定常与非定常传热

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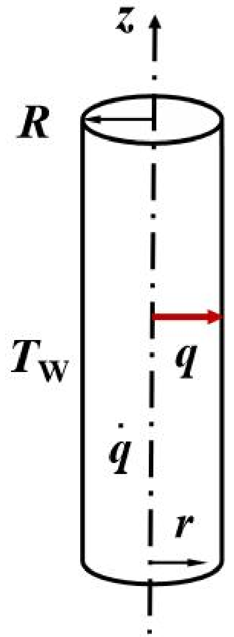
第十一讲. 定常与非定常传热

1. 通电导线内的温度分布
2. 半无限大平壁非定常导热
3. 小球非定常传热

1. 通电导线内的温度分布



柱坐标系下的对流传热微分方程：

$$\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}=a\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{\dot{q}}{\rho C_P}$$

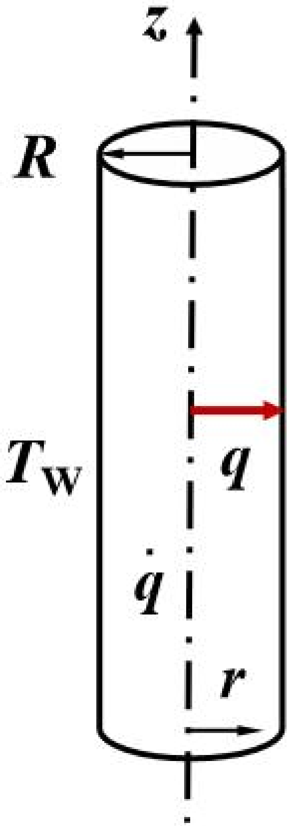
定常： $\frac{\partial T}{\partial t}=0$

导线内： $\begin{cases} u_r=0 \\ u_\theta=0 \\ u_z=0 \end{cases}$ $\begin{cases} \frac{\partial T}{\partial r}\neq 0 \\ \frac{\partial T}{\partial \theta}=0 \\ \frac{\partial T}{\partial z}=0 \end{cases}$ $\begin{cases} \frac{\partial^2 T}{\partial \theta^2}=0 \\ \frac{\partial^2 T}{\partial z^2}=0 \end{cases}$

有内热源： $\dot{q}\neq 0$

简化对流传热微分方程

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{q}}{k}$$



边界条件： $\begin{cases} r=0, & \dfrac{dT}{dr}=0 \\ r=R, & T=T_W \end{cases}$

积分得： $r\dfrac{dT}{dr}=-\dfrac{\dot{q}}{2k}r^2+C_1$

$$\because r=0,\ \frac{dT}{dr}=0;\quad \therefore C_1=0$$

通电导线内的温度分布： $T=T_W+\dfrac{\dot{q}}{4k}\left(R^2-r^2\right)$

过余温度分布：$$T-T_W=\frac{\dot{q}}{4k}\left(R^2-r^2\right)$$

$r=0$，$T=T_0$ 最大温升：$$T_0-T_W=\frac{\dot{q}}{4k}R^2$$

无量纲温度分布：$$\frac{T-T_W}{T_0-T_W}=1-\frac{r^2}{R^2}$$

截面平均温度：$$T_{av}=\frac{\int_0^R T\,2\pi r\,dr}{\pi R^2}=T_W+\frac{\dot{q}}{8k}R^2$$

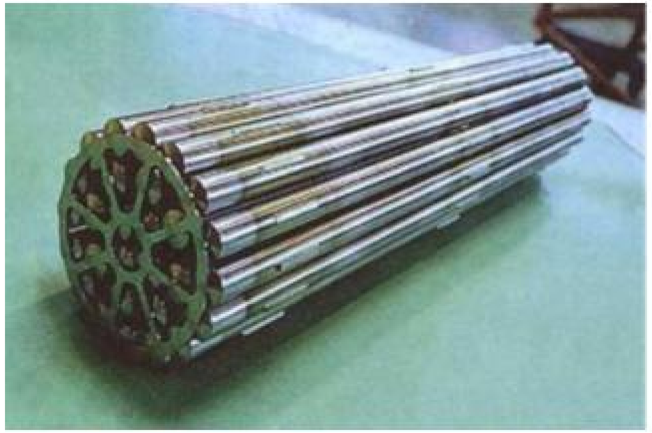
平均温度与最大温升关系：$$\frac{T_{av}-T_W}{T_0-T_W}=\frac{1}{2}$$

