传热边界层

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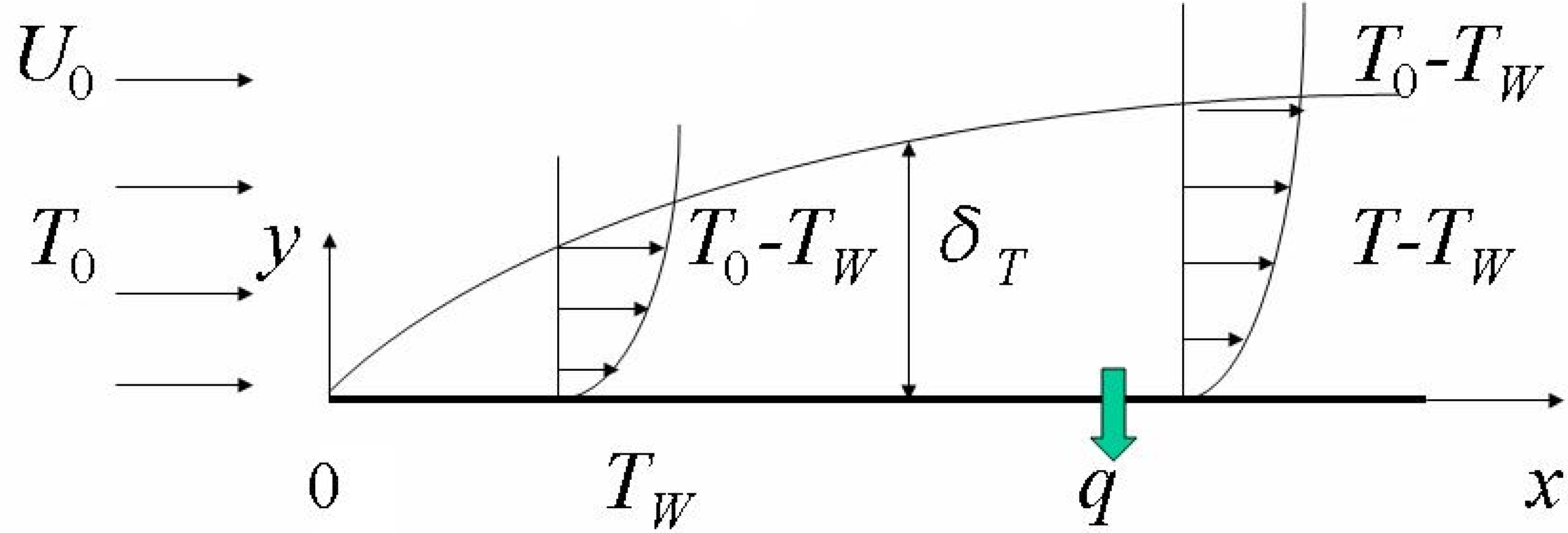
第十二讲. 传热边界层

1. 热边界层
2. 传热边界层能量积分方程
3. 平板传热边界层计算
4. 圆管传热进口段
5. 管内层流换热
6. 绕圆柱对流传热

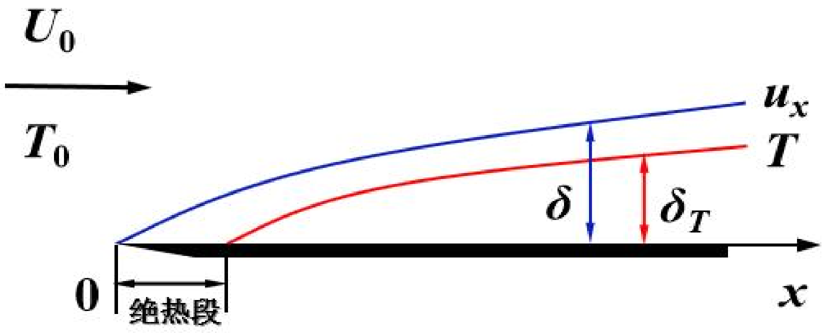
1. 热边界层

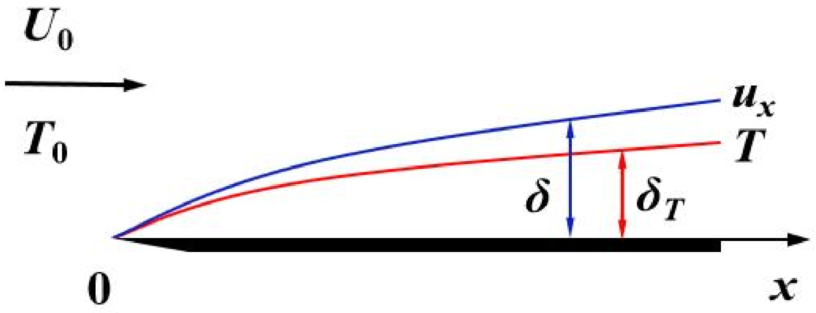
传热边界层的形成和特点

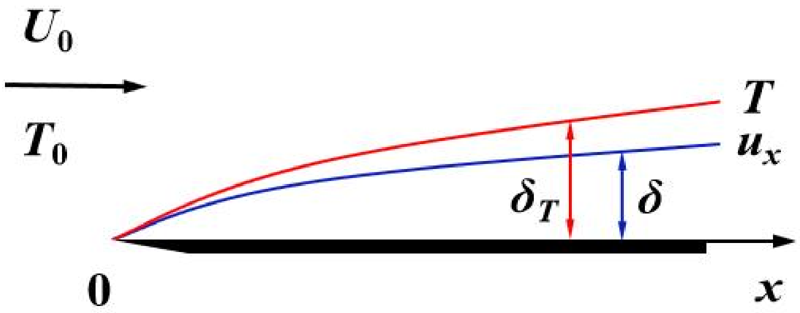


类似流动边界层，以 $T-T_W=99\%$（T_0-T_W）为界线。

传热边界层与流动边界层的关系







$$\frac{\delta}{\delta_T} = Pr^n = \left(\frac{\nu}{a}\right)^n = \left(\frac{\mu C_p}{k}\right)^n$$

