传递过程

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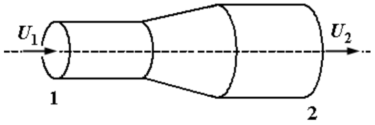
1.7 守恒原理

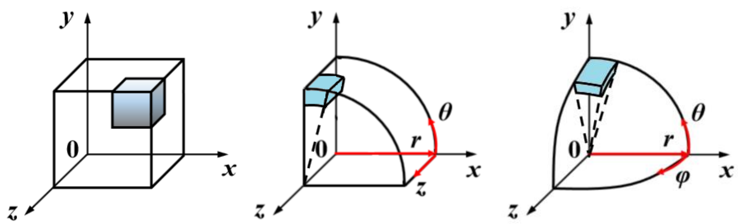
1.7.1控制面与控制体

确定对象及范围：控制面　控制体



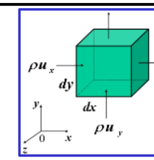
设备控制体

微元控制体



直角坐标系　　柱坐标系　　球坐标系

1.7.2 守恒原理的一般表达式



对于某一个选定的控制体

特征量变化速率 = 特征量输入速率 − 特征量输出速率 + 特征量生成速率

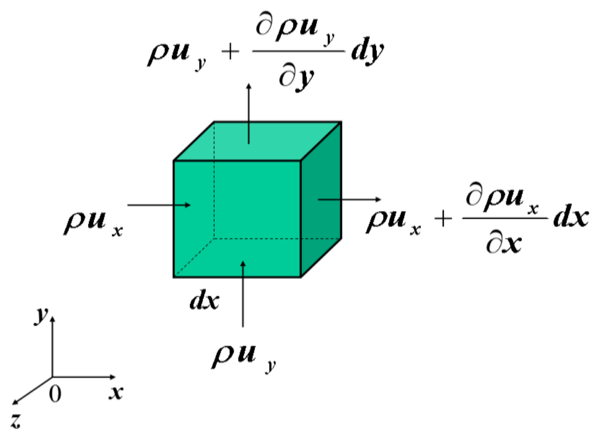
特征量变化速率是指控制体内特征量对时间的变化量，即 $\frac{\partial(\ )}{\partial t}$

特征量输入和输出速率是指单位时间从控制面输入和输出控制体的特征量

特征量生成速率是指单位时间控制体内因化学或物理过程产生或消失的特征量

1.7.3 连续性方程

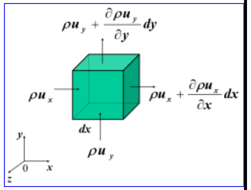
选取流场中一微元体（直角坐标系）



X方向：进： $\rho u_x dydz$

出： $(\rho u_x + \frac{\partial \rho u_x}{\partial x} dx) dydz$

进 - 出： $-\frac{\partial \rho u_x}{\partial x} dxdydz$



Y方向：进 - 出： $-\frac{\partial \rho u_y}{\partial y} dxdydz$

Z方向：进 - 出： $-\frac{\partial \rho u_z}{\partial z} dxdydz$

累积速率： $\frac{\partial \rho}{\partial t} dxdydz$

生成速率： $R = 0$

质量守恒： $\frac{\partial M}{\partial t} = W_1 - W_2 + R$

$$\frac{\partial \rho}{\partial t} dxdydz = \left(-\frac{\partial \rho u_x}{\partial x} - \frac{\partial \rho u_y}{\partial y} - \frac{\partial \rho u_z}{\partial z}\right) dxdydz$$

连续性方程

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_x}{\partial x} + \frac{\partial \rho u_y}{\partial y} + \frac{\partial \rho u_z}{\partial z} = 0$$

定常流动连续性方程 $\frac{\partial \rho}{\partial t} = 0$

$$\frac{\partial \rho u_x}{\partial x} + \frac{\partial \rho u_y}{\partial y} + \frac{\partial \rho u_z}{\partial z} = 0$$

