传递过程

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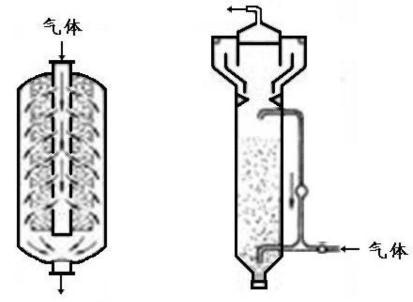
3.3.6小球非定常传热

传热原理

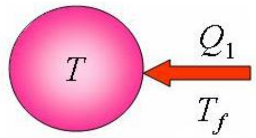
反应器中的球形催化剂颗粒体积V，表面积A，初始温度T_0，通入温度为T_f的热气流，颗粒温度将随时间升高。



固定床反应器　　流化床反应器

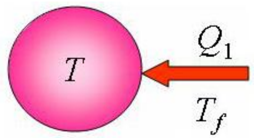
简化：忽略颗粒内部导热热阻，集总参数法。

毕奥数 $Bi<0.1$　　（Biot）



$$Bi=\frac{\frac{V}{A}\Big/k}{1/h}=\frac{\text{内部导热热阻}}{\text{外部对流热阻}}$$

球坐标系下的对流传热微分方程

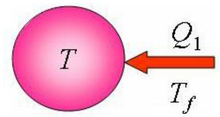


$$\frac{\partial T}{\partial t}+u_r\frac{\partial T}{\partial r}+\frac{u_\theta}{r}\frac{\partial T}{\partial \theta}+\frac{u_\varphi}{r\sin\theta}\frac{\partial T}{\partial \varphi}$$

$$=a\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \varphi^2}\right]+\frac{\dot{q}}{\rho C_P}$$

简化得： $\dfrac{dT}{dt}=\dfrac{\dot{q}}{\rho C_P}$ 　　式中： $\dot{q}=\dfrac{hA\left(T_f-T\right)}{V}$

得： $\dfrac{dT}{dt}=\dfrac{hA\left(T_f-T\right)}{\rho C_p V}$



$$\frac{dT}{dt}=\frac{hA\left(T_f-T\right)}{\rho C_p V}$$

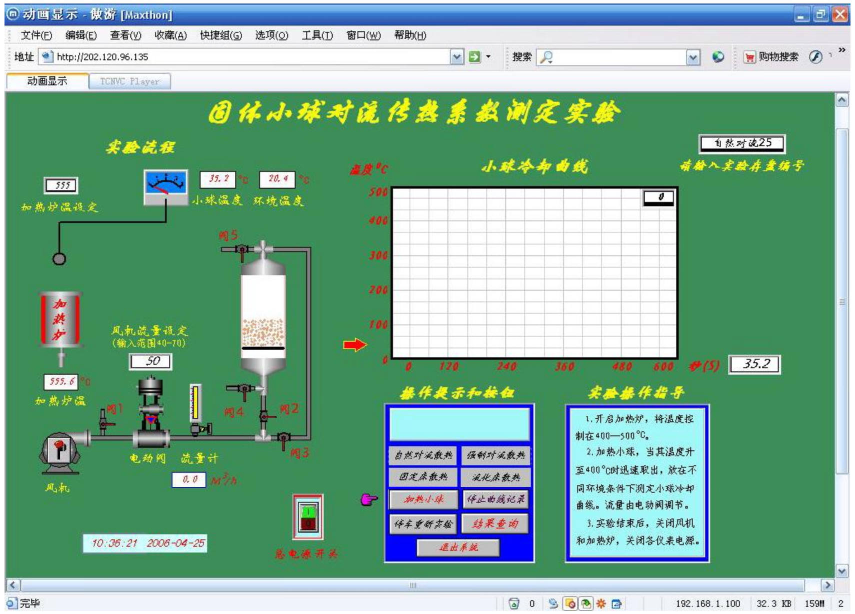
$$\int_{T_0-T_f}^{T-T_f}\frac{d(T-T_f)}{T-T_f}=-\frac{hA}{\rho C_p V}\int_0^t dt$$