气体 $Pr \approx 1$

粘性油 $Pr \to \infty$

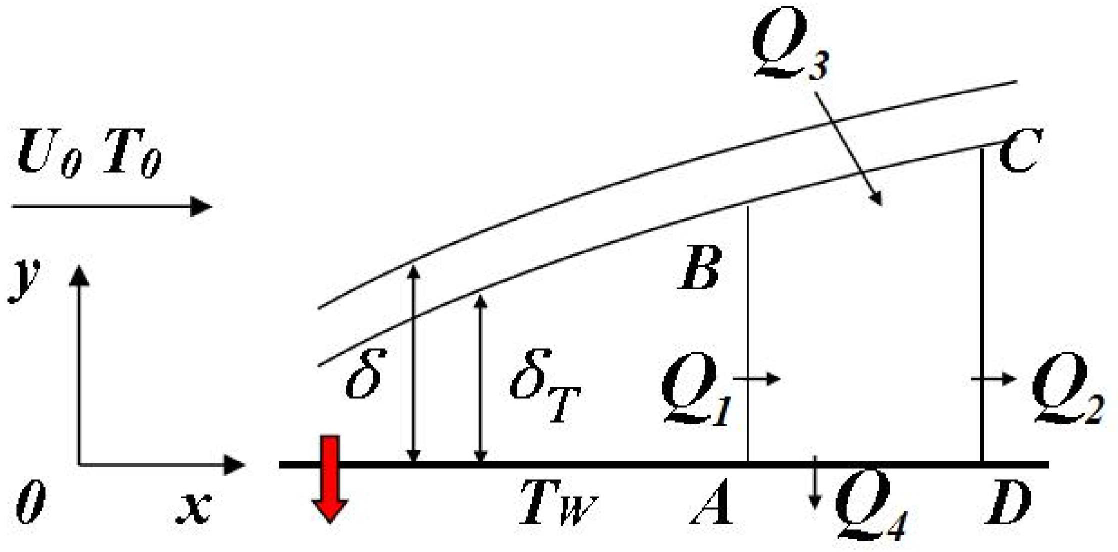
液态金属 $Pr \to 0$

对 $Pr = 0.6 \sim 15$ 内的层流：$n = 1/3$

湍流：$n = 0.585$

$$Pr = \frac{\nu}{a} = \frac{\text{分子动量扩散}}{\text{分子热量扩散}}$$

2. 传热边界层能量积分方程



选取控制体 $ABCD$，单位宽度，$\delta_T < \delta$

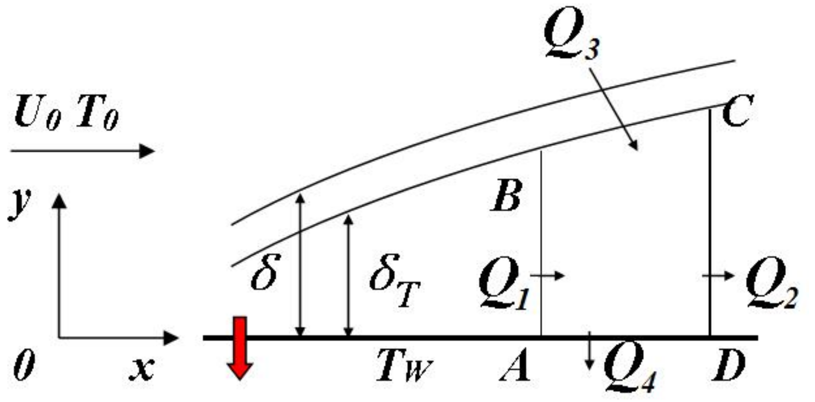
对定常流动传热： $Q_1 + Q_3 = Q_2 + Q_4$

$$Q_1 = \int_0^{\delta_T} \rho C_P T u_x dy$$

$$Q_2 = \int_0^{\delta_T} \rho C_P T u_x dy + \frac{\partial}{\partial x}\left(\int_0^{\delta_T} \rho C_P T u_x dy\right)dx$$

$$Q_3 = C_P T_0 \frac{\partial}{\partial x}\left(\int_0^{\delta_T} \rho u_x dy\right)dx$$

$$Q_4 = -k\frac{\partial T}{\partial y}\bigg|_{y=0} dx$$



根据能量守恒：

$$\frac{\partial}{\partial x}\int_0^{\delta_T}\left(T_0 - T\right)u_x dy = a\frac{\partial T}{\partial y}\bigg|_{y=0}$$

传热边界层能量积分方程

设温度分布:

$$\frac{T-T_W}{T_0-T_W}=a+b\left(\frac{y}{\delta_T}\right)+c\left(\frac{y}{\delta_T}\right)^2+d\left(\frac{y}{\delta_T}\right)^3$$

边界条件:

$$\begin{cases} y=0,\quad T=T_W & y=0,\quad \dfrac{\partial^2 T}{\partial y^2}=0 \\ y=\delta_T,\quad T=T_0 & y=\delta_T,\quad \dfrac{\partial T}{\partial y}=0 \end{cases}$$

