管内层流

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第五讲. 管内层流

1. 雷诺数
2. 管内层流速度分布
3. 管道沿程压降
4. 毛细管粘度计

1. 雷诺数

$$Re=\frac{\text{流体密度}\times\text{特征尺度}\times\text{特征速度}}{\text{流体粘度}}$$

管内流动： $$Re=\frac{\rho DU}{\mu}=\frac{DU}{\nu}$$

$$Re<2100 \quad \text{层流}$$
$$2100<Re<10000 \quad \text{过渡流}$$
$$Re>10000 \quad \text{湍流}$$

思考

流动状态判别 — 临界雷诺数： Re_{xc}=2100

层流具有稳定性

雷诺数物理含义

(经典)

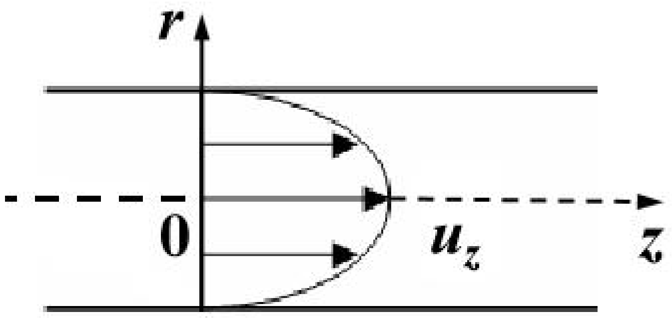
$$F_a = ma \propto \rho V \frac{U}{t} \propto \rho L^3 \frac{U}{L/U} = \rho U^2 L^2$$

$$F_\tau = \mu A \frac{dU}{dy} \propto \mu L^2 \frac{U}{L} = \mu UL$$

$$\frac{F_a}{F_\tau} = \frac{\rho UL}{\mu}$$

$$Re = \frac{\text{惯性力}}{\text{粘性力}}$$

2. 管内层流速度分布



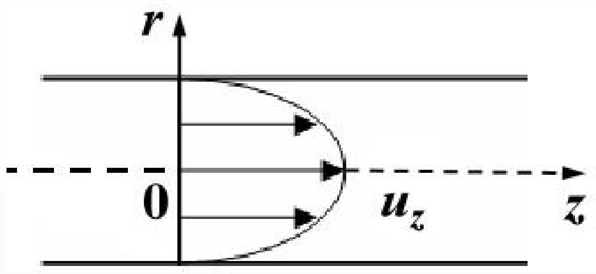
物理分析

定常： $\dfrac{\partial u_z}{\partial t} = 0$

一维流动： $\begin{cases} u_r = 0 \\ u_\theta = 0 \\ u_z \neq 0 \end{cases}$ $\begin{cases} \dfrac{\partial u_z}{\partial r} \neq 0 \\ \dfrac{\partial u_z}{\partial \theta} = 0 \\ \dfrac{\partial u_z}{\partial z} = 0 \end{cases}$ $\begin{cases} \dfrac{\partial^2 u_z}{\partial \theta^2} = 0 \\ \dfrac{\partial^2 u_z}{\partial z^2} = 0 \end{cases}$

z 方向无重力： $X_z = 0$

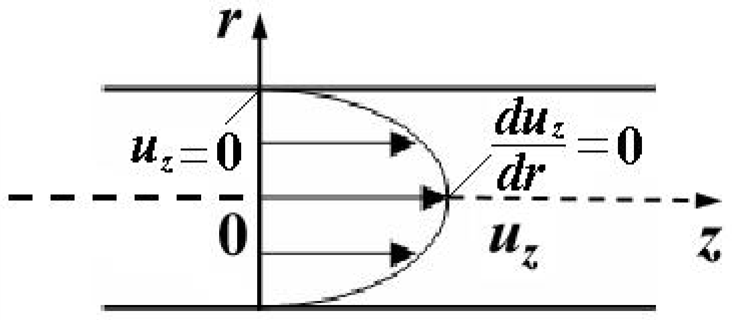
柱坐标系 z 方向奈维-斯托克斯方程



$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho X_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]$$

