北京化工大学

Beijing University of Chemical Technology

信号与系统

第三章 信号的频域表达-傅里叶变换

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主要内容 CONTENTS

16

傅里叶变换的基本性质

-- 对称性质、线性性质

-- 奇偶虚实性、尺度变换性质

-- 微分性质、积分性质

--时移特性、频移特性

傅里叶变换具有唯一性。傅氏变换的性质揭示了信号的时域特性和频域特性之间的确定的内在联系。讨论傅里叶变换的性质，目的在于：

- 了解特性的内在联系；
- 用性质求$F(\omega)$；
- 了解在通信系统领域中的实用。

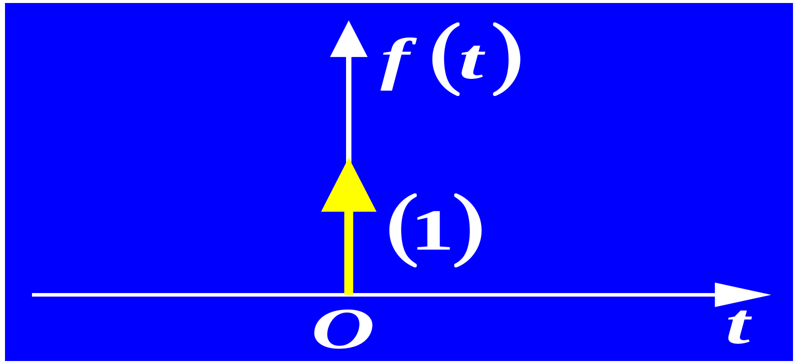
若$\boldsymbol{f(t)} \leftrightarrow \boldsymbol{F(\omega)}$　　则$\boldsymbol{F(t)} \leftrightarrow \boldsymbol{2\pi f(-\omega)}$

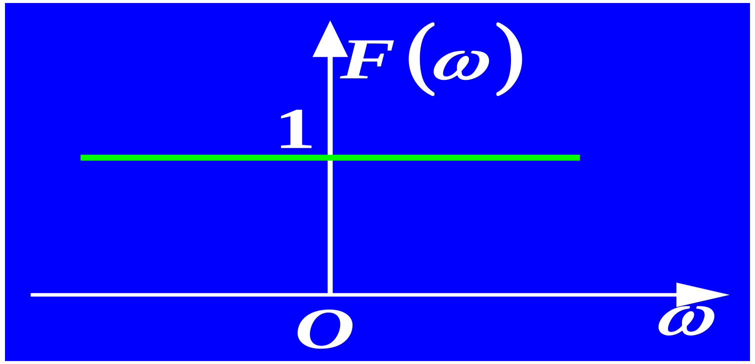
若$f(t)$为偶函数　则$F(t) \leftrightarrow 2\pi f(\omega)$

若$\boldsymbol{f(t)}$形状与$\boldsymbol{F(\omega)}$相同，　$(\boldsymbol{\omega \to t})$

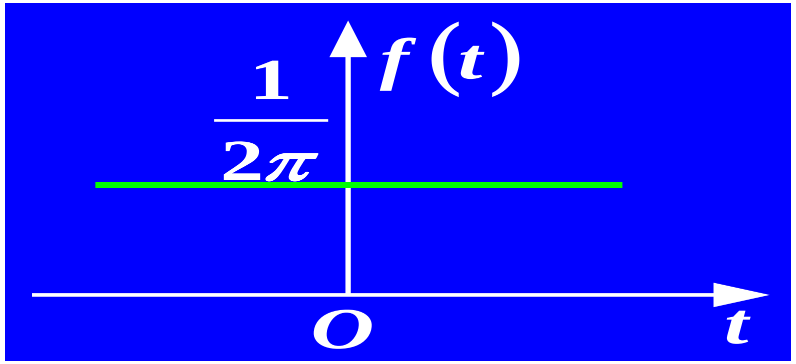
则$\boldsymbol{F(t)}$的频谱函数形状与　$\boldsymbol{f(t)}$形状相同，　$(\boldsymbol{t \to \omega})$，幅度差$\boldsymbol{2\pi}$

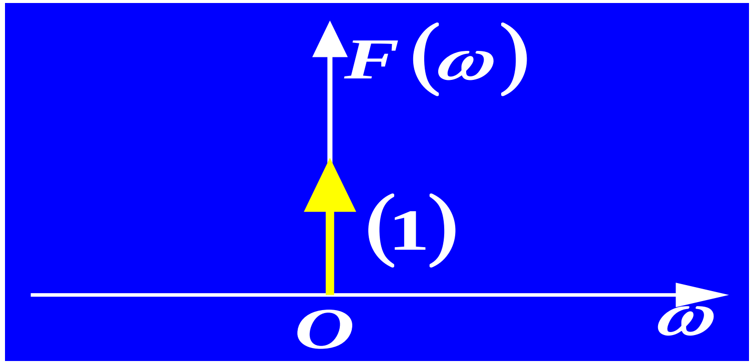
$$\delta(t) \leftrightarrow F(\omega) = 1$$