温度分布:

$$\frac{T-T_W}{T_0-T_W}=\frac{3}{2}\left(\frac{y}{\delta_T}\right)-\frac{1}{2}\left(\frac{y}{\delta_T}\right)^3$$

将 T， u_x 代入传热边界层能量积分方程求得：

$$\frac{\delta_T}{\delta} = \frac{1}{1.026} Pr^{-1/3} \approx Pr^{-1/3}$$

代入： $\frac{\delta}{x} = 4.64 Re_x^{-1/2}$ 　　得： $\frac{\delta_T}{x} = 4.64\, Re_x^{-1/2}\, Pr^{-1/3}$

壁面导热速率等于该处的对流换热速率:

$$h_x A\left(T_0 - T_W\right) = kA \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$\therefore\ h_x = \frac{k}{T_0 - T_W} \left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{3}{2}\frac{k}{\delta_T}$$

代入 δ_T 得局部对流传热系数 h_x:

$$h_x = 0.323\frac{k}{x}Re_x^{1/2}\,Pr^{1/3}$$

局部努塞尔数 Nu_x:

$$Nu_x = \frac{h_x x}{k} = 0.323\,Re_x^{1/2}\,Pr^{1/3}$$

平均对流传热系数 h_L:

$$h_L = \frac{1}{L}\int_0^L h_x dx = 0.646\,\frac{k}{L}Re_L^{1/2}\,Pr^{1/3}$$

平均努塞尔数 Nu_L:

$$Nu_L = \frac{h_L L}{k} = 0.646\,Re_L^{1/2}\,Pr^{1/3}$$

$$h_x A\left(T_0 - T_W\right) = kA \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

代入能量积分方程

$$h_x = \frac{\rho C_p}{T_0 - T_W}\frac{\partial}{\partial x}\int_0^{\delta_T}\left(T_0 - T\right)u_x dy$$

湍流时，设速度分布：

$$\frac{u_x}{U_0} = \left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

温度分布：

$$\frac{T - T_W}{T_0 - T_W} = \left(\frac{y}{\delta_T}\right)^{\frac{1}{7}}$$

可得：

$$h_x = \frac{7}{72}\rho C_p U_0 \frac{d}{dx}\left[\delta_T\left(\frac{\delta_T}{\delta}\right)^{1/7}\right]$$

已知层流： $\frac{\delta_T}{\delta} = Pr^{-1/3}$

对湍流，假定： $\frac{\delta_T}{\delta} = Pr^{-n}$

式中 n 由实验测定。

湍流边界层厚度： $\frac{\delta}{x} = \frac{0.376}{\sqrt[5]{Re_x}}$

湍流热边界层厚度： $\delta_T = \frac{0.376\, x}{\sqrt[5]{Re_x}} Pr^{-n}$

$$h_x = \frac{7}{72}\rho C_p U_0 Pr^{-n/7}\frac{d\delta_T}{dx} = 0.0292\rho C_p U_0 Re_x^{-1/5} Pr^{-8n/7}$$

$$h_x = 0.0292\rho C_p U_0 Re_x^{-1/5} Pr^{-8n/7} = 0.0292\frac{k}{x} Re_x^{4/5} Pr^{(7-8n)/7}$$

$$Nn_x = \frac{h_x x}{k} = 0.0292 Re_x^{4/5} Pr^{(7-8n)/7}$$

实验表明，湍流边界层传热时 $n = 0.585$，可得：

$$\delta_T = \frac{0.376\, x}{\sqrt[5]{Re_x}} Pr^{-0.585}$$

局部努塞尔数 Nu_x：

$$Nn_x = \frac{h_x x}{k} = 0.0292 Re_x^{4/5} Pr^{1/3}$$

平均对流传热系数 h_L：

$$h_L = \frac{1}{L}\int_0^L h_x dx = 0.0365\ \frac{k}{L} Re_L^{4/5}\ Pr^{1/3}$$

平均努塞尔数 Nu_L：

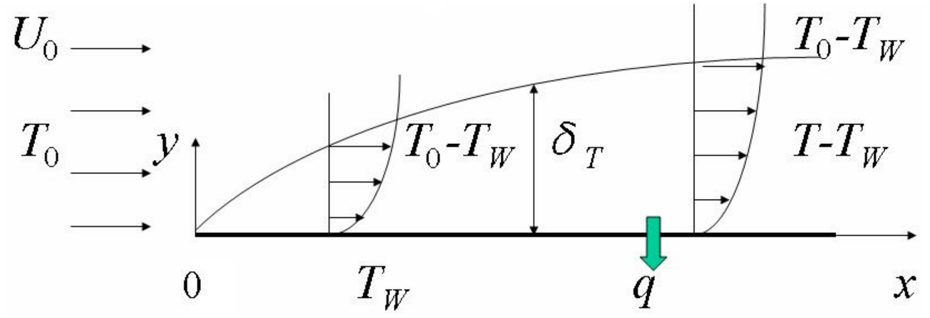
$$Nu_L = \frac{h_L L}{k} = 0.0365\ Re_L^{4/5}\ Pr^{1/3}$$

考虑到一开始始终有一段层流，

$$h_L = \frac{1}{L}\left(\int_0^{x_c} h_{x层} dx + \int_{x_c}^L h_{x湍} dx\right)$$