简化得：
$$\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)=\frac{\partial p}{\partial z}$$

$$\mu \frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right)=\frac{dp}{dz}$$



积分： $$r\frac{du_z}{dr}=\frac{1}{2\mu}\frac{dp}{dz}r^2+C_1$$

边界条件： $$\begin{cases} r=0, & \frac{du_z}{dr}=0 \\ r=R, & u_z=0 \end{cases}$$

$$\because r=0,\quad \frac{du_z}{dr}=0;\quad \therefore C_1=0$$

$$\frac{du_z}{dr}=\frac{1}{2\mu}\frac{dp}{dz}r$$

再积分，代入边界条件得： $$u_z=-\frac{1}{4\mu}\frac{dp}{dz}\left(R^2-r^2\right)$$ 管内层流速度分布

哈根-泊谡叶方程

速度分布：

$$u_z = -\frac{1}{4\mu}\frac{dp}{dz}\left(R^2 - r^2\right)$$

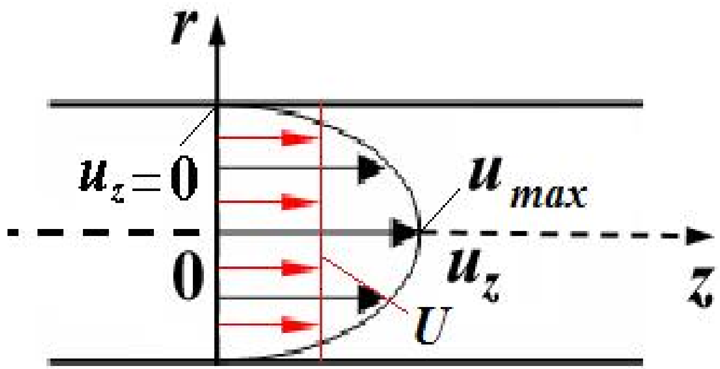
体积流率：

$$V = \int_A u_z dA = -\frac{\pi R^4}{8\mu}\frac{dp}{dz}$$

哈根-泊谡叶方程

平均速度：

$$U = \frac{V}{A} = -\frac{R^2}{8\mu}\frac{dp}{dz}$$



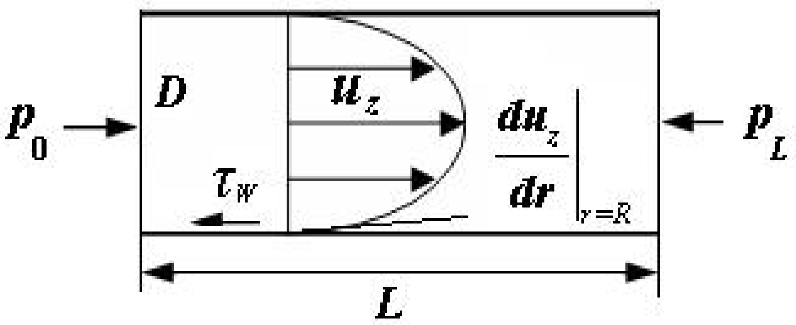
速度分布：

$$u_z = 2U\left(1 - \frac{r^2}{R^2}\right)$$

3. 管道沿程压降

$$\mu \longrightarrow u_z \longrightarrow \left.\frac{du_z}{dr}\right|_{r=R} \longrightarrow \tau_W \longrightarrow -\Delta p$$

$$u_z = 2U\left(1-\frac{r^2}{R^2}\right) \longrightarrow \left.\frac{du_z}{dr}\right|_{r=R} = -\frac{4U}{R} \longrightarrow \tau_W = -\mu\left.\frac{du_z}{dr}\right|_{r=R} = \frac{4\mu U}{R}$$