温度随时间变化关系：

$$\frac{T-T_f}{T_0-T_f}=e^{-\frac{hA}{\rho C_p V}\cdot t}$$

例3-8 小球传热专业实验

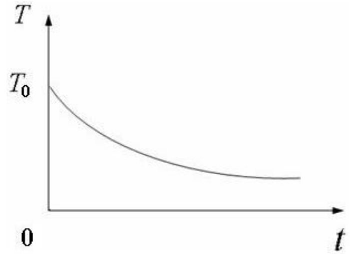
① 控制界面



② 实验装置

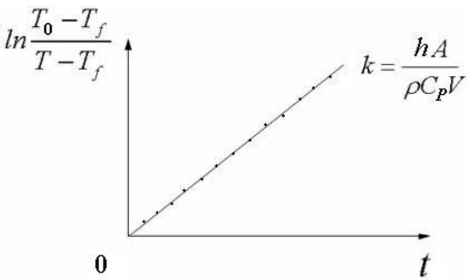
③ 实验结果讨论

小球温度随时间的变化关系



$$\frac{T-T_f}{T_0-T_f}=e^{-\frac{hA}{\rho C_P V}\cdot t}$$

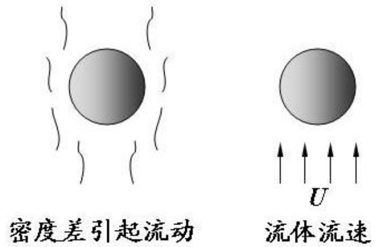
数据处理



$$ln\frac{T_0-T_f}{T-T_f}=\frac{hA}{\rho C_P V}\cdot t$$

单个颗粒

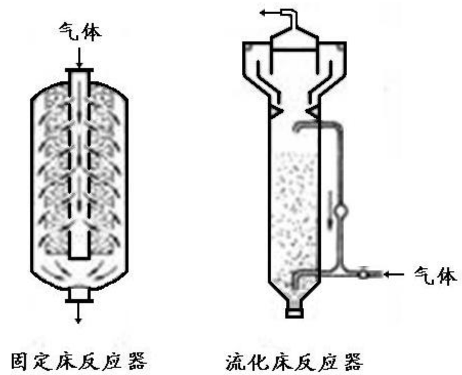
自然对流



强制对流

颗粒群

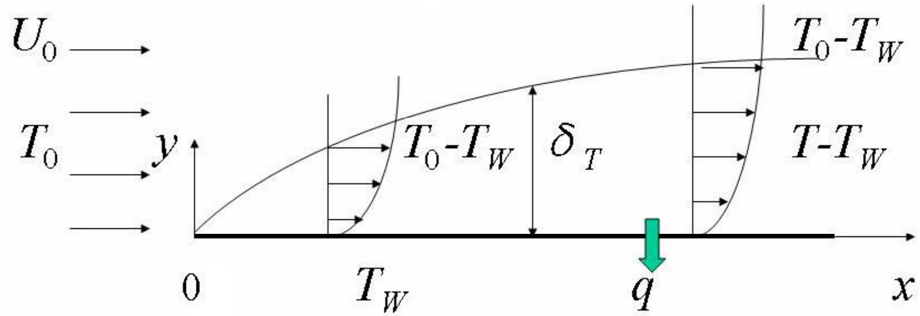
固定床



流化床

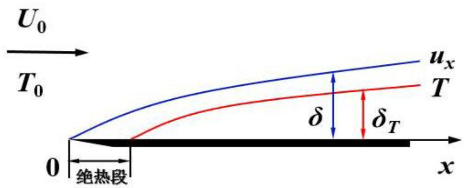
3.4传热边界层

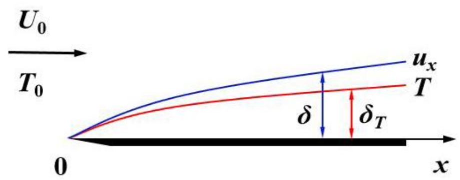
3.4.1传热边界层的形成和特点

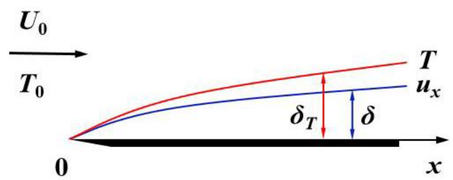


类似流动边界层，以 ***T*-*T*$_W$ = 99%（*T*$_0$-*T*$_W$）**为界线。

传热边界层与流动边界层的关系







$$\frac{\delta}{\delta_T} = Pr^n = \left(\frac{\nu}{a}\right)^n = \left(\frac{\mu C_p}{k}\right)^n$$

