Assignment Problems

2.2. A die is tossed twice and the number of dots facing up in each toss is counted and noted in the order of occurrence.

(a) Find the sample space.

(b) Find the set A corresponding to the event "number of dots in first toss is not less than number of dots in second toss."

(c) Find the set B corresponding to the event "number of dots in first toss is 6."

(d) Does A imply B or does B imply A?

(e) Find $A \cap B^c$ and describe this event in words.

(f) Let C correspond to the event "number of dots in dice differs by 2." Find $A \cap C$.

2.3. Two dice are tossed and the magnitude of the difference in the number of dots facing up in the two dice is noted.

(a) Find the sample space.

(b) Find the set A corresponding to the event "magnitude of difference is 3."

(c) Express each of the elementary events in this experiment as the union of elementary events from Problem 2.2.

2.8. A number U is selected at random from the unit interval. Let the events A and B be: A = "U differs from 1/2 by more than 1/4" and B = "$1 - U$ is less than 1/2." Find the events $A \cap B$, $A^c \cap B$, $A \cup B$.

2.23. A random experiment has sample space $S = \{a, b, c, d\}$. Suppose that $P[\{c, d\}] = 3/8$, $P[\{b, c\}] = 6/8$, and $P[\{d\}] = 1/8$, $P[\{c, d\}] = 3/8$. Use the axioms of probability to find the probabilities of the elementary events.

2.28. A hexadecimal character consists of a group of three bits. Let A_i be the event "ith bit in a character is a 1."

(a) Find the probabilities for the following events: $A_1, A_1 \cap A_3, A_1 \cap A_2 \cap A_3$ and $A_1 \cup A_2 \cup A_3$. Assume that the values of bits are determined by tosses of a fair coin.

(b) Repeat part a if the coin is biased.

2.29. Let M be the number of message transmissions in Problem 2.7. Find the probabilities of the events $A, B, C, C^c, A \cap B, A - B, A \cap B \cap C$. Assume the probability of successful transmission is 1/2.

2.30. Use Corollary 7 to prove the following:

(a) $P[A \cup B \cup C] \leq P[A] + P[B] + P[C]$.

(b) $P\left[\bigcup_{k=1}^{n} A_k\right] \leq \sum_{k=1}^{n} P[A_k]$.

2.62. A die is tossed twice and the number of dots facing up is counted and noted in the order of occurrence. Let A be the event "number of dots in first toss is not less than number of dots in second toss," and let B be the event "number of dots in first toss is 6." Find $P[A|B]$ and $P[B|A]$.

2.82. Let $S = \{1, 2, 3, 4\}$ and $A = \{1, 2\}$, $B = \{1, 3\}$, $C = \{1, 4\}$. Assume the outcomes are equiprobable. Are A, B, and C independent events?

2.87. Let A, B, and C be events with probabilities $P[A]$, $P[B]$, and $P[C]$.

(a) Find $P[A \cup B]$ if A and B are independent.

(b) Find $P[A \cup B]$ if A and B are mutually exclusive.

(c) Find $P[A \cup B \cup C]$ if A, B, and C are independent.

(d) Find $P[A \cup B \cup C]$ if A, B, and C are pairwise mutually exclusive.

Assignment Problems

3.8. An urn contains 9 \$1 bills and one \$50 bill. Let the random variable X be the total amount that results when two bills are drawn from the urn *with* replacement.

(a) Describe the underlying space S of this random experiment and specify the probabilities of its elementary events.

(b) Show the mapping from S to S_X, the range of X.

(c) Find the probabilities for the various values of X.

3.13. Let X be a random variable with pmf $p_k = c/k^2$ for $k = 1, 2, \ldots$.

(a) Estimate the value of c numerically. Note that the series converges.

(b) Find $P[X > 4]$.

(c) Find $P[6 \leq X \leq 8]$.

3.49. Let X be a binomial random variable that results from the performance of n Bernoulli trials with probability of success p.

(a) Suppose that $X = 1$. Find the probability that the single event occurred in the kth Bernoulli trial.

