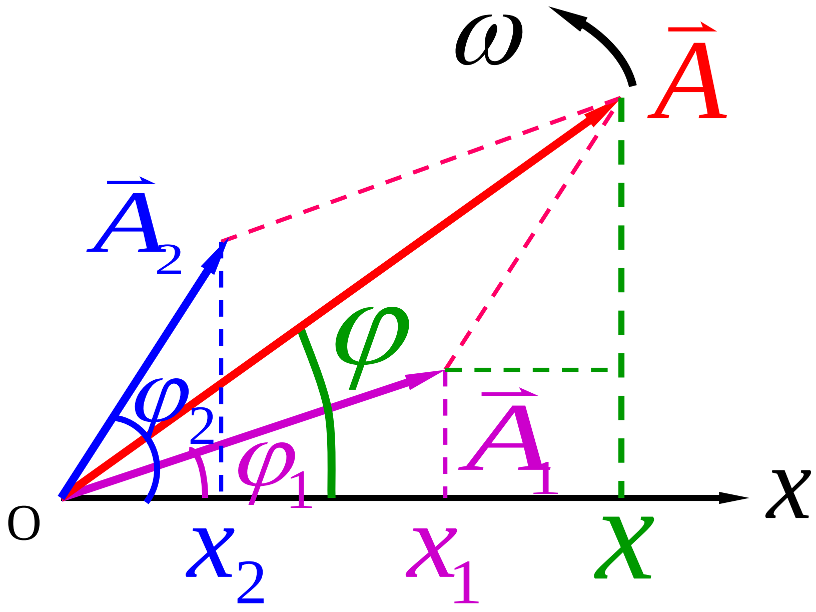
一、两个同方向同频率简谐运动的合成

$$\begin{cases} x_1 = A_1\cos(\omega t+\varphi_1) \\ x_2 = A_2\cos(\omega t+\varphi_2) \end{cases}$$

$$x = x_1 + x_2$$

$$\boxed{x = A\cos(\omega t+\varphi)}$$



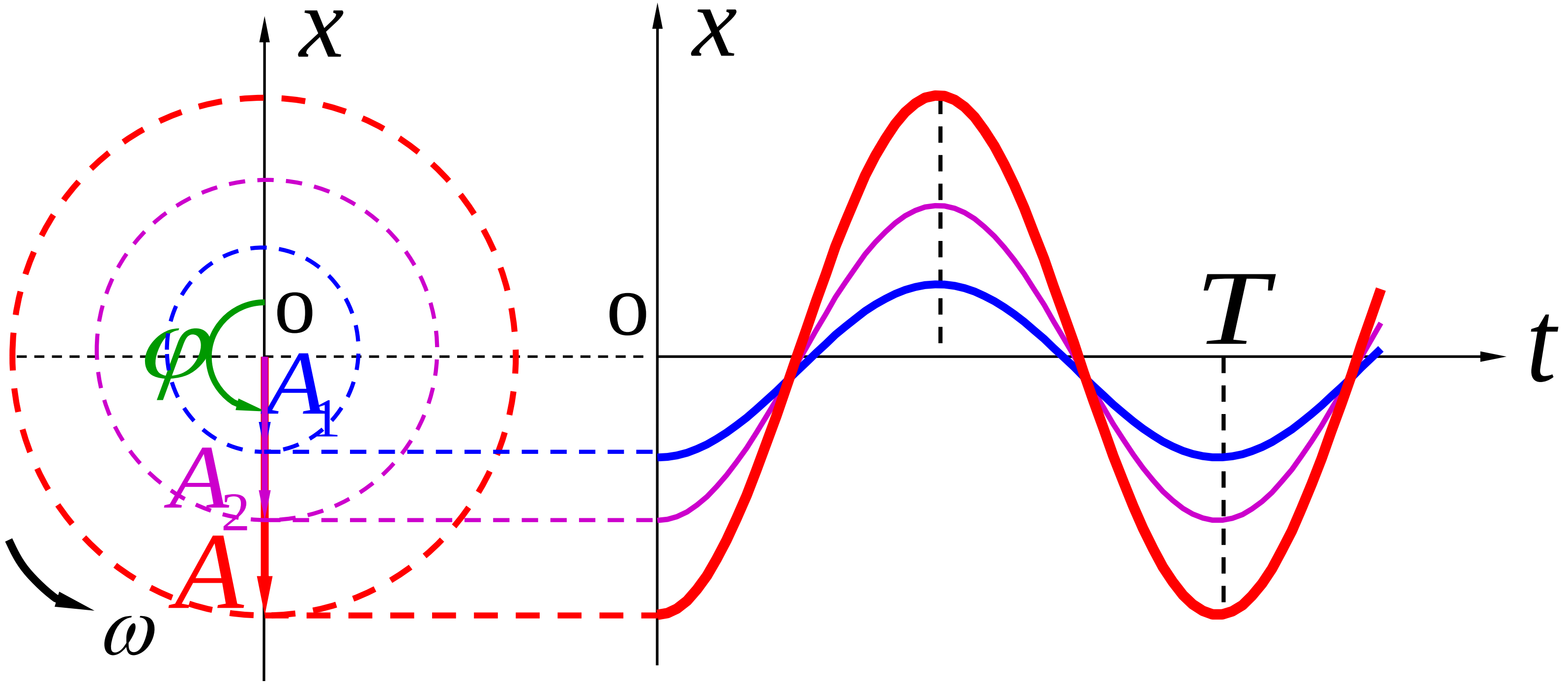
$$\begin{cases} A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2-\varphi_1)} \\ \tan\varphi = \dfrac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2} \end{cases}$$

