第六节 力系的平衡方程及其应用

一、力系的平衡方程

1、空间一般力系平衡的充要条件：

力系的主矢量和对任一点的主矩都等于零。

$$F_R' = \sum F = 0$$
$$M_O = \sum M_O(F) = 0$$

$$\sum F_x = 0 \ , \sum M_x(F) = 0$$
$$\sum F_y = 0 \ , \sum M_y(F) = 0$$
$$\sum F_z = 0 \ , \sum M_z(F) = 0$$

空间一般力系平衡的必要和充分条件是力系中各力在各坐标轴上的投影的代数和以及对各坐标轴之矩的代数和分别等于零。

- 平面任意力系的平衡方程

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M_z(F) = \sum M_O(F) = 0 \end{cases}$$

平面一般力系平衡的必要和充分条件是：

力系中各力在直角坐标系Oxy的各坐标轴上的投影的代数和及对任意点（如O点）力矩的代数和分别等于零。

- 特殊力系的平衡方程

	汇交力系	平行力系	力偶系
空间	$\sum F_x = 0$ $\sum F_y = 0$ $\sum F_z = 0$	$\sum F_z = 0$ $\sum M_x(F) = 0$ $\sum M_y(F) = 0$	$\sum M_x = 0$ $\sum M_y = 0$ $\sum M_z = 0$
平面	$\sum F_x = 0$ $\sum F_y = 0$	$\sum F_z = 0$ $\sum M_o(F) = 0$	$\sum M = 0$

二、 静力分析的基本方法及典型实例

(1)问题的要求 ⇨ 选取合适的研究对象

⇨ 受力分析 ⇨ 画出受力图

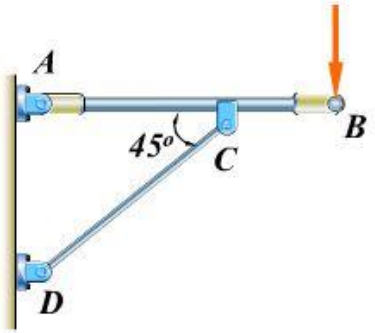
(2)受力图 ⇨ 确定受力系的类型 ⇨ 列出平衡方程

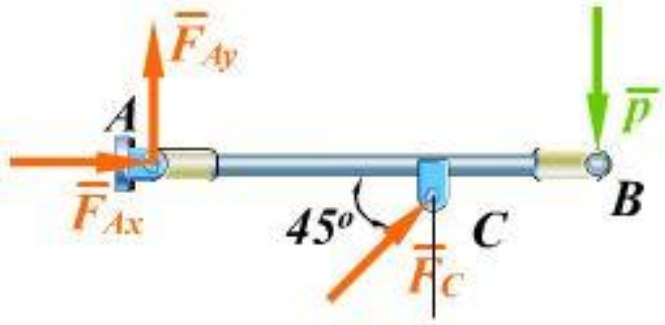
⇨ 求出未知力大小和方向

例1

已知：$AC=CB=l$，$P=10\text{kN}$;

求： 铰链A和DC杆受力.





解：取AB梁，画受力图.

$$\sum F_x = 0 \quad F_{Ax} + F_C \cos 45^\circ = 0$$

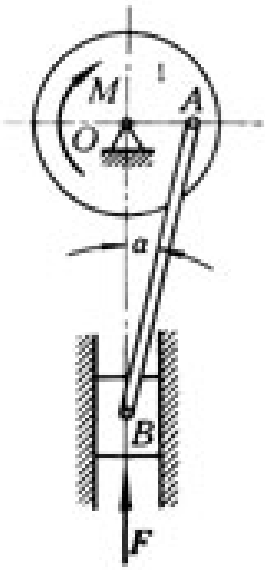
$$\sum F_y = 0 \quad F_{Ay} + F_C \sin 45^\circ - P = 0$$

$$\sum M_A = 0 \quad F_C \cos 45^\circ \cdot l - P \cdot 2l = 0$$

解得 $F_C = 28.28\text{kN}, F_{Ax} = -20\text{kN}, F_{Ay} = -10\text{kN}$

例2 已知：$OA=R, AB=l$, $\vec{F}$，不计物体自重与摩擦，系统在图示位置平衡；

求：力偶矩M的大小，轴承O处的约束力，连杆AB受力，冲头给导轨的侧压力.

