Challenging Exercises for Tutorial

Lecture II. Reflection and Refraction

Exercise 1. Fermat's Principle of Least Time.

Fermat's principle of least time states that among all possible paths between two points, the one actually taken by a ray of light is that for which the time of travel is a minimum.

Based on this principle, we can prove **the law of reflection**. As shown in figure.1, a ray of light traveling with speed c leaves point 1 and is reflected to point 2. The ray strikes the reflecting surface a horizontal distance x from point 1. The time required for the light to travel from 1 to 2 is

$$t = \frac{\sqrt{y_1^2 + x^2} + \sqrt{y_2^2 + (l - x)^2}}{c}$$

It is easy to prove that, when $\theta_1 = \theta_2$, this time reaches its minimum value.

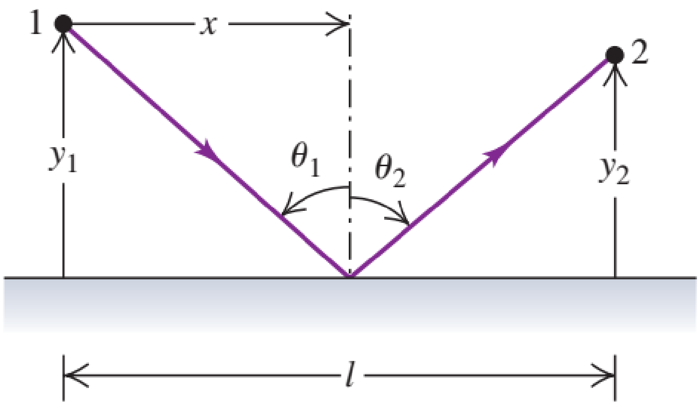

figure.1

Based on aforementioned knowledge, please give a proof to **the law of refraction** (Snell's law), as shown in the figure,, when $n_a sin\theta_a = n_b sin\theta_b$ corresponds to the actual path taken by the light.

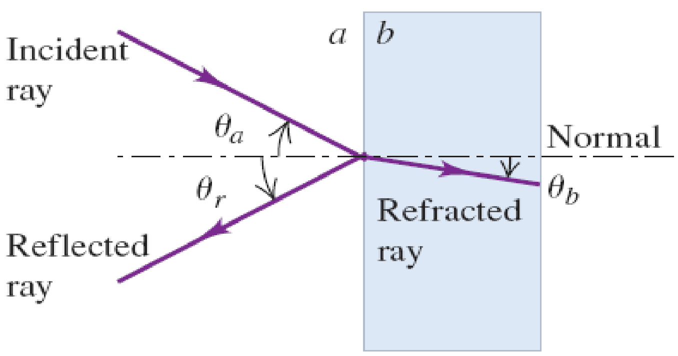

figure.2

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Exercise 2. Angle of Deviation.

The angle of deviation δ is defined as the difference between the directions of incoming ray and the outgoing ray. Please prove that when the incident angle θ_a is chosen to so that the light passes symmetrically through the prism, which has refractive index n and apex angle A, the angle of deviation is a minimum, and the refractive index of the prism can be calculated by

$$n = \frac{sin\frac{A+\delta}{2}}{sin\frac{A}{2}}$$

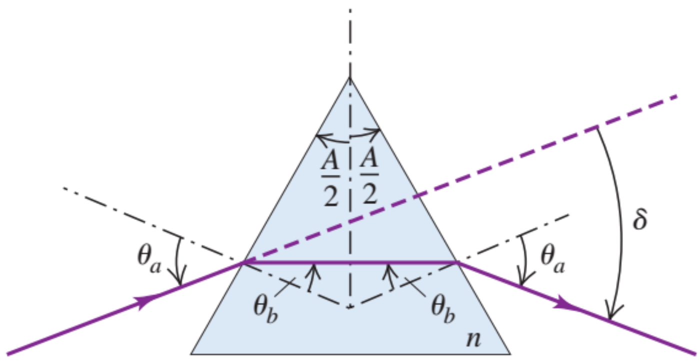

figure.1

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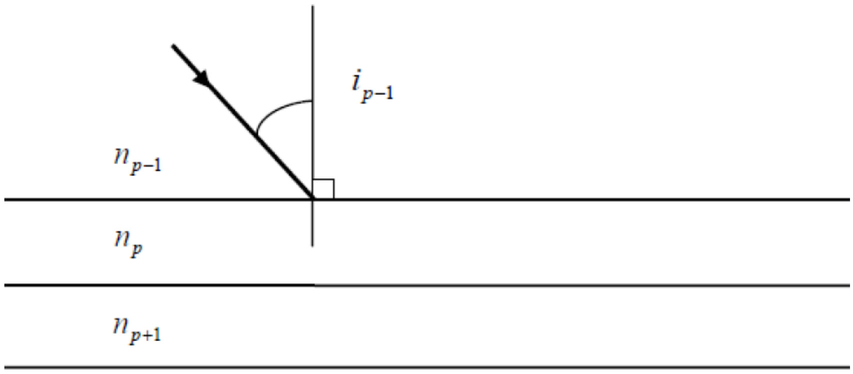
Exercise 3. Mirage

A light beam enter a transparent medium which is formed by layers (index n_p, $p \in$ □). The layers are separated by plane frontiers. $\forall p \in \mathbb{N}$ $n_{P+1} < n_p$



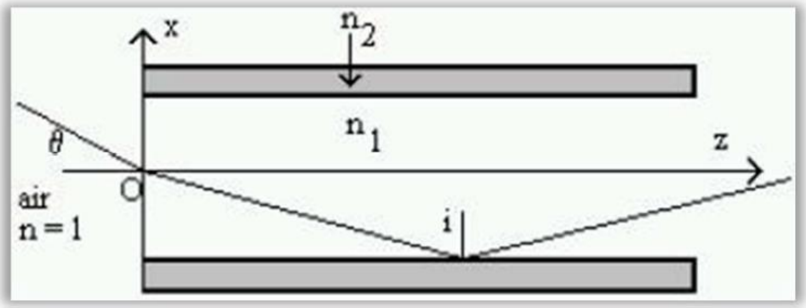
1. Give the relation between n_p, n_{p+1}, i_p and i_{p+1}.
2. Sketch the path followed by the beam through the layers by analyzing the previous relation.
3. Explain the mirage phenomenon.
4. Study the case $\forall p \in \mathbb{N}$ $n_{P+1} > n_p$

Challenging Exercises for Tutorial

Lecture III. Total Internal Reflection

Exercise 1. Optical fiber and Telecommunication

We consider an optical fiber along the *Oz* axis. The core is made of a transparent medium (refractive index n_1) and it surrounded by another medium (refractive index $n_2 < n_1$). $n_1 = 1.50$; $n_2/n_1 = 0.99$



1. Prove that the light can propagate into the fiber only if the i angle is larger than a limit i_0 and calculate i_0 as a function of n_1 and n_2.
2. The input surface of the fiber is a plane surface which is perpendicular to the *Oz* axis. The light beam enter the fiber with a θ angle. Give the expression of θ_0 corresponding to i_0 as a function of n_1 and n (refractive index of the air).

Depending on the θ angle, the length that the light has to follow is variable. So do the time which is necessary to reach the end of the fiber. Thus a short light pulse which enter the fiber will be enlarged at the end of the fiber. This phenomenon limits the amount of information which can be transferred by the fiber.

3. Give the expression of the time delay between two light beams which propagate in the fiber (length *L*), the first one along the *Oz* axis and the other entering the fiber with angle.
4. How much information can be transferred by time unit using this fiber for *L* = 1 m, 100 m or 10 km?