# Asymptotically time-optimal smooth trajectory planning in dynamic environments

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*Abstract*—In this paper we proposed an algorithm for smooth trajectory generation in complex environments with dynamic obstacles and velocity constraints. The proposed algorithm Tube Space-Time RRT* (Tube ST-RRT*) is combined with the improved reformulation dynamic coordinate minimum snap (RDCMS) to generate smooth collision-free trajectories with asymptotic time Optimal.First, sample the space-time state space to obtain the time information for each node to complete the avoidance of moving obstacles.Then, to address the issue of non-smooth paths in ST-RRT*, generate a dynamic Tube for each node and combine it with the improved minisnap to create a smooth, collision-free trajectory Finally,Simulations in complex environments demonstrate the effectiveness of our proposed algorithm.

*Index Terms*—Tube Space-Time RRT*; asymptotic time Optimal; smooth collision-free trajectories

## I. INTRODUCTION

Trajectory planning is a fundamental challenge in robotics [1], as obstacles in the real world often change over time. Applications such as robotics and autonomous driving typically require a smooth, collision-free trajectory. Assuming the obstacle trajectories are known a priori, the problem can be modeled as navigation in a dynamic environment. Mathematically, this is expressed as planning through a space-time state space [2].

Planning in dynamic environments has been studied for a long time and has yielded significant research results. These results can be broadly categorized into two approaches. For example, RRTX [3] and Real-time RRT* [4] require fast re-planning when previously computed paths become invalid during execution. However, as the dimensionality increases, the replanning time becomes longer, making it difficult to respond to moving obstacles. RiskRRT [5] combines predictions of obstacle movements and computes partial motion paths to keep the collision probability below a given threshold. However, since only partial paths are returned, frequent replanning is still necessary.Another approach assumes that the trajectories of moving obstacles are unknown, while another assumes that the trajectories of the obstacles are completely known. For example, Time-Based RRT [6] extends the configuration state space through the time dimension and unidirectionally plans to a set of known target states. However, it requires the assumption of knowing the specific time for each target

configuration. Additionally, due to the random sampling nature of RRT, the resulting trajectories are usually not smooth.

In this paper, we proposes a Tube-based space-time sampling planning algorithm, Tube-ST-RRT*, and an improved reformulation dynamic coordinate minimum snap (RDCMS) method. Through a two-step method of planning and optimizing, it achieves asymptotic time-optimal collision-free trajectory planning in dynamic environments, while ensuring high trajectory smoothness to facilitate easier tracking control. The Tube-ST-RRT* algorithm adds a time dimension to the configuration space so that each node contains time information, allowing it to respond to time-varying obstacles. Additionally, the improved RDCMS method combines with each node's tube information and time information to generate smooth, collision-free trajectories. Finally, comparative simulations verify the effectiveness of the algorithm

The rest of this paper is organized as follows: At Section II, explains the problem of time-optimal planning in dynamic environments. In Section III,presents the main results of this paper, including the principles and steps of the Tube ST-RRT* planning algorithm and the RDCMS trajectory optimization algorithm. Section IV provides simulation results and comparative experiments to demonstrate the feasibility and superiority of the proposed method. Finally, Section V summarizes the paper.

## II. PREPARE YOUR PAPER BEFORE STYLING

Before you begin to format your paper, first write and save the content as a separate text file. Complete all content and organizational editing before formatting. Please note sections II-A to II-H below for more information on proofreading, spelling and grammar.

Keep your text and graphic files separate until after the text has been formatted and styled. Do not number text heads—LaTeX will do that for you.

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Define abbreviations and acronyms the first time they are used in the text, even after they have been defined in the abstract. Abbreviations such as IEEE, SI, MKS, CGS, ac, dc, and rms do not have to be defined. Do not use abbreviations in the title or heads unless they are unavoidable.

### B. Units

- Use either SI (MKS) or CGS as primary units. (SI units are encouraged.) English units may be used as secondary units (in parentheses). An exception would be the use of English units as identifiers in trade, such as "3.5-inch disk drive".
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Number equations consecutively. To make your equations more compact, you may use the solidus ( / ), the exp function, or appropriate exponents. Italicize Roman symbols for quantities and variables, but not Greek symbols. Use a long dash rather than a hyphen for a minus sign. Punctuate equations with commas or periods when they are part of a sentence, as in:

$$a + b = \gamma \qquad (1)$$

Be sure that the symbols in your equation have been defined before or immediately following the equation. Use "(1)", not "Eq. (1)" or "equation (1)", except at the beginning of a sentence: "Equation (1) is . . ."

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Please don't use the `{eqnarray}` equation environment. Use `{align}` or `{IEEEeqnarray}` instead. The `{eqnarray}` environment leaves unsightly spaces around relation symbols.