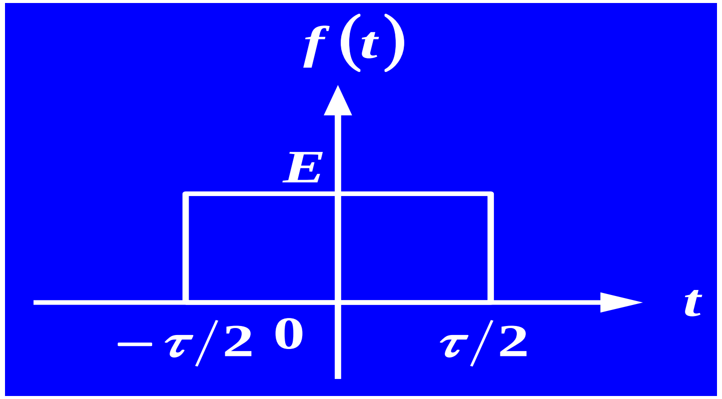


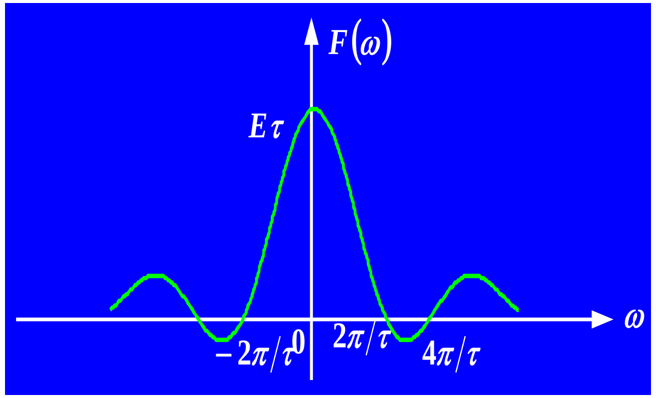
$$f(t) = \frac{1}{2\pi} \leftrightarrow \delta(\omega)$$



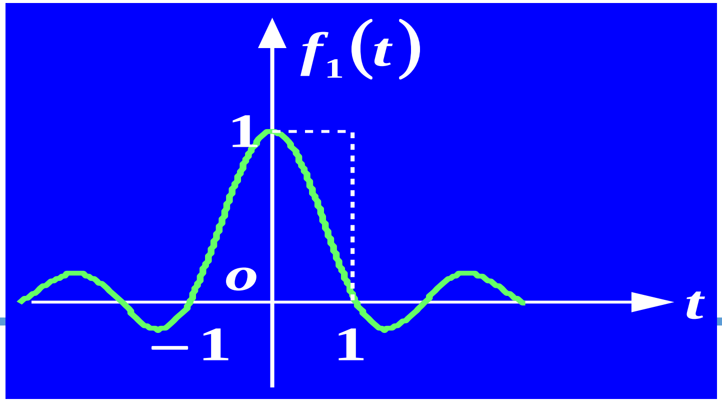


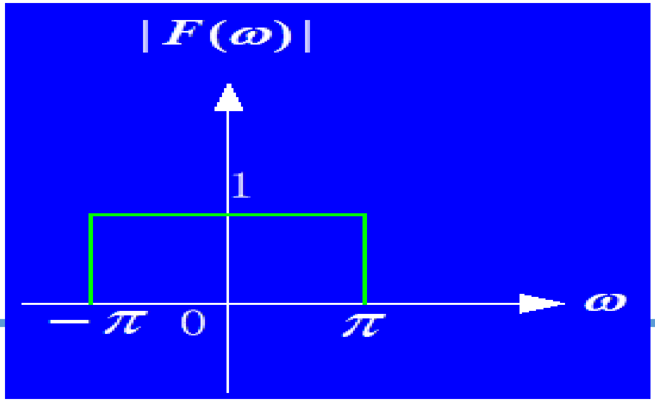
$$f(t)=E\left[u\left(t+\frac{\tau}{2}\right)-u\left(t-\frac{\tau}{2}\right)\right] \quad \leftrightarrow \quad F(\omega)=E\tau Sa\left(\frac{\omega\tau}{2}\right)$$





$$F(t)=\frac{1}{2\pi}E\omega_c\left(\frac{\omega_c t}{2}\right) \quad \leftrightarrow \quad \left[u\left(\omega+\frac{\omega_c}{2}\right)-u\left(\omega-\frac{\omega_c}{2}\right)\right]$$





