CHAPTER : 15

ELECTRIC CHARGE AND ELECTRIC FIELD

So far you have learnt about mechanical, thermal and optical systems and various phenomena exhibited by them. The importance of electricity in our daily life is too evident. The physical comforts we enjoy and the various devices used in daily life depend on the availability of electrical energy. An electrical power failure demonstrates directly our dependence on electric and magnetic phenomena; the lights go off, the fans, coolers and air-conditioners in summer and heaters and gysers in winter stop working. Similarly, radio, TV, computers, microwaves can not be operated. Water pumps stop running and fields cannot be irrigated. Even train services are affected by power failure. Machines in industrial units can not be operated. In short, life almost comes to a stand still, sometimes even evoking public anger. It is, therefore, extremely important to study electric and magnetic phenomena.

In this lesson, you will learn about two kinds of electric charges, their behaviour in different circumstances, the forces that act between them, the behaviour of the surrounding space etc. Broadly speaking, we wish to study that branch of physics which deals with electrical charges at rest. This branch is called **electrostatics**.

OBJECTIVES

After studying this lesson, you should be able to :

- *state the basic properties of electric charges;*
- *explain the concepts of quantisation and conservation of charge;*
- *explain Coulomb's law of force between electric charges;*
- *define electric field due to a charge at rest and draw electric lines of force;*
- *define electric dipole, dipole moment and the electric field due to a dipole;*

- *state Gauss' theorem and derive expressions for the electric field due to a point charge, a long charged wire, a uniformly charged spherical shell and a plane sheet of charge; and*
- *describe how a van de Graaff generator functions.*

15.1 FRICTIONAL ELECTRICITY

The ancient Greeks observed electric and magnetic phenomena as early as 600 B.C. They found that a piece of amber, when rubbed, becomes electrified and attracts small pieces of feathers. The word **electric** comes from Greek word for amber meaning **electron**.

You can perform simple activities to demonstrate the existence of charges and forces between them. If you run a comb through your dry hair, you will note that the comb begins to attract small pieces of paper. Do you know how does it happen? Let us perform two simple experiments to understand the reason.

ACTIVITY 15.1

Take a hard rubber rod and rub it with fur or wool. Next you take a glass rod and rub it with silk. Suspend them (rubber rod and a glass rod) separately with the help of non-metallic threads, as shown in Fig. 15.1.

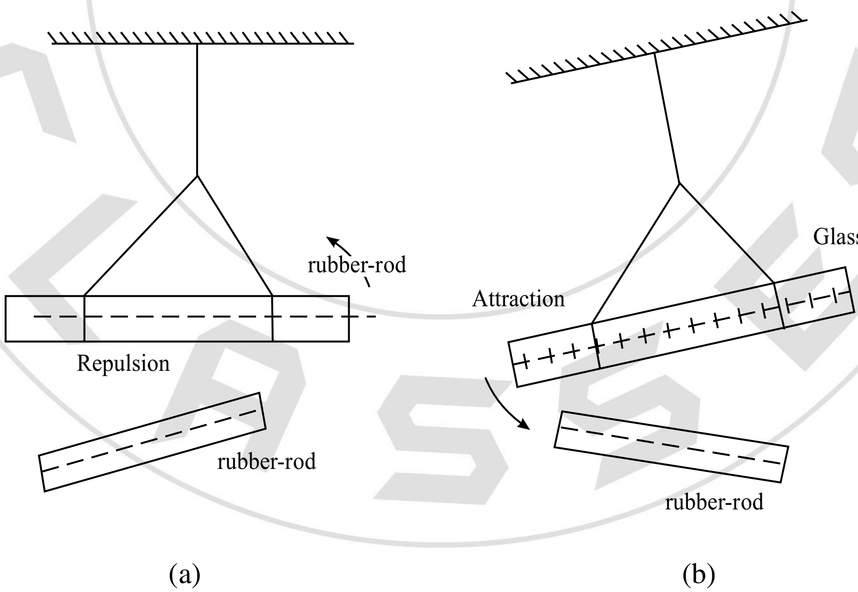


Fig. 15.1: Force of attraction/repulsion between charges: a) a charged rubber rod repels another charged rubber rod : like charges repel each other; and b) a charged glass rod attracts a charged rubber rod : unlike charges attract each other.

Now bring rubber rod rubbed with wool near these rods one by one. What do you observe? You will observe that

- when a charged rubber rod is brought near the charged (suspended) rubber rod, they show repulsion [Fig. 15.1(a)]; and
- when the charged rubber rod is brought near the (suspended) charged glass rod, they show attraction [Fig 15.1(b)].

Similar results will be obtained by bringing a charged glass rod.

On the basis of these observations, we can say that

- A charged rubber rod attracts a charged glass rod but repels a charged rubber rod.
- A charged glass rod repels a charged glass rod but attracts a charged rubber rod.

From these activities we can infer that the rubber rod has acquired one kind of electricity and the glass rod has acquired another kind of electricity. Moreover, **like charges repel and unlike charges attract each other.**

Franklin (Benjamin Franklin, 1706 -1790) suggested that the charge on glass rod is to be called **positive** and that on the rubber rod is to be called **negative.** We follow this convention since then.

Once a body is charged by friction, it can be used to charge other conducting bodies by

conduction, i.e., by touching the charged body with an uncharged body; and

induction, i.e., by bringing the charged body close to an uncharged conductor and earthing it. Subsequently, the charged body and the earthing are removed simultaneously.

15.1.1 Conservation of Charge

In Activity 15.1, you have seen that when a glass rod is rubbed with silk, the rod acquires positive charge and silk acquires negative charge. Since both materials in the normal state are neutral (no charge), the positive charge on the glass rod should be equal in magnitude to the negative charge on silk. This means that the total charge of the system (glass + silk) is conserved. It is neither created nor destroyed. It is only transferred from one body of the system to the other. The transfer of charges takes place due to increase in the thermal energy of the system when the glass rod is rubbed; the less tightly bound electrons from the glass rod are transferred to silk. The glass rod (deficient in electrons) becomes positively charged and silk, which now has excess electrons, becomes negatively charged. When rubber is rubbed with fur, electrons from the fur are transferred to rubber.

That is, rubber gains negative charge and fur gains an equal amount of positive charge. Any other kind of charge (other than positive and negative) has not been found till today.

15.1.2 Quantisation of Charge

In 1909, Millikan (Robert Millikan, 1886-1953) experimentally proved that charge always occurs as some integral multiple of a fundamental unit of charge, which is taken as the charge on an electron. This means that if Q is the charge on an object, it can be written as $Q = Ne$, where N is an integer and e is charge on an electron. Then we say that charge is quantised. It means that a charged body cannot have $2.5e$ or $6.4e$ amount of charge. In units 24-26, you will learn that an electron has charge $-e$ and a proton has charge $+e$. Neutron has no charge. Every atom has equal number of electrons and protons and that is why it is neutral. From this discussion, we can draw the following conclusions :

- There are only two kinds of charges in nature; positive and negative.
- Charge is conserved.
- Charge is quantised.

INTEXT QUESTIONS 15.1

1. A glass rod when rubbed with silk cloth acquires a charge $q = +3.2\times10^{-17}$ C.

   i) Is silk cloth also charged?

   ii) What is the nature and magnitude of the charge on silk cloth?

2. There are two identical metallic spheres A and B. A is given a charge $+Q$. Both spheres are then brought in contact and then separated.

   (i) Will there be any charge on B ?

   (ii) What will the magnitude of charge on B, if it gets charged when in contact with A.

3. A charged object has $q = 4.8\times10^{-16}$ C. How many units of fundamental charge are there on the object? (Take $e = 1.6\times10^{-19}$ C).

15.2 COULOMB'S LAW

You have learnt that two stationary charges either attract or repel each other. The force of attraction or repulsion between them depends on their nature. Coulomb studied the nature of this force and in 1785 established a fundamental law governing it. From experimental observations, he showed that the electrical force between two static point charges q_1 and q_2 placed some distance apart is

- directly proportional to their product ;
- inversely proportional to the square of the distance r between them;
- directed along the line joining the two charged particles ; and
- repulsive for same kind of charges and attractive for opposite charges.