(b) Suppose that $X = 2$. Find the probability that the two events occurred in the jth and kth Bernoulli trials where $j < k$.

(c) In light of your answers to parts a and b in what sense are the successes distributed "completely at random" over the n Bernoulli trials?

3.53. Let N be a geometric random variable with $S_N = \{1, 2, \ldots\}$.

(a) Find $P[N = k | N \leq m]$.

(b) Find the probability that N is odd.

Assignment Problems

4.6. A dart is equally likely to land at any point inside a circular target of radius 2. Let R be the distance of the landing point from the origin.

(a) Find the sample space S and the sample space of R, S_R.

(b) Show the mapping from S to S_R.

(c) The "bull's eye" is the central disk in the target of radius 0.25. Find the event A in S_R corresponding to "dart hits the bull's eye." Find the equivalent event in S and $P[A]$.

(d) Find and plot the cdf of R.

4.11. The random variable X is uniformly distributed in the interval $[-1, 2]$.

(a) Find and plot the cdf of X.

(b) Use the cdf to find the probabilities of the following events: $\{X \le 0\}$, $\{|X - 0.5| < 1\}$, and $C = \{X > -0.5\}$.

4.13. A random variable X has cdf:

$$F_X(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 - \frac{1}{4}e^{-2x} & \text{for } x \ge 0. \end{cases}$$

(a) Plot the cdf and identify the type of random variable.

(b) Find $P[X \le 2]$, $P[X = 0]$, $P[X < 0]$, $P[2 < X < 6]$, $P[X > 10]$.

4.17. A random variable X has pdf:

$$f_X(x) = \begin{cases} c(1 - x^2) & -1 \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find c and plot the pdf.

(b) Plot the cdf of X.

(c) Find $P[X = 0]$, $P[0 < X < 0.5]$, and $P[|X - 0.5| < 0.25]$.

4.18. A random variable X has pdf:

$$f_X(x) = \begin{cases} cx(1 - x^2) & 0 \le x \le 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Find c and plot the pdf.

(b) Plot the cdf of X.

(c) Find $P[0 < X < 0.5]$, $P[X = 1]$, $P[.25 < X < 0.5]$.

Assignment Problems

5.8. For the pair of random variables (X, Y) sketch the region of the plane corresponding to the following events. Identify which events are of product form.

(a) $\{X + Y > 3\}$.

(b) $\{e^X > Ye^3\}$.

(c) $\{\min(X, Y) > 0\} \cup \{\max\{X, Y) < 0\}$.

(d) $\{|X - Y| \geq 1\}$.

5.12. A modem transmits a two-dimensional signal (X, Y) given by:

$$X = r\cos(2\pi\Theta/8) \quad \text{and} \quad Y = r\sin(2\pi\Theta/8)$$

where Θ is a discrete uniform random variable in the set $\{0, 1, 2, \ldots, 7\}$.

(a) Show the mapping from S to S_{XY}, the range of the pair (X, Y).

(b) Find the joint pmf of X and Y.

5.17. A point (X, Y) is selected at random inside a triangle defined by $\{(x, y) : 0 \leq y \leq x \leq 1\}$. Assume the point is equally likely to fall anywhere in the triangle.

(a) Find the joint cdf of X and Y.

(b) Find the marginal cdf of X and of Y.

(c) Find the probabilities of the following events in terms of the joint cdf: $A = \{X \leq 1/2, Y \leq 3/4\}$; $B = \{1/4 < X \leq 3/4, 1/4 < Y \leq 3/4\}$.

5.18. A dart is equally likely to land at any point (X_1, X_2) inside a circular target of unit radius. Let R and Θ be the radius and angle of the point (X_1, X_2).

(a) Find the joint cdf of R and Θ.

(b) Find the marginal cdf of R and Θ.

5.26. Let X and Y have joint pdf:

$$f_{X,Y}(x, y) = k(x + y) \qquad \text{for } 0 \leq x \leq 1, 0 \leq y \leq 1.$$

(a) Find k.

(b) Find the joint cdf of (X, Y).

(c) Find the marginal pdf of X and of Y.