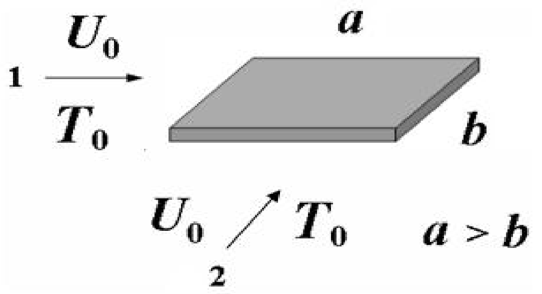
$$h_L = \frac{k}{L}\left(0.0365\ Re_L^{4/5} - 866\right) Pr^{1/3}$$

3. 平板传热边界层计算



临界雷诺数
Re_{xc}=5×10^5

	传热边界层厚度	平均对流传热系数
层流	$\dfrac{\delta_T}{\delta}=Pr^{-1/3}$	$h_L=0.646\dfrac{k}{L}Re_L^{1/2}\;Pr^{1/3}$
湍流	$\dfrac{\delta_T}{\delta}=Pr^{-0.585}$	$h_L=\dfrac{k}{L}\left(0.0365\,Re_L^{4/5}-866\right)Pr^{1/3}$



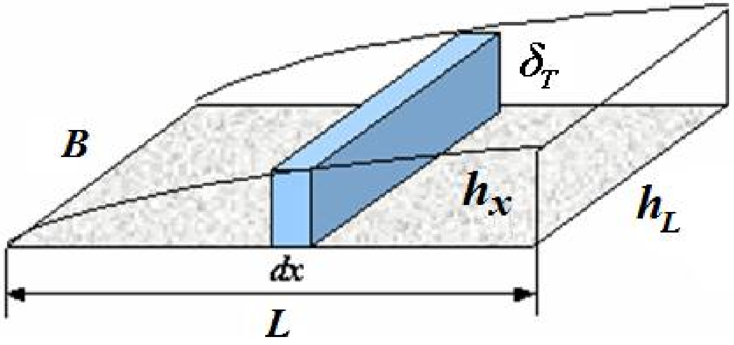
问题探讨

平板冷却速率

课后自学

1.平板层流传热边界层精确解。

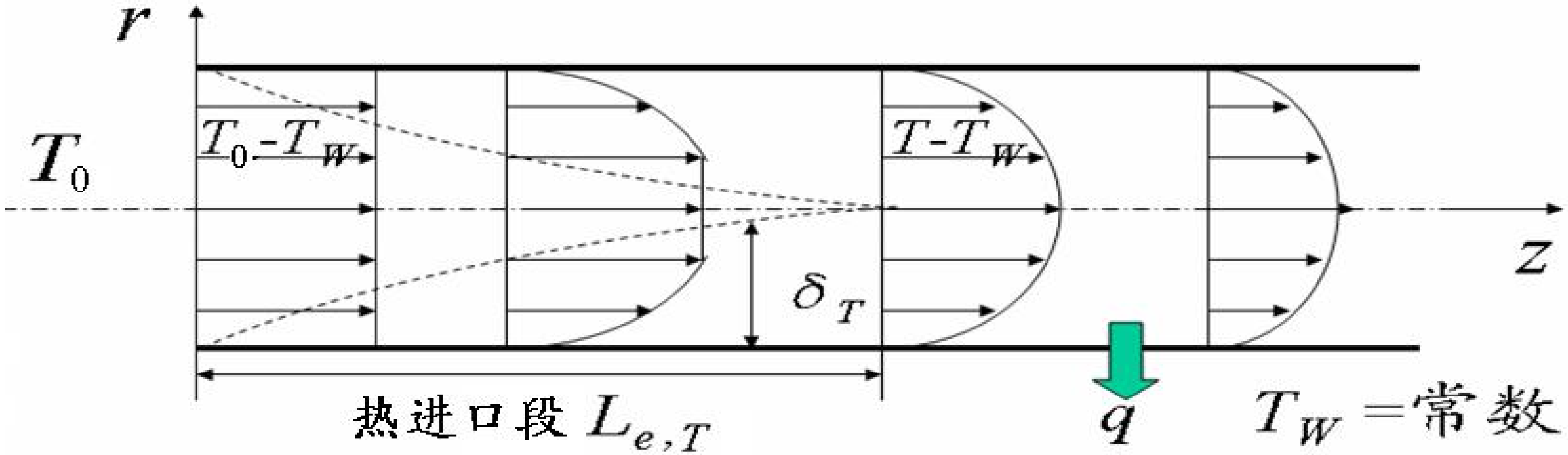
$$\frac{\delta_T}{x} = \frac{5.0}{\sqrt{Re_x}} Pr^{-1/3} \qquad \frac{\delta}{\delta_T} = Pr^{1/3}$$