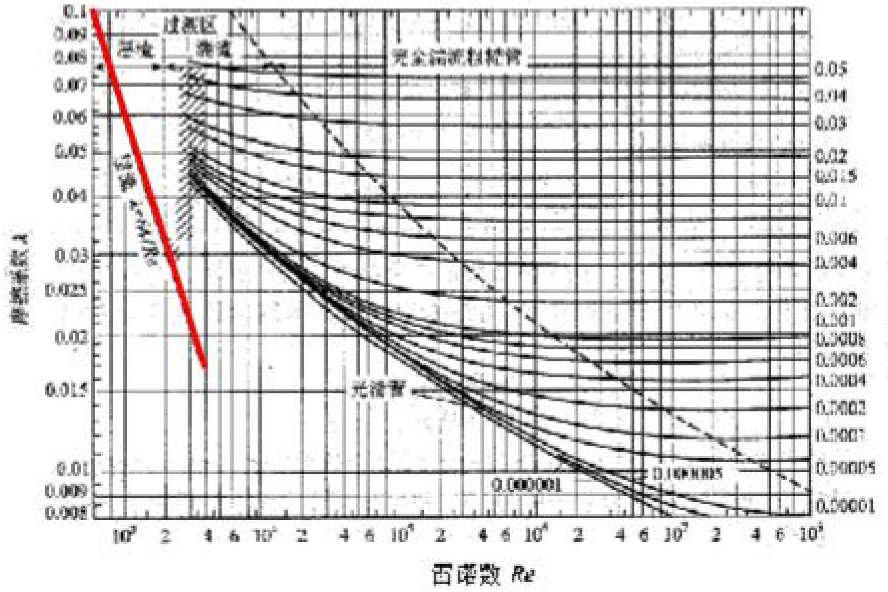
$$\left(p_0 - p_L\right)\cdot\frac{1}{4}\pi D^2 = \tau_W \cdot \pi DL$$

$$-\Delta p = p_0 - p_L = \frac{4L}{D}\tau_W = \frac{32\mu UL}{D^2} = \frac{64}{Re}\frac{L}{D}\frac{1}{2}\rho U^2$$

管道沿程压降定义式：

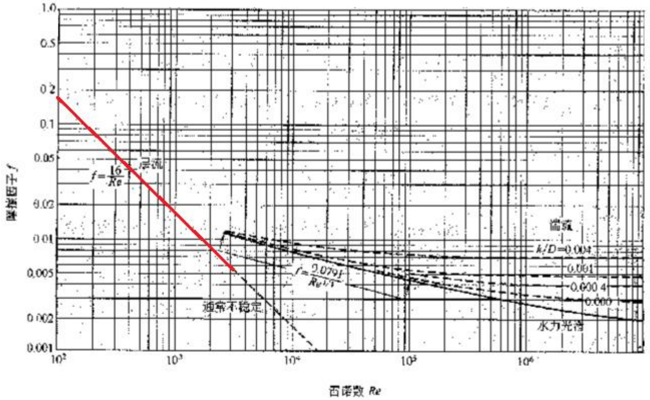
$$-\Delta p = \lambda \frac{L}{D}\frac{1}{2}\rho U^2$$

摩擦阻力系数 $\lambda = \dfrac{64}{Re}$



范宁摩擦系数

圆管层流壁面切应力为： $\tau_W = \dfrac{4\mu U}{R}$



$$\tau_W = \frac{4\mu U}{R} = \frac{16\mu}{\rho D U}\frac{1}{2}\rho U^2 = \frac{16}{Re}\frac{1}{2}\rho U^2$$