若$f(t) \leftrightarrow F(\omega)$

则$f(-t) \leftrightarrow F(-\omega)$

$$F[f(-t)] = \int_{-\infty}^{\infty} f(-t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} f(u)e^{-j(-\omega)u}du = F(-\omega)$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} f(t)\cos\omega t dt - j\int_{-\infty}^{\infty} f(t)\sin\omega t dt$$

$$R(\omega) = \int_{-\infty}^{\infty} f(t)\cos\omega t dt$$ 关于ω 的偶函数 $R(\omega) = R(-\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} f(t)\sin\omega t dt$$ 关于ω 的奇函数$X(\omega) = -X(-\omega)$

$\therefore F(-\omega) = F^*(\omega)$

若$f(t) \leftrightarrow F(\omega)$，则$f(-t) \leftrightarrow F^*(\omega)$

$\therefore F[f(-t)] = F^*(\omega)$

尺度变换性：若$f(t) \leftrightarrow F(\omega)$，则$f(at) \leftrightarrow \dfrac{1}{|a|}F\left(\dfrac{\omega}{a}\right)$，$a$为非零函数

$$F[f(at)] = \int_{-\infty}^{\infty} f(at)\, e^{-j\omega t} dt$$

当$a > 0$，令$x = at$

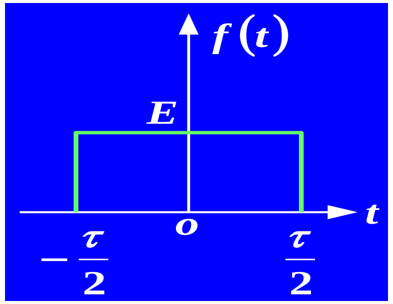
$$F[f(at)] = \frac{1}{a}\int_{-\infty}^{\infty} f(x)\, e^{-j\omega\frac{x}{a}} dx = \frac{1}{a}F\left(\frac{\omega}{a}\right)$$

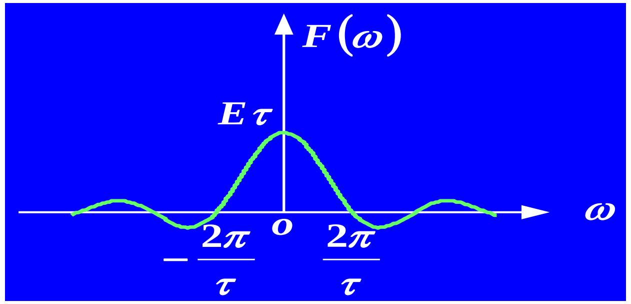
$$F[f(at)] = \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$$

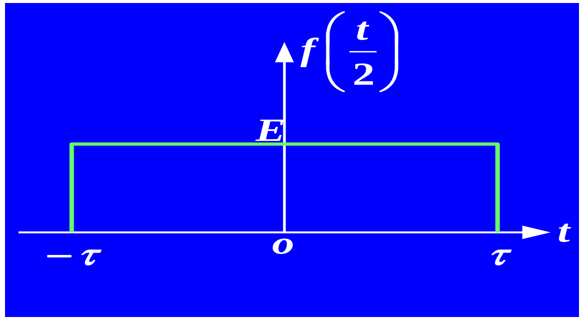
当$a < 0$，令$x = -|a|t$

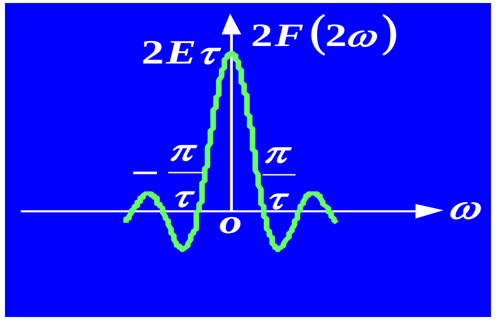
$$F[f(at)] = \frac{-1}{|a|}\int_{+\infty}^{-\infty} f(x)\, e^{-j\omega\frac{x}{a}} dx = \frac{1}{|a|}\int_{-\infty}^{\infty} f(x)\, e^{-j\frac{\omega}{a}x} dx = \frac{1}{|a|}F\left(\frac{\omega}{a}\right)$$