不可压缩流体连续性方程 ρ为常数

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

解读： 不可压缩流体定常流动，速度散度为零（流速在各方向的变化率之和为零），意味着无源场，即满足连续性方程

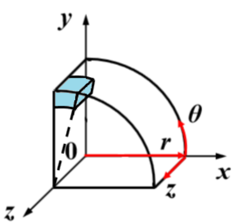
1.7.4 柱坐标系和球坐标系中连续性方程的形式

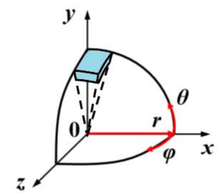
柱坐标系

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial \rho r u_r}{\partial r}+\frac{1}{r}\frac{\partial \rho u_\theta}{\partial \theta}+\frac{\partial \rho u_z}{\partial z}=0$$

若定常且ρ为常数

$$\frac{1}{r}\frac{\partial r u_r}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial u_z}{\partial z}=0$$





球坐标系

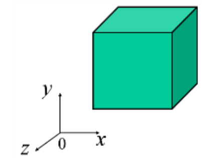
$$\frac{\partial \rho}{\partial t}+\frac{1}{r^2}\frac{\partial \rho r^2 u_r}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial \rho u_\theta \sin\theta}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial \rho u_\varphi}{\partial \varphi}=0$$

若定常且ρ为常数

$$\frac{1}{r^2}\frac{\partial r^2 u_r}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial u_\theta \sin\theta}{\partial \theta}+\frac{1}{r\sin\theta}\frac{\partial u_\varphi}{\partial \varphi}=0$$

第二章 动量传递

2.1粘性流体运动方程



选取流场中一微元体（直角坐标系）

对非定常流体的动量守恒：

$$\frac{\partial(m\bar{u})}{\partial t}=(w\bar{u})_1-(w\bar{u})_2+\Sigma\vec{F}$$

x，y，z 方向的分量守恒式：

$$\frac{\partial(m\bar{u}_x)}{\partial t}=(w\bar{u})_{1x}-(w\bar{u})_{2x}+\Sigma\vec{F}_x$$

$$\frac{\partial(m\bar{u}_y)}{\partial t}=(w\bar{u})_{1y}-(w\bar{u})_{2y}+\Sigma\vec{F}_y$$

$$\frac{\partial(m\bar{u}_z)}{\partial t}=(w\bar{u})_{1z}-(w\bar{u})_{2z}+\Sigma\vec{F}_z$$

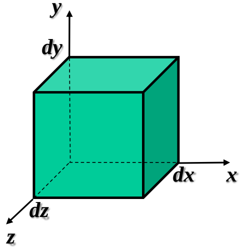
ΣF包括压力p、剪切应力τ、体积力X、其它外力。

粘性流体运动方程（动量守恒原理）

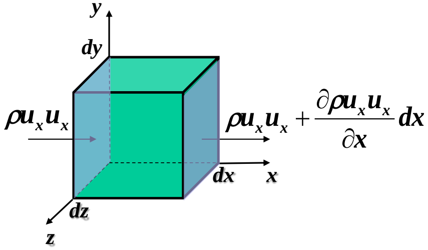
x方向： $$\frac{\partial(m\bar{u}_x)}{\partial t}=(w\bar{u})_{1x}-(w\bar{u})_{2x}+\Sigma\vec{F}_x$$

微元体内动量累积速率：

$$\frac{\partial(m\bar{u}_x)}{\partial t}=\frac{\partial(\rho u_x)}{\partial t}dxdydz$$



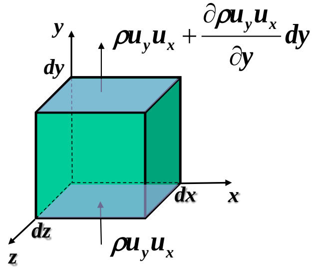
对流传递产生的动量净速率：



x方向流动产生的x方向动量净速率：

$$\rho u_x u_x dydz-\left(\rho u_x u_x+\frac{\partial \rho u_x u_x}{\partial x}dx\right)dydz=-\frac{\partial \rho u_x u_x}{\partial x}dxdydz$$

