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2011-9-4

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谢勇，王国建，唐杰，高路，陈一鸣，郑智聪，尹旭波
王金胜，黄日强，梁卓，林伟池，陈秋茹，钟巧思，刘正旺

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A000001

COMMENTS

Also, number of nonisomorphic subgroups of order n in symmetric group S_n. - Lekraj Beedassy (blekraj(AT)yahoo.com), Dec 16 2004

Also, number of nonisomorphic primitives of the combinatorial species Lin[n] - Nicolae Boicu (nicolae.boicu(AT)gmail.com), April 29 2011

FORMULA

Formulae from Mitch Harris (Harris.Mitchell(AT)mgh.harvard.edu), Oct 25 2006

(Start) For p, q, r primes:

a(p) = 1, a(p^2) = 2, a(p^3) = 5, a(p^4) = 14, if p = 2, otherwise 15.

a(p^5) = 61 + 2p + 2gcd(p-1,3) + gcd(p-1,4), p>=5, a(2^5)=51, a(3^5)=67.

a(p^e) ~ p^((2/27)e^3 + O(e^(8/3)))

a(pq) = 1 if gcd(p,q-1) = 1, 2 if gcd(p,q-1) = p. (p < q)

a(pq^2) = one of the following:

* 5, p=2, q odd,
* (p+9)/2, q=1 mod p, p odd,
* 5, p=3, q=2,
* 3, q = -1 mod p, p and q odd.
* 4, p=1 mod q, p > 3, p != 1 mod q^2
* 5, p=1 mod q^2
* 2, q != +/-1 mod p and p != 1 mod q,

a(pqr) (p < q < r) = one of the following:

* q==1 mod p r==1 mod p r==1 mod q a(pqr)
* No..........No..........No..........1
* No..........No..........Yes.........2
* No..........Yes.........No..........2
* No..........Yes.........Yes.........4
* Yes.........No..........No..........2
* Yes.........No..........Yes.........3
* Yes.........Yes.........No..........p+2
* Yes.........Yes.........Yes.........p+4 (table from Derek Holt) (End)

A000002

COMMENTS

It is an unsolved problem to show that the density of 1's is equal to 1/2.

The sequence is cube-free and all square subwords have lengths which are one of 2, 4, 6, 18 and 54.

This is a fractal sequence: replace each run by its length and recover the original sequence. - Kerry Mitchell (lkmitch(AT)gmail.com), Dec 08 2005

Kupin and Rowland write: We use a method of Goulden and Jackson to bound freq_1(K), the limiting frequency of 1 in the Kolakoski word K. We prove that |freq_1(K) - 1/2| <= 17/762, assuming the limit exists and establish the semi-rigorous bound |freq_1(K) - 1/2| <= 1/46. [From Jonathan Vos Post (jvospost3(AT)gmail.com), Sep 16 2008]

FORMULA

Omit the initial 1 (so this remark is really about A078880). Then the sequence can be generated by starting with 22 and applying the block-substitution rules 22 -> 2211, 21 -> 221, 12 -> 211, 11 -> 21 (Lagarias)

These two formulae define completely the sequence: a(1)=1, a(2)=2, a(a(1)+a(2)+...+a(k))=(3+(-1)^k)/2 and a(a(1)+a(2)+...+a(k)+1)=(3-(-1)^k)/2 - Benoit Cloitre (benoit7848c(AT)orange.fr), Oct 06 2003

a(n+2)*a(n+1)*a(n)/2 = a(n+2)+a(n+1)+a(n)-3 (this formula doesn't define the sequence, just a consequence of definition) - Benoit Cloitre (benoit7848c(AT)orange.fr), Nov 17 2003

a(n+1)=3-a(n)+[a(n)-a(n-1)]*[a(b(n))-1], where b(n) is the sequence A156253 [From Jean-Marc Fedou and Gabriele Fici (fici(AT)i3s.unice.fr), Mar 18 2010]

A000004

FORMULA

a(n)=A*sin(n*Pi) for any A. [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Apr 02 2010]

A000005

COMMENTS

If the canonical factorization of n into prime powers is Product p^e(p) then d(n) = Product (e(p) + 1). More generally, for k>0, sigma_k(n) = Product_p ((p^((e(p)+1)*k))-1)/(p^k-1).

Number of ways to write n as n = x*y, 1 <= x <= n, 1 <= y <= n. For number of unordered solutions to x*y=n, see A038548.

Note that d(n) is not the number of Pythagorean triangles with radius of the inscribed circle equal to n (that is A078644). For number of primitive Pythagorean triangles having inradius n, see A068068(n).

Number of factors in the factorization of the polynomial x^n-1 over the integers. - T. D. Noe, Apr 16 2003

If d(n) is odd, n is a perfect square. If d(n) = 2, n is prime. - Donald Sampson (Marsquo(AT)hotmail.com), Dec 10 2003

Number of even divisors of n = d(2*n) * (1 - n mod 2). - Reinhard Zumkeller, Dec 28 2003

Also equal to the number of partitions p of n such that all the parts have the same cardinality, i.e. max(p)=min(p). - Giovanni Resta, Feb 06 2006

Equals A127093 as an infinite lower triangular matrix * the harmonic series, [1/1, 1/2, 1/3,...]. - Gary W. Adamson, May 10 2007

Sum_{n>0} d(n)/(n^n) = Sum_{n>0, m>0} 1/(n*m). - Gerald McGarvey, Dec 15 2007

For odd n, this is the number of partitions of n into consecutive integers. Proof: For n = 1, clearly true. For n = 2k + 1, k >= 1, map each (necessarily odd) divisor to such a partition as follows: For 1 and n, map k + (k+1) and n, respectively. For any remaining divisor d <= sqrt(n), map (n/d - (d-1)/2) + ... + (n/d - 1) + (n/d) + (n/d + 1) + ... + (n/d + (d-1)/2) {i.e., n/d plus (d-1)/2 pairs each summing to 2n/d)}. For any remaining divisor d > sqrt(n), map ((d-1)/2 - (n/d - 1)) + ... + ((d-1)/2 - 1) + (d-1)/2 + (d+1)/2 + ((d+1)/2 + 1) + ... + ((d+1)/2 + (n/d - 1)) {i.e., n/d pairs each summing to d}. As all such partitions must be of one of the above forms, the 1-to-1 correspondence and proof is complete. - Rick L. Shepherd, Apr 20 2008

Number of subgroups of the cyclic group of order n. - Benoit Jubin (benoit_jubin(AT)yahoo.fr), Apr 29 2008

Equals row sums of triangle A143319 [From Gary W. Adamson, Aug 07 2008]

Contribution from Gary W. Adamson, Apr 26 2009: (Start)

Equals row sums of triangle A159934, equivalent to generating a(n) by convolving A000005 prefaced with a 1; (1, 1, 2, 2, 3, 2,...) with the INVERTi transform of A000005, (A159933): (1, 1,-1, 0, -1, 2,...):

Example: a(6) = 4 = (1, 1, 2, 2, 3, 2) dot (2, -1, 0, -1, 1, 1) = (2, -1, 0, -2, 3, 2) = 4. (End)

a(n) = A048691(n) - A055205(n). [From Reinhard Zumkeller, Dec 08 2009]

For n>0, a(n) = 1 + Sum(s=2..n, cos(Pi*n/s)^2). - Eric Desbiaux (moongerms(AT)wanadoo.fr), Mar 09 2010, corrected Apr 16 2011

Number of times n appears in an n X n multiplication table. [From Dominick Cancilla, Aug 02 2010]

FORMULA

If n is written as 2^z*3^y*5^x*7^w*11^v*... then d(n)=(z+1)*(y+1)*(x+1)*(w+1)*(v+1)*...

Multiplicative with a(p^e) = e+1. - David W. Wilson, Aug 01, 2001.

G.f.: Sum_{n >= 1} d(n) x^n = Sum_{k>0} x^k/(1-x^k). This is usually called THE Lambert series (see Knopp, Titchmarsh).

d(n) <= 2 sqrt(n) [see Mitrinovich, p. 39, also A046522].

a(n) is odd iff n is a square. - Reinhard Zumkeller, Dec 29, 2001

a(n) = sum(k=1, n, f(k, n)) where f(k, n) = 1 if k divides n, 0 otherwise. Equivalently, f(k, n) = (1/k)*sum(l=1, k, z(k, l)^n) with z(k, l) the k-th roots of unity. - Ralf Stephan (ralf(AT)ark.in-berlin.de), Dec 25 2002

G.f.: Sum_{n>0} ((-1)^(n+1) x^(n(n+1)/2) / ((1-x^n)*Product(1-x^i, i=1..n))).

a(n)=n-sum(k=1, n, ceil(n/k)-floor(n/k)) - Benoit Cloitre (benoit7848c(AT)orange.fr), May 11 2003

a(n) = A032741(n)+1 = A062011(n)/2 = A054519(n)-A054519(n-1) = A006218(n)-A006218(n-1) = sum(k=0, n-1, A051950(k)). - R. Stephan, Mar 26 2004

G.f.: Sum_{k>0} x^(k^2)*(1+x^k)/(1-x^k). Dirichlet g.f.: zeta(s)^2. - Michael Somos, Apr 05 2003

Sequence = M*V where M = A129372 as an infinite lower triangular matrix and V = ruler sequence A001511 as a vector: [1, 2, 1, 3, 1, 2, 1, 4,...]. - Gary W. Adamson, Apr 15 2007

A000005 = M*V, where M = A115361 is an infinite lower triangular matrix and V = A001227, the number of odd divors of n, is a vector: [1, 1, 2, 1, 2, 2, 2,...]. - Gary W. Adamson, Apr 15 2007

Row sums of triangle A051731 - Gary W. Adamson, Nov 02 2007

a(n)=sum(k=1, n, floor(n/k)-floor((n-1)/k) [From Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Aug 27 2009]

a(s)=2^omega(s), if s>1 is a squarefree number (A005117) and omega(s) is: A001221 [From Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Sep 08 2009]

a(n)=1+sum(k=1,n,mod(floor(2^n/(2^k-1)),2)) For every n [From Fabio Civolani (civox(AT)tiscali.it), Mar 12 2010]

1)(Sum{d|n}a(d))^2=Sum{d|n}(a(d))^3 (J.Liouville); 2) Sum{d|n}A008836(d)*(a(d))^2=A008836(n)*Sum{d|n}a(d). [From Vladimir Shevelev, May 22 2010]

$d(n)=\sigma_{0}(n)=1+\sum_{m=2}^{\infty}\sum_{r=1}^{\infty}\frac{1}{m^{r+1}} \sum_{j=1}^{(m-1)} \sum_{k=0}^{(m^{r+1}-1)}e^{\frac{2k\pi i(n+(m-j)m^{r})}{m^{r+1}}}$ [From A.Neves (ammdneves(AT)gmail.com), Oct 04 2010]

A000007

COMMENTS

Changing the offset to 1 gives the arithmetical function a(1)=1, a(n)=0 for n>1, the identity function for Dirichlet multiplication (see Apostol).

Hankel transform (see A001906 for definition) of : A000007 (powers of 0), A000012 (powers of 1), A000079 (powers of 2), A000244 (powers of 3), A000302 (powers of 4), A000351 (powers of 5), A000400 (powers of 6), A000420 (powers of 7), A001018 (powers of 8), A001019 (powers of 9), A011557 (powers of 10), A001020 (powers of 11), etc. ... - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jul 07 2005

This is the identity sequence with respect to convolution. - David W. Wilson (davidwwilson(AT)comcast.net), Oct 30 2006

a(A000004(n)) = 1; a(A000027(n)) = 0. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Oct 12 2008]

The alternating sum of the nth row of [[Pascal's triangle]] gives the characteristic function of 0, a(n) = 0^n. [From Daniel Forgues (squid(AT)zensearch.com), May 25 2010]

The number of maximal self-avoiding walks from the NW to SW corners of an 1-by-n grid.

FORMULA

Multiplicative with a(p^e) = 0. - David W. Wilson, Sep 01, 2001

a(n)= floor(1/(n+1)). - Franz Vrabec (franz.vrabec(AT)aon.at), Aug 24 2005

a(n)=((n+1)!^2 mod (n+2))*((n+2)!^2 mod (n+3)), with n>=0 - Paolo P. Lava (ppl(AT)spl.at), Apr 24 2007

a(n)=1-{[(n+1)!+1] mod (n+1)}, with n>=0. - Paolo P. Lava (ppl(AT)spl.at), May 22 2007

a(n)=1-[(n+2) mod (n+1)], with n>=0. - Paolo P. Lava (ppl(AT)spl.at), Jun 27 2007

a(n)=C(2*n,n) mod 2 - Paolo P. Lava (ppl(AT)spl.at), Aug 31 2007

a(n)=((-1)^A000040(n)+1)/2. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 25 2009]

A000009

COMMENTS

Partitions into distinct parts are sometimes called "strict partitions".

The number of different ways to run up a staircase with m steps, taking steps of odd sizes (or taking steps of distinct sizes), where the order is not relevant and there is no other restriction on the number or the size of each step taken is the coefficient of x^m.

Ramanujan theta functions: f(q) (see A121373), phi(q) (A000122), psi(q) (A010054), chi(q) (A000700).

The result that number of partitions of n into distinct parts = number of partitions of n into odd parts is due to Euler.

Bijection: given n = l1 * 1 + l2 * 3 + l3 * 5 + l7 * 7 + ..., a partition into odd parts, write each li in binary, li = 2^a1 + 2^a2 + 2^a3 + ... where the aj's are all different, then expand n = (2^a1 * 1 + ...)*1 + ... by removing the brackets and we get a partition into distinct parts. For the reverse operation, just keep splitting any even number into halves until no evens remain.

Euler transform of period 2 sequence [1,0,1,0,...]. - Michael Somos, Dec 16, 2002

Number of different partial sums 1+[1,2]+[1,3]+[1,4]+..., where [1,x] indicates a choice. e.g. a(6)=4, as we can write 1+1+1+1+1+1, 1+2+3, 1+2+1+1+1, 1+1+3+1. - Jon Perry (perry(AT)globalnet.co.uk), Dec 31 2003 a(n) is the sum of the number of partitions of x_j into at most j parts, where j is the index for the j-th triangular number and n-T(j)=x_j. For example; a(12)=partitions into <=4 parts of 12-T(4)=2 + partitions into <=3 parts of 12-T(3)=6 + partitions into <=2 parts of 12-T(2)=9 + partitions into 1 part of 12-T(1)=11 =(2)(11)+(6)(51)(42)(411)(33)(321)(222)+(9)(81)(72)(63)(54)+(11) =2+7+5+1 =15 - Jon Perry (perry(AT)globalnet.co.uk), Jan 13 2004

Number of partitions of n into parts where if k is the largest part, all parts 1..k are present - Jon Perry (perry(AT)globalnet.co.uk), Sep 21 2005

a(n) = Sum(A117195(n,k): 0<=k<n) = A117192(n)+A117193(n) for n>0. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 03 2006

The number of connected threshold graphs having n edges. - Michael D. Barrus (mbarrus2(AT)uiuc.edu), Jul 12 2007

Starting with offset 1 = row sums of triangle A146061 and the INVERT transform of A000700 starting: (1, 0, 1, -1, 1, -1, 1, -2, 2, -2, 2, -3, 3, -3, 4, -5,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 26 2008]

Number of partitions of n in which the largest part occurs an odd number of times and all other parts occur an even number of times. (Such partitions are the duals of the partitions with odd parts.) [From David Wasserman (dwasserm(AT)earthlink.net), Mar 04 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 11 2009: (Start)

Equals A035363 convolved with A010054. The convolution square of A000009 = A022567 = A000041 convolved with A010054.

A000041 = A000009 convolved with A035363. (End)

Considering all partitions of n into distinct parts: there are A140207(n) partitions of maximal size which is A003056(n), and A051162(n) is the greatest number occurring in these partitions. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jun 13 2009]

a(A004526(n)) = A172033(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jan 23 2010]

Equals left border of triangle A091602 starting with offset 1. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 13 2010]

FORMULA

G.f.: Product_{m >= 1} (1 + x^m) = 1/Product_{m >= 0} (1-x^(2m+1)) = Sum_{k>=0} Product_{i=1..k} x^i/(1-x^i).

G.f.: sum(n>=0, x^n*prod(k=1,n-1,1+x^k)) = 1+sum(n>=1, x^n*prod(k>=n+1,1+x^k)) - Joerg Arndt, Jan 29 2011

Asymptotics: a(n) ~ exp(pi l_n / sqrt(3)) / (4 3^(1/4) l_n^(3/2)) where l_n = (n-1/24)^(1/2) (Ayoub).

Product_{k=1..inf} (1+x^(2k)) = Sum_{k=0..inf} x^(k*(k+1))/Product_{i=1..k}(1-x^(2i)) - Euler (Hardy and Wright, Theorem 346).

For n>1, a(n)=(1/n)*Sum_{k=1..n} b(k)*a(n-k), with a(0)=1, b(n)= A000593(n)=sum of odd divisors of n; cf. A000700. - Vladeta Jovovic (vladeta(AT)eunet.rs), Jan 21 2002

a(n) = t(n, 0), t as defined in A079211.

a(n)=A026837(n)+A026838(n)=A118301(n)+A118302(n); a(A001318(n))=A051044(n); a(A118300(n))=A118303(n). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Apr 22 2006

Expansion of 1 / chi(-x) = chi(x) / chi(-x^2) = f(-x) / phi(x) = f(x) / phi(-x^2) = psi(x) / f(-x^2) =

f(-x^2) / f(-x) = f(-x^4) / psi(-x) in powers of x where phi(), psi(), chi(), f() are Ramanujan theta functions. - Michael Somos Mar 12 2011

G.f. is Fourier series which satisfies f(-1/ (1152 t)) = 2^(-1/2) / f(t) where q = exp(2 pi i t). - Michael Somos Aug 16 2007

Expansion of q^(-1/24) * eta(q^2) / eta(q) in powers of q.

Expansion of q^(-1/24) 2^(-1/2) f2(t) in powers of q = exp(2 Pi i t) where f2() is a Weber function. - Michael Somos Oct 18 2007

Given g.f. A(x), then B(x) = x * A(x^3)^8 satisfies 0 = f(B(x), B(x^2)) where f(u, v) = v - u^2 + 16*u*v^2 . - Michael Somos May 31 2005

Given g.f. A(x), then B(x) = x * A(x^8)^3 satisfies 0 = f(B(x), B(x^3)) where f(u, v) = (u^3 - v) * (u + v^3) - 9 * u^3 * v^3. - Michael Somos, Mar 25 2008

Contribution from Evangelos Georgiadis, Andrew V. Sutherland, Kiran S. Kedlaya (egeorg(AT)mit.edu), Mar 03 2009: (Start)

a(0)=1. a(n)= 2*(Sum_{k=1} (-1)^(k+1) a(n-k^2)) + sigma(n) where

sigma(n)= (-1)^(j) if (n=(j*(3*j+1))/2 OR n=(j*(3*j-1))/2)

otherwise sigma(n)=0. (End)

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 13 2009: (Start)

The product G.f. = (1/(1-x))*(1/(1-x^3)*(1/(1-x^5)*...; = (1,1,1,...)*

(1,0,0,1,0,0,1,0,0,1,...)*(1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,...) * ...; =

a*b*c*... where a, a*b, a*b*c,...converge to A000009:

1, 1, 1, 2, 2, 2, 3, 3, 3,..4,... = a*b

1, 1, 1, 2, 2, 3, 4, 4, 5,..6,... = a*b*c

1, 1, 1, 2, 2, 3, 4, 5, 6,..7,... = a*b*c*d

1, 1, 1, 2, 2, 3, 4, 5, 6,..8,... = a*b*c*d*e

1, 1, 1, 2, 2, 3, 4, 5, 6,..8,... = a*b*c*d*e*f

...(Cf. analogous example in A000041). (End)

a(n) = P(n)-P(n-2)-P(n-4)+P(n-10)+P(n-14)+...+(-1)^m P(n-2p_m)+..., where P(n) is the partition function (A000041) and p_m =m(3m-1)/2 is the m-th generalized pentagonal number (A001318). - Jerome Malenfant (jm@aps.org), Feb 16 2011

A000010

COMMENTS

Number of elements in a reduced residue system modulo n.

Degree of the n-th cyclotomic polynomial (cf. A013595). - Benoit Cloitre (benoit7848c(AT)orange.fr), Oct 12 2002

Number of distinct generators of a cyclic group of order n. Number of primitive n-th roots of unity.(A primitive n-th root x is such that x^k is not equal to 1 for k=1, 2, ..., n-1, but x^n=1) - Lekraj Beedassy (blekraj(AT)yahoo.com), Mar 31 2005

Also number of complex Dirichlet characters modulo n; sum(k=1,n,a(k)) is asymptotic to (3/pi^2)*n^2. - S. R. Finch (Steven.Finch(AT)inria.fr), Feb 16 2006

a(n) is the highest degree of irreducible polynomial dividing 1 + x + x^2 + ... + x^(n-1) = (x^n - 1)/(x - 1). - Alexander Adamchuk (alex(AT)kolmogorov.com), Sep 02 2006, corrected Sep 27 2006

a(p) = p - 1 for prime p. a(n) is even for n>2. For n>2 a(n)/2 = A023022(n) = number of partitions of n into 2 ordered relatively prime parts. - Alexander Adamchuk (alex(AT)kolmogorov.com), Jan

25 2007

Row sums of A127448. - Mats O. Granvik (mgranvik(AT)abo.fi), May 28 2008

Equals row sums of triangle A143239 (a consequence of the Dedekind-Liouville rule, see Concrete Mathematics p. 137). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 01 2008]

Number of automorphisms of the cyclic group of order n. [From Benoit Jubin (benoit_jubin(AT)yahoo.fr), Aug 09 2008]

Equals row sums of triangle A143353. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 10 2008]

a(n+2) equals the number of palindromic Sturmian words of length n which are 'bispecial', prefix or suffix of two Sturmian words of length n+1. [From Fred Lunnon (Fred.Lunnon(AT)may.ie), Sep 05 2010]

FORMULA

phi(n) = n*Product_{distinct primes p dividing n} (1-1/p).

Sum_{ d divides n } phi(d) = n.

phi(n) = Sum_{ d divides n } mu(d)*n/d, i.e., the Moebius transform of the natural numbers; mu() = Moebius function A008683().

Dirichlet generating function sum_{n >= 1} phi(n)/n^s = zeta(s-1)/zeta(s). Also Sum_{n >= 1} phi(n)*x^n/(1-x^n) = x/(1-x)^2.

Multiplicative with a(p^e) = (p-1)*p^(e-1). - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01 2001.

Sum_{n>=1} [phi(n)*ln(1-x^n)/n] = -x/(1-x) for -1<x<1 (cf. A002088) - Henry Bottomley (se16(AT)btinternet.com), Nov 16 2001

a(n)=binomial(n+1, 2) - sum{i=1, n-1, a(i)*floor(n/i)} (see A000217 for inverse) - Jon Perry (perry(AT)globalnet.co.uk), Mar 02 2004

Comment from Pieter Moree, Sep 10 2004: It is a classical result (certainly known to Landau, 1909) that lim inf n/phi(n)=1 (taking n to be primes), lim sup n/(phi(n) log log n)=e^{gamma}, with gamma = Euler's constant (taking n to be products of consecutive primes starting from 2 and applying Mertens' theorem). See e.g. Ribenboim, pp. 319-320.

a(n)=sum(i=1, n, | k(n, i) |) where k(n, i) is the Kronecker symbol. Also a(n)=#{ 1<=i<=n : k(n, i)=0} where k(n, i) is the Kronecker symbol. - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 06 2004

Conjecture : limit Sum((-1)^i/(i * phi(i)) 2<=i<=Infinity) exists and is ca. 0.558. - Orges Leka (oleka(AT)students.uni-mainz.de), Dec 23 2004

Equals row sums of triangle A143276 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 03 2008]

Contribution from Enrique Perez Herrero (psychgeometry(AT)gmail.com), Sep 07 2010: (Start)

a(n)=sum_{i=1}^{n}{floor(sigma_k(i*n)/sigma_k(i)*sigma_k(n))}, where sigma_2 is A001157

a(n)=sum_{i=1}^{n}{floor(tau_k(i*n)/tau_k(i)*tau_k(n))}, where tau_3 is A007425

a(n)=sum_{i=1}^{n}{floor(rad(i*n)/rad(i)*rad(n))}, where rad is A007947 (End)

a(n) = A173557(n) * A003557(n). - R. J. Mathar, Mar 30 2011

A000012

COMMENTS

Number of ways of writing n as a product of primes.

Number of ways of writing n as a sum of distinct powers of 2.

Continued fraction for golden ratio A001622.
Partial sums of A000007 (characteristic function of 0). - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Sep 08 2002
An example of an infinite sequence of positive integers whose distinct pairwise concatenations are all primes! - Don Reble, Apr 17 2005
Binomial transform of A000007; inverse binomial transform of A000079 . Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jul 07 2005
A063524(a(n)) = 1. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Oct 11 2008]
For n >= 0, let M(n) be the matrix with first row = (n n+1) and 2nd row = (n+1 n+2). Then a(n) = absolute value of det(M(n)). [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), Apr 11 2009]
The partial sums give the natural numbers (A000027). [From Daniel Forgues (squid(AT)zensearch.com), May 08 2009]
Contribution from Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Sep 04 2009: (Start)
a(n) is also tau_1(n) where tau_2(n) is A000005
a(n) is a completely multiplicative arithmetical function.
a(n) is both square free and a perfect square. See A005117 and A000290. (End)
Also smallest divisor of n. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Sep 07 2009].
Also decimal expansion of 1/9. [From Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Sep 18 2009; corrected by Klaus Brockhaus, Apr 02 2010]
a(n) is also the number of complete graphs on n nodes. [From Pablo Chavez (pchavez(AT)cmu.edu), Sep 15 2009]
Totally multiplicative sequence with a(p) = 1 for prime p. Totally multiplicative sequence with a(p) = a(p-1) for prime p. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Oct 18 2009]
n-th prime minus phi(prime(n)); number of divisors of n-th prime minus number of perfect partitions of n-th prime; the number of perfect partitions of n-th prime number; the number of perfect partitions of n-th non-composite number. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 26 2009]
Contribution from Harlan J. Brothers (harlan(AT)brotherstechnology.com), Nov 01 2009: (Start)
For all n>0, the sequence of limit values for a(n)=n!Sum[k=n..inf, k/(k+1)!]
Also, for all n != 0, a(n)=n^0 (End)
a(n) is also the number of 0-regular graphs on n vertices. [From Jason Kimberley (Jason.Kimberley(AT)newcastle.edu.au), Nov 07 2009]
Differences between consecutive n. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Dec 05 2009]
Triangle of numerators in Leibniz harmonic triangle. [From Paul Muljadi (paulmuljadi(AT)yahoo.com), Jan 21 2010]
Contribution from Matthew Vandermast (ghodges14(AT)comcast.net), Oct 31 2010: (Start)
1) When sequence is read as a regular triangular array, T(n,k) is the coefficient of the k-th power in the expansion of (x^(n+1)-1)/(x-1).
2) Sequence can also be read as a uninomial array with rows of length 1, analogous to arrays of binomial, trinomial, etc., coefficients. In a q-nomial array, T(n,k) is the coefficient of the k-th power in the expansion of ((x^q -1)/(x-1))^n, and row n has a sum of q^n and a length of (q-1)*n + 1. (End)
The number of maximal self-avoiding walks from the NW to SW corners of a 2-by-n grid.

When considered as a rectangular array, A000012 is a member of the chain of accumulation arrays that includes the multiplication table A003991 of the positive integers. The chain is ... < A185906 < A000007 < A000012 < A003991 < A098358 < A185904 < A185905 < ... (See A144112 for the definition of accumulation array.) [From Clark Kimberling, ck6(AT)evansville.edu Feb 6 2011]

FORMULA

G.f.: 1/(1-x); a(n)=1. E.g.f.: e^x.

G.f.: Product[(1+x^(2^k)),{k,0,Infinity}]. - Zak Seidov (zakseidov(AT)yahoo.com), Apr 06 2007

Multiplicative with a(p^e) = 1.

Dirichlet generating function: zeta(s). - Franklin T. Adams-Watters, Sep 11 2005.

Regarded as a square array by antidiagonals, g.f. 1/((1-x)(1-y)), e.g.f. sum T(n,m) x^n/n! y^m/m! = e^{x+y}, e.g.f. sum T(n,m) x^n y^m/m! = e^y/(1-x). Regarded as a triangular array, g.f. 1/((1-x)(1-xy)), e.g.f. sum T(n,m) x^n y^m/m! = e^{xy}/(1-x). - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Feb 06 2006

Contribution from Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Sep 04 2009: (Start)

a(n) = Sum(d|n,mu(n/d)*tau_2(d))=1, where tau_2(n)=A000005 and mu(n)=A008683

a(n) = |Sum(d|n,mu(d)*tau_2(d))|=1 (End)

a(n) = A002033(A000040(n)) = A002033(A008578(n)) = A000005(A000040(n)) -A002033(n) = A000027(A000040(n)) -A000010(A000040(n)). [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 26 2009]

A000014

FORMULA

G.f.: A(x) = ((x-1)/x)*f(x) + ((1+x)/x^2)*g(x) - (1/x^2)*g(x)^2 where f(x) is g.f. for A059123 and 1+g(x) is g.f. for A001678. [Harary and E. M. Palmer, p. 62, Eq. (3.3.10) with extra -(1/x^2)*Hbar(x)^2 term which should be there according to eq.(3.3.14), p. 63, with eq.(3.3.9)].

A000019

COMMENTS

A check found errors in Theissen's data (degree 121 and 125) as well as in Short's work (degree 169). - Alexander Hulpke (hulpke(AT)math.colostate.edu), Feb 19 2002

There is an error at n=574 in the Dixon-Mortimer paper. - Colva M. Roney-Dougal.

A000027

COMMENTS

a(n) is smallest positive integer which is consistent with sequence being monotonically increasing and satisfying a(a(n)) = n (cf. A007378).

Inverse Euler transform of A000219.

The rectangular array having A000027 as antidiagonals is the dispersion of the complement of the triangular numbers, A000217 (which triangularly form column 1 of this array). The array is also the transpose of A038722. - Clark Kimberling (ck6(AT)evansville.edu), Apr 05 2003

For nonzero x, define f(n)=floor(nx)-floor(n/x). Then f=A000027 if and only if x=tau or x=-tau. - Clark Kimberling (ck6(AT)evansville.edu), Jan 09 2005

Sum of powers of 2 (A007088) or algebraic sum of powers of 3 (A112867, A112952). - Lekraj Beedassy (blekraj(AT)yahoo.com), Mar 24 2006

Numbers of form (2^i)*k for odd k [i.e. n = A006519(n)*A000265(n)]; Thus n corresponds uniquely to an ordered pair (i,k) where i=A007814,k=A000265 {with

A007814(2n)=A001511(n),A007814(2n+1)=0 } - Lekraj Beedassy (blekraj(AT)yahoo.com), Apr 22 2006

If the offset were changed to 0, we would have the following pattern: a(n)=binomial(n,0) + binomial(n,1) for the present sequence (number of regions in 1-space defined by n points), A000124 (number of regions in 2-space defined by n straight lines), A000125 (number of regions in 3-space defined by n planes), A000127 (number of regions in 4-space defined by n hyperplanes), A006261, A008859, A008860, A008861, A008862 and A008863, where the last six sequences are interpreted analogously and in each "... by n ..." clause an offset of 0 has been assumed, resulting in a(0)=1 for all of them, which corresponds to the case of not cutting with a hyperplane at all and therefore having one region. - Peter C. Heinig (algorithms(AT)gmx.de), Oct 19 2006

Define a number of points lines on a straight line to be in general arrangement when no two points coincide. Then these are the numbers of regions defined by n points in general arrangement on a straight line, when an offset of 0 is assumed. For instance, a(0)=1, since using no point at all leaves one region. The sequence satisfies the following recursion a(n) = a(n-1) + 1. This has the following geometrical interpretation: Suppose there are already n-1 points in general arrangement, thus defining the maximal number of regions on a straight line obtainable by n-1 points and now one more point is added in general arrangement. Then it will coincide with no other point and act as a dividing wall thereby creating one new region in addition to the a(n-1)=(n-1)+1=n regions already there, hence a(n)=a(n-1)+1. Cf. the comments on A000124 for an analogous interpretation. - Peter C. Heinig (algorithms(AT)gmx.de), Oct 19 2006

The sequence a(n)=n (for n=1,2,3) and a(n)=n+1 (for n=4,5,...) gives to the rank (minimal cardinality of a generating set) for the semigroup I_n\S_n, where I_n and S_n denote the symmetric inverse semigroup and symmetric group on [n]. - James East (james.east(AT)latrobe.edu.au), May 03 2007

The sequence a(n)=n (for n=1,2), a(n)=n+1 (for n=3) and a(n)=n+2 (for n=4,5,...) gives the rank (minimal cardinality of a generating set) for the semigroup PT_n\T_n, where PT_n and T_n denote the partial transformation semigroup and transformation semigroup on [n]. - James East (james.east(AT)latrobe.edu.au), May 03 2007

Comment from Clark Kimberling (ck6(AT)evansville.edu), Jul 07 2007: (Start) "God made the integers; all else is the work of man." This famous quotation is a translation of "Die ganzen Zahlen hat der liebe Gott gemacht, alles andere ist Menschenwerk," spoken by Leopold Kronecker in a lecture at the Berliner Naturforscher-Versammlung in 1886.

It is not clear, nor important, whether the "ganzen Zahlen" means the whole numbers, A000027, or all the integers, A130472. What is more important is the adjective "liebe" in "liebe Gott." Walter Felscher explains that because "lieber Gott" is a colloquial phrase usually used only when speaking to children or illiterati, Kronecker's witticism was not intended as a theologico-philosophical statement.

Possibly the first publication of the statement is in Heinrich Weber's "Leopold Kronecker," Jahresberichte D.M.V. 2 (1893) 5-31. (End)

Binomial transform of A019590, inverse binomial transform of A001792 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 24 2007

Contribution from Clark Kimberling (ck6(AT)evansville.edu), Sep 11 2008: (Start)

Writing A000027 as N, perhaps the simplest one-to-one correspondence between

NxN and N is this: f(m,n)=((m+n)^2 - m - 3n + 2)/2. Its inverse is given

by I(k)=(g,h), where g = k - J(J-1)/2, h = J + 1 - g, J = floor((1 + sqrt(8k - 7))/2).
Thus I(1)=(1,1), I(2)=(1,2), I(3)=(2,1) and so on; the mapping I fills the
first-quadrant lattice by successive antidiagonals. (End)
A000007(a(n)) = 0; A057427(a(n)) = 1. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Oct 12 2008]
a(n) is also the mean of the first n odd integers. [From Ian Kent (abides(AT)bu.edu), Dec 23 2008]
Equals INVERTi transform of A001906, the even-indexed Fibonacci numbers starting (1, 3, 8, 21, 55,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 05 2009]
These are also the 2-rough numbers: positive integers that have no prime factors less than 2. [From Michael Porter (michael_b_porter(AT)yahoo.com), Oct 08 2009]
Totally multiplicative sequence with a(p) = p for prime p. Totally multiplicative sequence with a(p) = a(p-1) + 1 for prime p. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Oct 18 2009]
Triangle T(k,j) of natural numbers, read by rows, with T(k,j)=C(k,2)+j=.5(k^2-k)+j where 1<=j<=k. In other words, a(n)=n=C(k,2)+j where k is the largest integer such that C(k,2)<n and j=n-C(k,2). For example, T(4,1)=7, T(4,2)=8, T(4,3)=9, and T(4,4)=10. Note that T(n,n)=A000217(n), the n-th triangular number. [From Dennis Walsh (dwalsh(AT)mtsu.edu), Nov 19 2009]
pi(prime(n)). [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Nov 29 2009]
Hofstadter-Conway-like sequence (see A004001): a(n) = a(a(n-1)) + a(n-a(n-1)) with a(1) = 1, a(2) = 2. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Dec 11 2009]
a(n) is also the dimension of the irreducible representations of the Lie algebra sl(2) [From Leonid Bedratyuk (leonid.uk(AT)gmail.com), Jan 04 2010]
Floyd's triangle read by rows. [From Paul Muljadi (paulmuljadi(AT)yahoo.com), Jan 25 2010]
Number of numbers between k and 2k where k is a integer. [From Giovanni Teofilatto (g.teofilatto(AT)tiscalinet.it), Mar 26 2010]
Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), May 29 2010: (Start)
Generated from a(2n) = r*a(n), a(2n+1) = a(n) + a(n+1), r = 2; in an infinite
set, row 2 of the array shown in A178568. (End)
Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 15 2010: (Start)
1/n = continued fraction [n].
Let barover[n] = [n,n,n,...] = 1/k. Then k - 1/k = n.
Example: [2,2,2,...] = (sqrt(2) - 1) = 1/k, with k =
(sqrt(2) + 1). Then 2 = k - 1/k. (End)
Number of n-digit numbers the binary expansion of which contains one run of 1's. [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jul 30 2010]
Contribution from Clark Kimberling (ck6(AT)evansville.edu), Jan 29 2011: (Start)
Let T denote the "natural number array A000027":
1....2.....4....7...
3....5.....8...12...
6....9....13...18...
10...14...19...25...
T(n,k)=n+(n+k-2)(n+k-1)/2. See A185787 for a list of sequences based on T, such as rows, columns, diagonals, and sub-arrays.
(End)

The Stern polynomial B(n,x) evaluated at x=2. See A125184. - T. D. Noe, Feb 28 2011
FORMULA
Multiplicative with a(p^e) = p^e. - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.
Another g.f.: Sum_{n>0} phi(n)x^n/(1-x^n) (Apostol).
When seen as array: T(k, n) = n+1 + (k+n)*(k+n+1)/2. Main diagonal is 2n(n+1)+1 (A001844), antidiagonal sums are n(n^2+1)/2 (A006003). - Ralf Stephan, Oct 17 2004
Dirichlet generating function: zeta(s-1). - Franklin T. Adams-Watters, Sep 11 2005.
G.f.: x/(1-x)^2. E.g.f.: x*exp(x). a(n)=n. a(-n)=-a(n).
Series reversion of g.f. A(x) is x*C(-x)^2 where C(x) is g.f. A000108.- Michael Somos Sep 04 2006
Convolution of A000012 (the all-ones sequence) with itself. - Tanya Khovanova (tanyakh(AT)yahoo.com), Jun 22 2007
a(n)=2*a(n-1)-a(n-2); a(1)=1, a(2)=2. a(n)=1+a(n-1). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 03 2008]
a(n)=A000720(A000040(n)). [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Nov 29 2009]
A000029
FORMULA
Sum_{ d divides n } phi(d)*2^(n/d)/(2*n) + either 2^((n-1)/2) if n odd or 2^(n/2-1)+2^(n/2-2) if n even.
A000031
COMMENTS
Also a(n)-1 is number of 1's in truth table for lexicographically least de Bruijn cycle (Fredricksen).
FORMULA
a(n) = (1/n)*Sum_{ d divides n } phi(d)*2^(n/d).
A000032
COMMENTS
This is also the Horadam sequence (2,1,1,1). - Ross La Haye (rlahaye(AT)new.rr.com), Aug 18 2003
For distinct primes p,q, L(p) is congruent to 1 mod p, L(2p) is congruent to 3 mod p and L(pq) is congruent 1+q(L(q)-1) mod p. Also, L(m) divides F(2km) and L((2k+1)m), k,m >=0.
a(n)=sum(P(3;n-1-k,k),k=0..ceiling((n-1)/2)), n>=1, with a(0)=2. These are the sums over the SW-NE diagonals in P(3;n,k), the (3,1) Pascal triangle A093560. Observation by Paul Barry (pbarry(AT)wit.ie), Apr 29 2004. Proof via recursion relations and comparison of inputs. Also SW-NE diagonal sums of the (1,2) Pascal triangle A029635 (with T(0,0) replaced by 2).
Suppose psi=ln(phi). We get the representation L(n)=2*cosh(n*psi) if n is even; L(n)=2*sinh(n*psi) if n is odd. There is a similar representation for Fibonacci numbers (A000045). Many Lucas formulas now easily follow from appropriate sinh- and cosh-formulas. For example: the identity cosh^2(x)-sinh^2(x)=1 implies L(n)^2-5F(n)^2=4*(-1)^n (setting x=n*psi). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Apr 18 2007
Comments from John Blythe Dobson (j.dobson(AT)uwinnipeg.ca), Oct 02 2007, Oct 11 2007: (Start) The parity of L(n) follows easily from its definition, which shows that L(n) is even when n is a multiple of 3 and odd otherwise.

The first six multiplication formulae are:
L(2n) = (L(n))^2 - 2*(-1)^n
L(3n) = (L(n))^3 - 3*((-1)^n)*L(n)
L(4n) = (L(n))^4 - 4*((-1)^n)*(L(n))^2 + 2
L(5n) = (L(n))^5 - 5*((-1)^n)*(L(n))^3 + 5*L(n)
L(6n) = (L(n))^6 - 6*((-1)^n)*(L(n))^4 + 9*(L(n))^2 - 2*(-1)^n
Generally, L(n) | L(mn) iff m is odd. (End)
In the expansion of L(mn), where m represents the multiplier and n the index of a known value of L(n), the absolute values of the coefficients are the terms in the m-th row of the triangle A034807. When m=1 and n=1, L(n)=1 and all the terms are positive and so the row sums of A034807 are simply the Lucas numbers. (End)
The comments submitted by Miklos Kristof on Mar 19 2007 for the Fibonacci numbers (A00045) contain four important identities which have close analogues in the Lucas numbers: For a>=b and odd b, L(a+b) + L(a-b) = 5*F(a)*F(b). For a>=b and even b, L(a+b) + L(a-b) = L(a)*L(b). For a>=b and odd b, L(a+b) - L(a-b) = L(a)*L(b). For a>=b and even b, L(a+b) - L(a-b) = 5*F(a)*F(b). - John Blythe Dobson (j.dobson(AT)uwinnipeg.ca), Nov 15 2007. A particularly interesting instance of the difference identity for even b is L(a+30) - L(a-30) = 5*F(a)*832040, since 5*832040 is divisible by 100, proving that the last two digits of Lucas nu mbers repeat in a cycle of length 60.
Further comments from John Blythe Dobson (j.dobson(AT)uwinnipeg.ca), Nov 15 2007: (Start) The Lucas numbers satisfy remarkable difference equations, in some cases best expressed using Fibonacci numbers, of which representative examples are the following:
L(n) - L(n-3) = 2*L(n-2)
L(n) - L(n-4) = 5*F(n-2)
L(n) - L(n-6) = 4*L(n-3)
L(n) - L(n-12) = 40*F(n-6)
L(n) - L(n-60) = 4160200*F(n-30).
These formulae establish, respectively, that the Lucas numbers form a cyclic residue system of length 3 (mod 2), of length 4 (mod 5), of length 6 (mod 4), of length 12 (mod 40) and of length 60 (mod 4160200). The divisibility of the last modulus by 100 accounts for the fact that the last two digits of the Lucas numbers begin to repeat at L(60).
The divisibility properties of the Lucas numbers are very complex and still not fully understood, but several important criteria are established in Zhi-Hong Sun's 2003 survey of congruences for Fibonacci numbers. (End)
Sum_{n>0} a(n)/(n*2^n) = 2*log(2) [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Oct 11 2009]
A010888(a(n)) = A030133(n). [Reinhard Zumkeller, Aug 20 2011]
FORMULA
Conjecture: Let f(n) = Phi^n + Phi^(-n), then L(2n) = f(2n) and L(2n+1) = f(2n+1) - 2*Sum(k=0..infinity, C(k+1)/f(2n+1)^(2k+1)) where C(n) are Catalan numbers (A000108). - Gerald McGarvey (gerald.mcgarvey(AT)comcast.net), Dec 21 2007
G.f.: (2-x)/(1-x-x^2). L(n)=((1+sqrt(5))/2)^n + ((1-sqrt(5))/2)^n.
L(n) = L(n-1) + L(n-2) = (-1)^n L(-n).
E.g.f.: 2*exp(x/2)*cosh(sqrt(5)*x/2). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Nov 30 2001

L(n) = Fibonacci(2*n)/Fibonacci(n) for n > 0. - Jeff Burch (gburch(AT)erols.com)

L(n) = Fib(n) + 2*Fib(n-1) = Fib(n + 1) + Fib(n-1) - Henry Bottomley (se16(AT)btinternet.com), Apr 12 2000

a(n)=sqrt(F(n)^2+4*F(n+1)*F(n-1)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 06 2003 [Corrected by Gary Detlefs (gdetlefs(AT)aol.com) Jan 21 2011]

a(n)=2^(1-n)sum{k=0..floor(n/2), C(n, 2k)5^k}. a(n)=2T(n, i/2)(-i)^n with T(n, x) Chebyshev's polynomials of the first kind (see A053120) and i^2=-1. - Paul Barry (pbarry(AT)wit.ie), Nov 15 2003

L(n)=2*Fib(n+1)-Fib(n) - Paul Barry (pbarry(AT)wit.ie), Mar 22 2004

a(n)=floor((phi)^n+(-phi)^(-n)) - Paul Barry (pbarry(AT)wit.ie), Mar 12 2005

Comments from Miklos Kristof (kristmikl(AT)freemail.hu), Mar 19 2007: (Start)

Let F(n)=A000045=Fibonacci numbers, L(n)=a(n)=Lucas numbers:

L(n+m)+(-1)^m*L(n-m)=L(n)*L(m)

L(n+m)-(-1)^m*L(n-m)=8*F(n)*F(m)

L(n+m+k)+(-1)^k*L(n+m-k)+(-1)^m*(L(n-m+k)+(-1)^k*L(n-m-k))=L(n)*L(m)*L(k)

L(n+m+k)-(-1)^k*L(n+m-k)+(-1)^m*(L(n-m+k)-(-1)^k*L(n-m-k))=5*F(n)*L(m)*F(k)

L(n+m+k)+(-1)^k*L(n+m-k)-(-1)^m*(L(n-m+k)+(-1)^k*L(n-m-k))=5*F(n)*F(m)*L(k)

L(n+m+k)-(-1)^k*L(n+m-k)-(-1)^m*(L(n-m+k)-(-1)^k*L(n-m-k))=5*L(n)*F(m)*F(k) (End)

Inverse: floor(log_phi(a(n))+0.5)=n, for n>1. Also for n>=0, floor(1/2*log_phi(a(n)*a(n+1)))=n. Extension valid for all integers n: floor(1/2*sign(a(n)*a(n+1))*log_phi|a(n)*a(n+1)|)=n {where sign(x) = sign of x}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), May 02 2007

Starting (1, 3, 4, 7, 11,...) = row sums of triangle A131774. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 14 2007

a(n)=2*fibonacci(n-1)+fibonacci(n), n>=0 - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Oct 05 2007

a(n) = trace of the 2 X 2 matrix [0,1; 1,1]^n - Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 02 2008

Comments from Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jan 02 2009 (Start): For odd n: a(n)=floor(1/(fract(phi^n))); for even n>0: a(n)=ceiling(1/(1-fract(phi^n))). This follows from the basic property of the golden ratio phi, which suffices phi-phi^(-1)=1 (see general formula described in A001622).

a(n)=nint(1/(min(fract(phi^n), 1-fract(phi^n))), for n>1, where fract(x)=x-floor(x). (End)

E.g.f.: exp(phi*x) + exp(-x/phi) with phi:=(1+sqrt(5))/2 (golden section). 1/phi = phi-1. See another form given in the Smiley e.g.f. comment. [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), May 15 2010]

L(n))/L(n-1) -> (sqrt(5)+1)/2) =1,618033989.... [From Vincenzo Librandi (vincenzo.librandi(AT)tin.it), Jul 17 2010]

a(n)=2*a(n-2)+a(n-3),n>2 [From Gary Detlefs (gdetlefs(AT)aol.com), Sep 09 2010]

L(n)=floor(1/fract(Fib(n)*phi)), for n odd. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), October 20 2010

L(n)=ceiling(1/(1-fract(Fib(n)*phi))), for n even. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), October 20 2010

L(n)= 2^n*(cos(Pi/5)^n + cos(3*Pi/5)^n) [From Gary Detlefs (gdetlefs(AT)aol.com) Nov 29 2010]

L(n)= (Fibonacci(2*n-1)*Fibonacci(2*n+1)-1)/(Fibonacci(n)*Fibonacci(2*n)) [From Gary Detlefs (gdetlefs(AT)aol.com) Dec 13 2010]

L(n)= sqrt(5*Fibonacci(n)^2-4*(-1)^(n+1)) [From Gary Detlefs (gdetlefs(AT)aol.com) Dec 26 2010]

L(n)= floor(phi^n)+((-1)^n+1)/2, where phi=(1+sqrt(5))/2. [From Gary Detlefs (gdetlefs(AT)aol.com) Jan 20 2011]

L(n)= Fibonacci(n+6) mod Fibonacci(n+2),n>2. [From Gary Detlefs (gdetlefs(AT)aol.com) May 19 2011]

A000035

COMMENTS

Least significant bit of n, lsb(n).

Also decimal expansion of 1/99.

a(n) = ABS(A134451(n)). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Oct 27 2007

Characteristic function of odd numbers: a(A005408(n))=1, a(A005843(n))=0. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Sep 29 2008]

A102370(n) modulo 2 . [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Apr 04 2009]

Base b expansion of 1/(b^2-1) for any b>=2 is 0.0101... (A005563 has b^2-1). [From Rick L. Shepherd (rshepherd2(AT)hotmail.com), Sep 27 2009]

Let A be the Hessenberg n by n matrix defined by: A[1,j]=j mod 2, A[i,i]:=1, A[i,i-1]=-1, and A[i,j]=0 otherwise. Then, for n>=1, a(n)=(-1)^n*charpoly(A,1). [From Milan R. Janjic (agnus(AT)blic.net), Jan 24 2010]

Contribution from R. J. Mathar (mathar(AT)strw.leidenuniv.nl), Jul 15 2010: (Start)

The sequence is the principal Dirichlet character of the reduced residue system mod 2 or mod 4 or mod 8 or mod 16...

Associated Dirichlet L-functions are for example L(2,chi)= sum_{n>=1} a(n)/n^2 ==A111003,

or L(3,chi)= sum_{n>=1} a(n)/n^3 = 1.05179979... = 7*A002117/8,

or L(4,chi)= sum_{n>=1} a(n)/n^4 = 1.014678... = A092425/96. (End)

FORMULA

a(n)={1 - (-1)^n}/2. a(n) = n mod 2.

Multiplicative with a(p^e) = p%2. - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.

G.f.: x/(1-x^2). E.g.f.: sinh(x). a(n)=n mod 2. a(n)=1/2 - (-1)^n/2. - Paul Barry (pbarry(AT)wit.ie), Mar 11 2003

a(n)=(A000051(n)-A014551(n))/2. - Mario Catalani (mario.catalani(AT)unito.it), Aug 30 2003

a(n) = ceiling((-2)^(-n-1)). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Apr 19 2005

a(n)= [sin(n*Pi/2)]^2 = [cos(n*Pi/2 +/- Pi/2)]^2 with n>=0. - Paolo P. Lava (ppl(AT)spl.at), Sep 20 2006

Dirichlet g.f. (1-1/2^s)*zeta(s). - R. J. Mathar, Mar 04 2011

A000040

COMMENTS

A number n is prime if it is greater than 1 and has no positive divisors except 1 and n.

A number n is prime if and only if it has exactly two positive divisors.

A prime has exactly one proper positive divisor, 1.

The sum of an odd number > 1 (2i+1, i >= 1) of consecutive positive odd numbers centered on the j-th odd number >= 2i+1 (2j+1, j >= i) being (2i+1)*(2j+1) has 2 or more odd prime factors (odd semiprime iff 2i+1 and 2j+1 are primes). - Daniel Forgues (squid(AT)zensearch.com), Jul 15 2009

Comment from Pieter Moree, Oct 14 2004: The paper by Kaoru Motose starts as follows: "Let q be a prime divisor of a Mersenne number 2^p-1 where p is prime. Then p is the order of 2 (mod q). Thus p is a divisor of q-1 and q>p. This shows that there exist infinitely many prime numbers."

1 is not a prime, for if the primes included 1, then the factorization of a natural number n into a product of primes would not be unique, since n = n*1.

1 is the empty product (has 0 prime factors) whereas a prime has 1 prime factor (itself). - Daniel Forgues, Jul 23 2009

Prime(n) and pi(n) are inverse functions: A000720(a(n)) = n and a(n) is the least number m such that a(A000720(m)) = a(n). a(A000720(n)) = n if (and only if) n is prime.

Elementary primality test: If no prime =<sqrt(m) divides m, then m is prime.(since a prime is its own exclusive multiple, apart from 1) - Lekraj Beedassy (blekraj(AT)yahoo.com), Mar 31 2005

Second sequence ever computed by electronic computer, on EDSAC, May 9 1949 (see Renwick link). - Russ Cox (rsc(AT)swtch.com), Apr 20 2006

Every prime p is a linear combination of previous primes p(n) with nonzero coefficients c(n) and |c(n)| < p(n). - Amarnath Murthy, Franklin T. Adams-Watters and Joshua Zucker, May 17 2006.

Odd primes can only be written as a sum of two consecutive integers. Powers of 2 do not have a representation as a sum of k consecutive integers (other than the trivial n=n, for k=1). See A111774. - Jaap Spies (j.spies(AT)hccnet.nl), Jan 04 2007

There is a unique decomposition of the primes: provided the weight A117078(n) is > 0, we have prime(n) = weight * level + gap, or A000040(n) = A117078(n) * A117563(n) +A001223(n). - Remi Eismann (reismann(AT)free.fr), Feb 16 2007

Equals row sums of triangle A143350 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 10 2008]

Contribution from Eric Desbiaux (moongerms(AT)wanadoo.fr), Oct 28 2008: (Start). APSO (Alternating partial sums of sequence) a-b+c-d+e-f+g...=(a+b+c+d+e+f+g...)-2*(b+d+f...):

APSO(A000040) = A008347=A007504 - 2*(A077126 repeated)

(A007504-A008347)/2 = A077131 alternated with A077126. (End)

The Greek transliteration of 'Prime Number' is 'Proton Arithmon'. [From Daniel Forgues (squid(AT)zensearch.com), May 08 2009]

Numbers with exactly one prime divisor. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Nov 10 2009]

a(n) = A008864(n) - 1 = A052147(n) - 2 = A113395(n) - 3 = A175221(n) - 4 = A175222(n) - 5 = A139049(n) - 6 = A175223(n) - 7 = A175224(n) - 8 = A140353(n) - 9 =A175225(n) - 10 . [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Mar 06 2010]

Contribution from Daniel Forgues (squid(AT)zensearch.com), Mar 19 2010: (Start)

2 and 3 might be referred to as the two "forcibly prime numbers" since there are no integers greater than 1 and less than or equal to their respective square roots. Not a single trial division ever needs to be done for 2 or 3, so they are disqualified from the get go from any attempt to

belong to the set of composite numbers. 2 and 3 are thus the only consecutive primes. Since any further prime needs to be coprime to both 2 and 3, they can only be congruent to 5 or 1 (mod 2*3) and thus must all be of the form (2*3)*k -/+ 1 with k >= 1. When both (2*3)*k - 1 and (2*3)*k + 1 are prime for a given k >= 1, they are referred to as twin primes. (3 and 5 being the only twin primes of the form (2*2)*k - 1 and (2*2)*k + 1) (End)

For prime n, the sum of divisors of n > product of divisors of n. Sigma(n)==1 (mod n). [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru, Mar 12 2011]

FORMULA

The prime number theorem is the statement that a(n) ~ n * log n as n -> infinity (Hardy and Wright, page 10).

For n >= 2, n*(log n + log log n - 3/2) < a(n); for n >= 20, a(n) < n*(log n + log log n - 1/2). [Rosser and Schoenfeld]

For all n, a(n) > n log n. [Rosser]

n log(n) + n (log log n - 1) < a(n) < n log n + n log log n for n >= 6 [Dusart, quoted in the Wikipedia article]

a(n) = n log n + n log log n + (n/log n)*(log log n - log 2 - 2) + O(n (log log n)^2/ (log n)^2). [Cipoli, quoted in the Wikipedia article]

a(n) = 2 + sum_{k=2..floor(2n*log(n)+2)} (1-floor(pi(k)/n)), for n>1, where the formula for pi(k) is given in A000720 (Ruiz and Sondow 2002) - Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Mar 06 2004

I conjecture that Sum(1/(p(i)*log(p(i)))=Pi/2=1.570796327... Sum(1/(i=1..100000 p(i)*log(p(i)))=1.565585514... It converges very slowly. - Miklos Kristof (kristmikl(AT)freemail.hu), Feb 12 2007

The last conjecture has been discussed by the math.research newsgroup recently. The sum, which is greater than pi/2, is computed by Mathar in sequence A137245. [From T. D. Noe (noe(AT)sspectra.com), Jan 13 2009]

A000005(a(n))=2; A002033(a(n+1))=1 [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 17 2009]

A001222(a(n))=1. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Nov 10 2009]

Contribution from Gary Detlefs (gdetlefs(AT)aol.com), Sep 10 2010: (Start)

Conjecture:

a(n) ={n| n! mod n^2 = n(n-1)}, n<>4

a(n) ={n| n!*h(n) mod n = n-1},n<>4, where h(n) = sum(1/k,k=1..n) (End)

First 15 primes; a(n) = p + abs(p-3/2) + 1/2, where p = m + int((m-3)/2), and m = n + int((n-2)/8) + int((n-4)/8), 1<=n<=15. [From Timothy Hopper (timothyhopper(AT)hotmail.co.uk), Oct 23 2010]

A000041

COMMENTS

Also number of nonnegative solutions to b+2c+3d+4e+...=n and the number of nonnegative solutions to 2c+3d+4e+...<=n. - Henry Bottomley (se16(AT)btinternet.com), Apr 17 2001

a(n) is also the number of conjugacy classes in the symmetric group S_n (and the number of irreducible representations of S_n).

Also the number of rooted trees with n+1 nodes and height at most 2.

Coincides with the sequence of numbers of nilpotent conjugacy classes in the Lie algebras gl(n). A006950, A015128 and this sequence together cover the nilpotent conjugacy classes in the classical A,B,C,D series of Lie algebras. - Alexander Elashvili, Sep 08 2003

a(n)=a(0)b(n)+a(1)b(n-2)+a(2)b(n-4)+... where b=A000009.

Number of distinct Abelian groups of order p^n, where p is prime (the number is independent of p). - Lekraj Beedassy , Oct 16 2004

Number of graphs on n vertices that do not contain P3 as an induced subgraph. - Washington Bomfim, May 10 2005

It is unknown if there are infinitely many partition numbers divisible by 3, although it is known that there are infinitely many divisible by 2. - Jonathan Vos Post, Jun 21 2005

Numbers of terms to be added when expanding the n-th derivative of 1/f(x). - Thomas Baruchel (baruchel(AT)users.sourceforge.net), Nov 07 2005

a(n) = A114099(9*n). - Reinhard Zumkeller, Feb 15 2006

Comment from Maurice D. Craig (towenaar(AT)optusnet.com.au), Oct 30 2006: sequence agrees with expansion of Molien series for symmetric group S_n up to the term in x^n.

Also the number of nonnegative integer solutions to x_1+x_2+x_3+...+x_n=n such that n>=x_1>=x_2>=x_3>=...>=x_n>=0, because by letting y_k=x_k-x_(k+1)>=0 (where 0<k<n) we get y_1+2y_2+3y_3+...+(n-1)y_(n-1)+nx_n=n. - Werner Grundlingh (wgrundlingh(AT)gmail.com), Mar 14 2007

Let P(z):= Sum{j=0..inf} b_j z^j, b_0 != 0. Then 1/P(z) = Sum{j=0..inf} c_j z^j, where the c_j must be computed from the infinite triangular system b_0 c_0 = 1, b_0 c_1 + b_1 c_0 = 0 and so on (Cauchy products of the coefficients set to zero). The n-th partition number arises as the number of terms in the numerator of the expression for c_n: The coefficient c_n of the inverted power series is a fraction with b_0^(n+1) in the denominator and in its numerator having a(n) products of n coefficients b_i each. The partitions may be read off from the indices of the b_i. - Peter C. Heinig (algorithms(AT)gmx.de), Apr 09 2007

A026820(a(n),n) = A134737(n) for n>0. - Reinhard Zumkeller, Nov 07 2007

Equals row sums of triangle A137683 - Gary W. Adamson, Feb 05 2008

This is also the number of parts equal to 1 in the outer shell of the partitions of n+1 (see A138151). - Omar E. Pol (info(AT)polprimos.com), Apr 17 2008

a(n)= the number of different ways to run up a staircase with n steps, taking steps of sizes 1,2,3,... and r (r<=n), where the order is not important and there is no restriction on the number or the size of each step taken. - Mohammad K. Azarian (azarian(AT)evansville.edu), May 21 2008

Equals the eigenvector of triangle A145006 and row sums of the eigentriangle of the partition numbers, A145007. [From Gary W. Adamson, Sep 28 2008]

Contribution from Gary W. Adamson , Oct 05 2008: (Start)

Starting with offset 1 = INVERT transform of (1, 1, 0, 0, -1, 0, -1,...),

where A080995, the characteristic function of A001318 (1, 2, 5, 7, 12,...) is

signed (++ -- ++,...) as to 1's. This is equivalent to Lim__{n=1..inf}

A145006^n as a vector. The INVERT transform of (1, 1, 0, 0, -1,...) begins (1, 2,..)

then for each successive operation we take a dot product of (1, 1, 0, 0, -1,...) in reverse and the ongoing results of our series (1, 2, 3, 5, 7,...)

then add the result to the next term in (1, 1, 0, 0, -1,...). For example, a

(7) = 15 = (0, -1, 0, 0, 1, 1) dot (1, 2, 3, 5, 7, 11) = (0*1, (-1)*2, 0*3, 0*5, 1*7, 1*11)

= (-2 + 7 + 11) = 16, then add to (-1) = 15. (End)

Convolved with A147843 = A000203 prefaced with a zero: (0, 1, 3, 4, 7,...). [From Gary W. Adamson, Nov 15 2008]

Contribution from Gary W. Adamson , Jun 12 2009: (Start)

Equals an infinite convolution product_(1,1,1,...)*(1,0,1,0,1,...)*

(1,0,0,1,0,0,1,...)*(1,0,0,0,1,0,0,0,1,...)* ...; = a*b*c*...; where a =

(1/(1-x), b = (1/(1-x^2), c = (1/(1-x^3), ...etc. An array by rows: row 1 =

a, row 2 = a*b, row 3 = a*b*c,...; gives:

1, 1, 1, 1, 1, 1,. 1,. 1,. 1,..1,... = (a)................................

1, 1, 2, 2, 3, 3,. 4,..4,. 5,..5,... = (a*b)..............................

1, 1, 2, 3, 4, 5,. 7,..8,.10,.11,... = (a*b*c)............................

1, 1, 2, 3, 4, 5,. 6,..9,.11,.17,... = (a*b*c*d)..........................

1, 1, 2, 3, 5, 5,. 7,.10,.13,.18,... = (a*b*c*d*e)........................

1, 1, 2, 3, 5, 7,.11,.14,.20,.25,... = (a*b*c*d*e*f)......................

1, 1, 2, 3, 5, 7,.11,.15,.21,.27,... = (a*b*c*d*e*f*g)....................

1, 1, 2, 3, 5, 7,.11,.15,.22,.28,... = (a*b*c*d*e*f*g*h)..................

1, 1, 2, 3, 5, 7,.11,.15,.22,.29,... = (a*b*c*d*e*f*g*h*i)................

...with rows tending to A000041. Partition triangles A058398 = ascending

antidiagonals. Partition triangle A008284 reversal of A058398. (End)

a(n) is also the number of partitions of 2n into even parts. More generally, it appears that a(n) is also the number of partitions of k*n into parts divisible by k, for k>0. [From Omar E. Pol, Nov 20 2009, Nov 25 2009]

Starting with offset 1 = row sums of triangle A168532 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 28 2009]

Contribution from Reinhard Zumkeller, Jan 21 2010: (Start)

a(n) = A026820(n,n);

a(n) = A108949(n)+A045931(n)+A108950(n) = A130780(n)+A171966(n)-A045931(n) =A045931(n)+A171967(n). (End)

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 11 2010: (Start)

P(x) = A(x)/A(x^2) with P(x) = (1+x+2x^2+3x^3+5x^4+7x^5 + ...)

A(x) = (1+x+3x^2+4x^3+10x^4+13x^5 + ...),

A(x^2) = (1+x^2+3x^4+4x^6+10x^8+ ...), where A092119 = (1, 1, 3, 4, 10,...) =

Euler transform of the ruler sequence, A001511. (End)

Equals row sums of triangle A173304 [From Gary W. Adamson, Feb 15 2010]

p(x) = A(x)*A(x^2), A(x) = A174065; p(x) = B(x)*B(x^3), B(x) = A174068. Equals row sums of triangles A174066 and A174067 [From Gary W. Adamson, Mar 06 2010]

Contribution from Gary W. Adamson, Apr 11 2010: (Start)

Triangle A113685 is equivalent to p(x) = p(x^2) * A000009(x). Triangle

A176202 is equivalent to p(x) = p(x^3) * A000726(x). (End)

Contribution from Peter Luschny, Oct 24 2010: (Start)

A sequence of positive integers p = p_1...p_k is a descending partition of the positive integer n if p_1 +...+ p_k = n and p_1 >=...>= p_k. If formally needed p_j = 0 is appended to p for j > k. Let P_n denote the set of these partition for some n >= 1. Then

a(n) = 1 + sum_{p in P_n} floor((p_1-1)/(p_2+1)).

(Cf. A000065, where the formula reduces to the sum.) Proof in Kelleher and O'Sullivan (2009). For example a(6) = 1+0+0+0+0+1+0+0+1+1+2+5 = 11. (End)

a(n) is also A027293(n+k,k), the number of partitions of n+k that contain k as a part [From Omar E. Pol, Nov 27 2010]

Contribution from Jerome Malenfant, Feb 14. 2011: (Start)

Let n=sum(k_(p_m)) p_m)=k_1 + 2k_2 + 5k_5 + 7k_7 + ..., where p_m is the m-th generalized pentagonal number (A001318). Then a(n) is the sum over all such pentagonal partitions of n of (-1)^(k_5+k_7 + k_22 + . . .) (k_1+k_2+k_5 + . . .)! /(k_1! k_2! k_5! . . .), where the exponent of (-1) is the sum of all the k's corresponding to even-indexed GPN's. (End)

Contribution from Jerome Malenfant, Feb 14. 2011: (Start)

The matrix of a(n) values

a(0)
a(1) a(0)
a(2) a(1) a(0)
a(3) a(2) a(1) a(0)
....
a(n) a(n-1) a(n-2)...a(0)

is the inverse of the matrix

 1
-1 1
-1 -1 1
 0 -1 -1 1
....
-d_n -d_(n-1) -d_(n-2) ...-d_1 1

where d_q = (-1)^(m+1) if q = m(3m-1)/2 = the m-th generalized pentagonal number (A001318), = 0 otherwise. (End)

Equals row sums of triangle A187566 - Gary W. Adamson, Mar 21 2011.

Let k>0 be an integer, and let i_1,i_2,...,i_k be distinct integers such that 1<=i_1<i_2<...<i_k. Then, equivalently, a(n) equals the number of partitions of N=n+i_1+i_2+...+i_k in which each i_j (1<=j<=k) appears as a part at least once. To see this, note that the partitions of N of this class must be in 1-to-1 correspondence with the partitions of n, since N-i_1-i_2-...-i_k=n. - L. Edson Jeffery, April 16, 2011.

a(n) is the number of ordered degree sequences of all trees on vertex set {1,2,...,n+2}. Take a partition of the integer n, add 1 to each part and append 1's so that the total is 2n+2. Now you have a degree sequence of a tree with n+2 nodes. Example: The partition 3+2+1=6 corresponds to the degree sequence {4,3,2,1,1,1,1,1} of a tree with 8 vertices. *Geoffrey Critzer April 16, 2011.

a(n-1) is number of partitions of 2*n-1 into n parts [From Vladimir Kruchinin, May 03 2011]

FORMULA

G.f.: Product_{k>0} 1/(1-x^k) = Sum_{k>= 0} x^k Product_{i = 1..k} 1/(1-x^i) = 1+Sum_{k>0} x^(k^2)/(Product_{i = 1..k} (1-x^i))^2.