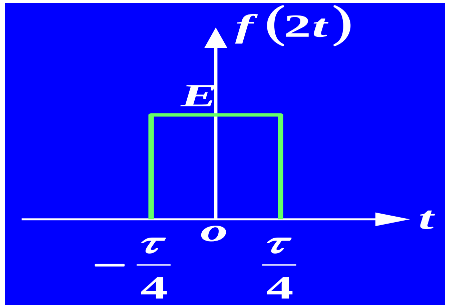
16.4 傅里叶变换的尺度变换性

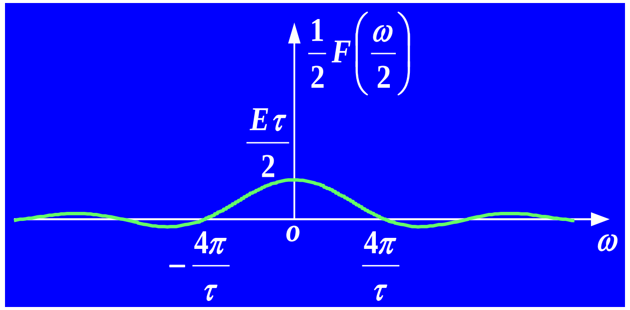












信号时域和频域的关系

信号的持续时间与信号占有频带成反比

(1) $0<a<1$ **时域扩展，频带压缩。**

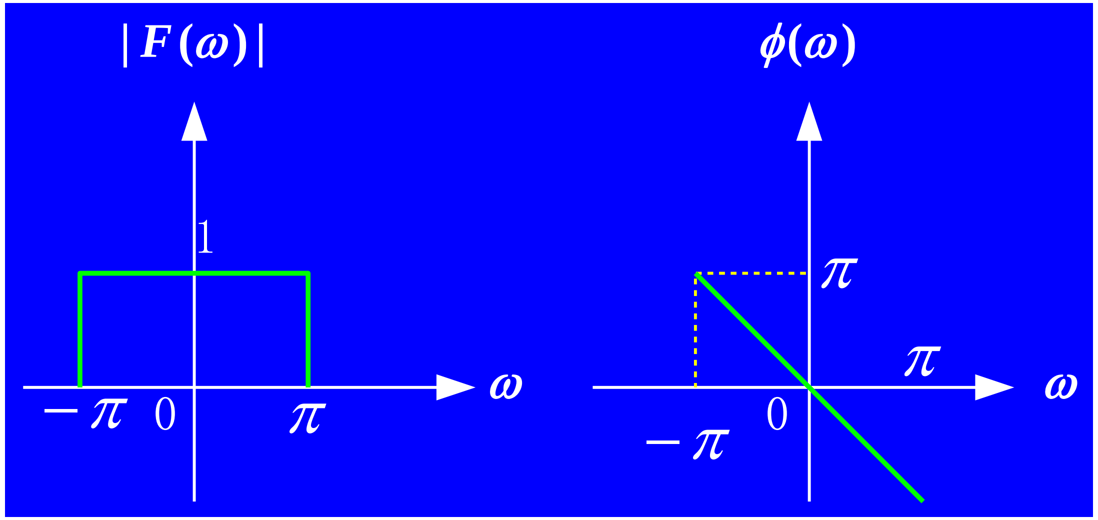
(2) $a>1$ **时域压缩，频域扩展a倍。**

若$f(t) \leftrightarrow F(\omega)$ 则$f(t-t_0) \leftrightarrow F(\omega)e^{-j\omega t_0}$;

若$F(\omega)=|F(\omega)|e^{j\varphi(\omega)}$ 则$f(t-t_0) \leftrightarrow |F(\omega)|\cdot e^{j[\Phi(\omega)-\omega t_0]}$

幅度频谱无变化，只影响相位频谱

$$\text{相移}\omega t_0 \begin{cases} \text{右} & -\omega t_0 \\ \text{左} & \omega t_0 \end{cases}$$



时移加尺度变换 $f(at+b) \leftrightarrow \frac{1}{|a|}F\left(\frac{\omega}{a}\right)\cdot e^{j\omega\frac{b}{a}}$