气体 $Pr \approx 1$
粘性油 $Pr \to \infty$
液态金属 $Pr \to 0$

对$Pr = 0.6 \sim 15$内的层流：$n = 1/3$
湍流：$n = 0.585$

$$Pr = \frac{\nu}{a} = \frac{\text{分子动量扩散}}{\text{分子热量扩散}}$$

回顾：

$$j_{Ay} = -D_{AB}\frac{d\rho_A}{dy}$$

$\left[\frac{\text{kg}}{\text{m}^2\cdot\text{s}}\right]$ 质量通量　$\left[\text{m}^2/\text{s}\right]$ 扩散系数　$\left[\frac{\text{kg}}{\text{m}^3}\right]$ 质量浓度

$$q_y = -k\frac{dT}{dy} = -\frac{k}{\rho C_P}\frac{d\left(\rho C_P T\right)}{dy} = -a\frac{d\left(\rho C_P T\right)}{dy}$$

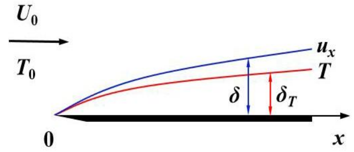
$\left[\frac{\text{J}}{\text{m}^2\cdot\text{s}}\right]$ 热量通量　$\left[\text{m}^2/\text{s}\right]$（热扩散系数）导温系数　$\left[\frac{\text{J}}{\text{m}^3}\right]$ 热量浓度

$$\tau_{yx} = -\mu\frac{du_x}{dy} = -\frac{\mu}{\rho}\frac{d\left(\rho u_x\right)}{dy} = -\nu\frac{d\left(\rho u_x\right)}{dy}$$

$\left[\frac{\text{kg}\cdot\text{m/s}}{\text{m}^2\cdot\text{s}}\right]$ 动量通量　$\left[\text{m}^2/\text{s}\right]$（运动粘度）粘性系数　$\left[\frac{\text{kg}\cdot\text{m/s}}{\text{m}^3}\right]$ 动量浓度

3.4.2平板传热边界层计算

3.4.2.1平板层流传热边界层精确解



对流传热微分方程

$$\rho C_p\left(\frac{\partial T}{\partial t}+u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}+u_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\dot{q}$$

简化可得：

$$u_x\frac{\partial T}{\partial x}+u_y\frac{\partial T}{\partial y}=a\frac{\partial^2 T}{\partial y^2}$$

引入无量纲温度：

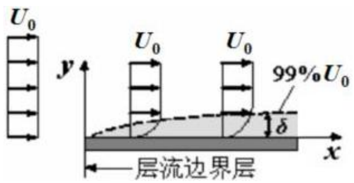
$$\Theta = \frac{T-T_W}{T_0-T_W}$$

$$u_x\frac{\partial \Theta}{\partial x}+u_y\frac{\partial \Theta}{\partial y}=a\frac{\partial^2 \Theta}{\partial y^2}$$

边界条件：

$$\begin{cases} y=0,\ \dfrac{u_x}{U_0}=0 \quad \Theta=0 \\ y\to\infty,\ \dfrac{u_x}{U_0}=1 \quad \Theta=1 \end{cases}$$

回顾：普朗特边界层计算



奈维-斯托克斯方程

x方向：

$$\rho\left(\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho X+\mu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)$$

量级比较简化可得普朗特边界层方程：

$$\begin{cases} u_x\dfrac{\partial u_x}{\partial x}+u_y\dfrac{\partial u_x}{\partial y}=\nu\dfrac{\partial^2 u_x}{\partial y^2} \\ \dfrac{\partial u_x}{\partial x}+\dfrac{\partial u_y}{\partial y}=0 \end{cases}$$

