传质边界层

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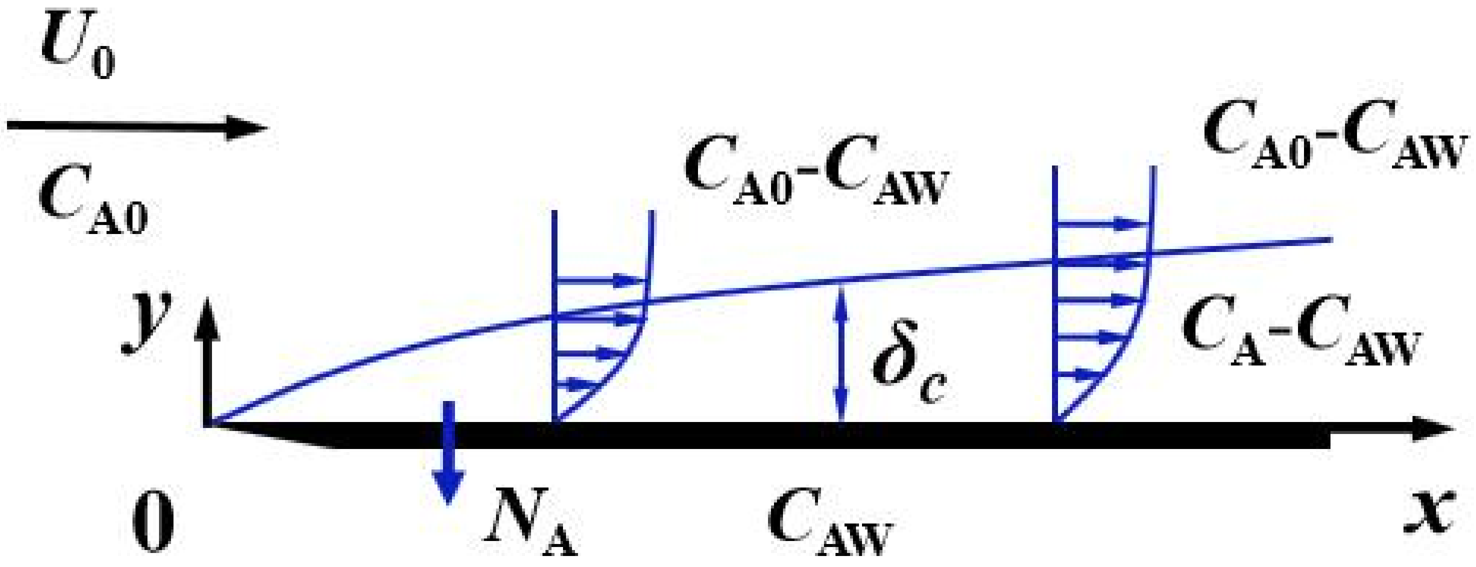
第十五讲. 传质边界层