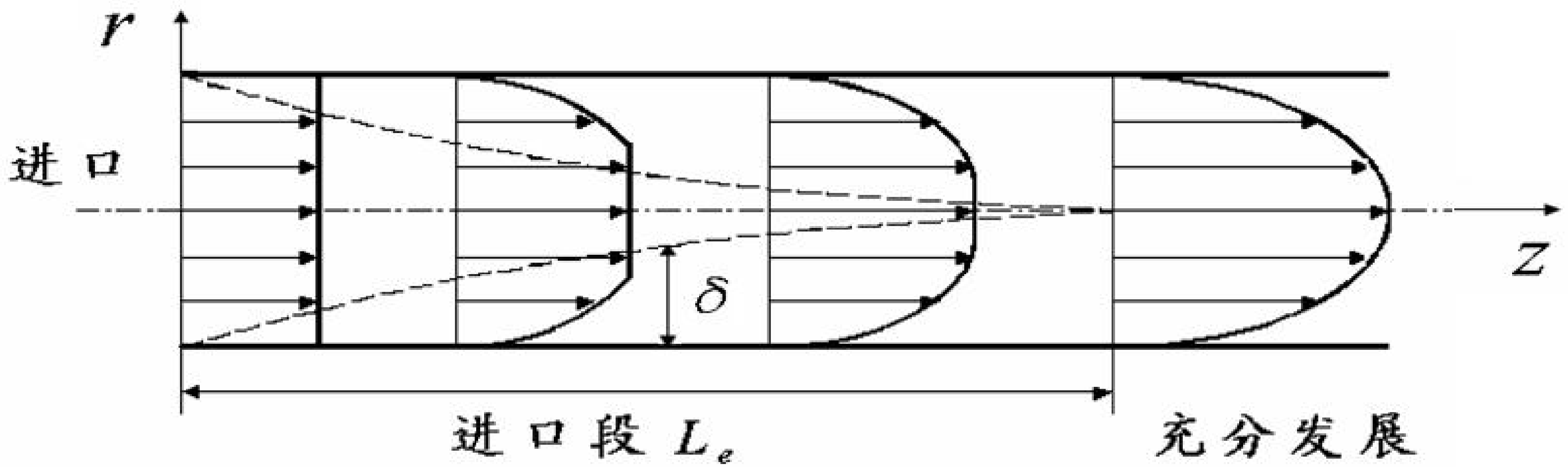


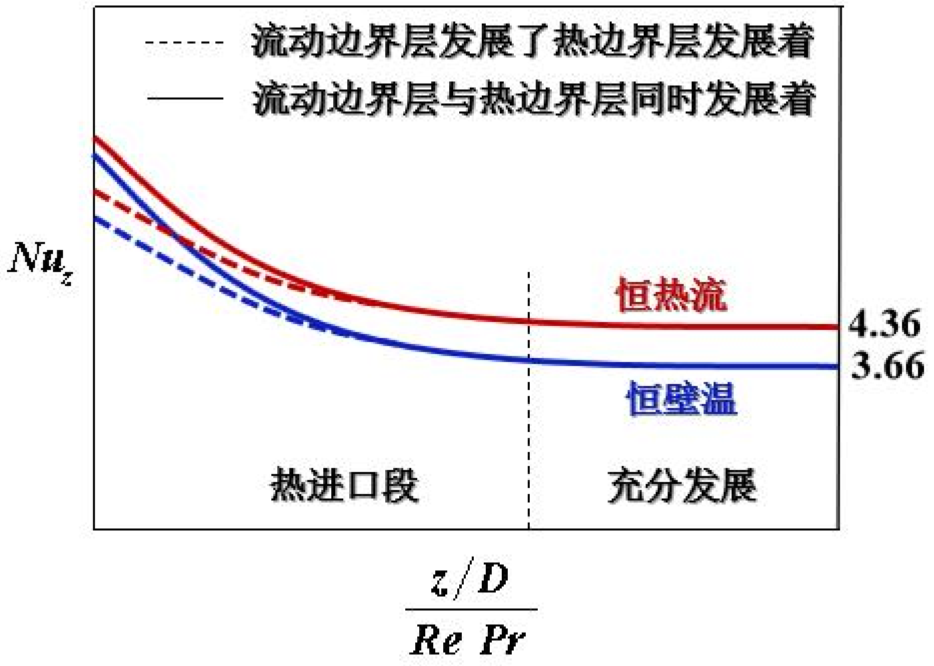
$$Nu_L = \frac{h_L L}{k} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$

适用条件： $Re_L < 5\times10^5 \quad Pr = 0.6 \sim 15$

4. 圆管传热进口段







传热进口段长度

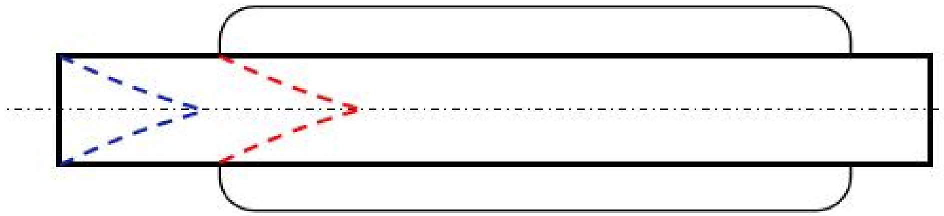
层流

恒热流： $$\frac{L_{e,T}}{D} = 0.07\,Re\,Pr$$

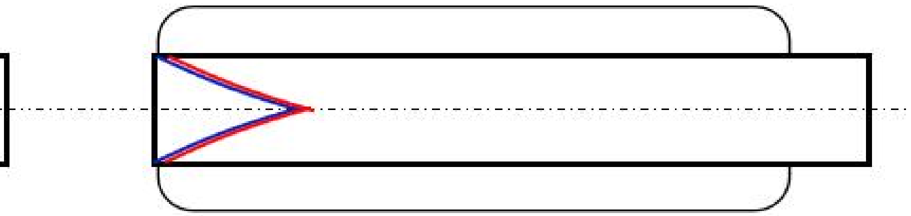
恒壁温： $$\frac{L_{e,T}}{D} = 0.055\,Re\,Pr$$