G.f.: 1+sum(n>=1, x^n/prod(k>=n ,1-x^k)) - Joerg Arndt, Jan 29 2011.

a(n) - a(n-1) - a(n-2) + a(n-5) + a(n-7) - a(n-12) - a(n-15) + ... = 0, where the sum is over n-k and k is a generalized pentagonal number (A001318) <= n and the sign of the k-th term is (-1)^([(k+1)/2]). See A001318 for a good way to remember this!

a(n) = (1/n) * Sum_{k=0, 1, ..., n-1} sigma(n-k)*a(k), where sigma(k) is the sum of divisors of k (A000203).

a(n) ~ 1/(4*n*sqrt(3)) * e^(Pi * sqrt(2n/3)) as n -> infinity (Hardy and Ramanujan).

a(n) < exp((2/3)^(1/2) pi sqrt(n)) (Ayoub, p. 197).

G.f.: Product (1+x^m)^A001511(m); m=1..inf. - Vladeta Jovovic (vladeta(AT)eunet.rs), Mar 26 2004

a(n)=sum(i=0, n-1, P(i, n-i)), where P(x, y) is the number of partitions of x into at most y parts and P(0, y)=1. - Jon Perry (perry(AT)globalnet.co.uk), Jun 16 2003

G.f. : product(i=1, oo, product(j=0, oo, (1+x^((2i-1)*2^j))^(j+1))) - Jon Perry (perry(AT)globalnet.co.uk), Jun 06 2004

G.f. e^{Sum_{k>0} (x^k/(1-x^k)/k)}. - Frank Adams-Watters, Feb 08 2006

Euler transform of all 1's sequence (A000012). Weighout transform of A001511. - Frank Adams-Watters, Mar 15 2006

a(n) = A027187(n)+A027193(n) = A000701(n)+A046682(n). - Reinhard Zumkeller, Apr 22 2006

1/(1-x) + sum(k>=1, x^k*(k+1)/ prod(i=1,k, (1-x^i)^2)*(1-x^k+1)) (the pronic equivalent of the Durfee Square GF) [From Jon Perry, Aug 02 2008]

Convolved with A152537 gives A000079, powers of 2. [From Gary W. Adamson, Dec 06 2008]

a(n) = Tr(n)/(24*n-1) = A183011(n)/A183010(n), n>=1. See the Bruinier-Ono paper in the link. [From Omar E. Pol, Jan 23 2011]

Contribution from Jerome Malenfant, Feb 14. 2011: (Start)

a(n) = determinant of the n by n Toeplitz matrix:

1 -1

1   1 -1

0   1   1 -1

0   0   1   1 -1

-1   0   0   1   1 -1

. . . .

d_n d_(n-1) d_(n-2)...1

where d_q = (-1)^(m+1) if q = m(3m-1)/2 = p_m, the m-th generalized pentagonal number (A001318), otherwise d_q = 0. Note that the 1's run along the diagonal and the -1's are on the superdiagonal. The (n-1) row, (not written), would end with ... 1 -1. (End)

Empirical: let F*(x)=Sum(p(n)*exp(-Pi*x*(n+1)),n=0..infinity),

then F*(2/5) = 1/sqrt(5) to a precision of 13 digits.

F*(4/5) = 1/2+3/2/sqrt(5)-sqrt(1/2*(1+3/sqrt(5))) to a precision of 28 digits. These are the only values found for a/b when a/b is from F60, Farey fractions up to 60. The number for F*(4/5) is one of the real roots of 25*x^4 - 50*x^3 - 10*x^2 - 10*x + 1. Note here the exponent (n+1) compared to the standard notation with n starting at 0. Simon Plouffe, Feb. 23 2011.

The constant (2^(7/8)*GAMMA(3/4))/(exp(Pi/6)*Pi^(1/4)) = 1.0000034873... when expanded in base exp(4*Pi) will give the first 52 terms of a(n), n>0, the precision needed is 300 decimal digits. Simon Plouffe, March 2 2011.

a(n) = A035363(2n). [From Omar E. Pol, Nov 20 2009]

A000043

COMMENTS

It is believed (but unproved) that this sequence is infinite. The data suggests that the number of terms up to exponent N is roughly K log N for some constant K.

Length of prime repunits in base 2.

The associated perfect number N=2^(p-1)*M(p) (=A019279*A000668=A000396), has 2p (=A061645) divisors with harmonic mean p (and geometric mean sqrt(N)). - Lekraj Beedassy (blekraj(AT)yahoo.com), Aug 21 2004

In one of his first publications Euler found the numbers up to 31 but erroneously included 41 and 47.

Equals number of bits in binary expansion of n-th Mersenne prime (A117293). - Artur Jasinski (grafix(AT)csl.pl), Feb 09 2007

Number of divisors of n-th even perfect number, divided by 2. Number of divisors of n-th even perfect number that are powers of 2. Number of divisors of n-th even perfect number that are multiples of n-th Mersenne prime A000668(n). - Omar E. Pol (info(AT)polprimos.com), Feb 24 2008

Number of divisors of n-th even superperfect number A061652(n). Numbers of divisors of n-th superperfect number A019279(n), assuming there are no odd superperfect numbers. - Omar E. Pol (info(AT)polprimos.com), Mar 01 2008

Differences between exponents when the even perfect numbers are represented as differences of powers of 2, for example: The 5th even perfect number is 33550336 = 2^25 - 2^12 then a(5)=25-12=13 (see A135655, A133033, A090748). - Omar E. Pol (info(AT)polprimos.com), Mar 01 2008

Base 2 logarithm of (1 + n-th Mersenne prime A000668(n)). - Omar E. Pol (info(AT)polprimos.com), Mar 02 2008

Base 2 logarithm of A075398(n). - Omar E. Pol (info(AT)polprimos.com), Apr 17 2008

Number of 1's in binary expansion of n-th even perfect number (see A135650). Number of 1's in binary expansion of divisors of n-th even perfect number that are multiples of n-th Mersenne prime A000668(n) (see A135652, A135653, A135654,A135655). - Omar E. Pol (info(AT)polprimos.com), May 04 2008

Indices of the numbers A006516 that are also even perfect numbers. [From Omar E. Pol (info(AT)polprimos.com), Aug 30 2008]

Indices of Mersenne numbers A000225 that are also Mersenne primes A000668. [From Omar E. Pol (info(AT)polprimos.com), Aug 31 2008]

A modification of the Eberhart's conjecture proposed by Wagstaff (1983) which proposes that if q_n is the nth prime such that M_(q_n) is a Mersenne prime, then q_n is approximately (2^(e^(-gamma)))^n, where gamma is the Euler-Mascheroni constant. [From Jonathan Vos Post (jvospost3(AT)gmail.com), Sep 10 2010]

The (prime) number p appears in this sequence if and only if there is no prime q<2^p-1 such that the order of 2 modulo q equals p; a special case is that if p=4k+3 is prime and also q=2p+1 is prime then the order of 2 modulo q is p so p is not a term of this sequence. [From Joerg Arndt, Jan 16 2011]

FORMULA

A000043(n)=Log[(1/2)(1+Sqrt[1+8*A000396(n)])]/Log[2] [From Artur Jasinski (grafix(AT)csl.pl), Sep 23 2008]

a(n) = A000005(A061652(n)). [From Omar E. Pol (info(AT)polprimos.com), Aug 26 2009]

A000045

COMMENTS

Also called Lam{\'e}'s sequence.

F(n+2) = number of binary sequences of length n that have no consecutive 0's.

F(n+2) = number of subsets of {1,2,...,n} that contain no consecutive integers.

F(n+1) = number of tilings of a 2 X n rectangle by 2 X 1 dominoes.

F(n+1) = number of matchings in a path graph on n vertices: F(5)=5 because the matchings of the path graph on the vertices A, B, C, D are the empty set, {AB}, {BC}, {CD} and {AB, CD}. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Jun 18 2001

F(n) = number of compositions of n+1 with no part equal to 1 [Grimaldi]

Positive terms are the solutions to z = 2*x*y^4 + (x^2)*y^3 - 2*(x^3)*y^2 - y^5 - (x^4)*y + 2*y for x,y >= 0 (Ribenboim, page 193). When x=F(n), y=F(n + 1) and z>0 then z=F(n + 1).

For Fibonacci search see Knuth, Vol. 3; Horowitz and Sahni; etc.

F(n) is the diagonal sum of the entries in Pascal's triangle at 45 degrees slope. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Dec 29 2001

F(n+1) is the number of perfect matchings in ladder graph L_n = P_2 X P_n, - Sharon Sela (sharonsela(AT)hotmail.com), May 19 2002

F(n+1) = number of (3412,132)-, (3412,213)- and (3412,321)-avoiding involutions in S_n.

This is also the Horadam sequence (0,1,1,1). - Ross La Haye (rlahaye(AT)new.rr.com), Aug 18 2003

An INVERT transform of A019590. INVERT([1,1,2,3,5,8,...]) gives A000129. INVERT([1,2,3,5,8,13,21,...]) gives A028859. - Antti Karttunen, Dec 12, 2003

Number of meaningful differential operations of the k-th order on the space R^3. - Branko Malesevic (malesevic(AT)kiklop.etf.bg.ac.yu), Mar 02 2004

F(n)=number of compositions of n-1 with no part greater than 2. Example: F(4)=3 because we have 3 = 1+1+1=1+2=2+1.

F(n) = number of compositions of n into odd parts; e.g. F(6) counts 1+1+1+1+1+1, 1+1+1+3, 1+1+3+1, 1+3+1+1, 1+5, 3+1+1+1, 3+3, 5+1. - Clark Kimberling (ck6(AT)evansville.edu), Jun 22 2004

F(n) = number of binary words of length n beginning with 0 and having all runlengths odd; e.g. F(6) counts 010101, 010111, 010001, 011101, 011111, 000101, 000111, 000001. - Clark Kimberling (ck6(AT)evansville.edu), Jun 22 2004

F(n) = number of Catalan paths between the lines y = 0 and y = 3 from (0,0) to (n, GCD(n,2)). - Clark Kimberling (ck6(AT)evansville.edu), Jun 22 2004

The number of sequences (s(0),s(1),...s(n)) such that 0<s(i)<5, |s(i)-s(i-1)|=1 and s(0)=1 is F(n+1); e.g. F(5+1) = 8 corresponds to 121212, 121232, 121234, 123212, 123232, 123234, 123432, 123434. - Clark Kimberling (ck6(AT)evansville.edu), Jun 22 2004. [Corrected by Neven Juric, Jan 09 2009]

Likewise F(6+1) = 13 corresponds to these thirteen sequences with seven numbers: 1212121, 1212123, 1212321, 1212323, 1212343, 1232121, 1232123, 1232321, 1232323, 1232343, 1234321, 1234323, 1234343. - Neven Juric, Jan 09, 2008.

A relationship between F(n) and the Mandelbrot set is discussed in the link 'Le nombre d'or dans l'ensemble de Mandelbrot' (in French). - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Sep 19 2004

For n>0, the continued fraction for F(2n-1)*Phi=[F(2n);L(2n-1),L(2n-1),L(2n-1),...] and the continued fraction for F(2n)*Phi=[F(2n+1)-1;1,L(2n)-2,1,L(2n)-2,...]. Also true: F(2n)*Phi=[F(2n+1);-L(2n),L(2n),-L(2n),L(2n),...] where L(i) is the i-th Lucas number (A000204).... - Clark Kimberling (ck6(AT)evansville.edu), Nov 28 2004. [Correct by Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Oct 20 2010]

F(n) = number of permutations p of 1,2,3,...,n such that |k-p(k)|<=1 for k=1,2,...,n. (For <=2 and <=3, see A002524 and A002526.). - Clark Kimberling (ck6(AT)evansville.edu), Nov 28 2004

The ratios F(n+1)/F(n) for n>0 are the convergents to the simple continued fraction expansion of the golden section. - Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Dec 19 2004

Lengths of successive words (starting with a) under the substitution: {a -> ab, b -> a} - J. F. J. Laros (jlaros(AT)liacs.nl), Jan 22 2005

The Fibonacci sequence, like any additive sequence, naturally tends to be geometric with common ratio not a rational power of 10; consequently, for a sufficiently large number of terms, Benford's law of first significant digit {i.e., first digit 1 =< d =< 9 occurring with probability log_10(d+1) - log_10(d)} holds. - Lekraj Beedassy (blekraj(AT)yahoo.com), Apr 29 2005

a(n) = Sum(abs(A108299(n, k)): 0 <= k <= n). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jun 01 2005

a(n) = A001222(A000304(n)).

Fib(n+2)=sum(k=0..n, binomial(floor((n+k)/2),k)), row sums of A04685 4. - Paul Barry (pbarry(AT)wit.ie), Mar 11 2003

Number of order ideals of the "zig-zag" poset. See vol. 1, ch. 3, prob. 23 of Stanley. - Mitch Harris (Harris.Mitchell (AT) mgh.harvard.edu), Dec 27, 2005

F(n+1)/F(n) is also the Farey fraction sequence (see A097545 for explanation) for the golden ratio, which is the only number whose Farey fractions and continued fractions are the same. - Joshua Zucker (joshua.zucker(AT)stanfordalumni.org), May 08 2006

a(n+2) is the number of paths through 2 plates of glass with n reflections (reflections occurring at plate/plate or plate/air interfaces). Cf.A006356-A006359. - Mitch Harris (Harris.Mitchell(AT)mgh.harvard.edu), Jul 06 2006

F(n+1) equals the number of downsets (i.e. decreasing subsets)of an n-element fence, i.e. an ordered set of height 1 on {1,2,...,n} with 1 > 2 < 3 > 4 < ... n and no other comparabilities. Alternatively, F(n+1) equals the number of subsets A of {1,2,...,n} with the property that, if k is in A, then the adjacent elements of {1,2,...,n} belong to A, i.e. both k - 1 and k + 1 are in A (provided they are in {1,2,...,n}). - Brian A. Davey (B.Davey(AT)latrobe.edu.au), Aug 25 2006

Number of Kekule structures in polyphenanthrenes. See the paper by Lukovits and Janezic for details. - Parthasarathy Nambi (PachaNambi(AT)yahoo.com), Aug 22 2006

Inverse: With phi = (sqrt(5) + 1)/2, round(log_phi(sqrt((sqrt(5) a(n) + sqrt(5 a(n)^2 - 4))(sqrt(5) a(n) + sqrt(5 a(n)^2 + 4)))/2)) = n for n >= 3, obtained by rounding the arithmetic mean of the inverses given in A001519 and A001906. - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Feb 19 2007

Comment from Larry Gerstein (gerstein(AT)math.ucsb.edu), Mar 30 2007: A result of Jacobi from 1848 states that every symmetric matrix over a p.i.d. is congruent to a triple-diagonal matrix. Consider the maximal number T(n) of summands in the determinant of an n X n triple-diagonal matrix. This is the same as the number of summands in such a determinant in which the main-, sub- and super-diagonal elements are all nonzero. By expanding on the first row we see that the sequence of T(n)'s is the Fibonacci sequence without the initial stammer on the 1's.

Suppose psi=ln(phi). We get the representation F(n)=(2/sqrt(5))*sinh(n*psi) if n is even; F(n)=(2/sqrt(5))*cosh(n*psi) if n is odd. There is a similar representation for Lucas numbers (A000032). Many Fibonacci formulas now easily follow from appropriate sinh- and cosh-formulas. For example: the de Moivre theorem (cosh(x)+sinh(x))^m=cosh(mx)+sinh(mx) produces L(n)^2+5F(n)^2=2L(2n) and L(n)F(n)=F(2n) (setting x=n*psi and m=2). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Apr 18 2007

Inverse: floor(log_phi(sqr(5)*Fib(n))+0.5)=n, for n>1. Also for n>0, floor(1/2*log_phi(5*Fib(n)*Fib(n+1)))=n. Extension valid for integer n, except n=0,-1: floor(1/2*sign(Fib(n)*Fib(n+1))*log_phi|5*Fib(n)*Fib(n+1)|)=n {where sign(x) = sign of x}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), May 02 2007

F(n+2) = The number of Khalimsky-continuous functions with a two-point codomain. - Shiva Samieinia (shiva(AT)math.su.se), Oct 04 2007

From Kauffman and Lopes, Proposition 8.2, p. 21: "The sequence of the determinants of the Fibonacci sequence of rational knots is the Fibonacci sequence (of numbers)." - Jonathan Vos Post (jvospost3(AT)gmail.com), Oct 26 2007

This is a_1(n) in the Doroslovacki reference.

Let phi = 1.6180339...; then phi^n = (1/phi)*a(n) + a(n+1). Example: phi^4 = 6.8541019...= (.6180339...)*3 + 5. Also phi = 1/1 + 1/2 + 1/(2*5) + 1/(5*13) + 1/(13*34) + 1/(34*89),... - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 15 2007

The sequence of first differences, fib(n+1)-fib(n), is essentailly the same sequence: 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... - Colm Mulcahy, Mar 03 2008

a(n)= the number of different ways to run up a staircase with n steps, taking steps of odd sizes where the order is relevant and there is no other restriction on the number or the size of each step taken. - Mohammad K. Azarian (azarian(AT)evansville.edu), May 21 2008

Equals row sums of triangle A144152. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 12 2008]

Contribution from Cino Hilliard (hillcino368(AT)gmail.com), Sep 15 2008: (Start)
Except for the initial term, the numerator of the convergents to the recursion x
= 1/(x+1). (End)

Contribution from Ross Drewe (rd(AT)labyrinth.net.au), Oct 05 2008: (Start)
F(n) is the number of possible binary sequences of length n that obey the
sequential construction rule: if last symbol is 0, add the complement (1);
else add 0 or 1. Here 0,1 are metasymbols for any 2-valued symbol set. This
rule has obvious similarities to JFJ Laros's rule, but is based on addition
rather than substitution and creates a tree rather than a single sequence. (End)

F(n) = PRODUCT_{k=1, (n-1)/2} (1 + 4*Cos^2 k*pi/n); where terms = roots to the Fibonacci product polynomials, A152063. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 22 2008]

Fp == 5^((p-1)/2) mod p, p = prime; [Schroeder, p. 90]. [From Gary W. Adamson & Alexander Povolotsky (qntmpkt(AT)yahoo.com), Feb 21 2009]

(Ln)^2 - 5*(Fn)^2 = 4*(-1)^n. Example: 11^2 - 5*5 = -4. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 11 2009]

Output of Kasteleyn's formula for the number of perfect matchings of an m x n grid specializes to the Fibonacci sequence for m=2. [From Sarah-Marie Belcastro (smbelcas(AT)toroidalsnark.net), Jul 04 2009]

(Fib(n),Fib(n+4)) satisfies the Diophantine equation: X^2 + Y^2 - 7XY = 9*(-1)^n. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 06 2009]

(Fib(n),Fib(n+2)) satisfies the Diophantine equation: X^2 + Y^2 - 3XY = (-1)^n. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 08 2009]

a(n+2)=A083662(A131577(n)). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Sep 26 2009]

Difference between of number of closed walks of length n+1 from a node on a pentagon and number of walks of length n+1 between two adjacent nodes on a pentagon. [From Henry Bottomley (se16(AT)btinternet.com), Feb 10 2010]

F(n+1) = number of Motzkin paths of length n having exactly one weak ascent. A Motzkin path of length n is a lattice path from (0,0) to (n,0) consisting of U=(1,1), D=(1,-1) and H=(1,0) steps and never going below the x-axis. A weak ascent in a Motzkin path is a maximal sequence of consecutive U and H steps. Example: a(5)=5 because we have (HHHH), (HHU)D, (HUH)D, (UHH)D, and (UU)DD (the unique weak ascent is shown between parentheses; see A114690). [From Emeric Deutsch (deutsch(AT)duke.poly.edu), Mar 11 2010]

(F(n-1)+F(n+1))^2 - 5F(n-2)*F(n+2) = 9*(-1)^n. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Mar 31 2010]

Pinter and Ziegler show that essentially the Fibonacci sequence is the unique binary recurrence which contains infinitely many three-term arithmetic progressions. A criterion for general linear recurrences having infinitely many three-term arithmetic progressions is also given. [From Jonathan Vos Post (jvospost3(AT)gmail.com), May 22 2010]

F(n+1) = number of paths of length n starting at initial node on the path graph P_4. [From Johannes W. Meijer (meijgia(AT)hotmail.com), May 27 2010]

F(k) = Number of cyclotomic polynomials in denominator of generating function for number of ways to place k nonattacking queens on an n X n board (Vaclav Kotesovec, 31.5.2010) [From Vaclav Kotesovec (kotesovec(AT)chello.cz), Jun 07 2010]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 16 2010: (Start)

As n-> inf. (a(n)/a(n-1) - (a(n-1)/a(n)) tends to 1.0. Example: a(12)/a(11)

- a(11)/a(12) = 144/89 - 89/144 = .99992197... (End)

Comments from Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), October 20 2010 (Start)

Fibonacci numbers are those numbers m such that m*phi is closer to an integer than k*phi for all k, 1<=k<m. More formally: a(0)=0, a(1)=1, a(2)=1, a(n+1)=minimal m>a(n) such that m*phi is closer to an integer than a(n)*phi.

For all numbers 1<=k<Fib(n) the inequality |k*phi-nint(k*phi)| > |Fib(n)*phi-nint(Fib(n)*phi)| holds.

Fib(n)*phi-nint(Fib(n)*phi) = -(-phi)^(-n)), for n>1.

fract(0.5+Fib(n)*phi) =0.5 -(-phi)^(-n), for n>1.

fract(Fib(n)*phi) = (1/2)*(1+(-1)^n)-(-phi)^(-n), n>1.
Inverse: n=-log_phi |0.5-fract(0.5+Fib(n)*phi)|. (End)
Contribution from Paul Barry (pbarry(AT)wit.ie), Nov 03 2010: (Start)
The sequence 1,1,1,2,3,5,... has g.f. 1/(1-x/(1-x^2)), INVERT transform of A059841.
It is an eigensenquence for the sequence array for A059841. (End)
F(A001177(n)*k) mod n = 0, for any integer k.[From Gary Detlefs (gdetlefs(at)aol.com) Nov 27 2010]
F(n+k)^2-F(n)^2= F(k)*F(2n+k), for even k [From Gary Detlefs (gdetlefs(at)aol.com) Dec 04 2010]
F(n+k)^2+F(n)^2 = F(k)*F(2n+k), for odd k [From Gary Detlefs (gdetlefs(at)aol.com) Dec 04 2010]
"Even the Fibonacci sequence - 1,1,2,3,5,8,13 - follows Benford's law." see Pickover.
FORMULA
G.f.: x/(1-x-x^2).
F(n)=((1+sqrt(5))^n-(1-sqrt(5))^n)/(2^n*sqrt(5)).
Alternatively, F(n) = ((1/2+sqrt(5)/2)^n-(1/2-sqrt(5)/2)^n)/sqrt(5).
F(n) = F(n-1) + F(n-2) = -(-1)^n F(-n).
F(n) = round(phi^n/sqrt(5)).
F(n+1) = Sum(0 <= j <= [n/2]; binomial(n-j, j))
E.g.f.: (2/sqrt(5))*exp(x/2)*sinh(sqrt(5)*x/2). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Nov 30 2001
[0 1; 1 1]^n [0 1] = [F(n); F(n+1)]
x | F(n) ==> x | F(kn).
A sufficient condition for F(m) to be divisible by a prime p is (p - 1) divides m, if p == 1 or 4 (mod 5); (p + 1) divides m, if p == 2 or 3 (mod 5); or 5 divides m, if p = 5. (This is essentially Theorem 180 in Hardy and Wright.) - Fred W. Helenius (fredh(AT)ix.netcom.com), Jun 29, 2001
a(n)=F(n) has the property: F(n)*F(m) + F(n+1)*F(m+1) = F(n+m+1) - Miklos Kristof (kristmikl(AT)freemail.hu), Nov 13 2003
Kurmang. Aziz. Rashid (Kurmang.Rashid(AT)Btopenworld.com), Feb 21 2004, makes 4 conjectures and gives 3 theorems:
Conjecture 1: for n>=2 sqrt{F(2n+1)+F(2n+2)+F(2n+3)+F(2n+4)+2*(-1)^n}={F(2n+1)+2*(-1)^n}/F(n-1). Conjecture 2: for n>=0, {F(n+2)* F(n+3)}-{F(n+1)* F(n+4)}+ (-1)^n = 0.
Conjecture 3: for n>=0, F(2n+1)^3 - F(2n+1)*[(2*A^2) -1] - [A + A^3]=0, where A= {F(2n+1)+sqrt{5*F(2n+1)^2 +4}}/2
Conjecture 4: for x>=5, if x is a Fibonacci number >= 5 then g*x*[{x+sqrt{5*(x^2) +- 4}}/2]*[2x+{{x+sqrt{5*(x^2) +- 4}}/2}]*[2x+{{3x+3*sqrt {5*(x^2) +- 4}}/2}]^2+[2x+{{x+sqrt{5*(x^2) +- 4}}/2}] +- x*[2x+{{3x+3*sqrt{5*(x^2) +- 4}}/2}]^2 -x*[2x+{{x+sqrt{5*(x^2) +- 4}}/2}]*[x+{{x+sqrt{5*(x^2) +- 4}}/2}]* [2x+ {{3x+3*sqrt{5*(x^2) +- 4}}/2}]^2= 0, where g = {1 + sqrt 5 /2}.
Theorem 1: for n>=0, {F(n+3)^ 2 - F(n+1)^ 2}/F(n+2)={F(n+3)+ F(n+1)}. Theorem 2: for n>=0, F(n+10) = 11* F(n+5) + F(n). Theorem 3: for n>=6, F(n) = 4* F(n-3) + F(n-6).

Conjecture 2 of Rashid is actually a special case of the general law F(n)*F(m) + F(n+1)*F(m+1) = F(n+m+1) (take n <- n+1 and m <- -(n+4) in this law). - Harmel Nestra (harmel.nestra(AT)ut.ee), Apr 22 2005

Conjecture: for all c such that 2-Phi <= c < 2*(2-Phi) we have F(n) = floor(Phi*a(n-1)+c) for n > 2 - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Jul 21 2004

|2*Fib(n) - 9*Fib(n+1)| = 4*A000032(n) + A000032(n+1). - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Aug 13 2004

For x > Phi, Sum n=0..inf F(n)/x^n = x/(x^2 - x - 1) - Gerald McGarvey (gerald.mcgarvey(AT)comcast.net), Oct 27 2004

F(n+1) = exponent of the n-th term in the series f(x, 1) determined by the equation f(x, y) = xy + f(xy, x). - Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Dec 19 2004

a(n-1)=sum(k=0, n, (-1)^k*binomial(n-ceil(k/2), floor(k/2))) - Benoit Cloitre (benoit7848c(AT)orange.fr), May 05 2005

F(n+1)=sum{k=0..n, binomial((n+k)/2, (n-k)/2)(1+(-1)^(n-k))/2}; - Paul Barry (pbarry(AT)wit.ie), Aug 28 2005

Fibonacci(n)=Product(1 + 4[cos(j*Pi/n)]^2, j=1..ceil(n/2)-1). [Bicknell and Hoggatt, pp. 47-48] - Emeric Deutsch, Oct 15 2006

F(n)=2^-(n-1)*sum{k=0..floor((n-1)/2), binomial(n,2*k+1)*5^k}; - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Feb 07 2006

a(n)=(b(n+1)+b(n-1))/n where {b(n)} is the sequence A001629 - Sergio Falcon (sfalcon(AT)dma.ulpgc.es), Nov 22 2006

F(n*m) = Sum{k = 0..m, binomial(m,k)*F(n-1)^k*F(n)^(m-k)*F(m-k)}. The generating function of F(n*m) (n fixed, m = 0,1,2...) is G(x) = F(n)*x / ((1-F (n-1)*x)^2-F(n)*x*(1-F(n-1)*x)-(F(n)*x)^2). E.g. F(15) = 610 = F(5*3) = binomial(3,0)* F(4)^0*F(5)^3*F(3) + binomial(3,1)* F(4)^1*F(5)^2*F(2) + binomial(3,2)* F(4)^2*F(5)^1*F(1) + binomial(3,3)* F(4)^3*F(5)^0*F(0) = 1*1*125*2 + 3*3*25*1 + 3*9*5*1 + 1*27*1*0 = 250 + 225 + 135 + 0 = 610 - Miklos Kristof, Feb 12 2007

Comments from Miklos Kristof (kristmikl(AT)freemail.hu), Mar 19 2007 (Start)

Let L(n)=A000032=Lucas numbers. Then:

For a>=b and odd b, F(a+b)+F(a-b)=L(a)*F(b).

For a>=b and even b, F(a+b)+F(a-b)=F(a)*L(b).

For a>=b and odd b, F(a+b)-F(a-b)=F(a)*L(b).

For a>=b and even b, F(a+b)-F(a-b)=L(a)*F(b).

F(n+m)+(-1)^m*F(n-m)=F(n)*L(m);

F(n+m)-(-1)^m*F(n-m)=L(n)*F(m);

F(n+m+k)+(-1)^k*F(n+m-k)+(-1)^m*(F(n-m+k)+(-1)^k*F(n-m-k))=F(n)*L(m)*L(k);

F(n+m+k)-(-1)^k*F(n+m-k)+(-1)^m*(F(n-m+k)-(-1)^k*F(n-m-k))=L(n)*L(m)*F(k);

F(n+m+k)+(-1)^k*F(n+m-k)-(-1)^m*(F(n-m+k)+(-1)^k*F(n-m-k))=L(n)*F(m)*L(k);

F(n+m+k)-(-1)^k*F(n+m-k)-(-1)^m*(F(n-m+k)-(-1)^k*F(n-m-k))=5*F(n)*F(m)*F(k). (End)

Fib(n)=b(n)+(p-1)*sum{1<k<n, floor(b(k)/p)*Fib(n-k+1)} where b(k) is the digital sum analogue of the Fibonacci recurrence, defined by b(k)=ds_p(b(k-1))+ds_p(b(k-2)), b(0)=0, b(1)=1, ds_p=digital sum base p. Example for base p=10: Fib(n)=A010077(n)+9*sum{1<k<n, A059995(A010077(k))*Fib(n-k+1)}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jul 01 2007

Fib(n)=b(n)+p*sum{1<k<n, floor(b(k)/p)*Fib(n-k+1)} where b(k) is the digital product analogue of the Fibonacci recurrence, defined by b(k)=dp_p(b(k-1))+dp_p(b(k-2)), b(0)=0, b(1)=1, dp_p=digital product base p. Example for base p=10: Fib(n)=A074867(n)+10*sum{1<k<n, A059995(A074867(k))*Fib(n-k+1)}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jul 01 2007

a(n) = denominator of continued fraction [1,1,1,...], (with n ones); e.g. 2/3 = continued fraction [1,1,1]; where barover[1] = [1,1,1...] = .6180339,... - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 29 2007

F(n + 3) = 2F(n + 2) - F(n), F(n + 4) = 3F(n + 2) - F(n), F(n + 8) = 7F(n + 4) - F(n), F(n + 12) = 18F(n + 6) - F(n). - Paul Curtz (bpcrtz(AT)free.fr), Feb 01 2008

1 = 1/(1*2) + 1/(1*3) + 1/(2*5) + 1/(3*8) + 1/(5*13) + ... = 1/2 + 1/3 + 1/10 + 1/24 + 1/65 + 1/168 + ...; where A059929 = (0, 2, 3, 10, 24, 65, 168,...). - Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 16 2008

a(2^n) = prod{i=0}^{n-2}B(i) where B(i) is A001566. Example 3*7*47 = Fib(16) - Kenneth J Ramsey (Ramsey2879(AT)msn.com), Apr 23 2008

F(n) = (1/(n-1)!) * [n^(n-1) - { C(n-2,0) +4*C(n-2,1) +3*C(n-2,2) }*n^(n-2) + { 10*C(n-3,0) +49*C(n-3,1) +95*C(n-3,2) +83*C(n-3,3) +27*C(n-3,4) }*n^(n-3) - { 90*C(n-4,0) +740*C(n-4,1) +2415*C(n-4,2) +4110*C(n-4,3) +3890*C(n-4,4) +1950*C(n-4,5) +405*C(n-4,6) }*n^(n-4) + ...]. - Andre F. Labossiere (boronali(AT)laposte.net), Nov 24 2004

a(n+1)=Sum_{k, 0<=k<=n} A109466(n,k)*(-1)^(n-k). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 26 2008]

Formula from Thomas Wieder (wieder.thomas(AT)t-online.de), Feb 25 2009:

a(n) = sum_{l_1=0}^{n+1} sum_{l_2=0}^{n}...sum_{l_i=0}^{n-i}...sum_{l_n=0}^{1}

delta(l_1,l_2,...,l_i,...,l_n)

where delta(l_1,l_2,...,l_i,...,l_n) = 0 if any l_i + l_(i+1) >= 2 for i=1..n-1

and delta(l_1,l_2,...,l_i,...,l_n) = 1 otherwise.

2^n (\prod _{k=1}^n \sqrt[4]{\cos^2(k\pi/(n+1))+1/4})^2 (Kasteleyn's formula specialized) [From Sarah-Marie Belcastro (smbelcas(AT)toroidalsnark.net), Jul 04 2009]

a(n+1) =sum_{k=floor[n/2] mod 5} C(n,k) - sum_{k=floor[(n+5)/2] mod 5} C(n,k) =A173125(n)-A173126(n) =|A054877(n)-A052964(n-1)| [From Henry Bottomley (se16(AT)btinternet.com), Feb 10 2010]

If p[i]=modp(i,2) and if A is Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n)=det A. [From Milan R. Janjic (agnus(AT)blic.net), May 02 2010]

Limit(F(k+n)/F(k), k = infinity) = (L(n) + F(n)*sqrt(5))/2 with the Lucas numbers L(n)= A000032(n). [From Johannes W. Meijer (meijgia(AT)hotmail.com), May 27 2010]

For n>=1, F(n)=round(log_2(2^{\phi*F(n-1)}+2^{\phi*F(n-2)})), where \phi is Golden ratio. [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jun 24 2010, Jun 27 2010]

a(n+1)=ceil(phi*a(n)), if n is even and a(n+1)=floor(phi*a(n)), if n is odd (phi=golden ratio). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jul 01 2010]

a(n)=2*a(n-2)+a(n-3), n>2 [Gary Detlefs(gdetlefs(AT)aol.com), Sep 08 2010]

a(2^n)=Prod{i=0,...,n-1} A000032(2^i). [From Vladimir Shevelev(shevelev(AT)bgu.ac.il), Nov 28 2010].

a(n)^2 - a(n-1)^2 = a(n+1)*a(n-2), see A121646.
a(n) = sqrt((-1)^k*(a(n+k)^2-a(k)*a(2n+k))), for any k [From Gary Detlefs (gdetlefs(at)aol.com) Dec 03 2010]
F(2*n) = F(n+2)^2 - F(n+1)^2 - 2*F(n)^2 [From Rick Forberg (rrforberg(at)yahoo.com) June 4, 2011

A001519

COMMENTS

If the initial term is deleted, this is a bisection of the Fibonacci sequenceA000045.

Number of ordered trees with n+1 edges and height at most 3 (height=number of edges on a maximal path starting at the root). Number of directed column-convex polyominoes of area n+1. Number of nondecreasing Dyck paths of length 2n+2. - Emeric Deutsch, Jul 11 2001

Terms for n>1 are the solutions to : 5x^2-4 is a square. - Benoit Cloitre (benoit7848c(AT)orange.fr), Apr 07 2002

a(1) = 1, a(n+1) = smallest Fibonacci number greater than the n-th partial sum. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Oct 21 2002

The fractional part of tau*a(n) decreases monotonically to zero. - Benoit Cloitre (benoit7848c(AT)orange.fr), Feb 01 2003

n such that floor(phi^2*n^2)-floor(phi*n)^2 = 1 where phi=(1+sqrt(5))/2 - Benoit Cloitre (benoit7848c(AT)orange.fr), Mar 16 2003

Number of leftist horizontally convex polyominoes with area n+1.

Number of 31-avoiding words of length n on alphabet {1,2,3} which do not end in 3. (e.g. n=3, we have 111,112,121,122,132,211,212,221,222,232,321,322 and 332). SeeA028859. - Jon Perry, Aug 04 2003

Appears to give all solutions >1 to the equation : x^2=ceiling(x*r*floor(x/r)) where r=phi=(1+sqrt(5))/2. - Benoit Cloitre, Feb 24, 2004

a(1) = 1, a(2) = 2, then the least number such that the square of any term is just less than the geometric mean of its neighbors. a(n+1)*a(n-1)> a(n)^2. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Apr 06 2004

All positive integer solutions of Pell equation b(n)^2 - 5*a(n)^2 = -4 together with b(n)=A002878(n), n>=0. - Wolfdieter Lang, Aug 31 2004

a(n) = L(n,3), where L is defined as in A108299; see also A002878 for L(n,-3). - Reinhard Zumkeller, Jun 01 2005

Essentially same as Pisot sequence E(2,5).

Number of permutations of [n+1] avoiding 321 and 3412. E.g. a(3) = 13 because the permutations of [4] avoiding 321 and 3412 are: 1234, 2134, 1324, 1243, 3124, 2314, 2143, 1423, 1342, 4123, 3142, 2413, 2341. - Bridget Eileen Tenner (bridget(AT)math.mit.edu), Aug 15 2005

Number of 1324-avoiding circular permutations on [n+1].

A subset of the Markoff numbers (A002559). - Robert G. Wilson v, Oct 05 2005.

(x,y)=(a(n),a(n+1)) are the solutions of x/(yz)+y/(xz)+z/(xy)=3 with z=1. - Floor van Lamoen (fvlamoen(AT)hotmail.com), Nov 29 2001

Number of (s(0), s(1), ..., s(2n)) such that 0 < s(i) < 5 and |s(i) - s(i-1)| = 1 for i = 1,2,....,2n, s(0) = 1, s(2n) = 1. - Herbert Kociemba (kociemba(AT)t-online.de), Jun 10 2004

With interpolated zeros, counts closed walks of length n at the start or end node of P_4. a(n) counts closed walks of length 2n at the start or end node of P_4. The sequence 0,1,0,2,0,5,.. counts walks of length n between the start and second node of P_4. - Paul Barry, Jan 26 2005

a(n) = number of ordered trees on n edges containing exactly one non-leaf vertex all of whose children are leaves (every ordered tree must contain at least one such vertex). For example, a(0) = 1 because the root of the tree with no edges is not considered to be a leaf and the condition "all children are leaves" is vacuously satisfied by the root and a(4) = 13 counts all 14 ordered trees on 4 edges (A000108) except (ignore dots)

|..|

.\/.

which has two such vertices. - David Callan (callan(AT)stat.wisc.edu), Mar 02 2005

Number of directed column-convex polyominoes of area n. Example: a(2)=2 because we have the 1 X 2 and the 2 X 1 rectangles. - Emeric Deutsch, Jul 31 2006

Same as the number of Kekule structures in polyphenanthrene in terms of the number of hexagons in extended (1,1)-nanotubes. See Table 1 on page 411 of I. Lukovits and D. Janezic. - Parthasarathy Nambi (PachaNambi(AT)yahoo.com), Aug 22 2006

Number of free generators of degree n of symmetric polynomials in 3-noncommuting variables. - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Oct 24 2006

Inverse: With phi = (sqrt(5) + 1)/2, log_phi((sqrt(5) a(n) + sqrt(5 a(n)^2 - 4))/2) = n for n >= 1. - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Feb 19 2007

Consider a teacher who teaches one student, then he finds he can teach two students while the original student learns to teach a student. And so on with every generation an individual can teach one more student then he could before. a(n) starting at a(2) gives the total number of new students/teachers (see program). - Ben Thurston (benthurston27(AT)yahoo.com), Apr 11 2007

The Diophantine equation a(n)=m has a solution (for m>=1) iff ceiling(arsinh(sqr(5)*m/2)/log(phi))) != ceiling(arcosh(sqr(5)*m/2)/log(phi))) where phi is the golden ratio. An equivalent condition is A130255(m)=A130256(m). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), May 24 2007

a(n+1)= B^(n)(1), n>=0, with compositions of Wythoff's complementary A(n):=A000201(n) and B(n)=A001950(n) sequences. See the W. Lang link under A135817 for the Wythoff representation of numbers (with A as 1 and B as 0 and the argument 1 omitted). E.g. 2=`0`, 5=`00`, 13=`000`,..., in Wythoff code.

The sequence starting (1, 2, 5, 13,...) = row sums of triangle A140068. - Gary W. Adamson, May 04 2008

Bisection of the Fibonacci sequence into odd indexed non-zero terms (1, 2, 5, 13,...) and even indexed terms (1, 3, 8, 21,...) may be represented as row sums of companion triangles A140068 and A140069. - Gary W. Adamson, May 04 2008

a(n) = number of partitions pi of [n] (in standard increasing form) such that Flatten[pi] is a (2-1-3)-avoiding permutation. Example: a(4)=13 counts all 15 partitions of [4] except 13/24 and 13/2/4. Here "standard increasing form" means the entries are increasing in each block and the blocks are arranged in increasing order of their first entries. Also number that avoid 3-1-2. - David Callan (callan(AT)stat.wisc.edu), Jul 22 2008

Equals row sums of triangle A152251 starting with offset 1. [From Gary W. Adamson, Nov 30 2008]

From Gary W. Adamson, Dec 27 2008: (Start)
Let P = the partial sum operator, A000012: (1; 1,1; 1,1,1;...) and A153463
= M, the partial sum & shift operator. It appears that beginning with any
randomly taken sequence S(n), iterates of the operations M * S(n), -> M * ANS,
-> P * ANS,...etc, (or starting with P) will rapidly converge upon a two-
sequence limit cycle of (1, 2, 5, 13, 34,...) and (1, 1, 3, 8, 21,...). (End)

Row sums of triangle A153342 = odd indexed Fibonacci numbers. [From Gary W. Adamson, Dec 24 2008]

a(n) = 0.25*(A153266(n) + A153267(n)), apart from initial terms [From Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Jan 02 2009]

Sum_{n>=0} atan(1/a(n)) = (3/4)Pi [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Feb 27 2009]

Syntactically similar to A001906 in the sense that both have the n-th term t(n) given by t(n) = 3t(n-1) + t(n-2), for n > 1. [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), Apr 24 2009]

Summation of the squares of fibonacci numbers taken 2 at a time. Offset 1. a(3)=5. [From Al Hakanson (hawkuu(AT)gmail.com), May 27 2009]

Number of musical compositions of Rhythm-music over a time period of n-1 units. Example: a(4)=13; indeed, denoting by R a rest over a time period of 1 unit and by N[j] a note over a period of j units, we have (writing N for N[1]): NNN, NNR, NRN, RNN, NRR, RNR, RRN, RRR, N[2]R, RN[2], NN[2], N[2]N, N[3]) (see the J. Groh reference, pp. 43-48). [From Juergen K. Groh (juergen.groh(AT)lhsystems.com), Jan 17 2010]

From Gary W. Adamson, Feb 18 2010: (Start)
Given an infinite lower triangular matrix M with (1, 2, 3,...) in every
column but the leftmost column shifted upwards one row. Then (1, 2, 5,...)
= Lim_{n->inf} M^n. (Cf. A144257). (End)

As a fraction: 8/71 = 0.112676 or 98/9701 = 0.010102051334... (Fraction 9/71 or 99/9701 for sequence without initial term). 19/71 or 199/9701 for sequence in reverse. [From M. Dols (markdols99(AT)yahoo.com), May 18 2010]

Starting (1, 2, 5, 13,...) = sums of A179806 row terms. [From Gary W. Adamson, Jul 28 2010]

Starting (1, 2, 5, 13,...) = row sums of triangle A179745 [From Gary W. Adamson, Jul 25 2010]

For n>=1, a(n) is the number of compositions (ordered integer partitions) of 2n-1 into an odd number of odd parts. O.g.f.:(x-x^3)/(1-3x^2+x^4)= A(A(x))where A(x)=
1/(1-x)-1/(1-x^2)

For n>0, determinant of the n x n tridiagonal matrix with 1's in the super and subdiagonals, (1,3,3,3,...) in the main diagonal, and the rest zeros. - Gary W. Adamson, Jun 27 2011

The Hosoya index H(n) of the n-path graph P_n is given by H(2n-1) = 0 and H(2n) = a(n+1). [Eric W. Weisstein, Jul 11 2011]

The Gi3 sums, see A180662, of the triangles A108299 and A065941 equal the terms of this sequence without a(0). [Johannes W. Meijer, Aug 14 2011]

FORMULA

G.f.: (1-2*x)/(1-3*x+x^2).

a(n) = 3*a(n-1) - a(n-2).

a(n) = a(1-n).

a(1)=1, a(2)=2, a(n+2)=(a(n+1)^2+1)/a(n) - Benoit Cloitre, Aug 29 2002 a(n)=(phi^(2n-1)+phi^(1-2n))/sqrt(5) where phi=(1+sqrt(5))/2. - Michael Somos, Oct 28 2002

a(n) = A007598(n-1)+A007598(n) = A000045(n-1)^2+A000045(n)^2 = F(n)^2+F(n+1)^2 - Henry Bottomley (se16(AT)btinternet.com), Feb 09 2001

a(n)=sum(binomial(n+k, 2k), k=0..n). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Dec 09 2001

a(n) ~ (1/5)*sqrt(5)*phi^(2*n+1) - Joe Keane (jgk(AT)jgk.org), May 15 2002

a(n) = Sum(k=0, n, C(n, k)*F(k+1)) - Benoit Cloitre, Sep 03 2002

Let q(n, x)=sum(i=0, n, x^(n-i)*binomial(2*n-i, i)); then q(n, 1)=a(n) (this comment is essentially the same as that of L. Smiley) - Benoit Cloitre, Nov 10 2002

a(n) = (1/2)*(3*a(n-1)+sqrt(5*a(n-1)^2-4)) - Benoit Cloitre, Apr 12 2003

Main diagonal of array defined by T(i, 1)=T(1, j)=1, T(i, j)=Max(T(i-1, j)+T(i-1, j-1); T(i-1, j-1)+T(i, j-1)) - Benoit Cloitre, Aug 05 2003

Hankel transform of A002212. E.g. Det([1, 1, 3;1, 3, 10;3, 10, 36])= 5 . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jan 25 2004

Solutions x > 0 to equation floor(x*r*floor(x/r)) = floor(x/r*floor(x*r)) when r=phi - Benoit Cloitre, Feb 15 2004

a(n)=sum(i=0, n, binomial(n+i, n-i)) - Jon Perry, Mar 08 2004

a(n)= S(n, 3) - S(n-1, 3) = T(2*n+1, sqrt(5)/2)/(sqrt(5)/2) with S(n, x)=U(n, x/2), resp. T(n, x), Chebyshev's polynomials of the second, resp. first kind. See triangle A049310, resp. A053120. - Wofdieter Lang, Aug 31 2004

a(n)= ((-1)^n)*S(2*n, I), with the imaginary unit I and S(n, x)=U(n, x/2) Chebyshev's polynomials of the second kind, A049310. - Wolfdieter Lang, Aug 31 2004

a(n)=sum_{0<=i_1<=i_2<=n}binomial(i_2, i_1)*binomial(n, i_1+i_2) - Benoit Cloitre, Oct 14 2004

a(n) = a(n - 1) + SUM[i = 0 to n - 1] a(i) a(n) = Fib(2n + 1) SUM[i = 0 to n - 1] a(i) = Fib(2n) - Andras Erszegi (erszegi.andras(AT)chello.hu), Jun 28 2005

The i-th term of the sequence is the entry (1, 1) of the i-th power of the 2 by 2 matrix M=((1, 1), (1, 2)). - Simone Severini, Oct 15 2005

a(n-1)=(1/n)*sum_{k=0...n}B(2k)*F(2n-2k)*binomial(2n, 2k) where B(2k) is the (2k)-th Bernoulli number - Benoit Cloitre, Nov 02 2005

a(n) = A055105(n,1)+A055105(n,2)+A055105(n,3) = A055106(n,1)+A055106(n,2) - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Oct 24 2006

a(n)=2/sqrt(5)*cosh((2n-1)*psi), where psi=ln(phi) and phi=(1+sqrt(5))/2. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Apr 24 2007

a(n) = (phi+1)^n - phi*A001906(n) with phi=(1+sqrt(5))/2. - Reinhard Zumkeller, Nov 22 2007

a(n) = 2*a(n-1)+2*a(n-2)-a(n-3); a(n) = (sqrt(5.0) + 5.0)/10.0*(3.0/2.0 + sqrt(5.0)/2.0)^(n-2) + (-sqrt(5.0) + 5.0)/10.0*(3.0/2.0 - sqrt(5.0)/2.0)^(n-2). - Antonio A. Olivares (olivares14031(AT)yahoo.com), Mar 21 2008

a(n)=A147703(n,0). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 29 2008]

a(n) = -a(n-1)+11*a(n-2)-4*a(n-3), this formula (it is one of two 3rd order linear recurrence relations given for this sequence) is a result of the generating floretion Z = X*Y with X = 1.5'i + 0.5i' + .25(ii + jj + kk + ee) and Y = 0.5'i + 1.5i' + .25(ii + jj + kk + ee). [From Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Jan 02 2009]

With X,Y defined as X = (F(n) F(n+1)), Y = (F(n+2) F(n+3)), where F(n) is the n-th Fibonacci number (A000045), it follows a(n+2) = X.Y', where Y' is the transpose of Y (n >= 0). [From Kailasam Viswanathan Iyer, Apr 24 2009]

a(n)= Fibonacci(2n+2) mod Fibonacci(2n), n>1 [From Gary Detlefs, Nov 22 2010]

a(n) = (Fibonacci(n-1)^2+Fibonacci(n)^2+Fibonacci(2n-1))/2 [From Gary Detlefs, Nov 22 2010]

A001906

COMMENTS

n such that 5*n^2 + 4 is a square. - Gregory V. Richardson (omomom(AT)hotmail.com), Oct 13 2002

Apart from initial terms, also Pisot sequences E(3,8), P(3,8), T(3,8). See A008776for definitions of Pisot sequences.

a(n)=n+sum(k=0,n-1,sum(i=0,k,a(i))). - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 26 2003

Binomial transform of A000045. - Paul Barry (pbarry(AT)wit.ie), Apr 11 2003

Number of walks of length 2n+1 in the path graph P_4 from one end to the other one. Example: a(2)=3 because in the path ABCD we have ABABCD, ABCBCD and ABCDCD. - Emeric Deutsch, Apr 02 2004

Simplest example of a second-order recurrence with the sixth term a square.

Number of (s(0), s(1), ..., s(2n)) such that 0 < s(i) < 5 and |s(i) - s(i-1)| = 1 for i = 1,2,....,2n, s(0) = 1, s(2n) = 3. - Lekraj Beedassy, Jun 11 2004

a(n) (for n>0) is the smallest positive integer that cannot be created by summing at most n values chosen among the previous terms (with repeats allowed). - Andrew Weimholt, Jul 20 2004

a(n+1) = (A005248(n+1) - A001519(n))/2. - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Aug 15 2004

All nonnegative integer solutions of Pell equation b(n)^2 - 5*a(n)^2 = +4 together with b(n)=A005248(n), n>=0. - Wolfdieter Lang, Aug 31 2004

a(n+1) is a Chebyshev transform of 3^n (A000244), where the sequence with g.f. G(x) is sent to the sequence with g.f. (1/(1+x^2))G(x/(1+x^2)) - Paul Barry, Oct 25 2004

a(n) = the number of unique products of matrices A, B, C, in (A+B+C)^n where commutator [A,B]= 0 but C does not commute with A or B. - Paul D. Hanna and Max Alekseyev, Feb 01 2006

Number of binary words with exactly k-1 strictly increasing runs. Example: a(3)=F(6)=8 because we have 0|0,1|0,1|1,0|01,01|0,1|01,01|1 and 01|01. Column sums of A119900. - Emeric Deutsch, Jul 23 2006

See Table 1 on page 411 of Lukovits and Janezic paper. - Parthasarathy Nambi, Aug 22 2006

Inverse: With phi = (sqrt(5) + 1)/2, log_phi((sqrt(5) a(n) + sqrt(5 a(n)^2 + 4))/2) = n. - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Feb 19 2007

[1,3,8,21,55,144,...] is the Hankel transform of [1,1,4,17,75,339,1558,...](seeA026378). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Apr 13 2007

The Diophantine equation a(n)=m has a solution (for m>=1) iff floor(arsinh(sqr(5)*m/2)/ln(phi))<>floor(arcosh(sqr(5)*m/2)/ln(phi)) where phi is the golden ratio. An equivalent condition is A130259(m)=A130260(m). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), May 25 2007

a(n+1)= AB^(n)(1), n>=0, with compositions of Wythoff's complementary A(n):=A000201(n) and B(n)=A001950(n) sequences. See the W. Lang link under A135817for the Wythoff representation of numbers (with A as 1 and B as 0 and the argument 1 omitted). E.g. 1=`1`, 3=`10`, 8=`100`, 21=`1000`,..., in Wythoff code.

Bisection of the Fibonacci sequence into odd indexed non-zero terms (1, 2, 5, 13,...) and even indexed terms (1, 3, 8, 21,...) may be represented as row sums of companion triangles A140068 and A140069. - Gary W. Adamson, May 25 2008

Equals row sums of triangles A140069, A140736 and A140737. - Gary W. Adamson, May 25 2008

a(n) is also the number of idempotent order-preserving partial transformations (of an n-element chain) of width n (width(alpha) = max(Im(alpha))). Equivalently, it is the number of idempotent order-preserving full transformations (of an n-element chain). [From A. Umar (aumarh(AT)squ.edu.om), Sep 08 2008]

Contribution from Udita Katugampola (SIU) (uditanalin(AT)yahoo.com), Sep 24 2008: (Start)

a(n) is the number of ways that a string of 0,1 and 2 of size n can be arranged with no 12-pairs.

Here a(1)=3, a(2)=8, a(3)=21 and so on. a(n)=1/sqrt(5){phi^(2*n+2)-phi^(-2*n-2)}, where phi=(1+sqrt(5))/2 - The Golden Ratio

Thus it gives every other Fibonacci number starting with 3. (End)

Starting with offset 1 = row sums of triangle A175011 [From Gary W. Adamson, Apr 03 2010]

As a fraction: 1/71 = 0.01408450... or 1/9701 = 0.0001030821.... [From M. Dols (markdols99(AT)yahoo.com), May 18 2010]

Sum of the products of the elements in the compositions of n (example for n=3: the compositions are 1+1+1, 1+2, 2+1, and 3; a(3) = 1*1*1 + 1*2 + 2*1 + 3 = 8). [From Dylon Hamilton, Jun 20 2010, Geoffrey Critzer, Joerg Arndt, Dec 06 2010]

Contribution from Gary W. Adamson, Aug 15 2010: (Start)

a(n) relates to regular polygons with even numbers of edges such that

PRODUCT_{k=1..(n-2)/2} (1 + 4*Cos^2 k*Pi/n) = even indexed Fibonacci numbers

with a(n) relating to the 2*n-Gons. The constants as products = roots to even

indexed rows of triangle A152063.

For example: a(5) = 55 satisfies the product formula relating to the 10-Gon,

Alternatively, product of roots to x^4 - 12x^3 + 51x^2 - 90x + 55, (10-th row

of triangle A152063) = (4.618...)*(3.618...)*(2.381...)*(1.381...) = 55. (End)

a(n) is the number of generalized compositions of n when there are i different types of i, (i=1,2,...). [From Milan R. Janjic (agnus(AT)blic.net), Aug 26 2010]

Contribution from Gary W. Adamson, Aug 28 2010: Starting with "1" = row sums of triangle A180339, and eigensequence of triangle A137710.

FORMULA

G.f.: x / (1 - 3*x + x^2).

a(n) = 3*a(n-1) - a(n-2).

a(n) = -a(-n).

a(n) = (ap^n - am^n)/(ap-am), with ap := (3+sqrt(5))/2, am := (3-sqrt(5))/2.

Invert transform of natural numbers: a(n)=Sum_{k=1..n} k*a(n-k), a(0)=1. - Vladeta Jovovic (vladeta(AT)eunet.rs), Apr 27 2001

a(n) = S(n-1, 3) with S(n, x)=U(n, x/2) Chebyshev's polynomials of the 2nd kind, seeA049310.

a(n) = Sum(k=0, n, C(n, k)*F(k)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Sep 03 2002

Lim. n-> Inf. a(n)/a(n-1) = 1 + phi = (3 + Sqrt(5))/2 This sequence includes all of the elements of A033888 combined with A033890.

a(0)=0, a(1)=1, a(2)=3, a(n)*a(n-2)+1=a(n-1)^2. - Benoit Cloitre, Dec 06 2002

a(n) = Sum_{k=1..n} binomial(n+k-1, n-k). - Vladeta Jovovic (vladeta(AT)eunet.rs), Mar 23 2003

E.g.f. (2/sqrt(5))*exp(3*x/2)*sinh(sqrt(5)*x/2) - Paul Barry (pbarry(AT)wit.ie), Apr 11 2003

Second diagonal of array defined by T(i, 1)=T(1, j)=1, T(i, j)=Max(T(i-1, j)+T(i-1, j-1); T(i-1, j-1)+T(i, j-1)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 05 2003

a(n)=F(n)*L(n)=A000045(n)*A000032(n). - Lekraj Beedassy, Nov 17 2003

Fib(2n+2)=1, 3, 8, ... is the binomial transform of Fib(n+2). - Paul Barry, Apr 24 2004

Partial sums of A001519(n). - Lekraj Beedassy, Jun 11 2004

a(n)=Sum(C(2n-1-i, i)5^(n-i-1)(-1)^i, i=0, .., n-1). - Mario Catalani (mario.catalani(AT)unito.it), Jul 23 2004

a(n)=sum{k=0..n, binomial(n+k, n-k-1)}=sum{k=0..n, binomial(n+k, 2k+1)}

a(n+1)=sum{k=0..floor(n/2), binomial(n-k, k)(-1)^k*3^(n-2k)} - Paul Barry, Oct 25 2004

a(n) = (1/5)*(n*L(n)-F(n)) = sum(k=0..n-1, (-1)^n*Lucas(2*n-2*k-1)).

The i-th term of the sequence is the entry (1, 2) in the i-th power of the 2 by 2 matrix M=((1, 1), (1, 2)). - Simone Severini, Oct 15 2005

Computation suggests that this sequence is the Hankel transform of A005807. The Hankel transform of {a(n)} is Det[{{a(1), ..., a(n)}, {a(2), ..., a(n+1)}, ..., {a(n), ..., a(2n-1)}}] - John W. Layman, Jul 21 2000

a(n+1) = Sum[i=0..n, Sum[j=0..n, C(n-i, j)*C(n-j, i)]]. - N. J. A. Sloane, Feb 20 2005

a(n)=2/sqrt(5)*sinh(2n*psi), where psi:=log(phi) and phi=(1+sqrt(5))/2. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Apr 24 2007

a(n) = ((phi+1)^n - A001519(n))/phi with phi=(1+sqrt(5))/2. - Reinhard Zumkeller, Nov 22 2007

Row sums of triangle A135871 such that F(2n) = sum of n-th row terms of A135871. Example: F(10) = 55 = (1 - 27 + 81). - Gary W. Adamson, Dec 02 2007

a(n)^2 = sum(k=1..n, a(2k-1)). This is a property of any sequence S(n) such that S(n) = B*S(n-1) - S(n-2) with S(0) = 0 and S(1) = 1 including {0,1,2,3,... where B = 2. - Kenneth J Ramsey, Mar 23 2008

a(n)=1/sqrt(5)*(phi^(2*n+2)-phi^(-2*n-2)), where phi=(1+sqrt(5))/2 - The Golden Ratio [From Udita Katugampola (SIU) (uditanalin(AT)yahoo.com), Sep 24 2008]

If p[i]=i and if A is Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n)=det A. [From Milan R. Janjic (agnus(AT)blic.net), May 02 2010]

If p[i]=stirling2(i,2) and if A is the Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n-1)= det A. [From Milan R. Janjic (agnus(AT)blic.net), May 08 2010]

a(n) = fib(2n+10) mod fib(2n+5)

a(n) = 2*a(n-1) + a(n-2) + a(n-3) + ... + a(1) + 1, with a(0)=0. - Gary W. Adamson, Feb 19 2011

a(n) is equal to the permanent of the (n-1) X (n-1) Hessenberg matrix with 3's along the main diagonal, i's along the superdiagonal and the subdiagonal (i is the imaginary unit), and 0's everywhere else. [From John M. Campbell, June 9 2011] a(n), n>1 is equal to the determinant of an (n-x)x(n-1) tridiagonal matrix with 3's in the main diagonal, 1's in the super and subdiagonals, and the rest zeros. - Gary W. Adamson, Jun 27 2011

a(n)=b such that Integral_{x=0..Pi/2} sin(n*x)/(3/2-cos(x)) dx = c + b*ln(3) [From Francesco Daddi (francesco.daddi(AT)libero.it), Aug 01 2011]

A000048

COMMENTS

Also 2n-bead balanced binary necklaces of fundamental period 2n that are equivalent to their complements; binary Lyndon words of length n with an odd number of 1's; number of binary irreducible polynomials of degree n having trace 1.

Also number of binary vectors (x_1,...x_n) satisfying Sum_{i=1..n} i*x_i = 1 (mod n+1) = size of Varshamov-Tenengolts code VT_1(n).

The trace of a polynomial of degree n is the coefficient of x^(n-1); the subtrace is the coefficient of x^(n-2).

Also number of binary Lyndon words with trace 1 over GF(2).

Number of self-reciprocal polynomials of degree 2n over GF(2).

Also the number of dynamical cycles of period 2n of a threshold Boolean automata network which is a quasi-minimal negative circuit of size nq where q is odd and which is updated in parallel. [Mathilde Noual (mathilde.noual(AT)ens-lyon.fr), Mar 03 2009]

Also the number of 3-elements orbits of the symmetric group S3 action on irreducible polynomials of degree 2n, n>1, over GF(2). [From Jean-Francis Michon, Philippe Ravache (philippe.ravache(AT)univ-rouen.fr), Oct 04 2009]

FORMULA

a(n) = 1/(2*n) * Sum_{odd d divides n} mu(d)*2^(n/d).

A000055

COMMENTS

Also, number of unlabeled 2-gonal 2-trees with n 2-gons.

Equals INVERTi transform of A157904: (1, 2, 4, 8, 17, 36, 78, 170,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 08 2009]

Equals left border of triangle A157905 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 08 2009]

Contribution from Robert Munafo (mrob27(AT)gmail.com), Jan 24 2010: (Start)

Also counts classifications of K items that require exactly N-1 binary partitions; see Munafo link at A005646, also A171871 and A171872.

The 11 trees for N=7 are illustrated at the Munafo web link.

Link to A171871/A171872 conjectured by Robert Munafo, then proved by Andrew Weimholt and Franklin T. Adams-Watters on Dec 29 2009. (End)

FORMULA

G.f.: A(x) = 1 + T(x)-T^2(x)/2+T(x^2)/2, where T(x) = x + x^2 + 2*x^3 + ... is g.f. for A000081

A000058

COMMENTS

Also called Euclid numbers.

Another version begins 1, 2, 3, 7, 43, 1807, ..., but the initial 1 seems artificial.

The greedy Egyptian representation of 1 is 1 = 1/2 + 1/3 + 1/7 + 1/43 + 1/1807 + ...

Take a square. Divide it into 2 equal rectangles by drawing a horizontal line. Divide the upper rectangle into 2 squares. Now you can divide the lower one into another 2 squares, but instead of doing so draw a horizontal line below the first one so you obtain a (2+1=3)x1 rectangle which can be divided in 3 squares. Now you have a 6x1 rectangle at the bottom. Instead of dividing it into 6 squares, draw another horizontal line so you obtain a (6+1=7)x1 rectangle and a 42x1 rectangle left. Etc... - Nestor Romeral Andres (nestor.romeral(AT)gmail.com), Oct 29 2001

More generally one may define f(1) = x_1, f(2) = x_2, ..., f(k) = x_k, f(n) = f(1)*...*f(n-1)+1 for n>k and natural numbers x_i (i = 1, ..., k) which satisfy GCD(x_i, x_j) = 1 for i<>j. By definition of the sequence we have that for each pair of numbers x, y from the sequence GCD(x, y) = 1. An interesting property of a(n) is that for n >= 2 1/a(1)+1/a(2)+...1/a(n-1) = (a(n)-2)/(a(n)-1). Thus we can also write a(n) = (1/a(1)+1/a(2)+...1/a(n-1)-2)/(1/a(1)+1/a(2)+...1/a(n-1)-1). - Frederick Magata (fmagata(AT)mi.uni-koeln.de), May 10 2001

A greedy sequence: a(n+1) is the smallest integer > a(n) such that 1/a(1) + 1/a(2) + ... + 1/a(n+1) doesn't exceed 1. - Ulrich Schimke, Nov 17, 2002

The sequence gives infinitely many ways of writing 1 as the sum of Egyptian fractions: Cut the sequence anywhere and decrement the last element. 1 = 1/2 + 1/3 + 1/6 = 1/2 + 1/3 + 1/7 + 1/42 = 1/2 + 1/3 + 1/7 + 1/43 + 1/1806 = ... - Ulrich Schimke, Nov 17, 2002

Consider the mapping f(a/b) = (a^3 + b)/(a +b^3). Taking a = 1 b = 2 to start with and carrying out this mapping repeatedly on each new (reduced) rational number gives the following sequence 1/2,1/3,4/28=1/7,8/344=1/43,... 1/2,1/3,1/7,1/43,1/1807,... Sequence contains the denominators. Also the sum of the series converges to 1. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 22 2003

a(1) = 2, then the smallest number == 1 (mod all previous terms). a(2n+6) == 443 (mod 1000) and a(2n+7) == 807 (mod 1000). - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Sep 24 2003

An infinite coprime sequence defined by recursion.

Apart from the initial 2, a subsequence of A002061. It follows that no term is a square.

It appears that a(k)^2 + 1 divides a(k+1)^2 + 1. - David W. Wilson (davidwwilson(AT)comcast.net), May 30 2004. This is true since a(k+1)^2 + 1 = (a(k)^2 - a(k) + 1)^2 +1 = (a(k)^2-2*a(k)+2)*(a(k)^2 + 1) (a(k+1)=a(k)^2-a(k)+1 by definition). - Pab Ter (pabrlos(AT)yahoo.com), May 31 2004

In general, for any m>0 coprime to a(0), the sequence a(n+1) = a(n)^2 -ma(n) + m is infinite coprime (Mohanty). This sequence has (m,a(0))=(1,2); (2,3) is A000215; (1,4) is A082732; (3,4) is A000289; (4,5) is A000324.

Any prime factor of a(n) has -3 as its quadratic residue (Granville, exercise 1.2.3c in Pollack).

Note that values need not be prime, the first composites being 1807 = 13 * 139 and 10650056950807 = 547 * 19569939581. [From Jonathan Vos Post (jvospost3(AT)gmail.com), Aug 03 2008]

Comment from Nick McClendon, May 14 2009: If one takes any subset of the sequence comprising the reciprocals of the first n terms, with the condition that the first term is negated, then this subset has the property that the sum of its elements equals the product of its elements.

Thus -1/2 = -1/2, -1/2 + 1/3 = -1/2 * 1/3, -1/2 + 1/3 + 1/7 = -1/2 * 1/3 * 1/7, -1/2 + 1/3 + 1/7 + 1/43 = -1/2 * 1/3 * 1/7 * 1/43, and so on.