1. 浓度边界层
2. 传质边界层质量积分方程
3. 平板传质边界层计算
4. 圆管传质进口段
5. 管内层流传质

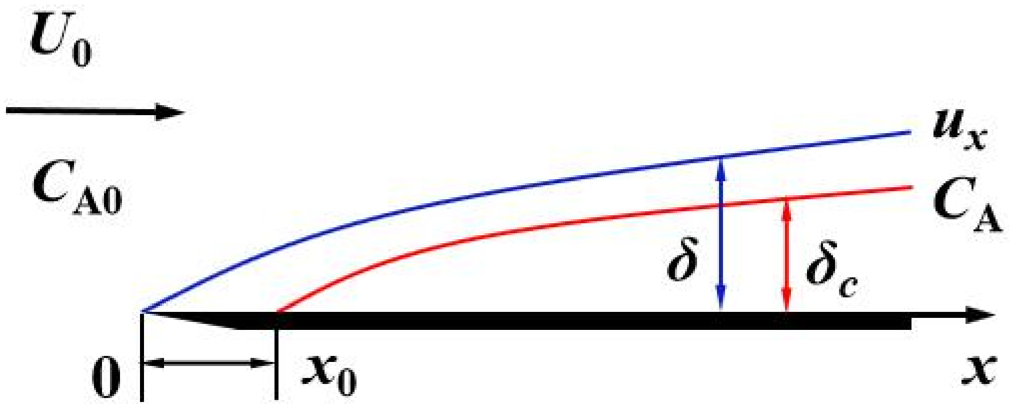
1. 浓度边界层

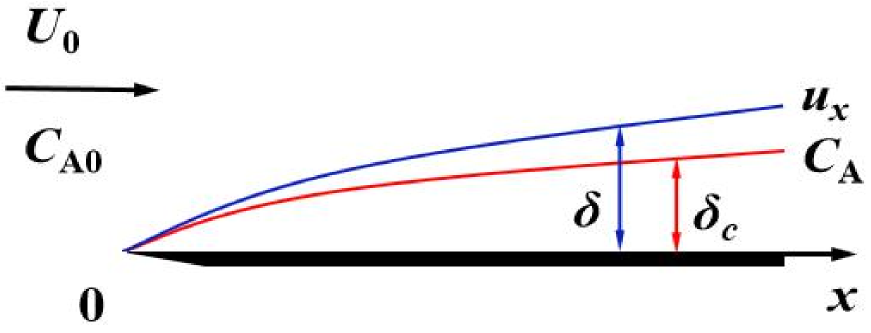
传质边界层的形成和特点

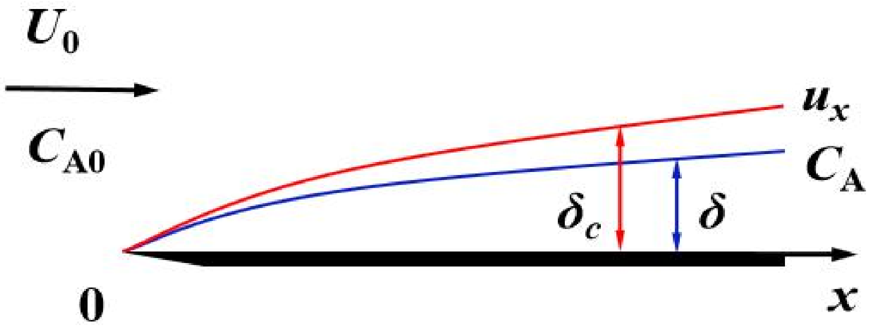


类似流动边界层，以 C-C_W= 99%（C_0-C_W）为界线。

传质边界层与流动边界层的关系





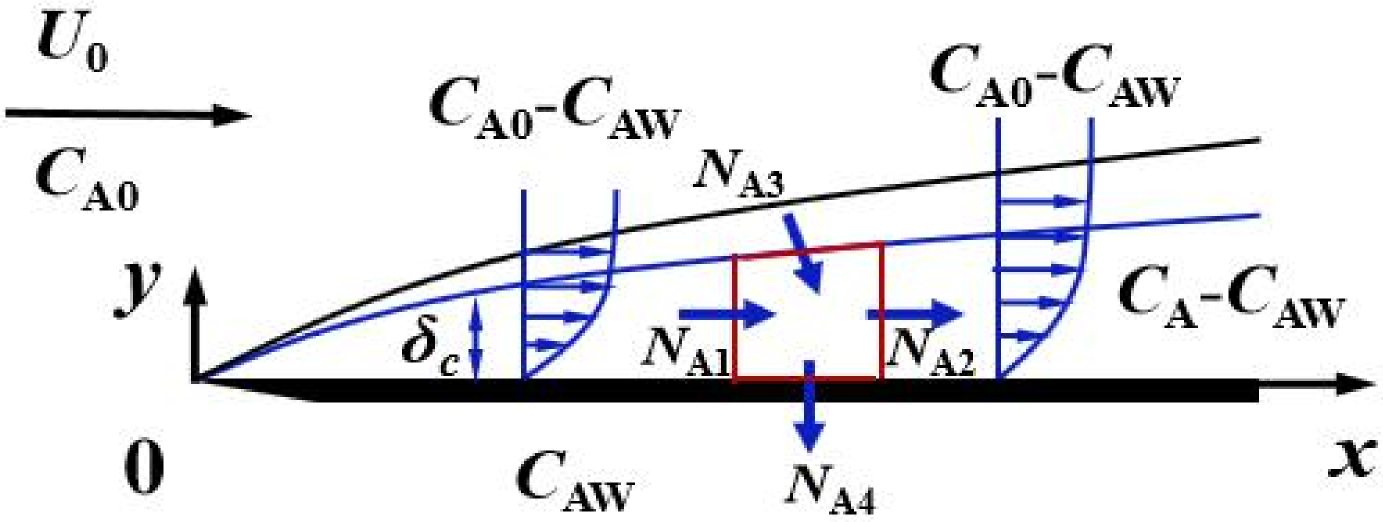


$$\frac{\delta}{\delta_c} = Sc^n = \left(\frac{v}{D_{AB}}\right)^n = \left(\frac{\mu}{\rho D_{AB}}\right)^n$$