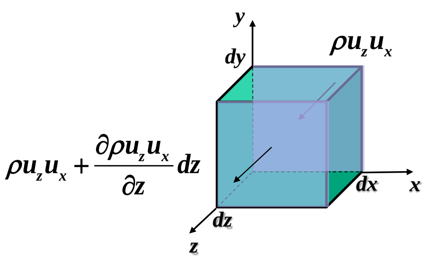
对流传递产生的动量净速率：



*y方向*流动产生的x方向动量净速率：

$$\rho u_y u_x dxdz - \left(\rho u_y u_x + \frac{\partial \rho u_y u_x}{\partial y} dy\right) dxdz = -\frac{\partial \rho u_y u_x}{\partial y} dxdydz$$

对流传递产生的动量净速率：



*z方向*流动产生的x方向动量净速率：

$$\rho u_z u_x dxdy - \left(\rho u_z u_x + \frac{\partial \rho u_z u_x}{\partial z} dz\right) dxdy = -\frac{\partial \rho u_z u_x}{\partial z} dxdydz$$

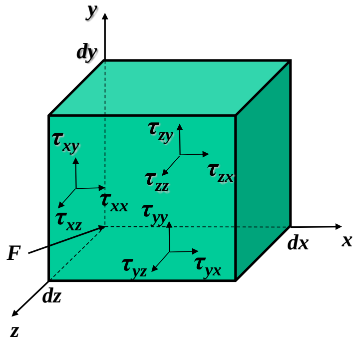
三者相加（前三页）

对流传递产生的x方向上动量净速率：

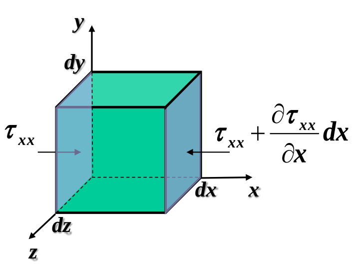
$$(w\vec{u})_{1x}-(w\vec{u})_{2x}=-\left(\frac{\partial\rho u_x u_x}{\partial x}+\frac{\partial\rho u_y u_x}{\partial y}+\frac{\partial\rho u_z u_x}{\partial z}\right)dxdydz$$

微元体上的作用力：$\Sigma\vec{F}_x=f(\tau)+f(p)+f(X)$

剪切应力τ：



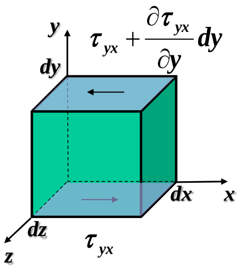
剪切应力τ产生的动量净速率：



x面上τ产生的x方向动量净速率：

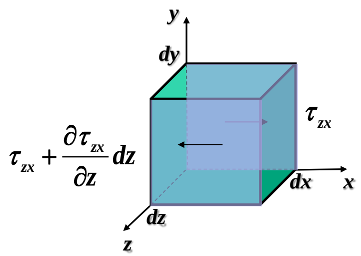
$$\tau_{xx}dydz-\left(\tau_{xx}+\frac{\partial\tau_{xx}}{\partial x}dx\right)dydz=-\frac{\partial\tau_{xx}}{\partial x}dxdydz$$

剪切应力τ产生的动量净速率：



y面上τ产生的x方向动量净速率：

$$\tau_{yx}dxdz-\left(\tau_{yx}+\frac{\partial\tau_{yx}}{\partial y}dy\right)dxdz=-\frac{\partial\tau_{yx}}{\partial y}dxdydz$$



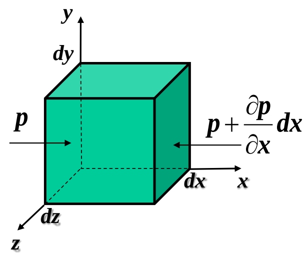
z面上τ产生的x方向动量净速率：

$$\tau_{zx}dxdy-\left(\tau_{zx}+\frac{\partial\tau_{zx}}{\partial z}dz\right)dxdy=-\frac{\partial\tau_{zx}}{\partial z}dxdydz$$

三者相加（前三页）

剪切应力τ产生的x方向上动量净速率：

$$f(\tau)=-\left(\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}\right)dxdydz$$