Apart the first term which is 2 we can easily prove that the number of units of a(n) is 3 or 7 alternately. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 01 2009]

(a(n)+a(n+1)) divides a(n)*a(n+1)-1 because a(n)*a(n+1) - 1 = a(n)*(a(n)^2 - a(n) + 1) - 1 = a(n)^3 - a(n)^2 + a(n) - 1 = (a(n)^2 + 1)*(a(n) - 1) = (a(n) + a(n)^2 - a(n) + 1)*(a(n) - 1) = (a(n) + a(n+1))*(a(n) - 1). [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Aug 29 2009]

FORMULA

a(n) = 1 + a(0)*a(1)*...*a(n-1).

a(n) = a(n-1)*(a(n-1)-1)+1; Sum(i=0 to oo) 1/a(i) = 1. - Nestor Romeral Andres (cashogor(AT)yahoo.com), Oct 29 2001

Vardi showed that a(n) = floor(c^(2^(n+1))+1/2) where c=1.2640847353053011130795995... - Benoit Cloitre (benoit7848c(AT)orange.fr), Nov 06 2002 (But see the Aho-Sloane paper!)

a(n) = A007018(n+1)+1 = A007018(n+1)/A007018(n) [A007018 is a(n)=a(n -1)^2+a(n-1), a(0)=1] - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Oct 11 2004

a(n) = SQRT(A174864(n+1)/A174864(n)). [From Giovanni Teofilatto (g.teofilatto(AT)tiscalinet.it), Apr 02 2010]

A000069

COMMENTS

This sequence and A001969 give the unique solution to the problem of splitting the nonnegative integers into two classes in such a way that sums of pairs of distinct elements from either class occur with the same multiplicities [Lambek and Moser]. Cf. A000028, A000379.

En francais: les nombres impies.

Has asymptotic density 1/2, since exactly 2 of the 4 numbers 4k, 4k+1, 4k+2, 4k+3 have an even sum of bits, while the other 2 have an odd sum. - J. O. Shallit, Jun 04, 2002

Nim-values for game of mock turtles played with n coins.

A115384(n) = number of odious numbers <= n; A000120(a(n))=A132680(n). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Aug 26 2007

Indices of ones in the Thue-Morse sequence A010060. [From Tanya Khovanova (tanyakh(AT)yahoo.com), Dec 29 2008]

Contribution from Pietro Majer (majer(AT)dm.unipi.it), Mar 15 2009: (Start)

For any positive integer m, the partition of the set of the first 2^m

positive integer numbers into evil ones E and odious ones O is a fair

division for any polynomial sequence p(k) of degree less than m, that is,

sum_{k in E}p(k)=sum_{k in O}p(k) holds for any polynomial p with deg(p)<m (End)

For n>1 let b(n) = a(n-1). Then b(b(n)) = 2b(n). - Benoit Cloitre, Oct 07 2010.

FORMULA

G.f.: 1+sum[k>=0, t(2+2t+5t^2-t^4)/(1-t^2)^2 * prod(l=0, k-1, 1-x^(2^l)), t=x^2^k]. - Ralf Stephan, Mar 25 2004

a(n) = 1/2 * (4n + 1 + (-1)^A000120(n)). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Sep 14 2003

Numbers n such that A010060(n)=1 - Benoit Cloitre (benoit7848c(AT)orange.fr), Nov 15 2003

a(2*n+1) + a(2*n) = A017101(n) = 8*n+3 . a(2*n+1) - a(2*n) gives the Thue-Morse sequence (1, 3 version): 1, 3, 3, 1, 3, 1, 1, 3, 3, 1, 1, 3, 1, ... A001969(n) +A000069(n) = A016813(n) = 4*n+1 . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 04 2004

(-1)^a(n)=2*A010060(n)-1 - Benoit Cloitre (benoit7848c(AT)orange.fr), Mar 08 2004

a(0) = 1, a(2n) = a(n) + 2n, a(2n+1) = -a(n) + 6n + 3.

A000079

COMMENTS 2^0 = 1 is the only odd power of 2.

Number of subsets of an n-set.

There are 2^(n-1) compositions (ordered partitions) of n - see for example Riordan. This is the unlabeled analogue of the preferential labelings sequence A000670.

This is also the number of weakly unimodal permutations of 1..n, that is, permutations with exactly one local maximum. E.g. a(5)=16: 12345, 12354, 12453, 12543, 13452, 13542, 14532 and 15432 and their reversals. - Jon Perry, Jul 27 2003. Proof: see next line! See also A087783.

Proof: n must appear somewhere and there are 2^(n-1) possible choices for the subset that precedes it. These must appear in increasing order and the rest must follow n in decreasing order. QED. - N. J. A. Sloane, Oct 26, 2003.

a(n+1) = smallest number that is not the sum of any number of (distinct) earlier terms.

Same as Pisot sequences E(1,2), L(1,2), P(1,2), T(1,2). See A008776 for definitions of Pisot sequences.

With initial 1 omitted, same as Pisot sequences E(2,4), L(2,4), P(2,4), T(2,4). - David W. Wilson.

Not the sum of two or more consecutive numbers. - Lekraj Beedassy, May 14 2004

Least deficient or near-perfect numbers (i.e. n such that sigma(n)=A000203(n)=2n-1). - Lekraj Beedassy, Jun 03 2004. Comment from Max Alekseyev, Jan 26 2005: All the powers of 2 are least deficient numbers but it is not known if there exists a least deficient number not a power of 2.

Almost-perfect numbers referred to as least deficient or slightly defective (Singh 1997) numbers. Does near-perfect numbers refer to both almost-perfect numbers (sigma(n) = 2n-1) and quasi-perfect numbers (sigma(n) = 2n+1)? There are no known quasi-perfect or least abundant or slightly excessive (Singh 1997) numbers.

The sum of the numbers in the n-th row of Pascal's triangle; the sum of the coefficients of x in the expansion of (x+1)^n.

The Collatz conjecture (the hailstone sequence will eventually reach the number 1, regardless of which positive integer is chosen initially) may be restated as (the hailstone sequence will eventually reach a power of 2, regardless of which positive integer is chosen initially.)

The only hailstone sequence which doesn't rebound (except "on the ground"). - Alexandre Wajnberg (alexandre.wajnberg(AT)ulb.ac.be), Jan 29 2005

With p(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n)=sum_{i=1}^{p(n)} p(i)!/(prod_{j=1}^{d(i)} m(i,j)!) - Thomas Wieder, May 18 2005

a(n+1) = a(n) XOR 3a(n) where XOR is binary exclusive OR operator. - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 19 2005

The number of binary relations on an n-element set that are both symmetric and antisymmetric. Also the number of binary relations on an n-element set that are symmetric, antisymmetric and transitive.

The first differences are the sequence itself. - Alexandre Wajnberg & Eric Angelini (alexandre.wajnberg(AT)ulb.ac.be), Sep 07 2005

a(n) = largest number with shortest addition chain involving n additions. - David W. Wilson, Apr 23 2006

Beginning with a(1) = 0, numbers not equal to the sum of previous distinct natural numbers. - Giovanni Teofilatto, Aug 06 2006

Smallest order of exactly p(n) nonisomorphic Abelian groups, where p(n)=A000041(n). {First occurrence of p(n) in A000688(n)} - Lekraj Beedassy, Jul 11 2006

For n>=1, a(n) is equal to the number of functions f:{1,2...,n}->{1,2} such that for a fixed x in {1,2,...,n} and a fixed y in {1,2]} we have f(x)<>y. - Aleksandar M. Janjic and Milan R. Janjic (agnus(AT)blic.net), Mar 27 2007

Let P(A) be the power set of an n-element set A. Then a(n) = the number of pairs of elements {x,y} of P(A) for which x = y. - Ross La Haye (rlahaye(AT)new.rr.com), Jan 09 2008 Ross La Haye

a(n)= the number of different ways to run up a staircase with n steps, taking steps of sizes 1,2,3,... and r (r<=n), where the order IS important and there is no restriction on the number or the size of each step taken. - Mohammad K. Azarian, May 21 2008

a(n)=number of permutations on [n+1] such that every initial segment is an interval of integers. Example: a(3) counts 1234, 2134, 2314, 2341, 3214, 3241, 3421, 4321. The map "p -> ascents of p" is a bijection from these permutations to subsets of [n]. An ascent of a permutation p is a position i such that p(i) < p(i+1). The permutations shown map to 123, 23, 13, 12, 3, 2, 1 and the empty set respectively. - David Callan, Jul 25 2008

2^(n-1) is the largest number having n divisors (in the sense of A077569); A005179(n) is the smallest. [From T. D. Noe, Sep 02 2008]

Contribution from Bill R McEachen, Oct 29 2008: (Start)

a(n) appears to match the number of divisors of the modified primorials (excluding 2,3and 5)

Very limited range examined, PARI example shown (End)

Successive k such that EulerPhi[k]/k = 1/2. [From Artur Jasinski, Nov 07 2008]

A classical transform consists (for general a(n)) in swapping a(2n) and a(2n+1);examples for Jacobsthal A001045 and successive differences: A092808,A094359,A140505. a(n)=A000079 leads to 2,1,8,4,32,16,=A135520. [From Paul Curtz, Jan 05 2009]

This is also the (L)-sieve transform of {2,4,6,8,...,2n,...}=A005843. (See A152079 for the definition of the (L)-sieve transform.) [From John W. Layman, Jan 23 2009]

a(n) = a(n-1)-th even natural numbers (A005843) for n > 1. [From Jaroslav Krizek, Apr 25 2009]

For n >= 0, a(n) is the number of leaves in a complete binary tree of height n. For n > 0, a(n) is the number of nodes in an n-cube. [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), May 04 2009]

Permutations of n+1 elements where no element is more than one position left of its original place. For example, there are 4 such permutations of three elements: 123, 132, 213, and 312. The 8 such permutations of four elements are 1234, 1243, 1324, 1423, 2134, 2143, 3124, and 4123. [From Joerg Arndt, June 24 2009]

Catalan transform of A099087. [From R. J. Mathar, Jun 29 2009]

a(n) written in base 2: 1,10,100,1000,10000,..., i.e. (n+1)times 1, n times 0 (A011557(n)).

[From Jaroslav Krizek, Aug 02 2009]

Or, phi(n) is equal to the number of perfect partitions of n. [From Juri-Stepan Gerasimov, Oct 10 2009]

These are the 2-smooth numbers, positive integers with no prime factors greater than 2. [From Michael Porter, Oct 04 2009]

A064614(a(n)) = A000244(n) and A064614(m) < A000244(n) for m<a(n). [From Reinhard Zumkeller, Feb 08 2010]

a(n) = the largest number m such that number of steps of iterations of {r - (largest divisor d < r)} needed to reach 1 starting at r = m is equal to n. Example (a(5) = 32): 32 - 16 = 16; 16 - 8 = 8; 8 - 4 = 4; 4 - 2 = 2; 2 - 1 = 1; number 32 has 5 steps and is the largest such number. See A105017, A064097, A175125. [From Jaroslav Krizek, Feb 15 2010]

a(n) = A173786(n,n)/2 = A173787(n+1,n). [From Reinhard Zumkeller, Feb 28 2010]

a(n) is the smallest multiple of a(n-1) [From Dominick Cancilla, Aug 09 2010]

The powers-of-2 triangle T(n,k), n>=0 and 0<=k<=n, begins with: {1}; {2, 4}; {8, 16, 32}; {64, 128, 256, 512}; ... . The first left hand diagonal T(n,0) = A006125(n+1), the first right hand diagonal T(n,n) = A036442(n+1) and the center diagonal T(2*n,n) = A053765(n+1). Some triangle sums, see A180662, are: Row1(n) = A122743(n) , Row2(n) = A181174(n), Fi1(n) = A181175(n), Fi2(2*n) = A181175(2*n) and Fi2(2*n+1) = 2*A181175(2*n+1). [From Johannes W. Meijer, Oct 10 2010]

Records in the number of prime factors. [From Juri-Stepan Gerasimov, Mar 12 2011]

Row sums of A152538 [From Gary W. Adamson, Dec 10 2008]

FORMULA a(n) = 2^n.

a(n) = 2*a(n-1).

G.f.: 1/(1-2*x).

E.g.f.: exp(2*x).

2^n = Sum_{k=0..n} binomial(n, k).

a(n) is the number of occurrences of n in A000523. a(n) = A001045(n) + A001045(n+1). a(n) = 1 + sum_{k=0..(n-1)} a(k). The Hankel transform of this sequence gives A000007 = [1, 0, 0, 0, 0, 0, ...]. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 25 2004

n such that phi(n)=n/2, for n>1, where phi is the Euler's totient (A000010). - Lekraj Beedassy (lbeedassy(AT)hotmail.com), Sep 07 2004

This sequence can be generated by the following formula: a(n) = a(n-1) + 2*a(n-2) when n > 2; a[1] = 1, a[2] = 2 - Alex Vinokur (alexvn(AT)barak-online.net), Oct 24 2004

a(n) = StirlingS2(n+1,2) + 1 - Ross La Haye (rlahaye(AT)new.rr.com), Jan 09 2008 Ross La Haye

This sequence can be generated by a(n+2)=6a(n+1)-8a(n), n=1,2,3,... with a(1)=1, a(2)=2. - Yosu Yurramendi (yosu.yurramendi(AT)ehu.es), Aug 06 2008

a(n)=ka(n-1)+(4-2k)a(n-2) for any integer k and n>1, with a(0)=1, a(1)=2. [From Jaume Oliver Lafont, Dec 05 2008]

Equals the partition numbers A000041 convolved with A152537. [From Gary W. Adamson, Dec 06 2008]

Formula from Thomas Wieder, Feb 25 2009:

a(n) = sum_{l_1=0}^{n+1} sum_{l_2=0}^{n}...sum_{l_i=0}^{n-i}...sum_{l_n=0}^{1}

delta(l_1,l_2,...,l_i,...,l_n)

where delta(l_1,l_2,...,l_i,...,l_n) = 0 if any l_i <= l_(i+1) and l_(i+1) <> 0

and delta(l_1,l_2,...,l_i,...,l_n) = 1 otherwise.

G.f.: exp(x)*cosh(x) . [From Zerinvary Lajos, Apr 05 2009]

a(0)=1, a(1)=2; a(n)=a(n-1)^2/a(n-2), n>=2 [From Jaume Oliver Lafont, Sep 22 2009]

If p[i]=i-1 and if A is Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n-1)=det A. [From Milan R. Janjic (agnus(AT)blic.net), May 02 2010]

If p[i]=fibonacci(i-2) and if A is the Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=2, a(n-2)=det A. [From Milan R. Janjic (agnus(AT)blic.net), May 08 2010]

The sum of reciprocals, 1/1+1/2+1/4+1/8+...+1/(2^n)+...=2. [From Mohammad K. Azarian, Dec. 29 2010]

a(n) = 2*A001045(n)+A078008(n) = 3*A001045(n)+(-1)^n. - Paul Barry (pbarry(AT)wit.ie), Feb 20 2003

a(n) = A118654(n, 2).

a(n) = A140740(n+1, 1).

a(n)= A131577(n) +A011782(n) = A024495(n) +A131708(n) +A024493(n) = A000749(n) +A038503(n) +A038504(n) +A038505(n-1) = A139761(n) + A139748(n) + A139714(n) + A133476(n) + A139398(n). - Paul Curtz, Jul 25 2011

A000081

COMMENTS Also, number of ways of arranging n-1 nonoverlapping circles: e.g. there are 4 ways to arrange 3 circles, as represented by ((O)), (OO), (O)O, OOO. (Of course the rules here are different from the usual counting parentheses problems - compare A000108, A001190, A001699.) See link below for proof.

Euler transform is sequence itself with offset -1.

Take a string of n x's and insert n-1 ^'s and n-1 pairs of parentheses in all possible legal ways (cf. A003018). Sequence gives number of distinct functions. The single node tree is "x". Making a node f2 a child of f1 represents f1^f2. Since (f1^f2)^f3 is just f1^(f2*f3) we can think of it as f1 raised to both f2 and f3, that is, f1 with f2 and f3 as children. E.g. for n=4 the distinct functions are ((x^x)^x)^x; (x^(x^x))^x; x^((x^x)^x); x^(x^(x^x)). - Edwin Clark (eclark(AT)math.usf.edu) and Russ Cox (rsc(AT)swtch.com) Apr 29, 2003; corrected by Keith Briggs (keith.briggs(AT)bt.com), Nov 14 2005

Triangle A144963: row sums = (1, 2, 4, 9, 20,...), right border = (1, 1, 2, 4, 9,...); and left border = A051573: (1, 1, 1, 2, 3, 8, 16,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 27 2008]

Also, number of connected multigraphs of order n without cycles except for one loop. See the Bomfim link for a picture showing the bijection between rooted trees and multigraphs of this kind. [From W. Bomfim (webonfim(AT)bol.com.br), Sep 04 2010]

FORMULA G.f. A(x) = x + x^2 + 2*x^3 + 4*x^4 + ... satisfies A(x) = x*exp(A(x)+A(x^2)/2+A(x^3)/3+A(x^4)/4+...) [Polya]

Also A(x) = Sum_{n >= 1} a(n)*x^n = x / Product_{n >= 1} (1-x^n)^a(n).

Recurrence: a(n+1) = (1/n) * sum_{k=1..n} (sum_{d|k} d*a(d)) * a(n-k+1).

A000085

COMMENTS a(n) is also the number of n X n symmetric permutation matrices.

a(n) is also the number of matchings in the complete graph K(n). - Ola Veshta (olaveshta(AT)my-deja.com), Mar 25 2001. Equivalently, this is the number of graphs on n labeled nodes with degrees at most 1. - D. E. Knuth, Mar 31 2008

a(n) is also the sum of the degrees of the irreducible representations of the symmetric group S_n - Avi Peretz (njk(AT)netvision.net.il), Apr 01 2001

a(n) is the number of partitions of a set of n distinguishable elements into sets of size 1 and 2. - Karol A. Penson (penson(AT)lptl.jussieu.fr), Apr 22 2003.

Number of tableaux on the edges of the star graph of order n, S_n (sometimes T_n) - Roberto E. Martinez II (remartin(AT)fas.harvard.edu), Jan 09 2002

The Hankel transform of this sequence is A000178 (superfactorials). Sequence is also binomial transform of the sequence 1, 0, 1, 0, 3, 0, 15, 0, 105, 0, 945, . . . (A001147 with interpolated zeros) . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 10 2005

Row sums of the exponential Riordan array (e^(x^2/2),x). - Paul Barry, Jan 12 2006

a(n) = number of nonnegative lattice paths of upsteps U = (1,1) and downsteps D = (1,-1) that start at the origin and end on the vertical line x = n in which each downstep (if any) is marked with an integer between 1 and the height of its initial vertex above the x-axis. For example, with the required integer immediately preceding each downstep, a(3) = 4 counts UUU, UU1D, UU2D, U1DU. - David Callan, Mar 07 2006

The descriptions in the Mathematica lines are due to w.meeussen (wouter.meeussen(AT)pandora.be).

Equals row sums of triangle A152736 starting with offset 1. [From Gary W. Adamson, Dec 12 2008]

Proof of the recurrence relation a(n)=a(n-1)+(n-1)*a(n-2): number of involutions of [n] containing n as a fixed point is a(n-1); number of involutions of [n] containing n in some cycle (j, n), where 1<=j<=n-1, is (n-1) times the number of involutions of [n] containing the cycle (n-1 n) = (n-1)*a(n-2). [From Emeric Deutsch, Jun 08 2009]

Number of ballot sequences (or lattice permutations) of length n. A ballot sequence B is a string such that, for all prefixes P of B, h(i)>=h(j) for i<j, where h(x) is the number of times x appears in P. For example, the ballot sequences of length 4 are 1111, 1112, 1121, 1122, 1123, 1211, 1212, 1213, 1231, and 1234. The string 1221 does not appear in the list because in the 3-prefix 122 there are two 2s but only one 1. (Cf. p.176 of Bruce E. Sagan: "The Symmetric Group"). [From Joerg Arndt, Jun 28 2009]

FORMULA   a(n) = a(n-1)+A013989(n-2) = A013989(n)/(n+1).

E.g.f.: exp(x+x^2/2). a(n) = a(n-1) + (n-1)*a(n-2), n>0. a(n)=Sum_{k=0..[n/2]} n!/((n-2*k)!*2^k*k!).

a(m+n) = Sum_{k>=0} k!*binomial(m, k)*binomial(n, k)*a(m-k)*a(n-k) . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Mar 05 2004

The e.g.f. y(x) satisfies y^2 = y''y' - (y')^2.

a(n) ~ c*(n/e)^(n/2)exp(n^(1/2)) where c=2^(-1/2)exp(-1/4). [Chowla]

Special values of Hermite polynomials. In Maple notation a(n)=HermiteH(n, 1/(sqrt(2)*I))/(-sqrt(2)*I)^n, n=0, 1..., - Karol A. Penson, May 16, 2002.

a(n)=sum{k=0..n, A001498((n+k)/2, (n-k)/2)(1+(-1)^(n-k))/2}; - Paul Barry, Jan 12 2006

For asymptotics see the Robinson paper.

a[n]=Sum[A0099174[n,m],{m,0,n}]. - Roger Bagula, Oct 06 2006

O.g.f.: A(x) = 1/(1-x-1*x^2/(1-x-2*x^2/(1-x-3*x^2/(1-... -x-n*x^2/(1- ...))))) (continued fraction). - Paul D. Hanna, Jan 17 2006

Contribution from Gary W. Adamson, Dec 29 2008: (Start)

a(n) = (n-1)*a(n-2) + a(n-1); e.g. a(7) = 232 = 6*26 + 76.

Starting with offset 1 = eigsensequence of triangle A128229. (End)

a(n)=(1/sqrt(2*Pi))*int(exp(-x^2/2)*(x+1)^(n),x=-infinity..infinity)- Groux Roland, Mar 14 2011.

A000088

COMMENTS Euler transform of the sequence A001349.

Also, number of equivalence classes of sign patterns of totally nonzero symmetric n X n matrices.

FORMULA a(n)=2^binomial(n, 2)/n!*(1+(n^2-n)/2^(n-1)+8*n!/(n-4)!*(3*n-7)*(3*n-9)/2^(2*n)+O(n^5/2^(5*n/2))) (see Harary, Palmer reference). - Vladeta Jovovic (vladeta(AT)eunet.rs) and Benoit Cloitre (benoit7848c(AT)orange.fr), Feb 01 2003

a(n)=2^binomial(n, 2)/n!*[1+2*n$2*2^{-n}+8/3*n$3*(3n-7)*2^{-2n}+64/3*n$4*(4n^2-34n+75)*2^{-3n}+O(n^8*2^{-4*n})] where n$k is the falling factorial: n$k=n(n-1)(n-2)...(n-k+1). - Keith Briggs (keith.briggs(AT)bt.com), Oct 24 2005

Contribution from David Pasino (davepasino(AT)yahoo.com), Jan 31 2009: (Start)

a(n) = a(n, 2) where a(n, t), the number of t-uniform hypergraphs on n

unlabeled nodes (cf. A000665 for t = 3 and A051240 for t = 4), is

a(n, t) = (sum on c: 1*c_1+2*c_2+...+n*c_n= n) per(c)*2^f(c), where

per(c) = 1/(prod on i=1 to n) c_i!*i^c_i and f(c) = (1/ord(c)) *

(sum on r=1 to ord(c)) (sum on x: 1*x_1+2*x_2...+t*x_t=t) (prod on k = 1 to t)

binom(y(r, k; c), x_k), where ord(c) = lcm{i : c_i > 0} and y(r, k; c) =

(sum on s|r with gcd(k, r/s) = 1) s*c_(k*s) (= the number of k-cycles of

the rth power of a permutation of type c). (End)

A000105

COMMENTS a(n) + A030228(n) = A000988(n) because the number of free polyominoes plus the number of polyominoes lacking bilateral symmetry equals the number of one-sided polyominoes. - Graeme McRae" (g_m(AT)mcraefamily.com), Jan 05 2006

The possible symmetry groups of a (nonempty) polyomino are the 10 subgroups of the dihedral group D_8 of order 8: D_8, 1, Z_2 (five times), Z_4, (Z_2)^2 (twice). - Benoit Jubin, Dec 30 2008

Names for first few polyominoes: "monomino", "domino", "tromino", "tetromino", "pentomino", "hexomino", "heptomino", "octomino", "enneomino", "decomino", "hendecomino", "dodecomino", ...

lim_{n->oo} a(n)^(1/n) = mu with 3.98 < mu < 4.64 (quoted by Castiglione et al., with a reference to Barequet et al., 2006, for the lower bound). Upper bound is due to Klarner and Rivest, 1973. By Madras, 1999, it is now known that this limit, also known as Klarner's constant, is equal to the limit growth rate lim_{n->oo} a(n+1)/a(n).

FORMULA

A000108

COMMENTS The solution to Schroeder's first problem. A very large number of combinatorial interpretations are known - see references, esp. Stanley, Enumerative Combinatorics, Volume 2.

Number of ways to insert n pairs of parentheses in a word of n+1 letters. E.g. for n=3 there are 5 ways: ((ab)(cd)), (((ab)c)d), ((a(bc))d), (a((bc)d)), (a(b(cd))).

Consider all the binomial(2n,n) paths on squared paper that (i) start at (0, 0), (ii) end at (2n, 0) and (iii) at each step, either make a (+1,+1) step or a (+1,-1) step. Then the number of such paths which never go never below the x-axis (Dyck paths) is C(n) [Chung-Feller]

Number of noncrossing partitions of the n-set. For example, of the 15 set partitions of the 4-set, only [{13},{24}] is crossing, so there are a(4)=14 noncrossing partitions of 4 elements. [Joerg Arndt, Jul 11 2011]

a(n-1) is the number of ways of expressing an n-cycle in the symmetric group S_n as a product of n-1 transpositions (u_1,v_1)*(u_2,v_2)*...*(u_{n-1},v_{n-1}) where uk<=uj and vk<=vj for k<j; see example. If the condition is dropped one obtains A000272. [Joerg Arndt and Greg Stevenson, Jul 11 2011]

a(n) is the number of ordered rooted trees with n nodes, not including the root. See the Conway-Guy reference where these rooted ordered trees are called plane bushes. See also the Bergeron et al. reference, Example 4, p. 167. W. Lang Aug 07 2007.

Shifts one place left when convolved with itself.

For n >= 1 a(n) is also the number of rooted bicolored unicellular maps of genus 0 on n edges. - Ahmed Fares (ahmedfares(AT)my-deja.com), Aug 15 2001

Ways of joining 2n points on a circle to form n nonintersecting chords. (If no such restriction imposed, then ways of forming n chords is given by (2n-1)!!=(2n)!/n!2^n=A001147(n).)

Arises in Schubert calculus - see Sottile reference.

Inverse Euler transform of sequence is A022553.

With interpolated zeros, the inverse binomial transform of the Motzkin numbers A001006. - Paul Barry (pbarry(AT)wit.ie), Jul 18 2003

The Hankel transforms of this sequence or of this sequence with the first term omitted give A000012 = 1, 1, 1, 1, 1, 1, ...; example : Det([1, 1, 2, 5; 1, 2, 5, 14; 2, 5, 14, 42; 5, 14, 42, 132]) = 1 and Det([1, 2, 5, 14; 2, 5, 14, 42; 5, 14, 42, 132; 14, 42, 132, 429]) = 1 . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Mar 04 2004

c(n) = C(2*n-2,n-1)/n = (1/n!) * [n^(n-1) + { C(n-2,1) +C(n-2,2) }*n^(n-2) + { 2*C(n-3,1) +7*C(n-3,2) +8*C(n-3,3) +3*C(n-3,4) }*n^(n-3) + { 6*C(n-4,1) +38*C(n-4,2) +93*C(n-4,3) +111*C(n-4,4) +65*C(n-4,5) +15*C(n-4,6) }*n^(n-4) + ...]. - Andre F. Labossiere (boronali(AT)laposte.net), Nov 10 2004

Sum_{n=0..infinity} 1/a(n) = 2 + 4*Pi/3^(5/2) = F(1,2;1/2;1/4) = 2.806133050770763... (see L'Universe de Pi link) - Gerald McGarvey and Benoit Cloitre, Feb 13 2005

a(n) equals sum of squares of terms in row n of triangle A053121, which is formed from successive self-convolutions of the Catalan sequence. - Paul D. Hanna (pauldhanna(AT)juno.com), Apr 23 2005

Comment from Donald D. Cross (cosinekitty(AT)hotmail.com), Feb 04 2005: Also coefficients of the Mandelbrot polynomial M iterated an infinite number of times. Examples: M(0) = 0 = 0*c^0 = [0], M(1) = c = c^1 + 0*c^0 = [1 0], M(2) = c^2 + c = c^2 + c^1 + 0*c^0 = [1 1 0], M(3) = (c^2 + c)^2 + c = [0 1 1 2 1], ... M(5) = [0 1 1 2 5 14 26 44 69 94 114 116 94 60 28 8

1], ...

The multiplicity with which a prime p divides C_n can be determined by first expressing n+1 in base p. For p=2, the multiplicity is the number of 1 digits minus 1. For p an odd prime, count all digits greater than (p+1)/2; also count digits equal to (p+1)/2 unless final; and count digits equal to (p-1)/2 if not final and the next digit is counted. For example, n=62, n+1 = 223_5, so C_62 is not divisible by 5. n=63, n+1 = 224_5, so 5^3 | C_63. - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Feb 08 2006

Koshy and Salmassi give an elementary proof that the only prime Catalan numbers are a(2) = 2 and a(3) = 5. Is the only semiprime Catalan number a(4) = 14? - Jonathan Vos Post (jvospost3(AT)gmail.com), Mar 06 2006

Comment from Franklin T. Adams-Watters, Apr 14 2006: The answer is yes. Using the formula C_n = C(2n,n)/(n+1), it is immediately clear that C_n can have no prime factor greater than 2n. For n >= 7, C_n > (2n)^2, so it cannot be a semiprime. Given that the Catalan numbers grow exponentially, the above consideration implies that the number of prime divisors of C_n, counted with multiplicity, must grow without limit. The number of distinct prime divisors must also grow without limit, but this is more difficult. Any prime between n+1 and 2n (exclusive) must divide C_n. That the number of such primes grows without limit follows from the prime number theorem.

The number of ways to place n indistinguishable balls in n numbered boxes B1,...,Bn such that at most a total of k balls are placed in boxes B1,...,Bk for k=1,...,n. For example, a(3)=5 since there are 5 ways to distribute 3 balls among 3 boxes such that (i) box 1 gets at most 1 ball and (ii)box 1 and box 2 together get at most 2 balls:(O)(O)(O), (O)()(OO), ()(OO)(O), ()(O)(OO), ()()(OOO). - Dennis P. Walsh (dwalsh(AT)mtsu.edu), Dec 04 2006

a(n) is also the order of the semigroup of order-decreasing and order-preserving full transformations (of an n-element chain) - now known as the Catalan monoid [From A. Umar (aumarh(AT)squ.edu.om), Aug 25 2008]

a(n) is the number of trivial representations in the direct product of 2n spinor (the smallest) representations of the group SU(2) (A(1)). [From Rutger Boels (boels(AT)nbi.dk), Aug 26 2008]

The invert transform appears to converge to the catalan numbers when applied infinitely many times to any starting sequence. [From Mats O. Granvik, Gary W. Adamson and Roger L. Bagula (mgranvik(AT)abo.fi), Sep 09 2008, Sep 12 2008]

lim(a(n)/a(n-1): n->infinity) = 4 [From Francesco Antoni (francesco_antoni(AT)yahoo.com), Nov 24 2008]

Starting with offset 1 = row sums of triangle A154559 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 11 2009]

Contribution from Benji Fisher (benji(AT)FisherFam.org), Mar 05 2009: (Start)

C(n) is the degree of the Grassmanian G(1,n+1): the set of lines in
(n+1)-dimensional projective space, or the set of planes through the origin in
(n+2)-dimensional affine space. The Grassmanian is considered a subset of
N-dimensional projective space, N = binomial(n+2,2) - 1. If we choose 2n
general (n-1)-planes in projective (n+1)-space, then there are C(n) lines that
meet all of them. (End)

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), May 01 2009: (Start)

Starting with offset 1 = A068875: (1, 2, 4, 10, 18, 84,...) convolved with

Fine numbers, A000957: (1, 0, 1, 2, 6, 18,...). a(6) = 132 =

(1, 2, 4, 10, 28, 84) dot (18, 6, 2, 1, 0, 1) = (18 + 12 + 8 + 10 + 0 + 84) = 132. (End)

Convolved with A032443: (1, 3, 11, 42, 163,...) = powers of 4, A000302: (1, 4, 16,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), May 15 2009]

Sum{k=1...Infinity,c(k-1)/2^(2k-1)}=1. The k-th term in the summation is the probability that a random walk on the integers (begining at the origin) will arrive at positive one (for the first time) in exactly (2k-1) steps. [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Sep 12 2009]

C(p+q)-C(p)*C(q)=sum(C(i)*C(j)*C(p+q-i-j-1), i=0..(p-1), j=0..(q-1)) [From Groux roland (roland.groux(AT)orange.fr), Nov 13 2009]

Leonhard Euler used the formula C(n) = product_{i=3..n}(4*i-10)/(i-1) in his 'Betrachtungen, auf wie vielerley Arten ein gegebenes polygonum durch Diagonallinien in triangula zerschnitten werden k"onne' and computes by recursion C(n+2) for n = 1..8. (Berlin, 4th September 1751, in a letter to Goldbach). [From Peter Luschny (peter(AT)luschny.de), Mar 13 2010]

Let A179277 = A(x). Then C(x) is satisfied by A(x)/A(x^2). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 07 2010]

a(n)= A000680(n)/A006472(n) [From M.dols (markdols99(AT)yahoo.com), Jul 14 2010]

a(n) is also the number of quivers in the mutation class of type B_n or of type C_n. [From Christian Stump (christian.stump(AT)gmail.com), Nov 02 2010]

Consider a set of A000217(n) balls of n colors in which, for each integer k = 1 to n, exactly one color appears in the set a total of k times. (Each ball has exactly one color and is indistinguishable from other balls of the same color.) a(n+1) equals the number of ways to choose 0 or more balls of each color while satisfying the following conditions: 1. No two colors are chosen the same positive number of times. 2. For any two colors (c, d) that are chosen at least once, color c is chosen more times than color d iff color c appears more times in the original set than color d.

If the second requirement is lifted, the number of acceptable ways equals A000110(n+1). See related comments for A016098, A085082. [From Matthew Vandermast (ghodges14(AT)comcast.net), Nov 22 2010]

Deutsch and Sagan prove the Catalan number C_n is odd if and only if n = 2^a - 1 for some nonnegative integer a. Lin proves for every odd Catalan number C_n, we have C_n == 1 (mod 4). [Jonathan Vos Post (jvospost3(AT)gmail.com), Dec 09 2010]

a(n) is the number of functions f:{1,2,...,n}->{1,2,...,n} such that f(1)=1 and for all n>=1 f(n+1)<=f(n)+1. For a nice bijection between this set of functions and the set of length 2n Dyck words see page 333 of the fxtbook (see link below).

Complement of A092459; A010058(a(n)) = 1. [Reinhard Zumkeller, Mar 29 2011]

FORMULA   a(n) = binomial(2n, n)/(n+1) = (2n)!/(n!(n+1)!).

a(n) = binomial(2n, n)-binomial(2n, n-1)

a(n) = Sum_{k=0..n-1} a(k)a(n-1-k).

G.f.: A(x) = (1 - sqrt(1 - 4*x)) / (2*x). G.f. A(x) satisfies A = 1 + x*A^2.

a(n+1) = Sum_{i} binomial(n, 2*i)*2^(n-2*i)*a(i) - Touchard.

2(2n-1)a(n-1) = (n+1)a(n).

It is known that a(n) is odd if and only if n=2^k-1, k=1, 2, 3, ... - Emeric Deutsch, Aug 04 2002.

Using the Stirling approximation in A000142 we get the asymptotic expansion a(n) ~ 4^n /

(sqrt(Pi * n) * (n + 1)). - Dan Fux (dan.fux(AT)OpenGaia.com or danfux(AT)OpenGaia.com), Apr 13 2001

Integral representation: a(n)=int(x^n*sqrt((4-x)/x), x=0..4)/(2*Pi). - Karol A. Penson (penson(AT)lptl.jussieu.fr), Apr 12 2001

E.g.f.: exp(2x) (I_0(2x)-I_1(2x)), where I_n is Bessel function. - Karol A. Penson (penson(AT)lptl.jussieu.fr), Oct 07 2001

Polygorial(n, 6)/Polygorial(n, 3) - Daniel Dockery (peritus(AT)gmail.com) Jun 24, 2003

G.f. A(x) satisfies ((A(x)+A(-x))/2)^2 = A(4*x^2). - Michael Somos, Jun 27, 2003

G.f. A(x) satisfies Sum_{k>=1} k(A(x)-1)^k = Sum_{n >= 1} 4^{n-1} x^n. - Shapiro, Woan, Getu

a(n+m) = Sum_{k} A039599(n, k)*A039599(m, k). - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Dec 22 2003

a(n+1) = (1/(n+1))*sum_{k=0..n} a(n-k)*binomial(2k+1, k+1) . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jan 24 2004

a(n) = Sum_{k>=0} A008313(n, k)^2 . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 14 2004

a(m+n+1) = Sum_{k>=0} A039598(m, k)*A039598(n, k) . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 15 2004

a(n)=sum{k=0..n, (-1)^k*2^(n-k)*binomial(n, k)*binomial(k, floor(k/2))} - Paul Barry (pbarry(AT)wit.ie), Jan 27 2005

a(n) = Sum_{k=0..[n/2]} ((n-2*k+1)*C(n, n-k)/(n-k+1))^2, which is equivalent to: a(n) = Sum_{k=0..n} A053121(n, k)^2, for n>=0. - Paul D. Hanna (pauldhanna(AT)juno.com), Apr 23 2005

a((m+n)/2) = Sum_{k>=0} A053121(m, k)*A053121(n, k) if m+n is even . - Philippe DELEHAM, May 26 2005

E.g.f. Sum_{n>=0} a(n)*x^(2n)/(2n)! = BesselI(1, 2x)/x . - Michael Somos Jun 22 2005

Given g.f. A(x), then B(x)=x*A(x^3) satisfies 0=f(x, B(X)) where f(u, v)=u-v+(uv)^2 or B(x)=x+(x*B(x))^2 which implies B(-B(x))=-x and also (1+B^3)/B^2 = (1-x^3)/x^2 . - Michael Somos Jun 27 2005

a(n) = a(n-1)*(4-6/(n+1)). a(n) = 2a(n-1)*(8a(n-2)+a(n-1))/(10a(n-2)-a(n-1)). - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Feb 08 2006

Sum_{k=1}^{infinity} a(k)/4^k = 1. - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Jun 28 2006

a(n) = A047996(2*n+1,n) . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jul 25 2006

Binomial transform of A005043 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 20 2006

a(n)=Sum_{k, 0<=k<=n}(-1)^k*A116395(n,k) . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 07 2006

a(n)=[1/(s-n)]*sum_{k=0..n} (-1)^k (k+s-n)*binomial(s-n,k)*binomial(s+n-k,s) with s a nonnegative free integer [H. W. Gould].

a(k) = Sum_{i=1..k} |A008276(i,k)| * (k-1)^(k-i) / k! - Andre F. Labossiere (boronali(AT)laposte.net), May 29 2007

a(n)=Sum_{k, 0<=k<=n}A129818(n,k)*A007852(k+1). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 20 2007 a(n)=Sum_{k, 0<=k<=n}A109466(n,k)*A127632(k). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 20 2007

Row sums of triangle A124926 - Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 22 2007

For G.f. A(x), g(x)= x*A(x) is the compositional inverse of f(x) = x*(1-x) and this relates the Catalan numbers to the row sums of A125181. - Tom Copeland (tcjpn(AT)msn.com), Jan 13 2008

lim(1+Sum(a(k)/A004171(k): 0<=k<=n): n->infinity) = 4/pi. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Aug 26 2008]

a(n)=Sum_{k, 0<=k<=n}A120730(n,k)^2 and a(k+1)=Sum_{n, n>=k}A120730(n,k). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 18 2008]

Comment from Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 27 2008: Given an integer t >= 1 and initial values u = [a_0, a_1, ..., a_{t-1}], we may define an infinite sequence Phi(u) by setting a_n = a_{n-1} + a_0*a_{n-1} + a_1*a_{n-2} + ... + a_{n-2}*a_1 for n >= t. For example the present sequence is Phi([1]) (also Phi([1,1])).

Formula from Thomas Wieder (wieder.thomas(AT)t-online.de), Feb 25 2009:

a(n) = sum_{l_1=0}^{n+1} sum_{l_2=0}^{n}...sum_{l_i=0}^{n-i}...sum_{l_n=0}^{1}

delta(l_1,l_2,...,l_i,...,l_n)

where delta(l_1,l_2,...,l_i,...,l_n) = 0 if any l_i < l_(i+1) and l_(i+1) <> 0

for i=1..n-1 and delta(l_1,l_2,...,l_i,...,l_n) = 1 otherwise.

C(n) = (4 - 6/n) * C(n-1) with C(1) = 1 [From M. Dols (markdols99(AT)yahoo.com), Feb 14 2010]

G.f. A(x), B(x)=x*A(x) satisfies the differential equation B'(x)-2*B'(x)*B(x)-1=0 [From Vladimir Kruchinin (kru(AT)ie.tusur.ru), Jan 18 2011]

G.f.: 1/(1-x/(1-x/(1-x/(...)))) (continued fraction). [Joerg Arndt, Mar 18 2011]

A000109

COMMENTS

FORMULA

A000110

COMMENTS Number of partitions of a set of n labeled elements.

a(n-1) = number of nonisomorphic colorings of a map consisting of a row of n+1 adjacent regions. - David W. Wilson, Feb 22, 2005

If an integer is squarefree and has n distinct prime factors then a(n) is the number of ways of writing it as a product of its divisors - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Apr 23 2001

Consider rooted trees of height at most 2. Letting each tree 'grow' into the next generation of n means we produce a new tree for every node which is either the root or at height 1, which gives the Bell numbers. - Jon Perry (perry(AT)globalnet.co.uk), Jul 23 2003

Begin with [1,1] and follow the rule that [1,k] -> [1,k+1] and [1,k] k times, e.g. [1,3] is transformed to [1,4], [1,3], [1,3], [1,3]. Then a(n) is the sum of all components. [1,1]=2, [1,2],[1,1]=5, [1,3],[1,2],[1,2],[1,1],[1,2]=15, etc... - Jon Perry (perry(AT)globalnet.co.uk), Mar 05 2004

Number of distinct rhyme schemes for a poem of n lines: a rhyme scheme is a string of letters (eg, 'abba') such that the leftmost letter is always 'a' and no letter may be greater than one more than the greatest letter to its left. Thus 'aac' is not valid since 'c' is more than one greater than 'a'. For example, a(3)=5 because there are 5 rhyme schemes. aaa, aab, aba, abb, abc. - Bill Blewett

(BillBle(AT)microsoft.com), Mar 23 2004

Comment from Neven Juric, Oct 19 2009: (Start)

The case n=4: aaaa, aaab, aaba, aabb, aabc, abaa, abab, abac, abba, abbb, abbc, abca, abcb, abcc, abcd

The case n=5:

aaaaa, aaaab, aaaba, aaabb, aaabc, aabaa, aabab, aabac, aabba, aabbb,

aabbc, aabca, aabcb, aabcc, aabcd, abaaa, abaab, abaac, ababa, ababb,

ababc, abaca, abacb, abacc, abacd, abbaa, abbab, abbac, abbba, abbbb,

abbbc, abbca, abbcb, abbcc, abbcd, abcaa, abcab, abcac, abcad, abcba,

abcbb, abcbc, abcbd, abcca, abccb, abccc, abccd, abcda, abcdb, abcdc,

abcdd, abcde (End)

In other words, number of length-n restricted growth strings (RGS) [s(0),s(1),...,s(n-1)] where s(0)=0 and s(k)<=1+max(prefix) for k>=1, see example (cf. A080337 and A189845). [Joerg Arndt, Apr 30 2011]

Also the number of equivalence relations in (alternatively, or the number of partitions of) a set of n elements. - Federico Arboleda (federico.arboleda(AT)gmail.com), Mar 09 2005

Number of partitions of {1, ...,n+1} into subsets of nonconsecutive integers, including the partition 1|2|...|n+1. E.g. a(3)=5: there are 5 partitions of {1,2,3,4} into subsets of nonconsecutive integers namely 13|24, 13|2|4, 14|2|3, 1|24|3, 1|2|3|4. - A. O. Munagi (amunagi(AT)yahoo.com), Mar 20 2005

Triangle (addition) scheme to produce terms, derived from the recurrence, from Oscar Arevalo (loarevalo(AT)sbcglobal.net), May 11 2005:

1

1 2

2 3 5

5 7 10 15

15 20 27 37 ... [This is Aitken's array A011971]

With p(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, p(j,i) = the j-th part of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n)=sum_{i=1}^{p(n)} (n!/(prod_{j=1}^{p(i)}p(i,j)!)) * (1/(prod_{j=1}^{d(i)} m(i,j)!)) - Thomas Wieder (wieder.thomas(AT)t-online.de), May 18 2005

a(n+1) = the number of binary relations on an n-element set that are both symmetric and transitive. - Justin Witt (justinmwitt(AT)gmail.com), Jul 12 2005

If Jon Perry's rule is used, i.e. "Begin with [1,1] and follow the rule that [1,k] -> [1,k+1] and [1,k] k times, e.g. [1,3] is transformed to [1,4], [1,3], [1,3], [1,3]. Then a(n) is the sum of all components. [1,1]=2, [1,2],[1,1]=5, [1,3],[1,2],[1,2],[1,1],[1,2]=15, etc..." then a(n-1) = [number of components used to form a(n)] / 2 - Daniel Kuan (dkcm(AT)yahoo.com), Feb 19 2006

a(n) is the number of functions f from {1,...,n} to {1,...,n,n+1} that satisfy the following two conditions for all x in the domain: (1) f(x)>x; (2)f(x)=n+1 or f(f(x))=n+1.E.g. a(3)=5 because there are exactly five functions that satisfy the two conditions: f1={(1,4),(2,4),(3,4)}, f2={(1,4),(2,3),(3,4)}, f3={(1,3),(2,4),(3,4)}, f4={(1,2),(2,4),(3,4)} and f5={(1,3),(2,3),(3,4)}. - Dennis P. Walsh (dwalsh(AT)mtsu.edu), Feb 20 2006

Number of asynchronic siteswap patterns of length n which have no zero-throws (i.e. contain no 0's) and whose number of orbits (in the sense given by Allen Knutson) is equal to the number of balls. E.g. for n=4 the condition is satisfied by the following 15 siteswaps 4444, 4413, 4242, 4134, 4112, 3441, 2424, 1344, 2411, 1313, 1241, 2222, 3131, 1124, 1111. Also number of ways to choose n permutations from identity and cyclic permutations (1 2), (1 2 3), ..., (1 2 3 ... n) so that their composition is identity. For n=3 we get the following five: id o id o id, id o (1 2) o (1 2), (1 2) o id o (1 2), (1 2) o (1 2) o id, (1 2 3) o (1 2 3) o (1 2 3). (To see the bijection, look at Ehrenborg and Readdy paper.) - Antti Karttunen (his-firstname.his-surname(AT)gmail.com), May 01 2006.

a(n) = number of permutations on [n] in which a 3-2-1 (scattered) pattern occurs only as part of a 3-2-4-1 pattern. Example. a(3) = 5 counts all permutations on [3] except 321. See "Eigensequence for Composition" reference a(n) = number of permutation tableaux of size n (A000142) whose first row contains no 0's. Example: a(3)=5 counts {{}, {}, {}}, {{1}, {}}, {{1}, {0}}, {{1}, {1}}, {{1, 1}}. - David Callan (callan(AT)stat.wisc.edu), Oct 07 2006

Take the series 1^n/1! + 2^n/2! + 3^n/3! + 4^n/4! ... If n=1 then the result will be e, about 2.71828. If n=2, the result will be 2e. If n=3, the result will be 5e. This continues, following the pattern of the Bell numbers: e, 2e, 5e, 15e, 52e, 203e, etc. - Jonathan R. Love (japanada11(AT)yahoo.ca), Feb 22 2007

Comment from Gottfried Helms (helms(AT)uni-kassel.de), Mar 30 2007. (Start) This sequence is also the first column in the matrix-exponential of the (lower triangular) Pascal-matrix, scaled by exp(-1): PE = exp(P) / exp(1) =

....1..............................
....1......1........................
....2......2......1.................
....5......6......3.....1...........
...15.....20.....12.....4.....1.....
...52.....75.....50....20.....5....1
..203....312....225...100....30....6
..877...1421...1092...525...175...42

First 4 columns are A000110, A033306, A105479, A105480. The general case is mentioned in the two latter entries. PE is also the Hadamard-product Toeplitz(A000110) (X) P:

....1.........................
....1.....1.................
....2.....1....1............
....5.....2....1....1........
...15.....5....2....1...1.... (X) P
...52....15....5....2...1...1
..203....52...15....5...2...1
..877...203...52...15...5...2 (End)

The terms can also be computed with finite steps and precise integer arithmetic. Instead of exp(P)/exp(1) one can compute A = exp(P - I) where I is the identity-matrix of appropriate dimension since (P-I) is nilpotent to the order of its dimension. Then a(n)=A[n,1] where n is the row-index starting at 1. - Gottfried Helms helms(at)uni-kassel.de, Apr 10 2007.

Comment from David W. Wilson (davidwwilson(AT)comcast.net), Aug 04 2007 and Sep 24 2007: Define a Bell pseudoprime to be a composite number n such that a(n) == 2 (mod n). W. F.

Lunnon recently found the Bell pseudoprimes 21361 = 41*521 and C46 = 3*23*16218646893090134590535390526854205539989357 and conjectured that Bell pseudoprimes are extremely scarce. So the second Bell pseudoprime is unlikely to be known with certainty in the near future. I confirmed that 21361 is the first.

This sequence and A000587 form a reciprocal pair under the list partition transform described in A133314. - Tom Copeland (tcjpn(AT)msn.com), Oct 21 2007

Starting (1, 2, 5, 15, 52,...), equals row sums and right border of triangle A136789. Also row sums of triangle A136790. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 21 2008

This is the exp transform of A000012. [From Thomas Wieder (thomas.wieder(AT)t-online.de), Sep 09 2008]

Contribution from A. Umar (aumarh(AT)squ.edu.om), Oct 12 2008: (Start)

a(n) is also the number of idempotent order-decreasing full transformations (of an n-chain).

a(n) is also the number of nilpotent partial one-one order-decreasing transformations (of an n-chain).

a(n+1) is also the number of partial one-one order-decreasing transformations (of an n-chain). (End)

Contribution from Peter Bala (pbala(AT)toucansurf.com), Oct 19 2008: (Start)

Bell(n) is the number of n-pattern sequences [Cooper & Kennedy]. An n-pattern sequence is a sequence of integers (a_1,...,a_n) such that a_i = i or a_i = a_j for some j < i. For example, Bell(3) = 5 since the 3-pattern sequences are (1,1,1), (1,1,3), (1,2,1), (1,2,2) and (1,2,3).

Bell(n) is the number of sequences of positive integers (N_1,...,N_n) of length n such that N_1 = 1 and N_(i+1) <= 1 + max{j = 1..i} N_j for i >= 1 (see the comment by B. Blewett above). It is interesting to note that if we strengthen the latter condition to N_(i+1) <= 1 + N_i we get the Catalan numbers A000108 instead of the Bell numbers.

(End)

Equals the eigensequence of Pascal's triangle, A007318; and starting with offset 1, = row sums of triangles A074664 and A152431. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 04 2008]

Contribution from David Pasino (davpas(AT)charter.net), Dec 04 2008: (Start)

The entries f(i, j) in the exponential of the infinite lower-triangular matrix

of binomial coefficients b(i, j) are f(i, j) = b(i, j) e a(i - j). (End)

Equals Lim_{k->inf.} A071919^k [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 02 2009]

Equals A154107 convolved with A014182, where A014182 = expansion of exp(1-x-exp(-x)), the eigensequence of A007318^(-1). Starting with offset 1 = A154108 convolved with (1,2,3,...) = row sums of triangle A154109. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 04 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 14 2009: (Start)

Repeated iterates of (binomial transform of [1,0,0,0,...]) will converge upon

(1, 2, 5, 15, 52,...) when each result is prefaced with a "1"; such that the

final result is the fixed limit: (binomial transform of [1,1,2,5,15,...] = (1,2,5,15,52,...). (End)

Contribution from Karol A. Penson (penson(AT)lptl.jussieu.fr), May 03 2009: (Start)

Relation between the Bell numbers B(n) and the n-th derivative of 1/Gamma(1+x) of such derivatives through seq(subs(x=0, simplify(diff(GAMMA(1+x)^(-1),x$n))), n=1..6);

b) leave them expressed in terms of digamma (Psi(k)) and polygamma (Psi(k,n)) functions and unevaluated ;

Examples of such expressions, for n=1..5, are :

n=1: -Psi(1),

n=2: -(-Psi(1)^2+Psi(1,1)),

n=3: -Psi(1)^3+3*Psi(1)*Psi(1,1)-Psi(2,1),

n=4: -(-Psi(1)^4+6*Psi(1)^2*Psi(1,1)-3*Psi(1,1)^2-4*Psi(1)*Psi(2,1)+Psi(3, 1)),

n=5: -Psi(1)^5+10*Psi(1)^3*Psi(1,1)-15*Psi(1)*Psi(1,1)^2-10*Psi(1)^2*Psi(2,1)

+10*Psi(1,1)*Psi(2,1)+5*Psi(1)*Psi(3,1)-Psi(4,1) ;

c) for a given n read off the sum of absolute values of coefficients of every term involving digamma or polygamma functions.

This sum is equal to B(n). Examples : B(1)=1, B(2)=1+1=2, B(3)=1+3+1=5, B(4)=1+6+3+4+1=15, B(5)=1+10+15+10+10+5+1=52;

d) Observe that this decomposition of the Bell number B(n) apparently does not involve the Stirling numbers of the second kind explicitly. (End)

The numbers given above by Penson lead to the multinomial coefficients A036040. - Johannes W. Meijer, Aug 14 2009

Column 1 of A162663. [From Franklin T. Adams-Watters (FrankTAW(AT)Netscape.net), Jul 09 2009]

Asymptotic expansions (0!+1!+2!+...+(n-1)!)/(n-1)! = a(0) + a(1)/n + a(2)/n^2 + ... and (0!+1!+2!+...+n!)/n! = 1 + a(0)/n + a(1)/n^2 + a(2)/n^3 + ... - Michael Somos, Jun 28 2009

Starting with offset 1 = row sums of triangle A165194 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 06 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 06 2009: (Start)

a(n+1) = A165196(2^n); where A165196 begins: (1, 2, 4, 5, 7, 12, 14, 15,...).

such that A165196(2^3) = 15 = A000110(4). (End)

Contribution from Johannes W. Meijer (meijgia(AT)hotmail.com), Oct 16 2009: (Start)

The divergent series g(x=1,m) = 1^m*1! - 2^m*2! + 3^m*3! - 4^m*4! + ..., m=>-1, which for m=-1 dates back to Euler, is related to the Bell numbers. We discovered that g(x=1,m) = (-1)^m * (A040027(m) - A000110(m+1) * A073003). We observe that A073003 is Gompertz's constant and that A040027 was published by Gould, see for more information A163940.

(End)

a(n)= E(X^n), i.e. the n-th moment about the origin of a random variable X that has a Poisson distribution with (rate) parameter, lambda = 1. [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Nov 30 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 09 2010: (Start)

Let A000110 = S(x), then S(x) = A(x)/A(x^2) when A(x) = A173110;

or (1, 1, 2, 5, 15, 52,...) = (1, 1, 3, 6, 20, 60,...) / (1, 0, 1, 0, 3, 0, 6, 0, 20,...). (End)

The Bell numbers serve as the upper limit for the number of unique homomorphic images from any given finite universal algebra. Every algebra homomorphism is determined by its kernel, which must be a congruence relation. As the number of possible congruence relations with respect to a finite universal algebra must be a subset of its possible equivalence classes (given by the bell numbers), it follows naturally. [From Max Sills (sillsm(AT)gmail.com), Jun 01 2010]

For a proof of the o.g.f. given in the R. Stephan comment see e.g. the W. Lang link under A071919. [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Jun 23 2010]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 08 2010: (Start)

Let B(x) = (1 + x + 2x^2 + 5x^3 + ...). Then B(x) is satisfied by A(x)/A(x^2)

where A(x) = polcoeff A173110: (1 + x + 3x^2 + 6x^3 + 20x^4 + 60x^5 + ...) =

B(x) * B(x^2) * B(x^4) * B(x^8) * ... (End)

Consider a set of A000217(n) balls of n colors in which, for each integer k = 1 to n, exactly one color appears in the set a total of k times. (Each ball has exactly one color and is indistinguishable from other balls of the same color.) a(n+1) equals the number of ways to choose 0 or more balls of each color without choosing any two colors the same positive number of times. (See related comments for A000108, A008277, A016098.) [From Matthew Vandermast(ghodges14(AT)comcast.net), Nov 22 2010]

A binary counter with faulty bits starts at value 0 and attempts to increment by 1 at each step. Each bit that should toggle may or may not do so. a(n) is the number of ways that the counter can have the value 0 after n steps. e.g. n=3, the 5 trajectories are 0,0,0,0; 0,1,0,0; 0,1,1,0; 0,0,1,0; 0,1,3,0 - [from David Scambler (dscambler(AT)bmm.com) Jan 24 2011]

No Bell number is divisible by 8, see the table 1.7 by Lunnon, Pleasants and Stephens. [From Jon Perry, 07 Feb 2011]

a(n+1) is the number of (symmetric) positive semidefinite n x n 0-1 matrices. These correspond to equivalence relations on {1,...,n+1}, where matrix element M[i,j] = 1 if and only if i and j are equivalent to each other but not to n+1. [from Robert Israel (israel(AT)math.ubc.ca) Mar 16 2011]

FORMULA E.g.f.: exp (exp(x)- 1). Recurrence: a(n+1) = Sum a(k)C(n, k). Also a(n) = Sum Stirling2(n, k), k=1..n.

a(n) = SUM(j = 0 to n-1, (1/(n-1)!) * A000166(j) * C(n-1, j) * (n-j)^(n-1)). - Andre F. Labossiere (boronali(AT)laposte.net), Dec 01 2004

G.f.: sum(1/((1-k*x)*k!), k = 0 .. infinity)/exp(1) = hypergeom([-1/x], [(x-1)/x], 1)/exp(1)=((1-2*x)+LaguerreL(1/x, (x-1)/x, 1)+x*LaguerreL(1/x, (2*x-1)/x, 1))*Pi/(x^2*sin(Pi*(2*x-1)/x)), where LaguerreL(mu, nu, z) =(GAMMA(mu+nu+1)/GAMMA(mu+1)/GAMMA(nu+1))* hypergeom([-mu], [nu+1], z) is the Laguerre function, the analytic extension of the Laguerre polynomials, for mu not equal to a nonnegative integer. This generating function has an infinite number of poles accumulating in the neighborhood of x=0.- Karol A. Penson (penson(AT)lptl.jussieu.fr), Mar 25, 2002.

a(n) = exp(-1)*sum(k=>0, k^n/k!) [Dobinski] - Benoit Cloitre (benoit7848c(AT)orange.fr), May 19 2002

a(n) is asymptotic to n!*(2 Pi r^2 exp(r))^(-1/2) exp(exp(r)-1) / r^n, where r is the positive root of r exp(r) = n. - see e.g. the Odlyzko reference.

a(n) is asymptotic to b^n*exp(b-n-1/2)/sqrt(ln(n)) where b satisfies b*ln(b) = n - 1/2 (see Graham, Knuth and Patashnik, Concrete Mathematics, 2nd ed., p. 493) - Benoit Cloitre (benoit7848c(AT)orange.fr), Oct 23 2002

G.f.: sum{k>=0, x^k/prod[l=1..k, 1-lx]}. - R. Stephan, Apr 18 2004

a(n+1) = exp(-1)*sum(k=>0, (k+1)^(n)/k!) - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Jun 03 2004

For n>0, a(n) = Aitken(n-1, n-1) [i.e. a(n-1, n-1) of Aiken's array (A011971)] - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Jun 26 2004

a(n)=sum{k=1..n, (1/k!)*sum{i=1..k, (-1)^(k-i)*binomial(k, i)*i^n}}+0^n; - Paul Barry (pbarry(AT)wit.ie), Apr 18 2005

a(n) = A032347(n) + A040027(n+1) - Jon Perry (perry(AT)globalnet.co.uk), Apr 26 2005

a(n) = 2*n!/(pi*e)*Im(integral_{0}^{pi} e^(e^(e^(ix))) sin(nx) dx) where Im denotes imaginary part [Cesaro]. - David Callan (callan(AT)stat.wisc.edu), Sep 03 2005

O.g.f.: A(x) = 1/(1-x-x^2/(1-2*x-2*x^2/(1-3*x-3*x^2/(1-... -n*x-n*x^2/(1- ...))))) (continued fraction). - Paul D. Hanna (pauldhanna(AT)juno.com), Jan 17 2006

Contribution by Karol A. Penson (penson(AT)lptl.jussieu.fr), Jan 14 2007 (Start)

Representation of Bell numbers B(n), n=1,2..., as special values of hypergeometric function of type (n-1)F(n-1), in Maple notation: B(n)=exp(-1)*hypergeom([2,2,...,2],[1,1,...,1],1), n=1,2..., i.e. having n-1 parameters all equal to 2 in the numerator, having n-1 parameters all equal to 1 in the denominator and the value of the argument equal to 1.

Examples:

B(1)=exp(-1)*hypergeom([],[],1)= 1

B(2)=exp(-1)*hypergeom([2],[1],1)= 2

B(3)=exp(-1)*hypergeom([2,2],[1,1],1)= 5

B(4)=exp(-1)*hypergeom([2,2,2],[1,1,1],1)= 15

B(5)=exp(-1)*hypergeom([2,2,2,2],[1,1,1,1],1)=52

(Warning: this formula is correct but its application by a computer may not yield exact results, especially with large number of parameters)

(End)

a(n+1) = 1+sum(sum((binomial(k,i))*(2^(k-i))*(a(i)),i = 0..k),k = 0..n-1) - Yalcin Aktar (aktaryalcin(AT)msn.com), Feb 27 2007 [There was an error in this formula, so it should be checked carefully]

a(n) = [1,0,0,...,0] T^(n-1) [1,1,1,...,1]', where T is the n X n matrix with main diagonal {1,2,3,...,n}, 1's on the diagonal immediately above and 0's elsewhere. [Meier]

a(n) = ((2*n!)/(pi * e)) * ImaginaryPart(Integral[from 0 to pi](e^e^e^(i*theta))*sin(n*theta) dtheta). - Jonathan Vos Post (jvospost3(AT)gmail.com), Aug 27 2007

Formulae and comments from Tom Copeland, Oct 10 2007 (Start): a(n) = T(n,1) = sum(j=0,...,n) S2(n,j) = sum(j=0,...,n) E(n,j) * Lag(n,-1,j-n) = sum(j=0,...,n) [E(n,j)/n!] * [n!*Lag(n,-1, j-n)] where T(n,x) are the Bell / Touchard / exponential polynomials; S2(n,j), the Stirling numbers of the second kind; E(n,j), the Eulerian numbers; and Lag(n,x,m), the associated Laguerre polynomials of order m. Note that E(n,j)/n! = E(n,j)/ {sum(k=0,..,n) E(n,k)}.

The Eulerian numbers count the permutation ascents and the expression [n!*Lag(n,-1, j-n)] is A086885 with a simple combinatorial interpretation in terms of seating arrangements, giving a combinatorial interpretation to n!*a(n) = sum(j=0,...,n) {E(n,j) * [n!*Lag(n,-1, j-n)]} . (End)

Define f_1(x),f_2(x),... such that f_1(x)=e^x and for n=2,3,... f_{n+1}(x)=diff(x*f_n(x),x). Then for Bell numbers B_n we have B_n=1/e*f_n(1). - Milan R. Janjic (agnus(AT)blic.net), May 30 2008

a(n) = (n-1)! sum_{k=1}^{n} a(n-k)/((n-k)! (k-1)!) where a(0)=1. [From Thomas Wieder (thomas.wieder(AT)t-online.de), Sep 09 2008]

a(n+k) = Sum_{m=0..n} Stirling2(n,m) Sum_{r=0..k} binomial(k,r) m^r a(k-r). - David Pasino (davepasino(AT)yahoo.com), Jan 25 2009. (Umbrally, this may be written as a(n+k) = Sum_{m=0..n} Stirling2(n,m) (a+m)^k. - N. J. A. Sloane (njas(AT)research.att.com), Feb 07 2009.)

Formula from Thomas Wieder (wieder.thomas(AT)t-online.de), Feb 25 2009:

a(n) = sum_{l_1=0}^{n+1} sum_{l_2=0}^{n}...sum_{l_i=0}^{n-i}...sum_{l_n=0}^{1}

delta(l_1,l_2,...,l_i,...,l_n)

where delta(l_1,l_2,...,l_i,...,l_n) = 0 if any l_i > l_(i+1) and l_(i+1) <> 0

and delta(l_1,l_2,...,l_i,...,l_n) = 1 otherwise.

Let A be the upper Hessenberg matrix of order n defined by: A[i,i-1]=-1, A[i,j]:=binomial(j-1,i-1), (i<=j), and A[i,j]=0 otherwise. Then, for n>=1, a(n)=det(A). [From Milan R. Janjic (agnus(AT)blic.net), Jul 08 2010]

A000111

COMMENTS Number of linear extensions of the "zig-zag" poset. See ch. 3, prob. 23 of Stanley. - Mitch Harris (Harris.Mitchell (AT) mgh.harvard.edu), Dec 27, 2005

Nnumber of increasing 0-1-2 trees on n vertices. [David Callan (callan(AT)stat.wisc.edu), Dec 22 2006]

Also the number of refinements of partitions. [Heinz-Richard Halder (halder.bichl(AT)t-online.de), Mar 07 2008]

The ratio a(n)/n! is also the probability that n numbers x1,x2,..,xn randomly choosen uniformly and independently in [0,1] satisfy x1>x2<x3>x4<...xn. [Pietro Majer (majer(AT)dm.unipi.it), Jul 13 2009]

For n >= 2, a(n-2) = number of permutations w of an ordered n-set {x_1 < ... x_n} with the following properties: w(1)=x_n, w(n)=x_{n-1}, w(2)>w(n-1), and neither any subword of w, nor its reversal, has the first three properties. The count is unchanged if the third condition is replaced with w(2)<w(n-1). [From Jeremy Martin (jmartin(AT)math.ku.edu), Mar 26 2010]

A partition of zigzag permutations of order n+1 by the smallest or the largest, whichever is behind. This partition has the same recurrent relation as increasing 1-2 trees of order n, by induction the bijection follows. - Wenjin Woan, May 06 2011

FORMULA E.g.f.: (1+sin(x))/cos(x) = tan(x) + sec(x).

E.g.f. for a(n+1) is 1/(cos(x/2)-sin(x/2))^2 = 1/(1-sin(x)) = d/dx(sec(x)+tan(x)).

E.g.f. A(x)=-log(1-sin(x), for a(n+1) [From Kruchinin Vladimir, Aug 09 2010]

O.g.f.: A(x) = 1+x/(1-x-x^2/(1-2*x-3*x^2/(1-3*x-6*x^2/(1-4*x-10*x^2/(1-... -n*x-(n*(n+1)/2)*x^2/(1- ...)))))) (continued fraction). - Paul D. Hanna, Jan 17 2006

O.g.f. A(x)=y satisfies 2y'=1+y^2. - Michael Somos Feb 03 2004

a(n) = P_n(0) + Q_n(0) (see A155100 and A104035), defining Q_{-1} = 0. Cf. A156142.

2*a(n+1) = Sum_{k=0..n} binomial(n, k)*a(k)*a(n-k).