层流：$n = 1/3$
湍流：$n = 0.585$

$$Sc = \frac{v}{D_{AB}} = \frac{\text{分子动量扩散}}{\text{分子质量扩散}}$$

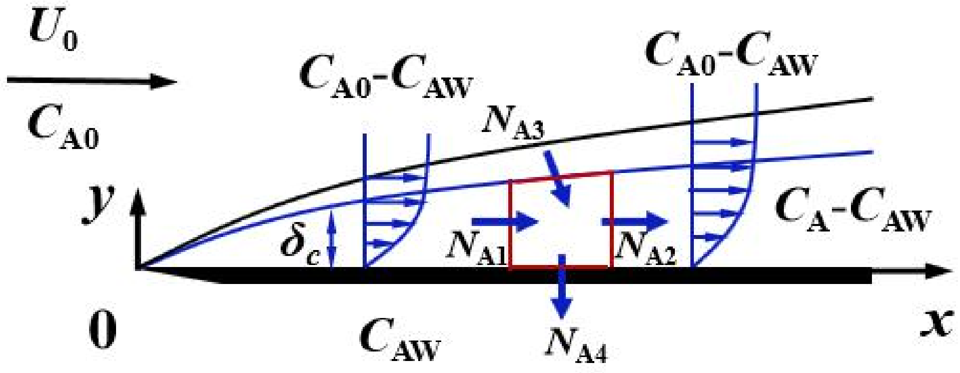
2. 传质边界层质量积分方程



选取控制体浓度边界层，单位宽度， $\delta_c < \delta$

对定常流动传质： $N_{A1} + N_{A3} = N_{A2} + N_{A4}$

$$N_{A1} = \int_0^{\delta_c} C_A u_x dy$$

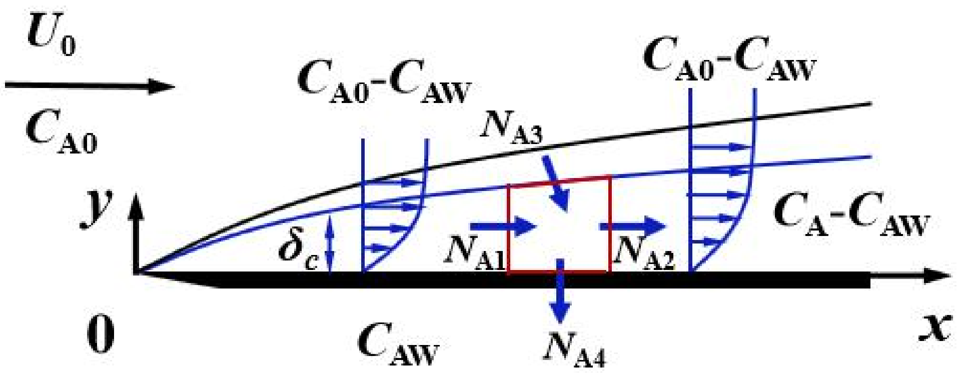


$$N_{A2} = \int_0^{\delta_c} C_A u_x dy + \frac{\partial}{\partial x}\left(\int_0^{\delta_c} C_A u_x dy\right)dx$$

$$N_{A3} = C_{A0}\left[\frac{\partial}{\partial x}\left(\int_0^{\delta_c} u_x dy\right)dx + u_{yw}dx\right]$$

$$N_{A4} = -D_{AB}\frac{\partial C_A}{\partial y}\bigg|_{y=0} dx + C_{AW}u_{yw}dx$$

根据质量守恒：



$$N_{A1} + N_{A3} = N_{A2} + N_{A4}$$

$$\frac{\partial}{\partial x}\int_0^{\delta_c}\left(C_{A0} - C_A\right)u_x dy = D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0} + \left(C_{A0} - C_{AW}\right)u_{yw}$$

设 $u_{yw}=0$

传质边界层质量积分方程

$$\frac{\partial}{\partial x}\int_0^{\delta_c}\left(C_{A0} - C_A\right)u_x dy = D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0}$$

设浓度分布：

$$\frac{C_A - C_{AW}}{C_{A0} - C_{AW}} = a + b\left(\frac{y}{\delta_c}\right) + c\left(\frac{y}{\delta_c}\right)^2 + d\left(\frac{y}{\delta_c}\right)^3$$

边界条件：

$$\begin{cases} y = 0,\quad C_A = C_{AW} & y = 0,\quad \dfrac{\partial^2 C_A}{\partial y^2} = 0 \\ y = \delta_c,\quad C_A = C_{A0} & y = \delta_c,\quad \dfrac{\partial C_A}{\partial y} = 0 \end{cases}$$

求得浓度分布：

$$\frac{C_A - C_{AW}}{C_{A0} - C_{AW}} = \frac{3}{2}\left(\frac{y}{\delta_c}\right) - \frac{1}{2}\left(\frac{y}{\delta_c}\right)^3$$

将 C_A，u_x 代入传质边界层质量积分方程求得：

$$\frac{\delta_c}{\delta} = Sc^{-1/3}$$

代入： $\frac{\delta}{x} = 4.64 Re_x^{-1/2}$ 得： $\frac{\delta_c}{x} = 4.64 Re_x^{-1/2} Sc^{-1/3}$

壁面 A 组分的扩散速率等于该处的对流传质速率：

$$k_{cx}^{0} A\left(C_{A0} - C_{AW}\right) = D_{AB} A \left.\frac{\partial C_A}{\partial y}\right|_{y=0}$$

$$\therefore \; k_{cx}^{0} = \frac{D_{AB}}{C_{A0} - C_{AW}} \left.\frac{\partial C_A}{\partial y}\right|_{y=0} = \frac{3}{2}\frac{D_{AB}}{\delta_c}$$

代入 δ_c 得局部对流传质系数：

$$k_{cx}^{0}=0.323\frac{D_{AB}}{x}Re_{x}^{1/2}Sc^{1/3}$$

局部 Sh_x：

$$sh_{x}=\frac{k_{cx}^{0}x}{D_{AB}}=0.323\,Re_{x}^{1/2}Sc^{1/3}$$

平均对流传质系数：

$$k_{cL}^{0}=\frac{1}{L}\int_{0}^{L}k_{cx}^{0}dx=0.646\frac{D_{AB}}{L}Re_{L}^{1/2}Sc^{1/3}$$

平均 Sh_L：

$$sh_{L}=\frac{k_{cL}^{0}L}{D_{AB}}=0.646\,Re_{L}^{1/2}Sc^{1/3}$$

湍流，设速度分布： $$\frac{u_x}{U_0}=\left(\frac{y}{\delta}\right)^{\frac{1}{7}}$$

浓度分布： $$\frac{C_A-C_{AW}}{C_{A0}-C_{AW}}=\left(\frac{y}{\delta_c}\right)^{\frac{1}{7}}$$

