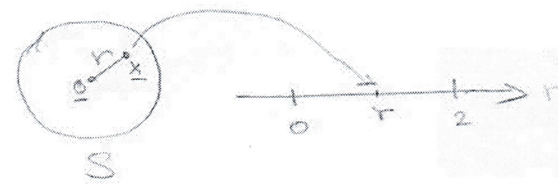
Assign. 4 Chap. 4 Solutions 6, 11, 13, 17, 18, 52

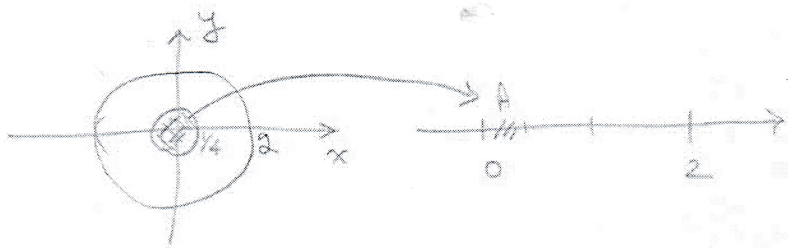
Assign. #4

4.6 $S = \{(x,y): x^2+y^2 \le 4\}$ $S_R = \{r: 0 \le r \le 2\}$

$R = \sqrt{x^2+y^2}$

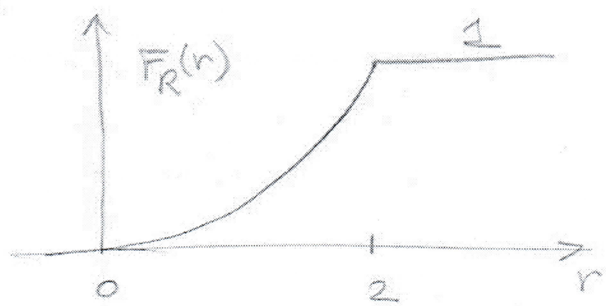


$P[A] = P[R \le \frac{1}{4}] = \frac{\pi(\frac{1}{4})^2}{\pi(2)^2} = \frac{1}{64}$

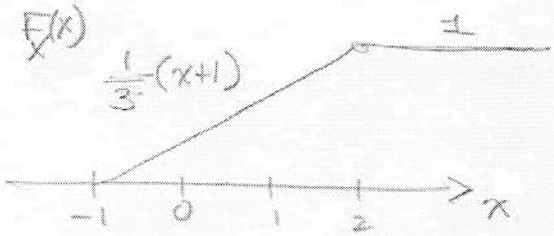


for $0 \le r \le 2$

$F_R(r) = P[R \le r] = \frac{\pi r^2}{\pi 2^2} = \left(\frac{r}{2}\right)^2$



4.11



$P[X<0] = F_X(0) = \frac{1}{3}$

$P[|X-\frac{1}{2}|<1] = P[-1<X-\frac{1}{2}<1] = P[-\frac{1}{2}<X<\frac{3}{2}]$

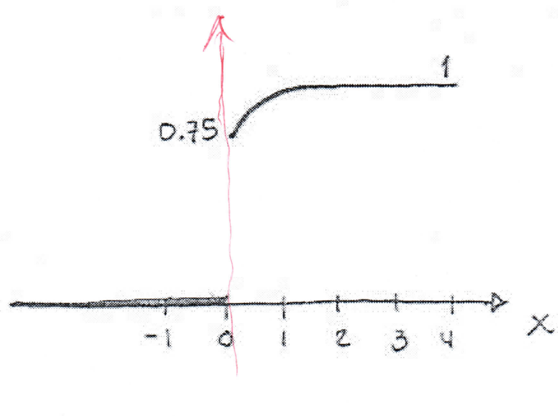
$= \frac{1}{3}(\frac{3}{2}+1) - \frac{1}{3}(-\frac{1}{2}+1)$

$= \frac{1}{3}(\frac{3}{2}+1+\frac{1}{2}-1) = \frac{2}{3}$

$P[X>-\frac{1}{2}] = 1-P[X\le-\frac{1}{2}] = 1-\frac{1}{3}(-\frac{1}{2}+1) = \frac{5}{6}$

4.13

a)



Mixed type random variable

b) $P[X \leq 2] = 1 - \frac{1}{4} e^{-2(2)}$

$= 0.9954$

$P[X = 0] = 1 - \frac{1}{4} e^{-2(0)}$

$= 0.75$

$P[X < 0] = 0$

$P[2 < X < 6] = P[X \leq 6] - P[X \leq 2]$

$= 1 - \frac{1}{4} e^{-2(6)} - 1 + \frac{1}{4} e^{-2(2)}$

$= 0.0046$

$P[X > 10] = 1 - P[X \leq 10]$

$= 1 - \left(1 - \frac{1}{4} e^{-2(10)}\right)$

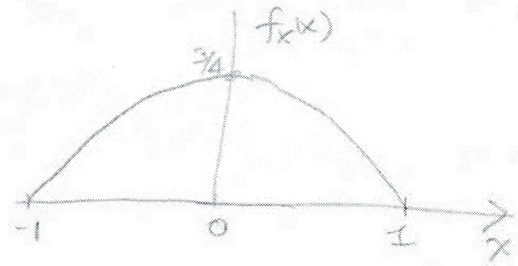
$= 5.15 \times 10^{-10}$

4.17

$$1 = c\int_{-1}^{1}(1-x^2)\,dx = c\left[x\Big|_{-1}^{1} - \frac{x^3}{3}\Big|_{-1}^{1}\right] = c\left[2-\frac{1}{3}2\right] = \frac{4}{3}c$$