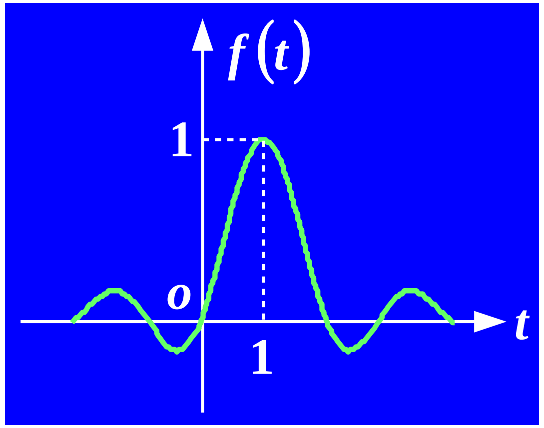
例16.1求图(a)所示函数的傅里叶变换。

$$2\pi f_1(\omega) = 2\pi Sa(\pi\omega) \to G_{2\pi}(t) = F_1(t)$$

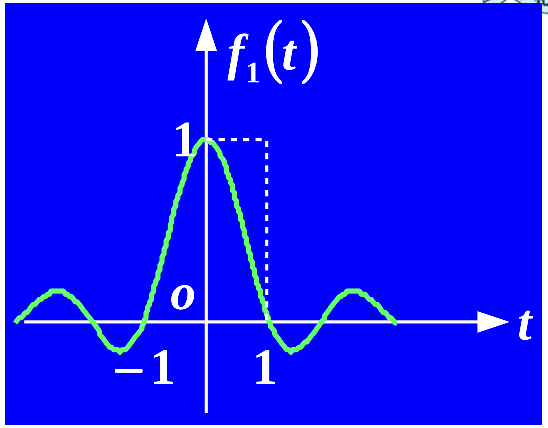
$$f_1(t) = Sa(\pi t) \quad F_1(\omega) = G_{2\pi}(\omega)$$

$$f(t) = f_1(t-1)$$

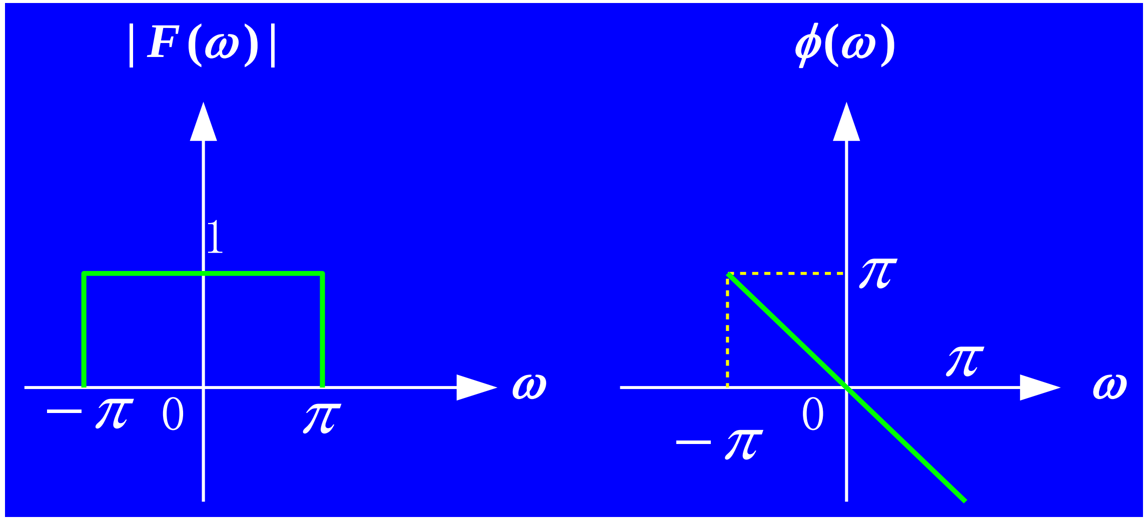
$$F(\omega) = F_1(\omega)\cdot e^{-j\omega} = G_{2\pi}(\omega)\cdot e^{-j\omega}$$



(a)



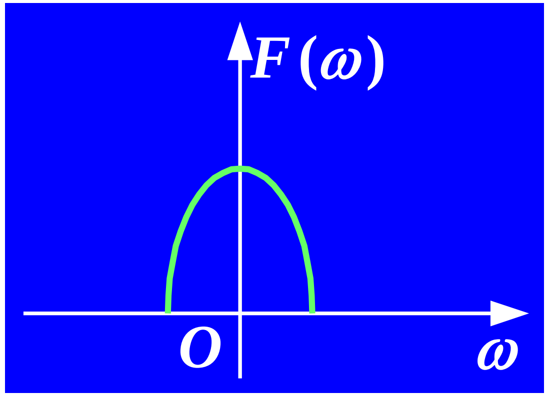
(b)