类似传热边界层有： $$\delta_c=\frac{0.376\,x}{\sqrt[5]{Re_x}}Sc^{-0.585}$$

$$k_{cx}^0=0.0292\frac{D_{AB}}{x}Re_x^{4/5}\,Sc^{1/3}\qquad Sh_x=0.0292\,Re_x^{4/5}\,Sc^{1/3}$$

$$k_{cL}^0=0.0365\frac{D_{AB}}{L}Re_L^{4/5}\,Sc^{1/3}\qquad Sh_L=0.0365\,Re_L^{4/5}\,Sc^{1/3}$$

3. 平板传质边界层计算

考虑到一开始始终有一段层流

$$k_{cL}^{0}=\frac{1}{L}\left(\int_{0}^{x_c}k_{cx层}^{0}dx+\int_{x_c}^{L}k_{cx湍}^{0}dx\right)$$

临界雷诺数

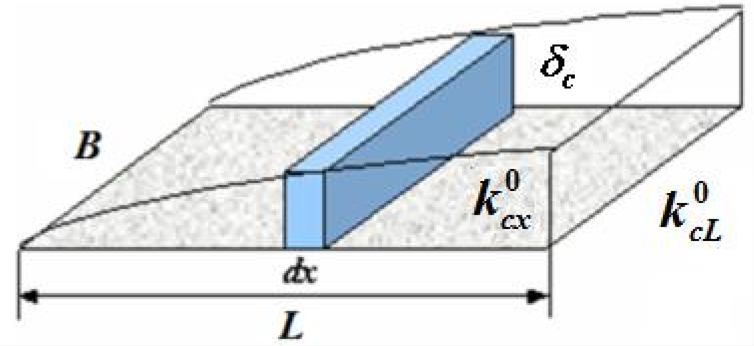
$Re_{xc}=5\times10^5$

$$k_{cL}^{0}=\frac{D_{AB}}{L}\left(0.0365\ Re_{L}^{4/5}-866\right)Sc^{1/3}$$

	传质边界层厚度	平均对流传质系数
层流	$\frac{\delta_c}{\delta}=Sc^{-1/3}$	$k_{cL}^{0}=0.646\frac{D_{AB}}{L}Re_{L}^{1/2}Sc^{1/3}$
湍流	$\frac{\delta_c}{\delta}=Sc^{-0.585}$	$k_{cL}^{0}=\frac{D_{AB}}{L}\left(0.0365Re_{L}^{4/5}-866\right)Sc^{1/3}$

课后自学

1.平板层流传质边界层精确解。



$$\frac{\delta_c}{x} = \frac{5.0}{\sqrt{Re_x}} Sc^{-1/3} \qquad \frac{\delta}{\delta_c} = Sc^{1/3}$$