The magnitude of force F can then be expressed as

$$F = k\,\frac{q_1 \times q_2}{r^2} \tag{15.1}$$

For free space, we write

$$F = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 \times q_2}{r^2} \tag{15.2}$$

where constant of proportionality $k = \frac{1}{4\pi\varepsilon_0}$ for free space (vacuum) and $k = \frac{1}{4\pi\varepsilon}$ for a material medium. ε_0 is called **permittivity** of free space and ε is the permittivity of the medium. It means that if the same system of charges is kept in a material medium, the magnitude of Coulomb force will be different from that in free space.

The constant k has a value which depends on the units of the quantities involved. The unit of charge in SI system is coulomb (C). The coulomb is defined in terms of the unit of current, called **ampere.** (You will learn about it later.) In SI system of units, the value of k is

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \frac{\text{N m}^2}{\text{C}^2} \tag{15.3}$$

since $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$.

Thus in terms of force, one coulomb charge can be defined as ***: If two equal charges separated by one metre experience a force of* 9 × 10⁹ N*, each charge has a magnitude of one coulomb.*** The value of electronic charge *e is* 1.60×10^{-19} C.

Note that

- Coulomb's law is also an inverse square law just like Newton's law of Gravitation, which you studied in lesson 6.
- Coulomb's law holds good for point charges only.
- Coulomb's force acts at a distance, unlike mechanical force.

How Big is One Coulomb?

The unit of electrical charge is coulomb. Have you ever thought : How big a coulomb is? To know this, let us calculate the magnitude of force between two charges, each of one coulomb, placed at a distance of one metre from one another:

$$|\mathbf{F}| = k \times \frac{q_1 \times q_2}{r^2}$$

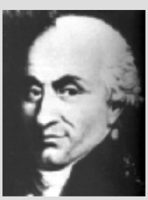
$$= 9.0 \times 10^9 \times \frac{1 \times 1}{1^2}$$

$$= 9.0 \times 10^9 \approx 10^{10}\,\text{N}$$

If the mass of a loaded passenger bus is 5000 kg, its weight mg = (5000 × 10) N (assume $g \approx 10$ m s^{-2}) = 5×10^4 N.

Let us assume that there are 10,000 such loaded buses in Delhi. The total weight of all these buses will be $5 \times 10^4 \times 10{,}000 = 5 \times 10^8$ N. If there are 10 cities having same number of buses as those in Delhi, the total weight of all these loaded buses will be 5×10^9 N. It means that the force between two charges, each of 1C and separated by on metre is equivalent to the weight of about two hundred thousand buses, each of mass 5000 kg.

Charles Augustin de Coulomb (1736–1806)



A French physicist, Coulomb started his career as military engineer in West Indies. He invented a torsional balance and used it to perform experiments to determine the nature of interaction forces between charges and magnets. He presented the results of these experiments in the form of Coulomb's law of electrostatics and Coulomb's law of magnetostatics. The SI unit of charge has been named in his honour.

You now know that the ratio of forces between two point charges q_1 and q_2 separated by a distance r, when kept in free space (vacuum) and material medium, is equal to $\varepsilon/\varepsilon_0$:

$$\frac{F_0 \text{ (in vaccum)}}{F \text{ (in medium)}} = \frac{\varepsilon}{\varepsilon_0} = \varepsilon_r$$

where ε_r is known as relative permittivity or **dielectric constant.** Its value is always greater than one. We will define dielectric constant in another form later.

15.2.1 Vector Form of Coulomb's Law

You know that force is a vector quantity. It means that force between two charges should also be represented as a vector. That is, Eqn. (15.1) should be expressed in vector form. Let us learn to do so now.

Let there be two point charges q_1 and q_2 separated by a distance r (Fig. 15.3). Suppose that $\mathbf{F}_{12}$ denotes the force experienced by q_1 due to the charge q_2 and $\mathbf{F}_{21}$ denotes the force on q_2 due to charge q_1. We denote the unit vector pointing from q_1 to q_2 by $\hat{\mathbf{r}}_{12}$. Then from Fig. 15.3 (a), it follows that

$$\mathbf{F}_{12} = k\,\frac{q_1\,q_2}{|\,r_{12}^2\,|}\,\hat{\mathbf{r}}_{12} \tag{15.4}$$

Similarly, for charges shown in Fig. 15.3 (b), we can write

$$\mathbf{F}_{21} = -\,k\,\frac{q_1\,q_2}{|\,r_{12}^2\,|}\,\hat{\mathbf{r}}_{12} \tag{15.5}$$

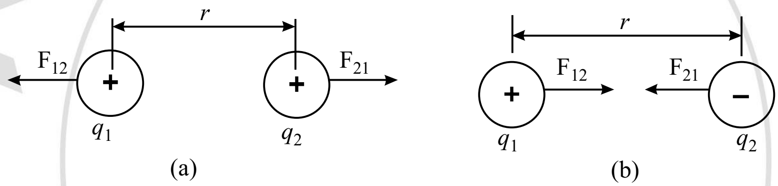


Fig. 15.3 : Two point charges q_1 and q_2 separated by a distance r : a) the direction of forces of repulsion between two positive charges, and b) the direction of forces of attraction between a positive and a negative charge.

The positive sign in Eqn. (15.4) indicates that the force is repulsive and the negative sign in Eqn. (15.5) indicates that the force is attractive.

The Coulomb's law obeys the principle of action and reaction between two charges q_1 and q_2. Therefore,

$$\mathbf{F}_{12} = -\,\mathbf{F}_{21} \tag{15.6}$$

In general, we can write the expression for force between two charges as

$$\mathbf{F}_{12} = k \times \frac{q_1\,q_2}{r^2}\;\hat{\mathbf{r}}_{12} \tag{15.7}$$

15.2.2 Principle of Superposition

If there are more than two charges, we can calculate the force between any two charges using Eqn. (15.7). Suppose now that there are several charges q_1, q_2, q_3, q_4, etc. The force exerted on q_1 due to all other charges is given by Eqn. (15.7):

$$\mathbf{F}_{12} = k\,\frac{q_1\, q_2}{|r_{12}^2|}\,\hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{13} = k\,\frac{q_1\, q_3}{|r_{13}^2|}\,\hat{\mathbf{r}}_{13}$$

and $$\mathbf{F}_{14} = k\,\frac{q_1\, q_4}{|r_{14}^2|}\,\hat{\mathbf{r}}_{14} \qquad (15.8)$$

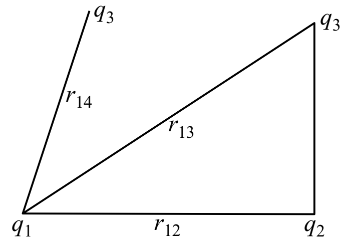


Fig. 15.4: Principle of superposition

The resultant of all these forces, i.e., the total force **F** experienced by q_1 is their vector sum:

$$\mathbf{F} = \mathbf{F}_{12} + \mathbf{F}_{13} + \mathbf{F}_{14} + \ldots \qquad (15.9)$$

This is known as principle of superposition.

Example 15.1 : A charge $+q_1 = 12$C is placed at a distance of 4.0 m from another charge $+q_2 = 6$C, as shown in the Fig. 15.5. Where should a negative charge q_3 be placed on the line joining q_1 and q_2 so that the charge q_3 does not experience any force?