湍流

$$\frac{L_{e,T}}{D} = 50$$



流动边界层发展了热边界层发展着



流动边界层与热边界层同时发展着

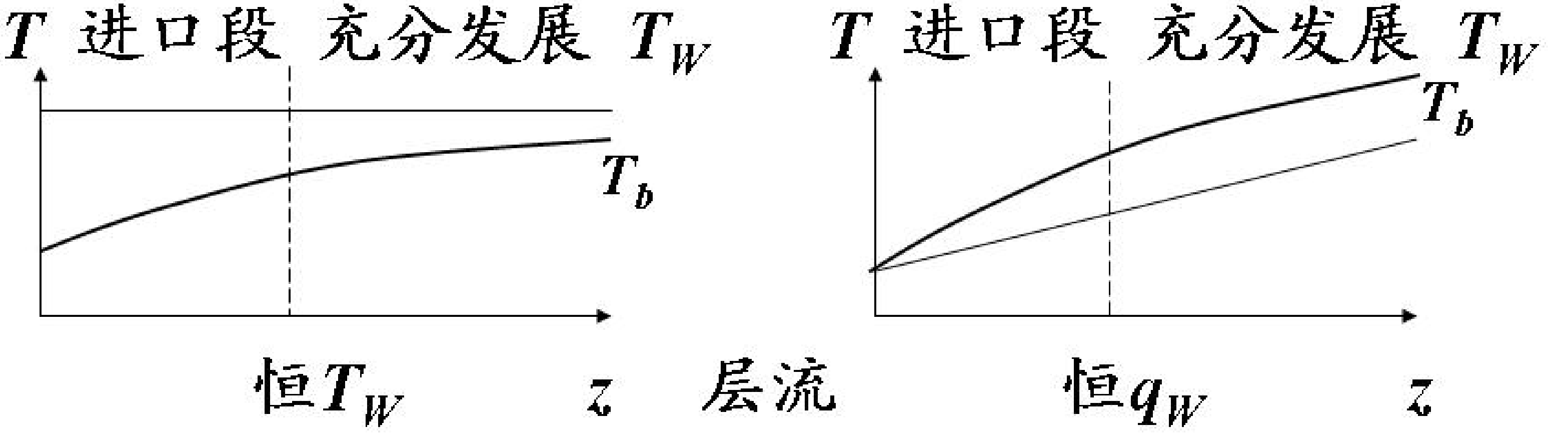
5. 管内层流换热

工业上常见的圆管加热方式有两种：

①. 恒壁温（夹套蒸气加热）T_W = 常数

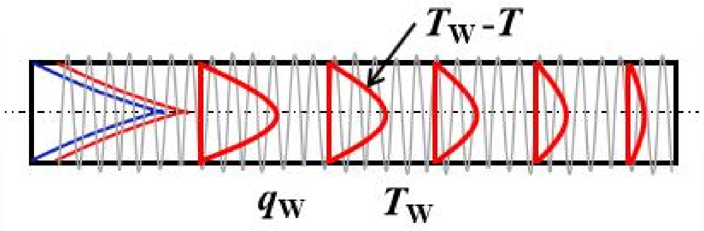
②. 恒热流（电加热）q_W = 常数

截面平均温度T_b随 z的变化如下图：



截面的温度分布T决定换热效果

恒热流（电加热）$q_W=$常数



管内层流传热过程中，速度边界层和温度边界层均充分发展后。

柱坐标系下的对流传热微分方程

$$\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+u_z\frac{\partial T}{\partial z}=a\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{\dot{q}}{\rho C_P}$$

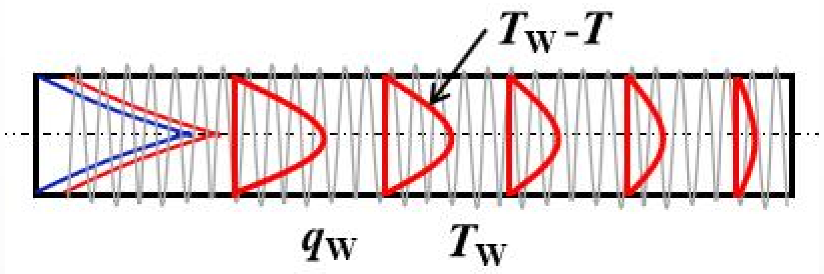
定常：$\frac{\partial T}{\partial t}=0$

管内流体：$\begin{cases} u_r=0 \\ u_\theta=0 \\ u_z\neq 0 \end{cases}$ $\begin{cases} \frac{\partial T}{\partial r}\neq 0 \\ \frac{\partial T}{\partial \theta}=0 \\ \frac{\partial T}{\partial z}\neq 0 \end{cases}$ $\begin{cases} \frac{\partial^2 T}{\partial \theta^2}=0 \\ \frac{\partial^2 T}{\partial z^2}\ll u_z\frac{\partial T}{\partial z} \end{cases}$

无内热源：$\dot{q}=0$

简化对流传热微分方程得：

$$u_z \frac{\partial T}{\partial z} = a \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r} \right)$$



管内层流：

$$u_z = 2U\left(1 - \frac{r^2}{R^2} \right)$$

恒热流（电加热）q_W = 常数

$$\frac{\partial T}{\partial z} = \frac{\partial T_W}{\partial z} = \frac{\partial T_b}{\partial z} = \text{常数}$$

$$T_b = \frac{\int_0^R \rho u_z C_p T 2\pi r dr}{\rho U \pi R^2 C_p}$$

可得：

$$2U\left(1-\frac{r^2}{R^2}\right)\frac{\partial T}{\partial z}=a\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)=\frac{2U}{a}\left(1-\frac{r^2}{R^2}\right)r\frac{\partial T}{\partial z}$$

边界条件：

$$\begin{cases} r=0, \quad \dfrac{dT}{dr}=0 \\ r=R, \quad T=T_W\,, q_W=k\left.\dfrac{dT}{dr}\right|_{r=R} \end{cases}$$