Asymptotics: a(n) ~ 2^(n+2)*n!/Pi^(n+1).

a(n) = (n-1)*a(n-1) - sum{i=2, n-2, (i-1)*E(n-1, i)}, where E are the Entringer numbers A008280. - Jon Perry (perry(AT)globalnet.co.uk), Jun 09 2003

a(2*k) = (-1)^k euler(2k) and a(2k-1) = (-1)^(k-1)2^(2k)(2^(2k)-1) bernoulli(2k)/(2k) - C. Ronaldo (aga_new_ac(AT)hotmail.com), Jan 17 2005

|a(n+1)-2*a(n)|=A000708(n) . - Philippe DELEHAM, Jan 13 2007

a(n) = 2^n|E(n,1/2)+E(n,1)| where E(n,x) are the Euler polynomials. [Peter Luschny, Jan 25 2009]

a(n) = 2^{n+2}*n!*S(n+1)/(Pi)^{n+1}, where S(n)=Sum(1/(4k+1)^n, k=-inf..inf) (see the Elkies reference). [From Emeric Deutsch, Aug 17 2009]

a(n) = i^{n+1} sum_{k=1..n+1} sum_{j=0..k} binomial(k,j)(-1)^j (k-2j)^{n+1} (2i)^{-k} k^{-1} [From Ross Tang (ph.tchaa(AT)gmail.com), Jul 28 2010]

a(n)=sum((if evenp(n+k) then (-1)^((n+k)/2)*sum(j!*stirling2(n,j)*2^(1-j)*(-1)^(n+j-k)*binomial(j-1,k-1),j,k,n) else 0),k,1,n), n>0. [From Vladimir Kruchinin, Aug 19 2010]

If n==1(mod 4) is prime, then a(n)==1(mod n); if n==3(mod 4) is prime, then a(n)==-1(mod n). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Aug 31 2010]

For m>=0, a(2^m)==1(mod 2^m); If p is prime, then a(2*p)==1(mod 2*p). [From Vladimir Shevelev, Sep 03 2010]

From Peter Bala, Jan 26 2011: (Start)

a(n) = A(n,I)/(1+I)^(n-1), where I = sqrt(-1) and {A(n,x)}n>=1 = [1,1+x,1+4*x+x^2,1+11*x+11*x^2+x^3,...] denotes the sequence of Eulerian polynomials.

Equivalently, a(n) = I^(n+1)*sum {k=1..n} (-1)^k*k!*Stirling2(n,k) * ((1+I)/2)^(k-1) = I^(n+1)*sum {k = 1..n} (-1)^k*((1+I)/2)^(k-1)* sum{j = 0..k} (-1)^(k-j)*binomial(k,j)*j^n.

This explicit formula for a(n) can be used to obtain congruence results. For example, for odd prime p, a(p) = (-1)^((p-1)/2) (mod p), as noted by Vladimir Shevelev above.

For the corresponding type B results see A001586. For the corresponding results for plane increasing 0-1-2 trees see A080635.

For generalized Eulerian, Stirling and Bernoulli numbers associated with the zigzag numbers see A145876, A147315 and A185424, respectively. For a recursive triangle to calculate a(n) see A185414.

(End)

a(n) = I^(n+1)*2*Li_{-n}(-I) for n > 0. Li_{s}(z) is the polylogarithm. [Peter Luschny, Jul 29 2011]

a(n)=2*sum(m=0..(n-2)/2, 4^m*(sum(i=m..(n-1)/2, (i-(n-1)/2)^(n-1)*binomial(n-2*m-1,i-m)*(-1)^(n-i-1)))), n>1, a(0)=1, a(1)=1. [From Vladimir Kruchinin, Aug 09 2011]

A000112

COMMENTS Also fixed effects ANOVA models with n factors, which may be both crossed and nested.

[a(15)-a(16) are from Brinkmann's and McKay's paper] - Vladeta Jovovic (vladeta(AT)eunet.rs), Jan 04 2006

FORMULA

A000120

COMMENTS The binary weight of n is also called Hamming weight of n.

a(n) is also the largest integer such that 2^a(n) divides binomial(2n,n)=A000984(n) - Benoit Cloitre (benoit7848c(AT)orange.fr), Mar 27 2002

To construct the sequence, start with 0 and use the rule: If k>=0 and a(0),a(1),...,a(2^k-1) are the 2^k first terms, then the next 2^k terms are a(0)+1,a(1)+1,...,a(2^k-1)+1. - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 30 2003

An example of a fractal sequence. That is, if you omit every other number in the sequence, you get the original sequence. And of course this can be repeated. So if you form the sequence a(0 * 2^n), a(1 * 2^n), a(2 * 2^n), a(3 * 2^n), ... (for any integer n > 0), you get the original sequence. - Christopher.Hills(AT)sepura.co.uk, May 14, 2003

The n-th row of Pascal's triangle has 2^k distinct odd binomial coefficients where k=a(n)-1. -

Lekraj Beedassy (blekraj(AT)yahoo.com), May 15 2003

Fixed point of the morphism 0 -> 01, 1 -> 12, 2 -> 23, 3 -> 34, 4 -> 45, etc., starting from a(0) = 0. - Robert G. Wilson v, Jan 24 2006. - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Jan 25 2006

a(n) = number of times n appears among the mystery calculator sequences: A005408, A042964, A047566, A115419, A115420, A115421. - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Jan 25 2006

a(n) = number of solutions of the Diophantine equation 2^m*k+2^(m-1)+i=n, where m>=1, k>=0, 0<=i<2^(m-1); a(5)=2 because only (m,k,i)=(1,2,0) [2^1*2+2^0+0=5] and (m,k,i)=(3,0,1) [2^3*0+2^2+1=5] are solutions. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jan 31 2006

The first appearance of k, k=>0, is at a(2^k-1). - Robert G. Wilson v Jul 27 2006

a(n) = A138530(n,2) for n > 1. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 26 2008

Sequence is given by T^(infty)(0) where T is the operator transforming any word w=w(1)w(2)...w(m) into T(w)=w(1)(w(1)+1)w(2)(w(2)+1)...w(m)(w(m)+1). i.e. T(0)=01, T(01)=0112, T(0112)=01121223. [From Benoit Cloitre (benoit7848c(AT)orange.fr), Mar 04 2009]

a(A077436(n))=A159918(A077436(n)); a(A000290(n))=A159918(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Apr 25 2009]

For n>=2, the minimal k for which a(k(2^n-1)) is not multiple of n is 2^n+3. [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jun 05 2009]

a(n) = A063787(n) - A007814(n). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 04 2009]

Triangle inequality: a(k+m)<=a(k)+a(m). Equality holds iff C(k+m,m) is odd. [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jul 19 2009]

Conjecture: The sequence where a(n) is the sum of digits of (n written in base b), can be written as a triangle T(r,k) in which all positive terms in column k are equal. Row 0 is a(0)=0 and row r lists a(b^(r-1)) .. a(b^r - 1), for r>=1 and b>=2. [From Omar E. Pol (info(AT)polprimos.com), Feb 20 2010]

The number of occurrences of value k in the first 2^n terms of A000120 is equal to the sum of the first n-k terms of the sequence T(k,i), where T(0,i) = 1,0,0,0,0,0,...(A000007), T(0,i) = 1,1,1,1,1,1,...(A000012), T(2,i) = 1,2,3,4,5,6,...(A000027), T(3,i) = 1,3,6,10,15,...(A000217), T(4,i) = 1,4,10,20,35...(A000292), and in general T(u,1) = 1; T(1,v) = 0 for v > 1; T(u,v) = T(u,v-1) + T(u-1, v) for u, v > 1 [From Brent Spillner (spillner(AT)acm.org), Sep 01 2010]

FORMULA a(0) = 0, a(2*n) = a(n), a(2*n+1) = a(n) + 1.

a(0) = 0, a(2^i) = 1; otherwise if n = 2^i + j with 0 < j < 2^i, a(n) = a(j) + 1.

G.f.: Product_{k >= 0} (1 + y*x^(2^k)) = Sum_{n >= 0} y^a(n)*x^n. - N. J. A. Sloane, Jun 04 2009

a(n) = a(n-1)+1-A007814(n) = log2[A001316(n)] = 2n-A005187(n) = A070939(n)-A023416(n). - Henry Bottomley (se16(AT)btinternet.com), Apr 04 2001; corrected by Ralf Stephan (ralf(AT)ark.in-berlin.de), Apr 15 2002

a(n)=log2(A000984(n)/A001790(n)). - Benoit Cloitre, Oct 02 2002

For n>0, a(n)=n-sum(k=1, n, A007814(k)). - Benoit Cloitre (benoit7848c(AT)orange.fr), Oct 19 2002 a(n)=n-sum(k>0, floor(n/2^k))=n-A011371(n). - Benoit Cloitre, Dec 19 2002

G.f.: 1/(1-x) * Sum(k>=0, x^(2^k)/(1+x^(2^k))). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Apr 19 2003

a(0)=0, a(n)=a(n-2^log_2(floor(n)))+1. Examples: a(6)=a(6-2^2)+1=a(2)+1=a(2-2^1)+1+1=a(0)+2=2; a(101)=a(101-2^6)+1=a(37)+1=a(37-2^5)+2=a(5-2^2)+3=a(1-2^0)+4=a(0)+4=4; a(6275)=a(6275-2^12)+1=a(2179-2^11)+2=a(131-2^7)+3=a(3-2^1)+4=a(1-2^0)+5=5; a(4129)=a(4129-2^12)+1=a(33-2^5)+2=a(1-2^0)+3=3; - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jan 22 2006

A fixed point of the mapping 0 -> 01, 1 -> 12, 2 -> 23, 3 -> 34, 4 -> 45, ... With f(i) = floor(n/2^i), a(n) is the number of odd numbers in the sequence f(0), f(1), f(2), f(3), f(4), f(5), ... - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jan 04 2004

When read mod 2 gives the Morse-Thue sequence A010060.

Let floor_pow4(n) denote n rounded down to the next power of four, floor_pow4(n) = 4 ^ floor(log4 n). Then a(0) = 0, a(1) = 1, a(2) = 1, a(3) = 2, a(n) = a(floor(n / floor_pow4(n))) + a(n % floor_pow4(n)) - Stephen K. Touset (stephen(AT)touset.org), Apr 04 2007

a(n)=n-sum{2<=k<=n, sum{j|n,j>=2, floor(log_2(j))-floor(log_2(j-1))}}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jun 18 2007

a(n)=A007814(C(2n,n))=1+A007814(C(2n-1,n)). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jul 20 2009]

For odd m>=1, a((4^m-1)/3)=a((2^m+1)/3)+(m-1)/2 (mod 2). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Sep 03 2010]

a(n) - a(n-1) = { 1 - a(n-1) iff A007814(n) = a(n-1), 1 iff A007814(n) = 0, -1 for all other A007814(n) } [From Brent Spillner (spillner(AT)acm.org), Sep 01 2010]

a(A001317(n))=2^a(n). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Oct 25 2010]

A000123

COMMENTS Also, a(n) = number of "non-squashing" partitions of 2n (or 2n+1), that is, partitions 2n=p_1+p_2+...+p_k with 1 <= p_1 <= p_2 <= ... <= p_k and p_1 + p_2 + ... + p_i <= p_{i+1} for all 1 <= i < k. [Hirschhorn and Sellers]

Row sums of A101566. - Paul Barry , Jan 03 2005

Equals infinite convolution product of [1,2,2,2,2,2,2,2,2,2] aerated A000079 - 1 times, i.e. [1,2,2,2,2,2,2,2,2,2] * [1,0,2,0,2,0,2,0,2] * [1,0,0,0,2,0,0,0,2]. [From Mats Granvik, Gary W. Adamson, Aug 04 2009]

Contribution from Gary W. Adamson, Dec 16 2009: (Start)

Given A018819 = A000123 with repeats, polcoeff (1, 1, 2, 2, 4, 4,...)

* (1, 1, 1,...) = (1, 2, 4, 6, 10,...) = (1, 0, 2, 0, 4, 0, 6,...)

* (1, 2, 2, 2,...). (End)

Contribution from Gary W. Adamson, Dec 06 2009: (Start)

Let M = an infinite lower triangular matrix with (1, 2, 2, 2,...) in every

column shifted down twice. A000123 = Lim:_{n..inf.} M^n, the left-shifted

vector considered as a sequence. Replacing (1, 2, 2, 2, ...) with

(1, 3, 3, 3,...) and following the same procedure, we obtain A171370:

(1, 3, 6, 12, 18, 30, 42, 66, 84, 120,...). (End)

FORMULA a(n)=a(n-1)+a([n/2]). For proof see A018819.

2 * a(n) = a(n+1) + a(n-1) if n is even. - Michael Somos, Jan 07 2011

G.f.: (1-x)^(-1) Product_{n=0..inf} (1 - x^(2^n))^(-1).

a(n) = Sum_{i=0..n} a([i/2]). [O'Shea]

a(n)=(1/n)*Sum_{k=1..n} (A038712(k)+1)*a(n-k), n > 1, a(0)=1. - Vladeta Jovovic (vladeta(AT)eunet.rs), Aug 22 2002

Conjecture: lim n ->infinity (log(n)*a(2n))/(n*a(n)) = c = 1.63... - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 26 2003

G.f. A(x) satisfies A(x^2)=((1-x)/(1+x))A(x). - Michael Somos, Aug 25 2003

G.F.: prod(k=0, inf, (1+x^(2^k))/(1-x^(2^k)), or prod(k=0, inf, 1+x^(2^k))^(k+1))/(1-x), or prod(k=0, inf, (1+x^(2^k))^(k+2)) - Joerg Arndt, Apr 24 2005

Comment from Philippe Flajolet (Philippe.Flajolet(AT)inria.fr), Sep 06 2008 (Start): The asymptotic rate of growth is known precisely - see De Bruijn's paper. With p(n) the number of partitions of n into powers of two, the asymptotic formula of de Bruijn is:

log(p(2*n)) = 1/(2*L2)*(log(n/log(n)))^2 + (1/2 + 1/L2 + LL2/L2)*log(n) - (1 + LL2/L2)*log(log(n)) + Phi(log(n/log(n))/L2),

where L2=log(2), LL2=log(log(2)) and Phi(x) is a certain periodic function with period 1 and a tiny amplitude.

Numerically, Phi(x) appears to have a mean value around 0.66. An expansion up to O(1) term had been obtained earlier by Kurt Mahler. (End)

A000124

COMMENTS These are Hogben's central polygonal numbers with the (two-dimensional) symbol

2

.P

1 n

m=(n-1)(n-2)/2+1 is also the smallest number of edges such that all graphs with n nodes and m edges are connected. - Keith M. Briggs, May 14 2004.

Also maximal number of grandchildren of a binary vector of length n+2. E.g. a binary vector of length 6 can produce at most 11 different vectors when 2 bits are deleted.

This is also the order dimension of the (strong) Bruhat order on the finite Coxeter group B_{n+1}. - Nathan Reading (reading(AT)math.umn.edu), Mar 07 2002

Number of 132- and 321-avoiding permutations of {1,2,...,n+1}. - Emeric Deutsch, Mar 14 2002

For n>=1 a(n) is the number of terms in the expansion of (x+y)*(x^2+y^2)*(x^3+y^3)*...*(x^n+y^n) - Yuval Dekel (dekelyuval(AT)hotmail.com), Jul 28 2003

Also the number of terms in (1)(x+1)(x^2+x+1)...(x^n+...+x+1); see A000140.

Narayana transform (analogue of the binomial transform) of vector [1, 1, 0, 0, 0...] = A000124; using the infinite lower Narayana triangle of A001263 (as a matrix), N; then N * [1, 1, 0, 0, 0...] = A000124. - Gary W. Adamson, Apr 28 2005

a(n) = A108561(n+3,2). - Reinhard Zumkeller, Jun 10 2005

Number of interval subsets of {1,2,3,...,n} (cf. A002662). - Jose Luis Arregui (arregui(AT)unizar.es), Jun 27 2006

Define a number of straight lines in the plane to be in general arrangement when (1) no two lines are parallel, (2) there is no point common to three lines. Then these are the maximal numbers of regions defined by n straight lines in general arrangement in the plane. - Peter C. Heinig (algorithms(AT)gmx.de), Oct 19 2006

Note that a(n) = a(n-1) + A000027(n-1). This has the following geometrical interpretation: Suppose there are already n-1 lines in general arrangement, thus defining the maximal number of regions in the plane obtainable by n-1 lines and now one more line is added in general arrangement. Then it will cut each of the n-1 lines and acquire intersection points which are in general arrangement. (See the comments on A000027 for general arrangement with points.) These points on the new line define the maximal number of regions in 1-space definable by n-1 points, hence this is A000027(n-1), where for A000027 an offset of 0 is assumed, that is, A000027(n-1)=(n+1)-1=n. Each of these regions acts as a dividing wall, thereby creating as many new regions in addition to the a(n-1) regions already there, hence a(n)=a(n-1)+A000027(n-1). Cf. the comments on A000125 for an analogous interpretation. - Peter C. Heinig (algorithms(AT)gmx.de), Oct 19 2006

When constructing a zonohedron, one zone at a time, out of (up to) 3-d non-intersecting parallelepipeds, the n-th element of this sequence is the number of edges in the n-th zone added with the n-th "layer" of parallelepipeds. (Verified up to 10-zone zonohedron, the enneacontahedron). E.g. adding the 10th zone to the enneacontahedron requires 46 parallel edges (edges in the 10th zone) by looking directly at a 5-valence vertex and counting visible vertices. - Shel Kaphan (skaphan(AT)gmail.com), Feb 16 2006

If Y is a 2-subset of an n-set X then, for n>=3, a(n-3) is the number of (n-2)-subsets of X which have no exactly one element in common with Y. - Milan R. Janjic (agnus(AT)blic.net), Dec 28 2007

Equals row sums of triangle A144328 [From Gary W. Adamson, Sep 18 2008]

It appears that a(n) is the number of distinct values among the fractions F(i+1)/F(j+1) as j ranges from 1 to n and, for each fixed j, i ranges from 1 to j, where F(i) denotes the i-th Fibonacci number. [From John W. Layman, Dec 02 2008]

a(n) is the number of subsets of {1,2,...,n} that contain at most two elements. [From Geoffrey Critzer, Mar 10 2009]

Contribution from Srikanth K S (sriperso(AT)gmail.com), Oct 22 2009: (Start)

For n >= 2, a(n) gives the number of sets of subsets $A_1,A_2,\dots A_n$

of $[n]=\{1,2,\dots,n\}$ so that $\cap_{i=1}^{n} A_i=\emptyset$ and the sum

$\sum_{\forall j\in [n]}\left (|\cap_{i=1,i\ne j}^{n} A_i|\right)$ is maximum (End)

The numbers along the left edge of Floyd's triangle. [From Paul Muljadi, Jan 25 2010]

Let A be the Hessenberg matrix of order n, defined by: A[1,j]=A[i,i]:=1, A[i,i-1]=-1, and A[i,j]=0 otherwise. Then, for n>=1, a(n-1)=(-1)^(n-1)*coeff(charpoly(A,x),x). [From Milan R. Janjic (agnus(AT)blic.net), Jan 24 2010]

Also the number of deck entries of Euler's ship. See the Meijer-Nepveu link. [From Johannes W. Meijer, June 21, 2010]

(1 + x^2 + x^3 + x^4 + x^5 + ...)*(1 + 2x + 3x^2 + 4x^3 + 5x^4 + ...) = (1 + 2x + 4x^2 + 7x^3 + 11x^4 + ...). [Gary W. Adamson, Jul 27 2010]

The number of length n binary words that have no 0-digits between any pair of consecutive 1-digits. [From Jeffrey Liese, Dec 23 2010]

Let b(0)=b(1)=1; b(n)=max(b(n-1)+n-1,b(n-2)+n-2) then a(n)=b(n+1). [Yalcin Aktar, Jul 28 2011]

FORMULA G.f.: (1-x+x^2)/(1-x)^3.

G.f.: (1-x^6)/((1-x)^2*(1-x^2)*(1-x^3)). a(-1-n)=a(n). - Michael Somos Sep 04 2006

a(n+3)=3*a(n+2)-3*a(n+1)+a(n) and a(1)=1, a(2)=2, a(3)=4. [From Artur Jasinski, Oct 21 2008]

a(n) = A000217(n)+1.

a(n)=a(n-1)+n. E.g.f.:(1+x+x^2/2)*exp(x) [From Geoffrey Critzer, Mar 10 2009]

a(n)=sum{k=0..n+1, binomial(n+1, 2(k-n))} - Paul Barry, Aug 29 2004

Euler transform of length 6 sequence [2, 1, 1, 0, 0, -1]. - Michael Somos Sep 04 2006

binomial(n+2,1)-2*binomial(n+1,1)+binomial(n+2,2). - Zerinvary Lajos, May 12 2006

Binomial transform of (1, 1, 1, 0, 0, 0,...) and inverse binomial transform of A072863: (1, 3, 9, 26, 72, 192,...). - Gary W. Adamson, Oct 15 2007

a(n) = A086601(n)^(1/2). - Zerinvary Lajos, Apr 25 2008

From Thomas Wieder, Feb 25 2009: (Start)

a(n) = sum_{l_1=0}^{n+1} sum_{l_2=0}^{n}...sum_{l_i=0}^{n-i}...sum_{l_n=0}^{1}

delta(l_1,l_2,...,l_i,...,l_n)

where delta(l_1,l_2,...,l_i,...,l_n) = 0 if any l_i <> l_(i+1) and l_(i+1) <> 0

and delta(l_1,l_2,...,l_i,...,l_n) = 1 otherwise. (End)

a(n) = A034856(n+1) - A005843(n) = A000217(n) + A005408(n) - A005843(n). [From Jaroslav Krizek, Sep 05 2009]

a(n) = 2*a(n-1)-a(n-2)+1. [From Eric Werley, Jun 27 2011]

A000129 Pell numbers: a(0) = 0, a(1) = 1; for n > 1, a(n) = 2*a(n-1) + a(n-2).

COMMENTS Sometimes also called lambda numbers.

Also denominators of continued fraction convergents to sqrt(2): 1, 3/2, 7/5, 17/12, 41/29, 99/70, 239/169, 577/408, 1393/985, 3363/2378, 8119/5741, 19601/13860, 47321/33461, 114243/80782, ... = A001333/A000129

Number of lattice paths from (0,0) to the line x=n-1 consisting of U=(1,1), D=(1,-1) and H=(2,0) steps (i.e. left factors of Grand Schroeder paths); for example, a(3)=5, counting the paths H, UD, UU, DU and DD. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Oct 27 2002

a(2*n) with b(2*n) := A001333(2*n), n>=1, give all (positive integer) solutions to Pell equation b^2 - 2*a^2 = +1 (see Emerson reference). a(2*n+1) with b(2*n+1) := A001333(2*n+1), n>=0, give all (positive integer) solutions to Pell equation b^2 - 2*a^2 = -1.

Bisection: a(2*n+1)= T(2*n+1,sqrt(2))/sqrt(2)= A001653(n), n>=0 and a(2*n)= 2*S(n-1,6)= 2*A001109(n),n>=0, with T(n,x), resp. S(n,x), Chebyshev's polynomials of the first,resp. second kind. S(-1,x)=0. See A053120, resp. A049310. - W. Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Jan 10 2003

Consider the mapping f(a/b) = (a + 2b)/(a + b). Taking a = b = 1 to start with and carrying out this mapping repeatedly on each new (reduced) rational number gives the following sequence 1/1, 3/2,7/5,17/12,41/29,... converging to 2^(1/2). Sequence contains the denominators. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 22 2003

This is also the Horadam sequence (0,1,1,2). a(n) / a(n-1) converges to 2^1/2 + 1 as n approaches infinity. - Ross La Haye (rlahaye(AT)new.rr.com), Aug 18 2003

Number of 132-avoiding two-stack sortable permutations.

For n>0, the number of (s(0), s(1), ..., s(n)) such that 0 < s(i) < 4 and |s(i) - s(i-1)| <= 1 for i = 1,2,....,n, s(0) = 2, s(n) = 3. - Herbert Kociemba (kociemba(AT)t-online.de), Jun 02 2004

Number of (s(0), s(1), ..., s(n)) such that 0 < s(i) < 4 and |s(i) - s(i-1)| <= 1 for i = 1,2,....,n, s(0) = 1, s(n) = 2. - Herbert Kociemba (kociemba(AT)t-online.de), Jun 02 2004

Counts walks of length n from a vertex of a triangle to another vertex to which a loop has been added. - Mario Catalani (mario.catalani(AT)unito.it), Jul 23 2004

Apart from initial terms, Pisot sequence P(2,5). See A008776 for definition of Pisot sequences. - David W. Wilson (davidwwilson(AT)comcast.net)

Sums of antidiagonals of A038207 [Pascal's triangle squared] - Ross La Haye (rlahaye(AT)new.rr.com), Oct 28 2004

The Pell primality test is "If N is an odd prime, then P(N)-kronecker(2,N) is divisible by N". "Most" composite numbers fail this test, so it makes a useful pseudoprimality test. The odd composite numbers which are Pell pseudoprimes (i.e. that pass the above test) are in A099011. - Jack Brennen (jb(AT)brennen.net), Nov 13, 2004

a(n) = sum of n-th row of triangle in A008288 = A094706(n)+A000079(n). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Dec 03 2004

Pell trapezoids (cf. A084158); for n>0, A001109(n)= (a(n-1)+a(n+1))*a(n)/2; e.g. 1189=(12+70)*29/2 - Charlie Marion (charliemath(AT)optonline.net), Apr 1 2006

(0!a(1),1!a(2),2!a(3),3!a(4),...) and (1,-2,-2,0,0,0,...) form a reciprocal pair under the list partition transform and associated operations described in A133314. - Tom Copeland (tcjpn(AT)msn.com), Oct 29 2007

Let C = (sqrt(2)+1) = 2.414213562..., then for n>1, C^n = a(n)*(1/C) + a(n+1). Example: C^3 = 14.0710678... = 5*(.414213562...) + 12. Let X = the 2 X 2 matrix [0, 1; 1, 2]; then X^n * [1, 0] = [a(n-1), a(n); a(n), a(n+1)]. a(n) = numerator of n-th convergent to (Sqrt(2)-1) = .41421356... = [2, 2, 2,...], the convergents being [1/2, 2/5, 5/12,...]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 21 2007

A = sqrt(2) = 2/2 + 2/5 + 2/(5*29) + 2/(29*169) + 2/(169*985) + ...; B = ((5/2) - sqrt(2)) = 2/2 + 2/(2*12) + 2/(12*70) + 2/(70*408) + 2/(408*2378) + ...; A+B = 5/2. C = 1/2 = 2/(1*5) + 2/(2*12) + 2/(5*29) + 2/(12*70) + 2/(29*169) + ... - Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 16 2008

Prime Pell numbers with an odd index gives the RMS value (A141812) of prime RMS numbers (A140480). [From Ctibor O. Zizka (ctibor.zizka(AT)seznam.cz), Aug 13 2008]

Comment from Clark Kimberling (ck6(AT)evansville.edu), Aug 27 2008 (Start): Related convergents (numerator/denominator):

lower principal convergents: A002315/A001653

upper principal convergents: A001541/A001542

intermediate convergents: A052542/A001333

lower intermediate convergents: A005319/A001541

upper intermediate convergents: A075870/A002315

principal and intermediate convergents: A143607/A002965

lower principal and intermediate convergents: A143608/A079496

upper principal and intermediate convergents: A143609/A084068 (End)

Equals row sums of triangle A143808 starting with offset 1. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 01 2008]

Binomial transform of the sequence:= 0,1,0,2,0,4,0,8,0,16,..., powers of 2 alternating with zeros. [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 28 2008] a(n) is also the sum of the n-th row of the triangle formed by starting with the top two rows of Pascal's triangle and then each next row has a 1 at both ends and the interior values are the sum of the three numbers in the triangle above that position. [From Patrick Costello (pat.costello(AT)eku.edu), Dec 07 2008]

Starting with offset 1 = eigensequence of triangle A135387 (an infinite lower triangular matrix with (2,2,2,...) in the main diagonal and (1,1,1,...) in the subdiagonal. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 29 2008]

Starting with offset 1 = row sums of triangle A153345 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 24 2008]

Contribution from Charlie Marion (charliemath(AT)optonline.net), Jan 07 2009: (Start)

In general, denominators, a(k,n) and numerators, b(k,n), of continued

fraction convergents to sqrt((k+1)/k) may be found as follows:

let a(k,0) = 1, a(k,1) = 2k; for n>0, a(k,2n) = 2*a(k,2n-1)+a(k,2n-2)

and a(k,2n+1)=(2k)*a(k,2n)+a(k,2n-1);

let b(k,0) = 1, b(k,1) = 2k+1; for n>0, b(k,2n) = 2*b(k,2n-1)+b(k,2n-2)

and b(k,2n+1)=(2k)*b(k,2n)+b(k,2n-1).

For example, the convergents to sqrt(2/1) start 1/1, 3/2, 7/5,

17/12, 41/29.

In general, if a(k,n) and b(k,n) are the denominators and numerators,

respectively, of continued fraction convergents to sqrt((k+1)/k)

as defined above, then

k*a(k,2n)^2-a(k,2n-1)*a(k,2n+1)=k=k*a(k,2n-2)*a(k,2n)-a(k,2n-1)^2 and

b(k,2n-1)*b(k,2n+1)-k*b(k,2n)^2=k+1=b(k,2n-1)^2-k*b(k,2n-2)*b(k,2n);

for example, if k=1 and n=3, then a(1,n)=a(n+1) and

1*a(1,6)^2-a(1,5)*a(1,7)=1*169^2-70*408=1;

1*a(1,4)*a(1,6)-a(1,5)^2=1*29*169-70^2=1;

b(1,5)*b(1,7)-1*b(1,6)^2=99*577-1*239^2=2;

b(1,5)^2-1*b(1,4)*b(1,6)=99^2-1*41*239=2.

Cf. A001333, A142238-A142239, A153313-153318.

[From Charlie Marion (charliemath(AT)optonline.net), Jan 07 2009]

(End)

Starting with offset 1 = row sums of triangle A155002, equivalent to the statement that the Fibonacci series convolved with the Pell series prefaced with a "1": (1, 1, 2, 5, 12, 29,...) = (1, 2, 5, 12, 29,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 18 2009]

It appears that P(p) == 8^((p-1/2)) mod p, p = prime; analogous to [Schroeder, p.90]: Fp == 5^((p-1)/2)) mod p. Example: Given P(11) = 5741, == 8^5 mod 11. Given P(17) = 11336689, == 8^8 mod 17 since 17 divides (8^8 - P(l7)). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 21 2009]

Equals eigensequence of triangle A154325 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 12 2009]

Another combinatorial interpretation of a(n+1) arises from a simple tiling scenario. Namely, a(n+1) gives the number of ways of tiling a 1 by n rectangle with indistinguishable 1 by 2 rectangles and 1 by 1 squares that come in two varieties, A and B say. For example, with C representing the 1 by 2 rectangle, we obtain a(4)=12 from AAA, AAB, ABA, BAA, ABB, BAB, BBA, BBB, AC, BC, CA and CB. [From Martin Griffiths (griffm(AT)essex.ac.uk), Apr 25 2009]

a(n+1)=2*a(n)+ a(n-1) a(1=1,a(2)=2 was used by Theon from Smyrna. [From Sture Sjoestedt (sture.sjostedt(AT)spray.se), May 29 2009]

The n-th Pell number counts the perfect matchings of the edge-labeled graph C_2 x P_(n-1), or equivalently, the number of domino tilings of a 2 x (n-1) cylindrical grid. [From Sarah-Marie Belcastro (smbelcas(AT)toroidalsnark.net), Jul 04 2009]

Number of units of a(n) belongs to a periodic sequence: 0, 1, 2, 5, 2, 9, 0, 9, 8, 5, 8, 1. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 04 2009]

As a fraction: 1/79 = 0.0126582278481... or 1/9799 = 0.000102051229...(1/119 and 1/10199 for sequence in reverse). [From M. Dols (markdols99(AT)yahoo.com), May 18 2010]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 16 2010: (Start)

As n->inf. (a(n)/a(n-1)) - (a(n-1)/a(n)) tends to 2.0. Example: a(7)/a(6) -

a(6)/a(7) = 169/70 - 70/169 = 2.0000845... (End)

Numbers n such that 2*n^2+-1 is a square. [From Vincenzo Librandi (vincenzo.librandi(AT)tin.it), Jul 18 2010]

Starting (1, 2, 5,...) = INVERTi transform of A006190: (1, 3, 10, 33, 109,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 06 2010]

[u,v] = [a(n), a(n-1)] generates all Pythagorean triples [u^2-v^2, 2uv, u^2+v^2] whose legs differ by 1. [From James Buddenhagen (jbuddenh(AT)gmail.com), Aug 14 2010]

Contribution from Johannes W. Meijer (meijgia(AT)hotmail.com), Aug 15 2010: (Start)

An elephant sequence, see A175654. For the corner squares six A[5] vectors, with decimal values between 21 and 336, lead to this sequence (without the leading 0). For the central square these vectors lead to the companion sequence A078057.

(End)

Let the 2x2 square matrix A=[2, 1; 1, 0] then a(n) = the (1,1) element of A^(n-1). [Carmine Suriano, Jan 14 2011]

FORMULA G.f.: x/(1-2*x-x^2).

a(n) = 2*a(n-1)+a(n-2), a(0)=0, a(1)=1.

a(n)=((1+sqrt(2))^n -(1-sqrt(2))^n)/(2*sqrt(2))

a(n) = integer nearest a(n-1)/(sqrt(2) - 1), where a(0) = 1 - from Clark Kimberling (ck6(AT)evansville.edu)

a(n)= Sum_{i, j, k >= 0: i+j+2k=n} (i+j+k)!/(i!*j!*k!).

a(n)^2 + a(n+1)^2 = a(2n+1) (1999 Putnam examination).

a(2n) = 2*a(n)*A001333(n). - John McNamara, Oct 30, 2002

a(n) = ((-i)^(n-1))*S(n-1, 2*i), with S(n, x) := U(n, x/2) Chebyshev's polynomials of the second kind. See A049310. S(-1, x)=0, S(-2, x)= -1.

Binomial transform of expansion of sinh(sqrt(2)x)/sqrt(2). E.g.f.: exp(x)sinh(sqrt(2)x)/sqrt(2). - Paul Barry (pbarry(AT)wit.ie), May 09 2003

a(n)=sum{k=0, ..floor(n/2), C(n, 2k+1)2^k}. - Paul Barry (pbarry(AT)wit.ie), May 13 2003

a(n-2) + a(n) = (1 + sqrt2)^(n-1) + (1 - sqrt2)^(n-1) = A002203(n-1). [A002203(n)]^2 - 8[a(n)]^2 = 4(-1)^n - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 15 2003

G.f. : x(1+x)/(1-x-3x^2-x^3); a(n)=a(n-1)+3a(n-2)+a(n-2); - Mario Catalani (mario.catalani(AT)unito.it), Jul 23 2004

a(n+1)=Sum(C(n-k, k)2^(n-2k), k=0, .., Floor[n/2]). - Mario Catalani (mario.catalani(AT)unito.it), Jul 23 2004

Apart from initial terms, inverse binomial transform of A052955. - Paul Barry, May 23 2004

a(n)^2+a(n+2k+1)^2=A001653(k)*A001653(n+k);e.g., 5^2+70^2=5*985 - Charlie Marion (charliemath(AT)optonline.net) Aug 03 2005

a(n+1)=sum{k=0..n, binomial((n+k)/2, (n-k)/2)(1+(-1)^(n-k))2^k/2}; - Paul Barry (pbarry(AT)wit.ie), Aug 28 2005

a(n) = a(n - 1) + A001333(n - 1) = A001333(n) - a(n - 1) = A001109(n)/A001333(n) = sqrt(A001110(n)/A001333(n)^2) = ceiling(sqrt(A001108(n)/2)) - Henry Bottomley (se16(AT)btinternet.com), Apr 18 2000

a(n)=F(n, 2), the n-th Fibonacci polynomial evaluated at x=2. - T. D. Noe (noe(AT)sspectra.com), Jan 19 2006

Define c(2n) = -A001108(n), c(2n+1) = -A001108(n+1) and d(2n) = d(2n+1) = A001652(n), then ((-1)^n)*(c(n) + d(n)) = a(n). - Proof given by Max Alekseyev (maxale(AT)gmail.com) - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Jul 21 2005

a(r+s) = a(r)*a(s+1) + a(r-1)*a(s). - Lekraj Beedassy (blekraj(AT)yahoo.com), Sep 03 2006

a(n)=(b(n+1)+b(n-1))/n where {b(n)} is the sequence A006645 - Sergio Falcon (sfalcon(AT)dma.ulpgc.es), Nov 22 2006

Comments from Miklos Kristof (kristmikl(AT)freemail.hu), Mar 19 2007: (Start)

Let F(n)=a(n)=Pell numbers, L(n)=A002203=companion Pell numbers (A002203):

For a>=b and odd b F(a+b)+F(a-b)=L(a)*F(b).

For a>=b and even b F(a+b)+F(a-b)=F(a)*L(b).

For a>=b and odd b F(a+b)-F(a-b)=F(a)*L(b).

For a>=b and even b F(a+b)-F(a-b)=L(a)*F(b).

F(n+m)+(-1)^m*F(n-m)=F(n)*L(m)

F(n+m)-(-1)^m*F(n-m)=L(n)*F(m)

F(n+m+k)+(-1)^k*F(n+m-k)+(-1)^m*(F(n-m+k)+(-1)^k*F(n-m-k))=F(n)*L(m)*L(k)

F(n+m+k)-(-1)^k*F(n+m-k)+(-1)^m*(F(n-m+k)-(-1)^k*F(n-m-k))=L(n)*L(m)*F(k)

F(n+m+k)+(-1)^k*F(n+m-k)-(-1)^m*(F(n-m+k)+(-1)^k*F(n-m-k))=L(n)*F(m)*L(k)

F(n+m+k)-(-1)^k*F(n+m-k)-(-1)^m*(F(n-m+k)-(-1)^k*F(n-m-k))=8*F(n)*F(m)*F(k) (End)

a(n+1)*a(n)=2*sum{k=0..n, a(k)^2} (a similar relation holds for A001333) - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Aug 28 2007

a(n+1) = sum(k=0,...,n) binomial(n+1,2k+1) * 2^k = sum(k=0,...,n) A034867(n,k) * 2^k = (1/n!)sum(k=0,...,n) A131980(n,k) * 2^k . - Tom Copeland (tcjpn(AT)msn.com), Nov 30 2007

Equals row sums of unsigned triangle A133156. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Apr 21 2008

a(n) (n>=3) is the determinant of the (n-1) by (n-1) tridiagonal matrix with diagonal entries 2, superdiagonal entries 1 and subdiagonal entries -1. [From Emeric Deutsch (deutsch(AT)duke.poly.edu), Aug 29 2008]

a(n) = 5*a(n-2)+2*a(n-3), a(n) = 6*a(n-2)-a(n-4). - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 04 2008.

Comments from Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jan 02 2009 (Start): fract((1+sqrt(2))^n)) = (1/2)*(1+(-1)^n)-(-1)^n*(1+sqrt(2))^(-n) = (1/2)*(1+(-1)^n)-(1-sqrt(2))^n.

See A001622 for a general formula concerning the fractional parts of powers of numbers x>1, which suffice x-x^(-1)=floor(x).

a(n) = nint((1+sqrt(2))^n) for n>0. (End)

a(n)=((4+sqrt18)*(1+sqrt2)^n)+(4-sqrt18)*(1-sqrt2)^n)/4 offset 0. [From Al Hakanson (hawkuu(AT)gmail.com), Aug 08 2009]

If p[i]=fibonacci(i) and if A is the Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n)=det A. [From Milan R. Janjic (agnus(AT)blic.net), May 08 2010]

a(n)= 3*a(n-1)-a(n-2)-a(n-3),n>2; a(n)= a(n-1)+3*a(n-2)+a(n-3), n>2.- Gary Detlefs (gdetlefs(AT)aol.com), Sep 09 2010

a(n)=2*(a(2k-1)+a(2k))*a(n-2k)-a(n-4k).

a(n)=2*(a(2k)+a(2k+1))*a(n-2k-1)+a(n-4k-2). - Charlie Marion, Apr 13 2011

A000140 Kendall-Mann numbers: the maximal number of inversions in a permutation on n letters is floor(n(n-1)/4); a(n) = number of permutations with this many inversions.

COMMENTS Row maxima of A008302, see example.

The term a(0) would be 1: the empty product is one and there is just one coefficient 1=x^0, corresponding to the 1 empty permutation (which has 0 inversions).

Comments from Ryen Lapham and Anant Godbole (zrcl9(AT)imail.etsu.edu), Dec 12 2006: (Start) "Also, the number of permutations on {1,2,....,n} for which the number A of monotone increasing subsequences of length 2 and the number D of monotone decreasing 2-subsequences are as close to each other as possible, i.e. 0 or 1. We call such permutations 2-balanced.

"If 4|n(n-1) then (with A and D as above) the feasible values of A-D are {n choose 2}, {n choose 2}-2,....,2,0,-2,.....-{n choose 2}, whereas if 4 does not divide n(n-1), A-D may equal {n choose 2}, {n choose 2}-2,....,1,-1,.....-{n choose 2}. Let a_n(i) equal the number of permutations with A-D the i-th highest feasible value.

"The sequence in question gives the number of permutations for which A-D=0 or A-D=1, i.e. it equals A_n(j) where j=floor(({n choose 2}+2)/2). Here is the recursion: a_n(i)=a_n(i-1)+a_{n-1}(i) for 1 <= i <= n and a_n(n+k)=a_n(n+k-1)+a_{n-1}(n+k)-a_n(k) for k>= 1." (End)

The only two primes found < 301 are for n = 3 & 6.

FORMULA Largest coefficient of (1)(x+1)(x^2+x+1)...(x^(n-1)+...+x+1) (David W. Wilson).

The number of terms is given in A000124.

A000142 Factorial numbers: n! = 1*2*3*4*...*n (order of symmetric group S_n, number of permutations of n letters).

COMMENTS For n >= 1 a(n) is the number of n X n (0,1) matrices with each row and column containing exactly one entry equal to 1.

This sequence is the BinomialMean transform of A000354. (See A075271 for definition.) - John W. Layman (layman(AT)math.vt.edu), Sep 12 2002. This is easily verified from the Paul Barry formula for A000354, by interchanging summations and using the formula: Sum_k (-1)^k C(n-i,k) = KroneckerDelta(i,n). - David Callan (callan(AT)stat.wisc.edu), Aug 31 2003

Number of distinct subsets of T(n-1) elements with 1 element A, 2 elements B,..., n-1 elements X (e.g. n=5, we consider the distinct subsets of ABBCCCDDDD and there are 5!=120.) - Jon Perry (perry(AT)globalnet.co.uk), Jun 12 2003

n! is the smallest number with that prime signature. E.g. 720 = 2^4*3^2*5. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Jul 01 2003 a(n) is the permanent of the n X n matrix M with M(i,j) = 1 . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Dec 15 2003

Given n objects of distinct sizes (e.g. areas, volumes) such that each object is sufficiently large simultaneously to contain all previous objects, then n! is the total number of essentially different arrangements using all n objects. Arbitrary levels of nesting of objects is permitted within arrangements. (...sequence inspired by considering left-over moving boxes.). If the restriction exists that each object is only able or permitted to contain at most one smaller (but possibly nested) object at a time, the resulting sequence begins 1,2,5,15,52 (Bell Numbers?). Sets of nested wooden boxes or traditional nested Russian dolls come to mind here. - Rick L. Shepherd (rshepherd2(AT)hotmail.com), Jan 14 2004

From Michael Somos, Mar 04 2004: (Start)

Stirling transform of a(n) = [2,2,6,24,120,...] is A052856(n) = [2, 2, 4, 14, 76, ...].

Stirling transform of a(n) = [1,2,6,24,120,...] is A000670(n) = [1, 3, 13, 75, ...].

Stirling transform of a(n) = [0,2,6,24,120,...] is A052875(n) = [0, 2, 12, 74, ...].

Stirling transform of a(n-1) = [1,1,2,6,24,...] is A000629(n-1) = [1, 2, 6, 26, ...].

Stirling transform of a(n-1) = [0,1,2,6,24,...] is A002050(n-1) = [0, 1, 5, 25, 140, ...].

Stirling transform of -(-1)^n * A089064(n) = [1, 0, 1, -1, 8, -26, 194, ...] is a(n-1) = [1,1,2,6,24,120,...]. (End)

First Eulerian transform of 1,1,1,1,1,1... The first Eulerian transform transforms a sequence s to a sequence t by the formula t(n) = Sum[e(n,k)s(k), k=0...n], where e(n,k) is a first-order Eulerian number [A008292]. - Ross La Haye (rlahaye(AT)new.rr.com), Feb 13 2005

1, 6, 120 are the only numbers which are both triangular and factorial. - Christopher M. Tomaszewski (cmt1288(AT)comcast.net), Mar 30 2005

n! is the n-th finite difference of consecutive n-th powers. E.g. for n=3, [0, 1, 8, 27, 64, ...] -> [1, 7, 19, 37, ...] -> [6, 12, 18, ...] -> [6, 6, ...] - Bryan Jacobs (bryanjj(AT)gmail.com), Mar 31 2005

a(n+1)=(n+1)!=1,2,6,.. has e.g.f. 1/(1-x)^2. - Paul Barry (pbarry(AT)wit.ie), Apr 22 2005

Write numbers 1 to n on a circle. Then a(n) = sum of the products of all n-2 adjacent numbers. E.g. a(5) = 1*2*3 + 2*3*4 + 3*4*5 + 4*5*1 +5*1*2 = 120. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Jul 10 2005

The number of chains of maximal length in the power set of {1,2,...,n} ordered by the subset relation. - Rick L. Shepherd (rshepherd2(AT)hotmail.com), Feb 05 2006

The number of circular permutations of n letters for n >= 0 is 1,1,1,2,6,24,120,720,5040,40320,... - Xavier Noria (fxn(AT)hashref.com), Jun 04 2006

a(n)=number of deco polyominoes of height n (n>=1; see definitions in the Barcucci et al. references). - Emeric Deutsch (deutsch(AT)duke.poly.edu), Aug 07 2006

a(n) = number of partition tableaux of size n. See Steingrimsson/Williams link for the definition. - David Callan (callan(AT)stat.wisc.edu), Oct 06 2006

Consider the n! permutation of the integer sequence [n]=1,2,...,n. The i-th permutation consists of ncycle(i) permutation cycles. Then, if the sum Sum_{i=1}^{n!} 2^ncycle(i) runs from 1 to n!, we have Sum_{i=1}^{n!} 2^ncycle(i) = (n+1)!. E.g. for n=3 we have ncycle(1)=3, ncycle(2)=2, ncycle(3)=1, ncycle(4)=2, ncycle(5)=1, ncycle(6)=2 and 2^3+2^2+2^1+2^2+2^1+2^2 = 8+4+2+4+2+4 = 24 = (n+1)!. - Thomas Wieder (thomas.wieder(AT)t-online.de), Oct 11 2006 a(n) = number of set partitions of {1,2,...,2n-1,2n} into blocks of size 2 (perfect matchings) in which each block consists of one even and one odd integer. For example, a(3)=6 counts 12-34-56, 12-36-45, 14-23-56, 14-25-36, 16-23-45, 16-25-34. - David Callan (callan(AT)stat.wisc.edu), Mar 30 2007

Consider the multiset M = [1,2,2,3,3,3,4,4,4,4,...] = [1,2,2,...,n x 'n'] and form the set U (where U is a set in the strict sense) of all subsets N (where N may be a multiset again) of M. Then the number of elements |U| of U is equal to (n+1)!. E.g. for M = [1,2,2] we get U = [[],[2],[2,2],[1],[1,2],[1,2,2]] and |U| = 3! = 6. This observation is a more formal version of the comment given already by Rick L. Shepherd (rshepherd2(AT)hotmail.com), Jan 14 2004. - Thomas Wieder (thomas.wieder(AT)t-online.de), Nov 27 2007

For n >= 1, a(n) = 1, 2, 6, 24, ... are the positions corresponding to the 1's in decimal expansion of Liouville's constant (A012245). - Paul Muljadi (paulmuljadi(AT)yahoo.com), Apr 15 2008

Number of terms in a determinant when writing down all multiplication permutations. [From Mats O. Granvik (mgranvik(AT)abo.fi), Sep 12 2008]

Triangle A144107 has row sums = n!(n>0) with right border n! and left border A003319, the INVERTi transform of (1, 2, 6, 24,...) [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 11 2008]

Equals INVERT transform of A052186: (1, 0, 1, 3, 14, 77,...) and row sums of triangle A144108. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 11 2008]

Contribution from A. Umar (aumarh(AT)squ.edu.om), Oct 12 2008: (Start)

a(n) is also the number of order-decreasing full transformations (of an n-chain).

a(n-1) is also the number of nilpotent order-decreasing full transformations (of an n-chain). (End)

Contribution from Calin D. Morosan (cd_moros(AT)alumni.concordia.ca), Nov 28 2008: (Start)

n! is also the number of optimal broadcast schemes in

the complete graph K_{n}, equivalent to the number of binomial

trees embedded in K_{n} (see Calin D. Morosan, Information

Processing Letters, 100 (2006), 188-193). (End)

Sum_{n>=0} 1/a(n) = e [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Mar 03 2009]

Let S_{n} denote the n-star graph. The S_{n} structure consists of n S_{n-1} structures. This sequence gives the number of edges between the vertices of any two specified S_{n+1} structures in S_{n+2} (n >=1). [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), Mar 18 2009]

Chromatic invariant of the sun graph S_{n-2}

It appears that a(n+1) is the inverse binomial transform of A000255. [From Timothy Hopper (timothyhopper(AT)hotmail.co.uk), Aug 20 2009]

a(n) is also the determinant of an square matrix, An, whose coefficients are the reciprocals of beta function: a{i,j}=1/beta(i,j), det(An)=n! [From Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Sep 21 2009]

Contribution from Johannes W. Meijer (meijgia(AT)hotmail.com), Oct 20 2009: (Start)

The asymptotic expansions of the exponential integrals E(x,m=1,n=1) ~ exp(-x)/x*(1 - 1/x + 2/x^2 - 6/x^3 + 24/x^4 + ...) and E(x,m=1,n=2) ~ exp(-x)/x*(1 - 2/x + 6/x^2 - 24/x^3 + ...) lead to the factorial numbers. See A163931 and A130534 for more information.

(End)

Satisfies A(x)/A(x^2), A(x) = A173280. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 14 2010]

a(n) = A173333(n,1). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Feb 19 2010]

a(n) = G^n where G = geometric mean of the first n positive integers. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), May 28 2010]

Increasing colored 1-2 trees with choice of two colors for the rightmost branch of nonleaves.[Wenjin Woan, May 23, 2011]

FORMULA Exp(x) = Sum_{m >= 0} x^m/m!. [From Mohammad K. Azarian, Dec 28 2010]

Sum((-1)^i * i^n * binomial(n, i), i=0..n) = (-1)^n * n! - Yong Kong (ykong(AT)curagen.com), Dec 26 2000

Sum((-1)^i * (n-i)^n * binomial(n, i), i=0..n) = n! - Peter C. Heinig (algorithms(AT)gmx.de), Apr 10 2007

The sequence trivially satisfies the recurrence a(n+1) = sum(binomial(n,k)*a(k)*a(n-k),k=0..n). - Robert Ferreol, Dec 05 2009

a(n)=n*a(n-1), n >= 1. n! ~ sqrt(2*Pi) * n^(n+1/2) / e^n (Stirling's approximation).

a(0)=1, a(n)=subs(x=1, diff(1/(2-x), x$n)), n=1, 2... - Karol A. Penson (penson(AT)lptl.jussieu.fr), Nov 12 2001

E.g.f.: 1/(1-x).

a(n) = Sum_{k = 0..n, (-1)^(n-k)*A000522(k)*binomial(n, k)} = Sum_{k = 0..n, (-1)^(n-k)*(x+k)^n*binomial(n, k)} . - DELEHAM Philippe, Jul 08 2004

Binomial transform of A000166. - Ross La Haye (rlahaye(AT)new.rr.com), Sep 21 2004

a(n)=sum(i=1, n, (-1)^(i-1) * sum of 1..n taken n-i at a time) - e.g. 4! = (1*2*3+1*2*4+1*3*4+2*3*4) - (1*2+1*3+1*4+2*3+2*4+3*4) + (1+2+3+4) - 1 = (6+8+12+24) - (2+3+4+6+8+12) + 10 - 1 = 50 - 35 + 10 - 1 = 24 - Jon Perry (perry(AT)globalnet.co.uk), Nov 14 2005

a(n)=(n-1)*(a(n-1)+a(n-2)), n >= 2. - Matthew J. White (mattjameswhite(AT)hotmail.com), Feb 21 2006

a(n) = 1/Det[Table[(i+j)!/i!/(j+1)!,{i,1,n},{j,1,n}]] for n>0. This is a matrix with Catalan numbers on diagonal. - Alexander Adamchuk (alex(AT)kolmogorov.com), Jul 04 2006

Hankel transform of A074664 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 21 2007

For n>=2, a(n-2)=(-1)^n*sum((j+1)*stirling1(n,j+1),j=0..n-1); [From Milan R. Janjic (agnus(AT)blic.net), Dec 14 2008]

Contribution from Paul Barry (pbarry(AT)wit.ie), Jan 15 2009: (Start)

G.f.: 1/(1-x-x^2/(1-3x-4x^2/(1-5x-9x^2/(1-7x-16x^2/(1-9x-25x^2....(continued fraction), hence Hankel transform is A055209.

G.f. of (n+1)! is 1/(1-2x-2x^2/(1-4x-6x^2/(1-6x-12x^2/(1-8x-20x^2.... (continued fraction), hence Hankel transform is A059332. (End)

a(n) = Prod_{p prime} p^{Sum_{k>0} [n/p^k]} by Legendre's formula for the highest power of a prime dividing n!. [From Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Jul 24 2009] a(n) = A053657(n)/A163176(n) for n > 0. [From Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Jul 26 2009]

It appears that a(n)=(1/0!)+(1/1!)*n+(3/2!)*n*(n-1)+(11/3!)*n*(n-1)*(n-2)+...+(b(n)/n!)*n*(n-1)*...*2*1, where a(n)=(n+1)! and b(n)=A000255. [From Timothy Hopper (timothyhopper(AT)hotmail.co.uk), Aug 12 2009]

a(n)= a(n-1)^2/a(n-2) +a(n-1), n>=2 [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Sep 21 2009]

a(n)=Gamma(n+1) [From Barbarel Tres Mil (barbarel3000(AT)yahoo.es), Sep 21 2009]

a(n) = A_{n}(1) where A_{n}(x) are the Eulerian polynomials. [From Peter Luschny (peter(AT)luschny.de), Aug 03 2010]

a(n)= n*(2*a(n-1) - (n-1)*a(n-2)), n>1 [From Gary Detlefs (gdetlefs(AT)aol.com), Sep 16 2010]

1/a(n) = -sum_{k=1..n+1} (-2)^k*(n+k+2)*a(k)/(a(2*k+1)*a(n+1-k)). [From Groux Roland, Dec 08 2010]

Contribution by Vladimir Shevelev (shevelev(AT)bgu.ac.il), Feb 21 2011. (Start)

a(n) = prod{p prime, p <=n} p^(sum{i>=1} floor(n/p^i);

The infinitary analog of this formula is

a(n) = prod{q terms of A050376 <=n} q^((n)_q), where (n)_q denotes the number of those numbers <=n for which q is an infinitary divisor (for the definition see comment in A037445). (End)

A000161 Number of partitions of n into 2 squares.

COMMENTS Number of ways of writing n as a sum of 2 squares when order does not matter.

Number of similar sublattices of square lattice with index n.

Comment from Charles R Greathouse IV, Mar 08 2010: Let Pk = the number of partitions of n into k nonzero squares. Then we have A000161 = P0 + P1 + P2, A002635 = P0 + P1 + P2 + P3 + P4, A025427 = P3, A025428 = P4.

a(A022544(n))=0; a(A001481(n))>0; a(A125022(n))=1; a(A118882(n))>1. [Reinhard Zumkeller, Aug 16 2011]

FORMULA a(n) = card { { a,b } c N | a^2+b^2 = n } - M. F. Hasler (Maximilian.Hasler(AT)gmail.com), Nov 23 2007

Let f(n)= the number of divisors of n that are congruent to 1 modulo 4 minus the number of its divisors that are congruent to 3 modulo 4, and define delta(n) to be 1 if n is a perfect square and 0 otherwise. Then a(n)=1/2 (f(n)+delta(n)+delta(1/2 n)). [From Ant King (mathstutoring(AT)ntlworld.com), Oct 05 2010]

A000166 Subfactorial or rencontres numbers, or derangements: number of permutations of n elements with no fixed points.

COMMENTS Euler not only gives the first ten or so terms of the sequence, he also proves both recurrences a(n)=(n-1)(a(n-1)+a(n-2)) and a(n)=na(n-1)+(-1)^n.

a(n) is the permanent of the matrix with 0 on the diagonal and 1 elsewhere. - Yuval Dekel, Nov 01 2003

a(n) is the number of desarrangements of length n. A desarrangement of length n is a permutation p of {1,2,...,n} for which the smallest of all the ascents of p (taken to be n if there are no ascents) is even. Example: a(3)=2 because we have 213 and 312 (smallest ascents at i=2). See the J. D\'esarm\'enien link and the Bona reference (p. 118). - Emeric Deutsch (deutsch(AT)duke.poly.edu), Dec 28 2007

a(n) is the number of deco polyominoes of height n and having in the last column an even number of cells. A deco polyomino is a directed column-convex polyomino in which the height, measured along the diagonal, is attained only in the last column. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Dec 28 2007

Attributed to Nicholas Bernoulli in connection with a probability problem that he presented. See Problem #15, p. 494, in "History of Mathematics" by David M. Burton, 6th edition. - Mohammad K. Azarian (azarian(AT)evansville.edu), Feb 25 2008

a(n) is the number of permutations p of {1,2,...,n} with p(1)!=1 and having no right-to-left minima in consecutive positions. Example a(3)=2 because we have 231 and 321. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Mar 12 2008

a(n) is the number of permutations p of {1,2,...,n} with p(n)!=n and having no left to right maxima in consecutive positions. Example a(3)=2 because we have 312 and 321. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Mar 12 2008

Number of wedged (n-1)-spheres in the homotopy type of the Boolean complex of the complete graph K_n. - Bridget Eileen Tenner (bridget(AT)math.depaul.edu), Jun 04 2008

The only prime number in the sequence is 2. [From Howard Berman (howard_berman(AT)hotmail.com), Nov 08 2008]

Contribution from Emeric Deutsch (deutsch(AT)duke.poly.edu), Apr 02 2009: (Start)

a(n) is the number of permutations of {1,2,...,n} having exactly one small ascent. A small ascent in a permutation (p_1,p_2,...,p_n) is a position i such that p_{i+1} - p_i =1. (Example: a(3)=2 because we have 312 and 231 (see the Charalambides reference, pp. 176-180).

a(n) is the number of permutations of {1,2,...,n} having exactly one small descent. A small descent in a permutation (p_1,p_2,...,p_n) is a position i such that p_i - p_{i+1} =1. (Example: a(3)=2 because we have 132 and 213. (End)

For n>2, a(n) + a(n-1) = A000255(n-1); where A000255 = (1, 1, 3, 11, 53,...) [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Apr 16 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Apr 17 2009: (Start)

Connection to A002469 (game of mousetrap with n cards): A002469(n) =

(n-2)*A000255(n-1) + A000166(n). (Cf. triangle A159610). (End)

Contribution from Emeric Deutsch (deutsch(AT)duke.poly.edu), Jul 18 2009: (Start)

a(n) is the sum of the values of the largest fixed points of all non-derangements of length n-1. Example: a(4)=9 because the non-derangements of length 3 are 123, 132, 213, and 321, having largest fixed points 3, 1, 3, and 2, respectively.

a(n) is the number of non-derangements of length n+1 for which the difference between the largest and smallest fixed point is 2. Example: a(3)=2 because we have 1'43'2 and 32'14'; a(4)=9 because we have 1'23'54, 1'43'52, 1'53'24, 52'34'1, 52'14'3, 32'54'1, 213'45', 243'15', and 413'25' (the extreme fixed points are marked).

(End)

a(n), n>=1, is also the number of unordered necklaces with n beads, labeled differently from 1 to n, where each necklace has >=2 beads. This produces the M2 multinomial formula involving partitions without part 1 given below. Because M2(p) counts the permutations with cycle structure given by partition p, this formula gives the number of permutations without fixed points (no 1-cycles), i.e., the derangements, hence the subfactorials with their recurrence relation and inputs. Each necklace with no beads is assumed to contribute a factor 1 in the counting, hence a(0)=1. This comment derives from a family of recurrences found by Malin Sjodahl for a combinatorial problem for certain quark and gluon diagrams (Febr 27 2010). [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Jun 01 2010]

Contribution from Emeric Deutsch (deutsch(AT)duke.poly.edu), Sep 06 2010: (Start)

a(n) is the number of permutations of {1,2,...,n, n+1} starting with 1 and having no successions. A succession in a permutation (p_1,p_2,...,p_n) is a position i such that p_{i+1} - p_i =1. Example: a(3)=2 because we have 1324 and 1432.

a(n) is the number of permutations of {1,2,...,n} that do not start with 1 and have no successions. A succession in a permutation (p_1,p_2,...,p_n) is a position i such that p_{i+1} - p_i =1. Example: a(3)=2 because we have 213 and 321.

(End)

Increasing colored 1-2 trees with choice of two colors for the rightmost branch of nonleave except on the leftmost path, there is no vertex of outdegree one on the leftmost path. [Wenjin Woan, May 23, 2011]

FORMULA (this_sequence + A000522)/2 = A009179, (this_sequence - A000522)/2 = A009628.

The termwise sum of this sequence and A003048 gives the factorial numbers - D. G. Rogers, Aug 26 2006

a(n) = {(n-1)!)/exp(1)}, n > 1, and {x} is the nearest integer function. Simon Plouffe, March 1993

a(0) = 1, a(n) = [n!/e + 1/2] for n > 0.

a(n) = n!*Sum((-1)^k/k!, k=0..n).

a(n) = (n-1)*(a(n-1)+a(n-2)), n>0.

a(n) = n*a(n-1)+(-1)^n.

E.g.f.: exp(-x)/(1-x).

O.g.f. for number of permutations with exactly k fixed points is (1/k!)*Sum_{i>=k} i!*x^i/(1+x)^(i+1). - Vladeta Jovovic (vladeta(AT)eunet.rs), Aug 12 2002

E.g.f. for number of permutations with exactly k fixed points is x^k/(k!*exp(x)*(1-x)). - Vladeta Jovovic (vladeta(AT)eunet.rs), Aug 25 2002

a(n)=sum{k=0..n, binomial(n, k)(-1)^(n-k)k!}=sum{k=0..n, (-1)^(n-k)*n!/(n-k)!} - Paul Barry (pbarry(AT)wit.ie), Aug 26 2004

The e.g.f. y(x) satisfies y' = x*y/(1-x).

Inverse binomial transform of A000142. - Ross La Haye (rlahaye(AT)new.rr.com), Sep 21 2004

Subf(n) = n^(n-1) - { 2*C(n-2, 0) +2*C(n-2, 1) +C(n-2, 2) }*n^(n-2) + { 4*C(n-3, 0) +11*C(n-3, 1) +16*C(n-3, 2) +11*C(n-3, 3) +3*C(n-3, 4) }*n^(n-3) - { 10*C(n-4, 0) +55*C(n-4, 1) +147*C(n-4, 2) +215*C(n-4, 3) +179*C(n-4, 4) +80*C(n-4, 5) +15*C(n-4, 6) }*n^(n-4) + ... - Andre F. Labossiere (boronali(AT)laposte.net), Dec 06 2004

In Maple notation, representation as n-th moment of a positive function on [-1, infinity] : a(n)= int(x^n*exp(-x-1), x=-1..infinity), n=0, 1... . a(n) is the Hamburger moment of the function exp(-1-x)*Heaviside(x+1) . From Karol A. Penson - penson(AT)lptl.jussieu.fr, Jan 21 2005 a(n) = A001120(n) - n! . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Sep 04 2005

a(n) = Integral_{x=0..infinity} (x-1)^n*exp(-x) dx - Gerald McGarvey (gerald.mcgarvey(AT)comcast.net), Oct 14 2006

a(n)=Sum(T(n,k),k=2,4,...), where T(n,k)=A092582(n,k)=kn!/(k+1)! for 1<=k<n and T(n,n)=1. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Feb 23 2008

a(n) = n!/e + (-1)^n*(1/(n+2 - 1/(n+3 - 2/(n+4 - 3/(n+5 - ...))))). Asymptotic result (Ramanujan): (-1)^n*(a(n) - n!/e) ~ 1/n - 2/n^2 + 5/n^3 - 15/n^4 + ..., where the sequence [1,2,5,15,...] is the sequence of Bell numbers A000110. - Peter Bala (pbala(AT)toucansurf.com), Jul 14 2008

Contribution from William Vaughn (wvaughn(AT)cvs.rochester.edu), Apr 13 2009: (Start)

a(n) = Integral_{p=0..1} (log(1/(1-p)) - 1)^n dp

Proof: Using the substitutions 1=log(e) and y = e(1-p) the above integral can be converted to: ((-1)^n/e) Integral_{y=0..e} (log(y))^n dy.

From CRC Integral tables we find the antiderivative of

(log(y))^n is (-1)^n n! Sum_{k=0..n} (-1)^k y(log(y))^k / k!.

Using the fact that e(log(e))^r = e for any r>=0 and

0(log(0))^r = 0 for any r>=0 the Integral becomes

n! Sum_{k=0..n} (-1)^k / k!, which is line 9 of the "formula" section.

Q.E.D. (End)

a(n) = exp(-1)*GAMMA(n+1,-1) (incomplete Gamma function) [From Mark van Hoeij (hoeij(AT)math.fsu.edu), Nov 11 2009]

G.f.: 1/(1-x^2/(1-2x-4x^2/(1-4x-9x^2/(1-6x-16x^2/(1-8x-25x^2/(1-... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Nov 27 2009]

a(n) = sum(M2(p), p from Pano1(n)), n>=1, with Pano1(n) the set of partitions without part 1, and the multinomial M2 numbers. See the characteristic array for partitions without part 1 given by A145573 in Abramowitz-Stegun (A-S) order, with A002865(n) the total number of such partitions. The M2 numbers are given for each partition in A-St order by the array A036039. [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Jun 01 2010]

a(n) = row sum of A008306(n), n>1 [From Gary Detlefs (gdetlefs(AT)aol.com), Jul 14 2010]

a(n)= ((-1)^n)*(n-1)*hypergeom([-n+2, 2], [], 1), n>=1; 1 for n=0. [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Aug 16 2010]

a(n)= hypergeom([-n, 1], [], 1), n>=1; 1 for n=0. From the binomial convolution due to the e.g.f. [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Aug 26 2010]

int(x^n*exp(x),x=0..1)=(-1)^n*(a(n)*e-n!)

A000169 Number of labeled rooted trees with n nodes: n^(n-1).