Solution : Let q_3 be placed between q_1 and q_2 at a distance of x metre from q_1. (It can be easily seen that on placing q_3 on the left of q_1 or on the right of q_2 or at any position other than the one between the line joining q_1 and q_2, the resultant force can not be zero.) The force exerted on q_3 by q_1 will be

$$\mathbf{F}_{31} = k\,\frac{q_1\, q_3}{r_{31}^2}\,\hat{\mathbf{r}}_{31} \text{ towards } q_1$$

$$\therefore \qquad |\mathbf{F}_{31}| = k\,\frac{q_3\, q_1}{x^2}$$

The magnitude of force on q_3 due to q_2 is given by

$$|\mathbf{F}_{32}| = k\,\frac{q_3\, q_2}{(4-x)^2} \text{ towards } q_2$$

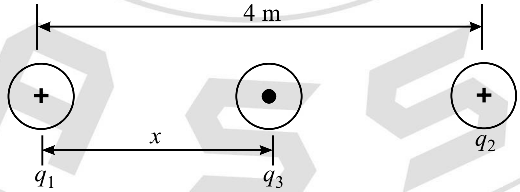


Fig. 15.5 : Three point charges q_1, q_2 and q_3 placed in a straight line

The resultant force on q_3 will be zero when $\mathbf{F}_{31} = \mathbf{F}_{32}$. Therefore, on substituting the numerical values, we get

$$k \times \frac{12q_3}{x^2} = k \times \frac{6q_3}{(4-x)^2}$$

Note that $6q_3k$ is common on both sides and cancels out. Therefore, on simplification, we get

$$\frac{2}{x^2} = \frac{1}{(4-x)^2}$$

or $\quad 2(4-x)^2 = x^2$

$\Rightarrow \quad x^2 - 16x + 32 = 0$

On solving this, we get two values of x : 2.35 m and 13.65 m. The latter value is inadmissible because it goes beyond q_2. Therefore, the charge q_3 should be placed at a distance of 2.35 m from q_1.

It is a reasonable solution qualitatively also. The charge q_1 is stronger than q_2. Hence the distance between q_1 and q_3 should be greater than that between q_2 and q_3.

> The roots of a quadratic equation of the form
>
> $$ax^2 + bx + c = 0$$
>
> are given by
>
> $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
>
> In this case, $a = 1$, $b = -16$ and $c = 32$.
>
> $$\therefore \quad x = \frac{16 \pm \sqrt{256 - 4 \times 32}}{2}$$
>
> $$= 2.35,\ 13.65$$

Example 15.2 : Two charges, each of 6.0×10^{-10} C, are separated by a distance of 2.0 m. Calculate the magnitude of Coulomb force between them.

Solution : We know that the magnitude of Coulomb force between two charges is given by Eqn. (15.2) :

$$F = k\frac{q_1 \cdot q_2}{r^2}$$

Given, $q_1 = q_2 = 6.0 \times 10^{-10}$C and $r = 2.0$ m, Therefore on putting these values, we get

$$F = \frac{(9\times10^{9}\ \text{N m}^2\text{C}^{-2})\times(6.0\times10^{-10}\text{C})^2}{2^2\ \text{m}^2}$$

$$= \frac{9 \times 10^{9} \times 36.0 \times 10^{-20}}{4}\ \text{N}$$

$$= 81 \times 10^{-11}\ \text{N}$$

INTEXT QUESTIONS 15.2

1. Two charges $q_1 = 16\mu$C and $q_2 = 9\ \mu$C are separated by a distance 12m. Determine the magnitude of the force experienced by q_1 due to q_2 and also the direction of this force. What is the direction of the force experienced by q_2 due to q_1?

2. There are three point charges of equal magnitude q placed at the three corners of a right angle triangle, as shown in Fig. 15.2. $AB = AC$. What is the magnitude and direction of the force exerted on $-q$?

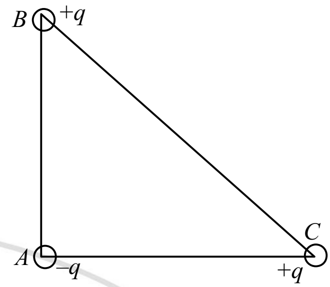


Fig. 15.2 : Three charges placed at the three corners of a right angle triangle.

15.3 ELECTRIC FIELD

To explain the interaction between two charges placed at a distance, Faraday introduced the concept of electric field. The electric field **E** at a point is defined as the electric force **F** experienced by a positive test charge q_0 placed at that point divided by the magnitude of the test charge. Mathematically, we write

$$\mathbf{E} = \frac{\mathbf{F}}{q_0} \tag{15.10}$$

This is analogous to the definition of acceleration due to gravity, $\mathbf{g} = \mathbf{F}/m_0$, experienced by mass m_0 in the gravitational field **F**.

The electric field **E** is a vector quantity and has the same direction as the electric force **F**. Note that the electric field is due to an external charge and not due to the test charge. The test charge q_0 should, therefore, be so small in magnitude that it does not disturb the field due to external charge. (In practice, however, even the smallest test charge will disturb the external field.) Strictly speaking, mathematical definition given below is more accurate :

$$\mathbf{E} = \lim_{q_0 \to 0} \frac{\mathbf{F}}{q_0} \tag{15.11}$$

In SI system, the force is in newton and the charge is in coulomb. Therefore, according to Eqn.(15.10), the electric field has the unit newton per coulomb. The direction of **E** is same as that of **F**. Note that *the action of electric force is mediated through electric field*.

Let us now examine why the test charge q_0 should be infinitesimally small.

Refer to Fig. 15.6. It shows a uniformly charged metallic sphere with charge q and a test charge $q_0 (< < q)$. It means that charge density per unit area is same around points A, B, C and D. The test charge q_0 must measure the force **F** without disturbing the charge distribution on the sphere. Fig. 15.6 (b) shows the situation when $q \simeq q_0$. In this case, the presence of the test charge modifies the surface

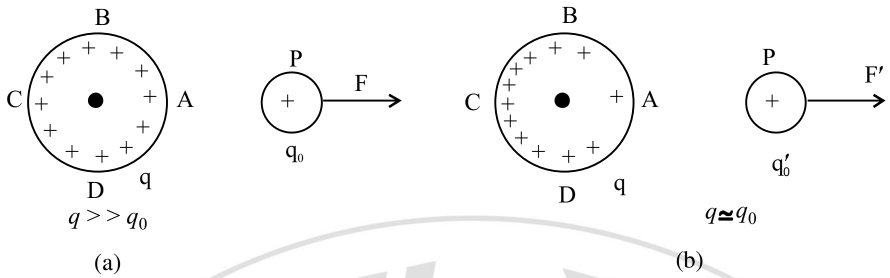


Fig. 15.6 : **a)** uniformly charged metallic sphere and a test charge, and **b)** redistribution of charge on the sphere when another charge is brought near it.

charge density. As a result, the electrical force experienced by the test charge q_0 will also change, say from **F** to **F′**. That is, the force in the presence of test charge is different from that in its absence. But without q_0, the force cannot be measured. If q_0 is infinitesimally small in comparison to q, the charge distribution on the sphere will be minimally affected and the results of measurement will have a value very close to the true value. That is, **F′** will be very nearly equal to **F**. We hope you now appreciate the point as to why the test charge should be infinitesimally small.

Let there be a point charge q. A test charge q_0 is placed at a distance r from q. The force experienced by the test charge is given by

$$\mathbf{F} = k\,\frac{qq_0}{r^2}\,\hat{\mathbf{r}} \tag{15.12}$$

The electric field is defined as the force per unit charge. Hence

$$\mathbf{E} = k \times \frac{q}{r^2}\,\hat{\mathbf{r}} \tag{15.13}$$

If q is positive, the field **E** will be directed away from it. If q is negative, the field **E** will be directed towards it. This is shown in Fig. 15.7.

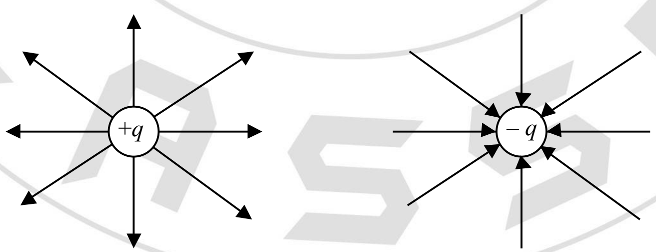


Fig. 15.7 : Direction of electric field due to positive and negative charges

The principle of superposition applies to electric field also. If there are a number of charges q_1, q_2, q_3, ..., the corresponding fields at a point P according to Eqn. (15.13) are

$$\mathbf{E_1} = k \times \frac{q_1}{r_1^2}\ \hat{\mathbf{r}}_1, \quad \mathbf{E_2} = k \times \frac{q_2}{r_2^2}\ \hat{\mathbf{r}}_2 \text{ and } \mathbf{E_3} = k \times \frac{q_3}{r_3^2}\ \hat{\mathbf{r}}_3$$

The total field at point P due to all charges is the vector sum of all fields. Thus,

$$\mathbf{E} = \mathbf{E_1} + \mathbf{E_2} + \mathbf{E_3} + \ldots$$

or

$$\mathbf{E} = k\sum_{i=1}^{N} \frac{q_i\ \hat{\mathbf{r}}_i}{r_i^2} \tag{15.15}$$

where r_i is the distance between P and charge q_i and $\hat{\mathbf{r}}_i$ is the unit vector directed from q_i to P. The force on a charge q in an electric field **E** is

$$\mathbf{F} = q\,\mathbf{E} \tag{15.16}$$

Example 15.3 : The electric force at some point due to a point charge $q = 3.5\mu C$ is 8.5×10^{-4} N. Calculate the strength of electric field at that point.