问题探讨

其与管内层流规律类似？

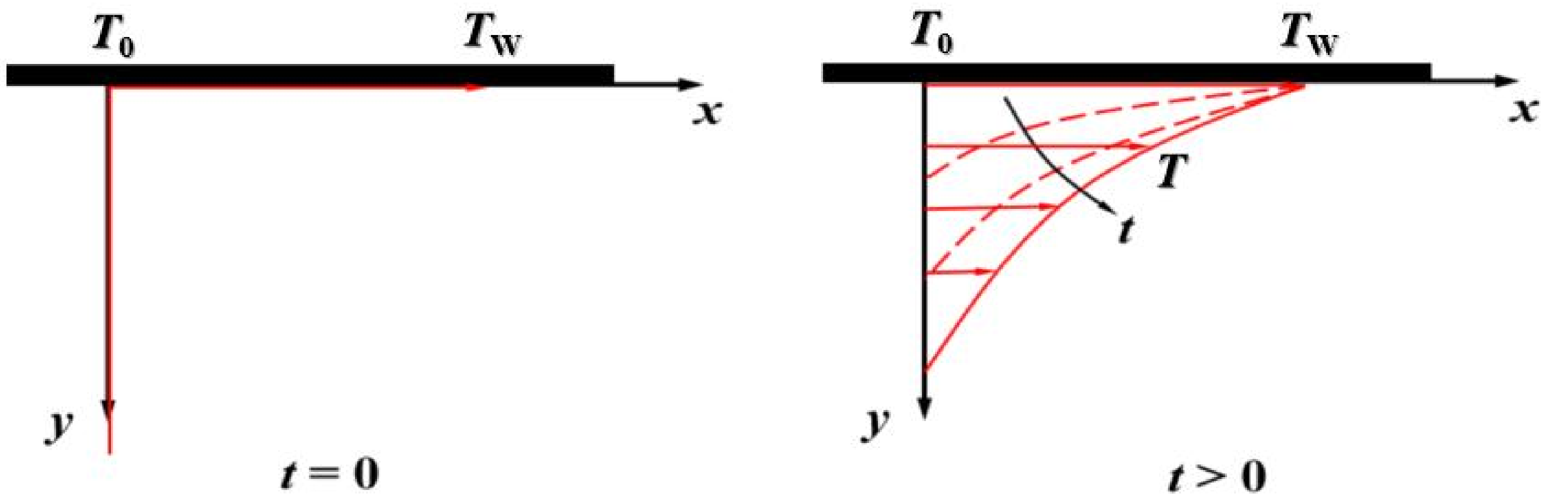
课后思考

1.电热棒、管式固定床反应器、核燃料棒的温度分布规律?



2. 半无限大平壁非定常导热

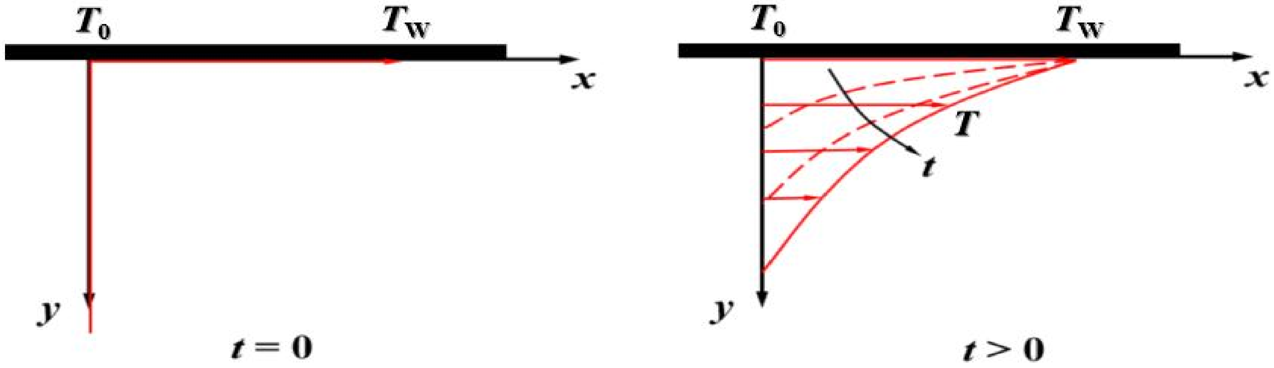
一半无限大平壁，初始温度为 T_0，突然壁面温度变为 T_W，并维持不变。平壁内的温度分布 T 随时间也发生变化。