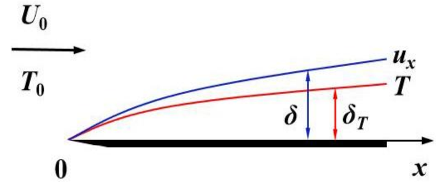
边界条件：$\begin{cases} y=0, u_x=u_y=0 \\ y\to\infty, u_x=U_0 \end{cases}$

流函数 $u_x=\dfrac{\partial \psi}{\partial y}$； $u_y=-\dfrac{\partial \psi}{\partial x}$

$$\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=\nu\frac{\partial^3 \psi}{\partial y^3}$$

边界条件：$\begin{cases} y=0, \dfrac{\partial \psi}{\partial y}=\dfrac{\partial \psi}{\partial x}=0 \\ y\to\infty, \dfrac{\partial \psi}{\partial y}=U_0 \end{cases}$

类似流动边界层求解



$$\Theta=\varphi(\eta)=\varphi\left(y\sqrt{\frac{U_0}{\nu x}}\right)$$

其中：$\eta=y\sqrt{\dfrac{U_0}{\nu x}}$

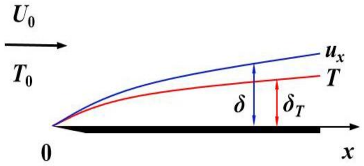
$$\begin{cases} \dfrac{\partial \Theta}{\partial x}=\dfrac{\partial \Theta}{\partial \eta}\dfrac{\partial \eta}{\partial x}=-\dfrac{\eta}{2x}\dfrac{\partial \Theta}{\partial \eta} \\ \dfrac{\partial \Theta}{\partial y}=\dfrac{\partial \Theta}{\partial \eta}\dfrac{\partial \eta}{\partial y}=\sqrt{\dfrac{U_0}{\nu x}}\dfrac{\partial \Theta}{\partial \eta} \\ \dfrac{\partial^2 \Theta}{\partial y^2}=\dfrac{\partial}{\partial \eta}\left(\dfrac{\partial \Theta}{\partial y}\right)\dfrac{\partial \eta}{\partial y}=\dfrac{U_0}{\nu x}\dfrac{\partial^2 \Theta}{\partial \eta^2} \end{cases}$$

$$\psi=\sqrt{\nu x U_0}\,f(\eta)$$

$$\begin{cases} u_x=\dfrac{\partial \psi}{\partial y}=U_0 f'(\eta) \\ u_y=-\dfrac{\partial \psi}{\partial x}=-\dfrac{1}{2}\sqrt{\dfrac{\nu U_0}{x}}\left[f(\eta)-\eta f'(\eta)\right] \end{cases}$$

代入整理得：

$$\frac{d^2\Theta}{d\eta^2}+\frac{1}{2}Pr\,f(\eta)\frac{d\Theta}{d\eta}=0$$



边界条件：$\begin{cases}\eta=0,T=T_W & \Theta=0\\ \eta\to\infty,T=T_0 & \Theta=1\end{cases}$

采用变量置换法求解

令 $\dfrac{d\Theta}{d\eta}=p$ $\qquad \dfrac{dp}{d\eta}+\dfrac{1}{2}Pr\,f(\eta)p=0$

积分： $p=C_1e^{-\frac{1}{2}Pr\int f(\eta)d\eta}$

$$\frac{d\Theta}{d\eta}=C_1e^{-\frac{1}{2}Pr\int f(\eta)d\eta}$$

再积分： $\displaystyle\int_0^{\Theta}d\Theta=\int_0^{\eta}C_1e^{-\frac{1}{2}Pr\int_0^{\eta}f(\eta)d\eta}d\eta$

$$\Theta=C_1\int_0^{\eta}e^{-\frac{1}{2}Pr\int_0^{\eta}f(\eta)d\eta}d\eta$$

边界条件： $\eta\to\infty,T=T_0\quad\Theta=1$

$$C_1=\frac{1}{\int_0^{\infty}e^{-\frac{1}{2}Pr\int_0^{\infty}f(\eta)d\eta}d\eta}$$