Solution : From Eq. (15.16) we can write

$$E = \frac{F}{q} = \frac{8.5\times10^{-4}\,\text{N}}{3.5\times10^{-6}\,\text{C}}$$

$$= 2.43 \times 10^{2}\ \text{NC}^{-1}$$

Example 15.4 : Three equal positive point charges are placed at the three corners of an equilateral triangle, as shown in Fig. 15.8. Calculate the electric field at the centroid P of the triangle.

Solution : Suppose that a test charge q_0 has been placed at the centroid P of the triangle. The test charge will experienced force in three directions making same angle between any two of them. The resultant of these forces at P will be zero. Hence the field at P is zero.

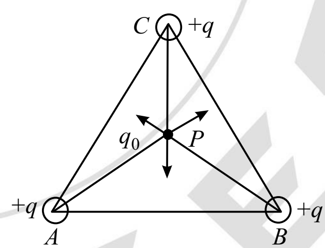


Fig. 15.8 : Electric field at the centroid of an equilateral triangle due to equal charges at its three corners is zero.

INTEXT QUESTIONS 15.3

1. A charge + Q is placed at the origin of co-ordinate system. Determine the direction of the field at a point P located on

   a) + x-axis b) + y-axis c) x = 4 units and y = 4 units

2. The $\Delta\, ABC$ is defined by $AB = AC = 40$ cm. And angle at A is 30°. Two charges, each of magnitude 2×10^{-6} C but opposite in sign, are placed at B and C, as shown in Fig. 15.9. Calculate the magnitude and direction of the field at A.

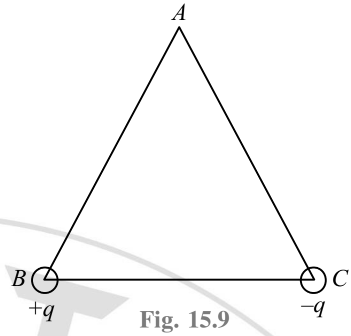


Fig. 15.9

3. A negative charge is located in space and the electric field is directed towards the earth. What is the direction of the force on this charge?

4. Two identical charges are placed on a plane surface separated by a distance d between them. Where will the resultant field be zero?

15.3.1 Electric Field due to a Dipole

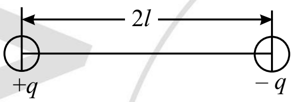


Fig. 15.10: Two unlike charges of equal magnitude separated by a small distance form a dipole.

If two equal and opposite charges are separated by a small distance, the system is said to form a dipole**.** The most familiar example is H_2O. Fig 15.10 shows charges $+q$ and $-q$ separated by a small distance $2l$. The product of the magnitude of charge and separation between the charges is called **dipole moment,** p **:**

$$p = q \times 2l \qquad (15.17)$$

Its SI unit is coulomb-metre.

The dipole moment is a vector quantity. Eqn. (15.17) gives its magnitude and its direction is from negative charge to positive charge along the line joining the two charges (axis of the dipole). Having defined a dipole and dipole moment, we are now in a position to calculate the **electric field due to a dipole.** The calculations are particularly simple in the following cases.

CASE I : *Electric field due to a dipole at an axial point : End–on position*

To derive an expression for the electric field of a dipole at a point P which lies on the axis of the dipole, refer to Fig. 15.11.This is known as **end-on position.** The point charges $-q$ and $+q$ at points A and B are separated by a distance $2l$. The point O is at the middle of AB. Suppose that point P is at a distance r from the mid point O. Then electric field at P due to $+q$ at B is given by

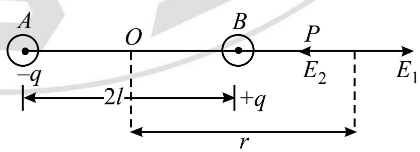


Fig. 15.11 : Field at point P on the dipole axis

$$\mathbf{E_1} = k \times \frac{q}{(r-l)^2} \text{ in the direction } AP$$

Similarly, the electric field $\mathbf{E}_2$ at P due to $-q$ is given by

$$\mathbf{E_2} = k \times \frac{q}{(r+l)^2} \text{ in the direction } PA$$

The resultant field $\mathbf{E}$ at P will be in the direction of $\mathbf{E}_1$, since $\mathbf{E}_1$ is greater than $\mathbf{E}_2$ [as $(r - l)$ is less than $(r + l)$]. Hence

$$\mathbf{E} = \frac{kq}{(r-l)^2} - \frac{kq}{(r+l)^2}$$

$$= kq\left[\frac{1}{(r-l)^2} - \frac{1}{(r+l)^2}\right]$$

$$= kq\left[\frac{(r+l)^2-(r-l)^2}{(r^2-l^2)^2}\right]$$

$$= kq \times \frac{4lr}{(r^2-l^2)^2}$$

$(a + b)^2 - (a - b)^2 = 4ab$
$(a + b)\,(a - b) = a^2 - b^2$

$$= k\,\frac{(2lq)\,2r}{(r^2-l^2)^2}$$

$$= k\frac{2\mathbf{p}r}{(r^2-l^2)^2}$$

where dipole moment $\mathbf{p} = 2lq$. Since $k = 1/4\pi\varepsilon_0$, we can rewrite it as

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\varepsilon_0} \times \frac{r}{r^4\,(1-l^2/r^2)^2}$$

If $r >> l$, l^2/r^2 will be very small compared to 1. It can even be neglected and the expression for electric field then simplifies to

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\varepsilon_0 r^3} \qquad (15.18)$$

It shows that electric field is in the direction of **p** and its magnitude is inversely proportional to the third power of distance of the observation point from the centre of the dipole.

CASE II : *Electric field due to a dipole at a point on the perpendicular bisector : Broad-on position*

Suppose that point P lies on the perpendicular bisector of the line joining the charges shown in Fig. 15.12. Note that $AB = 2l$, $OP = r$, and $AO = OB = l$.

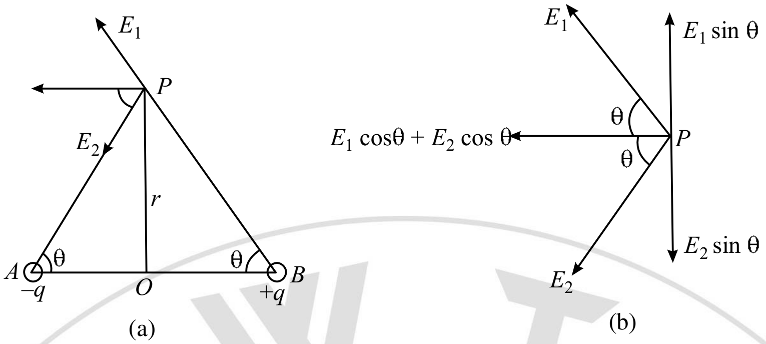


Fig. 15.12 : a) Field at point *P* on the perpendicular bisector of the line joining the charges, and b) resolution of field in rectangular components.

The angle θ is shown in Fig. 15.12(a). From right angled Δs *PAO* and *PBO*, we can write

$$AP = BP = \sqrt{l^2 + r^2}$$

The field at *P* due to charge + *q* at *B* in the direction of *BP* can be written as

$$\mathbf{E}_1 = k\frac{q}{l^2 + r^2}$$

Similarly, the field at *P* due to charge at *A* in the direction of *PA* is given as

$$\mathbf{E}_2 = k\frac{q}{l^2 + r^2}$$

Note that the magnitudes of $\mathbf{E}_1$ and $\mathbf{E}_2$ are equal.