导热微分方程：$$\rho C_p\frac{\partial T}{\partial t}=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{q}$$

课后自学

1.类似静止流体中的平板启动，推导半无限大平壁非定常导热数学模型为：



$$\frac{d^2T}{d\eta^2}+2\eta\frac{dT}{d\eta}=0$$

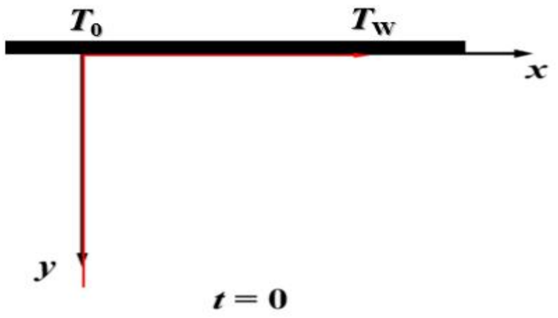
边界条件： $\begin{cases}\eta=0, T=T_W\\ \eta\to\infty, T=T_0\end{cases}$

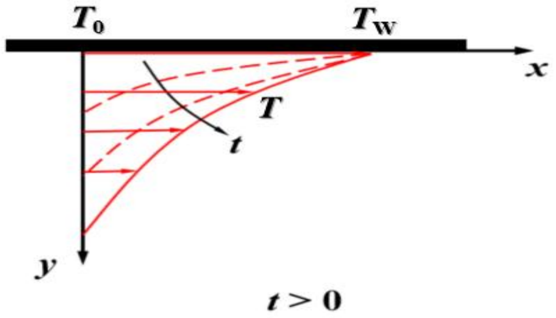
非定常： $\dfrac{\partial T}{\partial t} \neq 0$

一维导热：

$$\begin{cases} \dfrac{\partial T}{\partial x} = 0 \\ \dfrac{\partial T}{\partial y} \neq 0 \\ \dfrac{\partial T}{\partial z} = 0 \end{cases} \qquad \begin{cases} \dfrac{\partial^2 T}{\partial x^2} = 0 \\ \dfrac{\partial^2 T}{\partial y^2} \neq 0 \\ \dfrac{\partial^2 T}{\partial z^2} = 0 \end{cases}$$

无内热源： $\dot{q} = 0$





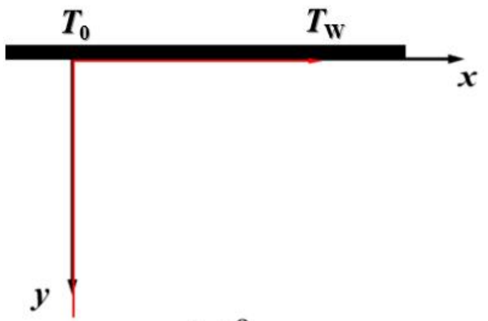
$$\rho C_p \frac{\partial T}{\partial t} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + \dot{q}$$

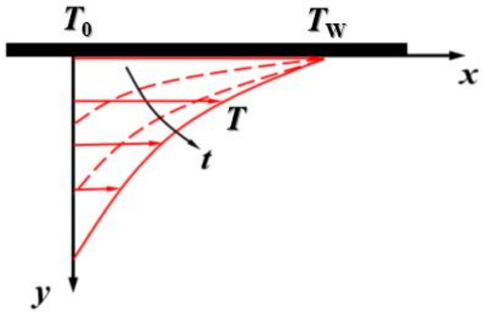
简化导热微分方程，可得：

$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial y^2}$$

初始条件： $t = 0,\ \ T = T_0$

边界条件： $t > 0,\ \begin{cases} y = 0, T = T_W \\ y \to \infty, T = T_0 \end{cases}$