定义 $\tau_W = f\dfrac{1}{2}\rho U^2$　　**范宁摩擦系数：** $f = \dfrac{16}{Re}$

摩擦阻力系数 $\lambda = \dfrac{64}{Re} = 4\times\dfrac{16}{Re} = 4f$　**范宁摩擦系数**

4. 毛细管粘度计

奥氏粘度计

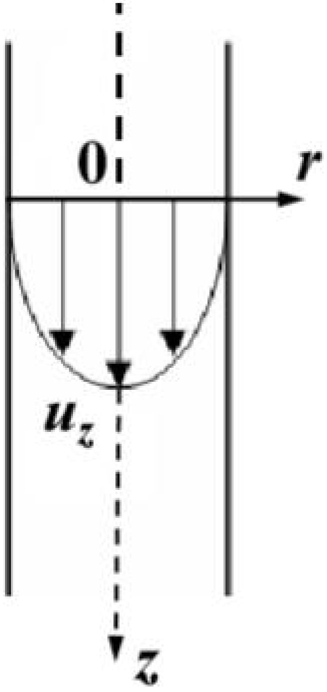
乌氏粘度计

奥氏粘度计和乌氏粘度计的毛细直管中

物理分析　**定常：** $\dfrac{\partial u_z}{\partial t}=0$

一维流动：

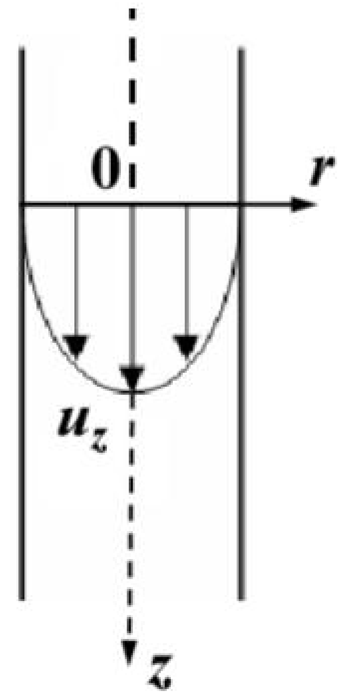
$$\begin{cases} u_r=0 \\ u_\theta=0 \\ u_z\neq 0 \end{cases} \qquad \begin{cases} \dfrac{\partial u_z}{\partial r}\neq 0 \\ \dfrac{\partial u_z}{\partial \theta}=0 \\ \dfrac{\partial u_z}{\partial z}=0 \end{cases} \qquad \begin{cases} \dfrac{\partial^2 u_z}{\partial \theta^2}=0 \\ \dfrac{\partial^2 u_z}{\partial z^2}=0 \end{cases}$$



z 方向无压差力： $\dfrac{\partial p}{\partial z}=0$　　**z 方向有重力：** $X_z=g$

简化柱坐标系中的奈维-斯托克斯方程得：

$$\mu \frac{1}{r}\frac{d}{dr}\left(r\frac{du_z}{dr}\right) = -\rho g$$



边界条件：

$$\begin{cases} r = 0, & \dfrac{du_z}{dr} = 0 \\ r = R, & u_z = 0 \end{cases}$$

解得速度分布：

$$u_z = \frac{\rho g}{4\mu}\left(R^2 - r^2\right)$$

$$v = \frac{\mu}{\rho}$$

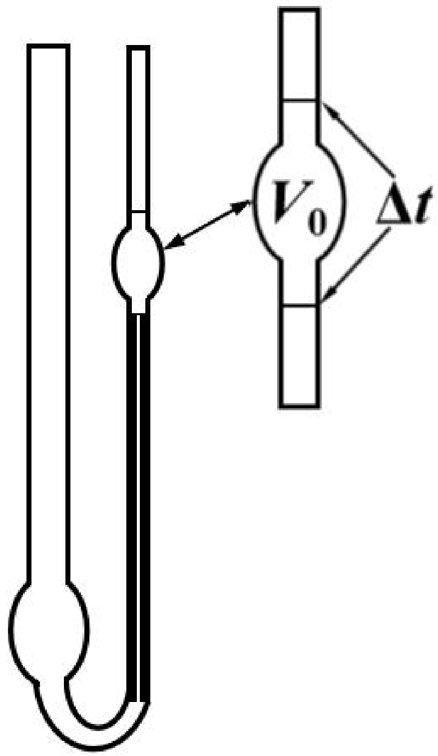
$$u_z = \frac{g}{4v}\left(R^2 - r^2\right)$$

思考

与水平管的区别

流量：$V=\int_A u_z dA=\dfrac{\pi g R^4}{8\nu}$

哈根-泊谡叶方程



$$V_0=V\Delta t=\frac{\pi g R^4\Delta t}{8\nu}$$

标准样ν_1测定：流完V_0需要 Δt_1 时间

样品样ν_2测定：流完V_0需要 Δt_2 时间

$$\frac{\dfrac{\pi g R^4\Delta t_1}{8\nu_1}}{\dfrac{\pi g R^4\Delta t_2}{8\nu_2}}=1$$

$$\nu_2=\frac{\nu_1}{\Delta t_1}\Delta t_2$$

粘度计算公式

问题

测定时间 Δt 长短影响实验结果吗？