温度分布：

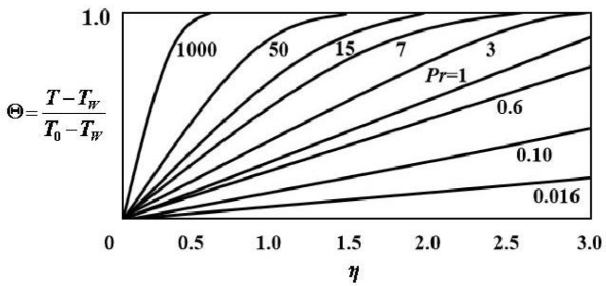
$$\Theta=\frac{\int_0^{\eta}e^{-\frac{1}{2}Pr\int_0^{\eta}f(\eta)d\eta}d\eta}{\int_0^{\infty}e^{-\frac{1}{2}Pr\int_0^{\infty}f(\eta)d\eta}d\eta}$$

还记得速度分布吗？

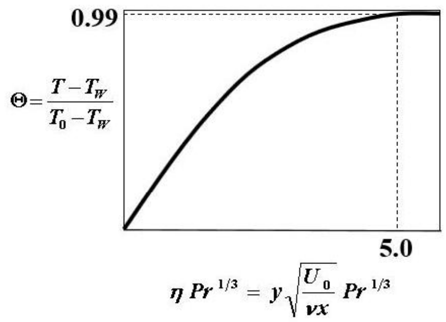
$$f'(\eta)=\frac{u_x}{U_0}=0.332\eta-2.2963\times10^{-3}\eta^4+1.9967\times10^{-5}\eta^7-1.5694\times10^{-7}\eta^{10}+\cdots$$

$$f(\eta)=\frac{0.332}{2}\eta^2-\frac{2.2963\times10^{-3}}{5}\eta^5+\frac{1.9967\times10^{-5}}{8}\eta^8-\frac{1.5694\times10^{-7}}{11}\eta^{11}+\cdots$$

Pohlhausen 采用数值法求解上式，对于范围$Pr=0.016\sim1000$内的层流流动，其解如图所示：



Pohlhausen 进一步对于范围在$Pr=0.6\sim15$内的层流流动进行研究，绘制$\Theta\sim\eta Pr^{1/3}$曲线，其解如图所示：



当 $\Theta=\frac{T-T_W}{T_0-T_W}=0.99$ 时，

$\eta\,Pr^{1/3}=5.0$ 则有：

$$\eta=y\sqrt{\frac{U_0}{\nu x}}\Rightarrow y=\eta\sqrt{\frac{\nu x}{U_0}}\Rightarrow\delta_T=\frac{5.0}{Pr^{1/3}}\sqrt{\frac{\nu x}{U_0}}$$

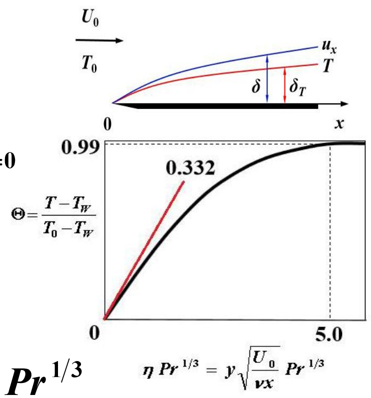
$$\frac{\delta_T}{x}=\frac{5.0}{\sqrt{Re_x}}Pr^{-1/3}$$

对比 $\frac{\delta}{x}=\frac{4.96}{\sqrt{Re_x}}$ 则有：$\frac{\delta}{\delta_T}=Pr^{1/3}$

局部对流传热系数 $h_x\left(T_0-T_W\right)=k\left.\dfrac{\partial T}{\partial y}\right|_{y=0}$

$$h_x=k\left.\frac{\partial\dfrac{T-T_W}{T_0-T_W}}{\partial y}\right|_{y=0}=k\left.\frac{\partial\Theta}{\partial y}\right|_{y=0}=k\sqrt{\frac{U_0}{\nu x}}\left.\frac{\partial\Theta}{\partial\eta}\right|_{\eta=0}$$

$$=k\sqrt{\frac{U_0}{\nu x}}\,Pr^{1/3}\left.\frac{\partial\Theta}{\partial\left(\eta\,Pr^{1/3}\right)}\right|_{\eta=0}$$