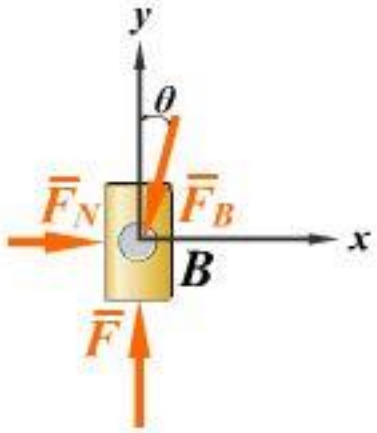


解：取冲头B，画受力图.

$$\sum F_y = 0 \quad F - F_B \cos\theta = 0$$

$$\sum F_x = 0 \quad F_N - F_B \sin\theta = 0$$

$$F_B = \frac{F}{\cos\theta} = \frac{Fl}{\sqrt{l^2 - R^2}} \qquad F_N = F\tan\theta = \frac{FR}{\sqrt{l^2 - R^2}}$$



取轮,画受力图.

$$\sum F_x = 0 \qquad F_{Ox} + F_A \sin\theta = 0$$

$$\sum F_y = 0 \qquad F_{Oy} + F_A \cos\theta = 0$$

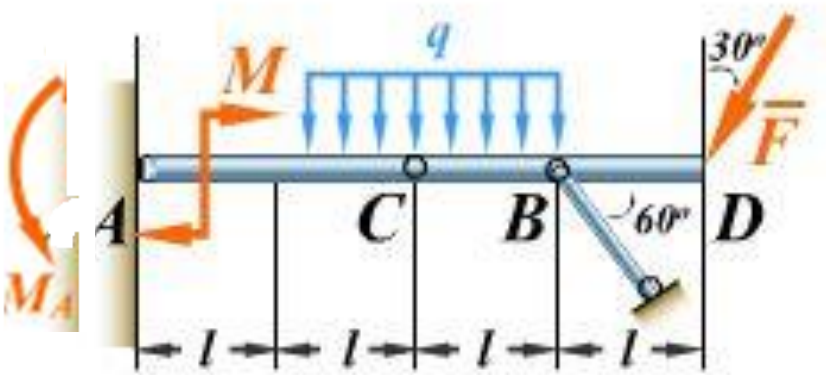
$$\sum M_O(F) = 0 \qquad F_A \cos\theta \cdot R - M = 0$$

$$F_{Ox} = -\frac{FR}{\sqrt{l^2 - R^2}} \qquad F_{Oy} = -F$$

$$M = FR$$

例3　已知: F=20kN, q=10kN/m, $M=20\text{kN}\cdot\text{m}$, l=1m;

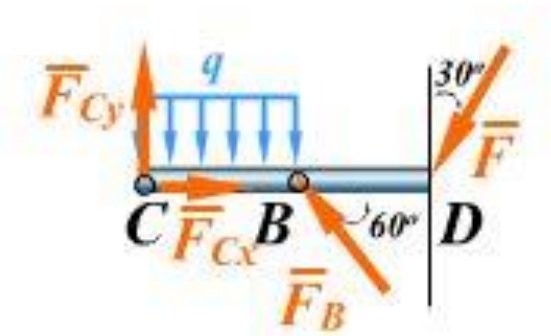
求: A,B处的约束力.



解: 取CD梁,画受力图.

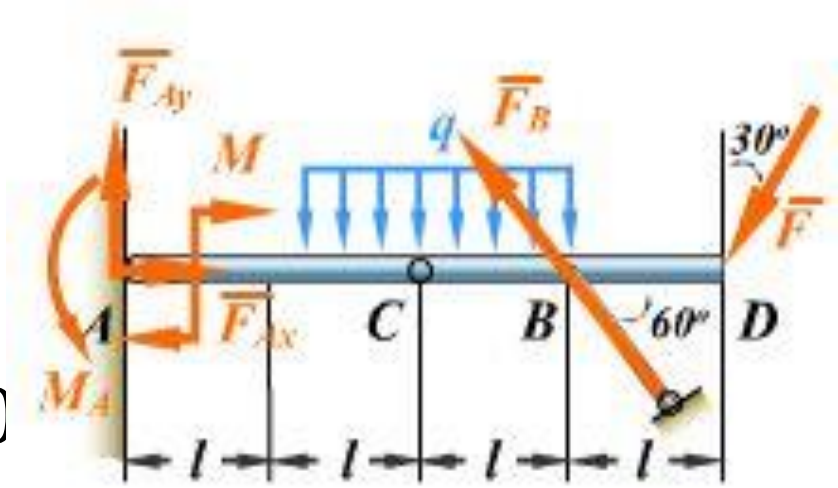
$$\sum M_C = 0$$

$$F_B \sin 60^\circ \cdot l - ql \cdot \frac{l}{2} - F\cos 30^\circ \cdot 2l = 0$$



$\Rightarrow$ F_B=45.77kN

取AD梁,画受力图.



$$\sum F_x = 0$$

$$F_{Ax} - F_B \cos 60^\circ - F \sin 30^\circ = 0$$

$$\sum F_y = 0$$

$$F_{Ay} + F_B \sin 60^\circ - 2ql - F \cos 30^\circ = 0$$

$$\sum M_A = 0$$

$$M_A - M - 2ql \cdot 2l + F_B \sin 60^\circ \cdot 3l - F \cos 30^\circ \cdot 4l = 0$$

$$\Rightarrow \quad M_A = 10.37\,\mathrm{kN \cdot m} \qquad F_{Ax} = 32.89\,\mathrm{kN} \qquad F_{Ay} = -2.32\,\mathrm{kN}$$