$$Sh_L = \frac{k_{cL}^0 L}{D_{AB}} = 0.664\, Re_L^{1/2}\, Sc^{1/3}$$

适用条件： $u_{yw} = 0 \qquad Re_L < 5 \times 10^5 \quad Sc = 0.6 \sim 15$

课后思考

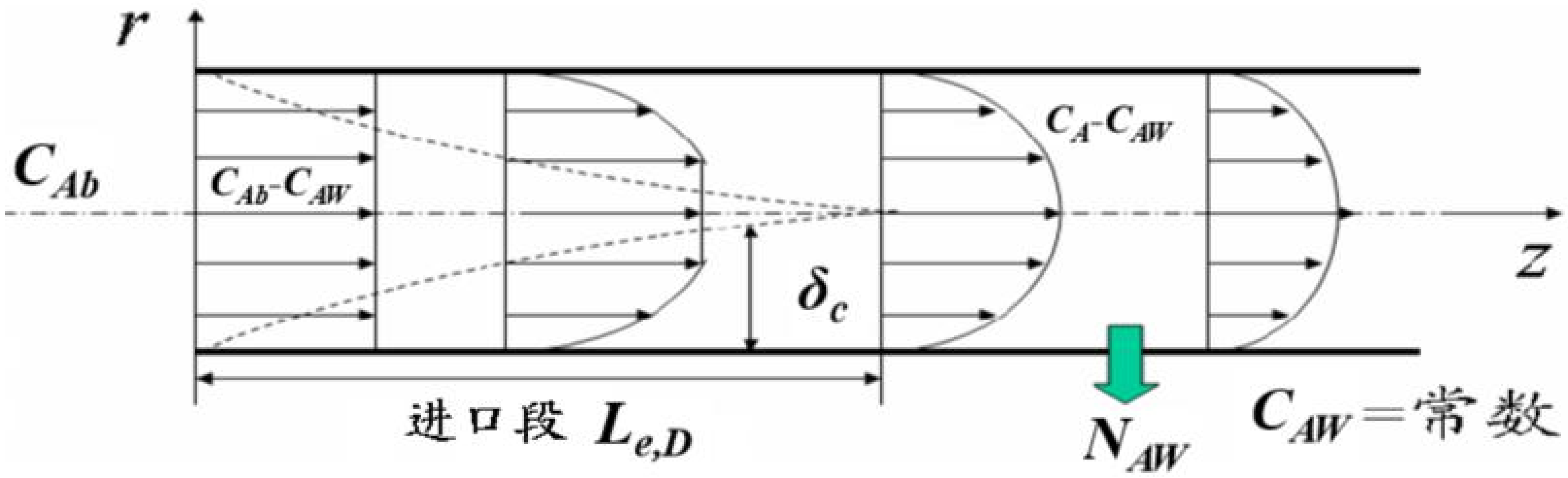
1.对比平板边界层动量、能量和质量积分方程，体会传递现象的类似性。

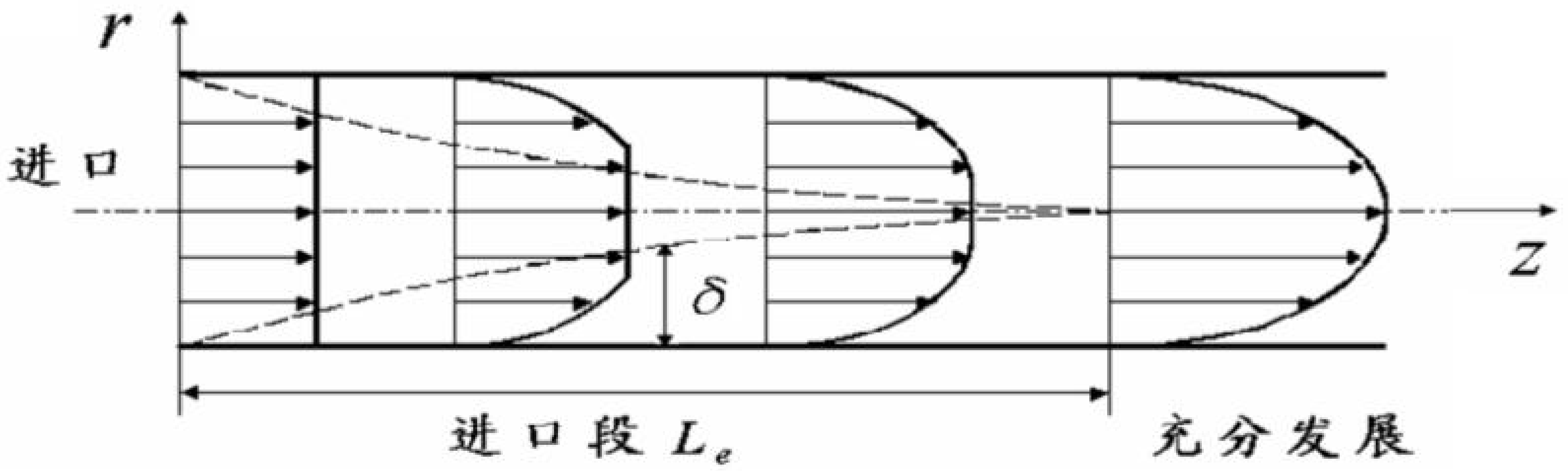
$$\rho\frac{\partial}{\partial x}\int_0^{\delta}(U_0-u_x)u_x dy=\tau_w$$

$$\frac{\partial}{\partial x}\int_0^{\delta_T}(T_0-T)u_x dy=a\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

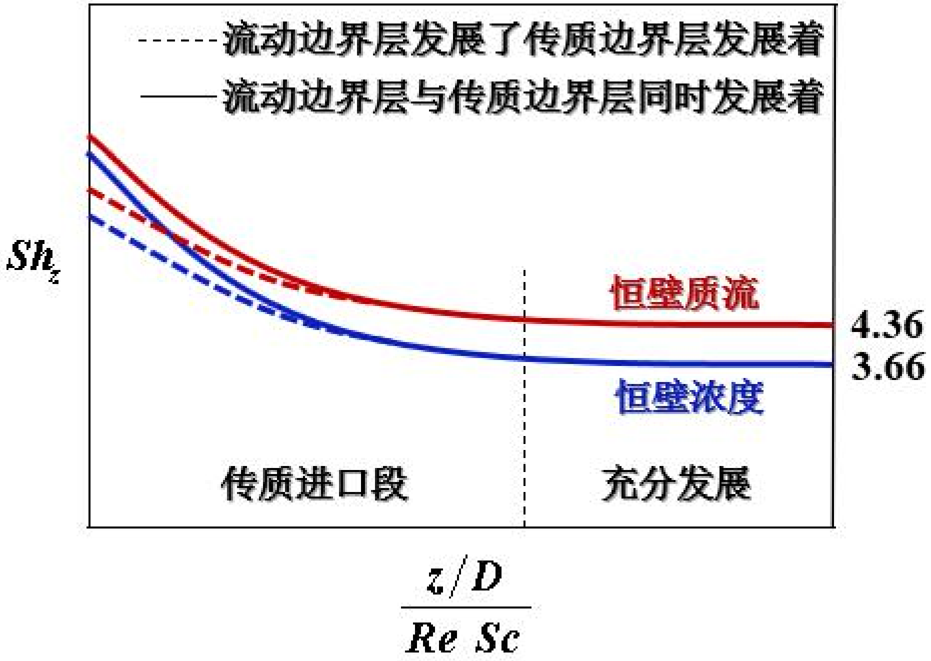
$$\frac{\partial}{\partial x}\int_0^{\delta_c}(C_{A0}-C_A)u_x dy=D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0}+(C_{A0}-C_{AW})u_{yw}$$