两个同方向同频率简谐运动合成后仍为简谐运动。

讨论

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

（1）相位差 $\Delta\varphi=\varphi_2-\varphi_1=2k\pi \quad (k=0,\pm1,\pm2,\cdots)$



$$\begin{cases} A=A_1+A_2 \\ \varphi=\varphi_2=\varphi_1+2k\pi \end{cases}$$

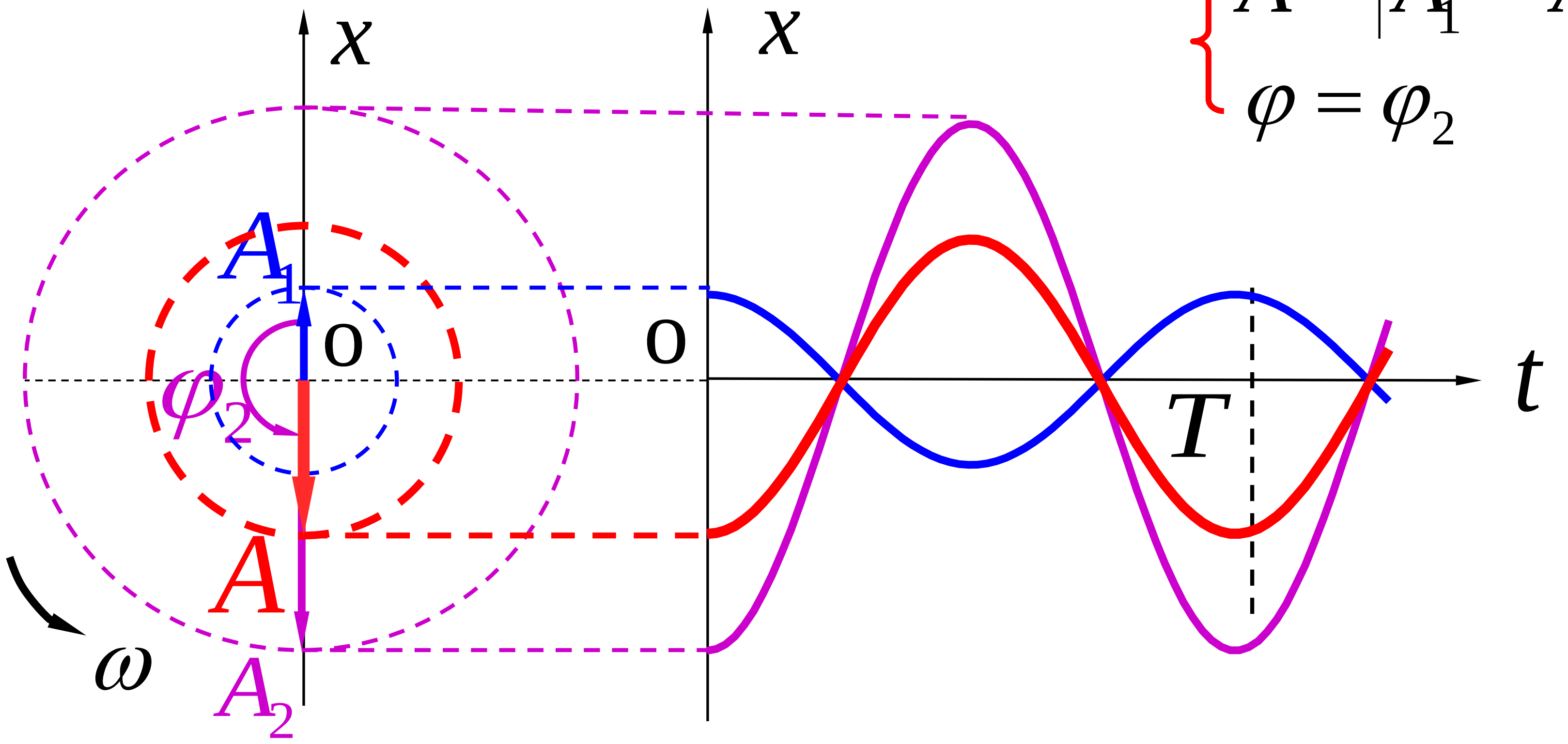
$$\boxed{x=(A_1+A_2)\cos(\omega t+\varphi)}$$