COMMENTS Also the number of connected transitive subtree acyclic digraphs on n vertices. - Robert Castelo (rcastelo(AT)imim.es), Jan 06 2001

For any given integer k a(n) is also is the number of functions from {1,2,...,n} to {1,2,...,n} such that the sum of the function values is k mod n. - Sharon Sela (sharonsela(AT)hotmail.com), Feb 16 2002

The n-th term of a geometric progression with first term 1 and common ratio n: a(1) = 1 -> 1,1,1,1,... a(2) = 2 -> 1,2,... a(3) = 9 -> 1,3,9,... a(4) = 64 -> 1,4,16,64,... - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 25 2004

All rational solutions to the equation x^y = y^x, with x < y, are given by x =

A000169(n+1)/A000312(n), y = A000312(n+1)/A007778(n), where n = 1, 2, 3, ... - Nick Hobson Nov 30 2006

a(n+1) is also the number of partial functions on n labeled objects. - Franklin T. Adams-Watters, Dec 25 2006

In other words, if A is a finite set of size n-1, then a(n) is the number of binary relations on A that are also functions. Note that a(n)=sum(binomial(n-1,k)*(n-1)^k, k=0..n-1)=n^(n-1), where binomial(n-1,k) is the number of ways to select a domain D of size k from A and n^k is the number of functions from D to A. [From Dennis P. Walsh, April 21 2011]

More generally, consider the class of sequences of the form a(n)=[n*c(1)*...*c(i)]^(n-1). This sequence has c(1)=1. A052746 has a(n) = [2*n]^(n-1), A052756 has a(n)=[3*n]^(n-1),A052764 has a(n)=[4*n]^(n-1), A052789 has a(n)=[5*n]^(n-1). These sequences have a combinatorial structure like simple grammars. - Ctibor O. ZIZKA, Feb 23 2008

a(n) is equal to the logarithmic transform of the sequence b(n) = n^(n-2) starting at b(2). [From Kevin Hu (10thsymphony(AT)gmail.com), Aug 23 2010]

Also, number of labeled connected multigraphs of order n without cycles except one loop. See link below to have a picture showing the bijection between rooted trees and multigraphs of this kind. (Note that there are no labels in the picture, but the bijection remains true if we label the nodes.) [From W. Bomfim, Sep 04 2010]

a(n) is also the number of functions f:{1,2,...,n} -> {1,2,...,n} such that f(1) = 1.

FORMULA The e.g.f. T(x) = Sum_{n=1..infinity} n^(n-1)*x^n/n! satisfies T(x) = x*exp(T(x)), so T(x) is the functional inverse (series reversion) of x*exp(-x).

Also T(x) = -LambertW(-x) where W(x) is the principal branch of Lambert's function.

T(x) is sometimes called Euler's tree function.

a(n) = A000312(n-1)*A128434(n,1)/A128433(n,1). - Reinhard Zumkeller, Mar 03 2007

A000182 Tangent (or "Zag") numbers: expansion of tan(x), also expansion of tanh(x).

COMMENTS Number of Joyce trees with 2n-1 nodes. Number of tremolo permutations of {0,1,...,2n}. - Ralf Stephan, Mar 28 2003

The Hankel transform of this sequence is A000178(n) for n odd = 1, 12, 34560, ...; example: det([1, 2, 16; 2, 16, 272, 16, 272, 7936]) = 34560. - DELEHAM Philippe, Mar 07 2004

a(n) = number of increasing labeled full binary trees with 2n-1 vertices. Full binary means every non-leaf vertex has two children, distinguished as left and right; labeled means the vertices are labeled 1,2,...,2n-1; increasing means every child has a label greater than its parent. - David Callan, Nov 29 2007

Contribution from Micha Hofri (hofri(AT)wpi.edu), May 27 2009: (Start)

a(n) was found to be the number of permutations of [2n] which when inserted in order, to form a binary search tree, yield the maximally full possible tree (with only one single-child node).

The e.g.f. is sec^2(x)=1+tan^2(x), and the same coefficients can be manufactured from the tan(x) itself, which is the e.g.f. for the number of trees as above for odd number of nodes. (End)

FORMULA E.g.f.: log(sec x) = Sum_{n > 0} a(n)*x^(2*n)/(2*n)!.

E.g.f.: tan x = Sum_{n >= 0} a(n+1)*x^(2*n+1)/(2*n+1)!.

E.g.f.: (sec x)^2 = Sum_{n >= 0} a(n+1)*x^(2*n)/(2*n)!.

2/(exp(2x)+1) = 1 + Sum_{n>=1} (-1)^(n+1) a(n) x^(2n-1)/(2n-1)! = 1 - x + x^3/3 - 2*x^5/15 + 17*x^7/315 - 62*x^9/2835 + ...

a(n) = 2^(2*n) (2^(2*n) - 1) |B_(2*n)| / (2*n) where B_n are the Bernoulli numbers

(A000367/A002445 or A027641/A027642).

Asymptotics: a(n) ~ 2^(2*n+1)*(2*n-1)!/Pi^(2*n).

Sum[2^(2*n + 1 - k)*(-1)^(n + k + 1)*k!*StirlingS2[2*n + 1, k], {k, 1, 2*n + 1}]. - Victor Adamchik, Oct 05 2005

a(n) = abs[c(2*n-1)] where c(n)= 2^(n+1) * (1-2^(n+1)) * Ber(n+1)/(n+1) = 2^(n+1) * (1-2^(n+1)) * (-1)^n * Zeta(-n) = [-(1+EN(.))]^n = 2^n * GN(n+1)/(n+1) = 2^n * EP(n,0) = (-1)^n * E(n,-1) = (-2)^n * n! * Lag[n,-P(.,-1)/2] umbrally = (-2)^n * n! * C{T[.,P(.,-1)/2] + n, n} umbrally for the signed Euler numbers EN(n), the Bernoulli numbers Ber(n), the Genocchi numbers GN(n), the Euler polynomials EP(n,t), the Eulerian polynomials E(n,t), the Touchard / Bell polynomials T(n,t), the binomial function C(x,y) = x!/[(x-y)!*y!] and the polynomials P(j,t) of A131758. - Tom Copeland, Oct 05 2007

Contribution from Johannes W. Meijer, Jun 27 2009: (Start)

a(1) = A094665(0,0)*A156919(0,0) and a(n) = sum(2^(n-k-1)*A094665(n-1, k)*A156919(k,0), k = 1..n-1) for n = 2, 3, .., see A162005.

(End)

G.f.: 1/(1-1*2*x/(1-2*3*x/(1-3*4*x/(1-4*5*x/(1-5*6*x/(1-... (continued fraction) [From Paul Barry, Feb 24 2010]

Contribution from Paul Barry, Mar 29 2010: (Start)

G.f.: 1/(1-2x-12x^2/(1-18x-240x^2/(1-50x-1260x^2/(1-98x-4032x^2/(1-162x-9900x^2/(1-... (continued fraction);

coefficient sequences given by (2n+1)(2n)^2(2n-1) and 2(2n+1)^2 (see Van Fossen Conrad reference). (End)

E.g.f.: Sum_{n>=0} Product_{k=1..n} tanh(2k*x) = Sum_{n>=0} a(n)*x^n/n!. [From Paul D. Hanna, May 11 2010]

a(n)=sum(sum(binomial(k,r)*sum(sum(binomial(l,j)/2^(j-1)*sum((-1)^(n)*binomial(j,i)*(j-2 *i)^(2*n),i,0,floor((j-1)/2))*(-1)^(l-j),j,1,l)*(-1)^l*binomial(r+l-1,r-1),l,1,2*n)*(-1)^(1-r),r,1,k)/k,k ,1,2*n), n>0 [From Kruchinin Vladimir, Aug 23 2010]

a(n)=(-1)^(n+1)*sum(j!*stirling2(2*n+1,j)*2^(2*n+1-j)*(-1)^(j),j,1,2*n+1). n>=0. [From Kruchinin Vladimir, Aug 23 2010]

If n is odd such that 2*n-1 is prime, then a(n)==1(mod (2*n-1); if n is even such that 2*n-1 is prime, then a(n)==-1(mod (2*n-1). [From Vladimir Shevelev, Sep 01 2010]

Recursion: a(n) = (-1)^(n-1)+sum_{i=1..n-1}(-1)^{n-i+1)*C(2*n-1,2*i-1)* a(i). - Vladimir Shevelev, Aug 08 2011

A000203 sigma(n) = sum of divisors of n. Also called sigma_1(n).

COMMENTS Multiplicative: If the canonical factorization of n into prime powers is the product of p^e(p) then sigma_k(n) = Product_p ((p^((e(p)+1)*k))-1)/(p^k-1).

Sum_{d|n} 1/d^k is equal to sigma_k(n)/n^k. So sequences A017665-A017712 also give the numerators and denominators of sigma_k(n)/n^k for k = 1..24. The power sums sigma_k(n) are in sequences A000203 (this sequence) (k=1), A001157-A001160 (k=2,3,4,5), A013954-A013972 for k = 6,7,...,24. - Ahmed Fares (ahmedfares(AT)my-deja.com), Apr 05 2001

A number n is abundant if sigma(n) > 2n (cf. A005101), perfect if sigma(n) = 2n (cf. A000396), deficient if sigma(n) < 2n (cf. A005100).

a(n) = number of sublattices of index n in a generic 2-dimensional lattice - Avi Peretz (njk(AT)netvision.net.il), Jan 29 2001

The sublattices of index n are in one-one correspondence with matrices [a b; 0 d] with a>0, ad=n, b in [0..d-1]. The number of these is Sum_{d|n} = sigma(n), which is A000203. A sublattice is primitive if gcd(a,b,d) = 1; the number of these is n * product_{p|n} (1+1/p), which is A001615. [Cf. Grady reference.]

Sum of number of common divisors of n and m, where m runs from 1 to n. - Naohiro Nomoto (pcmusume(AT)m11.alpha-net.ne.jp), Jan 10 2004

a(n) is the cardinality of all extensions over Q_p with degree n in the algebraic closure of Q_p, where p>n. - Volker Schmitt (clamsi(AT)gmx.net), Nov 24 2004. Cf. A100976, A100977, A100978 (p-adic extensions).

Triangle A144736: row sums = sigma(n), right border = phi(n), left border = d(n). [From Gary W. Adamson, Sep 20 2008]

Regarding Euler's recurring sequence for sigma(n). Let s=sigma, then Euler states [Young, p.361]: "...I say that the value of s(n) can always be combined from some of the preceding as prescribed by the following formula: s(n) = s(n-1) + s(n-2) - s(n-5) - s(n-7) + s(n-12) + s(n-15) - s(n-22) - s(n-26) + ..." [From Gary W. Adamson, Oct 05 2008]

Prefaced with a zero: (0, 1, 3, 4, 7,...) = A147843 convolved with the partition numbers, A000041. [From Gary W. Adamson, Nov 15 2008]

Relating to the partition numbers, = row sums of triangle A174740 [From Gary W. Adamson, Mar 28 2010]

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Relating to the partition numbers, = row sums of triangle A174740 [From Gary W. Adamson, Mar 28 2010]

A000204 Lucas numbers (beginning with 1): L(n) = L(n-1) + L(n-2) with L(1) = 1, L(2) = 3.

COMMENTS See A000032 for the version beginning 2, 1, 3, 4, 7, ...

Also called Schoute's accessory series (see Jean, 1984). - N. J. A. Sloane, Jun 08 2011.

L(n) is the number of matchings in a cycle on n vertices: L(4)=7 because the matchings in a square with edges a,b,c,d (labeled consecutively) are the empty set,a,b,c,d,ac and bd. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Jun 18 2001

This comment covers a family of sequences which satisfy a recurrence of the form a(n) = a(n-1) + a(n-m), with a(n) = 1 for n = 1...m-1, a(m) = m+1. The generating function is (x+m*x^m)/(1-x-x^m). Also a(n) = 1 + n*sum(binomial(n-1-(m-1)*i, i-1)/i, i=1..n/m). This gives the number of ways to cover (without overlapping) a ring lattice (or necklace) of n sites with molecules that are m sites wide. Special cases: m=2: A000204, m=3: A001609, m=4: A014097, m=5: A058368, m=6: A058367, m=7: A058366, m=8: A058365, m=9: A058364.

L(n) is the number of points of period n in the golden mean shift. The number of orbits of length n in the golden mean shift is given by the n-th term of the sequence A006206 - Thomas Ward (t.ward(AT)uea.ac.uk), Mar 13 2001

Row sums of A029635 are 1,1,3,4,7,... - Paul Barry (pbarry(AT)wit.ie), Jan 30 2005

a(n) counts circular n-bit strings with no repeated 1's. E.g. for a(5): 00000 00001 00010 00100 00101 01000 01001 01010 10000 10010 10100. Note #{0...} = fib(n+1), #{1...} = fib(n-1), #{000..., 001..., 100...} = a(n-1), #{010..., 101...} = a(n-2). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Oct 14 2001

Contribution from Matthew Lehman (matt.comicopia(AT)gmail.com), Nov 17 2008: (Start)

In the Fibonacci sequence, F(n) = F(n-1) + F(n-2),

for every i-th number, F(n+i) = A(i)*F(n) + B(i)*F(n-i),

A(i) is given by this sequence,

Also, A(i) = F(2*i-1)/F(i-1).

B(i) = (-1)^(i+1).

For every Fibonacci number, F(n+1) = F(n) + F(n-1).

For every 2nd Fibonacci number, F(n+2) = 3*F(n) - F(n-2).

For every 3rd Fibonacci number, F(n+3) = 4*F(n) + F(n-3).

For every 4th Fibonacci number, F(n+4) = 7*F(n) - F(n-4).

For every 5th Fibonacci number, F(n+5) = 11*F(n) + F(n-5).

(End)

A014217(n+2)-A014217(n). A014217=1,1,2,4,6,11,17,29,. In A014217 L(n),Lucas, with L(0)=2,L(1)=1, A000032. See submitted A153263. [From Paul Curtz (bpcrtz(AT)free.fr), Dec 22 2008]

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(End)

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A000217 Triangular numbers: a(n) = C(n+1,2) = n(n+1)/2 = 0+1+2+...+n.

COMMENTS Number of edges in complete graph of order n, K_n.

Number of legal ways to insert a pair of parentheses in a string of n letters. E.g. there are 6 ways for three letters: (a)bc, (ab)c, (abc), a(b)c, a(bc), ab(c). [Proof: there are C(n+2,2) ways to choose where the parentheses might go, but n+1 of them are illegal because the parentheses are adjacent.] Cf. A002415.

For n >= 1 a(n)=n(n+1)/2 is also the genus of a nonsingular curve of degree n+2 like the Fermat curve x^(n+2) + y^(n+2) = 1 - Ahmed Fares (ahmedfares(AT)my_deja.com), Feb 21 2001

From Harnack's theorem (1876), the number of branches of a non-singuliar curve of order n is bounded by a(n) - Benoit Cloitre, Aug 29 2002

Number of tiles in the set of double-n dominoes. - Scott A. Brown (scottbrown(AT)neo.rr.com), Sep 24 2002

Number of ways a chain of n non-identical links can be be broken up. This is based on a similar problem in the field of proteomics: the number of ways a peptide of n amino acid residues can be be broken up in a mass spectrometer. In general each amino acid has a different mass, so AB and BC would have different masses. - James Raymond (raymond(AT)unlv.edu), Apr 08 2003

Maximum number of intersections of n+1 lines which may only have 2 lines per intersection point. Maximal number of closed regions when n+1 lines are maximally 2-intersected in given by T(n-1). Using n+1 lines with k>1 parallel lines, the maximum number of 2-intersections is given by T(n)-T(k-1). - Jon Perry, Jun 11 2003

Number of distinct straight lines that can pass through n points in 3-dimensional space. - Cino Hilliard (hillcino368(AT)gmail.com), Aug 12 2003

Triangular numbers - odd numbers = triangular numbers: 0,1,3,6,10,15,21... - 0,1,3,5,7,9,11... = 0,0,0,1,3,6,10... - Xavier Acloque Oct 31 2003

Centered polygonal numbers are the result of [number of sides * A000217 + 1]. E.g. centered pentagonal numbers (1,6,16,31...)= 5 * (0,1,3,6...) + 1. Centered heptagonal numbers (1,8,22,43...)= 7 * (0,1,3,6...) + 1. - Xavier Acloque Oct 31 2003

Maximum number of lines formed by the intersection of n+1 planes. - Ronald R. King (king_ron(AT)asdk12.org), Mar 29 2004

Number of permutations of [n] which avoid the pattern 132 and have exactly 1 descent. - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Aug 26 2004

a(n) == 1 mod (n+2) if n is odd and == n/2+2 mod (n+2) if n is even. - Jon Perry, Dec 16 2004

Number of ways two different numbers can be selected from the set {0,1,2,...,n} without repetition, or, number of ways two different numbers can be selected from the set {1,2,...,n} with repetition.

1, 6, 120 are the only numbers which are both triangular and factorial. - Christopher M. Tomaszewski (cmt1288(AT)comcast.net), Mar 30 2005

a(n) = A108299(n+3,4) = -A108299(n+4,5). - Reinhard Zumkeller, Jun 01 2005

A110560/A110561 = numerator/denominator of the coefficients of the exponential generating function. - Jonathan Vos Post, Jul 27 2005

Binomial transform is {0, 1, 5, 18, 56, 160, 432, ... }, A001793 with one leading zero . - Philippe DELEHAM, Aug 02 2005

a(n) = A111808(n,2) for n>1. - Reinhard Zumkeller, Aug 17 2005

Each pair of neighboring terms adds to a perfect square. - Zak Seidov, Mar 21 2006

a(n)*a(n+1) = A006011(n+1) = (n+1)^2*(n^2+2)/4 = 3*A002415(n+1) = 1/2*a(n^2+2*n). a(n-1)*a(n) = 1/2*a(n^2-1). - Alexander Adamchuk, Apr 13 2006 Corrected and edited by Charlie Marion, Nov 26 2010

Number of transpositions in the symmetric group of n+1 letters i.e. the number of permutations that leave all but two elements fixed. - Geoffrey Critzer, Jun 23 2006

With rho(n):=exp(i*2*Pi/n) (an n-th root of 1) one has, for n>=1, rho(n)^a(n)=(-1)^(n+1). Just use the triviality a(2*k+1)=0(mod (2*k+1)) and a(2*k)=k(mod 2*k).

a(n) = A126890(n,0). - Reinhard Zumkeller, Dec 30 2006

a(n) is the number of terms in the expansion of (a_1+a_2+a_3)^n - Sergio Falcon (sfalcon(AT)dma.ulpgc.es), Feb 12 2007

(sqrt(8 a(n) + 1) - 1)/2 = n. - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Feb 26 2007

The number of distinct handshakes in a room with n people (n>=2). - Mohammad K. Azarian, Apr 12 2007

Equal to the rank (minimal cardinality of a generating set) of the semigroup PT_n\S_n, where PT_n and S_n denote the partial transformation semigroup and symmetric group on [n]. - James East (james.east(AT)latrobe.edu.au), May 03 2007

Gives the total number of triangles found when cevians are drawn from a single vertex on a triangle to the side opposite that vertex, where n=the number of cevians drawn+1. For instance, with 1 cevian drawn, n=1+1=2 and a(n)=2(2+1)/2=3 so there is a total of 3 triangles in the figure. If 2 cevians are drawn from one point to the opposite side, then n=1+2=3 and a(n)=3(3+1)/2=6 so there is a total of 6 triangles in the figure. - Noah Priluck (npriluck(AT)gmail.com), Apr 30 2007

a(n), n>=1, is the number of ways in which n-1 can be written as a sum of three positive integers if representations differing in the order of the terms are considered to be different. In other words a(n),n>=1, is the number of positive integral solutions of the equation x + y + z = n-1. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Apr 22 2001

a(n+1), n>=0, is the number of levels with energy n+3/2 (in units of h*f0, with Planck's constant h and the oscillator frequency f0) of the three dimensional isotropic harmonic quantum oscillator. See the comment by A. Murthy above: n=n1+n2+n3 with positive integers and ordered. Proof from the o.g.f. See the A. Messiah reference. W. Lang, Jun 29 2007.

Numbers m>=0 such that round(sqrt(2m+1))-round(sqrt(2m))=1. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Aug 06 2007

Numbers m>=0 such that ceiling(2*sqrt(2m+1))-1=1+floor(2*sqrt(2m)). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Aug 06 2007

Numbers m>=0 such that fract(sqrt(2m+1))>1/2 and fract(sqrt(2m))<1/2, where fract(x) is the fractional part of x (i.e. x-floor(x), x>=0). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Aug 06 2007

Sequence allows us to find X values of the equation: 8*X^3 + X^2 = Y^2. To find Y values: b(n)=n(n+1)(2n+1)/2. - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Nov 06 2007

If Y and Z are 3-blocks of an n-set X then, for n>=6, a(n-1) is the number of (n-2)-subsets of X intersecting both Y and Z. - Milan R. Janjic (agnus(AT)blic.net), Nov 09 2007

Equals row sums of triangle A143320, n>0. [From Gary W. Adamson, Aug 07 2008]

a(n) is also a perfect number A000396 when n is a Mersenne prime A000668. [From Omar E. Pol, Sep 05 2008]

a(n) = A022264(n) - A049450(n). [From Reinhard Zumkeller, Oct 09 2008]

Equals row sums of triangle A152204 [From Gary W. Adamson, Nov 29 2008]

The number of matches played in a round robin tournament: n*(n-1)/2 gives the number of matches needed for n players. Everyone plays against everyone else exactly once. [From Georg Wrede (georg(AT)iki.fi), Dec 18 2008]

-a(n+1) = E(2)*C(n+2,2) (n>=0) where E(n) are the Euler number in the enumeration A122045 and C(n,k) are the binomial coefficients A007318. Viewed this way a(n) is the special case k=2 in the sequence of diagonals in the triangle A153641. [From Peter Luschny (peter(AT)luschny.de), Jan 06 2009]

4a(x)+4a(y)+1=(x+y+1)^2+(x-y)^2 [From Vladimir Shevelev , Jan 21 2009]

Equivalent to the first differences of successive tetrahedral numbers. See A000292. [From Jeremy Cahill (jcahill(AT)inbox.com), Apr 15 2009]

Contribution from Peter Luschny, Jul 12 2009: (Start)

The general formula for alternating sums of powers is in terms of the Swiss-Knife polynomials P(n,x) A153641 2^(-n-1)(P(n,1)-(-1)^k P(n,2k+1)). Thus

a(k) = |2^(-3)(P(2,1)-(-1)^k P(2,2k+1))|. (End)

a(n) is the smallest number > a(n-1) such that gcd(n,a(n)) = gcd(n,a(n-1)). If n is odd this gcd is n; if n is even it is n/2. [From Franklin T. Adams-Watters, Aug 06 2009]

a(A006894(n)) = a(A072638(n-1)+1) = A072638(n) = A006894(n+1)-1 for n >= 1. For n=4, a(11) = 66. [From Jaroslav Krizek, Sep 12 2009]

Partial sums of nonnegative numbers. [From Juri-Stepan Gerasimov, Jan 25 2010]

The numbers along the right edge of Floyd's triangle are 1, 3, 6, 10, 15, ... [From Paul Muljadi, Jan 25 2010]

a(n) = A001477(n)*n - A001477(n-1) - A001477(n-2) - A001477(n-3) - .... - A001477(1) - A001477(0), i.e. a(n) = n^2 - sum_{k=0}^{n-1} k [for d=0 in the general formula a(n) = n^2*(d*n-d+2)/2-sum_{k=0}^{n-1} k*(d*k-d+2)/2]. [From Bruno Berselli, Apr 21 2010]

a(n) = 4*a(floor(n/2))+(-1)^(n+1)*floor((n+1)/2). For n=15, a(15)=4*a(7)+8=4*28+8=120. [From Bruno Berselli, May 23 2010]

It is well known that a(n) + a(n-1) = n^2. Less well known is that

a(n)+2a(n-1)+a(n-2) = n^2+(n-1)^2; e.g., 10+2*6+3=25=4^2+3^2 and

a(n)+3a(n-1)+3a(n-2)+a(n-3)= n^2+2*(n-1)^2+(n-2)^2;

e.g., 15+3*10+3*6+3=66=5^2+2*4^2+3^2.

In general, for n>=m>2,sum_{k=0,...,m}binomial(m,m-k)*a(n-k)=

sum_{k=0,...,m-1}binomial(m-1,m-1-k)*(n-k)^2 For example,

1*28+5*21+10*15+10*10+5*6+1*3=416=1*7^2+4*6^2+6*5^2+4*4^2+1*3^2.

It is also well known that a(n) - a(n-1) = n. Less well known is that

a(n) - 2a(n-1) + a(n-2) = 1, a(n) - 3a(n-1) + 3a(n-2) - a(n-3) = 0 and

a(n) - 4a(n-1) + 6a(n-2) - 4(a-3) + a(n-4) = 0.

In general, for n>=m>2, sum_{k=0,...,m}(-1)^k*binomial(m,m-k)*a(n-k)=0.

For example, 1*28 - 5*21 + 10*15 - 10*10 + 5*6 - 1*3 = 0.

- Charlie Marion (charliemath(AT)optonline.net), Oct 15 2010

More generally, a(2k+1) == j(2j-1) mod (2k+2j+1) and

a(2k) == [-k + 2j(j-1)] mod (2k+2j)

Column sums of:

```
1 3 5  7  9...
   1  3  5...
         1...
.............
--------------
1 3 6 10 15...
```

Sum(n=1..infinity, 1/a(n)^2)=4*pi^2/3-12; equivalently,

sum(n=1..infinity, 1/a(n)^2)= 12 less than the volume of a sphere with radius pi^(1/3). - Charlie Marion (charliemath(AT)optonline.net), Dec 03 2010

A004201(a(n)) = A000290(n); A004202(a(n)) = A002378(n). [Reinhard Zumkeller, Feb 12 2011]

FORMULA G.f.: x/(1-x)^3.

E.g.f.: exp(x)*(x+x^2/2).

a(n)=a(-1-n).

a(n) = a(n-1)+n. - Zak Seidov, Mar 06 2005

a(n) + a(n-1)*a(n+1) = a(n)^2. - Terry Trotter (ttrotter(AT)telesal.net), Apr 08, 2002

a(n) = (-1)^n*sum(k=1, n, (-1)^k*k^2) - Benoit Cloitre, Aug 29 2002

a(n) = ((n+2)/n)*a(n-1)

Sum(n=1..infinity, 1/a(n)) = 2. - Jon Perry, Jul 13 2003

For n>0, a(n)=A001109(n)-(sum_{k=0...n-1}((2k+1)*A001652(n-1-k))) e.g. 10=204-(1*119+3*20+5*3+7*0) - Charlie Marion, Jul 18 2003

With interpolated zeros, this is n(n+2)/8*(1+(-1)^n)/2=sum{k=0..n, sum{j=0..k, floor(k^2/4)}}. - Benoit Cloitre, Aug 19 2003

a(n+1) is the determinant of the n X n symmetric Pascal matrix M_(i, j)=C(i+j+1, i) - Benoit Cloitre, Aug 19 2003

a(n)=[(n^3-(n-1)^3)-(n^1-(n-1)^1)]/(2^3-2^1)= (n^3-(n-1)^3-1)/6 - Xavier Acloque Oct 24 2003

a(n) = a(n-1) + (1 + sqrt[1 + 8*a(n-1)])/2. E.g. a(4) = a(3) + (1 + sqrt[1 + 8*a(3)])/2 = 6 + (1 + sqrt[49])/2 = 6+8/2 = 10. This recursive relation is inverted when taking the negative branch of the square root, i.e. a(n) is transformed into a(n-1) rather than a(n+1). - Carl R. White, Nov 04 2003

a(n)+a(n+1) = (n+1)^2.

a(n) = a(n-2)+2n-1. - Paul Barry (pbarry(AT)wit.ie), Jul 17 2004

a(n) = Sqrt[Sum[Sum[(i*j), {i, 1, n}], {j, 1, n}]] - Alexander Adamchuk, Oct 24 2004

a(n) = Sqrt[Sqrt[Sum[Sum[(i*j)^3, {i, 1, n}], {j, 1, n}]]]. a(n) = Sum[Sum[Sum[(i*j*k)^3, {i, 1, n}], {j, 1, n}], {k, 1, n}]^(1/6) - Alexander Adamchuk, Oct 26 2004

a(0) = 0, a(1) = 1, a(n) = 2*a(n-1)-a(n-2)+1 - Miklos Kristof, Mar 09 2005

a(n) = Sum_{k = 1...n} phi(k)*floor(n/k) = Sum{k = 1...n} A000010(k)*A010766(n, k) (R. Dedekind). - Vladeta Jovovic (vladeta(AT)eunet.rs), Feb 05 2004

a(n) = floor((2n+1)^2/8) - Paul Barry, May 29 2006

For positive n, we have a(8*a(n))/a(n) = 4*(2n+1)^2 = (4n+2)^2, i.e., a(A033996(n))/a(n) = 4*A016754(n) = (A016825(n))^2 = A016826(n). - Lekraj Beedassy, Jul 29 2006

[a(n)]^2+[a(n+1)]^2 = a((n+1)^2) [R B Nelsen, Math Mag 70 (2) (1997) p 130]. - R. J. Mathar, Nov 22 2006

a(n) = A023896(n) + A067392(n). - Lekraj Beedassy, Mar 02 2007

Sum_{k, 0<=k<=n}a(k)*A039599(n,k)=A002457(n-1), for n>=1 . - Philippe DELEHAM, Jun 10 2007

A general formula for polygonal numbers is: P(k,n) = (k-2)(n-1)n/2 + n, where P(k,n) is the n-th k-gonal number. - Omar E. Pol, Apr 28 2008

If we define f(n,i,a)=sum(binomial(n,k)*stirling1(n-k,i)*product(-a-j,j=0..k-1),k=0..n-i), then a(n)=-f(n,n-1,1), for n>=1. [From Milan R. Janjic (agnus(AT)blic.net), Dec 20 2008]

a(n) = A000124(n-1) + (n-1) for n >= 2. a(n) = A000124(n) - 1. A000124(n) = central polygonal numbers. [From Jaroslav Krizek, Jun 16 2009]

An exponential generating function for the inverse of this sequence is given by sum((pochhammer(1, m)*pochhammer(1, m))*x^m/(pochhammer(3, m)*factorial(m)), m = 0 .. infinity)=((2-2*x)*ln(1-x)+2*x)/x^2; The n-th derivative of which has a closed form which must be evaluated by taking the limit x=0. A000217[n+1]=limit(Diff(((2-2*x)*ln(1-x)+2*x)/x^2, x$n),x=0)^-1=limit((2*GAMMA(n)*(-1/x)^n*(n*(x/(-1+x))^n*(-x+1+n)*LerchPhi(x/(-1+x), 1, n)+(-1+x)*(n+1)*(x/(-1+x))^n+n*(ln(1-x)+ln(-1/(-1+x)))*(-x+1+n))/x^2),x=0)^-1 [From Stephen Crowley, Jun 28 2009]

a(n) = A034856(n+1) - A005408(n) = A005843(n) + A000124(n) - A005408(n) = A000124(n) - 1. [From Jaroslav Krizek, Sep 05 2009]

With offset 1, a(n)=1/2*floor(n^3/(n+1)) [From Gary Detlefs, Feb 14 2010]

a(n)=3*a(n-1)-3*a(n-2)+a*(n-3); a(0)=0,a(1)=1. [From M. Dols (markdols99(AT)yahoo.com), Aug 20 2010]

a(n)=sqrt((sum(i^3,{i,1,n}))) [Zak Seidov, Dec 07 2010]

a(n)*a(n+k) +a(n+1)*a(n+1+k) = a((n+1)*(n+1+k)). Charlie Marion, Feb 04 2011

For n>0 a(n)=1/(Integral_{x=0..Pi/2} 4*(sin(x))^(2*n-1)*(cos(x))^3). [From Francesco Daddi, Aug 02 2011]

a(n) = A110654(n) * A008619(n). [Reinhard Zumkeller, Aug 24 2011]

A000219 Number of planar partitions of n.

COMMENTS Two-dimensional partitions of n in which no row or column is longer than the one before it (compare A001970). E.g. a(4) = 13:

```
4.31.3.22.2.211.21..2.1111.111.11.11.1 but not 2
.....1....2.....1...1......1...11.1..1........ 11
....................1............1..1
.....................................1
```

Can also be regarded as number of "safe pilings" of cubes in the corner of a room: the height should not increase away from the corner - Wouter Meeussen (wouter.meeussen(AT)pandora.be).

Also number of partitions of n objects of 2 colors, each part containing at least one black object. - (Christian G. Bower (bowerc(AT)usa.net), Jan 08 2004)

Number of partitions of n into 1 type of part 1, 2 types of part 2, ..., k types of part k. e.g. n=3 gives 111, 12, 12', 3, 3', 3". - Jon Perry (perry(AT)globalnet.co.uk), May 27 2004

Can also be regarded as the number of Jordan canonical forms for an nxn matrix. (i.e. a 5x5 matrix has 24 distinct Jordan canonical forms, dependent on the algebraic and geometric multiplicity of each eigenvalue.) [From Aaron Gable (agable(AT)hmc.edu), May 26 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 13 2009: (Start)

(1/n) * convolution product of n terms * A001157 (sum of squares of divisors of n): (1, 5, 10, 21, 26, 50, 50, 85,...) = a(n). As shown by [Bressoud, p.12]: 1/6 * [1*24 + 5*13 + 10*6 + 21*3 + 26*1 + 50*1] = 288/6 = 48.

Convolved with the aerated version (1, 0, 1, 0, 3, 0, 6, 0, 13,...) = A026007: (1, 1, 2, 5, 8, 16, 28, 49, 83,...). (End)

Starting with offset 1 = row sums of triangle A162453 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 03 2009]

Also number of functions from an n-set to itself, up to permutation of the set (compare to partition function, which is number of conjugacy classes in S_n) [From Harry Altman (haltman(AT)umich.edu), Nov 21 2009]

FORMULA G.f.: Product_{k >= 1} 1/(1 - x^k)^k. - MacMahon, 1912.

Euler transform of sequence [1, 2, 3, ...].

a(n) ~ (c_2 / n^(25/36)) * exp(c_1 * n^(2/3)), where c_1 = 2.00945... and c_2 = 0.23151... - Wright, 1931. Corrected Jun 01 2010 by Rod Canfield - see Mutafchiev and Kamenov. The exact value of c_2 is exp(2c) 2^{-11/36} \zeta(3)^{7/36} (3\pi)^{-1/2}, where c = \int_0^{\infty}

\frac{y\log y}{exp(2\pi y)-1}dy = (1/2) \zeta'(-1).

a(n)=(1/n)*Sum_{k=1..n} a(n-k)*sigma_2(k), n > 0, a(0)=1, where sigma_2(n)=A001157(n)=sum of squares of divisors of n. - Vladeta Jovovic (vladeta(AT)eunet.rs), Jan 20 2002

G.f.: exp(Sum_{n>0} sigma_2(n)*x^n/n). a(n) = Sum_{pi} Product_{i=1..n} binomial(k(i)+i-1, k(i)) where pi runs through all nonnegative solutions of k(1)+2*k(2)+..+n*k(n)=n. - Vladeta Jovovic (vladeta(AT)eunet.rs), Jan 10 2003

A000225 2^n - 1. (Sometimes called Mersenne numbers, although that name is usually reserved for A001348.)

COMMENTS This is the Gaussian binomial coefficient [n,1] for q=2.

Number of rank-1 matroids over S_n.

Numbers n such that central binomial coefficient is odd : Mod[A001405[A000225(n)],2]=1 - Labos E. (labos(AT)ana.sote.hu), Mar 12 2003

This gives the (zero-based) positions of odd terms in the following convolution sequences: A000108, A007460, A007461, A007463, A007464, A061922.

Also solutions (with minimum number of moves) for the problem of Benares Temple, i.e. three diamond needles with n discs ordered by decreasing size on the first needle to place in the same order on the third one, without ever moving more than one disc at a time and without ever placing one disc at the top of a smaller one. - Xavier Acloque Oct 18 2003

a(0) = 0, a(1) = 1; a(n) = smallest number such that a(n)-a(m) == 0 (mod (n-m+1)), for all m. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Oct 23 2003

Binomial transform of [1, 1/2, 1/3...] = [1/1, 3/2, 7/3...]; (2^n - 1)/n, n=1,2,3... - Gary W. Adamson (qntmpkt(AT)yahoo.com), Apr 28 2005

Numbers whose binary representation is 111...1. E.g. the 7th term is (2^7)-1=127=1111111 (in base 2). - Alexandre Wajnberg (alexandre.wajnberg(AT)ulb.ac.be), Jun 08 2005

a(n) = A099393(n-1) - A020522(n-1) for n>0. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Feb 07 2006

Numbers n for which the expression 2^n/(n+1) is an integer. - Paolo P. Lava (ppl(AT)spl.at), May 12 2006

Number of nonempty subsets of a set with n elements. - Michael Somos Sep 03 2006

For n>=2, a(n) is the least Fibonacci n-step number that is not a power of 2. - Rick L. Shepherd (rshepherd2(AT)hotmail.com), Nov 19 2007

Let P(A) be the power set of an n-element set A. Then a(n) = the number of pairs of elements {x,y} of P(A) for which x and y are disjoint and for which either x is a subset of y or y is a subset of x. - Ross La Haye (rlahaye(AT)new.rr.com), Jan 10 2008

Also, let P(A) be the power set of an n-element set A. Then a(n+1) = the number of pairs of elements {x,y} of P(A) for which either 0) x and y are disjoint and for which either x is a subset of y or y is a subset of x, or 1) x = y. - Ross La Haye (rlahaye(AT)new.rr.com), Jan 10 2008

2^n-1 is the sum of the elements in a Pascal triangle of depth n. - Brian Lewis (bsl04(AT)uark.edu), Feb 26 2008

Sequence generalized : a(n)=(A^n -1)/(A-1), n>=1, A integer >=2. This sequence has A=2; A003462 has A=3; A002450 has A=4; A003463 has A=5; A003464 has A=6; A023000 has A=7; A023001 has A=8; A002452 has A=9; A002275 has A=10; A016123 has A=11; A016125 has A=12; A091030 has A=13; A135519 has A=14; A135518 has A=15; A131865 has A=16; A091045 has A=17; A064108 has A=20. - Ctibor O. Zizka (ctibor.zizka(AT)seznam.cz), Mar 03 2008

a(n) is also a Mersenne prime A000668 when n is a prime number A000043. [From Omar E. Pol (info(AT)polprimos.com), Aug 31 2008]

a(n) is also a Mersenne number A001348 when n is prime. [From Omar E. Pol (info(AT)polprimos.com), Sep 05 2008]

With offset 1, = row sums of triangle A144081; and INVERT transform of A009545 starting with offset 1; where A009545 = expansion of sin(x)*exp(x). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 10 2008]

Numbers n such that A000120(n)/A070939(n) = 1 [From Ctibor O.Zizka (c.zizka(AT)email.cz), Oct 15 2008]

a(n) = A024036(n)/A000051(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Feb 14 2009]

For n > 0, sequence is equal to partial sums of A000079 ; a(n) = A000203(A000079(n-1)). [From Lekraj Beedassy (blekraj(AT)yahoo.com), May 02 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), May 23 2009: (Start)

Starting with offset 1 = the Jacobsthal sequence, A001045,

(1, 1, 3, 5, 11, 21,...) convolved with (1, 2, 2, 2,...). (End)

Numbers n such that n=2*phi(n+1)-1. [From Farideh Firoozbakht (mymontain(AT)yahoo.com), Jul 23 2009]

a(n) = (a(n-1)+1) th odd numbers = A005408(a(n-1)) for n >= 1. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Sep 11 2009]

a(n) = sum of previous terms + n = (Sum_(i=0...n-1) a(i)) + n for n >= 1. Partial sums of a(n) for n >= 0 are A000295(n+1). Partial sums of a(n) for n >= 1 are A000295(n+1) and A130103(n+1). a(n) = A006127(n) - (n+1). [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Oct 16 2009]

If n is even a(n) mod 3 = 0. This follows from the congruences 2^(2k) - 1 ~ 2*2* ... *2 - 1 ~ 4*4* ... *4 - 1 ~ 1*1* ... *1 - 1 ~ 0 (mod 3). (Note that 2*2* ... *2 has an even number of terms.) [From W. Bomfim (webonfim(AT)bol.com.br), Oct 31 2009]

Let A be the Hessenberg matrix of order n, defined by: A[1,j]=1, A[i,i]:=2,(i>1), A[i,i-1]=-1, and A[i,j]=0 otherwise. Then, for n>=1, a(n)=det(A). [From Milan R. Janjic (agnus(AT)blic.net), Jan 26 2010]

a(2*n) = a(n)*A000051(n); a(n) = A173787(n,0). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Feb 28 2010]

For n>0: A179857(a(n))=A024036(n) and A179857(m)<A024036(n) for m<a(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jul 31 2010]

This is the sequence A(0,1;1,2;2) = A(0,1;3,-2;0) of the family of sequences [a,b:c,d:k] considered by G. Detlefs, and treated as A(a,b;c,d;k) in the W. Lang link given below. [From Wolfdieter Lang (wolfdieter.lang(AT)physik.uni-karlsruhe.de), Oct 18 2010]

a(n)=S(n+1,2), a Stirling number of the second kind.

See the example below. [From Dennis Walsh (dwalsh@mtsu.edu), March 29 2011]

Entries of row a(n) in Pascal's triangle are all odd, while entries of row a(n)-1 have alternating parities of the form odd, even, odd, even, ..., odd.

Define the bar operation as an operation on signed permutations that flips the sign of each entry. Then a(n+1) is the number of signed permutations of length 2n that are equal to the bar of their reverse-complements and avoid the set of patterns {(-2,-1), (-1,+2), (+2,+1)}. (See the Hardt and Troyka reference.) - Justin M. Troyka, Aug 13 2011.

FORMULA G.f.: x/((1-2*x)*(1-x)). E.g.f.: exp(2*x)-exp(x).

E.g.f. if offset 1: ((exp(x)-1)^2)/2.

a(n)=sum{k=0..n-1, 2^k} - Paul Barry (pbarry(AT)wit.ie), May 26 2003

a(n)=a(n-1)+2a(n-2)+2, a(0)=0, a(1)=1. - Paul Barry (pbarry(AT)wit.ie), Jun 06 2003

Let b(n)=(-1)^(n-1)a(n). Then b(n)=Sum(i!i Stirling2(n, i)(-1)^(i-1), i=1, .., n). E.g.f. of b(n): (exp(x)-1)/exp(2x). - Mario Catalani (mario.catalani(AT)unito.it), Dec 19 2003

a(n+1) = 2*a(n) + 1, a(0) = 0.

Sum_{k=1..n} C(n, k).

a(n) = n + sum(i=0, n-1, a(i)); a(0) = 0. - Rick L. Shepherd (rshepherd2(AT)hotmail.com), Aug 04 2004

a(n+1)=(n+1)sum{k=0..n, binomial(n, k)/(k+1)} - Paul Barry (pbarry(AT)wit.ie), Aug 06 2004

a(n+1)=sum{k=0..n, binomial(n+1, k+1)} - Paul Barry (pbarry(AT)wit.ie), Aug 23 2004

Inverse binomial transform of A001047. Also U sequence of Lucas sequence L(3, 2). - Ross La Haye (rlahaye(AT)new.rr.com), Feb 07 2005

a(n) = A119258(n,n-1) for n>0. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), May 11 2006

a(n) = 3*a(n-1) - 2*a(n-2); a(0)=0,a(1)=1 - Lekraj Beedassy (blekraj(AT)yahoo.com), Jun 07 2006

Sum_{n=1..inf}1/a(n) = 1.606695152...(Erdos-Borwein constant;see A065442, A038631) . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 27 2006

Stirling_2[n-k,2] starting from n=k+1. - Artur Jasinski (grafix(AT)csl.pl), Nov 18 2006

a(n) = A125118(n,1) for n>0. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Nov 21 2006

a(n) = StirlingS2(n+1,2) - Ross La Haye (rlahaye(AT)new.rr.com), Jan 10 2008

a(n) = A024088(n)/A001576(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Feb 15 2009]

Contribution from Enrique Perez Herrero (psychgeometry(AT)gmail.com), Aug 21 2010: (Start)

a(n)=J_n(2), where J_n is the nth Jordan Totient function: (A007434, is J_2)

a(n)=sum(d|2, d^n*mu(2/d)) (End)

A000244 Powers of 3.

COMMENTS Same as Pisot sequences E(1,3), L(1,3), P(1,3), T(1,3). Essentially same as Pisot sequences E(3,9), L(3,9), P(3,9), T(3,9). See A008776 for definitions of Pisot sequences.

Number of (s(0), s(1), ..., s(2n+2)) such that 0 < s(i) < 6 and |s(i) - s(i-1)| = 1 for i = 1,2,....,2n+2, s(0) = 1, s(2n+2) = 3. - Herbert Kociemba (kociemba(AT)t-online.de), Jun 10 2004

a(1) = 1, a(n+1) is the least number so that there are a(n) even numbers between a(n) and a(n+1). Generalization for the sequence of powers of k: 1,k,k^2, k^3, k^4,... There are a(n) multiples of k-1 between a(n) and a(n+1). - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Nov 28 2004

a(n) = sum of (n+1)-th row in Triangle A105728. - Reinhard Zumkeller, Apr 18 2005

With p(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n)=sum_{i=1}^{p(n)} (p(i)!/(prod_{j=1}^{d(i)} m(i,j)!))*2^(p(i)-1) - Thomas Wieder (wieder.thomas(AT)t-online.de), May 18 2005

For any k>1 in the sequence,k is the first prime power appearing in the prime decomposition of repunit R_k, i.e. of A002275(k). - Lekraj Beedassy, Apr 24 2006

a(n-1) is the number of compositions of compositions. In general, (k+1)^(n-1) is the number of k-levels nested compositions (e.g., 4^(n-1) is the number of compositions of compositions of compositions, etc.). Each of the n-1 spaces between elements can be a break for one of the k levels, or not a break at all. - Franklin T. Adams-Watters, Dec 06 2006

Let S be a binary relation on the power set P(A) of a set A having n = |A| elements such that for every element x, y of P(A), xSy if x is a subset of y. Then a(n) = |S|. - Ross La Haye (rlahaye(AT)new.rr.com), Dec 22 2006

If X_1, X_2, ..., X_n is a partition of the set {1,2,...,2*n} into blocks of size 2 then, for n>=1, a(n) is equal to the number of functions f : {1,2,..., 2*n}->{1,2} such that for fixed y_1,y_2,...,y_n in {1,2} we have f(X_i)<>{y_i}, (i=1,2,...,n). - Milan R. Janjic (agnus(AT)blic.net), May 24 2007

1/1 + 1/3 + 1/9 + ... = 3/2 [From Gary W. Adamson, Aug 29 2008]

Equals row sums of triangle A125076 [From Gary W. Adamson, Dec 18 2008]

Equals row sums of triangle A153279 [From Gary W. Adamson, Dec 23 2008]

This is a general comment on all sequences of the form a(n)=[(2^k)-1]^n for all positive integers k. Example 1.1.16 of Stanley's "Enumerative Combinatorics" offers a slightly different version. a(n) in the number of functions f:[n] into P([k])-{}. a(n) is also the number of funtions f:[k] into P([n]) such that the generalized intersection of f(i) for all i in [k] is the empty set. Where [n]={1,2,...n},P([n]) is the power set of [n] and {} is the empty set. [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Feb 28 2009]

a(n) = A064614(A000079(n)) and A064614(m)<a(n) for m < A000079(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Feb 08 2010]

Contribution from Gary W. Adamson, May 17 2010: (Start)

3^(n+1) = (1, 2, 2, 2,...) dot (1, 1, 3, 9,...3^n); e.g. 3^3 = 27 =

(1, 2, 2, 2) dot (1, 1, 3, 9) = (1 + 2 + 6 + 18) (End)

a(n) is the number of generalized compositions of n when there are 3*2^i different types of i, (i=1,2,...). [From Milan R. Janjic (agnus(AT)blic.net), Sep 24 2010]

For n>=1, a(n-1) is the number of generalized compositions of n when there are 2^(i-1) different types of i, (i=1,2,...). [From Milan R. Janjic (agnus(AT)blic.net), Sep 24 2010]

The sequence in question ("Powers of 3") also describes the number of moves of the k-th disk solving the [RED ; BLUE ; BLUE] or [RED ; RED ; BLUE] pre-colored Magnetic Tower of Hanoi puzzle (Cf. A183111 - A183125).

a(n) is the number of Stern polynomials of degree n. See A057526. - T. D. Noe, Mar 01 2011

Positions of records in the number of odd prime factors, A087436. [From Juri-Stepan Gerasimov, Mar 17 2011]

Sum of coefficients of the expansion of (1+x+x^2)^n. [From Adi Dani, Jun 21 2011]

a(n) is the number of compositions of n elements among {0,1,2}; e.g., a(2)=9 since there are the 9 compositions 0+0, 0+1, 1+0, 0+2, 1+1, 2+0, 1+2, 2+1, and 2+2. [From Adi Dani, Jun 21 2011, modified by editors.]

Except the first two terms, these are odd numbers n such that no x with 2<=x<=n-2 satisfy x^(n-1) == 1 (mod n). [From Arkadiusz Wesolowski, Jul 03 2011]

FORMULA a(n) = 3^n.

a(n) = 3*a(n-1).

G.f.: 1/(1-3*x).

E.g.f.: exp(3*x).

a(n)=n!*Sum_{i+j+k=n, i, j, k >= 0} 1/(i!*j!*k!). - Benoit Cloitre (benoit7848c(AT)orange.fr), Nov 01 2002

a(n) = Sum_{k=0..n} 2^k*binomial(n, k).

a(n) = A090888(n, 2). - Ross La Haye (rlahaye(AT)new.rr.com), Sep 21 2004

a(n) = 2^(2n) - A005061(n). - Ross La Haye (rlahaye(AT)new.rr.com), Sep 10 2005

a(n) = A112626(n, 0). - Ross La Haye (rlahaye(AT)new.rr.com), Jan 11 2006

Hankel transform of A007854 = [1, 3, 12, 51, 222, 978, 4338, ...] . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 26 2006

Binomial transform of the powers of two: (1, 2, 4, 8,...). - Gary W. Adamson, Sep 20 2007

a(n) = 2*StirlingS2(n+1,3) + StirlingS2(n+2,2) = 2*(StirlingS2(n+1,3) + StirlingS2(n+1,2)) + 1. - Ross La Haye (rlahaye(AT)new.rr.com), Jun 26 2008

a(n) = 2*StirlingS2(n+1,3) + StirlingS2(n+2,2) = 2*(StirlingS2(n+1,3) + StirlingS2(n+1,2)) + 1. - Ross La Haye (rlahaye(AT)new.rr.com), Jun 09 2008

If p[i]=fibonacci(2i-2) and if A is the Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n-1)= det A. [From Milan R. Janjic (agnus(AT)blic.net), May 08 2010]

G.f. A(x)=M(x)/(1-M(x))^2, M(x) - o.g.f for Motzkin numbers (A0001006) [From Kruchinin Vladimir, Aug 18 2010]

a(n) = A133494(n+1). [Arkadiusz Wesolowski, Jul 27 2011]

A000262 Number of "sets of lists": number of partitions of {1,..,n} into any number of lists, where a list means an ordered subset.

COMMENTS Determinant of n X n matrix M=[m(i,j)] where m(i,i)=i, m(i,j)=1 if i>j, m(i,j)=i-j if j>i. - Vladeta Jovovic (vladeta(AT)eunet.rs), Jan 19 2003

a(n) = Sum_{k=0..n} |A008275(n,k)| * A000110(k). - Vladeta Jovovic (vladeta(AT)eunet.rs), Feb 01 2003

a(n) = (n-1)!*LaguerreL(n-1,1,-1). - Vladeta Jovovic (vladeta(AT)eunet.rs), May 10 2003

With p(n) = the number of integer partitions of n, d(i) = the number of different parts of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n) = sum_{i=1}^{p(n)} n!/(prod_{j=1}^{d(i)} m(i,j)!) - Thomas Wieder, May 18 2005

Consider all n! permutations of the integer sequence [n] = 1,2,3,...,n. The i-th permutation, i=1,2,...,n!, consists of Z(i) permutation cycles. Such a cycle has the length lc(i,j), j=1,...,Z(i). For a given permutation we form the product of all its cycle lengths prod_{j=1}^{Z(i)} lc(i,j). Furthermore, we sum up all such products for all permutations of [n] which gives sum_{i=1 .. n!} prod_{j=1}^{Z(i)} lc(i,j) = A000262(n). For n=4 we have sum_{i=1 .. n!} prod_{j=1}^{Z(i)} lc(i,j) = 1*1*1*1 + 2*1*1 + 3*1 + 2*1*1 + 3*1 + 2*1 + 3*1 + 4 + 3*1 + 4 + 2*2 + 2*1*1 + 3*1 + 4 + 3*1 + 2*1*1 + 2*2 + 4 + 2*2 + 4 + 3*1 + 2*1*1 + 3*1 + 4 = 73 = A000262(4). - Thomas Wieder, Oct 06 2006

For a finite set S of size n, a chain gang G of S is a partially ordered set (S,<=) consisting only of chains. The number of chain gangs of S is a(n). For example, with S={a, b}and n=2, there are a(2)=3 chain gangs of S, namely, {(a,a),(b,b)}, {(a,a),(a,b),(b,b)} and {(a,a),(b,a),(b,b)}. - Dennis P. Walsh, Feb 22 2007

(-1)*A000262 with the first term set to 1 and A084358 form a reciprocal pair under the list partition transform and associated operations described in A133314. Cf. A133289. - Tom Copeland, Oct 21 2007

Consider the distribution of n unlabeled elements "1" onto n levels where empty levels are allowed. In addition, the empty levels are labeled. Their names are 0_1, 0_2, 0_3, etc. This sequence gives the total number of such distributions. If the empty levels are unlabeled ("0"), then the answer is A001700. Let the colon ":" separate two levels. Then, for example, for n=3 we have a(3)=13 arrangements: 111:0_1:0_2, 0_1:111:0_2, 0_1:0_2:111, 111:0_2:0_1, 0_2:111:0_1, 0_2:0_1:111, 11:1:0, 11:0:1, 0:11:1, 1:11:0, 1:0:11, 0:1:11, 1:1:1. - Thomas Wieder, May 25 2008

Row sums of exponential Riordan array [1,x/(1-x)]. - Paul Barry, Jul 24 2008

a(n) is the number of partitions of [n] (A000110) into lists of noncrossing sets. For example, a(3)=3 counts 12, 1-2, 2-1 and a(4) = 73 counts the 75 partitions of [n] into lists of sets (A000670) except for 13-24, 24-13 which fail to be noncrossing. - David Callan, Jul 25 2008

a(i-j)/(i-j)! gives the value of the non-null element (i, j) of the lower triangular matrix exp(S)/exp(1), where S is the lower triangular matrix - of whatever dimension - having all its (non-null) elements equal to one. [From Giuliano Ca rele (giulianoca rele(AT)tin.it), Aug 11 2008, Sep 07 2008]

a(n) is also the number of nilpotent partial one-one bijections (of an n-element set). Equivalently, it is the number of nilpotents in the symmetric inverse semigroup (monoid) [From A. Umar (aumarh(AT)squ.edu.om), Sep 14 2008]

A000262 is the exp transform of the factorial numbers A000142. [From Thomas Wieder, Sep 10 2008]

If n is a positive integer then the infinite continued fraction (1+n)/(1+(2+n)/(2+(3+n)/(3+...))) converges to the rational number A052852(n)/A000262(n). [David Angell (angell(AT)maths.unsw.edu.au), Dec 18 2008]

a(n) = exp(-1)*n!*M(n+1,2,1), n>=1, where M (=1F1) is the confluent hypergeometric function of the first kind. [Shai Covo (green355(AT)netvision.net.il), Jan 20 2010]

Jovovic's formula dated Sep 20 2006 can be restated as follows: a(n) is the n-th ascending factorial moment of the Poisson distribution with parameter (mean) 1. [Shai Covo (green355(AT)netvision.net.il), Jan 25 2010]

a(n) = n!* A067764(n) / A067653(n) [From Gary Detlefs, May 15 2010]

The umbral exponential generating function for a(n) is (1-x)^{-B}. In other words, write (1-x)^{-B} as a power series in x whose coefficients are polynomials in B, and then replace B^k with the Bell number B_k. We obtain a(0) + a(1)x + a(2)x^2/2! + ... [From Richard Stanley, Jun 07 2010]

a(n) is the number of Dyck n-paths (A000108) with its peaks labelled 1,2,…,k in some order, where k is the number of peaks. For example a(2)=3 counts U(1)DU(2)D, U(2)DU(1)D, UU(1)DD where the label at each peak is in parentheses. This is easy to prove using generating functions. [David Callan, Aug 23 2011]

FORMULA a(n) = (2*n-1)*a(n-1) - (n-1)*(n-2)*a(n-2).

E.g.f.: exp(x/(1-x)).

Representation as n-th moment of a positive function on positive half-axis, in Maple notation: a(n)=int(x^n*exp(-x-1)*BesselI(1, 2*x^(1/2))/x^(1/2), x =0..infinity), n=1, 2... - Karol A. Penson, Dec 4 2003.

a(n) = Sum_{k=0..n} A001263(n, k)*k!. - DELEHAM Philippe, Dec 10 2003

a(n) = n! Sum[Binomial[n-1, j]/(j+1)!, {j, 0, n-1}] for n=1, 2, 3, ... - Herbert S. Wilf (wilf(AT)math.upenn.edu), Jun 14 2005

Asymptotic expansion for large n: a(n)->(0.4289*n^(-1/4)+0.3574*n^(-3/4)-0.2531*n^(-5/4)+O(n^(-7/4)))*(n^n)*exp(-n+2*sqrt(n)) .- Karol A. Penson, Aug 28 2002

a(n) = exp(-1)*Sum_{m>=0} [m]^n/m!, where [m]^n = m*(m+1)*...*(m+n-1) is the rising factorial. - Vladeta Jovovic (vladeta(AT)eunet.rs), Sep 20 2006

Recurrence: D(n,k) = D(n-1,k-1) + (n-1+k) * D(n-1,k) n >= k >= 0; D(n,0)=0. From this, D(n,1) = n! and D(n,n)=1; a(n) = sum(i=0..n, D(n,i)). - Stephen Dalton (StephenMDalton(AT)gmail.com), Jan 05 2007

Proof: Notice two distinct subsets of the lists for [n]: 1) n is in its own list, then there are D(n-1,k-1); 2) n is in a list with other numbers. Denoting the separation of lists by |, it is not hard to see n has (n-1+k) possible positions, so (n-1+k) * D(n-1,k) - Stephen Dalton (StephenMDalton(AT)gmail.com), Jan 05 2007

Define f_1(x),f_2(x),... such that f_1(x)=exp(x), f_{n+1}(x)=diff(x^2*f_n(x),x), for n>=2. Then a(n-1)=exp(-1)*f_n(1). - Milan R. Janjic (agnus(AT)blic.net), May 30 2008

a(n) = (n-1)! * sum(k=1..n, (a(n-k)*k!)/((n-k)!*(k-1)!)) where a(0)=1. [From Thomas Wieder, Sep 10 2008]

A000272 Number of trees on n labeled nodes: n^(n-2).

COMMENTS Number of spanning trees in complete graph K_n on n labeled nodes.

Robert Castelo (rcastelo(AT)imim.es), Jan 06 2001, observes that n^(n-2) is also the number of transitive subtree acyclic digraphs on n-1 vertices.

a(n) is also the number of ways of expressing an n-cycle in the symmetric group S_n as a product of n-1 transpositions, see example. - Dan Fux (dan.fux(AT)OpenGaia.com or danfux(AT)OpenGaia.com), Apr 12 2001

Also counts parking functions, noncrossing partitions, critical configurations of the chip firing game, allowable pairs sorted by a priority queue [Hamel].

a(n+1) = sum(i * n^(n-1-i) * binomial(n, i), i=1..n) - Yong Kong (ykong(AT)curagen.com), Dec 28 2000

a(n+1) = number of endofunctions with no cycles of length > 1; number of forests of rooted labeled trees on n vertices. - Mitch Harris (Harris.Mitchell(AT)mgh.harvard.edu), Jul 06 2006

a(n) is also the number of nilpotent partial bijections (of an n-element set). Equivalently, the number of nilpotents in the partial symmetric semigroup, P sub n. [From A. Umar (aumarh(AT)squ.edu.om), Aug 25 2008]

a(n) is also the number of edge-labeled rooted trees on n nodes. [From Nikos Apostolakis (nikos.ap(AT)gmail.com), Nov 30 2008]

a(n+1) is the number of length n sequences on an alphabet of {1,2,...,n} that have a partial sum equal to n. For example a(4)=16 because there are 16 length 3 sequences on {1,2,3} in which the terms (beginning with the first term and proceeding sequentialy) sum to 3 at some point in the sequence. {1, 1, 1}, {1, 2, 1}, {1, 2, 2}, {1, 2, 3}, {2, 1, 1}, {2, 1, 2}, {2, 1, 3}, {3, 1, 1}, {3, 1, 2}, {3, 1, 3}, {3, 2, 1}, {3, 2, 2}, {3, 2, 3}, {3, 3, 1}, {3, 3, 2}, {3, 3, 3}} [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Jul 20 2009]

a(3) = 3 is the only prime value in the sequence. There are no semiprime values. Generally, the number of distinct primes dividing a(n) = omega(a(n)) = A001221(a(n)) = omega(n). Similarly, the number of prime divisors of a(n) (counted with multiplicity) = bigomega(a(n)) = A001222(a(n)) = Product (p_j^k_j) = Sum (k_j) where a(n) = Product (p_j^k_j), which is an obvious function of n and n-2. [From Jonathan Vos Post (jvospost3(AT)gmail.com), May 27 2010]

a(n) is the number of acyclic functions from {1,2,...,n-1} to {1,2,...,n}. An acyclic function f satisfies the following property: for any x in the domain, there exists a positive integer k such that (f^k)(x) is not in the domain. Note that f^k denotes the k-fold composition of f with itself, e.g., (f^2)(x)=f(f(x)). [From Dennis Walsh, March 2 2011]

FORMULA E.g.f.: T - (1/2)T^2; where T=T(x) is Euler's tree function (see A000169, also A001858). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Nov 19 2001

E.g.f.: ((W(-x)/x)^2)/(1+W(-x)), W(x): Lambert's function (principal branch).

Number of labeled k-trees on n nodes is binomial(n, k) * (k(n-k)+1)^(n-k-2).

Determinant of the symmetric matrix H generated for a polynomial of degree n by: for(i=1,n-1, for(j=1,i, H[i,j]=(n*i^3-3*n*(n+1)*i^2/2+n*(3*n+1)*i/2+(n^4-n^2)/2)/6-(i^2-(2*n+1)*i+n*(n+1))*(j-1)*j/4; H[j,i]=H[i,j];);); - Gerry Martens (GerryMrt(AT)aol.com), May 04 2007

For n>=1, a(n+1)= Sum(n^(n-i)*Binomial(n-1,i-1),i=1...n) [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Jul 20 2009]

E.g.f. for b(n)=a(n+1): exp(-W(-x)), where W is Lambert's function satisfying W(x)exp(W(x))=x. Proof is contained in link "Notes on acyclic functions..." [From Dennis Walsh, March 2 2011]

A000273 Number of directed graphs (or digraphs) with n nodes.

A000290 The squares: a(n) = n^2.

COMMENTS To test if a number is a square, see Cohen, p. 40. - N. J. A. Sloane, Jun 19 2011.

Zero followed by partial sums of A005408 (odd numbers). - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Aug 13 2002

Begin with n, add the next number, subtract the previous number and so on ending with subtracting a 1: a(n) = n + (n+1) - (n-1) +(n+2) -(n-2) +(n+3)-(n-3)...+(2n-1)-1 = n^2. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 24 2004

Sum of two consecutive triangular numbers A000217. - Lekraj Beedassy, May 14 2004

Numbers with an odd number of divisors: {d(n^2)=A048691(n); for the first occurrence of 2n+1 divisors, see A071571(n)}. - Lekraj Beedassy, Jun 30 2004. See also A000037.

First sequence ever computed by electronic computer, on EDSAC, May 6 1949 (see Renwick link). - Russ Cox, Apr 20 2006

Numbers n such that the imaginary quadratic field Q[Sqrt[-n]] has four units. - Marc LeBrun, Apr 12 2006

For n>0: number of divisors of (n-1)th power of any squarefree semiprime: a(n)=A000005(A006881(k)^(n-1)); a(n) = A000005(A000400(n-1)) = A000005(A011557(n-1)) = A000005(A001023(n-1)) = A000005(A001024(n-1)). - Reinhard Zumkeller, Mar 04 2007

For n>=1, a(n) is equal to the number of functions f:{1,2}->{1,2,...,n} such that for y_1, y_2 in {1,2,...,n} we have f(1)<>y_1 and f(2)<>y_2. - Milan R. Janjic (agnus(AT)blic.net), Apr 17 2007
If a 2-set Y and an (n-2)-set Z are disjoint subsets of an n-set X then a(n-2) is the number of 3-subsets of X intersecting both Y and Z. - Milan R. Janjic (agnus(AT)blic.net), Sep 19 2007
Also numbers a such that a^1/2 + b^1/2 = c^1/2 and a^2 + b = c. - Cino Hilliard (hillcino368(AT)hotmail.com), Feb 07 2008
Numbers n such that the geometric mean of the divisors of n is an integer. - Ctibor O. Zizka, Jun 26 2008
Equals row sums of triangle A143470. Example: 36 = sum of row 6 terms: (23 + 7 + 3 + 1 + 1 + 1). [From Gary W. Adamson , Aug 17 2008]
Equals row sums of triangles A143595 and A056944 [From Gary W. Adamson, Aug 26 2008]
Number of divisors of 6^(n-1) for n>0. - J. Lowell, Aug 30 2008
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Totally multiplicative sequence with a(p) = p^2 for prime p. [From Jaroslav Krizek, Nov 01 2009]
Satisfies A(x)/A(x^2), A(x) = A173277: (1, 4, 13, 32, 74,...) [From Gary W. Adamson, Feb 14 2010]
a(n) = 1 (mod n+1). [From Bruno Berselli, Jun 03 2010]
Positive members are the integers with an odd number of odd divisors and an even number of even divisors. See also A120349, A120359, A181792, A181793, A181795. [From Matthew Vandermast, Nov 14 2010]
A007968(a(n)) = 0. [Reinhard Zumkeller, Jun 18 2011]
A071974(a(n)) = n; A071975(a(n)) = 1. [Reinhard Zumkeller, Jul 10 2011]
FORMULA G.f.: x * (1 + x) / (1 - x)^3.
E.g.f.: exp(x) * (x + x^2).
Dirichlet g.f.: zeta(s-2).
a(n) = a(-n).
Multiplicative with a(p^e) = p^(2e). - David W. Wilson, Aug 01, 2001.
Sum of all matrix elements M(i, j) = 2*i/(i+j) (i, j = 1..n). a(n) = Sum[Sum[2*i/(i+j), {i, 1, n}], {j, 1, n}] - Alexander Adamchuk, Oct 24 2004
a(0)=0, a(1)=1, a(n)=2*a(n-1)-a(n-2)+2 - Miklos Kristof, Mar 09 2005
a(n)=sum of the odd numbers for i=1 to n. a(0)=0 a(1)=1 then a(n)=a(n-1)+2*n-1. - Pierre CAMI (pierrecami(AT)tele2.fr), Oct 22 2006
For n>0: a(n) = A130064(n)*A130065(n). - Reinhard Zumkeller, May 05 2007
a(n) = Sum(A002024(n,k): 1<=k<=n). - Reinhard Zumkeller, Jun 24 2007
Left edge of the triangle in A132111: a(n)=A132111(n,0). - Reinhard Zumkeller, Aug 10 2007

a(n) = {least common multiple of n and n-1} - (n-1). - Mats Granvik, Sep 16 2007

Binomial transform of [1, 3, 2, 0, 0, 0,...]. - Gary W. Adamson, Nov 21 2007

a(n) = binomial(n+1,2) + binomial(n,2).