Let us resolve the fields $\mathbf{E}_1$ and $\mathbf{E}_2$ parallel and perpendicular to *AB*. The components parallel to *AB* are $\mathbf{E}_1\cos\theta$ and $\mathbf{E}_2\cos\theta$, and both point in the same direction.

The components normal to *AB* are $\mathbf{E}_1\sin\theta$ and $\mathbf{E}_2\sin\theta$ and point in opposite directions. (Fig. 15.12b) Since these component are equal in magnitude but opposite in direction, they cancel each other. Hence, the magnitude of resultant electric field at *P* is given by

$$E = \mathrm{E}_1\cos\theta + \mathrm{E}_2\cos\theta$$

$$= k\frac{q}{l^2 + r^2}\cos\theta + k\frac{q}{l^2 + r^2}\cos\theta$$

But $\cos\theta = \dfrac{l}{\sqrt{(l^2 + r^2)}}$. Using this expression in the above result, the electric field at *P* is given by

$$E = \frac{kq}{(l^2 + r^2)} \times \frac{2l}{\sqrt{(l^2 + r^2)}}$$

$$= k\,\frac{2lq}{(l^2 + r^2)^{3/2}}$$

$$= k\,\frac{2lq}{r^3(1 + l^2/r^2)^{3/2}}$$

But $p = 2lq$. If $r^2 >> l^2$, the factor l^2/r^2 can be neglected in comparison to unity. Hence

$$E = \frac{p}{4\pi\varepsilon_0 r^3} \tag{15.19}$$

Note that electric field due to a dipole at a point in broad-on position is inversely proportional to the third power of the perpendicular distance between P and the line joining the charges.

If we compare Eqns. (15.18) and (15.19), we note that the electric field in both cases is proportional to $1/r^3$. But there are differences in details:

- The magnitude of electric field in end-on-position is twice the field in the broad-on position.
- The direction of the field in the end-on position is along the direction of dipole moment, whereas in the broad-on position, they are oppositely directed.

15.3.2 Electric Dipole in a Uniform Field

A uniform electric field has constant magnitude and fixed direction. Such a field is produced between the plates of a charged parallel plate capacitor. Pictorially, it is represented by equidistant parallel lines. Let us now examine the behaviour of an electric dipole when it is placed in a uniform electric field (Fig 15.13).

Let us choose x-axis such that the electric field points along it. Suppose that the dipole axis makes an angle θ with the field direction. A force $q\mathbf{E}$ acts on charge $+q$ along the $+x$ direction and an equal force acts on charge $-q$ in the $-x$ direction. Two equal, unlike and parallel forces form a couple and tend to rotate the dipole in clockwise direction. This couple tends to align the dipole in the direction of the external electric field $\mathbf{E}$. The magnitude of torque τ is given by

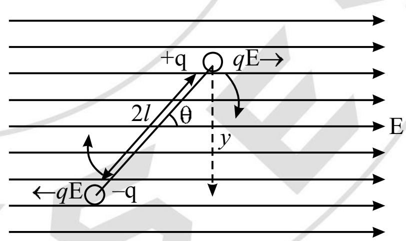


Fig. 15.13 : A dipole in a uniform electric field. The forces on the dipole form a couple and tend to rotate it.

$$\tau = \text{Force} \times \text{arm of the couple}$$

$$= qE \times y$$

$$= qE \times 2l \sin\theta$$

$$= pE \sin\theta$$

In vector form, we can express this result to

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} \tag{15.20}$$

We note that

– when $\theta = 0$, the torque is zero, and

– for $\theta = 90°$, the torque on the dipole is maximum, equal to pE. So we may conclude that the electric field tends to rotate the dipole and align it along its own direction.

Example 15.5 : Two charges $+q$ and $-q$, each of magnitude 6.0×10^{-6} C, form a dipole. The separation between the charges is 4×10^{-10} m. Calculate the dipole moment. If this dipole is placed in a uniform electric field $E = 3.0 \times 10^{2}\,\text{NC}^{-1}$ at an angle 30° with the field, calculate the value of torque on the dipole.

Solution : The dipole moment $p = qd$

$$= (6.0 \times 10^{-6}\text{C}) \times (4.0 \times 10^{-10}\,\text{m})$$

$$= 24 \times 10^{-16}\,\text{Cm}.$$

Since torque $\tau = p\text{E} \sin\theta$, we can write

$$\tau = (24 \times 10^{-16}\text{cm}) \times 3.0 \times 10^{2}\ \text{NC}^{-1})\sin 30^{\circ}$$

$$= \frac{72}{2} \times 10^{-14}\ \text{Nm}$$

$$= 36 \times 10^{-14}\,\text{Nm}$$

If a dipole is placed in a non-uniform electric field, the forces on the charges $-q$ and $+q$ will be unequal. Such as electric field will not only tend to rotate but also displace the dipole in the direction of the field.

15.3.3 Electric Lines of Force (Field Lines)

A very convenient method for depicting the electric field (or force) is to draw lines of force pointing in the direction of the field. The sketch of the electric field lines gives us an idea of the magnitude and direction of the electric field. **The number of field lines passing through a unit area of a plane placed perpendicular the direction of the field is proportional to the strength of the field.** A tangent at any point on the field lines gives the direction of the field at that point.

Note that the *electric field lines are only fictitious construction to depict the field. No such lines really exist*. But the behaviour of charges in the field and the interaction between charges can be effectively explained in terms of field lines. Some illustrative examples of electric field lines due to point charges are shown in Fig 15.14. The field lines of a stationary positive charge point radially in outward direction. But for stationary negative charge, the lines start from infinity and terminate at the point charge in radially inward direction (towards the point charge). You must understand that the electric field lines in both cases are in all directions in the space. Only those which are in the plane containing the charge are shown here.

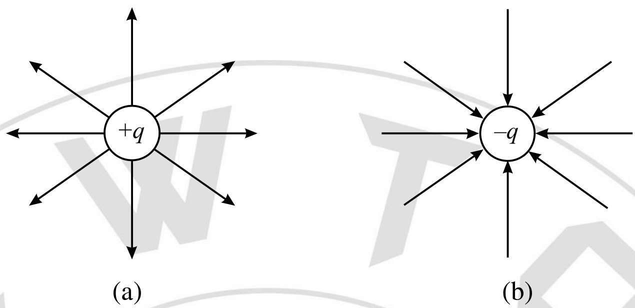


Fig. 15.14 : Electrical field lines of single point charges : a) The field lines of positive charge, and b) the field lines of negative charge.

Fig 15.15(a) shows a sketch of electric field lines of two equal and similar positive charges placed close to each other. The lines are almost radial at points very close to the positive charges and repel each other, bending outwards. There is a point *P* midway between the charges where no lines are present. The fields of the two charges at this point cancel each other and the resultant field at this point is zero.

Fig. 15.15(b) depicts the field lines due to a dipole. The number of lines leaving the positive charge is equal to the number of lines terminating on the negative charge.

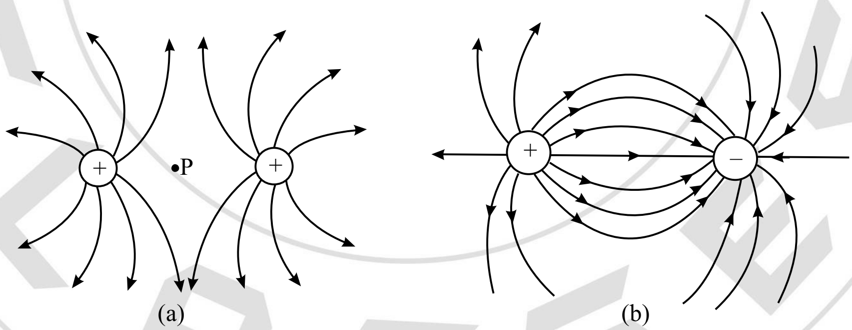


Fig. 15.15 : Electric field lines due to a system of two point charges : **a)** Two positive charges at rest, and **b)** The field lines due to a dipole start from the positive charge and terminate on the negative charge.