（2）相位差 $\Delta\varphi=\varphi_2-\varphi_1=(2k+1)\pi \quad (k=0,\pm1,\cdots)$

$$\begin{cases} x_1=A_1\cos\omega t \\ x_2=A_2\cos(\omega t+\pi) \end{cases}$$

$$\boxed{x=(A_2-A_1)\cos(\omega t+\pi)}$$

$$\begin{cases} A=\left|A_1-A_2\right| \\ \varphi=\varphi_2 \end{cases}$$



总结

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

（1）相位差 $\varphi_2-\varphi_1=2k\pi \qquad (k=0,\pm1,\cdots)$

$$A=A_1+A_2$$

相互加强

（2）相位差 $\varphi_2-\varphi_1=(2k+1)\pi \quad (k=0,\pm1,\cdots)$

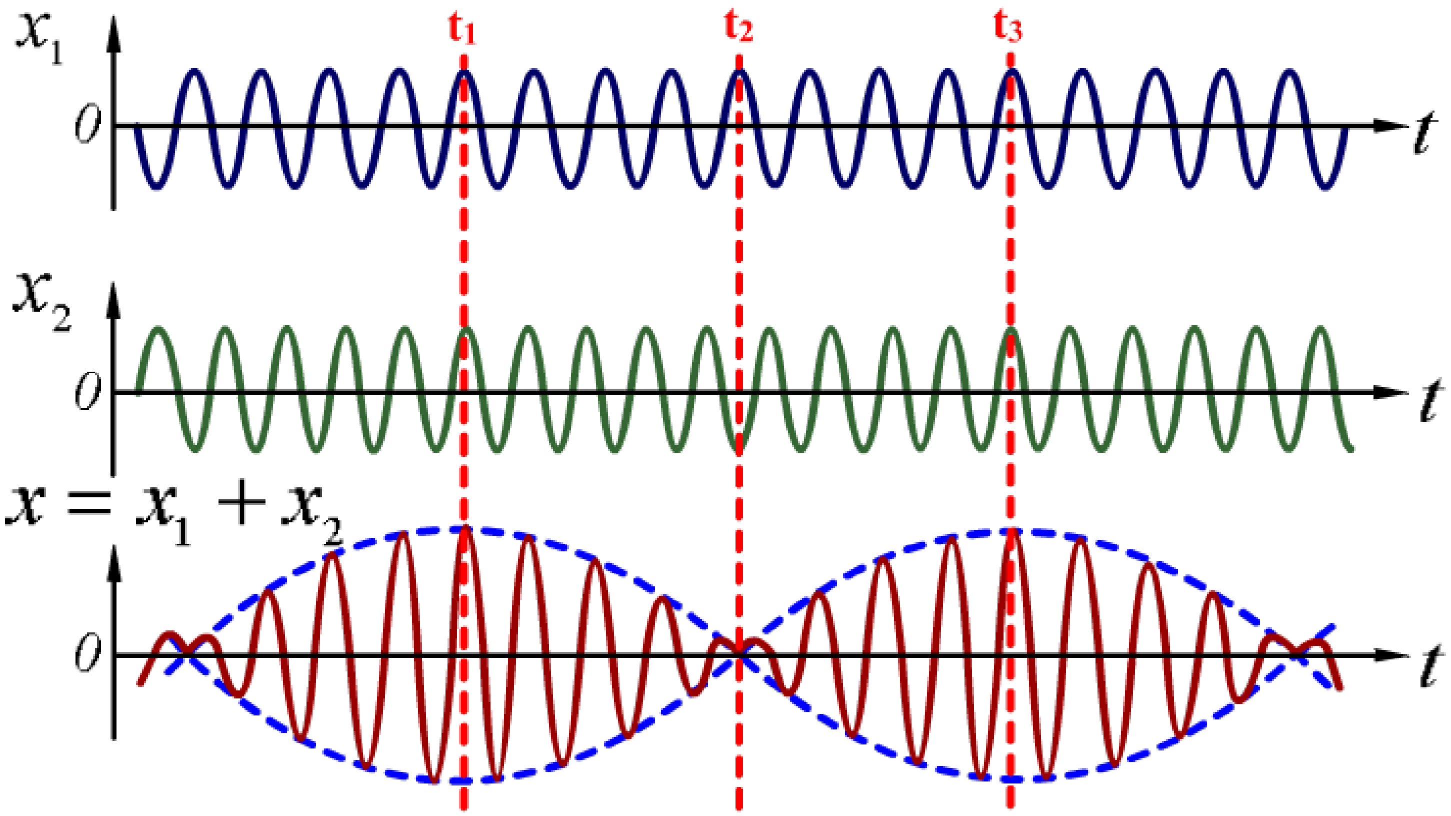
$$A=\left|A_1-A_2\right|$$

相互削弱

（3）一般情况

$$A_1+A_2>A>\left|A_1-A_2\right|$$