4. 圆管传质进口段


r
C_{Ab}
C_{Ab}-C_{AW}
C_A-C_{AW}
δ_c
z
进口段 $L_{e,D}$
N_{AW}
C_{AW}=常数

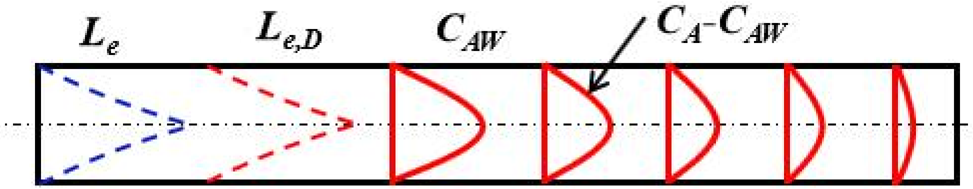

r
进口
δ
z
进口段 L_e
充分发展

传质进口段长度

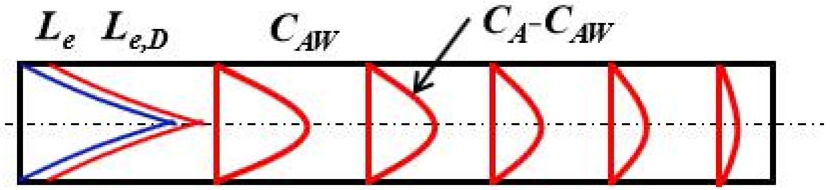


层流 $\dfrac{L_{e,D}}{D}=0.05ReSc$

湍流 $\dfrac{L_{e,T}}{D}=50$



流动边界层发展了传质边界层发展着



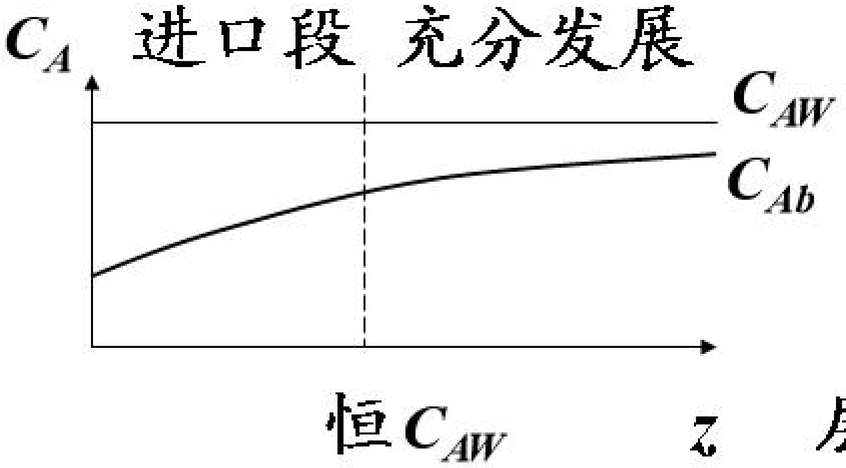
流动边界层与传质边界层同时发展着

5. 管内层流传质

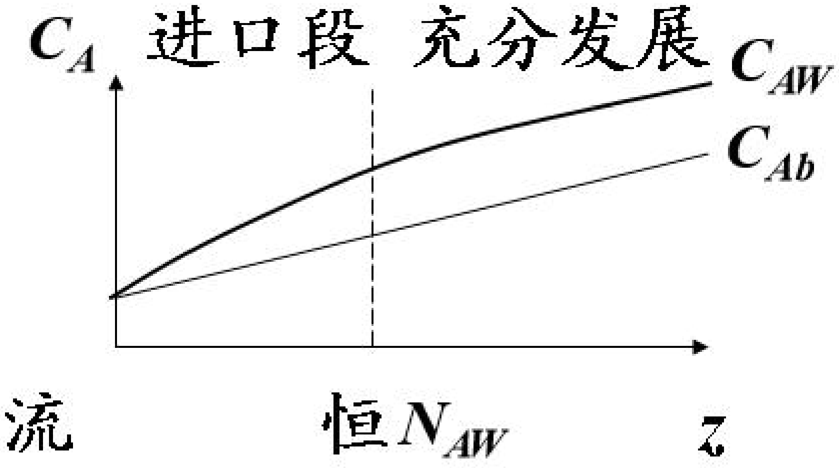
常见的圆管对流传质方式有两种：

①. 恒壁浓度 C_{AW} = 常数　②. 恒壁质流 N_{AW} = 常数

截面平均浓度 C_{Ab} 随 z 的变化如下图：



层流



截面的浓度分布 C_A 决定传质效果

恒壁质流 $N_{AW}=$ 常数

管内层流传质过程中，速度边界层和浓度边界层均充分发展后。

柱坐标系下的对流传质微分方程

$$\frac{\partial C_A}{\partial t}+u_{Mr}\frac{\partial C_A}{\partial r}+\frac{u_{M\theta}}{r}\frac{\partial C_A}{\partial \theta}+u_{Mz}\frac{\partial C_A}{\partial z}=D_{AB}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 C_A}{\partial \theta^2}+\frac{\partial^2 C_A}{\partial z^2}\right]+R_A$$