方程为一维非定常偏微分方程： $\dfrac{\partial T}{\partial t} = a\dfrac{\partial^2 T}{\partial y^2}$

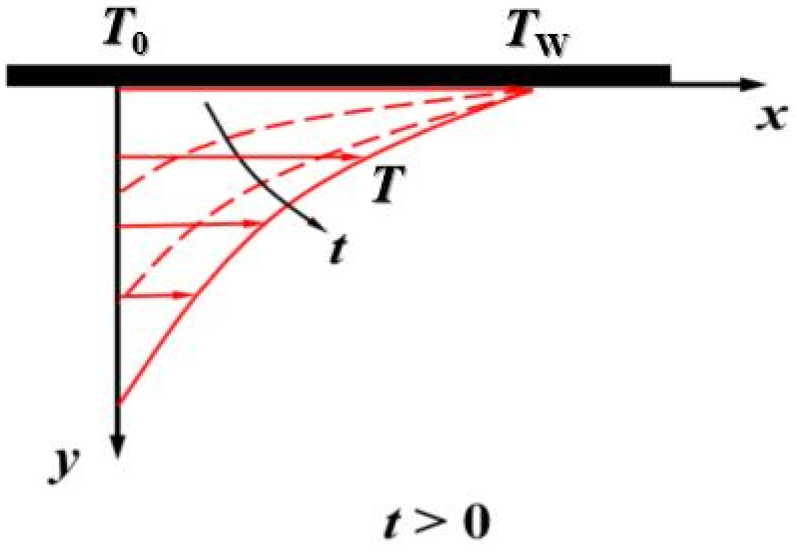
令 $\eta = \dfrac{y}{\sqrt{4at}}$ 则 $\dfrac{\partial T}{\partial t} = \dfrac{\partial T}{\partial \eta}\dfrac{\partial \eta}{\partial t} = -\dfrac{\eta}{2t}\dfrac{\partial T}{\partial \eta}$

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{1}{\sqrt{4at}}\frac{\partial T}{\partial \eta}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{1}{\sqrt{4at}}\frac{\partial \dfrac{\partial T}{\partial \eta}}{\partial \eta}\frac{\partial \eta}{\partial y} = \frac{1}{4at}\frac{\partial^2 T}{\partial \eta^2}$$

代入原方程可得： $\dfrac{d^2 T}{d\eta^2} + 2\eta\dfrac{dT}{d\eta} = 0$

$$\frac{d^2T}{d\eta^2}+2\eta\frac{dT}{d\eta}=0$$



边界条件：
$$\begin{cases}\eta=0, & T=T_W\\ \eta\to\infty, & T=T_0\end{cases}$$

解方程得温度分布：

$$\frac{T-T_W}{T_0-T_W}=\frac{2}{\sqrt{\pi}}\int_0^{\eta}e^{-\eta^2}d\eta=erf\left(\eta\right)$$

$$\frac{T-T_W}{T_0-T_W}=erf\left(\eta\right)$$ **高斯误差函数** $$\eta=\frac{y}{\sqrt{4at}}$$

t 时刻壁面处的导热通量：

$$q_{t,y=0} = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = -k\left(\frac{\partial T}{\partial \eta}\frac{\partial \eta}{\partial y}\right)\bigg|_{y=0} = \frac{k\left(T_W - T_0\right)}{\sqrt{\pi a t}}$$

$0 \sim t$ 时间内通过单位面积壁面的导热量：

$$Q = \int_0^t q_{t,y=0}\,dt = \int_0^t \frac{k\left(T_W - T_0\right)}{\sqrt{\pi a t}}dt = 2k\left(T_W - T_0\right)\sqrt{\frac{t}{\pi a}}$$

大地升温

若温度5°C的大地，已知：大地 $a=4.65\times10^{-7}\ \mathrm{m^2/s}$，表面突然升至37°C。试求：

(1). 1小时后地表面下0.05m处的温度?