This sequence could be derived from the following general formula (cf. A001286, A000330): n*(n+1)*...*(n+k)*[n+(n+1)+...+(n+k)]/((k+2)!*(k+1)/2) at k=0 Indeed, using the formula for the sum of the arithmetic progression [n+(n+1)+...+(n+k)]= (2*n + k)*(k + 1)/2 the general formula could be rewritten as: n*(n+1)*...*(n+k)*(2*n + k)/(k+2)! so for k=0 above general formula degenerates to n*(2*n + 0)/(0+2)!= n^2 - Alexander R. Povolotsky (pevnev(AT)juno.com), May 18 2008

From a(4) recurrence formula a(n+3)=3a(n+2)-3a(n+1)+a(n) and a(1)=1, a(2)=4, a(3)=9 [From Artur Jasinski, Oct 21 2008]

The recurrence a(n+3)=3*a(n+2)-3*a(n+1)+a(n) is satisfied by all k-gonal sequences from a(3), with a(0)=0, a(1)=1, a(2)=k. [From Jaume Oliver Lafont, Nov 18 2008]

a(n) = floor [n*(n+1)* [sum_{i=1..n} 1/(n*(n+1))]] [From Ctibor O. Zizka, Mar 07 2009]

Product_{i=2..infinity} (1-2/a(i)) = -sin(A063448)/A063448. [From R. J. Mathar, Mar 12 2009]

Let A000290=F(actor) then F*4=Q^2 always, where Q=2*n if n>=0 and n are the unique numbers of exact roots Q. [From David Scheers (dscheers(AT)webpoint.nl), Mar 15 2009]

a(n) = A002378(n-1) + n. [From Jaroslav Krizek, Jun 14 2009]

a(n) = n*A005408(n-1) - sum [i = 1 ... n-2] A005408(i) - (n-1) = n*A005408(n-1) - a(n-1) - (n-1) [From Bruno Berselli, May 04 2010]

a(n) = a(n-1)+a(n-2)-a(n-3)+4, n>2 [from Gary Detlefs, Sep 07 2010]

a(n+1) = integral{x=0..infinity} exp(-x)/((Pn(x)*exp(-x)*Ei(x)-Qn(x))^2 +(Pi*exp(-x)*Pn(x))^2), with Pn the Laguerre polynom of order n and Qn the secondary Laguerre polynom defined by Qn(x) = integral{t=0..infinity} (Pn(x)-Pn(t))*exp(-t)/(x-t) [From Groux Roland, Dec 08 2010]

Euler transform of length 2 sequence [4, -1]. - Michael Somos Feb 12 2011

A162395(n) = -(-1)^n * a(n). - Michael Somos Mar 19 2011

a(n) = A004201(A000217(n)); A007606(a(n)) = A000384(n); A007607(a(n)) = A001105(n). [Reinhard Zumkeller, Feb 12 2011]

A000292 Tetrahedral (or triangular pyramidal) numbers: a(n) = C(n+2,3) = n*(n+1)*(n+2)/6.

COMMENTS a(n) = number of balls in a triangular pyramid in which each edge contains n+1 balls. The sum of the first n triangular numbers (A000217).

One of the 5 Platonic polyhedral (tetrahedral, cube, octahedral, dodecahedral and icosahedral) numbers (cf. A053012).

Also (1/6)*(n^3+3*n^2+2*n) is the number of ways to color vertices of a triangle using <= n colors, allowing rotations and reflections. Group is the dihedral group D_6 with cycle index (x1^3+2*x3+3*x1*x2)/6.

Also the convolution of the natural numbers with themselves - Felix Goldberg (felixg(AT)tx.technion.ac.il), Feb 01 2001

Connected with the Eulerian numbers (1,4,1) via 1*a(x-2)+4*a(x-1)+1*a(x) = x^3. - Gottfried Helms (helms(AT)uni-kassel.de), Apr 15 2002

a(n) = sum |i-j| for all 1 <= i <= j <= n. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Aug 05 2002

a(n) = sum of the all possible products p*q where (p,q) are ordered pairs and p+q = n+1. a(5) = 5 + 8 + 9 + 8 + 5 = 35. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), May 29 2003

Number of labeled graphs on n+3 nodes that are triangles. - Jon Perry (perry(AT)globalnet.co.uk), Jun 14 2003

Number of permutations of n+3 which have exactly 1 descent and avoid the pattern 1324. - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Nov 05 2004

Schlaefli symbol for this polyhedron: {3,3}

Transform of n^2 under the Riordan array (1/(1-x^2),x). - Paul Barry, Apr 16 2005

a(n) = -A108299(n+5,6) = A108299(n+6,7). - Reinhard Zumkeller, Jun 01 2005

a(n) = -A110555(n+4,3). - Reinhard Zumkeller, Jul 27 2005

a(n) is a perfect square only for n = {1, 2, 48}. a(48) = 19600 = 140^2. - Alexander Adamchuk (alex(AT)kolmogorov.com), Nov 24 2006

a(n+1) is the number of terms in the expansion of (a_1+a_2+a_3+a_4)^n - Sergio Falcon (sfalcon(AT)dma.ulpgc.es), Feb 12 2007. (Corrected by Graeme McRae (g_m(AT)mcraefamily.com), Aug 28 2007)

This is also the average "permutation entropy", sum((pi(n)-n)^2)/n!, over the set of all possible n! permutations pi. - Jeff Boscole (jazzerciser(AT)hotmail.com), Mar 20 2007

a(n)=diff(S(n,x),x)|_{x=2}. First derivative of Chebyshev S-polynomials evaluated at x=2. See A049310. - Wolfdieter Lang, Apr 04 2007.

If X is an n-set and Y a fixed (n-1)-subset of X then a(n-2) is equal to the number of 3-subsets of X intersecting Y. - Milan R. Janjic (agnus(AT)blic.net), Aug 15 2007

Complement of A145397; A023533(a(n))=1; A014306(a(n))=0. [From Reinhard Zumkeller, Oct 14 2008]

Equals row sums of triangle A152205 [From Gary W. Adamson, Nov 29 2008]

a(n) is the number of gifts received from the lyricist's true love up to and including day n in the song "The Twelve Days of Christmas". a(12)=364, almost the number of days in the year. [From Bernard Hill (bernard(AT)braeburn.co.uk), Dec 05 2008]

From Johannes W. Meijer, Mar 07 2009: (Start)

Sequence of the absolute values of the z^1 coefficients of the polynomials in the GF2 denominators of A156925. See A157703 for background information.

(End)

Starting with 1 = row sums of triangle A158823 [From Gary W. Adamson, Mar 28 2009]

Wiener index of the path graph P_n [From Eric W. Weisstein, Apr 30 2009]

From Peter Luschny, Jul 14 2009: (Start)

This is a 'Matryoshka doll' sequence with alpha=0, the multiplicative counterpart is A000178

seq(add(add(i,i=alpha..k),k=alpha..n),n=alpha..50); (End)

a(n) is the number of non-decreasing, three-element permutations of n distinct numbers. [From Samuel Savitz, Sep 12 2009]

a(n+4) = Number of different partitions of number n on sum of 4 elements a(6)=a(2+4)becuse we have 10 different partionions 2 on sum of 4 elements 2=2+0+0+0=1+1+0+0=0+2+0+0=1+0+1+0=0+1+1+0=0+0+2+0=1+0+0+1=0+1+0+1=0+0+1+1= 0+0+0+2 [From Artur Jasinski (grafix(AT)csl.pl), Nov 30 2009]

a(n) corresponds to the total number of steps to memorize n verses by the technique described in A173564. [From Ibrahima Faye (ifaye2001(AT)yahoo.fr), Feb 22 2010]

a(n) is also given by a very small DERIVE-program: v(n) := VECTOR(k, k, 1, n) w(n) := VECTOR(n - k, k, 0, n - 1) a(n) := v(n) [nonascii characters here] cents w(n) [From Roland Schroeder (florola(AT)gmx.de), Jul 12 2010]

The number of (n+2)-bit numbers which contain two runs of 1's in their binary expansion. [Vladimir Shevelev, Jul 30 2010]

a(n) is also, starting at the second term, the number of triangles formed in n-gones by intersecting diagonals with three diagonal endpoints. Ref.: Steven E. Sommers in: Journ. of Integer Sequences, Vol. 1 (1998), Article 98.1.5 (see the first column of the table): http://www.cs.uwaterloo.ca/journals/JIS/sommars/newtriangle.html [Alexandre Wajnberg (alexandre.wajnberg(AT)skynet.be), Aug 21 2010.

Column sums of:

```
1 4 9 16 25...
  1  4  9...
       1...
.............
--------------
1 4 10 20 35...
```

From Johannes W. Meijer, May 20 2011: (Start)

The Ca3, Ca4, Gi3 and Gi4 triangle sums, for their definitions see A180662, of the Connell-Pol triangle A159797 are linear sums of shifted versions of the duplicated tetrahedral numbers, e.g. Gi3(n) = 17*a(n) + 19*a(n-1) and Gi4(n) = 5*a(n) + a(n-1).

Furthermore the Kn3, Kn4, Ca3, Ca4, Gi3 and Gi4 triangle sums of the Connell sequence A001614 as a triangle are also linear sums of shifted versions of the sequence given above. (End)

a(n-2)=:N_0(n),n>=1, with a(-1):=0, is the number of vertices of n planes in generic position in three-dimensional space. See a comment under A000125 for general arrangement. Comment to Arnold's problem 1990-11, see the Arnold reference, p.506. [From Wolfdieter Lang, May 27 2011]

We consider optimal proper vertex colorings of a graph G. Assume that the labeling i.e., coloring starts with 1. By optimality we mean that the maximum label used is the minimum of the maximum integer label used across all possible labelings of G. Let S=Sum of the differerences |l(v)-l(u)|, the sum being over all edges uv of G and l(w) is the label associated with a vertex w of G. We say G admits unique labeling if all possible labelings of G is S-invariant and yields the same integer partition of S. With an offset this sequence gives the S-values for the complete graph on n vertices, n=2,3, - - - . [Kailasam Viswanathan Iyer, July 8 2011]

FORMULA G.f.: x/(1-x)^4.

a(n)=4*a(n-1)-6*a(n-2)+4*a(n-3)-a(n-4) for n>=4. [Jaume Oliver Lafont, Nov 18 2008]

a(-4-n)=-a(n).

E.g.f.:((x^3)/6+x^2+x)*exp(x) [From Geoffrey Critzer, Feb 21 2009]

a(n) = sum_{k=1..n} k*(n-k+1). [Vladimir Shevelev, Jul 30 2010]

Partial sums of the triangular numbers (A000217).

a(n) = (n+3)*a(n-1)/n. - Ralf Stephan, Apr 26 2003

Sums of three consecutive terms give A006003. - Ralf Stephan, Apr 26 2003

a(n) = C(1,2)+C(2,2)+...+C(n-1,2)+C(n,2); e.g. for n=5: a(5)=0+1+3+6+10=20. - Labos E. (labos(AT)ana.sote.hu), May 09 2003

Determinant of the n X n symmetric Pascal matrix M_(i, j)=C(i+j+2, i) - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 19 2003

The sum of a series constructed by the products of the index and the length of the series (n) minus the index (i): a(n) = sum[i(n-i)]. Also the sum of n terms of A000217. - Martin Steven McCormick (mathseq(AT)wazer.net), Apr 06 2005

a(n)=sum{k=0..floor((n-1)/2), (n-2k)^2} [offset 0]; a(n+1)=sum{k=0..n, k^2*(1-(-1)^(n+k-1))/2} [offset 0]; - Paul Barry, Apr 16 2005

Values of the Verlinde formula for SL_2, with g=2: a(n)=sum(j=1, n-1, n/(2*sin^2(j*Pi/n))) - Simone Severini, Sep 25 2006

a(n) = Sum[Sum[k, {k,1,m}], {m,1,n}]. - Alexander Adamchuk, Oct 28 2006

a(n)=Sum{k=1..n} binomial(n*k+1,n*k-1), with a(0)=0. - Paolo P. Lava, Apr 13 2007

a(n-1) = 1/(1!*2!)*sum {1 <= x_1, x_2 <= n} |det V(x_1,x_2)| = 1/2*sum {1 <= i,j <= n} |i-j|, where V(x_1,x_2} is the Vandermonde matrix of order 2. Column 2 of A133112. - Peter Bala, Sep 13 2007

Starting with 1 = binomial transform of [1, 3, 3, 1,...]; e.g. a(4) = 20 = (1, 3, 3, 1) dot (1, 3, 3, 1) = (1 + 9 + 9 + 1). - Gary W. Adamson, Nov 04 2007

a(n) = A006503(n) - A002378(n). [From Reinhard Zumkeller, Sep 24 2008]

Sum_{n=1..infinity} 1/a(n) = 3/2, case x=1 in Gradstein-Ryshik 1.513.7. [From R. J. Mathar, Jan 27 2009]

Lim{n->oo} A171973(n)/a(n) = sqrt(2)/2. [From Reinhard Zumkeller, Jan 20 2010]

With offset 1, a(n) =(1/6)*floor(n^5/(n^2+1)) [From Gary Detlefs, Feb 14 2010]

a(n)= (3*n^2+6*n+2)/(6*(h(n+2)-h(n-1))),n>0, where h(n) is the n-th harmonic number. [From Gary Detlefs, Jul 01 2011]

a(n)=coefficient of x^2 in the Maclaurin expansion of 1+1/(x+1)+1/(x+1)^2+1/(x+1)^3+...+1/(x+1)^n. [From Francesco Daddi, Aug 02 2011]

a(n)=coefficient of x^4 in the Maclaurin expansion of sin(x)*exp((n+1)*x). [From Francesco Daddi, Aug 04 2011]

A000272 Number of trees on n labeled nodes: n^(n-2). (Formerly M3027 N1227)

COMMENTS Number of spanning trees in complete graph K_n on n labeled nodes.

Robert Castelo (rcastelo(AT)imim.es), Jan 06 2001, observes that n^(n-2) is also the number of transitive subtree acyclic digraphs on n-1 vertices.

a(n) is also the number of ways of expressing an n-cycle in the symmetric group S_n as a product of n-1 transpositions, see example. - Dan Fux (dan.fux(AT)OpenGaia.com or danfux(AT)OpenGaia.com), Apr 12 2001

Also counts parking functions, noncrossing partitions, critical configurations of the chip firing game, allowable pairs sorted by a priority queue [Hamel].

a(n+1) = sum(i * n^(n-1-i) * binomial(n, i), i=1..n) - Yong Kong (ykong(AT)curagen.com), Dec 28 2000

a(n+1) = number of endofunctions with no cycles of length > 1; number of forests of rooted labeled trees on n vertices. - Mitch Harris (Harris.Mitchell(AT)mgh.harvard.edu), Jul 06 2006

a(n) is also the number of nilpotent partial bijections (of an n-element set). Equivalently, the number of nilpotents in the partial symmetric semigroup, P sub n. [From A. Umar (aumarh(AT)squ.edu.om), Aug 25 2008]

a(n) is also the number of edge-labeled rooted trees on n nodes. [From Nikos Apostolakis

(nikos.ap(AT)gmail.com), Nov 30 2008]
a(n+1) is the number of length n sequences on an alphabet of {1,2,...,n} that have a partial sum equal to n. For example a(4)=16 because there are 16 length 3 sequences on {1,2,3} in which the terms (beginning with the first term and proceeding sequentialy) sum to 3 at some point in the sequence. {1, 1, 1}, {1, 2, 1}, {1, 2, 2}, {1, 2, 3}, {2, 1, 1}, {2, 1, 2}, {2, 1, 3}, {3, 1, 1}, {3, 1, 2}, {3, 1, 3}, {3, 2, 1}, {3, 2, 2}, {3, 2, 3}, {3, 3, 1}, {3, 3, 2}, {3, 3, 3}} [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Jul 20 2009]
a(3) = 3 is the only prime value in the sequence. There are no semiprime values. Generally, the number of distinct primes dividing a(n) = omega(a(n)) = A001221(a(n)) = omega(n). Similarly, the number of prime divisors of a(n) (counted with multiplicity) = bigomega(a(n)) = A001222(a(n)) = Product (p_j^k_j) = Sum (k_j) where a(n) = Product (p_j^k_j), which is an obvious function of n and n-2. [From Jonathan Vos Post (jvospost3(AT)gmail.com), May 27 2010]
a(n) is the number of acyclic functions from {1,2,...,n-1} to {1,2,...,n}. An acyclic function f satisfies the following property: for any x in the domain, there exists a positive integer k such that (f^k)(x) is not in the domain. Note that f^k denotes the k-fold composition of f with itself, e.g., (f^2)(x)=f(f(x)). [From Dennis Walsh, March 2 2011]
FORMULA E.g.f.: T - (1/2)T^2; where T=T(x) is Euler's tree function (see A000169, also A001858). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Nov 19 2001
E.g.f.: ((W(-x)/x)^2)/(1+W(-x)), W(x): Lambert's function (principal branch).
Number of labeled k-trees on n nodes is binomial(n, k) * (k(n-k)+1)^(n-k-2).
Determinant of the symmetric matrix H generated for a polynomial of degree n by: for(i=1,n-1, for(j=1,i,
H[i,j]=(n*i^3-3*n*(n+1)*i^2/2+n*(3*n+1)*i/2+(n^4-n^2)/2)/6-(i^2-(2*n+1)*i+n*(n+1))*(j-1)*j/4; H[j,i]=H[i,j];);); - Gerry Martens (GerryMrt(AT)aol.com), May 04 2007
For n>=1, a(n+1)= Sum(n^(n-i)*Binomial(n-1,i-1),i=1...n) [From Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Jul 20 2009]
E.g.f. for b(n)=a(n+1): exp(-W(-x)), where W is Lambert's function satisfying W(x)exp(W(x))=x. Proof is contained in link "Notes on acyclic functions..." [From Dennis Walsh, March 2 2011]
A000273 Number of directed graphs (or digraphs) with n nodes.
A000290 The squares: a(n) = n^2.
COMMENTS To test if a number is a square, see Cohen, p. 40. - N. J. A. Sloane, Jun 19 2011.
Zero followed by partial sums of A005408 (odd numbers). - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Aug 13 2002
Begin with n, add the next number, subtract the previous number and so on ending with subtracting a 1: a(n) = n + (n+1) - (n-1) +(n+2) -(n-2) +(n+3)-(n-3)...+(2n-1)-1 = n^2. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 24 2004
Sum of two consecutive triangular numbers A000217. - Lekraj Beedassy, May 14 2004
Numbers with an odd number of divisors: {d(n^2)=A048691(n); for the first occurrence of 2n+1 divisors, see A071571(n)}. - Lekraj Beedassy, Jun 30 2004. See also A000037.
First sequence ever computed by electronic computer, on EDSAC, May 6 1949 (see Renwick link). - Russ Cox, Apr 20 2006
Numbers n such that the imaginary quadratic field Q[Sqrt[-n]] has four units. - Marc LeBrun, Apr 12 2006

For n>0: number of divisors of (n-1)th power of any squarefree semiprime: a(n)=A000005(A006881(k)^(n-1)); a(n) = A000005(A000400(n-1)) = A000005(A011557(n-1)) = A000005(A001023(n-1)) = A000005(A001024(n-1)). - Reinhard Zumkeller, Mar 04 2007

For n>=1, a(n) is equal to the number of functions f:{1,2}->{1,2,...,n} such that for y_1, y_2 in {1,2,...,n} we have f(1)<>y_1 and f(2)<>y_2. - Milan R. Janjic (agnus(AT)blic.net), Apr 17 2007

If a 2-set Y and an (n-2)-set Z are disjoint subsets of an n-set X then a(n-2) is the number of 3-subsets of X intersecting both Y and Z. - Milan R. Janjic (agnus(AT)blic.net), Sep 19 2007

Also numbers a such that a^1/2 + b^1/2 = c^1/2 and a^2 + b = c. - Cino Hilliard (hillcino368(AT)hotmail.com), Feb 07 2008

Numbers n such that the geometric mean of the divisors of n is an integer. - Ctibor O. Zizka, Jun 26 2008

Equals row sums of triangle A143470. Example: 36 = sum of row 6 terms: (23 + 7 + 3 + 1 + 1 + 1). [From Gary W. Adamson , Aug 17 2008]

Equals row sums of triangles A143595 and A056944 [From Gary W. Adamson, Aug 26 2008]

Number of divisors of 6^(n-1) for n>0. - J. Lowell, Aug 30 2008

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a(n) is also the number of all partitions of the sum 2^2+2^2+...2^2, (n-1)-times, into powers of 2. [From Valentin Bakoev (v_bakoev(AT)yahoo.com), Mar 03 2009]

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Zero together with the numbers n such that 2=number of perfect partitions of n [From Juri-Stepan Gerasimov, Sep 26 2009]

Totally multiplicative sequence with a(p) = p^2 for prime p. [From Jaroslav Krizek, Nov 01 2009]

Satisfies A(x)/A(x^2), A(x) = A173277: (1, 4, 13, 32, 74,...) [From Gary W. Adamson, Feb 14 2010]

a(n) = 1 (mod n+1). [From Bruno Berselli, Jun 03 2010]

Positive members are the integers with an odd number of odd divisors and an even number of even divisors. See also A120349, A120359, A181792, A181793, A181795. [From Matthew Vandermast, Nov 14 2010]

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A071974(a(n)) = n; A071975(a(n)) = 1. [Reinhard Zumkeller, Jul 10 2011]

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E.g.f.: exp(x) * (x + x^2).

Dirichlet g.f.: zeta(s-2).

a(n) = a(-n).

Multiplicative with a(p^e) = p^(2e). - David W. Wilson, Aug 01, 2001.

Sum of all matrix elements M(i, j) = 2*i/(i+j) (i, j = 1..n). a(n) = Sum[Sum[2*i/(i+j), {i, 1, n}], {j, 1, n}] - Alexander Adamchuk, Oct 24 2004

a(0)=0, a(1)=1, a(n)=2*a(n-1)-a(n-2)+2 - Miklos Kristof, Mar 09 2005

a(n)=sum of the odd numbers for i=1 to n. a(0)=0 a(1)=1 then a(n)=a(n-1)+2*n-1. - Pierre CAMI (pierrecami(AT)tele2.fr), Oct 22 2006

For n>0: a(n) = A130064(n)*A130065(n). - Reinhard Zumkeller, May 05 2007
a(n) = Sum(A002024(n,k): 1<=k<=n). - Reinhard Zumkeller, Jun 24 2007
Left edge of the triangle in A132111: a(n)=A132111(n,0). - Reinhard Zumkeller, Aug 10 2007
a(n) = {least common multiple of n and n-1} - (n-1). - Mats Granvik, Sep 16 2007
Binomial transform of [1, 3, 2, 0, 0, 0,...]. - Gary W. Adamson, Nov 21 2007
a(n) = binomial(n+1,2) + binomial(n,2).
This sequence could be derived from the following general formula (cf. A001286, A000330): n*(n+1)*...*(n+k)*[n+(n+1)+...+(n+k)]/((k+2)!*(k+1)/2) at k=0 Indeed, using the formula for the sum of the arithmetic progression [n+(n+1)+...+(n+k)]= (2*n + k)*(k + 1)/2 the general formula could be rewritten as: n*(n+1)*...*(n+k)*(2*n + k)/(k+2)! so for k=0 above general formula degenerates to n*(2*n + 0)/(0+2)!= n^2 - Alexander R. Povolotsky (pevnev(AT)juno.com), May 18 2008
From a(4) recurrence formula a(n+3)=3a(n+2)-3a(n+1)+a(n) and a(1)=1, a(2)=4, a(3)=9 [From Artur Jasinski, Oct 21 2008]
The recurrence a(n+3)=3*a(n+2)-3*a(n+1)+a(n) is satisfied by all k-gonal sequences from a(3), with a(0)=0, a(1)=1, a(2)=k. [From Jaume Oliver Lafont, Nov 18 2008]
a(n) = floor [n*(n+1)* [sum_{i=1..n} 1/(n*(n+1))]] [From Ctibor O. Zizka, Mar 07 2009]
Product_{i=2..infinity} (1-2/a(i)) = -sin(A063448)/A063448. [From R. J. Mathar, Mar 12 2009]
Let A000290=F(actor) then F*4=Q^2 always, where Q=2*n if n>=0 and n are the unique numbers of exact roots Q. [From David Scheers (dscheers(AT)webpoint.nl), Mar 15 2009]
a(n) = A002378(n-1) + n. [From Jaroslav Krizek, Jun 14 2009]
a(n) = n*A005408(n-1) - sum [i = 1 ... n-2] A005408(i) - (n-1) = n*A005408(n-1) - a(n-1) - (n-1) [From Bruno Berselli, May 04 2010]
a(n) = a(n-1)+a(n-2)-a(n-3)+4, n>2 [from Gary Detlefs, Sep 07 2010]
a(n+1) = integral{x=0..infinity} exp(-x)/((Pn(x)*exp(-x)*Ei(x)-Qn(x))^2 +(Pi*exp(-x)*Pn(x))^2), with Pn the Laguerre polynom of order n and Qn the secondary Laguerre polynom defined by Qn(x) = integral{t=0..infinity} (Pn(x)-Pn(t))*exp(-t)/(x-t) [From Groux Roland, Dec 08 2010]
Euler transform of length 2 sequence [4, -1]. - Michael Somos Feb 12 2011
A162395(n) = -(-1)^n * a(n). - Michael Somos Mar 19 2011
a(n) = A004201(A000217(n)); A007606(a(n)) = A000384(n); A007607(a(n)) = A001105(n). [Reinhard Zumkeller, Feb 12 2011]

A000292 Tetrahedral (or triangular pyramidal) numbers: a(n) = C(n+2,3) = n*(n+1)*(n+2)/6.

COMMENTS a(n) = number of balls in a triangular pyramid in which each edge contains n+1 balls. The sum of the first n triangular numbers (A000217).
One of the 5 Platonic polyhedral (tetrahedral, cube, octahedral, dodecahedral and icosahedral) numbers (cf. A053012).
Also (1/6)*(n^3+3*n^2+2*n) is the number of ways to color vertices of a triangle using <= n colors, allowing rotations and reflections. Group is the dihedral group D_6 with cycle index (x1^3+2*x3+3*x1*x2)/6.
Also the convolution of the natural numbers with themselves - Felix Goldberg (felixg(AT)tx.technion.ac.il), Feb 01 2001
Connected with the Eulerian numbers (1,4,1) via 1*a(x-2)+4*a(x-1)+1*a(x) = x^3. - Gottfried Helms (helms(AT)uni-kassel.de), Apr 15 2002

a(n) = sum |i-j| for all 1 <= i <= j <= n. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Aug 05 2002

a(n) = sum of the all possible products p*q where (p,q) are ordered pairs and p+q = n+1. a(5) = 5 + 8 + 9 + 8 + 5 = 35. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), May 29 2003

Number of labeled graphs on n+3 nodes that are triangles. - Jon Perry (perry(AT)globalnet.co.uk), Jun 14 2003

Number of permutations of n+3 which have exactly 1 descent and avoid the pattern 1324. - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Nov 05 2004

Schlaefli symbol for this polyhedron: {3,3}

Transform of n^2 under the Riordan array (1/(1-x^2),x). - Paul Barry, Apr 16 2005

a(n) = -A108299(n+5,6) = A108299(n+6,7). - Reinhard Zumkeller, Jun 01 2005

a(n) = -A110555(n+4,3). - Reinhard Zumkeller, Jul 27 2005

a(n) is a perfect square only for n = {1, 2, 48}. a(48) = 19600 = 140^2. - Alexander Adamchuk (alex(AT)kolmogorov.com), Nov 24 2006

a(n+1) is the number of terms in the expansion of (a_1+a_2+a_3+a_4)^n - Sergio Falcon (sfalcon(AT)dma.ulpgc.es), Feb 12 2007. (Corrected by Graeme McRae (g_m(AT)mcraefamily.com), Aug 28 2007)

This is also the average "permutation entropy", sum((pi(n)-n)^2)/n!, over the set of all possible n! permutations pi. - Jeff Boscole (jazzerciser(AT)hotmail.com), Mar 20 2007

a(n)=diff(S(n,x),x)|_{x=2}. First derivative of Chebyshev S-polynomials evaluated at x=2. See A049310. - Wolfdieter Lang, Apr 04 2007.

If X is an n-set and Y a fixed (n-1)-subset of X then a(n-2) is equal to the number of 3-subsets of X intersecting Y. - Milan R. Janjic (agnus(AT)blic.net), Aug 15 2007

Complement of A145397; A023533(a(n))=1; A014306(a(n))=0. [From Reinhard Zumkeller, Oct 14 2008]

Equals row sums of triangle A152205 [From Gary W. Adamson, Nov 29 2008]

a(n) is the number of gifts received from the lyricist's true love up to and including day n in the song "The Twelve Days of Christmas". a(12)=364, almost the number of days in the year. [From Bernard Hill (bernard(AT)braeburn.co.uk), Dec 05 2008]

From Johannes W. Meijer, Mar 07 2009: (Start)

Sequence of the absolute values of the z^1 coefficients of the polynomials in the GF2 denominators of A156925. See A157703 for background information.

(End)

Starting with 1 = row sums of triangle A158823 [From Gary W. Adamson, Mar 28 2009]

Wiener index of the path graph P_n [From Eric W. Weisstein, Apr 30 2009]

From Peter Luschny, Jul 14 2009: (Start)

This is a 'Matryoshka doll' sequence with alpha=0, the multiplicative counterpart is A000178

seq(add(add(i,i=alpha..k),k=alpha..n),n=alpha..50); (End)

a(n) is the number of non-decreasing, three-element permutations of n distinct numbers. [From Samuel Savitz, Sep 12 2009]

a(n+4) = Number of different partitions of number n on sum of 4 elements a(6)=a(2+4)becuse we have 10 different partionions 2 on sum of 4 elements 2=2+0+0+0=1+1+0+0=0+2+0+0=1+0+1+0=0+1+1+0=0+0+2+0=1+0+0+1=0+1+0+1=0+0+1+1=0+0+0+2 [From Artur Jasinski (grafix(AT)csl.pl), Nov 30 2009] a(n) corresponds to the total number of steps to memorize n verses by the technique described in A173564. [From Ibrahima Faye (ifaye2001(AT)yahoo.fr), Feb 22 2010]

a(n) is also given by a very small DERIVE-program: v(n) := VECTOR(k, k, 1, n) w(n) := VECTOR(n - k, k, 0, n - 1) a(n) := v(n) [nonascii characters here] cents w(n) [From Roland Schroeder (florola(AT)gmx.de), Jul 12 2010]

The number of (n+2)-bit numbers which contain two runs of 1's in their binary expansion. [Vladimir Shevelev, Jul 30 2010]

a(n) is also, starting at the second term, the number of triangles formed in n-gones by intersecting diagonals with three diagonal endpoints. Ref.: Steven E. Sommers in: Journ. of Integer Sequences, Vol. 1 (1998), Article 98.1.5 (see the first column of the table): http://www.cs.uwaterloo.ca/journals/JIS/sommars/newtriangle.html [Alexandre Wajnberg (alexandre.wajnberg(AT)skynet.be), Aug 21 2010.

Column sums of:

```
1 4 9 16 25...
   1  4  9...
        1...
.............
--------------
1 4 10 20 35...
```

From Johannes W. Meijer, May 20 2011: (Start)

The Ca3, Ca4, Gi3 and Gi4 triangle sums, for their definitions see A180662, of the Connell-Pol triangle A159797 are linear sums of shifted versions of the duplicated tetrahedral numbers, e.g. Gi3(n) = 17*a(n) + 19*a(n-1) and Gi4(n) = 5*a(n) + a(n-1).

Furthermore the Kn3, Kn4, Ca3, Ca4, Gi3 and Gi4 triangle sums of the Connell sequence A001614 as a triangle are also linear sums of shifted versions of the sequence given above. (End)

a(n-2)=:N_0(n),n>=1, with a(-1):=0, is the number of vertices of n planes in generic position in three-dimensional space. See a comment under A000125 for general arrangement. Comment to Arnold's problem 1990-11, see the Arnold reference, p.506. [From Wolfdieter Lang, May 27 2011]

We consider optimal proper vertex colorings of a graph G. Assume that the labeling i.e., coloring starts with 1. By optimality we mean that the maximum label used is the minimum of the maximum integer label used across all possible labelings of G. Let S=Sum of the differerences |l(v)-l(u)|, the sum being over all edges uv of G and l(w) is the label associated with a vertex w of G. We say G admits unique labeling if all possible labelings of G is S-invariant and yields the same integer partition of S. With an offset this sequence gives the S-values for the complete graph on n vertices, n=2,3, - - - . [Kailasam Viswanathan Iyer, July 8 2011]

FORMULA G.f.: x/(1-x)^4.

a(n)=4*a(n-1)-6*a(n-2)+4*a(n-3)-a(n-4) for n>=4. [Jaume Oliver Lafont, Nov 18 2008]

a(-4-n)=-a(n).

E.g.f.:((x^3)/6+x^2+x)*exp(x) [From Geoffrey Critzer, Feb 21 2009]

a(n) = sum_{k=1..n} k*(n-k+1). [Vladimir Shevelev, Jul 30 2010]

Partial sums of the triangular numbers (A000217).

a(n) = (n+3)*a(n-1)/n. - Ralf Stephan, Apr 26 2003

Sums of three consecutive terms give A006003. - Ralf Stephan, Apr 26 2003

a(n) = C(1,2)+C(2,2)+...+C(n-1,2)+C(n,2); e.g. for n=5: a(5)=0+1+3+6+10=20. - Labos E.

(labos(AT)ana.sote.hu), May 09 2003

Determinant of the n X n symmetric Pascal matrix M_(i, j)=C(i+j+2, i) - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 19 2003

The sum of a series constructed by the products of the index and the length of the series (n) minus the index (i): a(n) = sum[i(n-i)]. Also the sum of n terms of A000217. - Martin Steven McCormick (mathseq(AT)wazer.net), Apr 06 2005

a(n)=sum{k=0..floor((n-1)/2), (n-2k)^2} [offset 0]; a(n+1)=sum{k=0..n, k^2*(1-(-1)^(n+k-1))/2} [offset 0]; - Paul Barry, Apr 16 2005

Values of the Verlinde formula for SL_2, with g=2: a(n)=sum(j=1, n-1, n/(2*sin^2(j*Pi/n))) - Simone Severini, Sep 25 2006

a(n) = Sum[Sum[k, {k,1,m}], {m,1,n}]. - Alexander Adamchuk, Oct 28 2006

a(n)=Sum{k=1..n} binomial(n*k+1,n*k-1), with a(0)=0. - Paolo P. Lava, Apr 13 2007

a(n-1) = 1/(1!*2!)*sum {1 <= x_1, x_2 <= n} |det V(x_1,x_2)| = 1/2*sum {1 <= i,j <= n} |i-j|, where V(x_1,x_2} is the Vandermonde matrix of order 2. Column 2 of A133112. - Peter Bala, Sep 13 2007

Starting with 1 = binomial transform of [1, 3, 3, 1,...]; e.g. a(4) = 20 = (1, 3, 3, 1) dot (1, 3, 3, 1) = (1 + 9 + 9 + 1). - Gary W. Adamson, Nov 04 2007

a(n) = A006503(n) - A002378(n). [From Reinhard Zumkeller, Sep 24 2008]

Sum_{n=1..infinity} 1/a(n) = 3/2, case x=1 in Gradstein-Ryshik 1.513.7. [From R. J. Mathar, Jan 27 2009]

Lim{n->oo} A171973(n)/a(n) = sqrt(2)/2. [From Reinhard Zumkeller, Jan 20 2010]

With offset 1, a(n) =(1/6)*floor(n^5/(n^2+1)) [From Gary Detlefs, Feb 14 2010]

a(n)= (3*n^2+6*n+2)/(6*(h(n+2)-h(n-1))),n>0, where h(n) is the n-th harmonic number. [From Gary Detlefs, Jul 01 2011]

a(n)=coefficient of x^2 in the Maclaurin expansion of 1+1/(x+1)+1/(x+1)^2+1/(x+1)^3+...+1/(x+1)^n. [From Francesco Daddi, Aug 02 2011]

a(n)=coefficient of x^4 in the Maclaurin expansion of sin(x)*exp((n+1)*x). [From Francesco Daddi, Aug 04 2011]

A000302 Powers of 4.

COMMENTS Same as Pisot sequences E(1,4), L(1,4), P(1,4), T(1,4). See A008776 for definitions of Pisot sequences.

The convolution square root of this sequence is A000984, the central binomial coefficients: C(2n,n). - T. D. Noe (noe(AT)sspectra.com), Jun 11 2002

a(n)=sum(k=0,n,C(2k,k)*C(2(n-k),n-k)). - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 26 2003

With p(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n)=sum_{i=1}^{p(n)} p(i)!/(prod_{j=1}^{d(i)} m(i,j)!) * 2^(n-1) - Thomas Wieder (wieder.thomas(AT)t-online.de), May 18 2005

Sums of rows of the triangle in A122366. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Aug 30 2006

A000005(a(n)) = A005408(n+1). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 04 2007

Hankel transform of A076035. [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Feb 28 2009]

Equals the Catalan sequence: (1, 1, 2, 5, 14,...), convolved with A032443: (1, 3, 11, 42,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), May 15 2009]

a(n) = A188915(A006127(n)). [Reinhard Zumkeller, Apr 14 2011]

Sum of coefficients of expansion of (1+x+x^2+x^3)^n.

a(n) is number of compositions of natural numbers into n parts <4.

a(2)=16 there are 16 compositions of natural numbers into 2 parts <4.

FORMULA a(n) = 4^n.

a(n) = 4*a(n-1).

G.f.: 1/(1-4*x).

E.g.f.: exp(4*x).

1 = sum(n>=1, 3/a(n)) = 3/4 + 3/16 + 3/64 + 3/256 + 3/1024...; with partial sums: 3/4, 15/16, 63/64, 255/256, 1023/1024... - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 16 2003

a(n)=A001045(2*n)+A001045(2*n+1). - Paul Barry (pbarry(AT)wit.ie), Apr 27 2004

a(n)=sum(2^(n-j)*binomial(n+j,j),j=0..n) - Peter C. Heinig (algorithms(AT)gmx.de), Apr 06 2007

Hankel transform of A115967 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 22 2007

a(n) = 6*StirlingS2(n+1,4) + 6*StirlingS2(n+1,3) + 3*StirlingS2(n+1,2) + 1 = 2*StirlingS2(2^n,2^n - 1) + StirlingS2(n+1,2) + 1. - Ross La Haye (rlahaye(AT)new.rr.com), Jun 26 2008

((2+sqrt(4))^n-(2-sqrt(4))^n)/4. Offset 1. a(3)=16. [From Al Hakanson (hawkuu(AT)gmail.com), Dec 31 2008]

a(n) = sum(k=0..n,C(2*n+1,k)). [Mircea Merca, Jun 25 2011]

A000311 Schroeder's fourth problem; also number of phylogenetic trees with n nodes; also number of total partitions of n.

COMMENTS a(n) = number of labeled series-reduced rooted trees with n leaves (root has degree 0 or >= 2); a(n-1) = number of labeled series-reduced trees with n leaves. Also number of series-parallel networks with n labeled edges, divided by 2.

Polynomials with coefficients in triangle A008517, evaluated at 2. - Ralf Stephan, Dec 13 2004

Row sums of unsigned A134685. [From Tom Copeland, Oct 11 2008]

FORMULA E.g.f. A(x) satisfies exp A(x) = 2*A(x) - x + 1.

a(0)=0, a(1)=a(2)=1; for n >= 2, a(n+1) = (n+2)*a(n) + 2*Sum_{k=2..n-1} binomial(n, k)*a(k)*a(n-k+1).

From the umbral operator L in A135494 acting on x^n comes, umbrally, (a(.) + x)^n = (n * x^(n-1) / 2) - (x^n / 2) + sum(j=1,...) [j^(j-1) * 2^(-j) / j! * exp(-j/2) * (x+j/2)^n] giving a(n) = 2^(-n) * sum(j=1,...) { j^(n-1) * [(j/2) * exp(-1/2)]^j / j! } for n > 1. - Tom Copeland, Feb 11 2008

A000312 Number of labeled mappings from n points to themselves (endofunctions): n^n.

COMMENTS Also number of labeled pointed rooted trees (or vertebrates) on n nods.

For n >= 1 a(n) is also the number of n X n (0,1) matrices in which each row contains exactly one entry equal to 1. - Avi Peretz (njk(AT)netvision.net.il), Apr 21 2001

Also the number of labeled rooted trees on (n+1) nodes such that the root is lower than its children. Also the number of alternating labeled rooted ordered trees on (n+1) nodes such that the root is lower than its children. - Cedric Chauve (chauve(AT)lacim.uqam.ca), Mar 27 2002

With p(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, p(j,i) = the j-th part of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n) = sum_{i=1}^{p(n)} (n!/(prod_{j=1}^{p(i)}p(i,j)!)) * ((n!/(n-p(i)))!/(prod_{j=1}^{d(i)} m(i,j)!)) - Thomas Wieder (wieder.thomas(AT)t-online.de), May 18 2005

All rational solutions to the equation x^y = y^x, with x < y, are given by x = A000169(n+1)/A000312(n), y = A000312(n+1)/A007778(n), where n = 1, 2, 3, ... - Nick Hobson Nov 30 2006

a(n) = total number of leaves in all (n+1)^(n-1) trees on {0,1,2,...,n} rooted at 0. For example, with edges directed away from the root, the trees on {0,1,2} are {0->1,0->2},{0->1->2},{0->2->1} and contain a total of a(2)=4 leaves. - David Callan (callan(AT)stat.wisc.edu), Feb 01 2007

lim_{n->infinity} A000169(n+1)/a(n) = exp(1). Convergence is slow, e.g., it takes n > 74 to get one decimal place correct and n > 163 to get two of them. - From Alonso del Arte, Jun 20 2011

FORMULA a(n-1) = -sum((-1)^i * i * n^(n-1-i)*binomial(n, i), i=1..n) - Yong Kong (ykong(AT)curagen.com), Dec 28 2000

E.g.f.: 1/(1+W(-x)), W(x) = principal branch of Lambert's function.

a(n) = Sum(k>=0, C(n, k)*Stirling2(n, k)*k!) = Sum(k>=0, A008279(n, k)*A048993(n, k)) = Sum(k>=0, A019538(n, k)*A07318(n, k)). - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Dec 14 2003

E.g.f.: 1/(1-T), where T=T(x) is Euler's tree function (see A000169).

a(n) = A000169(n+1)*A128433(n+1,1)/A128434(n+1,1). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 03 2007

Comment on power series with denominators a(n): Let f(x) = 1 + Sum_{n = 1..oo} x^n/n^n. Then as x -> oo, f(x) ~ exp(x/e)*sqrt(2*Pi*x/e). - Philippe Flajolet (Philippe.Flajolet(AT)inria.fr), Sep 11 2008

E.g.f.: 1-exp(W(-x)), W(x) - principal branch of Lambert's function. [From Kruchinin Vladimir (kru(AT)ie.tusur.ru), Sep 15 2010]

a(n)=(n-1)*a(n-1)+Sum{i=1,...,n}C(n,i)*a(i-1)*a(n-i). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Sep 30 2010]

A000326 Pentagonal numbers: n*(3*n-1)/2.

COMMENTS The average of the first n (n>0) pentagonal numbers is the n-th triangular number. - Mario Catalani (mario.catalani(AT)unito.it), Apr 10 2003

Partial sums of 1,4,7,10,13,16,... (1 mod 3), a(2k)=k(6k-1), a(2k-1)=(2k-1)(3k-2) - Jon Perry, Sep 10 2004

a(n) = A126890(n,n-1) for n>0. - Reinhard Zumkeller, Dec 30 2006

If Y is a 3-subset of an n-set X then, for n>=4, a(n-3) is the number of 4-subsets of X having at least two elements in common with Y. - Milan R. Janjic (agnus(AT)blic.net), Nov 23 2007

Solutions to the duplication formula 2*a(n)=a(k) are given by the index pairs (n,k) = (5,7), (5577,7887), (6435661,9101399), etc. The indices are integer solutions to the pair of equations 2(6n-1)^2=1+y^2, k=(1+y)/6, so these n can be generated from the subset of numbers [1+A001653(i)]/6, any i, where these are integers, confined to the cases where the associated k=[1+A002315(i)]/6 are also integers. - R. J. Mathar, Feb 01 2008

a(n) is a binomial coefficient C (n, 4) (A000332) if and only if n is a generalized pentagonal number (A001318). Also see A145920. [From Matthew Vandermast, Oct 28 2008]
Let P(n) = pentagonal number, T(n) = triangular number, then P(n)= T(n)+2*T(n-1) [From Vincenzo Librandi, Nov 20 2010]
Even octagonal numbers divided by 8. - Omar E. Pol, Aug 18 2011

FORMULA Product_{m>0} (1-q^m) = Sum_{k} (-1)^k*x^a(k). - Paul Barry, Jul 20 2003
G.f.: x*(1+2*x)/(1-x)^3.
E.g.f.: exp(x)*(x+3*x^2/2).
a(n) = n*(3*n-1)/2.
a(-n) = A005449(n).
a(n) = binomial(3*n, 2)/3 - Paul Barry, Jul 20 2003
a(n) is the sum of n integers starting from n, i.e. 1, 2+3, 3+4+5, 4+5+6+7, etc. - Jon Perry, Jan 15 2004
a(n) = A000290(n) + A000217(n-1). - Lekraj Beedassy, Jun 07 2004
a(0) = 0, a(1) = 1; for n >= 2, a(n) = 2*a(n-1)- a(n-2) + 3. - Miklos Kristof, Mar 09 2005
a(n) = sum{k=1..n, 2n-k}; - Paul Barry, Aug 19 2005
a(n) = 3*A000217(n) - 2*n . - Lekraj Beedassy, Sep 26 2006
a(n)=A049452(n)-A022266(n), a(n)=A033991(n)-A005476(n). - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Jun 12 2007
Equals A034856(n) + (n - 1)^2. Also equals A051340 * [1,2,3,...]. - Gary W. Adamson, Jul 27 2007
Starting with offset 1 = binomial transform of [1, 4, 3, 0, 0, 0,...]. Also, A004736 * [1, 3, 3, 3,...]. - Gary W. Adamson, Oct 25 2007
a(n) = C(n+1,2) + 2 C(n,2)
a(n)=A000290(n)+A000217(n-1). - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Feb 18 2008
a(n)=3*a(n-1)-3*a(n-2)+a(n-3), a(0)=0, a(1)=1, a(2)=5 [From Jaume Oliver Lafont, Dec 02 2008]
a(n)=a(n-1)+3*n-2 (with a(0)=0) [From Vincenzo Librandi, Nov 20 2010]
a(n) = A014642(n)/8. - Omar E. Pol, Aug 18 2011

A000330 Square pyramidal numbers: 0^2+1^2+2^2+...+n^2 = n(n+1)(2n+1)/6.

COMMENTS The sequence contains exactly one square greater than 1, namely 4900 (according to Gardner). - Jud McCranie (JudMcCranie(AT)ugaalum.uga.edu), Mar 19 2001, Mar 22 2007
Number of rhombi in an n X n rhombus. - Matti De Craene (Matti.DeCraene(AT)rug.ac.be), May 14 2000
Number of acute triangles made from the vertices of a regular n-polygon when n is odd (cf. A007290). - Sen-Peng You (giawgwan(AT)single.url.com.tw), Apr 05 2001
Gives number of squares formed from an n X n square. In a 1 X 1 square, one is formed. In a 2 X 2 square, five squares are formed. In a 3 X 3 square, 14 squares are formed and so on. - Kristie Smith (kristie10spud(AT)hotmail.com), Apr 16 2002
a(n-1)=B_3(n)/3 where B_3(x)=x(x-1)(x-1/2) is the third Bernoulli polynomial. - Michael Somos Mar 13 2004
Number of permutations avoiding 13-2 that contain the pattern 32-1 exactly once.
Since 3*r = (r+1)+r+(r-1) = T(r+1)-T(r-2), where T(r) = r-th triangular number r*(r+1)/2, we have 3*r^2 = r*{T(r+1)-T(r-2)} = f(r+1)-f(r-1)......(i), where f(r) = (r-1)*T(r) = (r+1)*T(r-1). Summing

over n, R.H.S. of relation (i) telescopes to f(n+1)+f(n) = T(n)*{(n+2)+(n-1)}, whence result sum_(1, n)r^2 = n*(n+1)*(2*n+1)/6 immediately follows. - Lekraj Beedassy (blekraj(AT)yahoo.com), Aug 06 2004

Also as a(n)=(1/6)*(2*n^3+3*n^2+n), n>0: structured trigonal diamond numbers (vertex structure 5) (Cf. A006003 = alternate vertex; A000447 = structured diamonds; A100145 for more on structured numbers). - James A. Record (james.record(AT)gmail.com), Nov. 7, 2004.

Number of triples of integers from {1,2,...,n} whose last component is greater than or equal to the others.

Kekule numbers for certain benzenoids. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Jun 12 2005

Sum of the first n squares, or square pyramidal numbers. - Cino Hilliard (hillcino368(AT)hotmail.com), Jun 18 2007

Maximal number of cubes of side 1 in a right pyramid with a square base of side n and height n - Pasquale CUTOLO (p.cutolo(AT)inwind.it), Jul 09 2007

If a 2-set Y and an (n-2)-set Z are disjoint subsets of an n-set X then a(n-3) is the number of 4-subsets of X intersecting both Y and Z. - Milan R. Janjic (agnus(AT)blic.net), Sep 19 2007

We also have the identity (1+(1+4)+(1+4+9)+..+(1+4+9+16+ .. + n^2)=n(n+1)(n+2)[n+(n+1)+(n+2)]/36; .. and in general the k-fold nested sum of squares can be expressed as n(n+1)...(n+k)[n+(n+1)+...+(n+k)]/((k+2)!(k+1)/2) - Alexander R. Povolotsky (pevnev(AT)juno.com), Nov 21 2007

The terms of this sequence are coefficients of the Engel expansion of the following converging sum: 1/(1^2) + (1/1^2)*(1/(1^2+2^2)) + (1/1^2)*(1/(1^2+2^2))*(1/(1^2+2^2+3^2)) + .. - Alexander R. Povolotsky (pevnev(AT)juno.com), Dec 10 2007

Starting (1, 5, 14, 30,...) = binomial transform of [1, 4, 5, 2, 0, 0, 0,...]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 13 2008

Starting (1,5,14,30,...) = second partial sums of binomial transform of [1,2,0,0,0,...]. a(n)=sum{i=0,n,C(n+2,i+2)*b(i)}, where b(i)=1,2,0,0,0,... [From Borislav St. Borisov (b.st.borisov(AT)abv.bg), Mar 05 2009]

Convolution of A001477 with A005408: a(n)=SUM((2*k+1)*(n-k):0<=k<=n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 07 2009]

Contribution from Johannes W. Meijer (meijgia(AT)hotmail.com), Mar 07 2009: (Start)

Sequence of the absolute values of the z^1 coefficients of the polynomials in the GF1 denominators of A156921. See A157702 for background information.

(End)

a(n) = A000217(n)*n - A000217(n-1) - A000217(n-2) - A000217(n-3) - .... - A000217(1)-A000217(0), i.e. a(n) = n^2*(n+1)/2 - sum_{k=0}^{n-1} k*(k+1)/2 [for d=1 in the general formula a(n) = n^2*(d*n-d+2)/2-sum_{k=0}^{n-1} k*(d*k-d+2)/2]. [From Bruno Berselli (berselli.bruno(AT)yahoo.it), Apr 21 2010]

a(n) / n = k^2 (k = integer) for n = 337; a(337) = 12814425, a(n) / n = 38025, k = 195, i.e. number k = 195 is quadratic mean (root mean square) of first 337 positive integers. There are other such numbers - see A084231 and A084232. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), May 23 2010]

Contribution from Carmine Suriano (surianonoi5(AT)libero.it), Sep 10 2010: (Start)

Also the number of moves to solve the "alternate coins game".

Given 2n+1 coins (n+1 Black, n White) set alternately in a row (BWBW...BWB) translate (not rotate) a pair of adjecent coins at a time (1 B and 1 W)
so that at the end the arrangement shall be BBBBB..BW...WWWWW (Blacks separated by Whites). Isolated coins cannot be moved. (End)
FORMULA a(n)=binomial(2(n+1), 3)/4 - Paul Barry (pbarry(AT)wit.ie), Jul 19 2003
a(n)=[(n^4-(n-1)^4)-(n^2-(n-1)^2)]/12 - Xavier Acloque Oct 16 2003
a(n) = Sqrt[Sum[Sum[(i*j)^2, {i, 1, n}], {j, 1, n}]. a(n) = Sum[Sum[Sum[(i*j*k)^2, {i, 1, n}], {j, 1, n}], {k, 1, n}]^(1/3). - Alexander Adamchuk (alex(AT)kolmogorov.com), Oct 26 2004
a(n)=sum(i=1, n, i*(2*n-2*i+1)) - sum of squares gives 1+(1+3)+(1+3+5)+... - Jon Perry (perry(AT)globalnet.co.uk), Dec 08 2004
a(n+1) = A000217(n+1) + 2*A000292(n-1) - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Mar 10 2005
Sum_{ n>0} 1/a(n) = 6(3 - 4log(2)) and Sum_{ n>0} = (-1)^(n+1)*1/a(n) = 6(Pi - 3) . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), May 31 2005
Sum of two consecutive tetrahedral (or pyramidal) numbers A000292: C(n+3,3) = (n+1)(n+2)(n+3)/6: a(n) = A000292(n-1) + A000292(n-2). - Alexander Adamchuk (alex(AT)kolmogorov.com), May 17 2006
Euler transform of length 2 sequence [5, -1]. - Michael Somos Sep 04 2006
G.f.: x(1+x)/(1-x)^4. E.g.f.: (x+3/2*x^2+1/3*x^3)*exp(x).
a(n)=n(n+1)(2n+1)/6=binomial(n+2, 3)+binomial(n+1, 3)=-a(-1-n).
a(n) = a(n-1) + n^2 - Rolf Pleisch (r.pleisch(AT)gmx.ch), Jul 22 2007
a(n) = A132121(n,0). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Aug 12 2007
Hankel transform of C(2n-3,n-1) is -a(n). - Paul Barry (pbarry(AT)wit.ie), Feb 12 2008
Convolution of A000290 with A000012 - Sergio Falcon (sfalcon(AT)dma.ulpgc.es), Feb 05 2008
Starting n (-1,0,1,2,...) a(n)=C(n+2,2)+2*C(n+2,3) [From Borislav St. Borisov (b.st.borisov(AT)abv.bg), Mar 05 2009]
A000364 Euler (or secant or "Zig") numbers: e.g.f. (even powers only) sech(x)=1/cosh(x).
COMMENTS Inverse Gudermannian gd^(-1)(x) = log(sec(x) + tan(x)) = log(tan(pi/4 + x/2)) = atanh(sin(x)) = 2 * atanh(tan(x/2)) = 2 * atanh(csc(x) - cot(x)). - Michael Somos, Mar 19 2011
FORMULA E.g.f.: Sum_{n >= 0} a(n) * x^(2*n) / (2*n)! = sec(x). - Michael Somos, Aug 15 2007
E.g.f.: Sum_{n >= 0} a(n) * x^(2*n+1) / (2*n+1)! = gd^(-1)(x). - Michael Somos, Aug 15 2007
E.g.f.: Sum_{n >= 0} a(n)*x^(2*n+1)/(2*n+1)! = 2*arctanh(cosec(x)-cotan(x)). - Ralf Stephan, Dec 16 2004
Pi/4 - [Sum_{k=0..n-1} (-1)^k/(2*k+1)] ~ (1/2)*[Sum_{k>=0} (-1)^k*E(k)/(2*n)^(2k+1)] for positive even n. [Borwein, Borwein, and Dilcher]
Let M_n be the n X n matrix M_n(i, j) = binomial(2*i, 2*(j-1)) = A086645(i, j-1); then for n>0, a(n) = det(M_n); example : det([1, 1, 0, 0; 1, 6, 1, 0; 1, 15, 15, 1; 1, 28, 70, 28]) = 1385 . - Philippe DELEHAM,, Sep 04 2005
This sequence is also (-1)^n*EulerE[2*n] or Abs[EulerE[2*n]]. - Paul Abbott

(paul(AT)physics.uwa.edu.au), Apr 14 2006

a(n) = 2^n * E_n(1/2), where E_n(x) is an Euler polynomial.

a(k) = a(l) (mod 2^n) if and only if k=l (mod 2^n) (k and l are even). [Stern; see also Wagstaff and Sun]

E_k(3^(k+1)+1)/4=(3^k/2)*sum(j=0..2^n-1, (-1)^(j-1)*(2j+1)^k*[(3j+1)/2^n] (mod 2^n) where k is even and [x] is the greatest integer function. [Sun]

a(n) ~ 2^(n+2)*n!/Pi^(n+1) as n -> infinity.

a(n) = sum(k=0..n, A094665(n, k)*2^(n-k)). - DELEHAM Philippe, Jun 10 2004

Recurrence: a(n) = -(-1)^n*sum(i=0..n-1, (-1)^i*a(i)*C(2*n, 2*i)). - Ralf Stephan, Feb 24 2005

O.g.f.: 1/(1-x/(1-4*x/(1-9*x/(1-16*x/(...-n^2*x/(1-...)))))) (continued fraction). - Paul D. Hanna, Oct 07 2005

a(n)=Integrate[Log[Tan[t/2]^2]^(2n),{t,0,Pi}]/Pi^(2n+1). - Logan Kleinwaks (kleinwaks(AT)alumni.princeton.edu), Mar 15 2007

Contribution from Peter Bala, Mar 24 2009: (Start)

Basic hypergeometric generating function: 2*exp(-t)*Sum {n = 0..inf} Product {k = 1..n} (1-exp(-(4*k-2)*t))*exp(-2*n*t)/Product {k = 1..n+1} (1+exp(-(4*k-2)*t)) = 1 + t + 5*t^2/2! + 61*t^3/3! + ... For other sequences with generating functions of a similar type see A000464, A002105, A002439, A079144 and A158690.

a(n) = 2*(-1)^n*L(-2*n), where L(s) is the Dirichlet L-function L(s) = 1 - 1/3^s + 1/5^s - + ... (End)

sum(n>=0, a(n)*z^(2*n)/(4*n)!!) = Beta(1/2-z/(2*Pi),1/2+z/(2*Pi))/Beta(1/2,1/2) with Beta(z,w) the Beta function. [Johannes W. Meijer, Jul 06 2009]

a(n)=sum(sum(binomial(k,m)*(-1)^(n+k)/(2^(m-1))*sum(binomial(m,j)*(2*j-m)^(2*n),j,0,m/2)*(-1)^(k-m),m,0,k),k,1,2*n), n>0 [From Kruchinin Vladimir, Aug 05 2010]

If n is prime, then a(n)==1 (mod 2*n). [From Vladimir Shevelev, Sep 04 2010]

From Peter Bala: (Start)

(1)... a(n) = (-1/4)^n*B(2*n,-1),

where {B(n,x)}n>=1 = [1,1+x,1+6*x+x^2,1+23*x+23*x^2+x^3,...] is the sequence of Eulerian polynomials of type B - see A060187. Equivalently,

(2)... a(n) = sum {k = 0..2*n} sum {j = 0..k} (-1)^(n-j) *binomial(2*n+1,k-j)*(j+1/2)^(2*n).

We also have

(3)... a(n) = 2*A(2*n,I)/(1+I)^(2*n+1),

where I = sqrt(-1) and where {A(n,x)}n>=1 = [x,x+x^2,x+4*x^2+x^3,...] denotes the sequence of Eulerian polynomials - see A008292. Equivalently,

(4)... a(n) = I*sum {k = 1..2*n} (-1)^(n+k)*k!*Stirling2(2*n,k) *((1+I)/2)^(k-1)

= I*sum {k = 1..2*n} (-1)^(n+k)*((1+I)/2)^(k-1) sum {j = 0..k}

(-1)^(k-j)*binomial(k,j)*j^(2*n).

Either this explicit formula for a(n) or (2) above may be used to obtain congruence results for a(n). For example, for prime p

(5a)... a(p) = 1 (mod p)

(5b)... a(2*p) = 5 (mod p)

and for odd prime p

(6a)... a((p+1)/2) = (-1)^((p-1)/2) (mod p)

(6b)... a((p-1)/2) = -1 + (-1)^((p-1)/2) (mod p).

(End)
It appears that a(n) = (2/Pi)^(2*n+1) * Integrate[EllipticF[x, 1]^(2*n), {x, 0, Pi/2}] (* From Vladimir Reshetnikov, Jan 24 2011 *)
a(n) = (-1)^n*2^(4*n+1)*(zeta(-2*n,1/4)-zeta(-2*n,3/4)) - [Gerry Martens, May 27 2011]
a(n) may be expressed as a sum of multinomials taken over all compositions of 2*n into even parts (Vella 2008): a(n) = sum {compositions 2*i_1+...+2*i_k = 2*n} (-1)^(n+k)* multinom(2*n,2*i_1,...,2*i_k). For example, there are 4 compositions of the number 6 into even parts, namely 6, 4+2, 2+4 and 2+2+2, and hence a(3) = 6!/6!-6!/(4!*2!)-6!/(2!*4!)+6!/(2!*2!*2!) = 61. A companion formula expressing a(n) as a sum of multinomials taken over the compositions of 2*n-1 into odd parts has been given by (Malenfant 2011). - Peter Bala, Jul 07 2011
a(n) = the upper left term in M^n, where M is an infinite square production matrix; M[i,j] = A000290(i) = i^2, i>=1 and 1<=j<=i+1, and M[i,j] = 0, i>=1 and j>=i+2, see the examples. [Gary W. Adamson, Jul 18 2011]
A000521 Coefficients of modular function j as power series in q = e^(2 Pi i t).
COMMENTS "The most natural normalization [of the j function] is to set the constant term equal to 24, the number given by Rademacher's infinite series for coefficients of the j function". [Borcherds]
Changing the term 744 to 24 gives A007240, the McKay-Thompson series of class 1A for Monster simple group.
sigma_3(n) is the sum of the cubes of the divisors of n (A001158).
Klein's absolute invariant J=j/1728 is Gamma-modular.
(n+1)*A000521(n)/24 yields integral values - see A161395 [From Alexander R. Povolotsky (pevnev(AT)juno.com), Jun 09 2009]
Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 07 2009: (Start)
Equals convolution square of A161361: (1, 372, 29250, -134120, 54261375,...)
and row sums of triangle A161362. (End)
FORMULA A007245(q)^3/q; or (1 + 240 sum sigma_3(n) q^n)^3 / (q prod (1-q^n)^24) (n=1..inf).
It appears that -n * a(n) = A035230(n). - Gerald McGarvey, Dec 21 2006
2 * a(2) = A028520(3). 2 * a(4) + a(1) = A028520(4). 2 * a(6) = A028520(5). - Gerald McGarvey, Dec 21 2006
Expansion of 128 * (theta_2(q)^8 + theta_3(q)^8 + theta_4(q)^8) * (theta_2(q)^-8 + theta_3(q)^-8 + theta_4(q)^-8) in powers of q^2. - Michael Somos Oct 02 2007
A000578 The cubes: a(n) = n^3.
COMMENTS a(n) = sum of the next n odd numbers; i.e. group the odd numbers so that the n-th group contains n elements like this (1), (3,5),(7,9,11),(13,15,17,19),(21,23,25,27,29,),... then each group sum = n^3 = a(n). Also the median of each group = n^2 = mean. As the sum of first n odd numbers is n^2 this gives another proof of the fact that the n-th partial sum = {n(n+1)/2}^2. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Sep 14 2002
Total number of triangles resulting from criss-crossing cevians within a triangle so that two of its sides are each n-partitioned. - Lekraj Beedassy (blekraj(AT)yahoo.com), Jun 02 2004
n^3 is the sum of the first n centered hexagonal numbers (A003215). - Alonso Delarte (alonso.delarte(AT)gmail.com), Jul 29 2004
Also structured triakis tetrahedral numbers (vertex structure 7) (Cf. A100175 = alternate vertex);

structured tetragonal prism numbers (vertex structure 7) (Cf. A100177 = structured prisms); structured hexagonal diamond numbers (vertex structure 7) (Cf. A100178 = alternate vertex; A000447 = structured diamonds); and structured trigonal anti-diamond numbers (vertex structure 7) (Cf. A100188 = structured anti-diamonds). Cf. A100145 for more on structured polyhedral numbers . - James A. Record (james.record(AT)gmail.com), Nov 07 2004.

Schlaefli symbol for this polyhedron: {4,3}

Least multiple of n such that every partial sum is a square. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Sep 09 2005

Draw a regular hexagon. Construct points on each side of the hexagon such that these points divide each side into equally-sized segments (i.e. a midpoint on each side or two points on each side placed to divide each side into three equally-sized segments or so on), do the same construction for every side of the hexagon so that each side is equally divided in the same way. Connect all such points to each other with lines that are parallel to at least one side of the polygon. The result is a triangular tiling of the hexagon and the creation of a number of smaller regular hexagons. The equation gives the total number of regular hexagons found where n=the number of points drawn+1. For example, if 1 point is drawn on each side then n=1+1=2 and a(n)=2^3=8 so there are 8 regular hexagons in total. If 2 points are drawn on each side then n=2+1=3 and a(n)=3^3=27 so there are 27 regular hexagons in total. - Noah Priluck (npriluck(AT)gmail.com), May 02 2007

a(n) = {least common multiple of n and (n-1)^2}-(n-1)^2. E.g.: {least common multiple of 1 and (1-1)^2}-(1-1)^2 = 0, {least common multiple of 2 and (2-1)^2}-(2-1)^2 = 1, {least common multiple of 3 and (3-1)^2}-(3-1)^2 = 8, ... - Mats Granvik (mgranvik(AT)abo.fi), Sep 24 2007

The solutions of the Diophantine equation: (X/Y)^2 - X*Y = 0 are of the form: (n^3, n) with n>=1. The solutions of the Diophantine equation: (m^2)*(X/Y)^2k - XY = 0 are of the form: (m*n^(2k+1), m*n^(2k-1)) with m>=1, k>=1 and n>=1. The solutions of the Diophantine equation: (m^2)*(X/Y)^(2k+1) - XY = 0 are of the form: (m*n^(k+1), m*n^k) with m>=1, k>=1 and n>=1. - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Oct 04 2007

Excepting for the first two terms, the sequence corresponds to the Wiener indices of C_{2n} i.e., the cycle on 2n vertices (n > 1). [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), Mar 16 2009]

Number of units of a(n) belongs to a periodic sequence: 0, 1, 8, 7, 4, 5, 6, 3, 2, 9. [From Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Sep 04 2009]

a(n) = A007531(n) + A000567(n). [From Reinhard Zumkeller, Sep 18 2009]

Totally multiplicative sequence with a(p) = p^3 for prime p. [From Jaroslav Krizek, Nov 01 2009]

Sums of rows of the triangle in A176271, n>0. [From Reinhard Zumkeller, Apr 13 2010]

One of the 5 Platonic polyhedral (tetrahedral, cube, octahedral, dodecahedral and icosahedral) numbers (cf. A053012). [From Daniel Forgues, May 14 2010]

Numbers n for which order of torsion subgroup t of the elliptic curve y^2=x^3-n is t=2. [From Artur Jasinski (grafix(AT)csl.pl), Jun 30 2010]

The sequence with the lengths of the Pisano periods mod k is 1, 2, 3, 4, 5, 6, 7, 8, 3, 10, 11, 12, 13, 14, 15, 16, 17, 6, 19, 20,.. for k>=1, apparently multiplicative and derived from A000027 by dividing every ninth term through 3. Cubic variant of A186646. - R. J. Mathar, Mar 10 2011

The number of atoms in a bcc (body-centered cubic) rhombic hexahedron with n shells is n^3 (T.P. Martin, Shells of atoms, eq.(8)). - Brigitte Stepanov, Jul 02 2011

FORMULA Multiplicative with a(p^e) = p^(3e). - David W. Wilson, Aug 01, 2001.

G.f.: x*(1+4*x+x^2)/(1-x)^4. - Michael Somos, May 06 2003

Dirichlet generating function: zeta(s-3). - Franklin T. Adams-Watters, Sep 11 2005. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Sep 09 2005

E.g.f.: (x+3*x^2+x^3)*exp(x). - Franklin T. Adams-Watters, Sep 11 2005. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Sep 09 2005

a(n) = Sum(Sum(A002024(j,i): i<=j<n+i): 1<=i<=n). - Reinhard Zumkeller, Jun 24 2007

Starting (1, 8, 27, 64, 125,...), = binomial tansform of [1, 7, 12, 6, 0, 0, 0,...]. - Gary W. Adamson, Nov 21 2007

a(n) = C(n+2,3) + 4 C(n+1,3) + C(n,3)

This sequence could be obtained from the general formula n*(n+1)*(n+2)*(n+3)* ...* (n+k) *(n*(n+k) + (k-1)*k/6)/((k+3)!/6) at k=0. - Alexander R. Povolotsky, May 17 2008

A000583 Fourth powers: a(n) = n^4.