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### E. Some Common Mistakes

- The word "data" is plural, not singular.
- The subscript for the permeability of vacuum $\mu_0$, and other common scientific constants, is zero with subscript formatting, not a lowercase letter "o".
- In American English, commas, semicolons, periods, question and exclamation marks are located within quotation marks only when a complete thought or name is cited, such as a title or full quotation. When quotation marks are used, instead of a bold or italic typeface, to highlight a word or phrase, punctuation should appear outside of the quotation marks. A parenthetical phrase or statement at the end of a sentence is punctuated outside of the closing parenthesis (like this). (A parenthetical sentence is punctuated within the parentheses.)
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- In your paper title, if the words "that uses" can accurately replace the word "using", capitalize the "u"; if not, keep using lower-cased.
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An excellent style manual for science writers is [7].

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**The class file is designed for, but not limited to, six authors.** A minimum of one author is required for all conference articles. Author names should be listed starting from left to right and then moving down to the next line. This is the author sequence that will be used in future citations and by indexing services. Names should not be listed in columns nor group by affiliation. Please keep your affiliations as succinct as possible (for example, do not differentiate among departments of the same organization).

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Headings, or heads, are organizational devices that guide the reader through your paper. There are two types: component heads and text heads.

Component heads identify the different components of your paper and are not topically subordinate to each other. Examples include Acknowledgments and References and, for these, the correct style to use is "Heading 5". Use "figure caption" for your Figure captions, and "table head" for your table title. Run-in heads, such as "Abstract", will require you to apply a style (in this case, italic) in addition to the style provided by the drop down menu to differentiate the head from the text.

Text heads organize the topics on a relational, hierarchical basis. For example, the paper title is the primary text head because all subsequent material relates and elaborates on this one topic. If there are two or more sub-topics, the next level head (uppercase Roman numerals) should be used and, conversely, if there are not at least two sub-topics, then no subheads should be introduced.

### H. Figures and Tables

*a) Positioning Figures and Tables:* Place figures and tables at the top and bottom of columns. Avoid placing them in the middle of columns. Large figures and tables may span across both columns. Figure captions should be below the figures; table heads should appear above the tables. Insert figures and tables after they are cited in the text. Use the abbreviation "Fig. 1", even at the beginning of a sentence.

TABLE I: Table Type Styles

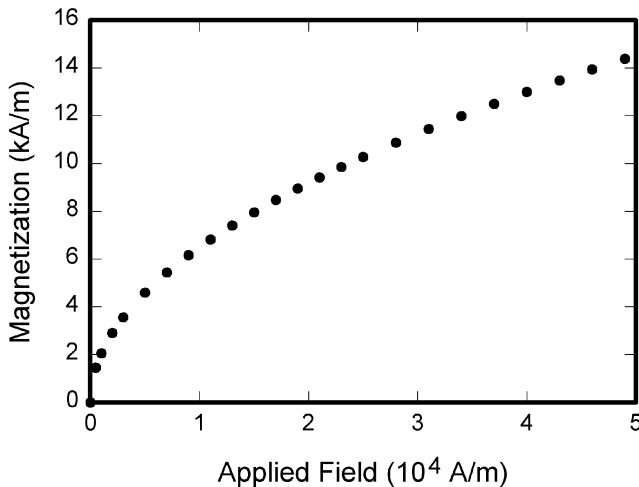| Table Head | Table Column Head | | |
|---|---|---|---|
| | *Table column subhead* | *Subhead* | *Subhead* |
| copy | More table copy[a] | | |

[a]Sample of a Table footnote.



Fig. 1: Example of a figure caption.

Figure Labels: Use 8 point Times New Roman for Figure labels. Use words rather than symbols or abbreviations when writing Figure axis labels to avoid confusing the reader. As an example, write the quantity "Magnetization", or "Magnetization, M", not just "M". If including units in the label, present them within parentheses. Do not label axes only with units. In the example, write "Magnetization (A/m)" or "Magnetization $\{A[m(1)]\}$", not just "A/m". Do not label axes with a ratio of quantities and units. For example, write "Temperature (K)", not "Temperature/K".

### ACKNOWLEDGMENT

The preferred spelling of the word "acknowledgment" in America is without an "e" after the "g". Avoid the stilted expression "one of us (R. B. G.) thanks …". Instead, try "R. B. G. thanks…". Put sponsor acknowledgments in the unnumbered footnote on the first page.

### REFERENCES

Please number citations consecutively within brackets [1]. The sentence punctuation follows the bracket [2]. Refer simply to the reference number, as in [3]—do not use "Ref. [3]" or "reference [3]" except at the beginning of a sentence: "Reference [3] was the first …"

Number footnotes separately in superscripts. Place the actual footnote at the bottom of the column in which it was cited. Do not put footnotes in the abstract or reference list. Use letters for table footnotes.

Unless there are six authors or more give all authors' names; do not use "et al.". Papers that have not been published, even if they have been submitted for publication, should be cited as "unpublished" [4]. Papers that have been accepted for publication should be cited as "in press" [5]. Capitalize only the first word in a paper title, except for proper nouns and element symbols.

For papers published in translation journals, please give the English citation first, followed by the original foreign-language citation [6].

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