二、两个同方向不同频率简谐运动的合成



频率较大而频率之差很小的两个同方向简谐运动的合成，其合振动的振幅时而加强时而减弱的现象叫拍。

$$\begin{cases} x_1 = A_1\cos\omega_1 t = A_1\cos 2\pi\,\nu_1 t \\ x_2 = A_2\cos\omega_2 t = A_2\cos 2\pi\,\nu_2 t \end{cases}$$

$$\boxed{x = x_1 + x_2}$$

讨论 $A_1 = A_2$， $|\nu_2 - \nu_1| << \nu_1 + \nu_2$ 的情况

$$x = x_1 + x_2 = A_1\cos 2\pi\,\nu_1 t + A_2\cos 2\pi\,\nu_2 t$$

$$x = \boxed{(2A_1\cos 2\pi\frac{\nu_2 - \nu_1}{2}t)}\cos 2\pi\boxed{\frac{\nu_2 + \nu_1}{2}}t$$

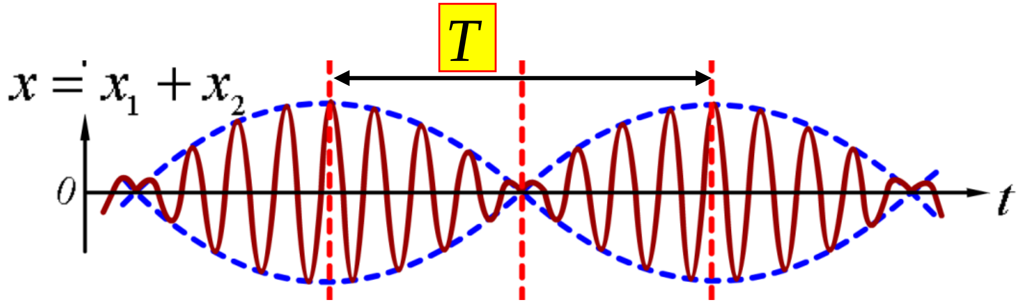
振幅部分

合振动频率

$$x = \left(2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t\right) \cos 2\pi \frac{\nu_2 + \nu_1}{2} t$$