压力p产生的x方向上动量净速率：

$$f(p) = p\,dydz - \left(p + \frac{\partial p}{\partial x}dx\right)dydz = -\frac{\partial p}{\partial x}dxdydz$$

体积力X产生的x方向上动量净速率：

$$f(X) = \rho X dxdydz$$

根据动量守恒原理

$$\frac{\partial(m\bar{u}_x)}{\partial t} = (w\bar{u})_{1x} - (w\bar{u})_{2x} + \Sigma\vec{F}_x$$

$$\frac{\partial(\rho u_x)}{\partial t} = -\left(\frac{\partial \rho u_x u_x}{\partial x} + \frac{\partial \rho u_y u_x}{\partial y} + \frac{\partial \rho u_z u_x}{\partial z}\right) - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho X$$

引入连续性方程 $\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$ ρ为常数

$$\frac{\partial \rho u_x}{\partial t} + u_x\frac{\partial \rho u_x}{\partial x} + u_y\frac{\partial \rho u_x}{\partial y} + u_z\frac{\partial \rho u_x}{\partial z} = -\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) - \frac{\partial p}{\partial x} + \rho X$$

应力形式的运动方程 $\rho\frac{Du_x}{Dt} = -\frac{\partial p}{\partial x} + \rho X - \left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right)$ ρ为常数

2.1.1奈维-斯托克斯方程

对于不可压缩流体，若应力各向同性以及应力与应变率符合线性关系，可引入牛顿粘性定律的一般表达式：

$$\tau_{xy}=\tau_{yx}=-\mu\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right) \qquad \tau_{xx}=-2\mu\frac{\partial u_x}{\partial x}$$

$$\tau_{yz}=\tau_{zy}=-\mu\left(\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right) \qquad \tau_{yy}=-2\mu\frac{\partial u_y}{\partial y}$$

$$\tau_{zx}=\tau_{xz}=-\mu\left(\frac{\partial u_z}{\partial x}+\frac{\partial u_x}{\partial z}\right) \qquad \tau_{zz}=-2\mu\frac{\partial u_z}{\partial z}$$

不可压缩流体的奈维-斯托克斯方程 （ρ为常数）

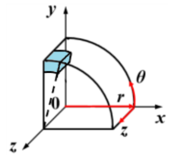
$$\rho\frac{Du_x}{Dt}=-\frac{\partial p}{\partial x}+\rho X+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$

$$\rho\frac{Du_y}{Dt}=-\frac{\partial p}{\partial y}+\rho Y+\mu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)$$

$$\rho\frac{Du_z}{Dt}=-\frac{\partial p}{\partial z}+\rho Z+\mu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)$$

惯性力　压力　体积力　粘性力

2.1.2柱坐标系和球坐标系中的形式



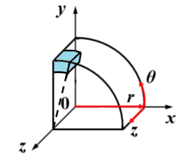
柱坐标系中剪切应力与形变的关系：

$$\tau_{r\theta}=\tau_{\theta r}=-\mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)+\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right] \qquad \tau_{rr}=-2\mu\frac{\partial u_r}{\partial r}$$

$$\tau_{\theta z}=\tau_{z\theta}=-\mu\left(\frac{\partial u_\theta}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial \theta}\right) \qquad \tau_{\theta\theta}=-2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r}{r}\right)$$

$$\tau_{zr}=\tau_{rz}=-\mu\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}\right) \qquad \tau_{zz}=-2\mu\frac{\partial u_z}{\partial z}$$

柱坐标系—奈维–斯托克斯方程

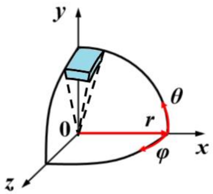


r方向：
$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta^2}{r}+u_z\frac{\partial u_r}{\partial z}\right)$$
$$=-\frac{\partial p}{\partial r}+\rho X_r+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r u_r}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}+\frac{\partial^2 u_r}{\partial z^2}\right]$$

