Homework 1

Sep 10, 2025

1 (30'). Prove

$$\frac{1}{2\ln 2} + \frac{1}{3\ln 3} + \ldots + \frac{1}{n\ln n} = \ln\ln n + O(1)$$

and

$$\frac{1}{1^{1.1}} + \frac{1}{2^{1.1}} + \ldots \frac{1}{n^{1.1}} = O(1).$$

2 (40'). Order the following functions by asymptotic growth. Place f before g if $f(n) = O(g(n))$.

- n^n
- $\sqrt{n}$
- $\log\log n$
- $0.001n^7$
- $5000n$
- e^n
- $2^{\log^2 n}$
- $10^{10^{10}}$
- $n!$
- 2^{2^n}

3 (30'). Let $\Sigma = \{0, 1\}$. Let language

$$L = \{w \in \{0,1\}^* : w \text{ has an unequal number of 0's and 1's}\}.$$

Prove $L^* = \Sigma^*$.