(2). [0，t]与[t,2t]内单位面积传热量之比。

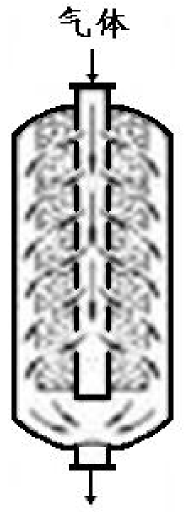
解：(1). $\eta=\dfrac{y}{\sqrt{4at}}=0.61$ 查表 $erf(\eta)\approx0.612=\dfrac{T-T_W}{T_0-T_W}$ $T=17.4°\mathrm{C}$

(2). $$\frac{Q_2}{Q_1}=\frac{2k\left(T_W-T_0\right)\dfrac{\sqrt{2t}-\sqrt{t}}{\sqrt{\pi a}}}{2k\left(T_W-T_0\right)\sqrt{\dfrac{t}{\pi a}}}=\sqrt{2}-1=41.4\%$$

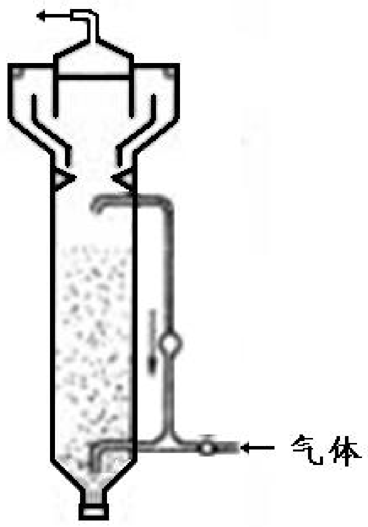
3. 小球非定常传热

传热原理

反应器中的球形催化剂颗粒体积 V，表面积 A，初始温度 T_0，通入温度为 T_f 的热气流，颗粒温度将随时间升高。



固定床反应器

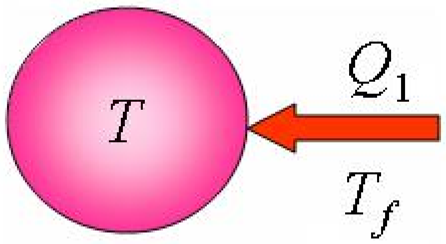


流化床反应器

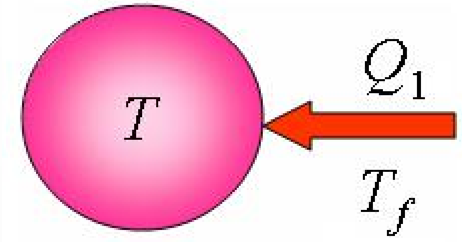
简化：忽略颗粒内部导热热阻，集总参数法。

毕奥数 $Bi < 0.1$

$$Bi = \frac{\frac{V}{A}\Big/k}{1/h} = \frac{\text{内部导热热阻}}{\text{外部对流热阻}}$$



球坐标系下的对流传热微分方程：



$$\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\varphi}{r\sin\theta}\frac{\partial T}{\partial \varphi}$$

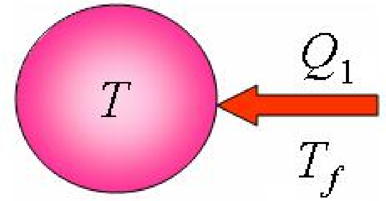
$$=a\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}\right]+\frac{\dot{q}}{\rho C_P}$$