θ方向：
$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r u_\theta}{r}+u_z\frac{\partial u_\theta}{\partial z}\right)$$
$$=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho X_\theta+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial r u_\theta}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}+\frac{\partial^2 u_\theta}{\partial z^2}\right]$$

z方向：
$$\rho\left(\frac{\partial u_z}{\partial t}+u_r\frac{\partial u_z}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_z}{\partial \theta}+u_z\frac{\partial u_z}{\partial z}\right)$$
$$=-\frac{\partial p}{\partial z}+\rho X_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 u_z}{\partial \theta^2}+\frac{\partial^2 u_z}{\partial z^2}\right]$$

球坐标系中剪切应力与形变的关系：

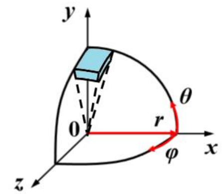


$$\tau_{r\theta}=\tau_{\theta r}=-\mu\left[r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right)+\frac{1}{r}\frac{\partial u_r}{\partial \theta}\right]\qquad \tau_{rr}=-2\mu\frac{\partial u_r}{\partial r}$$

$$\tau_{\theta\varphi}=\tau_{\varphi\theta}=-\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{u_\varphi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \varphi}\right]\qquad \tau_{\theta\theta}=-2\mu\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_r}{r}\right)$$

$$\tau_{\varphi r}=\tau_{r\varphi}=-\mu\left[\frac{1}{r\sin\theta}\frac{\partial u_r}{\partial \varphi}+r\frac{\partial}{\partial r}\left(\frac{u_\varphi}{r}\right)\right]\qquad \tau_{\varphi\varphi}=-2\mu\left(\frac{1}{r\sin\theta}\frac{\partial u_\theta}{\partial \varphi}+\frac{u_r}{r}+\frac{u_\theta\cot\theta}{r}\right)$$

球坐标系—奈维–斯托克斯方程

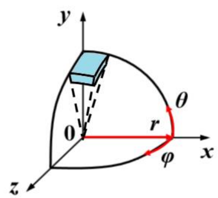


r 方向：

$$\rho\left(\frac{\partial u_r}{\partial t}+u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta}+\frac{u_\varphi}{r\sin\theta}\frac{\partial u_r}{\partial \varphi}-\frac{u_\theta^2+u_\varphi^2}{r}\right)$$

$$=-\frac{\partial p}{\partial r}+\rho X_r+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_r}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_r}{\partial \theta}\right)\right.$$

$$\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_r}{\partial \varphi^2}-\frac{2}{r^2}u_r-\frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta}-\frac{2}{r^2}u_\theta\cot\theta-\frac{2}{r^2\sin\theta}\frac{\partial u_\varphi}{\partial \varphi}\right]$$

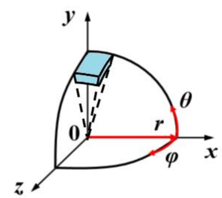


θ 方向：

$$\rho\left(\frac{\partial u_\theta}{\partial t}+u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta}+\frac{u_\varphi}{r\sin\theta}\frac{\partial u_\theta}{\partial \varphi}+\frac{u_r u_\theta}{r}-\frac{u_\varphi^2\cot\theta}{r}\right)$$

$$=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho X_\theta+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_\theta}{\partial \theta}\right)\right.$$

$$\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\theta}{\partial \varphi^2}+\frac{2}{r^2}\frac{\partial u_r}{\partial \theta}-\frac{u_\theta}{r^2\sin^2\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\varphi}{\partial \varphi}\right]$$



φ 方向：

$$\rho\left(\frac{\partial u_\varphi}{\partial t}+u_r\frac{\partial u_\varphi}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\varphi}{\partial \theta}+\frac{u_\varphi}{r\sin\theta}\frac{\partial u_\varphi}{\partial \varphi}+\frac{u_r u_\varphi}{r}+\frac{u_\theta u_\varphi\cot\theta}{r}\right)$$

$$=-\frac{1}{r\sin\theta}\frac{\partial p}{\partial \varphi}+\rho X_\varphi+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\varphi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial u_\varphi}{\partial \theta}\right)\right.$$

$$\left.+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 u_\varphi}{\partial \varphi^2}-\frac{u_\varphi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial u_r}{\partial \varphi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\theta}{\partial \varphi}\right]$$