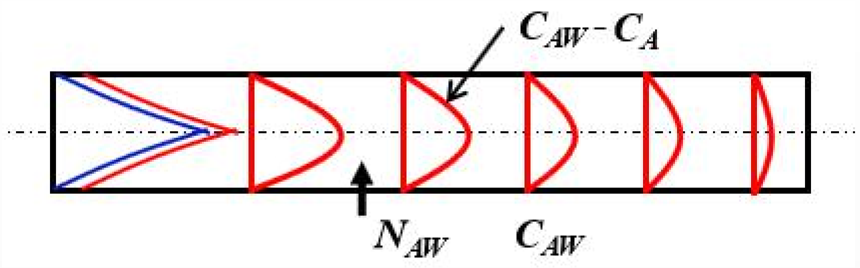
定常： $\dfrac{\partial C_A}{\partial t}=0$

管内流体： $\begin{cases} u_{Mr}=0 \\ u_{M\theta}=0 \\ u_{Mz}\neq 0 \end{cases}$ $\begin{cases} \dfrac{\partial C_A}{\partial r}\neq 0 \\ \dfrac{\partial C_A}{\partial \theta}=0 \\ \dfrac{\partial C_A}{\partial z}\neq 0 \end{cases}$ $\begin{cases} \dfrac{\partial^2 C_A}{\partial \theta^2}=0 \\ \dfrac{\partial^2 C_A}{\partial z^2}<<u_{Mz}\dfrac{\partial C_A}{\partial z} \end{cases}$

无化学反应： $R_A=0$

简化对流传质微分方程得：

$$u_{Mz}\frac{\partial C_A}{\partial z}=D_{AB}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right)$$



管内层流：

$$u_z=2U\left(1-\frac{r^2}{R^2}\right)$$

恒壁质流 $N_{AW}=$ 常数

$$\frac{\partial C_A}{\partial z}=\frac{\partial C_{AW}}{\partial z}=\frac{\partial C_{Ab}}{\partial z}=\text{常数}$$

$$C_{Ab}=\frac{\int_0^R u_z C_A 2\pi r dr}{U\pi\pi^2}$$

可得：

$$2U\left(1-\frac{r^2}{R^2}\right)\frac{\partial C_A}{\partial z}=D_{AB}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C_A}{\partial r}\right)$$

$$\frac{d}{dr}\left(r\frac{dC_A}{dr}\right)=\frac{2U}{D_{AB}}\left(1-\frac{r^2}{R^2}\right)r\frac{\partial C_A}{\partial z}$$

边界条件：

$$\begin{cases} r=0, \quad \dfrac{dC_A}{dr}=0 \\ r=R, \quad T=T_W, N_{AW}=k\left.\dfrac{dC_A}{dr}\right|_{r=R} \end{cases}$$

积分：

$$r\frac{dC_A}{dr}=\frac{2U}{D_{AB}}\frac{\partial C_A}{\partial z}\left(\frac{r^2}{2}-\frac{r^4}{4R^2}\right)+C_1$$

$$\because r=0, \quad \frac{dC_A}{dr}=0$$

$$\therefore C_1=0$$

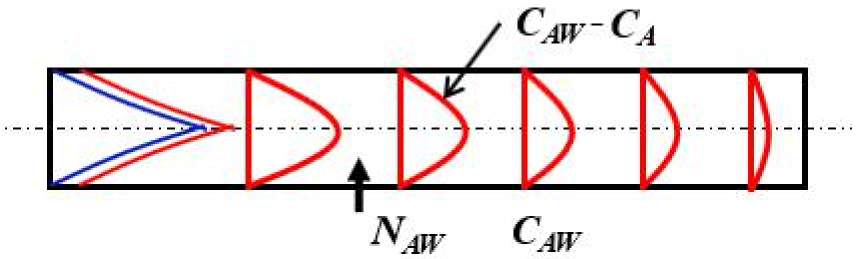
$$\frac{dC_A}{dr}=\frac{2U}{D_{AB}}\frac{\partial C_A}{\partial z}\left(\frac{r}{2}-\frac{r^3}{4R^2}\right)$$

再积分：

$$C_A=\frac{2U}{D_{AB}}\frac{\partial C_A}{\partial z}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)+C_2$$

$$\because r=R,\quad C_A=C_{AW}$$

$$\therefore C_2=C_{AW}-\frac{3U}{8D_{AB}}\frac{\partial C_A}{\partial z}R^2$$



浓度分布：

$$C_{AW}-C_A=\frac{3U}{8D_{AB}}\frac{\partial C_A}{\partial z}R^2-\frac{2U}{D_{AB}}\frac{\partial C_A}{\partial z}\left(\frac{r^2}{4}-\frac{r^4}{16R^2}\right)$$