COMMENTS Figurate numbers based on 4-dimensional regular convex polytope called the 4-measure polytope, 4-hypercube or tessaract with Schlaefli symbol {4,3,3}. - Michael J. Welch (mjw1(AT)ntlworld.com), Apr 01 2004

Sum(k>0,1/a(k))=Pi^4/90 [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Sep 20 2009]

Totally multiplicative sequence with a(p) = p^4 for prime p. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Nov 01 2009]

FORMULA Multiplicative with a(p^e) = p^(4e). - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.

G.f.: x*(1+11*x+11*x^2+x^3)/(1-x)^5. More generally, g.f. for n^m is Euler(m, x)/(1-x)^(m+1), where Euler(m, x) is Eulerian polynomial of degree m (cf. A008292).

Dirichlet generating function: zeta(s-4). - Franklin T. Adams-Watters, Sep 11 2005.

E.g.f.: (x+7x^2+6x^3+x^4)*e^x. More generally, the general form for the e.g.f. for n^m is phi_m(x)*e^x, where phi_m is the exponential polynomial of order n. - Franklin T. Adams-Watters, Sep 11 2005.

a(n)=sum(sum(sum(n, j=1..n),k=1..n),m=1..n), n>=0 . - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), May 09 2007

a(n) = {least common multiple of n and (n-1)^3}-(n-1)^3. E.g.: {least common multiple of 1 and (1-1)^3}-(1-1)^3 = 0, {least common multiple of 2 and (2-1)^3}-(2-1)^3 = 1, {least common multiple of 3 and (3-1)^3}-(3-1)^3 = 16, {least common multiple of 4 and (4-1)^3}-(4-1)^3 = 81, ... - Mats Granvik (mgranvik(AT)abo.fi), Sep 24 2007

a(n) = C(n+3,4) + 11 C(n+2,4) + 11 C(n+1,4) + C(n,4)

a(n) = n*A177342(n) - sum [i = 1..n-1] A177342(i) - (n-1), with n>0 . For n=5, a(5) = 5*A177342(5) - sum [i=1..4] A177342(i) - 4 = 5*149-(75+31+9+1)-4 = 625 [From Bruno Berselli (berselli.bruno(AT)yahoo.it), May 07 2010]

A000593 Sum of odd divisors of n.

FORMULA Inverse Moebius Transform of [0, 1, 0, 3, 0, 5, ...]

Dirichlet g.f.: zeta(s)*zeta(s-1)*(1-2^(1-s)).

a(2*n)=A000203(2*n)-2*A000203(n), a(2*n+1)=A000203(2*n+1) - Henry Bottomley (se16(AT)btinternet.com), May 16 2000

a(2*n) = A054785(2*n) - A000203(n). - from Reinhard Zumkeller, Apr 23 2008

Multiplicative with a(p^e) = 1 if p = 2, (p^(e+1)-1)/(p-1) if p > 2. - David W. Wilson, Aug 01, 2001.

a(n) = Sum_{d divides n} (-1)^(d+1)*n/d. - Vladeta Jovovic (vladeta(AT)eunet.rs), Sep 06 2002

Sum(k=1, n, a(k)) is asymptotic to c*n^2 where c=Pi^2/24. - Benoit Cloitre, Dec 29, 2002

G.f.: Sum_{n>0} n*x^n/(1+x^n). - Vladeta Jovovic (vladeta(AT)eunet.rs), Oct 11 2002

G.f.: (theta_3(q)^4 + theta_2(q)^4 -1)/24.

G.f.: Sum_{k>0} -(-x)^k/(1-x^k)^2. - Michael Somos, Oct 29 2005

a(n)=A050449(n)+A050452(n); a(A000079(n))=1; a(A005408(n))=A000203(A005408(n)). - Reinhard Zumkeller, Apr 18 2006

G.f.: sum(n=1, infinity, (2*n-1) * q^(2*n-1) / (1-q^(2*n-1))). G.f.: deriv(log(P))=deriv(P)/P where P=prod(n>=1, 1+q^n). [Joerg Arndt, Nov 09 2010]

Dirichlet convolution of A000203 with [1,-2,0,0,0,...]. Dirichlet convolution of A062157 with A000027. - R. J. Mathar, Jun 28 2011

A000594 Ramanujan's tau function (or tau numbers).

COMMENTS Coefficients of the cusp form of weight 12 for the full modular group.

It is conjectured that tau(n) is never zero.

Number of partitions of n into an even number of distinct parts - partitions of n into an odd number of distinct parts, with 24 types of each part. - Jon Perry (perry(AT)globalnet.co.uk), Apr 04 2004

M. J. Hopkins mentions that the only known primes p for which tau(p) == 1 mod p are 11, 23 and 691, that it is an open problem to decide if there are infinitely many such p and that no others are known below 35000. Simon Plouffe has now searched up to tau(314747) and found no other examples. - N. J. A. Sloane (njas(AT)research.att.com), Mar 25 2007

FORMULA G.f.: x*prod(k>=1, (1 - x^k)^24).

G.f. is a period 1 Fourier series which satisfies f(-1/t) = (t/i)^12*f(t) where q = exp(2*pi*i*t). - Michael Somos, Jul 04 2011

abs(a(n)) = O(n^(11/2 + epsilon)), abs(a(p)) <= 2 p^(11/2) if p is prime. These were conjectured by Ramanujan and proved by Deligne.

Zagier says: The proof of these formulae, if written out from scratch, has been estimated at 2000 pages; in his book Manin cites this as a probable record for the ratio: `length of proof:length of statement' in the whole of mathematics.

G.f. A(x) satisfies 0 = f(A(x), A(x^2), A(x^4)) where f(u, v, w) = u*w*(u + 48*v + 4096*w) - v^3. - Michael Somos, Jul 19 2004

A000602 Number of n-node unrooted quartic trees; number of n-carbon alkanes C(n)H(2n+2) ignoring stereoisomers.

COMMENTS Trees are unrooted, nodes are unlabeled. Every node has degree <= 4.

Ignoring stereoisomers means that the children of a node are unordered. They can be permuted in any way and it is still the same tree. See A000628 for the analogous sequence with stereoisomers counted.

In alkanes every carbon has valence exactly 4 and every hydrogen has valence exactly 1. But the trees considered here are just the carbon "skeletons" (with all the hydrogen atoms stripped off) so now each carbon bonds to 1 to 4 other carbons. The degree of each node is then <= 4.

FORMULA a(n) = A010372(n) + A010373(n/2) for n even, a(n) = A010372(n) for n odd.

Also equals A000022 + A000200 (n>0), both of which have known generating functions. Also g.f. = A000678(x)-A000599(x)+A000598(x^2) = (x+x^2+2x^3...)-(x^2+x^3+3x^4...)+(1+x^2+x^4+...) = 1+x+x^2+x^3+2x^4+3x^5...

A000609 Number of threshold functions of n or fewer variables.

A000670 Number of preferential arrangements of n labeled elements; or number of weak orders on n labeled elements.

COMMENTS Number of ways n competitors can rank in a competition, allowing for the possibility of ties.

Also number of asymmetric generalized weak orders on n points.

Also called the ordered Bell numbers, i.e. the number of ordered partitions of {1,..,n}.

A weak order is a relation that is transitive and complete.

Called Fubini numbers by Comtet: counts formulae in Fubini theorem when switching the order of summation in multiple sums. - Olivier Gerard, Sep 30, 2002

If the points are unlabeled then the answer is a(0) = 1, a(n) = 2^(n-1) (cf. A011782).

For n>0, a(n) is the number of elements in the Coxeter complex of type A_{n-1}. The corresponding sequence for type B is A080253 and there one can find a worked example as well as a geometric interpretation. - Tim Honeywill & Paul Boddington (tch(AT)maths.warwick.ac.uk), Feb 10 2003

Also number of labeled (1+2)-free posets. - Detlef Pauly, May 25 2003

Also the number of chains of subsets starting with the empty set and ending with a set of n distinct objects. - Andy Niedermaier (aniedermaier(AT)hmc.edu), Feb 20 2004

From Michael Somos, Mar 04 2004: (Start)

Stirling transform of A007680(n) = [3, 10, 42, 216, . . .] gives [3,13,75,541,...].

Stirling transform of a(n)=[1,3,13,75,...] is A083355(n)=[1,4,23,175,...].

Stirling transform of A000142(n)=[1,2,6,24,120,...] is a(n)=[1,3,13,75,...].

Stirling transform of A005359(n-1)=[1,0,2,0,24,0,...] is a(n-1)=[1,1,3,13,75,...].

Stirling transform of A005212(n-1)=[0,1,0,6,0,120,0,...] is a(n-1)=[0,1,3,13,75,...]. (End)

Unreduced denominators in convergent to log(2) = lim[n->inf, na(n-1)/a(n)].

a(n) is congruent to a(n+(p-1)p^(h-1)) (mod p^h) for n>=h (see Barsky).

Stirling-Bernoulli transform of 1/(1-x^2). - Paul Barry, Apr 20 2005

This is the sequence of moments of the probability distribution of the number of tails before the first head in a sequence of fair coin tosses. The sequence of cumulants of the same probability distribution is A000629. That sequence is twice the result of deletion of the first term of this sequence. - Michael Hardy (hardy(AT)math.umn.edu), May 01 2005

With p(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, p(j,i) = the j-th part of the i-th partition of n, m(i,j) = multiplicity of the j-th part of the i-th partition of n, sum_{i=1}^{p(n)} = sum over i and prod_{j=1}^{d(i)} = product over j one has: a(n) = sum_{i=1}^{p(n)} (n!/(prod_{j=1}^{p(i)}p(i,j)!)) * (p(i)!/(prod_{j=1}^{d(i)} m(i,j)!)) - Thomas Wieder, May 18 2005

The number of chains among subsets of [n]. The summed term in the new formula is the number of such chains of length k. - Micha Hofri (hofri(AT)wpi.edu), Jul 01 2006

Occurs also as first column of a matrix-inversion occurring in a sum-of-like-powers problem. Consider the problem for any fixed natural number m>2 of finding solutions to the equation sum(k=1,n,k^m) = (k+1)^m. Erdos conjectured that there are no solutions for n,m>2. Let D be the matrix of differences of D[m,n] := sum(k=1,n,k^m) - (k+1)^m. Then the generating functions for the rows of this matrix D constitute a set of polynomials in n (for varying n along columns) and the m-th polynomial defining the m-th row. Let GF_D be the matrix of the coefficients of this set of polynomials. Then the present sequence is the (unsigned) first column of GF_D^-1. - Gottfried Helms, Apr 01 2007

Assuming A=ln2, D is d/dx and f(x)=x/(exp(x)-1), we have a(n) = (n!/2A^(n+1)) sum(k=0 to n, (A^k/k!) D^n f(-A)) which gives Wilf's asymptotic value when n tends to infinity. Equivalently, D^n f(-a) = 2(A*a(n)-2*a(n-1)). - Martin Kochanski (mjk(AT)cardbox.com), May 10 2007

List partition transform (see A133314) of (1,-1,-1,-1,...). - Tom Copeland, Oct 24 2007

First column of A154921. [From Mats Granvik (mats.granvik(AT)abo.fi), Jan 17 2009]

A slightly more transparent interpretation of a(n) is as the number of 'factor sequences' of N for the case in which N is a product of n distinct primes. A factor sequence of N of length k is of the form 1=x(1),x(2),...,x(k)=N, where {x(i)} is an increasing sequence such that x(i) divides x(i+1), i=1,2,...,k-1. For example, N=70 has the 13 factor sequences {1,70}, {1,2,70}, {1,5,70}, {1,7,70}, {1,10,70}, {1,14,70}, {1,35,70}, {1,2,10,70}, {1,2,14,70}, {1,5,10,70}, {1,5,35,70}, {1,7,14,70}, {1,7,35,70}. [From Martin Griffiths (griffm(AT)essex.ac.uk), Mar 25 2009]

Starting (1, 3, 13, 75,...) = row sums of triangle A163204 [From Gary W. Adamson, Jul 23 2009]

Equals double inverse binomial transform of A007047: (1, 3, 11, 51,...). [From Gary W. Adamson, Aug 04 2009]

Contribution from Miklos Kristof, Nov 02 2009: (Start)

If f(x)=sum(c(n)*x^n,n=0..infinity) converges for every x, then

sum(f(n*x)/2^(n+1), n=0..infinity) = sum(c(n)*a(n)*x^n, n=0..infinity).

Example: sum(exp(n*x)/2^(n+1), n=0..infinity) = sum(a(n)*x^n/n!, n=0..infinity)= 1/(2-exp(x)) = E.g.f. (End)

Hankel transform is A091804. [From Paul Barry, Mar 30 2010]

It appears that the prime numbers greater than 3 in this sequence (13, 541, 47293, ...) are of the form 4n+1. [Paul Muljadi, Jan 28 2011]

The Fi1 and Fi2 triangle sums of A028246 are given by the terms of this sequence. For the definitions of these triangle sums see A180662. [From Johannes W. Meijer, Apr 20 2011]

The modified generating function A(x) = 1/(2-exp(x))-1 = x+3*x^2/2!+13*x^3/3!+... satisfies the autonomous differential equation A' = 1+3*A+2*A^2 with initial condition A(0) = 0. Applying [Bergeron et al, Theorem 1] leads to two combinatorial interpretations for this sequence: A) a(n) counts the number of plane increasing 0-1-2 trees on n vertices, where vertices of outdegree 1 come in 3 colors and vertices of outdegree 2 come in 2 colors. B) a(n) counts the number of non-plane increasing 0-1-2 trees on n vertices, where vertices of outdegree 1 come in 3 colors and vertices of outdegree 2 come in 4 colors. Examples are given below. - Peter Bala, Aug 31 2011

FORMULA G.f.: A(x) = Sum_{n>=0} n!*x^n / Product_{k=1..n} (1-k*x). [From Paul D. Hanna, Jul 20 2011]

a(n) = Sum_{ k=1 to n} k! StirlingS2(n, k) (whereas the Bell numbers A000110(n) = Sum_{ k=1 to n} StirlingS2(n, k)).

E.g.f.: 1/(2-exp(x)).

a(n)=sum(k>=2, binomial(n, k)*a(n-k)), a(0)=1.

The e.g.f. y(x) satisfies y' = 2*y^2 - y.

Contribution from Paul Barry, Mar 30 2010: (Start)

G.f.: 1/(1-x/(1-2x/(1-2x/(1-4x/(1-3x/(1-6x/(1-4x/(1-8x/(1-5x/(1-10x/(1-6x/(1-... (continued fraction);

Coefficients of continued fraction are given by floor((n+2)/2)*(3-(-1)^n)/2 (A029578(n+2)). (End)

G.f.: 1/(1-x-2x^2/(1-4x-8x^2/(1-7x-18x^2/(1-10x-32x^2/(1../(1-(3n+1)x-2(n+1)^2x^2/(1-... (continued fraction). [From Paul Barry, Jun 17 2010]

a(n) is asymptotic to (1/2)*n!*log_2(e)^(n+1), where log_2(e) = 1.442695... [Wilf]

For n >= 1, a(n) = (n!/2) * Sum from k=-infinity to infinity of (log(2) + 2 pi i k)^(-n-1) - from Dean Hickerson (dean.hickerson(AT)yahoo.com)

a(n) = ((x*d/dx)^n)(1/(2-x)) evaluated at x=1. - Karol A. Penson, Sep 24 2001

For n>=1, a(n) = sum(k>=1, (k-1)^n/2^k) = A000629(n)/2. - Benoit Cloitre, Sep 08 2002

Value of the n-th Eulerian polynomial (cf. A008292) at x=2. - Vladeta Jovovic (vladeta(AT)eunet.rs), Sep 26 2003

a(n) = A076726(n)/2.

a(n)=sum{k=0..n, (-1)^k*k!*S2(n+1, k+1)(1+(-1)^k)/2}; - Paul Barry, Apr 20 2005

a(n) + a(n+1) = 2*A005649(n) . - Philippe DELEHAM, May 16 2005 - Thomas Wieder, May 18 2005

Equals inverse binomial transform of A000629. - Gary W. Adamson, May 30 2005

First Eulerian transform of the powers of 2 [A000079]. See A000142 for definition of FET. - Ross La Haye (rlahaye(AT)new.rr.com), Feb 14 2005

a(n) = sum^{k=0}^n k!*(Stirling2(n+2,k+2) - Stirling2(n+1, k+2)). - Micha Hofri (hofri(AT)wpi.edu), Jul 01 2006

Recurrence: 2a(n)=(a+1)^n where superscripts are converted to subscripts after binomial expansion - reminiscent of Bernoulli numbers' B_n=(B+1)^n - Martin Kochanski (mjk(AT)cardbox.com), May 10 2007

a(n) = (-1)^n * n!*Laguerre(n,P((.),2)), umbrally, where P(j,t) are the polynomials in A131758. - Tom Copeland, Sep 27 2007

Formula in terms of the hypergeometric function, in Maple notation: a(n)=hypergeom([2,2...2],[1,1...1],1/2)/4, n=1,2..., where in the hypergeometric function there are n upper parameters all equal to 2 and n-1 lower parameters all equal to 1 and the argument is equal to 1/2. Example: a(4)=evalf(hypergeom([2,2,2,2],[1,1,1],1/2)/4)=75 - Karol A. Penson, Oct 04 2007

a(n)=Sum_{k, 0<=k<=n} A131689(n,k). [From Philippe DELEHAM, Nov 03 2008]

The adjusted e.g.f. A(x) := 1/(2-exp(x))-1, has inverse function A(x)^-1 = int {t = 0..x} 1/((1+t)*(1+2*t)). Applying [Dominici, Theorem 4.1] to invert the integral yields a formula for a(n): Let f(x) = (1+x)*(1+2*x). Let D be the operator f(x)*d/dx. Then a(n) = D^(n-1)(f(x)) evaluated at x = 0. Compare with A050351. - Peter Bala, Aug 31 2011

ANALOGY WITH THE BERNOULLI NUMBERS.

We enlarge upon the above comment of M. Kochanski.

The Bernoulli polynomials B_n(x), n = 0,1,..., are given by the formula

(1)... B_n(x) := sum {k = 0..n} binomial(n,k)*B(k)*x^(n-k),

where B(n) denotes the sequence of Bernoulli numbers B(0) = 1,

B(1) = -1/2, B(2) = 1/6, B(3) = 0, ...
By analogy, we associate with the present sequence an Appell sequence
of polynomials {P_n(x)}n>=0 defined by
(2)... P_n(x) := sum {k = 0..n} binomial(n,k)*a(k)*x^(n-k).
These polynomials have similar properties to the Bernoulli polynomials.
The first few values are P_0(x) = 1, P_1(x) = x + 1,
P_2(x) = x^2 + 2*x + 3, P_3(x) = x^3 + 3*x^2 + 9*x + 13 and
P_4(x) = x^4 + 4*x^3 + 18*x^2 + 52*x + 75. See A154921 for the
triangle of coefficients of these polynomials.
The e.g.f. for this polynomial sequence is
(3)... exp(x*t)/(2 - exp(t)) = 1 + (x + 1)*t + (x^2 + 2*x + 3)*t^2/2!
+ ....
The polynomials satisfy the difference equation
(4)... 2*P_n(x - 1) - P_n(x) = (x - 1)^n,
and so may be used to evaluate the weighted sums of powers of integers
(1/2)*1^m + (1/2)^2*2^m + (1/2)^3*3^m + ... + (1/2)^(n-1)*(n-1)^m
via the formula
(5)... sum {k = 1..n-1} (1/2)^k*k^m = 2*P_m(0) - (1/2)^(n-1)*P_m(n),
analogous to the evaluation of the sums 1^m + 2^m + ... + (n-1)^m in
terms of Bernoulli polynomials.
This last result can be generalised to
(6)... sum {k=1..n-1} (1/2)^k*(k+x)^m = 2*P_m(x)-(1/2)^(n-1)*P_m(x+n).
For more properties of the polynomials P_n(x) refer to A154921.
For further information on weighted sums of powers of integers and
the associated polynomial sequences see A162312.
The present sequence also occurs in the evaluation of another sum of
powers of integers. Define
(7)... S_m(n) := sum {k = 1..n-1} (1/2)^k*((n-k)*k)^m, m = 1,2,....
Then
(8)... S_m(n) = (-1)^m *[2*Q_m(-n) - (1/2)^(n-1)*Q_m(n)],
where Q_m(x) are polynomials in x given by
(9)... Q_m(x) = sum {k = 0..m} a(m+k)*binomial(m,k)*x^(m-k).
The first few values are Q_1(x) = x + 3, Q_2(x) = 3*x^2 + 26*x + 75
and Q_3(x) = 13*x^3 + 225*x^2 + 1623*x + 4683.
For example, m = 2 gives
(10)... S_2(n) := sum {k = 1..n-1} (1/2)^k*((n-k)*k)^2
= 2*(3*n^2 - 26*n + 75) - (1/2)^(n-1)*(3*n^2 + 26*n + 75).
(End)

a(n)=A074206(q_1*q_2*...*q_n), where {q_i} are distinct primes. - Vladimir Shevelev(shevelev(AT)bgu.ac.il) Aug 05 2011

A000688 Number of Abelian groups of order n; number of factorizations of n into prime powers.

COMMENTS Equivalently, number of Abelian groups with n conjugacy classes. - Michael Somos, Aug 10 2010 a(n) depends only on prime signature of n (cf. A025487). So a(24) = a(375) since 24=2^3*3 and 375=3*5^3 both have prime signature (3,1).

Also number of rings with n elements that are the direct product of fields; these are the commutative rings with n elements having no nilpotents; likewise the commutative rings where for every element x there is a k > 0 such that x^{k+1} = x. - Franklin T. Adams-Watters (FrankTAW(AT)Netscape.net), Oct 20 2006

Range is A033637.

FORMULA a(p^k) = number of partitions of k; a(mn)=a(m)a(n) if (m, n)=1.

Multiplicative with a(p^e) = A000041(e). - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.

a(n) = product(pa(e(j)),j=1..N(n)), n>=2, if

n = product(p(j)^e(j),j=1..N(n)) with p(j) = A000040(j) (primes), N(n) = A001221(n)(also called omega(n)), and pa(n) = A000041(n) (partition numbers). See the Richert reference, quoting A. Speiser's book on finite groups (in German, p. 51 in words). - Wolfdieter Lang, Jul 23 2011.

A000720 pi(n), the number of primes <= n. Sometimes called PrimePi(n) to distinguish it from the number 3.14159...

COMMENTS Partial sums of A010051 (characteristic function of primes). - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Aug 13 2002

pi(n) and prime(n) are inverse functions: a(A000040(n)) = n and A000040(n) is the least number m such that A000040(a(m)) = A000040(n). A000040(a(n)) = n if (and only if) n is prime. - Jonathan Sondow, Dec 27 2004

The g.f. -z*(-1-z-z**3-z**5+z**6+z**7)/((1+z)*(z**2-z+1)*(z**2+z+1)*(z-1)**2) conjectured by S. Plouffe in his 1992 dissertation is wrong.

See the additional references and links mentioned in A143227. [From Jonathan Sondow, Aug 03 2008]

Equals row sums of triangle A143538 [From Gary W. Adamson, Aug 23 2008]

a(n) = A036234(n)-1. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Mar 23 2009]

((10^n)^2)/(ln((10^n)!)) [From Eric Desbiaux (moongerms(AT)wanadoo.fr), Jul 15 2009]

A lower bound that gets better with larger N is that there are at least T prime numbers less than N, where the recursive function T is: T = N- N*sum(A005867(i)/A002110(i), i=0..T(sqrt(N))) [From Ben Thurston, Aug 23 2010]

FORMULA The prime number theorem gives the asymptotic expression a(n) ~ n/log(n).

For x > 1, pi(x) < (x / log x) * (1 + 3/(2 log x)). For x >= 59, pi(x) > (x / log x) * (1 + 1/(2 log x)). [Rosser and Schoenfeld]

For x >= 355991, pi(x) < (x / log(x)) * (1 + 1/log(x) + 2.51/(log(x))^2). For x >= 599, pi(x) > (x / log(x)) * (1 + 1/log(x)). [Dusart]

For x >= 55, x/(log(x)+2) < pi(x) < x/(log(x)-4). [Rosser]

For n>1: A138194(n) <= a(n) <= A138195(n) (Tschebyscheff, 1850). - Reinhard Zumkeller, Mar 04 2008

For n>=3, a(n)=1+sum_{j=3..n} ((j-2)!-j*floor((j-2)!/j)) (Hardy and Wright); for n>=1, a(n) = n - 1 + sum_{j=2..n} (floor((2 - sum_{i=1..j} (floor(j/i)-floor((j-1)/i)))/j)) (Ruiz and Sondow 2000) - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 31 2003

a(n)=A001221(A000142(n)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Jun 03 2005

G.f. sum_{p prime} x^p/(1-x) = b(x)/(1-x), where b(x) is the g.f. for A010051. - Frank Adams-Watters, Jun 15 2006

A recursive definition of PrimePi using the LegendrePhi function given in the Wagon_notes.nb: Pi(n) = Pi(Sqrt(n)) + Phi(n, Pi(Sqrt(n)) - 1, with Pi(0)=0, Pi(1)=0. - Roger L. Bagula, Mar 26 2008

Contribution from Enrique Perez Herrero, Jul 12 2010: (Start)

a(n)=sum_{i=2}^{n}{floor((i+1)/A000203(i))}

a(n)=sum_{i=2}^{n}{floor(A000010(n)/(i-1))}

a(n)=sum_{i=2}^{n}{floor(2/A000005(n))} (End)

Let pf(n) denote the set of prime factors of an integer n. Then a(n) = card(pf(n!/floor(n/2)!). - Peter Luschny, Mar 13 2011

A000793 Landau's function g(n): largest order of permutation of n elements. Equivalently, largest lcm of partitions of n.

COMMENTS Also the largest orbit size (cycle length) for the permutation A057511 acting on Catalan objects (e.g. planar rooted trees, parenthesizations) - Antti Karttunen Sep 07 2000

Grantham mentions that he computed a(n) for n <= 500000.

FORMULA Landau: lim_{n->infinity} (log a(n)) / sqrt(n log n) = 1.

A000796 Decimal expansion of Pi.

COMMENTS Sometimes called Archimedes's constant.

FORMULA Alexander R. Povolotsky came up with the following BBP-type formula: Pi= 2/3 * (-1 + Sum(7/(4*k+1) - 6/(4*k+3) - 1/(4*k+5),k = 0 .. infinity). J. Guillera noted: "There is an easy proof of that formula if to convert it into an integral. In doing the proof, observe that int_(0,1) x ^ (4n+a) = 1 / (4n+a+1). The proof is easy but it can be interesting if one does not know the method. The formula converges slowly because there is not a factor like for example 1/16^k." Roger Bagula tried to use this formula for generating high quality pseudo-random level results. He thinks that this formula's algorithm is faster and takes less computer memory for that (comparing with regular BBP) [From Alexander R. Povolotsky (pevnev(AT)juno.com), Nov 30 2008]

Pi = 2/3 * (-1 + Sum(7/(4*k+1) - 6/(4*k+3) - 1/(4*k+5),k = 0 .. infinity) [From Alexander R. Povolotsky (pevnev(AT)juno.com), Nov 30 2008]

Another (ugly) formula for Pi (in Maple syntax): Pi = 6/7*(1/3*sum((843*n + 4607)/((n+5)*(3*n+7)*(3*n+22)),n=0...infinity) - 655999/248976 - 7/2*ln(3))*sqrt(3) [From Alexander R. Povolotsky (pevnev(AT)juno.com), Dec 07 2008]

Pi = (4/5)*(Sum(7/(4*k+1) - 5/(4*k+3) - 2/(4*k+5),k = 0 .. infinity) -2) [From Alexander R. Povolotsky (pevnev(AT)juno.com), Dec 25 2008]

Pi = Sum(7/(4*k+1) - 4/(4*k+3) - 3/(4*k+5),k = 0 .. infinity) - 3 [From Alexander R. Povolotsky (pevnev(AT)juno.com), Dec 25 2008]

Pi = 4*Sum(1/(4*k+1) - 1/(4*k+3),k = 0 .. infinity) [From Alexander R. Povolotsky (pevnev(AT)juno.com), Dec 25 2008]

pi = c + sum(k>=0, (4-c)/(4k+1) -4/(4k+3) +c/(4k+5)) for any c. [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Jan 11 2009]

Pi=4*sqrt(-1*(sum((I^(2*n+1))/(2*n+1),n=0...infinity)^2)) [From Alexander R. Povolotsky (pevnev(AT)juno.com), Jan 25 2009]

Pi=2*n*A000111(n-1)/A000111(n) as n-->infinity (conjecture). [From Mats Granvik (mats.granvik(AT)abo.fi), Aug 12 2009]

A000798 Number of different quasi-orders (or topologies, or transitive digraphs) with n labeled elements.

COMMENTS a(17)-a(18) are from Brinkmann's and McKay's paper. - Vladeta Jovovic (vladeta(AT)eunet.rs), Jun 10 2007

FORMULA Related to A001035 by A000798(n) = Sum Stirling2(n, k)*A001035(k).

A000961 Prime powers p^k (p prime, k >= 0).

COMMENTS Since 1 = p^0 does not have a well defined prime base p, it is sometimes not regarded as a prime power.

These numbers are (apart from 1) the numbers of elements in finite fields. - Franz Vrabec (franz.vrabec(AT)aon.at), Aug 11 2004

Numbers whose divisors form a geometrical progression. The divisors of p^k are 1, p, p^2, p^3, ...p^k. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Jan 09 2002

a(n) = A025473(n)^A025474(n). - David Wasserman (wasserma(AT)spawar.navy.mil), Feb 16 2006

a(n) = A117331(A117333(n)). - Reinhard Zumkeller, Mar 08 2006

These are also precisely the orders of those finite affine planes that are known to exist as of today. (The order of a finite affine plane is the number of points in an arbitrarily chosen line of that plane. This number is unique for all lines comprise the same number of points.) - Peter C. Heinig (algorithms(AT)gmx.de), Aug 09 2006

Except for first term, the index of the second number divisible by n in A002378, if the index equals n. - Mats Granvik (mgranvik(AT)abo.fi), Nov 18 2007

These are precisely the numbers such that lcm(1,...,m-1)<lcm(1,...,m) (=A003418(m) for m>0; here for m=1, the l.h.s. is taken to be 0). We have a(n+1)=a(n)+1 if a(n) is a Mersenne prime or a(n)+1 is a Fermat prime; the converse is true except for n=7 (from Catalan's conjecture) and n=1, since 2^1-1 and 2^0+1 are not considered as Mersenne resp. Fermat prime. - M. F. Hasler, Jan 18 2007, Apr 18 2010

The sequence is A000015 without repetitions, or more formally, A000961=Union[A000015]. - Zak Seidov, Feb 06 2008

Except for a(1)=1, indices for which the cyclotomic polynomial Phi[k] yields a prime at x=1, cf. A020500. - M. F. Hasler, Apr 04 2008

Also, {A138929(k) ; k>1} = {2*A000961(k) ; k>1} = {4,6,8,10,14,16,18,22,26,32,34,38,46,50,54,58,62,64,74,82,86,94,98,...} are exactly the indices for which Phi[k](-1) is prime. - M. F. Hasler (www.univ-ag.fr/~mhasler), Apr 04 2008

A143201(a(n)) = 1. [From Reinhard Zumkeller, Aug 12 2008]

Number of distinct primes dividing n=omega(n)<2. [From Juri-Stepan Gerasimov, Oct 30 2009]

Or, prime numbers^nonnegative numbers (without repetition). Numbers n such that sum{p-1|p is prime and divisor of n}=product{p-1|p is prime and divisor of n}. A055631(n)=A173557(n-1). [From Juri-Stepan Gerasimov, Dec 09 2009, Mar 10 2010]

Numbers n such that A028236(n) = 1. [From Klaus Brockhaus, Nov 06 2010]

A188666(k) = a(k+1) for k: 2*a(k) <= k < 2*a(k+1), k > 0; notably a(n+1) = A188666(2*a(n)). [Reinhard Zumkeller, Apr 25 2011]

A003415(a(n)) = A192015(n); A068346(a(n)) = A192016(n); a(n)=A192134(n)+A192015(n). [Reinhard Zumkeller, Jun 26 2011]

FORMULA m=a(n) for some n <=> lcm(1,...,m-1)<lcm(1,...,m), where lcm(1...0):=0 as to include a(1)=1. a(n+1)=a(n)+1 <=> a(n+1)=A019434(k) or a(n)=A000668(k) for some k (by Catalan's conjecture), except for n=1 and n=7. - M. F. Hasler, Jan 18 2007, Apr 18 2010

A001221(a(n))<2. [From Juri-Stepan Gerasimov, Oct 30 2009]

A000984 Central binomial coefficients: C(2*n,n) = (2*n)!/(n!)^2.

COMMENTS Equal to the binomial coefficient sum Sum_{k=0..n} binomial(n,k)^2.

Number of possible interleavings of a program with n atomic instructions when executed by two processes - Manuel Carro (mcarro(AT)fi.upm.es), Sep 22 2001

Convolving a(n) with itself yields A000302, the powers of 4. - T. D. Noe (noe(AT)sspectra.com), Jun 11 2002

a(n)=Max{ (i+j)!/(i!j!) | 0<=i,j<=n } - Benoit Cloitre (benoit7848c(AT)orange.fr), May 30 2002

Number of ordered trees with 2n+1 edges, having root of odd degree and nonroot nodes of outdegree 0 or 2. - Emeric Deutsch, Aug 02 2002

Also number of directed, convex polyominoes having semiperimeter n+2.

Also number of diagonally symmetric, directed, convex polyominoes having semiperimeter 2n+2. - Emeric Deutsch, Aug 03 2002

Also Sum_{k=0..n} binomial(n+k-1,k). - Vladeta Jovovic (vladeta(AT)eunet.rs), Aug 28 2002

The second inverse binomial transform of this sequence is this sequence with interpolated zeros. Its G.f. is (1 - 4*x^2)^(-1/2), with n-th term C(n,n/2)(1+(-1)^n)/2. - Paul Barry (pbarry(AT)wit.ie), Jul 01 2003

Number of possible values of a 2*n bit binary number for which half the bits are on and half are off. - Gavin Scott (gavin(AT)allegro.com), Aug 09 2003

Ordered partitions of n with zeros to n+1, e.g. for n=4 we consider the ordered partitions of 11110 (5), 11200 (30), 13000 (20), 40000 (5) and 22000 (10), total 70 and a(4)=70. See A001700 (esp. Mambetov Bektur's comment). - Jon Perry (perry(AT)globalnet.co.uk), Aug 10 2003

Number of non-decreasing sequences of n integers from 0 to n: a(n) = sum_{i_{1}=0}^{n} sum_{i_{2}=i_{1}}^{n}...sum_{i_{n}=i_{n-1}}^{n}(1). - J. N. Bearden (jnb(AT)eller.arizona.edu), Sep 16 2003

Number of peaks at odd level in all Dyck paths of semilength n+1. Example: a(2)=6 because we have U*DU*DU*D, U*DUUDD, UUDDU*D, UUDUDD, UUU*DDD, where U=(1,1), D=(1,-1) and * indicates a peak at odd level. Number of ascents of length 1 in all Dyck paths of semilength n+1 (an ascent in a Dyck path is a maximal string of up steps). Example: a(2)=6 because we have uDuDuD, uDUUDD, UUDDuD, UUDuDD, UUUDDD, where an ascent of length 1 is indicated by a lower case letter. - Emeric Deutsch, Dec 05 2003

a(n-1)=number of subsets of 2n-1 distinct elements taken n at a time that contain a given element. e.g. n=4 -> a(3)=20 and if we consider the subsets of 7 taken 4 at a time with a 1 we get (1234, 1235, 1236, 1237, 1245, 1246, 1247, 1256, 1257, 1267, 1345, 1346, 1347, 1356, 1357, 1367, 1456, 1457, 1467, 1567) and there are 20 of them. - Jon Perry (perry(AT)globalnet.co.uk), Jan 20 2004

The dimension of a particular (necessarily existent) absolutely universal embedding of the unitary dual polar space DSU(2n,q^2) where q>2. - J. Taylor (jt_cpp(AT)yahoo.com), Apr 02 2004.

Number of standard tableaux of shape (n+1, 1^n). - Emeric Deutsch, May 13 2004

Erdos, Graham et al. conjectured that a(n) is never squarefree for sufficiently large n. Sarkozy showed that if s(n) is the square part of a(n), then s(n) is asymptotically (sqrt(2)-2)*(sqrt(n))*(Riemann Zeta Function(1/2)). Granville and Ramare proved that the only squarefree values are a(1)=2, a(2)=6 and a(4)=70. A000984(n)/(n+1) = A000108(n), that is, dividing by (n+1) scales the Central binomial coefficients to Catalan numbers also called Segner numbers. - Jonathan Vos Post, Dec 04 2004

p divides a((p-1)/2)-1=A030662[n] for prime p=5,13,17,29,37,41,53,61,73,89,97..=A002144[n] Pythagorean primes: primes of form 4n+1. - Alexander Adamchuk (alex(AT)kolmogorov.com), Jul 04 2006

The number of direct routes from my home to Granny's when Granny lives n blocks south and n blocks east of my home in Grid City. To obtain a direct route, from the 2n blocks, choose n blocks on which one travels south. For example, a(2)=6 because there are 6 direct routes: SSEE, SESE, SEES, EESS, ESES and ESSE. - Dennis P. Walsh (dwalsh(AT)mtsu.edu), Oct 27 2006

Inverse: With q = -log(log(16)/(pi a(n)^2)), ceiling((q + log(q))/log(16)) = n. - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Feb 26 2007

Number of partitions with Ferrers diagrams that fit in an n X n box (including the empty partition of 0). Example: a(2) = 6 because we have: empty, 1, 2, 11, 21 and 22. - Emeric Deutsch, Oct 02 2007

The number of walks of length 2n on an infinite linear lattice that begin and end at the origin. - Stefan Hollos (stefan(AT)exstrom.com), Dec 10 2007

The number of lattice paths from (0,0) to (n,n) using steps (1,0) and (0,1). [Joerg Arndt, Jul 01 2011]

Integral representation : C(2n,n)=1/Pi Integral [(2x)^(2n)/Sqrt[1 - x^2],{x,-1, 1}], i.e. C(2n,n)/4^n is the moment of order 2n of the arcsin distribution on the interval (-1,1). - Nour-Eddine Fahssi (fahssin(AT)yahoo.fr), Jan 02 2008

Define the array m(1,j)=1 ; m(i,1)=1 ; m(i,j)=m(i,j-1) + m(j,i-1), then a(n) = m(n,n) [From philippe lallouet (philip.lallouet(AT)orange.fr), Sep 15 2008]

Also the Catalan transform of A000079. [From R. J. Mathar, Nov 06 2008]

Straub, Amdeberhan and Moll: "... it is conjectured that there are only finitely many indices n such that C_n is not divisible by any of 3, 5, 7 and 11. Finally, we remark that Erdos et al. conjectured that the central binomial coefficients C_n are never squarefree for n > 4 which has been proved by Granville and Ramare." [From Jonathan Vos Post, Nov 14 2008]

Equals row sums of triangle A152229 [From Gary W. Adamson, Nov 29 2008]

Equals row sums of triangle A158815 [From Gary W. Adamson, Mar 27 2009]

This sequence appears in formulae in the link cited. [Oktay Haracci (timetunnel3(AT)hotmail.com), Apr 02 2009]

Equals INVERT transform of A081696: (1, 1, 3, 9, 29, 97, 333,...). [From Gary W. Adamson, May 15 2009]

Also, in sports, the number of ordered ways for a "Best of 2n-1 Series" to progress. For example, a(2) = 6 means there are six ordered ways for a "best of 3" series to progress. If we write A for a win by "team A" and B for a win by "team B" and if we list the played games chronologically from left to right then the six ways are AA, ABA, BAA, BB, BAB, and ABB. (Proof: To generate the a(n) ordered ways: Write down all a(n) ways to designate n of 2n games as won by team A. Remove the maximal suffix of identical letters from each of these.) [From Lee A. Newberg (integer(AT)quantconsulting.com), Jun 02 2009]

Index the central binomial coefficients with the natural numbers 1,2,3...,n. It appears that dividing the central binomial coefficients by their indexes yields the Catalan numbers (A000108). [Jason Richardson-White (coyoteworks(AT)gmail.com), Jun 15 2009]

Number of nXn binary arrays with rows, considered as binary numbers, in nondecreasing order, and columns, considered as binary numbers, in nonincreasing order. [From R. H. Hardin, Jun 27 2009]

Hankel transform is 2^n. [From Paul Barry (pbarry(AT)wit.ie), Aug 05 2009]

It appears that a(n) is also the number of quivers in the mutation class of twisted type BC_n for n>=2.

FORMULA G.f.: A(x) = (1 - 4*x)^(-1/2) = 1 + 2*x + 6*x^2 + 20*x^3 + ...

a(n) = 2^n/n! * product(k=0..n-1, (2*k+1)).

a(n) = a(n-1)*(4-2/n) = 4*a(n-1)+A002420(n) = A000142(2*n)/(A000142(n)^2) = A001813(n)/A000142(n) = sqrt(A002894(n)) = A010050(n)/A001044(n) = (n+1)*A000108(n) = -A005408(n-1)*A002420(n) - Henry Bottomley (se16(AT)btinternet.com), Nov 10 2000

Using Stirling's formula in A000142 it is easy to get the asymptotic expression a(n) ~ 4^n / sqrt(Pi * n) - Dan Fux (dan.fux(AT)OpenGaia.com or danfux(AT)OpenGaia.com), Apr 07 2001

Integral representation as n-th moment of a positive function on the interval[0, 4], in Maple notation: a(n)= int(x^n*((x*(4-x))^(-1/2))/Pi, x=0..4), n=0, 1, ... This representation is unique. - Karol A. Penson (penson(AT)lptl.jussieu.fr), Sep 17 2001

sum(n>=1, 1/a(n))=(2*Pi*sqrt(3)+9)/27 - Benoit Cloitre (benoit7848c(AT)orange.fr), May 01 2002

E.g.f.: exp(2*x)*I_0(2x), where I_0 is Bessel function. - Michael Somos, Sep 08 2002

E.g.f.: I_0(2*x)=sum a(n)*x^(2*n)/(2*n)!, where I_0 is Bessel function. - Michael Somos, Sep 09, 2002.

a(n) = sum(k=0, n, C(n, k)^2). - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 31 2003

Determinant of n X n matrix M(i, j)=binomial(n+i, j) - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 28 2003

Given m = C(2*n, n), let f be the inverse function, so that f(m) = n. Letting q denote -Log(Log(16)/(m^2*Pi)), we have f(m) = Ceiling((q + Log(q)) / Log(16)). - David W. Cantrell (DWCantrell(AT)sigmaxi.net), Oct 30 2003

a(n) = 2*Sum{k= 0...(n-1), a(k)*a(n-k+1)/(k+1)}. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jan 01 2004

a(n+1)=sum(j=n, n*2+1, binomial(j, n)). E.g. a(4)=C(7, 3)+C(6, 3)+C(5, 3)+C(4, 3)+C(3, 3)=35+20+10+4+1=70 - Jon Perry (perry(AT)globalnet.co.uk), Jan 20 2004

a(n) = (-1)^(n)*sum(j=0..(2*n), (-1)^j*binomial(2*n, j)^2) - Helena Verrill (verrill(AT)math.lsu.edu), Jul 12 2004

a(n)=sum{k=0..n, binomial(2n+1, k)*sin((2n-2k+1)*pi/2)}. - Paul Barry (pbarry(AT)wit.ie), Nov 02 2004

a(n-1)=(1/2)*(-1)^n*sum_{0<=i, j<=n}(-1)^(i+j)*binomial(2n, i+j) - Benoit Cloitre (benoit7848c(AT)orange.fr), Jun 18 2005

a(n) = C(2n, n-1) + C(n) = A001791(n) + A000108(n). a(n) = (n+1)*C(n) = (n+1)*A000108(n). - Lekraj Beedassy (blekraj(AT)yahoo.com), Aug 02 2005

G.f.: c(x)^2/(2*c(x)-c(x)^2) where c(x) is the g.f. of A000108; - Paul Barry (pbarry(AT)wit.ie), Feb 03 2006

a(n)=A006480(n)/A005809(n) - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Jun 28 2007

a(n)=Sum{k, 0<=k<=n}A106566(n,k)*2^k. - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Aug 25 2007

a(n)= Sum{k>=0, A039599(n, k)} . a(n)= Sum{k>=0, A050165(n, k)} . a(n)= Sum{k>=0, A059365(n, k)*2^k}, n>0 . a(n+1)= Sum{k>=0, A009766(n, k)*2^(n-k+1)}. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jan 01 2004

a(n)=4^n*sum{k=0..n, C(n,k)(-4)^(-k)*A000108(n+k)}; - Paul Barry (pbarry(AT)wit.ie), Oct 18 2007

Row sums of triangle A135091 - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 18 2007

a(n)=Sum_{k, 0<=k<=n}A039598(n,k)*A059841(k). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 12 2008]

A007814(a(n))=A000120(n). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jul 20 2009]

Contribution from Paul Barry (pbarry(AT)wit.ie), Aug 05 2009: (Start)

G.f.: 1/(1-2x-2x^2/(1-2x-x^2/(1-2x-x^2/(1-2x-x^2/(1-... (continued fraction);

G.f.: 1/(1-2x/(1-x/(1-x/(1-x/(1-... (continued fraction). (End)

a(n)=Product(k=1..n)[4-2/k] [From David Brown (thedabs(AT)gmail.com), Sep 19 2009]

If n>=3 is prime, then a(n)==2(mod 2*n). [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Sep 05 2010]

Let A(x) be the g.f. and B(x)=A(-x), then B(x)=sqrt(1-4*x*B(x)^2) [From Vladimir Kruchinin (kru(AT)ie.tusur.ru), Jan 16 2011]

a(n)=(-4)^n*sqrt(Pi)/(gamma((1/2-n))*gamma(1+n)) [Gerry Martens, May 3 2011]

a(n) = upper left term in M^n, M = the infinite square production matrix:

2, 2, 0, 0, 0, 0,...

1, 1, 1, 0, 0, 0,...

1, 1, 1, 1, 0, 0,...

1, 1, 1, 1, 1, 0,...

1, 1, 1, 1, 1, 1,....

- Gary W. Adamson, Jul 14 2011

A001003 Schroeder's second problem (generalized parentheses); also called super-Catalan numbers or little Schroeder numbers.

COMMENTS There are two schools of thought about the index for the first term. I prefer the indexing a(0) = a(1) = 1, a(2) = 3, a(3) = 11, etc.

a(n) = number of ways to insert parentheses in a string of n+1 symbols. The parentheses must be balanced but there is no restriction on the number of pairs of parentheses. The number of letters inside a pair of parentheses must be at least 2. Parentheses enclosing the whole string are ignored.

Also length of list produced by a variant of the Catalan producing iteration: replace each integer k by the list 0,1,..,k,k+1,k,...,1,0 and get the length a(n) of the resulting (flattened) list after n iterations. - Wouter Meeussen, Nov 11 2001

Stanley gives several other interpretations for these numbers.

Number of Schroeder paths of semilength n-1 (i.e. lattice paths from (0,0) to (2n-2,0), with steps H=(2,0), U=(1,1) and D(1,-1) and not going below the x-axis) with no peaks at level 1.

Example: a(3)=3 because among the six Schroeder paths of semilength two HH, UHD, UUDD, HUD, UDH and UDUD, only the first three have no peaks at level 1. - Emeric Deutsch, Dec 27 2003

a(n+1)=number of Dyck n-paths in which the interior vertices of the ascents are colored white or black. - David Callan, Mar 14 2004

Number of possible schedules for n time slots in the first-come first-served (FCFS) printer policy.

Also row sums of A086810, A033282 . - DELEHAM Philippe, May 09 2004

a(n+1) = number of pairs (u,v) of same-length compositions of n, 0's allowed in u but not in v and u dominates v (meaning u_1 >= v_1, u_1 + u_2 >= v_1 + v_2 and so on). For example, with n=2, a(3) counts (2,2), (1+1,1+1), (2+0,1+1). - David Callan, Jul 20 2005

The big Schroeder number (A006318) is the number of Schroeder paths from (0,0) to (n,n) (subdiagonal paths with steps (1,0) (0,1) and (1,1)). These paths fall in two classes: those with steps on the main diagonal and those without. These two classes are equinumerous and the number of paths in either class is the little Schroeder number a(n) (half the big Schroeder number). - Marcelo Aguiar (maguiar(AT)math.tamu.edu), Oct 14 2005

With offset 0, a(n) = number of (colored) Motzkin (n-1)-paths with each upstep U getting one of 2 colors and each flatstep F getting one of 3 colors. Example. With their colors immediately following upsteps/flatsteps, a(2) = 3 counts F1, F2, F3 and a(3)=11 counts U1D, U2D, F1F1, F1F2, F1F3, F2F1, F2F2, F2F3, F3F1, F3F2, F3F3. - David Callan, Aug 16 2006

Triangle A144156 has row sums = A006318 with left border A001003. [From Gary W. Adamson, Sep 12 2008]

Shifts left when INVERT transform applied twice. [From Alois P. Heinz, Apr 01 2009]

Georgiadis establishes a simple expression for the super Catalan numbers which appears to have been unnoticed before in the literature. [From Jonathan Vos Post]

FORMULA (n+1)*a(n)= (6*n-3)* a(n-1) -(n-2)*a(n-2) if n>1. a(0)= a(1)= 1.

a(n) = 3*a(n-1) + 2*A065096(n-2) (n>2). If g(x) = 1 + 3x + 11x^2 + 45x^3 + ... + a(n)*x^n + ..., then g(x) = 1 + 3(x*g(x)) + 2(x*g(x))^2, g(x)^2 = 1 + 6x + 31x^2 + 156x^3 + ... + A065096(n)*x^n + ... - Paul D. Hanna, Jun 10 2002

a(n+1) = -a(n) + 2*Sum_{k=1..n} a(k)*a(n+1-k). - DELEHAM Philippe, Jan 27 2004

G.f.: (1 + x - sqrt(1 - 6*x + x^2))/(4*x) = 2/(1 + x + sqrt(1 - 6*x + x^2)).

a(n) ~ W*(3+sqrt(8))^n*n^(-3/2) where W = (1/4)*sqrt((sqrt(18)-4)/Pi) [See Knuth I, p. 534, or Perez. Note that the formula on line 3, page 475 of Flajolet and Sedgewick seems to be wrong - it has to be multiplied by 2^(1/4).] - N. J. A. Sloane, Apr 10 2011

The Hankel transform of this sequence gives A006125 = 1, 1, 2, 8, 64, 1024, ...; example : det([1, 1, 3, 11; 1, 3, 11, 45; 3, 11, 45, 197; 11, 45, 197, 903]) = 2^6 = 64 . - DELEHAM Philippe, Mar 02 2004

a(n+1)=Sum_{k, 0, (n-1)/2} 2^k 3^(n-1-2k) binomial(n-1, 2k) CatalanNumber(k). This formula counts colored Dyck paths by the same parameter by which Touchard's identity counts ordinary Dyck paths: number of DDUs (U=up step, D=down step). See also Gouyou-Beauchamps reference. - David Callan, Mar 14 2004

a(n)=(1/(n+1))sum{k=0..n, C(n+1, k)C(2n-k, n)(-1)^k*2^(n-k)} [with offset 0]; a(n)=(1/(n+1))sum{k=0..n, C(n+1, k+1)C(n+k, k)(-1)^(n-k)*2^k} [with offset 0]; a(n)=sum{k=0..n, (1/(k+1))*C(n, k)C(n+k, k)(-1)^(n-k)*2^k} [with offset 0]; a(n)=sum{k=0..n, A088617(n, k)*(-1)^(n-k)*2^k} [with offset 0]; - Paul Barry, May 24 2005

E.g.f. of a(n+1) is exp(3*x)*BesselI(1, 2*sqrt(2)*x)/(sqrt(2)*x). - Vladeta Jovovic (vladeta(AT)eunet.rs), Mar 31 2004

Reversion of (x-2*x^2)/(1-x) is g.f. offset 1.

For n>=1, a(n)=sum(k=0, n, 2^k*N(n, k)) where N(n, k) =1/n*C(n, k)*C(n, k+1) are the Narayana numbers (A001263) - Benoit Cloitre, May 10 2003. This formula counts colored Dyck paths by number of peaks, which is easy to see because the Narayana numbers count Dyck paths by number of peaks and the number of peaks determines the number of interior ascent vertices.

a(n) = Sum_{k=0..n} A088617(n, k)*2^k*(-1)^(n-k). - DELEHAM Philippe, Jan 21 2004

For n > 0, a(n) = 1/(n+1) * sum_{k = 0 .. n-1} binomial(2*n-k, n)*binomial(n-1, k). This formula counts colored Dyck paths (as above) by number of white vertices. - David Callan, Mar 14 2004

a(n-1)=(diff(((1-x)/(1-2*x))^n, x$(n-1)))/n!|_{x=0}. (For a proof see the comment on the unsigned row sums of triangle A111785.)

a(n)= (1/n)*sum(binomial(n, k)*binomial(n+k, k-1), k=1..n) = hypergeom([1-n, n+2], [2], -1), n>=1. - Wolfdieter Lang, Sep 12 2005.

a(m+n+1) = Sum_{k>=0} A110440(m, k)*A110440(n, k)*2^k = A110440(m+n, 0) . - Philippe DELEHAM, Sep 14 2005

Sum over partitions formula (reference Schroeder paper p. 362, eq. (1) II). Number the partitions of n according to Abramowitz-Stegun p. 831-2 (see reference under A105805) with k=1..p(n)= A000041(n). For n>=1: a(n-1)=sum(A048996(n,k)*a(1)^e(k, 1)*a(1)^e(k, 2)*...*a(n-2)^e(k, n-1), k=2..p(n)) if the k-th partition of n in the mentioned order is written as (1^e(k, 1), 2^e(k, 2), ..., (n-1)e(k, n-1)). Note that the first (k=1) partition (n^1) has to be omitted. - Wolfdieter Lang, Aug 23 2005.

Starting (1, 3, 11, 45,...), = row sums of triangle A126216 = A001263 * [1, 2, 4, 8, 16,...]. - Gary W. Adamson, Nov 30 2007

Contribution from Paul Barry, May 15 2009: (Start)

G.f.: 1/(1+x-2x/(1+x-2x/(1+x-2x/(1+x-2x/(1-.... (continued fraction).

G.f.: 1/(1-x/(1-x-x/(1-x-x/(1-x-x/(1-... (continued fraction).

G.f.: 1/(1-x-2x^2/(1-3x-2x^2/(1-3x-2x^2/(1-... (continued fraction). (End)

a(n) = (LegendreP(n+1,3)-3*LegendreP(n,3))/(4*n) for n>0 [From Mark van Hoeij, Jul 12 2010]

a(n) = upper left term in M^n, where M is the production matrix:

1, 1, 0, 0, 0, 0,...

2, 2, 2, 0, 0, 0,...

1, 1, 1, 1, 0, 0,...

2, 2, 2, 2, 2, 0,...

1, 1, 1, 1, 1, 1,...

...

- Gary W. Adamson, Jul 08 2011

a(n) is the sum of top row terms of Q^(n-1), where Q is the infinite square production matrix:

1, 2, 0, 0, 0,...

1, 1, 2, 0, 0,...

1, 1, 1, 2, 0,...

1, 1, 1, 1, 2,...

... - Gary W. Adamson, Aug 23 2011

A001006 Motzkin numbers: number of ways of drawing any number of nonintersecting chords joining n (labeled) points on a circle. (Formerly M1184 N0456)

COMMENTS Number of (3412,2413)-, (3412,3142)- and (3412,3412)-avoiding involutions in S_n.

Number of sequences of length n-1 consisting of positive integers such that the opening and ending elements are 1 or 2 and the absolute difference between any 2 consecutive elements is 0 or 1. - Jon Perry (perry(AT)globalnet.co.uk), Sep 04 2003

Also number of Motzkin n-paths: paths from (0,0) to (n,0) in an n X n grid using only steps U = (1,1), F = (1,0) and D = (1,-1). - David Callan (callan(AT)stat.wisc.edu), Jul 15 2004

Number of Dyck n-paths with no UUU. (Given such a Dyck n-path, change each UUD to U, then change each remaining UD to F. This is a bijection to Motzkin n-paths. Example with n=5: U U D U D U U D D D -> U F U D D.) - David Callan (callan(AT)stat.wisc.edu), Jul 15 2004

Number of Dyck (n+1)-paths with no UDU. (Given such a Dyck (n+1)-path, mark each U that is followed by a D and each D that is not followed by a U. Then change each unmarked U whose matching D is marked to an F. Lastly, delete all the marked steps. This is a bijection to Motzkin n-paths. Example with n=6 and marked steps in small type: U U u d D U U u d d d D u d -> U U u d D F F u d d d D u d -> U U D F F D.) - David Callan (callan(AT)stat.wisc.edu), Jul 15 2004

a(n) is the number of strings of length 2n from the following recursively defined set: L contains the empty string and, for any strings a and b in L, we also find (ab) in L. The first few elements of L are e, (), (()), ((())), (()()), (((()))), ((()())), ((())()), (()(())) and so on. This proves that a(n) is less than or equal to C(n), the n-th Catalan number. - Saul Schleimer (saulsch(AT)math.rutgers.edu), Feb 23 2006

a(n) = number of Dyck n-paths all of whose valleys have even x-coordinate (when path starts at origin). For example, T(4,2)=3 counts UDUDUUDD, UDUUDDUD, UUDDUDUD. Given such a path, split it into n subpaths of length 2 and transform UU->U, DD->D, UD->F (there will be no DUs for that would entail a valley with odd x-coordinate). This is a bijection to Motzkin n-paths. - David Callan (callan(AT)stat.wisc.edu), Jun 07 2006

Also the number of standard tableaux of height less than or equal to 3. - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Mar 24 2007

a(n) is the number of RNA shapes of size 2n+2. RNA Shapes are essentially Dyck words without "directly nested" motifs of the form A[[B]]C, for A, B and C Dyck words. The first RNA Shapes are [], [][], [][][], [[][]], [][][][], [][[][]], [[][][]], [[][]][] ... - Yann Ponty (ponty(AT)lri.fr), May 30 2007

Equals right and left borders and row sums of triangle A144218 with offset variations. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 14 2008]

The sequence is self-generated from top row A going to the left starting (1,1) and bottom row = B, the same sequence but starting (0,1) and going to the right. Take dot product of A and B and add the result to n-th term of A to get the (n+1)-th term of A. Example: a(5) = 21 as follows: Take dot product of A = (9, 4, 2, 1, 1) and (0, 1, 1, 2, 4) = (0, + 4 + 2 + 2 + 4) = 12; which is added to 9 = 21. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 27 2008]

Equals A005773 / A005773 shifted: (i.e. (1,2,5,13,35,96,...) / (1,1,2,5,13,35,96,...)). [From

Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 21 2008]

Starting with offset 1 = iterates of M * [1,1,0,0,0,...], where M = a tridiagonal matrix with [0,1,1,1,...] in the main diagonal and [1,1,1,...] in the super and subdiagonals. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 07 2009]

Contribution from Emeric Deutsch (deutsch(AT)duke.poly.edu), May 29 2010: (Start)

a(n) is the number of involutions of {1,2,...,n} having genus 0. The genus g(p) of a permutation p of {1,2,...,n} is defined by g(p)=(1/2)[n+1-z(p)-z(cp')], where p' is the inverse permutation of p, c = 234...n1 = (1,2,...,n), and z(q) is the number of cycles of the permutation q. Example: a(4)=9; indeed, p=3412=(13)(24) is the only involution of {1,2,3,4} with genus >0. This follows easily from the fact that a permutation p of {1,2,...,n} has genus 0 if and only if the cycle decomposition of p gives a noncrossing partition of {1,2,...,n} and each cycle of p is increasing (see Lemma 2.1 of the Dulucq-Simion reference). [Also, redundantly, for p=3412=(13)(24) we have cp'=2341*3412=4123=(1432) and so g(p)=(1/2)(4+1-2-1)=1.]

(End)

Let w(i,j,n) denote walks in N^2 which satisfy the multivariate recurrence

w(i,j,n) = w(i, j + 1, n - 1) + w(i - 1, j, n - 1) + w(i + 1, j - 1, n - 1)

with boundary conditions w(0,0,0) = 1 and w(i,j,n) = 0 if i or j or n is < 0. Then a(n) = Sum{i = 0..n,j = 0..n} w(i,j,n) is the number of such walks of length n. - Peter Luschny, May 21 2011

a(n)/a(n-1) tends to 3.0 as Lim N->inf: (1+2*Cos 2Pi/N) relating to longest odd N regular polygon diagonals, by way of example, N=7: Using the tridiagonal generator [Cf. comment of Jan 07 2009], for polygon N=7, we extract a (N-1)/2 = 3X3 matrix, [0,1,0; 1,1,1; 0,1,1] with an e-val of 2.24697...; the longest Heptagon diagonal with edge = 1. As N tends to infinity, the diagonal lengths tend to 3.0, the convergent of the sequence. [From Gary W. Adamson, (qntmpkt(AT)yahoo.com), Jun 08 2011].

FORMULA G.f.: A(x) = (1 - x - (1-2*x-3*x^2)^(1/2))/(2*x^2). Satisfies A(x) = 1 + x*A(x) + x^2*A(x)^2.

a(n) = (-1/2) Sum (-3)^i C(1/2, i) C(1/2, j); i+j=n+2, i >= 0, j >= 0.

a(n) = (3/2)^(n+2) * Sum_{k >= 1} 3^(-k) * Catalan(k-1) * binomial(k, n+2-k) [Doslic et al.]

a(n) ~ 3^(n+1)sqrt(3)[1+1/(16n)]/[(2n+3)sqrt((n+2)Pi)]. [Barcucci, Pinzani and Sprugnoli]

lim(a(n)/a(n-1), n->infinity) = 3. [Aigner]

a(n+2) - a(n+1) = a(0)*a(n) + a(1)*a(n-1) + ... + a(n)*a(0) - Bernhart.

a(n) = (1/(n+1)) * Sum_{i} (n+1)!/(i!*(i+1)!*(n-2*i)!) - Bernhart.

a(n)=sum((-1)^(n-k)*binomial(n, k)*A000108(k+1), k=0..n). a(n)=sum(binomial(n+1, k)*binomial(n+1-k, k-1), k=0..ceil((n+1)/2))/(n+1); (n+2)a(n)=(2n+1)a(n-1)+(3n-3)a(n-2) - Len Smiley (smiley(AT)math.uaa.alaska.edu)

a(n)=sum{ k=0..n, C(n, 2k)*A000108(k) } - Paul Barry (pbarry(AT)wit.ie), Jul 18 2003

E.g.f.: exp(x)*BesselI(1, 2*x)/x. - Vladeta Jovovic (vladeta(AT)eunet.rs), Aug 20 2003

a(n) = A005043(n) + A005043(n+1).

The Hankel transform of this sequence gives A000012 = [1, 1, 1, 1, 1, 1, ...]. E;g. Det([1, 1, 2, 4; 1, 2, 4, 9; 2, 4, 9, 21; 4, 9, 21, 51]) = 1. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 23 2004

a(m+n) = Sum_{k>=0} A064189(m, k)*A064189(n, k) . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Mar 05 2004

a(n)=sum((-1)^j*binomial(n+1, j)*binomial(2n-3j, n), j=0..floor(n/3))/(n+1). - Emeric Deutsch

(deutsch(AT)duke.poly.edu), Mar 13 2004
a(n)=A086615(n)-A086615(n-1) (n>=1). - Emeric Deutsch (deutsch(AT)duke.poly.edu), Jul 12 2004
G.f.: A(x)=(1-y+y^2)/(1-y)^2 where (1+x)*(y^2-y)+x=0; A(x)=4*(1+x)/(1+x+sqrt(1-2*x-3*x^2))^2; a(n)=(3/4)*(1/2)^n*sum(k=0..2*n, 3^(n-k)*C(k)*C(k+1, n+1-k)) + 0^n/4 [after Doslic et al.] - Paul Barry (pbarry(AT)wit.ie), Feb 22 2005
G.f.: c(x^2/(1-x)^2)/(1-x), c(x) the g.f. of A000108; - Paul Barry (pbarry(AT)wit.ie), May 31 2006
Asymptotic formula : a(n) ~ sqrt(3/4/Pi)*3^(n+1)/n^(3/2) - Benoit Cloitre (benoit7848c(AT)orange.fr), Jan 25 2007
a(n) = A007971(n+2)/2. - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Feb 28 2007
a(n)=(1/(2*pi))*int(x^n*sqrt((3-x)*(1+x)),x,-1,3) is the moment representation; - Paul Barry (pbarry(AT)wit.ie), Sep 10 2007
Equals inverse binomial transform of A000108 starting (1, 2, 5, 14, 42,...). - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 10 2007
Given an integer t >= 1 and initial values u = [a_0, a_1, ..., a_{t-1}], we may define an infinite sequence Phi(u) by setting a_n = a_{n-1} + a_0*a_{n-1} + a_1*a_{n-2} + ... + a_{n-2}*a_1 for n >= t. For example Phi([1]) is the Catalan numbers A000108. The present sequence is Phi([0,1,1]), see the 6th formula. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 27 2008
G.f.: 1/(1-x-x^2/(1-x-x^2/(1-x-x^2/(1-x-x^2/(1-x-x^2/.... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Dec 06 2008]
G.f.: 1/(1-(x+x^2)/(1-x^2/(1-(x+x^2)/(1-x^2/(1-(x+x^2)/(1-x^2/(1-.... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Feb 08 2009]
a(n) = (-3)^(1/2)/(6*(n+2)) * (-1)^n*(3*hypergeom([1/2, n+1],[1],4/3) - hypergeom([1/2, n+2],[1],4/3)) [From Mark van Hoeij (hoeij(AT)math.fsu.edu), Nov 12 2009]
G.f.: 1/(1-x/(1-x/(1-x^2/(1-x/(1-x/(1-x^2/(1-x/(1-x/(1-x^2/(1-... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Mar 02 2010]
G.f.: 1/(1-x/(1-x/(1+x-x/(1-x/(1+x-x/(1-x/(1+x-x/(1-x/(1+x-x/(1-... (continued fraction). [From Paul Barry, Jan 26 2011. Adds apparently a third '1' in front - R. J. Mathar, Jan 29 2011]
Let A(x) be the g.f., then B(x)=1+x*A(x) = 1 +1*x +1*x^2 +2*x^3 +4*x^4 +9*x^5 +... = 1/(1-z/(1-z/(1-z/(...)))) where z=x/(1+x) (continued fraction); more generally B(x)=C(x/(1+x)) where C(x) is the g.f. for the Catalan numbers (A000108). [Joerg Arndt, Mar 18 2011]

EXAMPLE 1 + x + 2*x^2 + 4*x^3 + 9*x^4 + 21*x^5 + 51*x^6 + 127*x^7 + 323*x^8 + ...