振幅部分

合振动频率



振动频率 $\nu = (\nu_1 + \nu_2)/2$

振幅 $A = \left| 2A_1 \cos 2\pi \frac{\nu_2 - \nu_1}{2} t \right|$

$$\begin{cases} A_{\max} = 2A_1 \\ A_{\min} = 0 \end{cases}$$

$$2\pi \frac{\nu_2 - \nu_1}{2} T = \pi \qquad T = \frac{1}{\nu_2 - \nu_1}$$

$$\boxed{\nu = \nu_2 - \nu_1}$$

拍频（振幅变化的频率）

三、两个相互垂直的同频率简谐运动的合成

$$\begin{cases} x = A_1\cos(\omega t+\varphi_1) \\ y = A_2\cos(\omega t+\varphi_2) \end{cases}$$

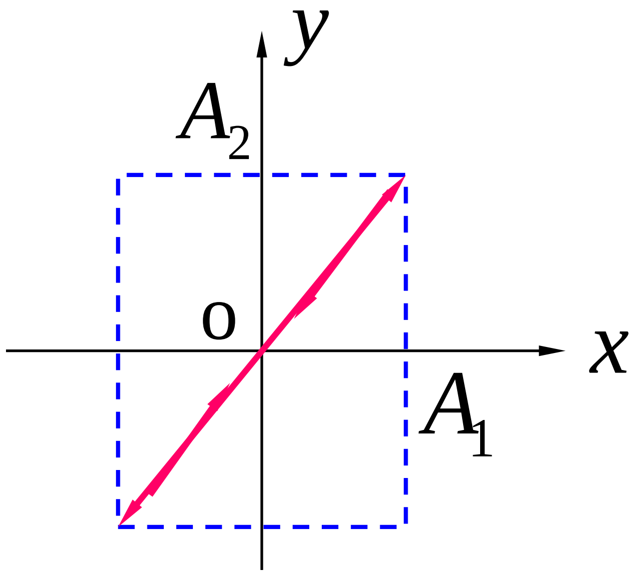
质点运动轨迹方程为：

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$（椭圆方程）

讨论

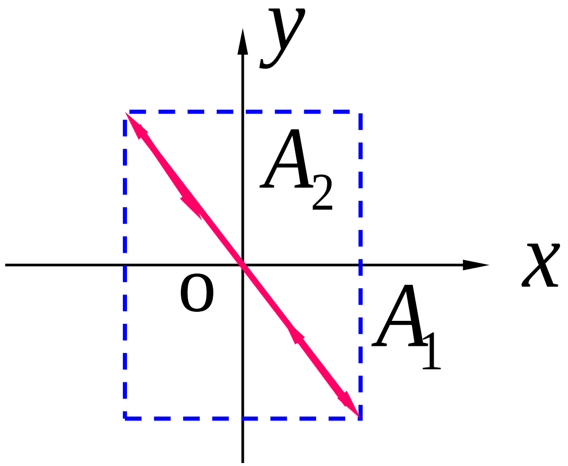
（1）$\varphi_2-\varphi_1=0$ 或 2π

$$y=\frac{A_2}{A_1}x$$



$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

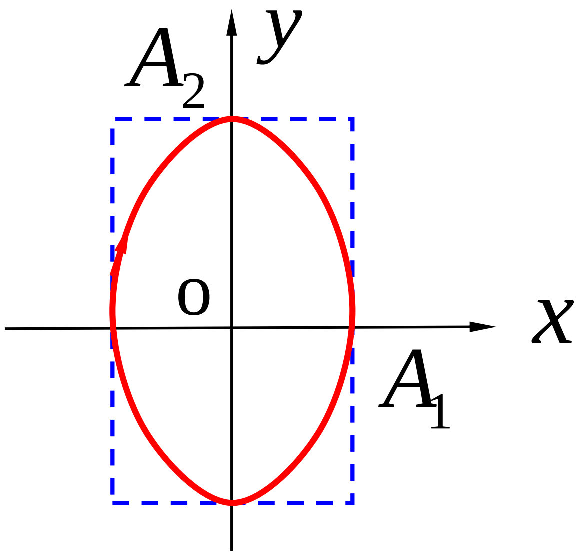
（2） $\varphi_2-\varphi_1=\pi$　　$y=-\dfrac{A_2}{A_1}x$