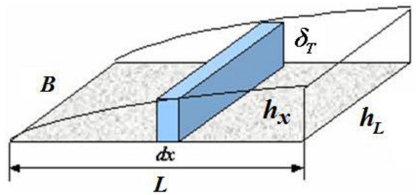


$$h_x=0.332k\sqrt{\frac{U_0}{\nu x}}\,Pr^{1/3}=0.332\frac{k}{x}Re_x^{1/2}\,Pr^{1/3}$$

局部努塞尔数 $Nu_x=\dfrac{h_x x}{k}=0.332\,Re_x^{1/2}\,Pr^{1/3}$

努塞尔数表示流体的导热阻力与对流传热阻力的比。

平均对流传热系数



长度为L、宽为 B 的平板的平均对流传热系数

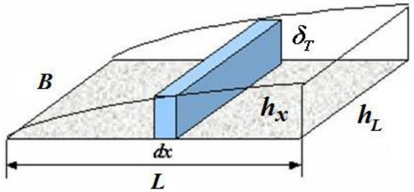
$$h_L=\frac{1}{LB}\int_0^L 0.332\frac{k}{x}Re_x^{1/2}\,Pr^{1/3}\,B\,dx=0.664\frac{k}{L}Re_L^{1/2}\,Pr^{1/3}$$

平均努塞尔数 $Nu_L=\dfrac{h_L L}{k}=0.664\,Re_L^{1/2}\,Pr^{1/3}$

适用条件： $Re_L<5\times10^5$ $\quad Pr=0.6\sim15$

汇总页：

平板层流传热边界层精确解

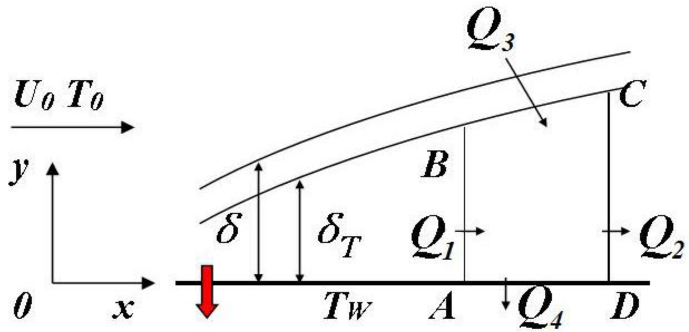


$$\frac{\delta_T}{x} = \frac{5.0}{\sqrt{Re_x}} Pr^{-1/3} \qquad \frac{\delta}{\delta_T} = Pr^{1/3}$$

$$Nu_L = \frac{h_L L}{k} = 0.664 Re_L^{1/2} Pr^{1/3}$$

适用条件： $Re_L < 5\times10^5 \quad Pr = 0.6 \sim 15$

3.4.2.2传热边界层能量积分方程

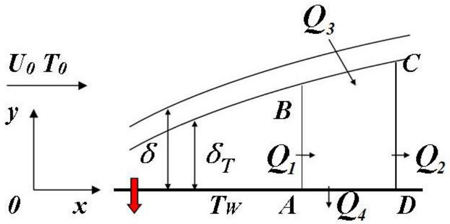


选取控制体*ABCD*，单位宽度，$\delta_T < \delta$

对定常流动传热： $Q_1 + Q_3 = Q_2 + Q_4$

相似性：冯卡门边界层动量积分方程

$$Q_1=\int_0^{\delta_T}\rho C_P T u_x dy$$



$$Q_2=\int_0^{\delta_T}\rho C_P T u_x dy+\frac{\partial}{\partial x}\left(\int_0^{\delta_T}\rho C_P T u_x dy\right)dx$$

$$Q_3=C_P T_0\frac{\partial}{\partial x}\left(\int_0^{\delta_T}\rho u_x dy\right)dx$$

$$Q_4=-k\frac{\partial T}{\partial y}\bigg|_{y=0}dx$$

根据能量守恒：

$$\frac{\partial}{\partial x}\int_0^{\delta_T}(T_0-T)u_x dy=a\frac{\partial T}{\partial y}\bigg|_{y=0}$$