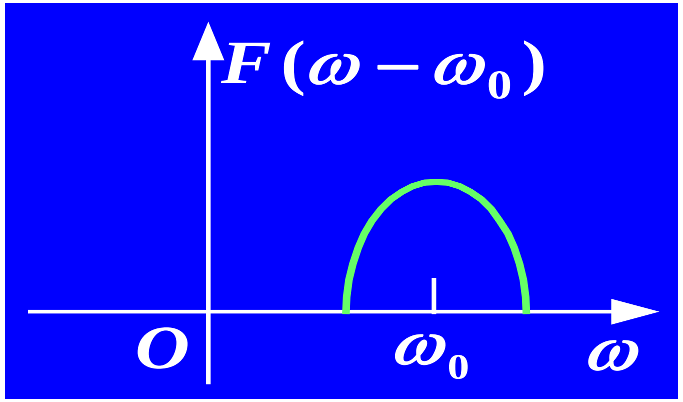


16.6 傅里叶变换的频移特性

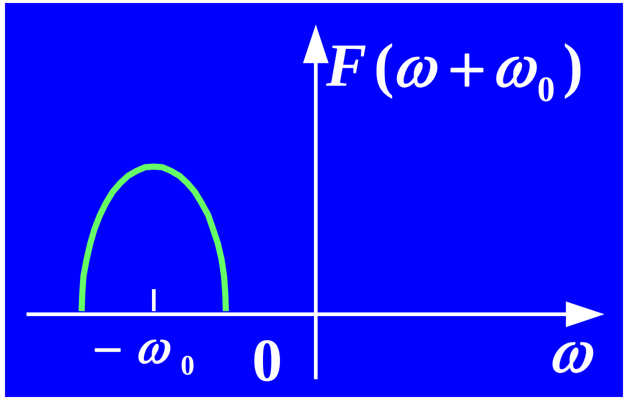
若 $f(t) \leftrightarrow F(\omega)$ 则 $\left.\begin{aligned} f(t)e^{j\omega_0 t} \leftrightarrow F(\omega-\omega_0) \\ f(t)e^{-j\omega_0 t} \leftrightarrow F(\omega+\omega_0) \end{aligned}\right\}$ ω_0为常数，注意±号

$$F\left[f(t)e^{j\omega_0 t}\right] = \int_{-\infty}^{\infty} \left[f(t)e^{j\omega_0 t}\right] e^{-j\omega t} \mathrm{d}t = \int_{-\infty}^{\infty} f(t)e^{-j(\omega-\omega_0)t} dt = F(\omega-\omega_0)$$





时域$f(t)$乘$e^{j\omega_0 t}$，
频域频谱搬移——右移ω_0



时域$f(t)$乘$e^{-j\omega_0 t}$
频域频谱搬移——左移ω_0

$$f(t) \leftrightarrow F(\omega)$$

时域微分性质：则$f'(t) \leftrightarrow j\omega F(\omega)$

$$f'(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega) j\omega e^{j\omega t} d\omega$$

频域微分性质：$-jtf(t) \leftrightarrow dF(\omega)/d\omega$

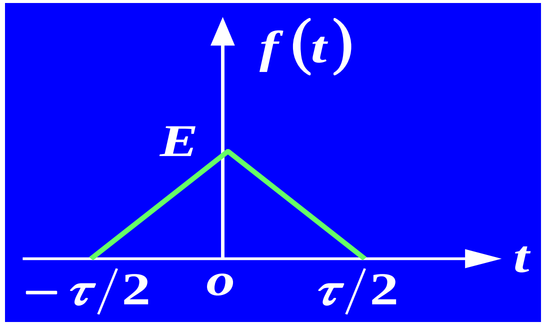
$tf(t) \leftrightarrow jdF(\omega)/d\omega$

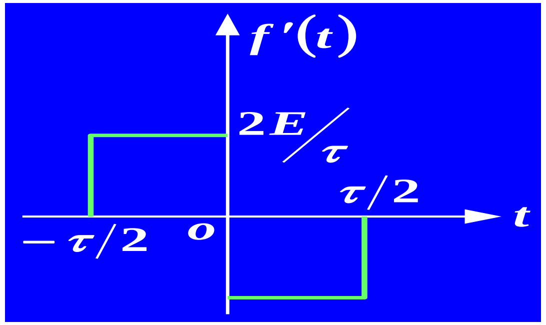
$$F'(\omega) = \int_{-\infty}^{\infty} -jte^{-j\omega t} dt$$

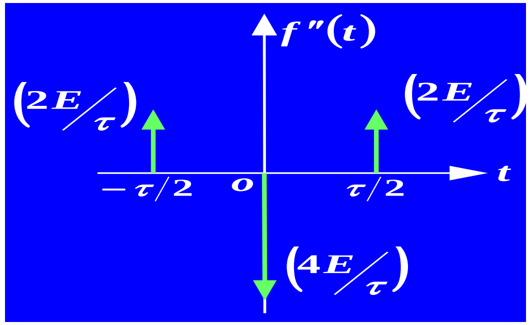
例16.2 已知$f(t) \leftrightarrow F(\omega)$，求$F[(t-2)f(t)] = ?$