You must remember the following properties of the electric field lines :

- The field lines start from a positive charge radially outward in all directions and terminate at infinity.
- The field lines start from infinity and terminate radially on a negative charge.

- For a dipole, field lines start from the positive charge and terminate on the negative charge.
- A tangent at any point on field line gives the direction of electric field at that point.
- The number of field lines passing through unit area of a surface drawn perpendicular to the field lines is proportional to the field strength on this surface.
- Two field lines never cross each other.

15.4 ELECTRIC FLUX AND GAUSS' LAW

Let us consider a sphere of radius r having charge $+q$ located at its center. The magnitude of electric field at every point on the surface of this sphere is given by

$$E = k \times \frac{q}{r^2}$$

The direction of the electric field is normal to the surface and points outward. Let us consider a small element of area Δs on the spherical surface. **Δs** is a vector whose magnitude is equal to the element of area Δs and its direction is perpendicular to this element (Fig.15.16). The electric flux Δϕ is defined as the scalar product of **Δs** and **E :**

$$\Delta\phi = \mathbf{E} \cdot \mathbf{\Delta s}$$

The total flux over the entire spherical surface is obtained by summing all such contributions:

$$\phi_E = \sum_{\Delta s_i \to 0} \mathbf{E_i} \cdot \mathbf{\Delta s_i} \qquad (15.21)$$

Since the angle between **E** and **Δs** is zero, the total flux through the spherical surface is given by

$$\phi_E = k \times \frac{q}{r^2} \Sigma \Delta s$$

The sum of all elements of area over the spherical surface is $4\pi r^2$. Hence the net flux through the spherical surface is

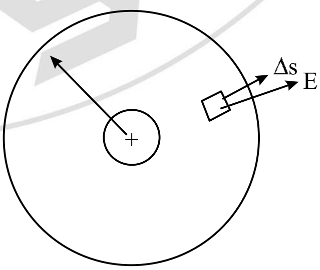


Fig. 15.16

$$\phi_E = k \times \frac{q}{r^2} \times 4\pi r^2$$

$$= 4\pi k \times q$$

On substituting for $k = 1/4\pi\varepsilon_0$, we get

$$\phi_E = \frac{1}{4\pi\varepsilon_0} \times 4\pi q$$
$$= q/\varepsilon_0 \qquad (15.22)$$

The spherical surface of the sphere is referred to as *Gaussian surface*. Eqn. (15.22) is known as Gauss' law. It states that *the net electric flux through a closed gaussian surface is equal to the total charge q inside the surface divided by* ε_0.

Gauss' law is a useful tool for determining the electric field. You must also note that gaussian surface is an imaginary mathematical surface. It may not necessarily coincide with any real surface.

Carl Friedrich Gauss
(1777 – 1855)

German genius in the field of physics and mathematics, Gauss has been one of the most influential mathematicians. He contributed in such diverse fields as optics, electricity and magnetism, astronomy, number theory, differential geometry, and mathematical analysis.

As child prodgy, Gauss corrected an error in his father's accounts when he was only three year old. In primary school, he stunned his teacher by adding the integers 1 to 100 within a second.

Though he shun interactions with scientific community and disliked teaching, many of his students rose to become top class mathematicians – Richard Dedekind, Berhard Riemann, Friedrich Bessel and Sophie Germain are a few among them. Germany issued three postal stamps and a 10 mark bank note in his honour. A crater on moon called Gauss crater, and asteroid 100 called Gaussia have been named after him.

15.4.1 Electric Field due to a Point Charge

Let us apply Gauss' law to calculate electric field due to a point charge. Draw a spherical surface of radius r with a point charge at the centre of the sphere, as shown in Fig. 15.17.

The electric field **E** is along the radial direction pointing away from the centre and normal to the surface of the

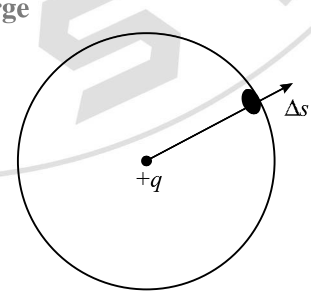


Fig. 15.17 : Electric field on a spherical surface due to a charge $+q$ at its centre

sphere at every point. The normal to the element of area $\mathbf{\Delta s}$ is parallel to $\mathbf{E}$. According to Gauss' law, we can write

$$\phi_E = \sum_i \mathbf{E}_i.\mathbf{\Delta s}_i = q/\varepsilon_0$$

Since $\cos\theta = 1$ and $\mathbf{E}$ is same on all points on the surface, we can write

$$\phi_E = E \times 4\pi r^2$$

or $$q/\varepsilon_0 = E \times 4\pi r^2$$

$\Rightarrow$ $$E = \frac{q}{4\pi\varepsilon_0 r^2} \tag{15.23}$$

If there is a second charge q_0 placed at a point on the surface of the sphere, the magnitude of force on this charge would be

$$F = q_0 \times E$$

so that

$$F = \frac{qq_0}{4\pi\varepsilon_0 r^2} \tag{15.24}$$

Do you recogmise this result? It is expression for Coulomb's force between two static point charges.

15.4.2 Electric Field due to a Long Line Charge

A line charge is in the form of a **thin charged wire of infinite length** with a uniform linear charge density σ_l (charge per unit length). Let there be a charge $+q$ on the wire. We have to calculate the electric field at a point P at a distance r. Draw a right circular cylinder of radius r with the long wire as the axis of the cylinder. The cylinder is closed at both ends. The surface of this cylinder is the gaussian surface and shown in Fig. 15.18. The magnitude of the electric field E is same at every point on the curved surface of the cylinder because all points are at the same distance from the charged wire. The electric field direction and the normal to area element Δs are parallel.

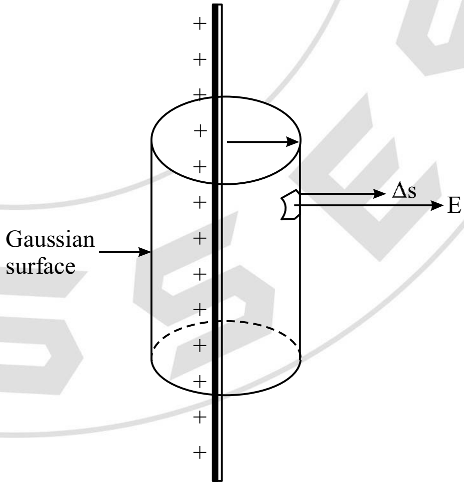


Fig. 15.18 : Electric field due to an infinite line of charges having uniform linear charge density. The gaussian surface is a right circular cylinder.

Let the length of the gaussian cylinder be l. The total charge enclosed in the cylinder is $q = \sigma_l l$. The area of the curved surface of the cylinder is $2\pi rl$.

For the flat surfaces at the top and bottom of the cylinder, the normals to these areas are perpendicular to the electric field ($\cos 90° = 0$). These surfaces, therefore, do not contribute to the total flux. Hence

$$\phi_E = \Sigma \mathbf{E}.\Delta\mathbf{s}$$

$$= E \times 2\pi rl$$

According to Gauss' law, $\phi_E = q/\varepsilon_0$. Hence

$$E \times 2\pi rl = q/\varepsilon_0 = \sigma_l l/\varepsilon_0$$

or

$$E = \frac{\sigma_l}{2\pi\varepsilon_0 r} \qquad (15.25)$$

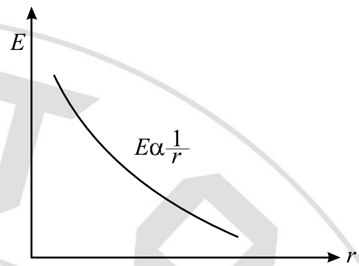
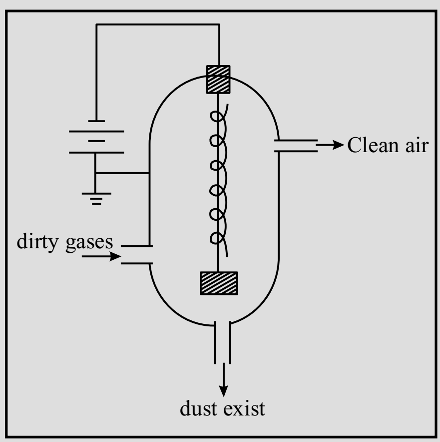


Fig. 15.19 : Variation of E with r for a line charge

This shows that electric field varies inversely with distance. This is illustrated in Fig. 15.19.