传热边界层能量积分方程

设温度分布：

$$\frac{T-T_W}{T_0-T_W}=a+b\left(\frac{y}{\delta_T}\right)+c\left(\frac{y}{\delta_T}\right)^2+d\left(\frac{y}{\delta_T}\right)^3$$

边界条件：
$$\begin{cases} y=0,T=T_W & y=0,\dfrac{\partial^2 T}{\partial y^2}=0 \\ y=\delta_T,T=T_0 & y=\delta_T,\dfrac{\partial T}{\partial y}=0 \end{cases}$$

求得温度分布：
$$\frac{T-T_W}{T_0-T_W}=\frac{3}{2}\left(\frac{y}{\delta_T}\right)-\frac{1}{2}\left(\frac{y}{\delta_T}\right)^3$$

回顾：

层流时，设速度分布：

$$\frac{u_x}{U}=a+b\left(\frac{y}{\delta}\right)+c\left(\frac{y}{\delta}\right)^2+d\left(\frac{y}{\delta}\right)^3$$

边界条件：
$$\begin{cases} y=0, u_x=0 & y=0, \dfrac{\partial^2 u_x}{\partial y^2}=0 \\ y=\delta, u_x=U_0 & y=\delta, \dfrac{\partial u_x}{\partial y}=0 \end{cases}$$

求得速度分布：$\dfrac{u_x}{U_0}=\dfrac{3}{2}\left(\dfrac{y}{\delta}\right)-\dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$

$$\tau_w=\mu\frac{\partial u_x}{\partial y}\bigg|_{y=0}=\mu\frac{3}{2}\frac{U_0}{\delta}$$

将T，u_x代入传热边界层能量积分方程求得：

$$\frac{\delta_T}{\delta}=\frac{1}{1.026}Pr^{-1/3}\approx Pr^{-1/3}$$

代入：$\dfrac{\delta}{x}=4.64Re_x^{-1/2}$　　得：$\dfrac{\delta_T}{x}=4.64\,Re_x^{-1/2}\,Pr^{-1/3}$

壁面导热速率等于该处的对流换热速率:

$$h_x A\left(T_0-T_W\right)=kA\frac{\partial T}{\partial y}\bigg|_{y=0}$$

$$\therefore\ h_x=\frac{k}{T_0-T_W}\frac{\partial T}{\partial y}\bigg|_{y=0}=\frac{3}{2}\frac{k}{\delta_T}$$

代入δ_T得局部对流传热系数h_x：

$$h_x = 0.323\frac{k}{x}Re_x^{1/2}\ Pr^{1/3}$$

局部努塞尔数Nu_x：

$$Nu_x = \frac{h_x x}{k} = 0.323\ Re_x^{1/2}\ Pr^{1/3}$$

平均对流传热系数h_L：

$$h_L = \frac{1}{L}\int_0^L h_x dx = 0.646\ \frac{k}{L}Re_L^{1/2}\ Pr^{1/3}$$

平均努塞尔数Nu_L：

$$Nu_L = \frac{h_L L}{k} = 0.646\ Re_L^{1/2}\ Pr^{1/3}$$

$$h_x A\left(T_0 - T_W\right) = kA\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

代入传热边界层能量积分方程

$$h_x = \frac{\rho C_p}{T_0 - T_W}\frac{\partial}{\partial x}\int_0^{\delta_T}\left(T_0 - T\right)u_x dy$$

湍流时，设速度分布：$\dfrac{u_x}{U_0} = \left(\dfrac{y}{\delta}\right)^{\frac{1}{7}}$

温度分布：$\dfrac{T - T_W}{T_0 - T_W} = \left(\dfrac{y}{\delta_T}\right)^{\frac{1}{7}}$

可得： $h_x=\frac{7}{72}\rho C_p U_0 \frac{d}{dx}\left[\delta_T\left(\frac{\delta_T}{\delta}\right)^{1/7}\right]$

