8. $\int \frac{x^2}{\sqrt{4-x^2}}dx \xlongequal{x=2\sin t} \int 4\sin^2 t\,dt = 2\int(1-\cos 2t)dt = 2t - \sin 2t + C$ ② ① ①

$= 2\arcsin\frac{x}{2} - \frac{x\cdot\sqrt{4-x^2}}{2} + C$ ①

9. $\int_{-1}^{1} \frac{x}{\sqrt{5-4x}}dx \xlongequal[x=\frac{5-t^2}{4}]{\sqrt{5-4x}=t} \int_1^3 \frac{1}{t}\cdot\frac{5-t^2}{4}\cdot\frac{t}{2}dt = \int_1^3 \frac{5-t^2}{8}dt = \frac{1}{6}$ ② 结果错扣1分

② 限对1分　被积函数表达对1分　令方出来②分

10. $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n^2}\ln(1+\frac{i}{n}) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{i}{n}\ln(1+\frac{i}{n})\cdot\frac{1}{n} = \int_0^1 x\ln(1+x)dx = \int_0^1 \ln(1+x)d\frac{x^2}{2}$ ③ 结果错扣1分

$= \frac{x^2}{2}\ln(1+x)\Big|_0^1 - \frac{1}{2}\int_0^1\frac{x^2}{1+x}dx = \frac{1}{2}\ln 2 - \frac{1}{2}\int_0^1(x-1+\frac{1}{1+x})dx = \frac{1}{2}\ln 2 - (\frac{-1}{4}+\frac{1}{2}\ln 2)$ ①

11. $V = \pi\int_{-\infty}^{+\infty}\frac{1}{1+2x^2}dx = \sqrt{2}\pi\int_0^{+\infty}\frac{1}{1+2x^2}d(\sqrt{2}x) = \sqrt{2}\pi\arctan\sqrt{2}x\Big|_0^{+\infty} = \frac{\sqrt{2}}{2}\pi^2$ ② ① ①

② 或∵ $\pi\int_{-\infty}^{0}\frac{1}{1+2x^2}dx = \frac{1}{\sqrt{2}}\int_{-\infty}^{0}\frac{1}{1+(\sqrt{2}x)^2}d(\sqrt{2}x) = \frac{1}{\sqrt{2}}\arctan\sqrt{2}x\Big|_{-\infty}^{0} = \frac{\pi}{2\sqrt{2}}$.

$\pi\int y^2dx$　$\int_0^{+\infty}\frac{1}{1+2x^2}dx = \frac{1}{\sqrt{2}}\int_0^{+\infty}\frac{1}{1+(\sqrt{2}x)^2}d(\sqrt{2}x) = \frac{1}{\sqrt{2}}\arctan\sqrt{2}x\Big|_0^{+\infty} = \frac{\pi}{2\sqrt{2}}$

② $\therefore V = 2\pi\cdot\frac{\pi}{2\sqrt{2}} = \frac{\pi^2}{\sqrt{2}} = \frac{\sqrt{2}}{2}\pi^2$

12. (1) 平均成本 $\overline{C}(x) = \frac{C}{x} = \frac{25000}{x} + 200 + \frac{x}{40}$，② 令 $\overline{C}'(x) = -\frac{25000}{x^2} + \frac{1}{40} = 0 \Rightarrow$

$x_1 = 1000$，① $x_2 = -1000$(舍). 因为 $\overline{C}''(x)\Big|_{x=1000} = 5\times10^4\cdot\frac{1}{x^3}\Big|_{x=1000} = 5\times10^{-5} > 0$.

所以当 $x=1000$ 时，$\overline{C}(x)$ 最小。

(2) $L(x) = R(x) - C(x) = 500x - (25000 + 200x + \frac{x^2}{40}) = 300x - 25000 - \frac{x^2}{40}$，②

令 $L'(x) = 300 - \frac{x}{20} = 0$　得 $x = 6000$，① 因为 $L''(x)\Big|_{x=6000} = -\frac{1}{20} < 0$

所以 $x=6000$ 时，利润最大。

三. 1. 当 $0<x\le1$ 时，$f(x) = \int_0^x(x^2-t^2)dt + \int_x^1(t^2-x^2)dt = \frac{4}{3}x^3 - x^2 + \frac{1}{3}$ ①

当 $x>1$ 时，$f(x) = \int_0^1(x^2-t^2)dt = x^2 - \frac{1}{3}$. ①

即 $f(x) = \begin{cases}\frac{4}{3}x^3 - x^2 + \frac{1}{3}, & 0<x\le1 \\ x^2 - \frac{1}{3}, & x>1\end{cases}$，$f'(x) = \begin{cases}4x^2-2x, & 0<x\le1 \\ 2x, & x>1.\end{cases}$ ①

令 $f'(x)=0$ 可得当 $0<x\le1$ 时，$x=\frac{1}{2}$ 为驻点；$f''(\frac{1}{2}) = (8x-2)\Big|_{x=\frac{1}{2}} = 2>0$.

② 所以 $x=\frac{1}{2}$ 为极小值点，极小值为 $f(\frac{1}{2}) = \frac{1}{4}$. 而 $f(1) = \frac{2}{3}$ ② 故 $f(x)$ 的最小值为 $\frac{1}{4}$。

2. 令 $F(x) = (b-x)^a f(x)$，则 $F(a)=0$，$F(b)=0$. 于是 $F(x)$ 在 $[a,b]$ 上

满足罗尔定理，所以 ∃一点 $\xi\in(a,b)$ 使得 $F'(\xi)=0$，即 $f(\xi) = \frac{b-\xi}{a}f'(\xi)$.

① $af(\xi) + (\xi-b)f'(\xi) = 0$. ②

9. $\int_{-1}^{1}\frac{x}{\sqrt{5-4x}}dx = -\frac{1}{4}\int_{-1}^{1}\frac{-4x+5-5}{\sqrt{5-4x}}dx = -\frac{1}{4}\int_{-1}^{1}\sqrt{5-4x}\,dx + \frac{5}{4}\int_{-1}^{1}(5-4x)^{-\frac{1}{2}}dx$

$= \frac{1}{16}\int_{-1}^{1}(5-4x)^{\frac{1}{2}}d(5-4x) - \frac{5}{16}\int_{-1}^{1}(5-4x)^{-\frac{1}{2}}d(5-4x)$

$= \frac{1}{16}\times\frac{2}{3}\times(5-4x)^{\frac{3}{2}}\Big|_{-1}^{1} - \frac{5}{16}\times2\times(5-4x)^{\frac{1}{2}}\Big|_{-1}^{1} = \frac{1}{24}(1-27) - \frac{5}{8}(1-3) = \frac{1}{6}$.

9. $\int_{-1}^{1}\frac{x}{\sqrt{5-4x}}dx = -\frac{1}{2}\int_{-1}^{1}x\,d\sqrt{5-4x} = -\frac{1}{2}\left[x\sqrt{5-4x}\Big|_{-1}^{1} - \int_{-1}^{1}\sqrt{5-4x}\,dx\right] = \frac{1}{6}$.