Electrostatic Filter

You must have seen black smoke and dirt particles coming out of a chimney of a thermal power station or brick klin. The smoke consists of not only gases but large quantities of small dust (coal) particles. The smoke along with the dirt is discharged into the atmosphere. The dust particles settle down on earth and pollute the soil. The gases contribute to global warming. These are extremely injurious to living systems (health). It is therefore essential that the dirt is removed from smoke before it is discharged into the atmosphere.



A very important application of electrical discharge in gases by application of high electric field is the construction of a device called *Electrostatic Filter or Precipitator*.

The basic diagram of the device is shown here. The central wire inside a metallic container is maintained at a very high negative potential (about 100 kV). The wall of the container is connected to the positive terminal of a high volt battery and is **earthed.** A weight W keeps the wire straight in the central part. The electric field thus created is from the wall towards the wire. The dirt and gases are passed through the container. An electrical discharge takes place because of the high field near the wire. Positive and negative ions and electrons are generated. These negatively charged particles are accelerated towards the wall. They collide with dust particles and charge them. Most of the dust particles become negatively charged because they capture electrons or negative ions. They are attracted towards the wall of the container. The container is periodically shaken so that the particles leave the surface and fall down at the bottom of the container. These are taken out through the exit pipe.

The undesirable dust particles are thus removed from the gases and the clean air goes out in the atmosphere. Most efficient systems of this kind are able to remove about 98% of the ash and dust from the smoke.

15.4.3 Electric Field due to a Uniformly Charged Spherical Shell

A spherical shell, by definition, is a hollow sphere having infinitesimal small thickness. Consider a spherical shell of radius R carrying a total charge Q which is uniformly distributed on its surface. We shall calculate the electric field due to the spherical charge distribution at points external as well as internal to the shell.

(a) Field at an external point

Let P be an external point distant r from the center O of the shell. Draw a spherical surface (called Gaussian surface) passing though P and concentric with the charge distribution. By symmetry, the electric field is radial, being directed outward as shown in Fig. 15.20.

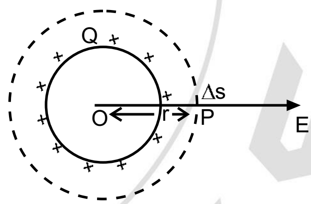


Fig. 15.20

The electric field $\vec{E}$ is normal to the surface element everywhere. Its magnitude at all points on the Gaussian surface has the same value E.

According to Gauss' law,

$$\Sigma E \Delta s \cos 0° = \frac{Q}{\varepsilon_0}$$

or $$\Delta E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

or $$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

From the result we can conclude that for a point external to the spherical shell, the entire charge on the shell can be treated as though located at its centre. The electric field decreases with distance.

Instead of a spherical shell if we had taken a charged solid conducting sphere, we would have obtained the same result. This is because the charge of a conductor always resides on its outer surface.

(b) Field at an Internal Point

Let P' be an internal point distant r from the centre of the shell. Draw a concentric sphere passing through the point P'.

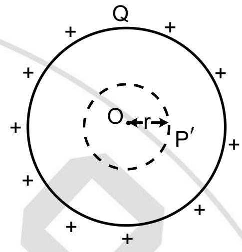


Fig. 15.21

Applying Gauss' Law,

$$\Sigma E \Delta s \cos 0° = \frac{Q}{\varepsilon_0}$$

or $$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$\Rightarrow$ $$E = 0 \text{ as } Q = 0$$

the electric field at an internal point of the shell is zero. The same result is applicable to a charged solid conducting sphere.

The variation of the electric field with the radial distance r has been shown in Fig 15.22.

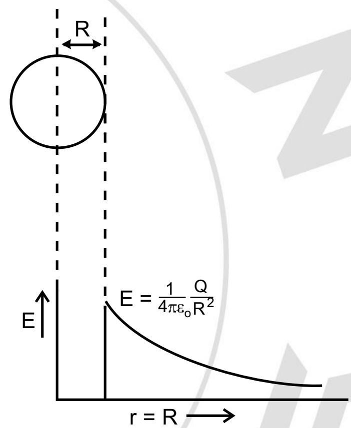


Fig. 15.22

15.4.4 Electric Field due to a Plane Sheet of Charge

Consider an infinite plane sheet of charge $ABCD$, charged uniformly with surface charge density σ.

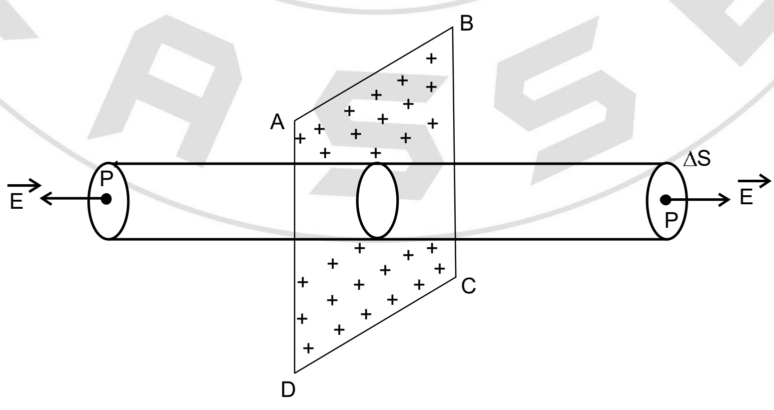


Fig. 15.23

For symmetry reasons, the electric field will be perpendicular to the sheet, directed away from it, if $\sigma > 0$. Let P be the point in front of the sheet where we want to find the electric field. Draw a Gaussian surface in the form of a cylinder with its axis parallel to the field and one of its circular caps passing through P. The other circular cap of the cylinder lies symmetrically opposite at P', on the other side of the sheet, being situated at the same distance as P.

The electric flux through both the circular caps is

$$\vec{E}\cdot\Delta\vec{s} + \vec{E}\cdot\Delta\vec{s} = E\Delta s + E\Delta s$$
$$= 2E\Delta s$$

The electric flux through the curved surface of the Gaussian surface is $\vec{E}\cdot\Delta\vec{s} = E\Delta s\cos 90° = 0.$ Hence, the total electric flux through the Gaussian cylinder is

$$\phi_E = \sum \vec{E}\cdot\Delta\vec{s}$$
$$= 2E\Delta s$$

As the charge enclosed by the Gaussian cylinder is $\sigma\Delta s$, using Gauss' Law we have

$$2E\Delta s = \frac{1}{\varepsilon_0}\sigma\Delta s$$

or

$$E = \frac{\sigma}{2\varepsilon_0}$$

Please note that the electric field is independent of the distance from the sheet.

15.5 VAN DE GRAAFF GENERATOR

Van de Graaff Generator is an electrostatic device that can produce potential differences of the order of a few million volts. It was named after its designer Robert J. van de Graaff.

It consists of a large hollow metallic sphere S mounted on an insulating stand. A long narrow belt, made of an insulating material, like rubber or silk, is wound around two pulleys P_1 and P_2 as shown in Fig. 15.5. The pulley P_2 is mounted at the centre of the sphere S while the pulley P_1 is mounted near the bottom. The belt is made to rotate continuously by driving the pulley P_1 by an electric motor M. Two comb-shaped conductors C_1 and C_2, having a number of sharp points in the shape of metallic needles, are mounted near the pulleys.