简化得： $$\frac{dT}{dt}=\frac{\dot{q}}{\rho C_P}$$

式中： $$\dot{q}=\frac{hA\left(T_f-T\right)}{V}$$

$$\frac{dT}{dt}=\frac{hA\left(T_f-T\right)}{\rho C_p V}$$

$$\frac{dT}{dt}=\frac{hA\left(T_f-T\right)}{\rho C_p V}$$



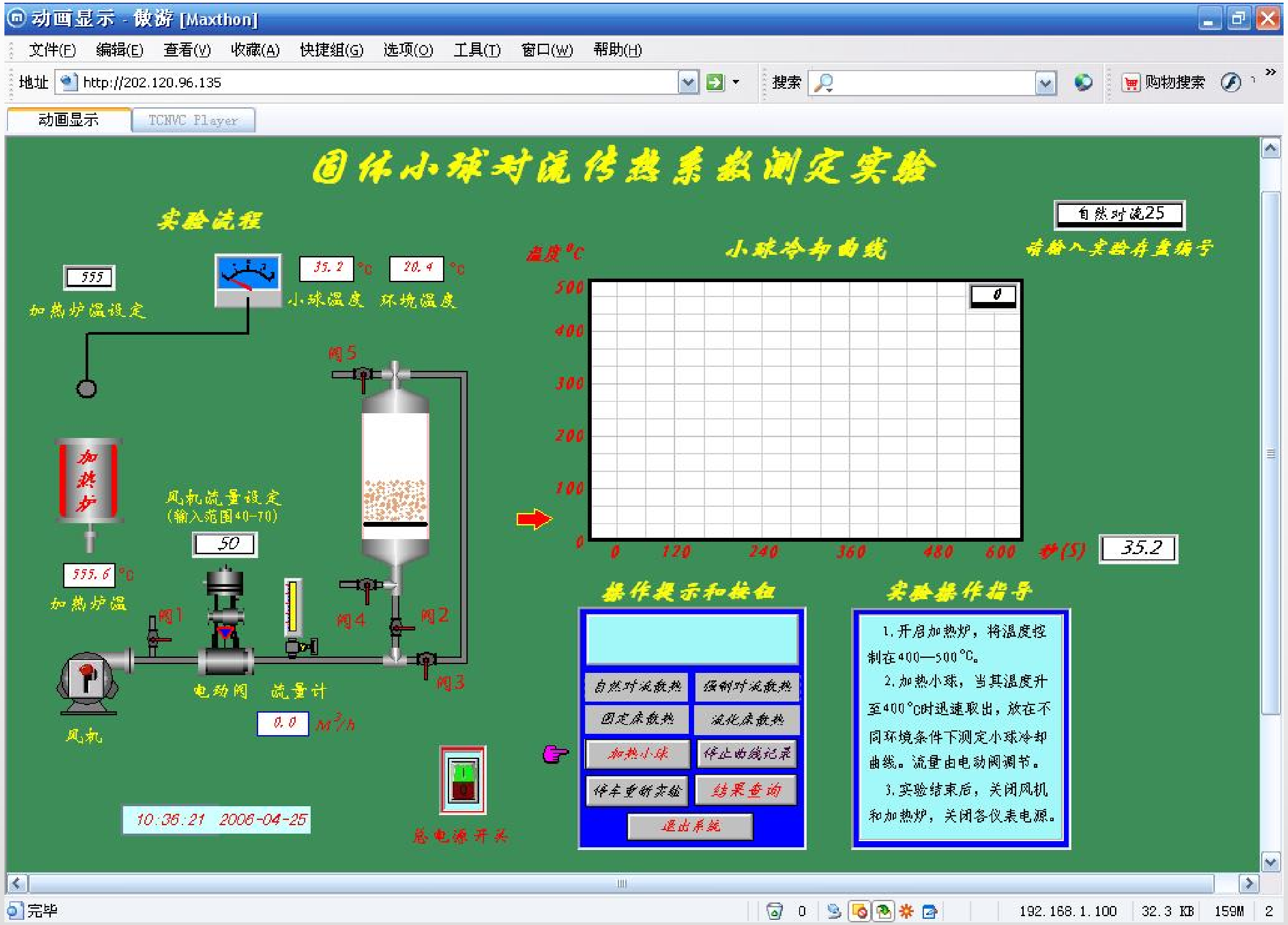
$$\int_{T_0-T_f}^{T-T_f}\frac{d\left(T-T_f\right)}{T-T_f}=-\frac{hA}{\rho C_p V}\int_0^t dt$$