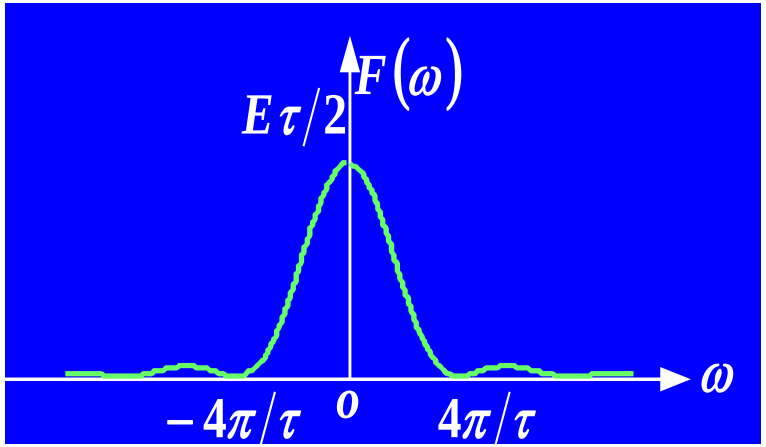
$$F[(t-2)f(t)] = F[tf(t) - 2f(t)] = j\frac{dF(\omega)}{d(\omega)} - 2F(\omega)$$

例16.2 求三角函数的频谱密度函数。









$$F[f''(t)] = \int_{-\infty}^{\infty}\left[\frac{2E}{\tau}\delta\left(t+\frac{\tau}{2}\right)-\frac{4E}{\tau}\delta(t)+\frac{2E}{\tau}\delta\left(t-\frac{\tau}{2}\right)\right]e^{-j\omega t}dt$$

$$=\frac{1}{-\omega^2}\frac{2E}{\tau}\left[e^{j\omega\tau/2}-2+e^{-j\omega\tau/2}\right] \quad =\frac{\tau E}{2}Sa\left(\frac{\omega\tau}{4}\right)^2$$

$$f(t) \leftrightarrow F(\omega)$$

$F(0)=0$时， $\int_{-\infty}^{t} f(\tau)d\tau \leftrightarrow \dfrac{F(\omega)}{j\omega}$

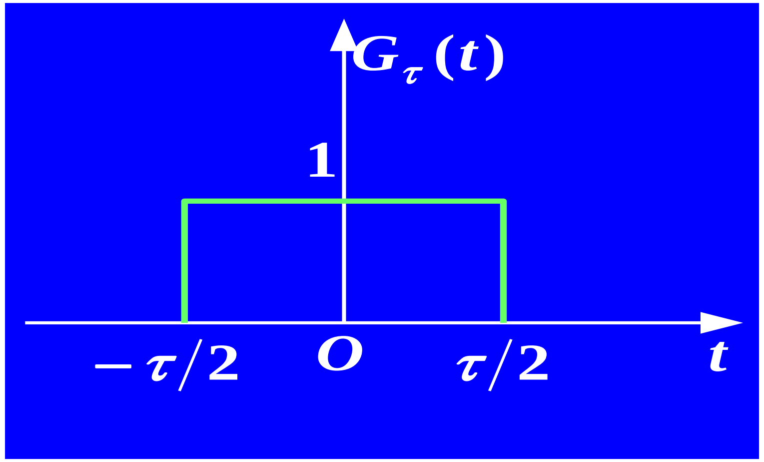
$F(0)\neq 0$时， $\int_{-\infty}^{t} f(\tau)d\tau \leftrightarrow \pi F(0)\delta(\omega)+\dfrac{F(\omega)}{j\omega}$

$$\int_{-\infty}^{t} f(\tau)d\tau \leftrightarrow F(\omega)\cdot\left[\frac{1}{j\omega}+\pi\delta(\omega)\right]$$