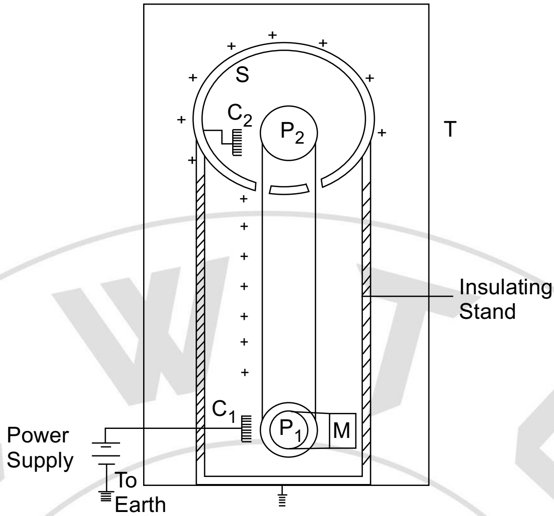


Fig. 15.24

The needles point towards the belt. The comb-shaped conductor C_1 is maintained at a high positive potential (~10^4 V) relative to the ground with the help of a power supply. The upper comb C_2 is connected to the inner surface of the metallic sphere S.

Near the sharp points of the comb-shaped conductor C_1, the charge density and electrostatic field are very high. Large electrostatic field near their pointed ends causes dielectric breakdown of the air, producing ions (both positive and negative) in the process. This phenomenon is known as corona discharge. The negative charges from the air move torwards the needles and the positive charge towards the belt. The negative charges neutralize some of the positive charges on the comb C_1. However, by supplying more positive charges to C_1, the power supply maintains its positive potential. As the belt carrying the positive charges moves towards C_2, the air near it becomes conducting due to corona discharge. The negative charges of the air move towards the belt neutralizing its positive charges while the positive charges of the air move towards the needles of the comb C_2. These positive charges are then transferred to the conducting sphere S which quickly moves them to its outer surface.

The process continues and positive charges keep on accumulating on the sphere S and it acquires a high potential.

As the surrounding air is at ordinary pressure, the leakage of charge from the sphere takes place. In order to prevent the leakage, the machine is surrounded by an earthed metallic chamber T whose inner space is filled with air at high pressure.

By using Van de Graaff generator, voltage upto 5 million volts (MV) have been achieved. Some generators have even gone up to creation of such high voltages as 20 MV.

Van de Graaff generator is used to accelerate the ion beams to very high energies which are used to study nuclear reactions.

INTEXT QUESTIONS 15.4

1. If the electric flux through a gaussian surface is zero, does it necessarily mean that
   (a) the total charge inside the surface is zero?
   (b) the electric field is zero at every point on the surface?
   (c) the electric field lines entering into the surface is equal to the number going out of the surface?
2. If the electric field exceeds the value 3.0×10^6 NC^{-1}, there will be sparking in air. What is the highest value of charge that a metallic sphere can hold without sparking in the surrounding air, if the radius of the sphere is 5.0 cm?
3. What is the magnitude and direction of the net force and net torque on a dipole placed along a a) uniform electric field, and b) non-uniform field.

WHAT YOU HAVE LEARNT

- Electric charge is produced when glass rod is rubbed with silk or rubber is rubbed with fur.
- By convention, the charge on glass rod is taken **positive** and that on rubber is taken **negative.**
- Like charges repel and unlike charges attract each other.
- Coulomb's law gives the magnitude and direction of force between two point charges :

$$\mathbf{F} = k\,\frac{q_1 \times q_2}{r^2}\,\hat{\mathbf{r}}$$

where $k = \dfrac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9$ Nm²C⁻².

- The smallest unit of charge in nature is the charge on an electron :

$$e = 1.60 \times 10^{-19}\,\text{C}$$

- Charge is conserved and quantised in terms of electronic charge.
- The electric field **E** due to a charge q at a point in space is defined as the force experienced by a unit test charge q_0 :

$$\mathbf{E} = \mathbf{F}/q_0 = k \times \frac{q}{r^2}\hat{\mathbf{r}}$$

- Superposition principle can be used to obtain the force experienced by a charge due to a group of charges. It is also applicable to electric field at a point due to a group of charges.
- Electric dipole is a system of two equal and unlike charges separated by a small distance. It has a dipole moment $|\mathbf{p}| = qr$; the direction of **p** is from negative charge to positive charge along the line joining the two charges.
- The electric field due to a dipole in end-on position and broad-on position is respectively given by

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\mathbf{p}}{r^3}$$

and $$\mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{r^3}.$$

- Electric field lines (line of force) are only a pictorial way of depicting field.
- Electric flux is the total number of electric lines of force passing through an area and is defined as $\phi_E = \mathbf{E}\cdot\mathbf{A}$.
- Gauss's law states that the total flux passing through a closed area is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by it.
- The electric field due to a line charge is given by $E = \frac{\sigma_l}{2\pi\varepsilon r}$.

ANSWERS TO INTEXT QUESTIONS

15.1

1. (i) Yes (ii) Charge = 3.2×10^{-17} C.

2. A has charge $+Q$. When A and B are brought in contact, charge wi distributed equally.
   (i) Yes., (ii) $+Q/2$
3. $q = 4.8 \times 10^{-16}$

   Since $Ne = q$, we get

   $$N = \frac{4.8\times10^{-16}}{1.6\times10^{-19}} = 3.0\times10^{3}\text{charges}$$

15.2

1. $Q_1 = 16\mu C$, $Q_2 = q\mu C$ and $r = 12m$

Since

$$F = \frac{1}{4\pi\varepsilon_0}\ \frac{q_1q_2}{r^2}$$

$$= \frac{(9\times10^{9}\,\text{Nm}^2\text{C}^{-2})(16\times10^{-6}\,\text{C})\times(12\times^{-6}\,\text{C})}{144\text{m}^2}$$

$= 9\times 10^{-3}$ N

(i) direction from q_2 to q_1

(ii) direction from q_1 to q_2

2. The force at A due to charge at B, $F_1 = k\dfrac{q^2}{a^2}$ where $AB = a$

   Since $AB = AC$, the force at A due to charge at B is

   $$F_2 = k\frac{q^2}{a^2}$$

   $R^2 = F_1^2 + F_2^2 = 2F^2$

   $R = F\sqrt{2}$ at 45°

15.3

1. (a) E along the $+x$ axis.
   (b) along the $+y$ axis.
   (c) at 45° with the x axis
2. $AB = AC = 40$ cm

   $$|\mathbf{E}_1| = \frac{kq}{r^2} = |\mathbf{E}_2| = \frac{9\times10^{9}\,\text{Nm}^2\text{C}^{-2}\times(2\times10^{-6}\,\text{C})}{(0.40\text{m})^2} = 1.125\times10^{5}\,\text{NC}^{-1}$$

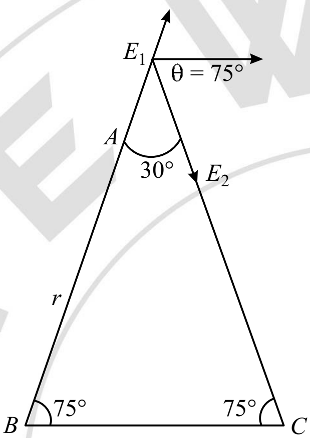
e resultant of $\mathbf{E}_1$ and $\mathbf{E}_2$ will be parallel to BC. Hence

$$= E_1^2 + E_2^2 + 2E_1 E_2 \cos 150$$

$$= 2E^2 + 2E^2 \cos(180\text{-}30)$$

$$= 2E^2 - 2E^2 \times \cos 30 = 2E^2\left(1+\frac{\sqrt{3}}{2}\right) = 4.723 \times 10^{10}\ \mathrm{N^2C^{-2}}.$$

rection will be parallel to BC in the direction $B \to C$.



s directed towards the earth. The force on –ve charge will be vertically wards.

e field will be zero at the mid point between the charge.

Yes (ii) not necessarily (iii) Yes.

$$= \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$= 4\pi\varepsilon_0 r^2 E$$

$$= (3 \times 10^6\ \mathrm{NC^{-1}}) \times \frac{1}{(9\times10^9\ \mathrm{Nm^2C^{-2}})} \times (25 \times 10^{-4}\mathrm{m^2})$$

$$= 8.3 \times 10^{-7}\ \mathrm{C}$$

$\mathbf{F} = 0, \boldsymbol{\tau} = 0$

$\mathbf{F} \neq 0\ \boldsymbol{\tau} = 0$