已知层流： $\frac{\delta_T}{\delta}=Pr^{-1/3}$

对湍流，假定： $\frac{\delta_T}{\delta}=Pr^{-n}$ 式中n由实验测定。

已知湍流流动边界层厚度： $\frac{\delta}{x}=\frac{0.376}{\sqrt[5]{Re_x}}$

$$\delta_T=\frac{0.376\,x}{\sqrt[5]{Re_x}}Pr^{-n}$$

$$h_x=\frac{7}{72}\rho C_p U_0\, Pr^{-n/7}\frac{d\delta_T}{dx}=0.0292\rho C_p U_0\, Re_x^{-1/5}\, Pr^{-8n/7}$$

$$h_x=0.0292\rho C_p U_0\, Re_x^{-1/5}\, Pr^{-8n/7}=0.0292\frac{k}{x}Re_x^{4/5}\, Pr^{(7-8n)/7}$$

$$Nu_x=\frac{h_x x}{k}=0.0292\, Re_x^{4/5}\, Pr^{(7-8n)/7}$$

实验表明，湍流边界层传热时 $n=0.585$，可得：

$$\delta_T=\frac{0.376\,x}{\sqrt[5]{Re_x}}Pr^{-0.585}$$

局部努塞尔数Nu_x：

$$Nu_x=\frac{h_x x}{k}=0.0292\, Re_x^{4/5}\, Pr^{1/3}$$

平均对流传热系数h_L：

$$h_L = \frac{1}{L}\int_0^L h_x dx = 0.0365\,\frac{k}{L} Re_L^{4/5}\, Pr^{1/3}$$

平均努塞尔数Nu_L：

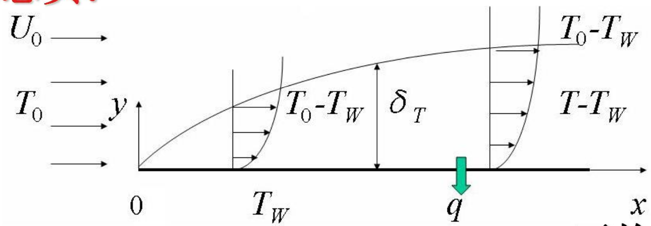
$$Nu_L = \frac{h_L L}{k} = 0.0365\, Re_L^{4/5}\, Pr^{1/3}$$

考虑到一开始始终有一段层流，

$$h_L = \frac{1}{L}\left(\int_0^{x_c} h_{x层} dx + \int_{x_c}^L h_{x湍} dx\right)$$

$$h_L = \frac{k}{L}\left(0.0365\, Re_L^{4/5} - 866\right) Pr^{1/3}$$

汇总页：



传热边界层厚度

临界雷诺数

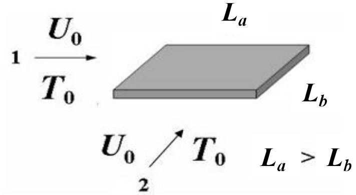
Re_{xc}=5×10^5

平均对流传热系数

层流	$\frac{\delta_T}{\delta} = Pr^{-1/3}$	$h_L = 0.646\,\frac{k}{L} Re_L^{1/2}\, Pr^{1/3}$
湍流	$\frac{\delta_T}{\delta} = Pr^{-0.585}$	$h_L = \frac{k}{L}\left(0.0365 Re_L^{4/5} - 866\right) Pr^{1/3}$

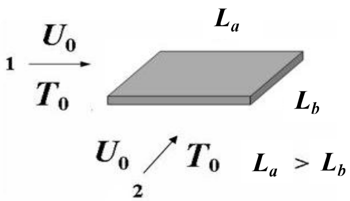
问题探讨

平板冷却速率



问题探讨

平板冷却速率



$$Q = h_L \cdot \Delta T \cdot A \quad (A = L_a L_b)$$

$$Q_1 = \frac{1}{L_a} \cdot L_a^{\frac{1}{2}} \cdot L_a \cdot L_b \cdot \text{constant} = L_a^{\frac{1}{2}} \cdot L_b \cdot \text{constant} = A^{\frac{1}{2}} \cdot L_b^{\frac{1}{2}} \cdot \text{constant}$$

同理

$$Q_2 = A^{\frac{1}{2}} \cdot L_a^{\frac{1}{2}} \cdot \text{constant}$$

所以

$$\frac{Q_1}{Q_2} = \frac{L_b^{\frac{1}{2}}}{L_a^{\frac{1}{2}}} < 1,$$

流动方向垂直于较宽边方向时，传热效率更高

（两种情况都是层流时）