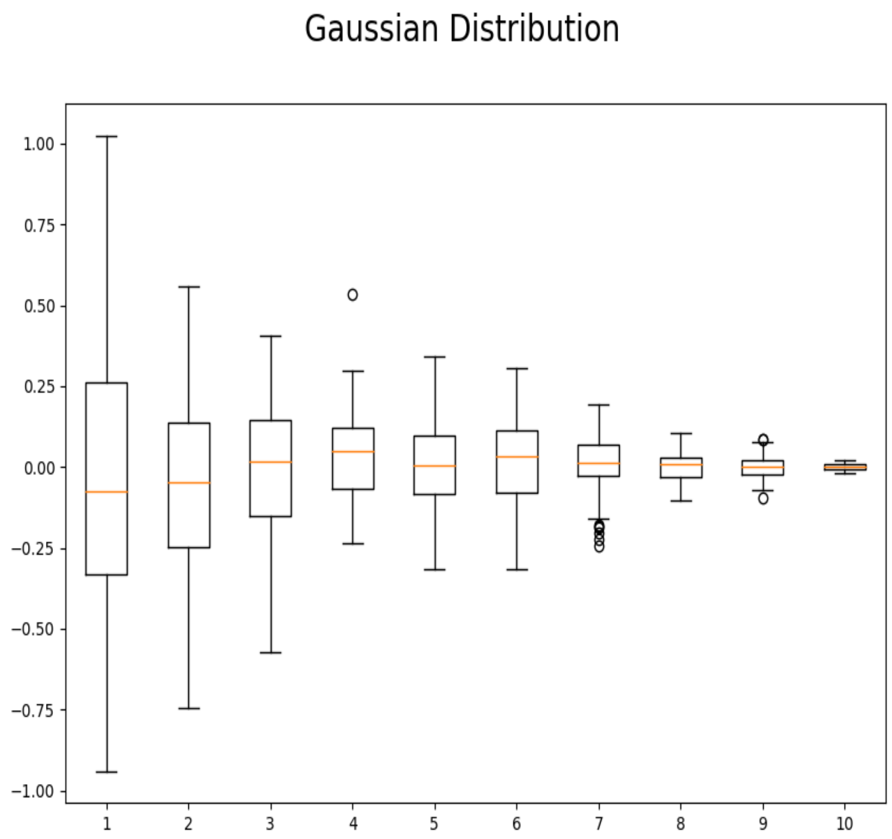

Figure 16: Error Boxplot

5.0.3 Interpretaion from Plot

Here boxplots represents the error distibution.

It can be clearly observed from the boxplots that the variance of the error distribution keeps on decreasing and keeps on converging to a smaller value. So as N increases and tends to ∞ the error and its variance converges to 0 for both **uniform** and **gaussian** distributions.

End of Report