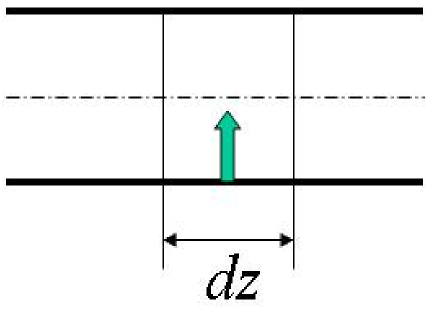
$$C_{Ab}=\frac{\int_0^R u_z C_A 2\pi r dr}{U\pi\pi^2}=C_{AW}-\frac{11}{48}\frac{UR^2}{D_{AB}}\frac{\partial C_A}{\partial z}$$

$$C_{AW}-C_{Ab}=\frac{11}{48}\frac{UR^2}{D_{AB}}\frac{\partial C_A}{\partial z}$$

在 dz 段上壁面处的扩散速率应等于流体和壁面之间的对流传质速率。

壁面处扩散速率： $$N_A=D_{AB}\left.\frac{dC_A}{dr}\right|_{r=R}2\pi R dz$$

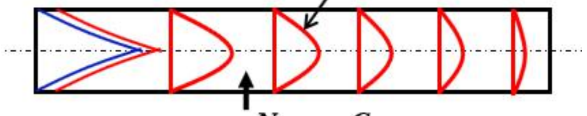
对流传质速率： $$N_A=k_c 2\pi R dz\left(C_{AW}-C_{Ab}\right)$$



$$\left.\frac{dC_A}{dr}\right|_{r=R} = \frac{UR}{2D_{AB}}\frac{\partial C_A}{\partial z} \qquad C_{AW} - C_{Ab} = \frac{11}{48}\frac{UR^2}{D_{AB}}\frac{\partial C_A}{\partial z}$$

$$k_c\left(C_{AW} - C_{Ab}\right) = D_{AB}\left.\frac{dC_A}{dr}\right|_{r=R}$$

$$\frac{k_c}{D_{AB}} = \frac{\left.\frac{dC_A}{dr}\right|_{r=R}}{C_{AW} - C_{Ab}} = \frac{\frac{UR}{2D_{AB}}\frac{\partial C_A}{\partial z}}{\frac{11}{48}\frac{UR^2}{D_{AB}}\frac{\partial C_A}{\partial z}} = \frac{48}{11\times 2R}$$

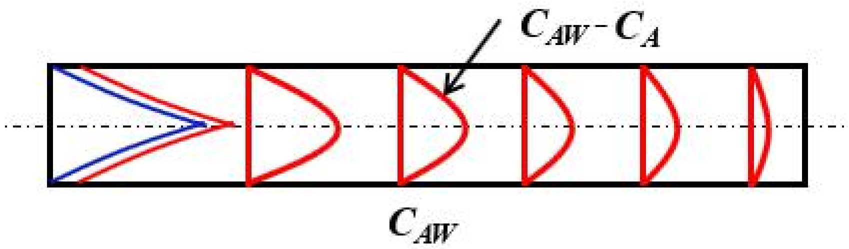


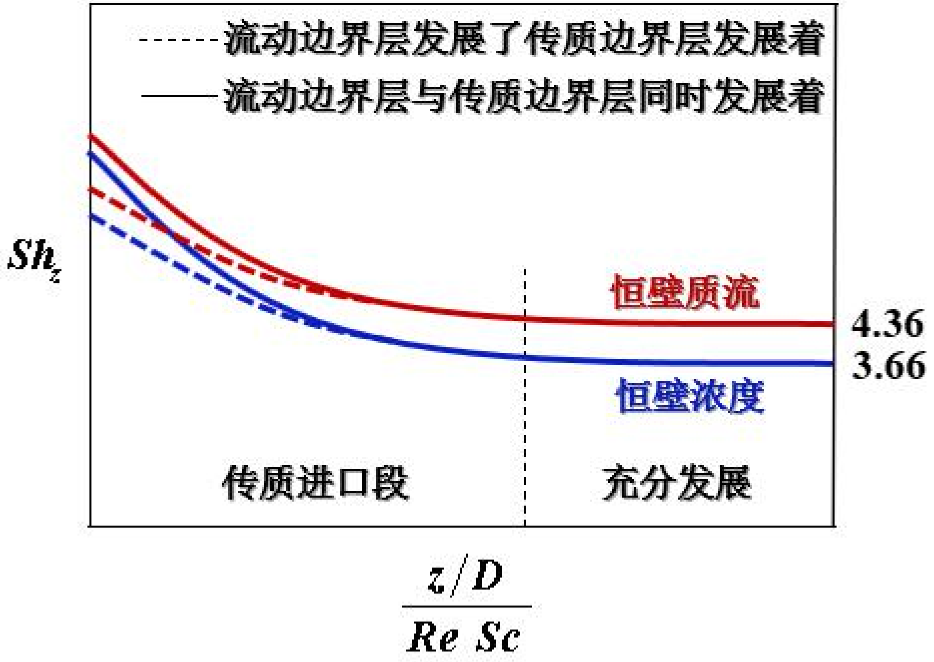
定义舍伍德数： $Sh = \frac{k_c D}{D_{AB}} = \frac{48}{11} = 4.36$

对圆管层流换热
恒 N_{AW}：Sh=4.36

对圆管层流恒壁浓度传质，类似恒壁温传热，可得结果为：

$$Sh=3.66$$





问题探讨

圆管层流传质
细管好,还是粗管好?