积分：

$$r\frac{dT}{dr}=\frac{2U}{a}\frac{\partial T}{\partial z}\left(\frac{r^2}{2}-\frac{r^4}{4R^2}\right)+C_1$$

$$\because r=0, \quad \frac{dT}{dr}=0$$

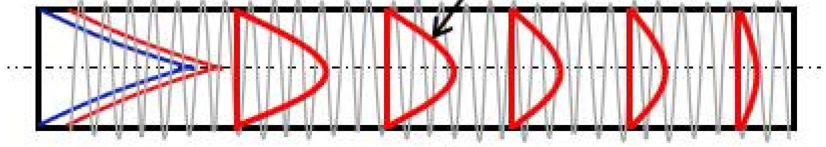
$$\therefore C_1=0$$

$$\frac{dT}{dr}=\frac{2U}{a}\frac{\partial T}{\partial z}\left(\frac{r}{2}-\frac{r^3}{4R^2}\right)$$

再积分： $$T=\frac{2U}{a}\frac{\partial T}{\partial z}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)+C_2$$

$$\because r=R,\quad T=T_W$$

$$\therefore C_2=T_W-\frac{3U}{8a}\frac{\partial T}{\partial z}R^2$$



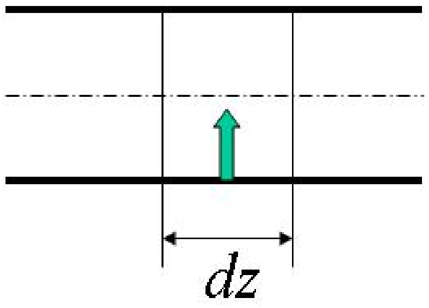
温度分布： $$T_W-T=\frac{3U}{8a}\frac{\partial T}{\partial z}R^2-\frac{2U}{a}\frac{\partial T}{\partial z}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)$$

$$T_b = \frac{\int_0^R \rho u_z C_p T 2\pi r dr}{\rho U \pi R^2 C_p} = T_W - \frac{11}{48}\frac{UR^2}{a}\frac{\partial T}{\partial z}$$

$$T_W - T_b = \frac{11}{48}\frac{UR^2}{a}\frac{\partial T}{\partial z}$$

在 dz 段上壁面处的导热速率应等于流体和壁面之间的对流换热速率。

壁面处导热速率： $$Q = k\left.\frac{dT}{dr}\right|_{r=R} 2\pi R dz$$



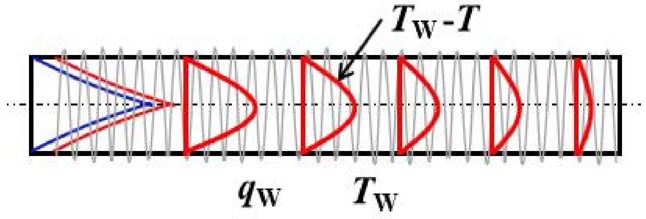
对流换热速率： $$Q = h 2\pi R dz\left(T_W - T_b\right)$$

$$\left.\frac{dT}{dr}\right|_{r=R} = \frac{UR}{2a}\frac{\partial T}{\partial z} \qquad T_W - T_b = \frac{11}{48}\frac{UR^2}{a}\frac{\partial T}{\partial z}$$

$$h\left(T_W - T_b\right) = k\left.\frac{dT}{dr}\right|_{r=R}$$

$$\frac{h}{k} = \frac{\left.\frac{dT}{dr}\right|_{r=R}}{T_W - T_b} = \frac{\frac{UR}{2a}\frac{\partial T}{\partial z}}{\frac{11}{48}\frac{UR^2}{a}\frac{\partial T}{\partial z}} = \frac{48}{11\times 2R}$$

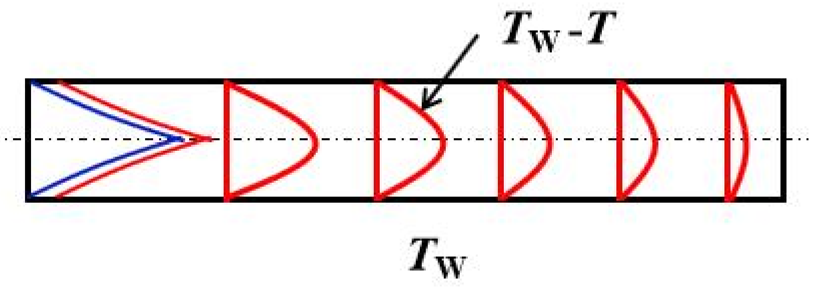
定义努塞尔数： $Nu = \frac{hD}{k} = \frac{48}{11} = 4.36$