温度随时间变化关系：

$$\frac{T-T_f}{T_0-T_f}=e^{-\frac{hA}{\rho C_p V}\cdot t}$$

小球传热专业实验

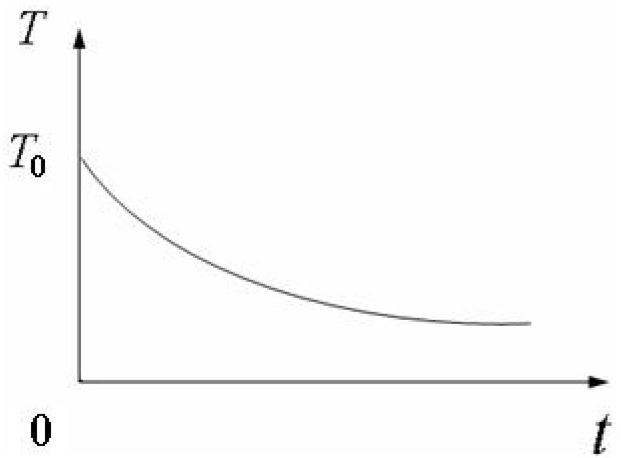
① 控制界面



② 实验装置

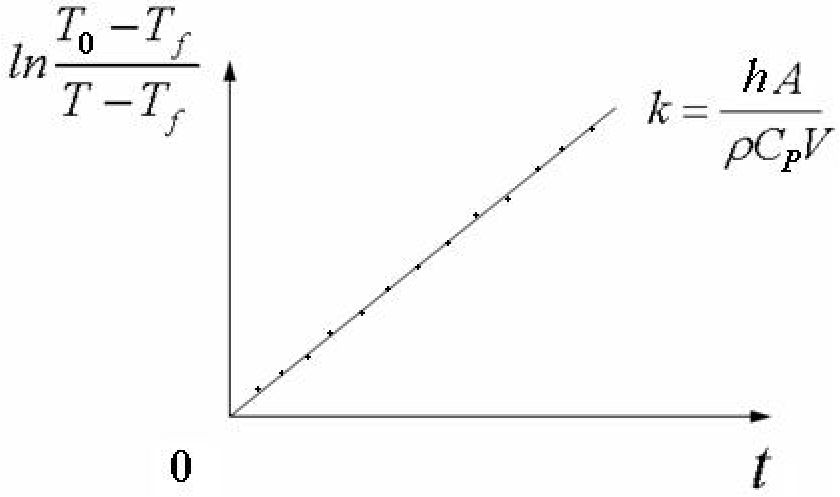
③ 实验结果讨论

小球温度随时间的变化关系



$$\frac{T-T_f}{T_0-T_f}=e^{-\frac{hA}{\rho C_p V}\cdot t}$$

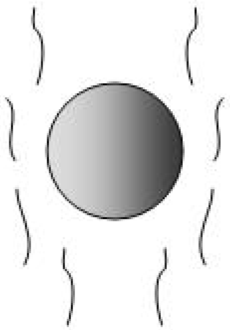
数据处理



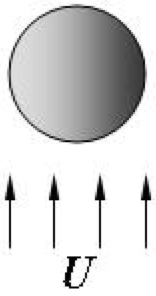
$$ln\frac{T_0-T_f}{T-T_f}=-\frac{hA}{\rho C_p V}\cdot t$$

单个颗粒

自然对流



密度差引起流动

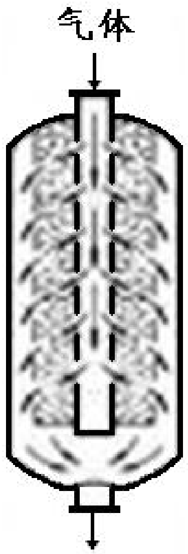


流体流速

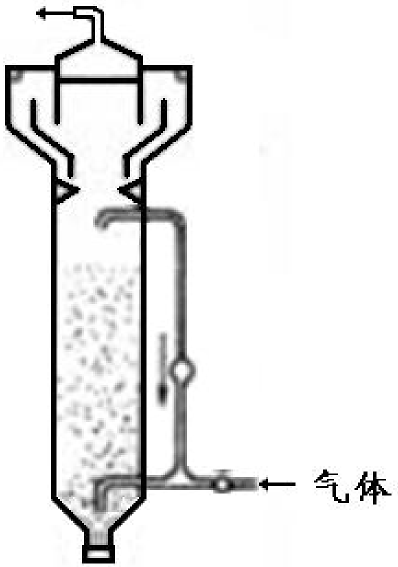
强制对流

颗粒群

固定床



固定床反应器



流化床反应器

流化床

课后思考

1.热电偶测温原理?

2.热电偶支架是不锈钢管，用焊接将其与小球连接，试分析对实验结果有何影响。