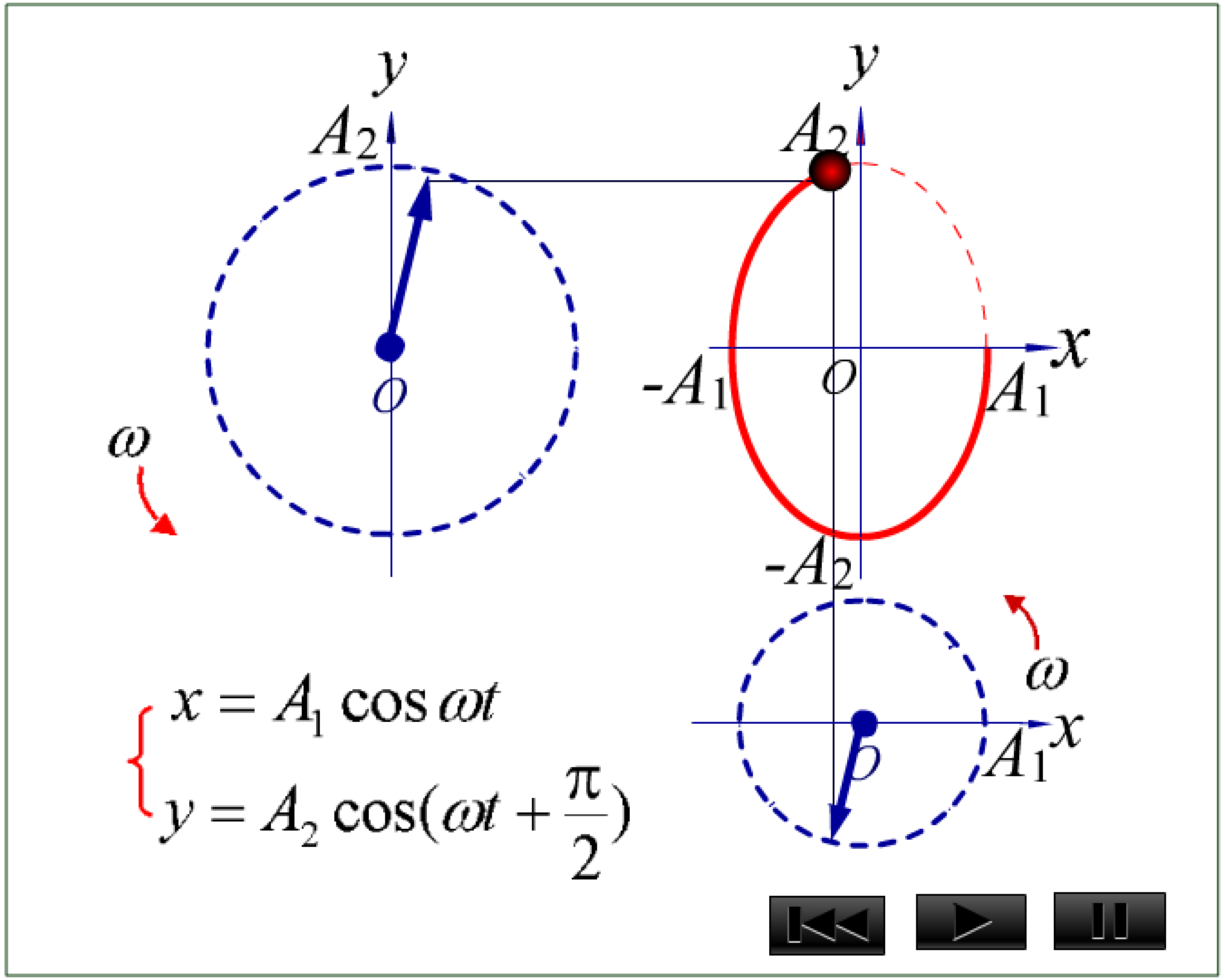
（3）$\varphi_2-\varphi_1=\pm\pi/2$

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}=1$$

$$\begin{cases} x=A_1\cos\omega t \\ y=A_2\cos(\omega t+\dfrac{\pi}{2}) \end{cases}$$