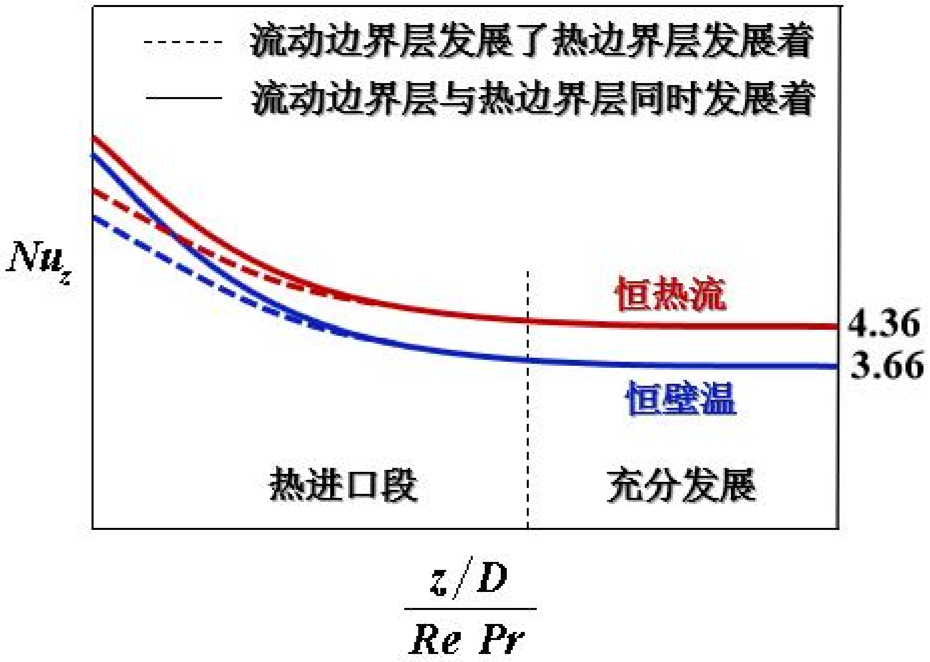


对圆管层流换热
恒 q_W：Nu=4.36

对圆管层流恒 T_W 换热，Greatz 分析求解的结果为：

$$Nu=3.66$$



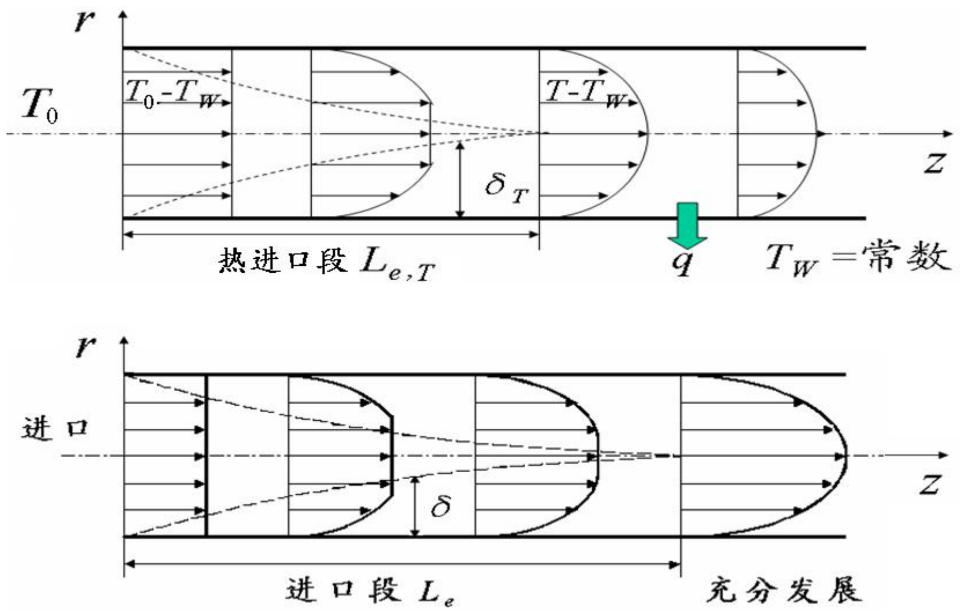


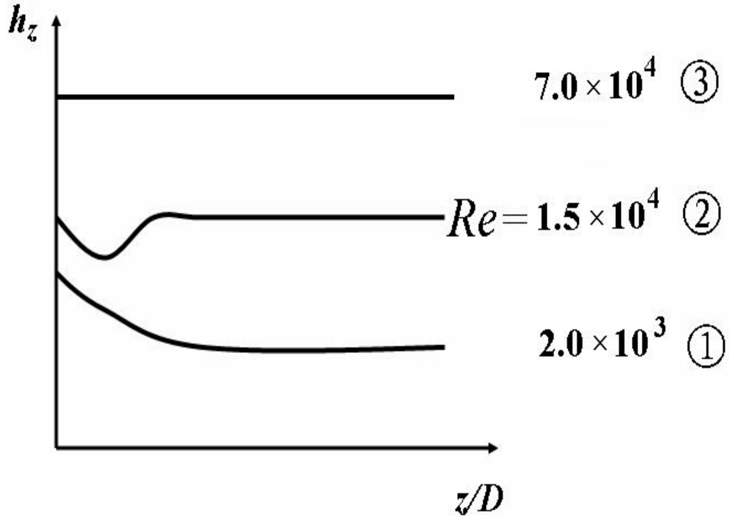
问题探讨

圆管层流换热
细管好,还是粗管好?

课后思考

1.用传热边界层分析圆管热进口段特点。并与流动进口段对比。
2. 图示圆管局部传热系数随 z 变化关系，讨论其规律。





6. 绕圆柱对流传热

Nu_θ 随 θ 的变化　　$Nu_\theta = \dfrac{h_\theta d}{k}$

①．层流边界层发展，$\delta_T\uparrow$， $h_\theta\downarrow$；至约81°处，边界层分离， $h_\theta\uparrow$；原因是旋涡冲刷表面。

②．先是层流边界层发展, $\delta_T\uparrow$, $h_\theta\downarrow$；层流→湍流， $h_\theta\uparrow\uparrow$，而后湍流边界层发展, $\delta_T\uparrow$, $h_\theta\downarrow$；至约130°处，湍流边界层分离，又促使$h_\theta\uparrow$。