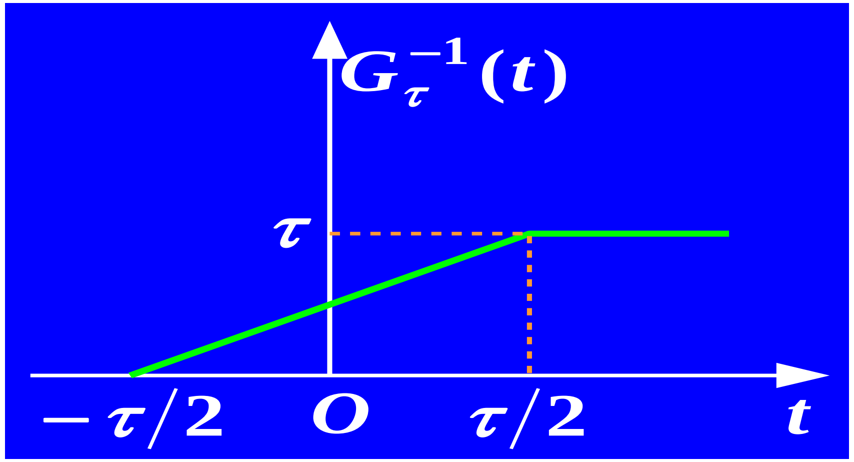
$$\int_{-\infty}^{\infty}\left[\int_{-\infty}^{t} f(\tau)d\tau\right]e^{-j\omega t}dt = \int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty} f(\tau)u(t-\tau)d\tau\right]e^{-j\omega t}dt = \int_{-\infty}^{\infty} f(\tau)\left[\int_{-\infty}^{\infty} u(t-\tau)e^{-j\omega t}dt\right]d\tau$$

$$= \int_{-\infty}^{\infty} f(\tau)\left(\pi\delta(\omega)+\frac{1}{j\omega}\right)e^{-j\omega\tau}d\tau = \left(\pi\delta(\omega)+\frac{1}{j\omega}\right)\int_{-\infty}^{\infty} f(\tau)e^{-j\omega\tau}d\tau = \pi F(0)\delta(\omega)+\frac{F(\omega)}{j\omega}$$

例1：已知 $u(t)=\int_{-\infty}^{t}\delta(t)dt$ $\quad\delta(t)\leftrightarrow 1$ $\quad u(t)\leftrightarrow\left[\frac{1}{j\omega}+\pi\delta(\omega)\right]\cdot 1=\frac{1}{j\omega}+\pi\delta(\omega)$



例2：

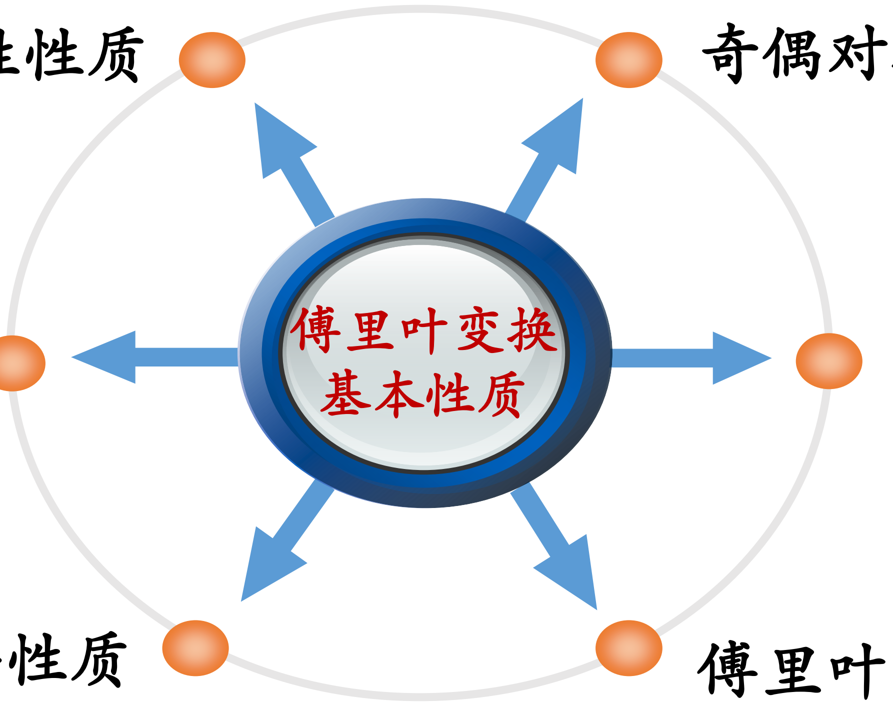


$$G_\tau(t)\leftrightarrow\tau Sa\left(\frac{\omega\tau}{2}\right)$$

由$Sa(0)=\tau$，知$F(0)\neq 0$

$$\therefore F\left[\int_{-\infty}^{t}G_\tau(\tau)d\tau\right]=\pi\tau\delta(\omega)+\frac{\tau}{j\omega}Sa\left(\frac{\omega\tau}{2}\right)$$

16.9 本讲小结



- 傅里叶变换的线性性质
- 奇偶对称性
- 傅里叶变换的尺度变换
- 傅里叶变换的对称性
- 傅里叶变换的时移和频移性质
- 傅里叶变换的微分和积分性质