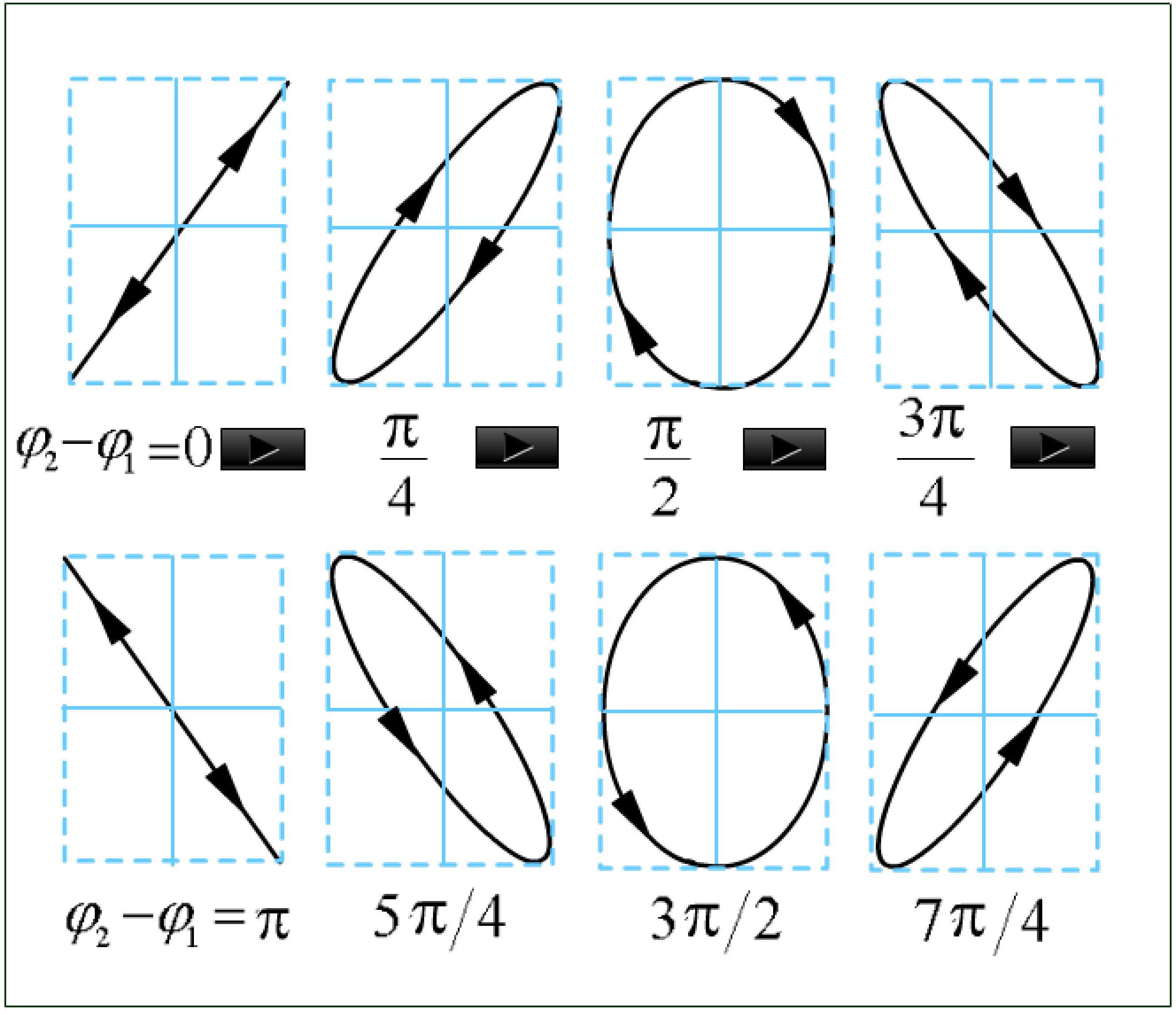


用旋转矢量描绘振动合成图



$$
\begin{cases}
x = A_1 \cos \omega t \\
y = A_2 \cos(\omega t + \dfrac{\pi}{2})
\end{cases}
$$

两相互垂直同频率不同相位差简谐运动的合成图



四、两相互垂直不同频率的简谐运动的合成

$$\begin{cases} x = A_1\cos(\omega_1 t + \varphi_1) \\ y = A_2\cos(\omega_2 t + \varphi_2) \end{cases}$$

$$\varphi_1 = 0$$

$$\varphi_2 = 0, \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$$

$$\frac{\omega_1}{\omega_2} = \frac{m}{n}$$

测量振动频率和相位的方法

李 萨 如 图