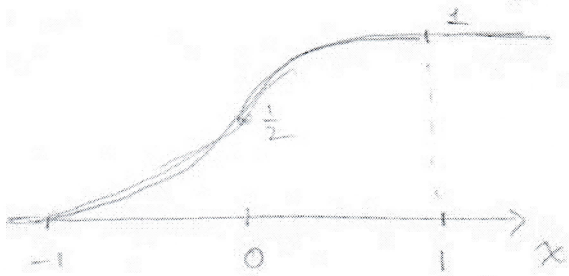
$$\Rightarrow c = \frac{3}{4}$$

$$f_X(x) = \frac{3}{4}(1-x^2) \quad -1 \le x \le 1$$



$$F_X(x) = \frac{3}{4}\int_{-1}^{x}(1-y^2)\,dy = \frac{3}{4}\left[y\Big|_{-1}^{x} - \frac{y^3}{3}\Big|_{-1}^{x}\right]$$

$$= \frac{3}{4}\left[(x+1) - \frac{1}{3}(x^3+1)\right]$$



$$P[X=0] = F_X(0) = 0$$

$$P[0<X<0.5] =$$

$$= \frac{3}{4}\left[\left(\frac{1}{2}+1\right) - \frac{1}{3}\left(\frac{1}{8}+1\right)\right]$$

$$- \frac{3}{4}\left[1-\frac{1}{3}\right]$$

$$= \frac{11}{32}$$

$$P\left[\left|X-\frac{1}{2}\right| < \frac{1}{4}\right] = P\left[\frac{1}{4} < X < \frac{3}{4}\right]$$

$$= \frac{3}{4}\left[\left(\frac{3}{4}+1\right) - \frac{1}{3}\left(\left(\frac{3}{4}\right)^3+1\right)\right] - \frac{3}{4}\left[\left(\frac{1}{4}+1\right) - \frac{1}{3}\left(\left(\frac{1}{4}\right)^3+1\right)\right]$$

$$= 0.2734$$

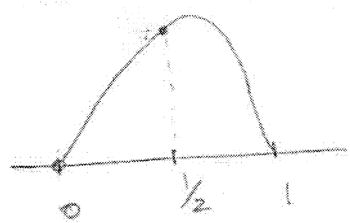
$$\begin{cases} x - \frac{1}{2} < \frac{1}{4} \rightarrow x < \frac{3}{4} \\ x - \frac{1}{2} > -\frac{1}{4} \rightarrow x > \frac{1}{4} \end{cases}$$

4.18

$$1 = c\int_0^1 x(1-x^2)\,dx = c\left[\frac{x^2}{2}\Big|_0^1 - \frac{x^4}{4}\Big|_0^1\right] = c\underbrace{\left[\frac{1}{2}-\frac{1}{4}\right]}_{\frac{1}{4}} =$$

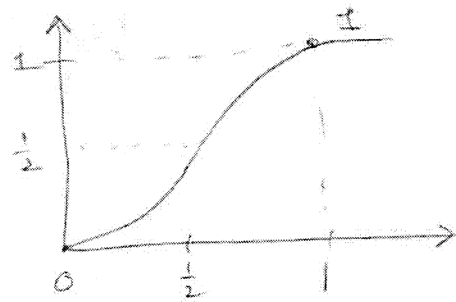
$$\Rightarrow c = 4$$

$$f_X(x) = 4x(1-x^2) \qquad 0 \le x \le 1$$



$$F_X(x) = 4\int_0^x (y - y^3)\,dy$$

$$= 4\left[\frac{x^2}{2} - \frac{x^4}{4}\right]$$



$$P\left[0 < X < \tfrac{1}{2}\right] = F_X\left(\tfrac{1}{2}\right)$$

$$= 7/16 = .4375$$

$$P[X = 1] = 0$$

$$P\left[\tfrac{1}{4} < X < \tfrac{1}{2}\right] = F_X\left(\tfrac{1}{2}\right) - F_X\left(\tfrac{1}{4}\right)$$

$$= \frac{7}{16} - 4\left[\frac{1}{32} - \frac{1}{1024}\right] = .3164$$

4.52
3.73

$$\mathcal{E}[c] = \int_{-\infty}^{\infty} c f_X(x)\,dx = c\int_{-\infty}^{\infty} f_X(x)\,dx = c$$

$$\mathcal{E}[c^2] = \int_{-\infty}^{\infty} c^2 f_X(x)\,dx = c^2$$

$$VAR[c] = \mathcal{E}[c^2] - \mathcal{E}[c]^2 = c^2 - c^2 = 0 \qquad 4.36\ (\cancel{3.68})$$

$$\begin{aligned} VAR[X+c] &= \mathcal{E}[((X+c) - \mathcal{E}[X+c])^2] \\ &= \mathcal{E}[(X + C - \mathcal{E}(X) - C)^2] \\ &= \mathcal{E}[(X - \mathcal{E}(X))^2] = VAR[X] \qquad 4.37\ (\cancel{3.69}) \end{aligned}$$

$$\begin{aligned} VAR[cX] &= \mathcal{E}[(cX - \mathcal{E}[cX])^2] \\ &= \mathcal{E}[c^2(X - \mathcal{E}[X])^2] \\ &= c^2 VAR[X] \qquad 4.38\ (\cancel{3.70}) \end{aligned}$$