MAPLE Three different Maple scripts for this sequence:
[seq(add(binomial(n+1, k)*binomial(n+1-k, k-1), k=0..ceil((n+1)/2))/(n+1), n=0..50)];
A001006 := proc(n) option remember; local k; if n <= 1 then 1 else procname(n-1) + add(procname(k)*procname(n-k-2), k=0..n-2); fi; end;
Order := 20: solve(series(x/(1+x+x^2), x)=y, x);
zl:=4*(1-z+sqrt(1-2*z-3*z^2))/(1-z+sqrt(1-2*z-3*z^2))^2/2: gser:=series(zl, z=0, 35): seq(coeff(gser, z, n), n=0..29); - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Feb 28 2007
n -> [a(0), a(1), .., a(n)]
A001006_list := proc(n) local w, m, j, i; w := proc(i, j, n) option remember;

if min(i, j, n) < 0 or max(i, j) > n then 0
elif n = 0 then if i = 0 and j = 0 then 1 else 0 fi else
w(i, j + 1, n - 1) + w(i - 1, j, n - 1) + w(i + 1, j - 1, n - 1) fi end:
[seq(add(add(w(i, j, m), i = 0..m), j = 0..m), m = 0..n)] end:
A001006(29)_list; # - Peter Luschny, May 21 2011

MATHEMATICA a[0]=1; a[n_Integer] := a[n]=a[n-1]+Sum[a[k]*a[n-2-k], {k, 0, n-2}]; Array[a[#]&, 30]

PROG (PARI) {a(n) = polcoeff((1 - x - sqrt((1 - x)^2 - 4 * x^2 + x^3 * O(x^n))) / (2 * x^2), n)} /* Michael Somos Sep 25 2003 */
(PARI) {a(n)= if(n<0, 0, n++; polcoeff(serreverse(x / (1 + x + x^2) + x * O(x^n)), n))} /* Michael Somos Sep 25 2003 */
(PARI) {a(n) = if(n<0, 0, n! * polcoeff(exp(x + x * O(x^n)) * besseli(1, 2 * x + x * O(x^n)), n))} /* Michael Somos Sep 25 2003 */
(Maxima) a[0]:1$
a[1]:1$
a[n]:=((2*n+1)*a[n-1]+(3*n-3)*a[n-2])/(n+2)$
makelist(a[n], n, 0, 12); /* Emanuele Munarini, Mar 02 2011 */

A001037 Number of degree-n irreducible polynomials over GF(2); number of n-bead necklaces with beads of 2 colors when turning over is not allowed and with primitive period n; number of binary Lyndon words of length n.
(Formerly M0116 N0046)
COMMENTS Also dimensions of free Lie algebras - see A059966, which is essentially the same sequence.
This sequence also represents the number N of cycles of length L in a digraph under x^2 seen modulo a Mersenne prime M_q=2^q-1. This number does not depend on q and L is any divisor of q-1. See Theorem 5 and Corollary 3 of the Shallit and Vasiga paper: N=sum(eulerphi(d)/order(d,2)) where d is a divisor of 2^(q-1)-1 such that order(d,2)=L. - Tony Reix (Tony.Reix(AT)laposte.net), Nov 17 2005
Except for a(0) = 1, Bau-Sen Du's [1985/2007] Table 1, p. 6, has this sequence as the 7th (rightmost) column. Other columns of the table include (but are not identified as) A006206-A006208. - Jonathan Vos Post (jvospost3(AT)gmail.com), Jun 18 2007
"Number of binary Lyndon words" means: number of binary strings inequivalent modulo rotation (cyclic permutation) of the digits and not having a period smaller than n. This provides a link to A103314, since these strings correspond to the inequivalent zero-sum subsets of U_m (m-th roots of unity) obtained by taking the union of U_n (n|m) with 0 or more U_d (n | d, d | m) multiplied by some power of exp(i 2pi/n) to make them mutually disjoint. (But not all zero-sum subsets of U_m are of that form.) - M. F. Hasler (Maximilian.Hasler(AT)gmail.com), Jan 14 2007
Contribution from Mathilde Noual (mathilde.noual(AT)ens-lyon.fr), Feb 25 2009: (Start)
Also the number of dynamical cycles of period n of a threshold Boolean automata
network which is a quasi-minimal positive circuit of size a multiple of n and which is updated in parallel. (End)
Also, the number of periodic points with (minimal) period n in the iteration of the tent map

f(x):=2min{x,1-x} on the unit interval. - Pietro Majer (majer(AT)dm.unipi.it), Sep 22 2009

FORMULA a(n) = (1/n) sum_{ d divides n } mu(n/d) 2^d.

A000031(n) = sum_{ d divides n } A001037(d); 2^n = sum_{ d divides n } d*A001037(d).

G.f.: 1 - Sum_{n>=1} moebius(n)*log(1 - 2*x^n)/n, where moebius(n)=A008683(n). [From Paul D. Hanna (pauldhanna(AT)juno.com), Oct 13 2010]

A001045 Jacobsthal sequence (or Jacobsthal numbers): a(n) = a(n-1) + 2*a(n-2), with a(0) = 0, a(1) = 1.

(Formerly M2482 N0983)

COMMENTS Number of ways to tile a 3 X (n-1) rectangle with 1 X 1 and 2 X 2 square tiles.

Also, number of ways to tile a 2 X (n-1) rectangle with 1 X 2 dominoes and 2 X 2 squares. - Toby Gottfried (toby(AT)gottfriedville.net), Nov 02, 2008.

Also a(n) counts each of the following four things: n-ary quasigroups of order 3 with automorphism group of order 3, n-ary quasigroups of order 3 with automorphism group of order 6, (n-1)-ary quasigroups of order 3 with automorphism group of order 2 and (n-2)-ary quasigroups of order 3. See the McKay-Wanless (2008) paper. - Ian Wanless (ian.wanless(AT)sci.monash.edu.au), Apr 28 2008

Also the number of ways to tie a necktie using n+2 turns. So three turns make an "oriental", four make a "four in hand" and for 5 turns there are 3 methods: "Kelvin", "Nicky" and "Pratt". The formula also arises from a special random walk on a triangular grid with side conditions (see Fink and Mao, 1999). - arne.ring(AT)epost.de, Mar 18 2001

Also the number of compositions of n+1 ending with an odd part (a(2)=3 because 3, 21, 111 are the only compositions of 3 ending with an odd part). Also the number of compositions of n+2 ending with an even part (a(2)=3 because 4, 22, 112 are the only compositions of 4 ending with an even part). - Emeric Deutsch (deutsch(AT)duke.poly.edu), May 08 2001

Arises in study of sorting by merge insertions and in analysis of a method for computing GCDs - see Knuth reference.

Roberto E. Martinez II (remartin(AT)fas.harvard.edu), Jan 07 2002: Number of perfect matchings of a 2 X n grid upon replacing unit squares with tetrahedra (C_4 to K_4):

```
o----o----o----o...
| \/ | \/ | \/ |
| /\ | /\ | /\ |
o----o----o----o...
```

Also the numerators of the reduced fractions in the alternating sum 1/2 - 1/4 + 1/8 - 1/16 + 1/32 - 1/64 + ... - Joshua Zucker (joshua.zucker(AT)stanfordalumni.org), Feb 07 2002

Also, if A(n),B(n),C(n) are the angles of the n-orthic triangle of ABC then A(1) = Pi - 2A, A(n) = s(n)*Pi + (-2)^n*A where s(n) = (-1)^(n-1) * a(n) [1-orthic triangle = the orthic triangle of ABC, n-orthic triangle = the orthic triangle of the (n-1)-orthic triangle] - Antreas P. Hatzipolakis (xpolakis(AT)otenet.gr), Jun 05 2002

Also the number of words of length n+1 in the two letters s and t that reduce to the identity 1 by using the relations sss=1, tt=1 and stst=1. The generators s and t and the three stated relations generate the group S3. - John W. Layman (layman(AT)math.vt.edu), Jun 14 2002

Sums of pair of consecutive terms give all powers of 2 in increasing order. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Aug 15 2002

Excess clockwise moves (over anti-clockwise) needed to move a tower of size n to the clockwise peg is -(-1)^n(2^n - (-1)^n)/3; a(n)=its unsigned version. - Wouter Meeussen (wouter.meeussen(AT)pandora.be), Sep 01 2002

Also the absolute value of the number represented in base -2 by the string of n 1's, the negabinary repunit. The Mersenne numbers (A000225 and its subsequences) are the binary repunits. - Rick L. Shepherd(AT)prodigy.net (rshepherd2(AT)hotmail.com), Sep 16 2002

Note that 3*a(n)+(-1)^n=2^n is significant for Pascal' triangle A007318. It arises from a Jacobsthal decomposition of Pascal's triangle illustrated by 1+7+21+35+35+21+7+1 = (7+35+1)+(1+35+7)+(21+21) = 43 + 43 + 42 = 3a(7)-1; 1+8+28+56+70+56+29+8+1 = (1+56+28)+(28+56+1)+(8+70+8) = 85 + 85 + 86 = 3a(8)+1. - Paul Barry (pbarry(AT)wit.ie), Feb 20 2003

Number of positive integers requiring exactly n signed bits in the non-adjacent form representation.

Counts walks between adjacent vertices of a triangle - Paul Barry (pbarry(AT)wit.ie), Nov 17 2003

Comment from Slavik Jablan, Dec 26, 2003: Every amphichiral rational knot written in Conway notation is a palindromic sequence of numbers, not beginning or ending with 1. For example, for 4 <= n <= 12, the amphichiral rational knots are: 2 2, 2 1 1 2, 4 4, 3 1 1 3, 2 2 2 2, 4 1 1 4, 3 1 1 1 1 3, 2 3 3 2, 2 1 2 2 1 2, 2 1 1 1 1 1 1 2, 6 6, 5 1 1 5, 4 2 2 4, 3 3 3 3, 2 4 4 2, 3 2 1 1 2 3, 3 1 2 2 1 3, 2 2 2 2 2 2, 2 2 1 1 1 1 2 2, 2 1 2 1 1 2 1 2, 2 1 1 1 1 1 1 1 1 2. The number of amphichiral knots for n=2k (k=1, 2, 3, ...) we obtain the 0, 1, 1, 3, 5, 11, 21, 43, 85, 171, 341, 683, ...

a(n+2) counts the binary sequences of total length n made up of codewords from C={0,10,11} - Paul Barry (pbarry(AT)wit.ie), Jan 23 2004

Number of permutations with no fixed points avoiding 231 and 132.

The n-th entry (n>1) of the sequence is equal to the 2,2-entry of the n-th power of the unnormalized 4 by 4 Haar matrix: [1 1 1 0 / 1 1 -1 0 / 1 1 0 1 / 1 1 0 -1]. - Simone Severini (simoseve(AT)gmail.com), Oct 27 2004

a(n) = number of Motzkin (n+1)-sequences whose flatsteps all occur at level 1 and whose height is <=2. For example, a(4)=5 counts UDUFD, UFDUD, UFFFD, UFUDD, UUDFD. - David Callan (callan(AT)stat.wisc.edu), Dec 09 2004

a(n+1) gives row sums of A059260. - Paul Barry (pbarry(AT)wit.ie), Jan 26 2005

If (m + n) is odd, then 3*(a(m) + a(n)) is always of the form a^2 + 2*b^2, where a and b both equal powers of 2; consequently every factor of (a(m) + a(n)) is always of the form a^2 + 2*b^2. - Matthew Vandermast (ghodges14(AT)comcast.net), Jul 12 2003

Number of "0,0" in f_{n+1}, where f_0 = "1" and f_{n+1} = a sequenece formed by changing all "1"s in f_n to "1,0" and all "0"s in f_n to "0,1" . - Fung Cheok Yin (cheokyin_restart(AT)yahoo.com.hk), Sep 22 2006

All prime Jacobsthal numbers A049883[n] = {3,5,11,43,683,2731,43691,...} have prime indices except a(4) = 5. All prime Jacobsthal numbers with prime indices (all but a(4) = 5) are of the form (2^p + 1)/3 - the Wagstaff primes A000979[n]. Indices of prime Jacobsthal numbers are listed in A107036[n] = {3,4,5,7,11,13,17,19,23,31,43,61,...}. For n>1 A107036[n] = A000978[n] Numbers n such that (2^n + 1)/3 is prime. - Alexander Adamchuk (alex(AT)kolmogorov.com), Oct 03 2006

Correspondence: a(n)=b(n)*2^(n-1), where b(n) is the sequence of the arithmetic means of previous two terms defined by b(n)=1/2*(b(n-1)+b(n-2)) with initial values b(0)=0, b(1)=1; The g.f. for b(n) is B(x):=x/(1-(x^1+x^2)/2), so the g.f. A(x) for a(n) suffices A(x)=B(2*x)/2. Because b(n) converges to the limit lim (1-x)*B(x)=1/3*(b(0)+2*b(1))=2/3 (for x-->1), it follows that a(n)/2^(n-1) also converges to 2/3 (see also A103770). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Feb 04 2006

Inverse: floor(log_2(a(n))=n-2 for n>=2. Also: log_2(a(n)+a(n-1))=n-1 for n>=1(see also A130249). Characterization: x is a Jacobsthal number if and only if there is a power of 4 (=c) such that x is a root of p(x)=9x(x-c)+(c-1)(2c+1) (see also the indicator sequence A105348). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), May 17 2007

This sequence counts the odd coefficients in the expansion of (1+x+x^2)^(2^n-1), n>=0. - Tewodros Amdeberhan (tewodros(AT)math.mit.edu), Oct 18 2007, Jan 08 2008

2^(n+1) = 2*A005578(n) + 2*a(n) + 2*A000975(n-1); e.g. 2^6 = 64 = 2*A005578(5) + 2*a(5) + 2*A000975(4) = (2*11 + 2*11 + 2*10). Let A005578(n), a(n), A000975(n-1) = triangle (a, b, c). Then ((S-c), (S-b), (S-a)) = (A005578(n-1), a(n-1), A000975(n-2)). Example: (a, b, c) = (11, 11, 10) = (A005578(5), a(5), A000975(4). Then ((S-c), (S-b), (S-a)) = (6, 5, 5) = (A005578(4), a(4), A000975(3)). - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 24 2007

Sequence is identical to the absolute values of its inverse binomial transform. A similar result holds for [0,A001045*2^n]. - Paul Curtz (bpcrtz(AT)free.fr), Jan 17 2008

From a(2) on (i.e., 1,3,5,11,21,...) also: least odd number such that the subsets of {a(2),...,a(n)} sum to 2^(n-1) different values, cf. A138000 and A064934. It is interesting to note the pattern of numbers occurring (or not occurring) as such a sum (A003158). - M. F. Hasler (www.univ-ag.fr/~mhasler), Apr 09 2008

a(n) = term (5,1) of n-th power of the 5x5 matrix shown in A121231 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 03 2008]

A147612(a(n)) = 1. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Nov 08 2008]

a(n+1) = Sum(A153778(i): 2^n <= i < 2^(n+1)). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jan 01 2009]

Contribution from John Fossaceca (john(AT)fossaceca.net), Jan 31 2009: (Start)

It appears that a(n) is also the number of integers between 2^n and 2^(n+1)

that are divisible by 3 with no remainder (End)

Number of pairs of consecutive odious (or evil) numbers between 2^(n+1) and 2^(n+2), inclusive. [From T. D. Noe (noe(AT)sspectra.com), Feb 05 2009]

Equals eigensequence of triangle A156319 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 07 2009]

Starting with offset 1 = row sums of triangle A156667. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 12 2009]

A three-dimensional interpretation of a(n+1) is that it gives the number of ways of filling a 2 by 2 by n hole with 1 by 2 by 2 bricks. [From Martin Griffiths (griffm(AT)essex.ac.uk), Mar 28 2009]

Starting with offset 1 = INVERTi transform of A002605: (1, 2, 6, 16, 44,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), May 12 2009]

Convolved with (1, 2, 2, 2,...) = A000225: (1, 3, 7, 15, 31,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), May 23 2009]

The product of a pair of successive terms is always a trianguler number. - Giuseppe Ottonello, Jun 14 2009

Let A be the Hessenberg matrix of order n, defined by: A[1,j]=1, A[i,i]:=-2, A[i,i-1]=-1, and A[i,j]=0 otherwise. Then, for n>=1, a(n)=(-1)^(n-1)det(A). [From Milan R. Janjic

(agnus(AT)blic.net), Jan 26 2010]
Let R denote the irreducible representation of the symmetric group S_3 of dimension 2, and let s and t denote respectively the sign and trivial irreducible representations of dimension 1. The decomposition of R^n into irreducible representations consists of a(n) copies of R and a(n-1) copies of each of s and t. [From Andrew Rupinski (rupinski(AT)math.upenn.edu), Mar 12 2010]
As a fraction: 1/88 = 0.0113636363.. or 1/9898 = 0.00010103051121.. [From M. Dols (markdols99(AT)yahoo.com), May 18 2010]
Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 28 2010: (Start)
Starting with "1" = the INVERT transform of (1, 0, 2, 0, 4, 0, 8,...); e.g.
a(7) = 43 = (1, 1, 1, 3, 5, 11, 21) dot (8, 0, 4, 0, 2, 0, 1) =
(8 + 4 + 10 + 21) = 43. (End)
Rule 28 elementary cellular automaton generates this sequence. [Paul Muljadi, Jan 27 2011]
This is a divisibility sequence. - Michael Somos Feb 06 2011
(Start) Let U be the unit-primitive matrix (see [Jeffery])
U=U_(6,2)=
(0 0 1)
(0 2 0)
(2 0 1).
Then a(n+1)=(Trace(U^n))/3, a(n+1)=((U^n)_{3,3})/3, a(n)=((U^n)_{1,3})/3 and a(n)=((U^(n+1))_{1,1})/2. - L. Edson Jeffery, April 4, 2011. (End)
FORMULA a(n) = 2^(n-1) - a(n-1). a(n) = 2*a(n-1) - (-1)^n = (2^n - (-1)^n)/3.
G.f.: x/(1-x-2*x^2).
E.g.f.: (exp(2*x)-exp(-x))/3.
a(2*n) = 2*a(2*n-1)-1 for n>=1, a(2*n+1) = 2*a(2*n)+1 for n>=0. - Lee Hae-hwang (mathmaniac(AT)empal.com), Oct 11 2002; corrected by Mario Catalani (mario.catalani(AT)unito.it), Dec 04 2002
Also a(n) is the coefficient of x^(n-1) in the bivariate Fibonacci polynomials F(n)(x, y)=x*F(n-1)(x, y)+y*F(n-2)(x, y), with y=2*x^2. - Mario Catalani (mario.catalani(AT)unito.it), Dec 04 2002
a(n) = sum(k=1..n, binomial(n, k)*(-1)^(n+k)*3^(k-1)). - Paul Barry (pbarry(AT)wit.ie), Apr 02 2003
The ratios a(n)/2^(n-1) converge to 2/3 and every fraction after 1/2 is the arithmetic mean of the two preceding fractions. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 05 2003
a(n) = U(n-1, i/(2*sqrt(2)))*(-i*sqrt(2))^(n-1) with i^2=-1 - Paul Barry (pbarry(AT)wit.ie), Nov 17 2003
a(n+1) = sum(k=0..ceil(n/2), 2^k*binomial(n-k, k)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Mar 06 2004
a(2*n) = A002450(n) = (4^n - 1)/3; a(2*n+1) = A007583(n) = (2^(2*n+1) + 1)/3. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Mar 27 2004
a(n) = round(2^n/3) = (2^n + (-1)^(n-1))/3 so lim n->inf 2^n/a(n) = 3 - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Jul 21 2004
a(0) = 0, a(n) = 2*a(n-1)-(-1)^n, n>0; a(n)=sum(k=0..n-1, (-1)^k*2^(n-k-1)) = sum(k=0..n-1, 2^k(-1)^(n-k-1)). - Paul Barry (pbarry(AT)wit.ie), Jul 30 2004
a(n+1) = sum(k=0..n, binomial(k, n-k)*2^(n-k)) - Paul Barry (pbarry(AT)wit.ie), Oct 07 2004

a(n) = sum(k=0..n-1, W(n-k, k)*(-1)^(n-k)*binomial(2*k,k)), W(n, k) as in A004070. - Paul Barry (pbarry(AT)wit.ie), Dec 17 2004

a(n) = sum(k=0..n, k*binomial(n-1, (n-k)/2)*(1+(-1)^(n+k))*floor((2*k+1)/3)); a(n+1)=sum(k=0..n, k*binomial(n-1, (n-k)/2)*(1+(-1)^(n+k))*(A042965(k)+0^k)); - Paul Barry (pbarry(AT)wit.ie), Jan 17 2005

a(n+1) = ceiling(2^n/3)+floor(2^n/3)=(ceiling(2^n/3))^2-(floor(2^n/3))^2; a(n+1) = A005578(n)+A000975(n-1)=A005578(n)^2-A000975(n-1)^2; - Paul Barry (pbarry(AT)wit.ie), Jan 17 2005

a(n+1) = sum(k=0..n, sum(j=0..n, (-1)^(n-j)*binomial(j, k))); - Paul Barry (pbarry(AT)wit.ie), Jan 26 2005

Let M=[1, 1, 0;1, 0, 1;0, 1, 1], then a(n) = (M^n)[2, 1], also matrix characteristic polynomial x^3 - 2*x^2 - x + 2 defines the three step recursion a(0)=0, a(1)=1, a(2)=1, a(n)=2a(n-1)+a(n-2)-2a(n-3) for n>2 - Lambert Klasen (lambert.klasen(AT)gmx.net), Jan 28 2005

a(n) = ceiling(2^(n+1)/3)-ceiling(2^n/3)=A005578(n+1)-A005578(n); - Paul Barry (pbarry(AT)wit.ie), Oct 08 2005

a(n) = floor(2^(n+1)/3)-floor(2^n/3)=A000975(n)-A000975(n-1); - Paul Barry (pbarry(AT)wit.ie), Oct 08 2005

a(n) = sum(k=0..floor(n, 3), binomial(n, f(n-1)+3*k)); a(n) = sum(k=0..floor(n/3), binomial(n, f(n-2)+3*k)), where f(n)=(0, 2, 1, 0, 2, 1, ...)=A080424(n). - Paul Barry (pbarry(AT)wit.ie), Feb 20 2003

a(2*n) = Product(d divides n, cyclotomic(d,4))/3; a(2*n+1) = Product(d divides 2*n+1, cyclotomic(2*d,2))/3. - Miklos Kristof (kristmikl(AT)freemail.hu), Mar 07 2007

From Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Apr 23 2007: (Start)

The a(n) are closely related to nested square roots; this is 2*sin(2^(-n)*pi/2*a(n))=sqr(2-sqr(2-sqr(2-sqr(...sqr(2)))...){n-times the '2', n>=0}.

Also 2*cos(2^(-n)*pi*a(n))=sqr(2-sqr(2-sqr(2-sqr(...sqr(2)))...){(n-1)-times the '2', n>=1} as well as

2*sin(2^(-n)*3/2*pi*a(n))=sqr(2+sqr(2+sqr(2+sqr(...sqr(2)))...){n-times the '2', n>=0} and

2*cos(2^(-n)*3*pi*a(n))=-sqr(2+sqr(2+sqr(2+sqr(...sqr(2)))...){(n-1)-times the '2', n>=1}.

a(n) = 2^(n+1)/pi*arcsin(b(n+1)/2) where b(n) is defined recursively by b(0)=2, b(n)=sqr(2-b(n-1)).

There is a similar formula regarding the arccos function, this is a(n) = 2^n/pi*arccos(b(n)/2).

With respect to the sequence c(n) defined recursively by c(0)=-2, c(n)=sqr(2+c(n-1)) the following fomulas hold true: a(n) = 2^n/3*(1-(-1)^n*(1-2/pi*arcsin(c(n+1)/2)); a(n) = 2^n/3*(1-(-1)^n*(1-1/pi*arccos(-c(n)/2)).

(End)

sum(k=0..n, A039599(n,k)*a(k)) = A049027(n), for n>=1 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 10 2007

sum(k=0..n, A039599(n,k)*a(k+1) = A067336(n). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 10 2007

Row sums of triangle A134317. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 19 2007

Let T = the 3 X 3 matrix [1,1,0; 1,0,1; 0,1,1]. Then T^n * [1,0,0,] = [A005578(n), a(n), A000975(n-1]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 24 2007

a(n) + a(n+5) = 11*2^n. - Paul Curtz (bpcrtz(AT)free.fr), Jan 17 2008 a(n) = sum(k=1..n, K(2, k)*a(n - k)), where K(n,k) = k if 0<=k<= n and K(n,k)=0 else. (When using such a K-coefficient several different arguments to K or several different definitions of K may lead to the same integer sequence. For example, the Fibonacci sequence can be generated in several ways using the K-coefficient.) - Thomas Wieder (thomas.wieder(AT)t-online.de), Jan 13 2008

a(n) + a(n+2*k+1) = a(2*k+1)*2^n. - Paul Curtz (bpcrtz(AT)free.fr), Feb 12 2008

a(n) = lower left term in the 2 X 2 matrix [0,2; 1,1]^n - Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 02 2008

a(n+1) = sum(k=0..n, A109466(n,k)*(-2)^(n-k)). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 26 2008]

For n > 0, a(n) = b(n) - b(n-1), where b(n) is defined by the sequence A000975. [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), May 12 2009]

a(n) = sqrt(8*a(n-1)*a(n-2) + 1). E.g. sqrt(3*5*8+1)=11, sqrt(5*11*8+1)=21. - Giuseppe Ottonello, Jun 14 2009

If p[i]=fibonacci(i-1) and if A is the Hessenberg matrix of order n defined by: A[i,j]=p[j-i+1], (i<=j), A[i,j]=-1, (i=j+1), and A[i,j]=0 otherwise. Then, for n>=1, a(n-1)=det(A). [From Milan R. Janjic (agnus(AT)blic.net), May 08 2010]

a(p-1) = p*A007663(n)/3 if n>1, and a(p-1) = p*A096060(n) if n>2, with p=prime(n). [From Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Jul 19 2010]

Algebraically equivalent to replacing the 5's with 9's in the explicit (Binet) formula for the n-th term in the Fibonacci sequence: The formula for the n-th term in the Fibonacci sequence is F(n)=((1+sqrt(5))^n-(1-sqrt(5))^n)/(2^n*sqrt(5)). Replacing the 5's with 9's gives ((1+sqrt(9))^n-(1-sqrt(9))^n)/(2^n*sqrt(9)) = (2^n+(-1)^(n+1))/3 = (2^n-(-1)^(n))/3 = a(n). [From Jeffrey R. Goodwin (jeff.r.goodwin(AT)gmail.com), May 27 2011]

A001065 Sum of proper divisors (or aliquot parts) of n: sum of divisors of n that are less than n. (Formerly M2226 N0884)

COMMENTS Equals row sums of triangle A141846 - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 11 2008

Equals row sums of triangle A176891 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), May 02 2010]

FORMULA G.f.: Sum_{k>0} k x^(2k)/(1-x^k) - Michael Somos Jul 05 2006

a(n) = sigma(n) - n = A000203(n) - n. - Lekraj Beedassy (blekraj(AT)yahoo.com), Jun 02 2005

Equals inverse Mobius transform of A051953 = A051731 * A051953. Example: a(6) = 6 = (1, 1, 1, 0, 0, 1) dot (0, 1, 1, 2, 1, 4) = (0 + 1 + 1 + 0 + 0 + 4), where A051953 = (0, 1, 1, 2, 1, 4, 1, 4, 3, 6, 1, 8,...) and (1, 1, 1, 0, 0, 1) = row 6 of A051731 where the 1's positions indicate the factors of 6. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 11 2008

COMMENTS e is sometimes called Euler's constant, also Napier's constant.

Also, decimal expansion of sinh(1)+cosh(1) - Mohammad K. Azarian (azarian(AT)evansville.edu), Aug 15 2006

If m and n are any integers with n > 1, then |e - m/n| > 1/(S(n)+1)!, where S(n) = A002034(n) is the smallest number such that n divides S(n)!. - Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Sep 04 2006

FORMULA e = Sum_{k >= 0} 1/k! = lim_{x -> 0} (1+x)^(1/x).

e is the unique positive root of the equation Integral_{u = 1..x} du/u = 1.

exp(1)= (16/31*(sum((1/2)^n*(1/2*n^3+1/2*n+1)/n!,n=1..infinity) +1))^2. Robert Israel confirmed that above formula is correct, saying: "In fact, sum(n^j*t^n/n!, n=0..infinity) = P_j(t)*exp(t) where P_0(t) = 1 and for j >= 1, P_j(t) = t (P_(j-1)'(t) + P_(j-1)(t)). Your sum is 1/2*P_3(1/2) + 1/2*P_1(1/2) + P_0(1/2) ." [From Alexander R. Povolotsky (pevnev(AT)juno.com), Jan 04 2009]

A001147 Double factorial numbers: (2*n-1)!! = 1*3*5*...*(2*n-1).

(Formerly M3002 N1217)

COMMENTS The solution to Schroeder's third problem.

a(n+2) is the number of full Steiner topologies on n points with n-2 Steiner points.

a(n) is also the number of perfect matchings in the complete graph K(2n) - Ola Veshta (olaveshta(AT)my-deja.com), Mar 25 2001

Number of ways to choose n disjoint pairs of items from 2*n items. - Ron Zeno (rzeno(AT)hotmail.com), Feb 06 2002

Also rational part of numerator of Gamma(n+1/2). Multiplying this sequence by sqrt(Pi) and dividing by 2^n gives the value of Gamma(n+1/2). - Yuriy Brun, Ewa Dominowska (brun(AT)mit.edu), May 12 2001

For n >= 1 a(n) is the number of permutations in the symmetric group S_(2n) whose cycle decomposition is a product of n disjoint transpositions. - Ahmed Fares (ahmedfares(AT)my-deja.com), Apr 21 2001

Number of fixed-point-free involutions in symmetric group S_{2n}.

a(n) is the number of distinct products of n+1 variables with commutative, nonassociative multiplication. - Andrew Walters (awalters3(AT)yahoo.com), Jan 17 2004. For example, a(3)=15 because the product of the four variables w, x, y and z can be constructed in exactly 15 ways, assuming commutativity but not associativity: 1. w(x(yz)) 2. w(y(xz)) 3. w(z(xy)) 4. x(w(yz)) 5. x(y(wz)) 6. x(z(wy)) 7. y(w(xz)) 8. y(x(wz)) 9. y(z(wx)) 10. z(w(xy)) 11. z(x(wy)) 12. z(y(wx)) 13. (wx)(yz) 14. (wy)(xz) 15. (wz)(xy)

a(n) = E(X^(2n)), where X is a standard normal random variable (i.e. X is normal with mean = 0, variance = 1). So for instance a(3) = E(X^6) = 15, etc. See Abramowitz and Stegun or Hoel, Port and Stone. - Jerome Coleman, Apr 06 2004

Second Eulerian transform of 1,1,1,1,1,1... The second Eulerian transform transforms a sequence s to a sequence t by the formula t(n) = Sum[E(n,k)s(k), k=0...n], where E(n,k) is a second-order Eulerian number [A008517]. - Ross La Haye (rlahaye(AT)new.rr.com), Feb 13 2005

Integral representation as n-th moment of a positive function on the positive axis, in Maple notation: a(n)=int(x^n*exp(-x/2)/sqrt(2*Pi*x), x=0..infinity), n=0,1... . - Karol A. Penson (penson(AT)lptl.jussieu.fr), Oct 10 2005.

Let PI be the set of all partitions of {1, 2, ..., 2n} into pairs without regard to order. There are (2n-1)!! such partitions. An element alpha in PI can be written as alpha = {(i_1, j_1), (i_2, j_2), ..., (i_n, j_n)} with i_k < j_k. Let pi be the corresponding permutation which maps 1 to i_1, 2 to j_1, 3 to i_2, 4 to j_2, ..., 2n to j_n. Define sgn(alpha) to be the signature of pi, which depends only on the partition alpha and not on the particular choice of pi. Let A = {a_ij} be a 2n x 2n skew-symmetric matrix. Given a partition alpha as above define A_alpha = sgn(alpha) a_(i_1,j_1)a_(i_2,j_2)...a_(i_n,j_n). We can then define the Pfaffian of A to be Pf(A) = SUM[alpha in PI]A_alpha. The Pfaffian of an n x n skew-symmetric matrix for n odd is defined to be zero. - Jonathan Vos Post, Mar 12 2006 a(n) is the number of binary total partitions (each non-singleton block must be partitioned into exactly two blocks) or, equivalently, the number of unordered full binary trees with labeled leaves (Stanley, ex 5.2.6) - Mitch Harris (Harris.Mitchell(AT)mgh.harvard.edu), Aug 01 2006

a(n) is the Pfaffian of the skew-symmetric 2n X 2n matrix whose (i,j) entry is i for i<j. - David Callan (callan(AT)stat.wisc.edu), Sep 25 2006

a(n) is the number of increasing ordered rooted trees on n+1 vertices where "increasing" means the vertices are labeled 0,1,2,...,n so that each path from the root has increasing labels. Increasing unordered rooted trees are counted by the factorial numbers A000142. - David Callan (callan(AT)stat.wisc.edu), Oct 26 2006

Number of perfect multi Skolem-type sequences of order n. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Nov 24 2006

a(n) = total weight of all Dyck n-paths (A000108) when each path is weighted with the product of the heights of the terminal points of its upsteps. For example with n=3, the 5 Dyck 3-paths UUUDDD, UUDUDD, UUDDUD, UDUUDD, UDUDUD have weights 1*2*3=6, 1*2*2=4, 1*2*1=2, 1*1*2=2, 1*1*1=1 respectively and 6+4+2+2+1=15. Counting weights by height of last upstep yields A102625. - David Callan (callan(AT)stat.wisc.edu), Dec 29 2006

a(n) is the number of increasing ternary trees on n vertices. Increasing binary trees are counted by ordinary factorials (A000142) and increasing quaternary trees by triple factorials (A007559). - David Callan (callan(AT)stat.wisc.edu), Mar 30 2007

This sequence is essentially self-reciprocal under the list partition transform and associated operations in A133314. More precisely, A001147 and -A001147 with a leading 1 attached are reciprocal. Therefore their e.g.f.'s are reciprocal. See A132382 for an extension of this result. - Tom Copeland (tcjpn(AT)msn.com), Nov 13 2007

Comments from Ross Drewe (rd(AT)labyrinth.net.au), Mar 16 2008: (Start) This is also the number of ways of arranging the elements of n distinct pairs, assuming the order of elements is significant but the pairs are not distinguishable, i.e. arrangements which are the same after permutations of the labels are equivalent.

If this sequence and A000680 are denoted by a(n) and b(n) respectively, then a(n) = b(n)/n! where n! = the number of ways of permuting the pair labels.

For example, there are 90 ways of arranging the elements of 3 pairs [1 1], [2 2], [3 3] when the pairs are distinguishable: A = { [112233], [112323], ... [332211] }.

By applying the 6 relabeling permutations to A, we can partition A into 90/6 = 15 subsets: B = { {[112233], [113322], [221133], [223311], [331122], [332211]}, {[112323], [113232], [221313], [223131], [331212], [332121]}, ...}

Each subset or equivalence class in B represents a unique pattern of pair relationships. For example, subset B1 above represents {3 disjoint pairs} and subset B2 represents {1 disjoint pair + 2 interleaved pairs}, with the order being significant (contrast A132101). (End)

A139541(n) = a(n) * a(2*n). - Reinhard Zumkeller, Apr 25 2008

a(n+1) = sum(j=0 to n) A074060(n,j) * 2^j. [From Tom Copeland (tcjpn(AT)msn.com), Sep 01 2008]

Contribution from Emeric Deutsch, Jun 05 2009: (Start)

a(n)=number of adjacent transpositions in all fixed-point-free involutions of {1,2,...,2n}. Example: a(2)=3 because in 2143=(12)(34), 3412=(13)(24), and 4321=(14)(23) we have 2 + 0 + 1 adjacent transpositions.

a(n)=Sum(k*A079267(n,k), k>=0).

(End)

Hankel transform is A137592. [From Paul Barry, Sep 18 2009]

(1, 3, 15, 105,...) = INVERT transform of A000698 starting (1, 2, 10, 74,...). [From Gary W. Adamson, Oct 21 2009]

a(n)=(-1)^(n+1)*H(2*n,0), where H(n,x) is the probabilists' Hermite polynomials. The generating function for the probabilists' Hermite polynomials is as follows: exp(x*t-t^2/2)=sum(H(i,x)*t^i/i!,i=0,1,...) [From Leonid Bedratyuk (leonid.uk(AT)gmail.com), Oct 31 2009]

Contribution from Paul Barry, Dec 04 2009: (Start)

The g.f. of a(n+1) is 1/(1-3x/(1-2x/(1-5x/(1-4x/(1-7x/(1-6x/(1-.... (continued fraction).

The Hankel transform of a(n+1) is A168467. (End)

a(n+1) is the number of ways to form n pairs from 2n people. [From Andrew (andrewkirk_17(AT)yahoo.co.uk), Sep 07 2010]

a(1)=1, a(n+1) is partial product of n-th odd number. [From Juri-Stepan Gerasimov, Oct 17 2010]

FORMULA E.g.f.: 1 / sqrt(1 - 2*x).

a(n) = A123023(2*n + 1). - Michael Somos, Jul 24 2011

a(n) = a(n-1)*(2*n-1) = (2*n)!/(n!*2^n) = A010050(n)/A000165(n).

a(n) ~ sqrt(2) * 2^n * (n/e)^n.

With interpolated zeros, the sequence has e.g.f. exp(x^2/2). - Paul Barry, Jun 27 2003

The Ramanujan polynomial psi(n+1, n) has value a(n). - R. Stephan, Apr 16 2004

a(n) = Sum_{k=0..n} (-2)^(n-k)*A048994(n, k) .- Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 29 2005

log(1+x+3*x^2+15*x^3+105*x^4+945*x^5+10395*x^6+...) = x+5/2*x^2+37/3*x^3+353/4*x^4+4081/5*x^5+55205/6*x^6+..., where [1, 5, 37, 353, 4081, 55205,...] = A004208 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 20 2006

1/3 + 2/15 + 3/105 +...= 1/2 1/1 + 1/3 + 2/15 + 6/105 + 24/945 +...= Pi/2 - Gary W. Adamson, Dec 21 2006

a(n)=(1/sqrt(2*pi))*int(x^n*exp(-x/2)/sqrt(x),x,0,infty); - Paul Barry, Jan 28 2008

a(n)=A006882(2n-1). [From R. J. Mathar, Jul 04 2009]

G.f.: 1/(1-x-2x^2/(1-5x-12x^2/(1-9x-30x^2/(1-13x-56x^2/(1- ... (continued fraction). [From Paul Barry, Sep 18 2009]

a(n)=(-1)^n*subs({ln(e)=1,x=0},coeff(simplify(series(e^(x*t-t^2/2),t,2*n+1)),t^(2*n))*(2*n)!) [From Leonid Bedratyuk (leonid.uk(AT)gmail.com), Oct 31 2009]

a(n)=2^n*gamma(n+1/2)/gamma(1/2) [From Jaume Oliver Lafont (joliverlafont(AT)gmail.com), Nov 09 2009]

G.f.: 1/(1-x/(1-2x/(1-3x/(1-4x/(1-5x/(1- ...(continued fraction) [From Aoife Hennessy (aoife.hennessy(AT)gmail.com), Dec 02 2009]

a(n)=Sum{i=1,...,n} C(n,i)*a(i-1)*a(n-i). [From Vladimir Shevelev, Sep 30 2010]

a(n) = Sum((-1)^k*binomial(2*n,n+k)*Stirling_1(n+k,k),k=0..n) [Kauers and Ko]

a(n) = A035342(n, 1), n >= 1 (first column of triangle).

a(n)=A001497(n, 0) = A001498(n, n), first column, resp. main diagonal, of Bessel triangle.

E.g.f.: A(x)=1-sqrt(1-2*x) satisfies the differential equation A'(x)-A'(x)*A(x)-1=0. [From Vladimir Kruchinin, Jan 17 2011]

Contribution from Gary W. Adamson, Jul 19 2011: (Start)
a(n) = upper left term of M^n and sum of top row terms of M^(n-1), where M = a variant of the (1,2) Pascal triangle (Cf. A029635) as the following production matrix:

1, 2, 0, 0, 0,...
1, 3, 2, 0, 0,...
1, 4, 5, 2, 0,...
1, 5, 9, 7, 2,...
... (end)

A001157 sigma_2(n): sum of squares of divisors of n.
(Formerly M3799 N1551)
COMMENTS If the canonical factorization of n into prime powers is the product of p^e(p) then sigma_k(n) = Product_p ((p^((e(p)+1)*k))-1)/(p^k-1).
sigma_2(n) is the sum of the squares of the divisors of n.
Sum_{d|n} 1/d^k is equal to sigma_k(n)/n^k. So sequences A017665-A017712 also give the numerators and denominators of sigma_k(n)/n^k for k = 1..24. The power sums sigma_k(n) are in sequences A000203 (k=1), A001157-A001160 (k=2,3,4,5), A013954-A013972 for k = 6,7,...,24. - comment from Ahmed Fares (ahmedfares(AT)my-deja.com), Apr 05 2001.
Row sums of triangles A134575 and A134559. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 02 2007
FORMULA G.f.: Sum_{k>0} k^2 x^k/(1-x^k). Dirichlet g.f.: zeta(s)*zeta(s-2). - Michael Somos, Apr 05 2003
Multiplicative with a(p^e) = (p^(2e+2)-1)/(p^2-1). - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.
G.f. for sigma_k(n): Sum_{m>0} m^k*x^m/(1-x^m). - Vladeta Jovovic (vladeta(AT)eunet.rs), Oct 18 2002
L.g.f.: -log(prod(j>=1, (1-x^j)^j)) = sum(n>=1, a(n)/n*x^n) - Joerg Arndt, Feb 4 2011
Equals A127093 * [1, 2, 3,...]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), May 10 2007
Equals A051731 * [1, 4, 9, 16, 25,...]. A051731 * [1/1, 1/2, 1/3, 1/4,...] = [1/1, 5/4, 10/9, 21/16, 26/25,...]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 02 2007
Row sums of triangle A134841 - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 12 2007
A001190 Wedderburn-Etherington numbers: binary rooted trees (every node has out-degree 0 or 2) with n endpoints (and 2n-1 nodes in all).
(Formerly M0790 N0298)
COMMENTS Also n-node binary rooted trees (every node has out-degree <= 2) where root has degree 0 or 1.
Number of interpretations of x^n (or number of ways to insert parentheses) when multiplication is commutative but not associative. E.g. a(4) = 2: x(x.x^2) and x^2.x^2. a(5) = 3: (x.x^2)x^2, x(x.x.x^2) and x(x^2.x^2).
Number of ways to place n stars in a single bound stable hierarchical multiple star system; i.e. taking only the configurations from A003214 where all stars are included in single outer parentheses. - Piet Hut, Nov 07 2003
FORMULA G.f.: A(x) = x + (1/2)*(A(x)^2 + A(x^2)).
G.f. A(x)=1-sqrt(1-2x-A(x^2)) satisfies A(x)^2-2*A(x)+2x+A(x^2)=0, A(0)=0. - Michael Somos, Sep 06 2003

a(2n-1)=a(1)a(2n-2)+a(2)a(2n-3)+...+a(n-1)a(n),

a(2n)=a(1)a(2n-1)+a(2)a(2n-2)+...+a(n-1)a(n+1)+a(n)(a(n)+1)/2.

Given g.f. A(x), then B(x)=-1+A(x) satisfies 0=f(B(x), B(x^2), B(x^4)) where f(u, v, w)= (u^2+v)^2 +2*(v^2+w) . - Michael Somos Oct 22 2006

A001221 Number of distinct primes dividing n (also called omega(n)).

(Formerly M0056 N0019)

COMMENTS Comments from Peter C. Heinig (algorithms(AT)gmx.de), Mar 08 2008: (Start) This is also the number of maximal ideals of the ring (Z/nZ,+,*). Since every finite integral domain must be a field, every prime ideal of Z/nZ is a maximal ideal and since in general each maximal ideal is prime, there are just as many prime ideals as maximal ones in Z/nZ, so the sequence gives the number of prime ideals of Z/nZ as well.

The reason why this number is given by the sequence is that the ideals of Z/nZ are precisely the subgroups of (Z/nZ,+). Hence for an ideal to be maximal it has form a maximal subgroup of (Z/nZ,+) and this is equivalent to having prime index in (Z/nZ) and this is equivalent to being generated by a single prime divisor of n.

Finally, all the groups arising in this way have different orders, hence are different, so the number of maximal ideals equals the number of distinct primes dividing n. (End)

Equals double inverse Mobius transform of A143519, where A051731 = the inverse Mobius transform. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 22 2008]

a(n) = number of prime divisors of n. a(n) = number of prime-power divisors of n. If n = Product (p_i^e_i), the prime-power divisors of n are p_1^e_1, p_2^e_2, ..., p_k^e_k, where k = number of distinct primes dividing n. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), May 04 2009]

sum_{k=0;inf} 1 / (10 ^ A000040(k) - 1) (see A073668) [From Eric Desbiaux (moongerms(AT)wanadoo.fr), Jun 24 2009]

FORMULA G.f.: sum(k>=1, x^prime(k)/(1-x^prime(k))) - Benoit Cloitre (benoit7848c(AT)orange.fr), Apr 21 2003

G.f.: sum(i=1, oo, isprime(i)/(1-x^i)) = sum(i=1, oo, isprime(i)*x^i/(1-x^i)), where isprime(n) returns 1 is n is prime, 0 otherwise. - Jon Perry (perry(AT)globalnet.co.uk), Jul 03 2004

Dirichlet generating function: zeta(s)*primezeta(s). - Franklin T. Adams-Watters, Sep 11 2005.

Additive with a(p^e) = 1.

a(1) = 0, a(p) = 1, a(pq) = 2, a(pq...z) = k, a(p^k) = 1, for p = primes (A000040), pq = product of two distinct primes (A006881), pq...z = product of k (k > 2) distinct primes p, q, ..., z (A120944), p^k = prime powers (A000961(n) for n > 1), k = natural numbers (A000027). [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), May 04 2009]

A001222 Number of prime divisors of n counted with multiplicity (also called bigomega(n) or Omega(n)).

(Formerly M0094 N0031)

COMMENTS Maximal number of terms in any factorization of n.

Number of prime powers (not including 1) that divide n.

Sum of exponents in prime-power factorization of n. [From Daniel Forgues, Mar 29 2009]

FORMULA n = Product (p_j^k_j) -> a(n) = Sum (k_j).

Dirichlet generating function: ppzeta(s)*zeta(s). Here ppzeta(s) = sum_{p prime} sum_{k=1}^{infinity} 1/(p^)k^s. Note that ppzeta(s) = sum_{p prime} 1/(p^s-1) and ppzeta(s) = sum_{k=1}^{infinity} primezeta(k*s). - Franklin T. Adams-Watters, Sep 11 2005.

Totally additive with a(p) = 1.

a(n) = if n=1 then 0 else a(n/A020639(n)) + 1. - Reinhard Zumkeller, Feb 25 2008

a(n) = sum(k=1..A001221(n), A124010(n,k)). [Reinhard Zumkeller, Aug 27 2011]

A001227 Number of odd divisors of n.

COMMENTS Also (1) number of ways to write n as difference of two triangular numbers (A000217); (2) number of ways to arrange n identical objects in a trapezoid. [Tom Verhoeff (Tom.Verhoeff(AT)acm.org)]

Also number of sums of sequences of consecutive positive integers including sequences of length 1 (e.g. 9 = 2+3+4 or 4+5 or 9 so a(9)=3). (Useful for cribbage players.) [Henry Bottomley, Apr 13 2000]

a(n) is also the number of factors in the factorization of the Chebyshev polynomial of thee first kind T_n(x). - Yuval Dekel (dekelyuval(AT)hotmail.com), Aug 28 2003

Number of even divisors of n = A000005(2*n) * (1 - n mod 2). - Reinhard Zumkeller, Dec 28 2003

Number of ways to present n as sum of consecutive integers. The trivial solution n=n is also counted. Equals 1 + A069283. - Alfred Heiligenbrunner (alfred.heiligenbrunner(AT)gmx.at), Jun 07 2004

Number of factors in the factorization of the polynomial x^n+1 over the integers. See also A000005. - T. D. Noe, Apr 16 2003

a(n)=1 for n=A000079. - Lekraj Beedassy, Apr 12 2005

For n odd, n is prime iff the n-th term of the sequence is 2. - George J. Schaeffer (gschaeff(AT)andrew.cmu.edu), Sep 10 2005

Also number of partitions of n such that if k is the largest part, then each of the parts 1,2,...,k-1 occurs exactly once. Example: a(9)=3 because we have [3,3,2,1],[2,2,2,2,1] and [1,1,1,1,1,1,1,1,1]. - Emeric Deutsch, Mar 07 2006

Also the number of factors of the n-th Lucas polynomial. - T. D. Noe, Mar 09 2006

FORMULA Dirichlet g.f.: zeta(s)^2*(1-1/2^s).

a(n) =A000005(n)/(A007814(n)+1) =A000005(n)/A001511(n).

Multiplicative with a(p^e) = 1 if p = 2; e+1 if p > 2. - David W. Wilson, Aug 01, 2001.

G.f.: Sum_{n>=1} x^n/(1-x^(2*n)). - Vladeta Jovovic (vladeta(AT)eunet.rs), Oct 16 2002

a(n)=A000005(A000265(n)). - Lekraj Beedassy, Jan 07 2005

G.f.: Sum_{k>0} x^(2k-1)/(1-x^(2k-1)) = Sum_{k>0} x^(k(k+1)/2)/(1-x^k). - Michael Somos Oct 30 2005

Moebius transform is period 2 sequence [1, 0, ...] = A000035, which means a(n) is the Dirichlet convolution of A000035 and A057427.

a(n) = A001826(n) + A001842(n). - Reinhard Zumkeller, Apr 18 2006

Sequence = M*V = A115369 * A000005, where M = an infinite lower triangular matrix and V = A000005, d(n); as a vector: [1, 2, 2, 3, 2, 4,...]. - Gary W. Adamson, Apr 15 2007

Number of occurrences of n in A049777. - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 19 2005

Equals A051731 * [1,0,1,0,1,...]; where A051731 is the inverse Mobius transform. - Gary W. Adamson, Nov 06 2007

G.f.: x/(1-x) + sum(n=1,infinity, x^(3*n)/(1-x^(2*n)), also L(x)-L(x^2) where L(x) = sum(n>=1, x^n/(1-x^n). [From Joerg Arndt, Nov 06 2010]

a(n) = A000005(n) - A183063(n).
a(n) = d(n) if n is odd, else d(n) - d(n/2). (See the Weisstein page). - Gary W. Adamson, Mar 15 2011
Dirichlet convolution of A000005 and A154955 (interpreted as a flat sequence). - R. J. Mathar, Jun 28 2011
A001285 Thue-Morse sequence: let A_k denote the first 2^k terms; then A_0 = 1 and for k >= 0, A_{k+1} = A_k B_k, where B_k is obtained from A_k by interchanging 1's and 2's.
(Formerly M0193 N0071)
COMMENTS Or, follow a(0), .., a(2^k-1) by its complement.
Equals convergent as an infinite string of A161175 row terms. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 05 2009]
Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 25 2010: (Start)
Parse A010060 into consecutive pairs: (01, 10, 10, 01, 10, 01,...); then apply
the rules: (01 -> 1; 10 ->2), obtaining (1, 2, 2, 1, 2, 1, 1,...). (End)
FORMULA a(2n)=a(n), a(2n+1)=3-a(n), a(0)=1. Also, a(k+2^m)=3-a(k) if 0<=k<2^m.
a(n) = 2-A010059(n) = 1/2*(3-(-1)^A000120(n)). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Jun 20 2003
a(n)=sum(k=0, n, binomial(n, k){mod 2}) {mod 3}=A001316(n) {mod 3} - Benoit Cloitre (benoit7848c(AT)orange.fr), May 09 2004
A001333 Numerators of continued fraction convergents to sqrt(2).
(Formerly M2665 N1064)
COMMENTS Number of n-step non-selfintersecting paths starting at (0,0) with steps of types (1,0), (-1,0) or (0,1) [Stanley].
Number of n steps one-sided prudent walks with east, west and north steps. [Shanzhen Gao, Apr 26 2011]
Number of symmetric 2n X 2 or (2n-1) X 2 crossword puzzle grids: all white squares are edge connected; at least 1 white square on every edge of grid; 180 degree rotational symmetry - Erich Friedman (erich.friedman(AT)stetson.edu)
a(n+1) is the number of ways to put molecules on a 2 X n ladder lattice so that the molecules do not touch each other.
Number of (n-1) X 2 binary arrays with a path of adjacent 1's from top row to bottom row. - R. H. Hardin (rhhardin(AT)att.net), Mar 16 2002
a(2*n+1) with b(2*n+1) := A000129(2*n+1), n>=0, give all (positive integer) solutions to Pell equation a^2 - 2*b^2 = -1.
a(2*n) with b(2*n) := A000129(2*n), n>=1, give all (positive integer) solutions to Pell equation a^2 - 2*b^2 = +1 (see Emerson reference).
Bisection: a(2*n)= T(n,3)=A001541(n), n>=0 and a(2*n+1)=S(2*n,2*sqrt(2))= A002315(n), n>=0, with T(n,x), resp. S(n,x), Chebyshev's polynomials of the first,resp. second kind. See A053120, resp. A049310.
Binomial transform of A077957. - Paul Barry (pbarry(AT)wit.ie), Feb 25 2003
For n>0, the number of (s(0), s(1), ..., s(n)) such that 0 < s(i) < 4 and |s(i) - s(i-1)| <= 1 for i = 1,2,....,n, s(0) = 2, s(n) = 2. - Herbert Kociemba (kociemba(AT)t-online.de), Jun 02 2004
For n>1, a(n) corresponds to the longer side of a near right-angled isosceles triangle, one of the equal sides being A000129(n). - Lekraj Beedassy (blekraj(AT)yahoo.com), Aug 06 2004

Exponents of terms in the series F(x,1), where F is determined by the equation F(x,y) = xy + F(x^2*y,x). - Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Dec 18 2004

Number of n-words from the alphabet A={0,1,2} which two neighbors differ by at most 1. - Fung Cheok Yin (cheokyin_restart(AT)yahoo.com.hk), Aug 30 2006

Consider the mapping f(a/b) = (a + 2b)/(a + b). Taking a = b = 1 to start with and carrying out this mapping repeatedly on each new (reduced) rational number gives the following sequence 1/1, 3/2,7/5,17/12,41/29,... converging to 2^(1/2). Sequence contains the numerators. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 22 2003 [Amended by Paul E. Black (paul.black(AT)nist.gov), Dec 18 2006]

a(n) mod 10 = A131707. See A131711. - Paul Curtz (bpcrtz(AT)free.fr), Apr 08 2008

Starting (1, 3, 7, 17,...) = row sums of triangle A140750 - Gary W. Adamson (qntmpkt(AT)yahoo.com), May 26 2008

Prime numerators with an odd index are prime RMS numbers(A140480) and also NSW primes(A088165). [From Ctibor O. Zizka (ctibor.zizka(AT)seznam.cz), Aug 13 2008]

a(2*n+1)=A002315(n); a(2*n)=A001541(n). [From Ctibor O. Zizka (ctibor.zizka(AT)seznam.cz), Aug 13 2008]

The intermediate convergents to 2^(1/2) begin with 4/3, 10/7, 24/17, 58/41; essentially, numerators=A052542 and denominators here. - Clark Kimberling (ck6(AT)evansville.edu), Aug 26 2008

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 06 2008: (Start)

Equals right border of triangle A143966. Starting (1, 3, 7,...) equals

INVERT transform of (1, 2, 2, 2,...) and row sums of triangle A143966. (End)

Inverse binomial transform of A006012 ; Hankel transform is := [1, 2, 0, 0, 0, 0, 0, 0, 0, ...] . [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Dec 04 2008]

Contribution from Charlie Marion (charliemath(AT)optonline.net), Jan 07 2009: (Start)

In general, denominators, a(k,n) and numerators, b(k,n), of continued fraction convergents to sqrt((k+1)/k) may be found as follows:

let a(k,0) = 1, a(k,1) = 2k; for n>0, a(k,2n) = 2*a(k,2n-1)+a(k,2n-2) and a(k,2n+1)=(2k)*a(k,2n)+a(k,2n-1);

let b(k,0) = 1, b(k,1) = 2k+1; for n>0, b(k,2n) = 2*b(k,2n-1)+b(k,2n-2) and b(k,2n+1)=(2k)*b(k,2n)+b(k,2n-1).

For example, the convergents to sqrt(2/1) start 1/1, 3/2, 7/5, 17/12, 41/29.

In general, if a(k,n) and b(k,n) are the denominators and numerators, respectively, of continued fraction convergents to sqrt((k+1)/k) as defined above, then

k*a(k,2n)^2 -a(k,2n-1)*a(k,2n+1) = k = k*a(k,2n-2)*a(k,2n)-a(k,2n-1)^2 and

b(k,2n-1)*b(k,2n+1) -k*b(k,2n)^2 = k+1 = b(k,2n-1)^2 -k*b(k,2n-2)*b(k,2n);

for example, if k=1 and n=3, then b(1,n)=a(n+1) and

1*a(1,6)^2-a(1,5)*a(1,7)=1*169^2-70*408=1;

1*a(1,4)*a(1,6)-a(1,5)^2=1*29*169-70^2=1;

b(1,5)*b(1,7)-1*b(1,6)^2=99*577-1*239^2=2;

b(1,5)^2-1*b(1,4)*b(1,6)=99^2-1*41*239=2.

Cf. A000129, A142238-A142239, A153313-A153318.

(End)

Equals row sums of triangle A160756 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), May

25 2009]
This sequence occurs in the lower bound of the order of the set of equivalent resistances of n equal resistors combined in series and in parallel (A048211 [From Sameen Ahmed KHAN (rohelakhan(AT)yahoo.com), Jun 28 2010]
Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 27 2010: (Start)
Let M = a triangle with the Fibonacci series in each column, but the leftmost
column is shifted upwards one row. A001333 = Lim_{n->inf} M^n, the
left-shifted vector considered as a sequence. (End)
a(n) is the number of compositions of n when there are 1 type of 1 and 2 types of other natural numbers. [From Milan R. Janjic (agnus(AT)blic.net), Aug 13 2010]
Equals the INVERTi transform of A055099. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 14 2010]
(Start) Let U be the unit-primitive matrix (see [Jeffery])
U=U_(8,2)=
(0 0 1 0)
(0 1 0 1)
(1 0 2 0)
(0 2 0 1).
Then a(n)=(1/4)*Trace(U^n). (See also A084130, A006012.) - L. Edson Jeffery, April 4, 2011. (End)
FORMULA a(n) = A055642(A125058(n)). - Reinhard Zumkeller, Feb 02 2007
a(n) = 2a(n-1) + a(n-2);
a(n) = ((1-sqrt(2))^n + (1+sqrt(2))^n)/2.
G.f.: (1-x)/(1-2*x-x^2).
A000129(2n) = 2*A000129(n)*a(n). - John McNamara, Oct 30, 2002
a(n) = (-i)^n *T(n, i), with T(n, x) Chebyshev's polynomials of the first kind A053120 and i^2 = -1.
a(n) = a(n-1)+A052542(n-1), n>1. a(n)/A052542(n) converges to sqrt(1/2). - Mario Catalani (mario.catalani(AT)unito.it), Apr 29 2003
E.g.f.: exp(x)cosh(x*sqrt(2)). - Paul Barry (pbarry(AT)wit.ie), May 08 2003
a(n) = sum{k=0..floor(n/2), C(n, 2k)2^k }. - Paul Barry (pbarry(AT)wit.ie), May 13 2003
For n > 0, a(n)^2 - (1 + (-1)^(n))/2 = sum_{k=0...n-1}((2k+1)*A001653(n-1-k)); e.g. 17^2-1=288=1*169+3*29+5*5+7*1; 7^2=49=1*29+3*5+5*1. - Charlie Marion (charliem(AT)bestweb.net), Jul 18 2003
a(n+2) = A078343(n+1) + A048654(n). - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Jan 19 2005
Conjecture: For prime p, a(p) congruent to 1 mod p (compare with similar comment for A000032) - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Oct 11 2005
a(n) = A000129(n)+A000129(n-1) = A001109(n)/A000129(n) = sqrt(A001110(n)/A000129(n)^2) = ceiling(sqrt(A001108(n))). - Henry Bottomley (se16(AT)btinternet.com), Apr 18 2000
Also the first differences of A000129 (the Pell numbers) because A052937(n) = A000129(n+1)+1. - Graeme McRae (g_m(AT)mcraefamily.com), Aug 03 2006
a(n) = Sum_{k,0<=k<=n} A122542(n,k). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 08 2006

For another recurrence see A000129.

Starting (1, 3, 7, 17, 41,...), = row sums of triangle A135837. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 01 2007

a(n) = Sum_{k, 0<=k<=n}A098158(n,k)*2^(n-k). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Dec 26 2007

a(n) = upper left and lower right terms of [1,1; 2,1]^n. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Mar 12 2008

A001349 Number of connected graphs with n nodes.

COMMENTS Inverse Euler transform of A000088.

A001358 Semiprimes (or biprimes): products of two primes.

COMMENTS Numbers of the form p*q where p and q are primes, not necessarily distinct.

These numbers are called semi-primes or 2-almost primes.

In this database the official spelling is "semiprime", not "semi-prime".

Numbers n such that OMEGA(n)=2 where OMEGA(n) is the sum of the exponents in the prime decomposition of n.

Complement of A100959; A064911(a(n)) = 1. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Nov 22 2004

Meng proved that for any sufficiently large odd integer n, the equation n = a + b + c has solutions where each of a, b, c are semiprimes (A001358). The number of such solutions, where lg x = log (base 2)(x), is (1/2)((lg n)/log n)^(1/3))(sigma(n))(n^2)(1+O(1/lg n)) where sigma(n) is a convergent series given by Meng which is > (1/2). - Jonathan Vos Post (jvospost3(AT)gmail.com), Sep 16 2005

The graph of this sequence appears to be a straight line with slope 4. However, the asymptotic formula shows that the linearity is an illusion and in fact a(n)/n ~ log n / log log n goes to infinity. See also the graph of A066265 = number of semiprimes < 10^n.

A174956(a(n)) = n. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Apr 03 2010]

For numbers between 33 and 15495, semiprimes are more plentiful than any other k-almost prime. See A125149.

FORMULA a(n) ~ n log n / log log n as n -> infinity [Landau, p. 211], [Ayoub].

Recurrence: a(1) = 4; for n > 1, a(n) = smallest composite number which is not a multiple of any of the previous terms. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com00), Nov 10 2002

A002033(a(n))=2 or 3. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Dec 01 2009]

A001405 Central binomial coefficients: C(n,floor(n/2)).

COMMENTS By symmetry, a(n)=C(n,ceiling(n/2)). - Labos E. (labos(AT)ana.sote.hu), Mar 20 2003

Sperner's theorem says that this is the maximal number of subsets of an n-set such that no one contains another.

When computed from index -1, [seq(binomial(n,floor(n/2)), n=-1..30)]; -> [1,1,1,2,3,6,10,20,35,70,126,...] and convolved with aerated Catalans [seq((n+1 mod 2)*binomial(n,n/2)/((n/2)+1), n=0..30)]; -> [1,0,1,0,2,0,5,0,14,0,42,0,132,0,...] shifts left by one: [1,1,2,3,6,10,20,35,70,126,252,...] and if again convolved with aerated Catalans, seems to give A037952 apart from the initial term. - Antti Karttunen, Jun 05 2001

Number of ordered trees with n+1 edges, having nonroot nodes of outdegree 0 or 2. - Emeric Deutsch, Aug 02 2002

Gives for n>=1 the maximum absolute column sum norm of the inverse of the Vandermonde matrix (a_ij) i=0..n-1, j=0..n-1 with a_00=1 and a_ij=i^j for (i,j)!=(0,0). - Torsten Muetze (torstenmuetze(AT)gmx.de), Feb 06 2004

Image of Catalan numbers A000108 under the Riordan array (1/(1-2x),-x/(1-2x)) or A065109. - Paul Barry, Jan 27 2005

Number of left factors of Dyck paths, consisting of n steps. Example: a(4)=6 because we have UDUD, UDUU, UUDD, UUDU, UUUD and UUUU, where U=(1,1) and D=(1,-1). - Emeric Deutsch, Apr 23 2005

Number of dispersed Dyck paths of length n; they are defined as concatenations of Dyck paths and (1,0)-steps on the x-axis; equivalently, Motzkin paths with no (1,0)-steps at positive height. Example: a(4)=6 because we have HHHH, HHUD, HUDH, UDHH, UDUD, and UUDD, where U=(1,1), H=(1,0), and D=(1,-1). [Emeric Deutsch, Jun 4 2011]

a(n) is odd iff n=2^k-1 - Jon Perry, May 05 2005

An inverse Chebyshev transform of binomial(1,n)=(1,1,0,0,0,...) where g(x)->(1/sqrt(1-4*x^2))*g(x*c(x^2)), with c(x) the g.f. of A000108. - Paul Barry, May 13 2005

In a random walk on the number line, starting at 0 and with 0 absorbing after the first step, number of ways of ending up at a positive integer after n steps. - Joshua Zucker, Jul 31 2005

Maximum number of sums of the form sum(0<i<=n, (e(i)*a(i))) that are congruent to 0 mod q, where e_i=0 or 1 and GCD(a_i,q)=1, provided that q>ceil(n/2). - Ralf Stephan, Apr 27 2003

Also the number of standard tableaux of height less than or equal to 2. - Mike Zabrocki (zabrocki(AT)mathstat.yorku.ca), Mar 24 2007

Hankel transform of this sequence forms A000012 = [1,1,1,1,1,1,1,...] . - Philippe DELEHAM, Oct 24 2007

A001263 * [1, -2, 3, -4, 5,...] = (1, -1, -2, 3, 6, -10, -20, 35, 70, -126,...] - Gary W. Adamson, Jan 02 2008

Equals right border of triangle A153585 [From Gary W. Adamson, Dec 28 2008]

Second binomial transform of A168491. [From Philippe DELEHAM, Nov 27 2009]

a(n) is also the number of distinct strings of length n, each of which is a prefix of a string of balanced parentheses; see example. [From Lee A. Newberg (integer(AT)quantconsulting.com), Apr 26 2010]

Number of symmetric balanced strings of n pairs of parentheses; see example. [Joerg Arndt, Jul 25 2011]

a(n) is the number of permutation patterns modulo 2. [Olivier Gerard, Feb 25 2011]

sum(n>=0, a(n)/10^n) = 0.1123724... = (sqrt(3)-sqrt(2))/(2*sqrt(2)); sum(n>=0 a(n)/100^n = 0.0101020306102035... = (sqrt(51)-sqrt(49))/(2*sqrt(49)) [From M. Dols (markdols99(AT)yahoo.com), Jul 15 2010]

For n>=2, a(n-1) is the number of incongruent two-color bracelets of 2*n-1 beads, n from them are black (A007123), having a diameter of symmetry [From Vladimir Shevelev, May 03 2011]

The number of permutations of n elements where p(k-2) < p(k) for all k. [Joerg Arndt, Jul 23 2011]

FORMULA a(n) = Max C(n, k), 1 <= k <= n.

Recurrence relation: a(0) = 1, a(1) = 1, and for n>=2, (n+1)*a(n) = 2*a(n-1) + 4*(n-1)*a(n-2) [Peter Bala, 28 Feb 2011].

G.f.: (1+x*c(x^2))/sqrt(1-4*x^2) = 1/(1-x-x^2*c(x^2)); where c(x) = g.f. for Catalan numbers A000108.

G.f.: (-1+2*x+sqrt(1-4*x^2))/(2*x-4*x^2). [From Lee A. Newberg (integer(AT)quantconsulting.com), Apr 26 2010]

G.f.: 1/(1-x-x^2/(1-x^2/(1-x^2/(1-x^2/(1-... (continued fraction). [From Paul Barry, Aug 12 2009]

a(0) = 1; a(2*m+2) = 2*a(2*m+1); a(2*m+1) = sum((-1)^k*a(k)*a(2m-k), k = 0..2*m). - Len Smiley (smiley(AT)math.uaa.alaska.edu), Dec 09 2001

G.f.: (sqrt((1+2*x)/(1-2*x))-1)/(2*x). - Vladeta Jovovic (vladeta(AT)eunet.rs), Apr 28 2003

The o.g.f. A(x) satisfies A(x)+x*A^2(x) = 1/(1-2*x) [Peter Bala, 28 Feb 2011].

E.g.f.: BesselI(0, 2*x)+BesselI(1, 2*x). - Vladeta Jovovic (vladeta(AT)eunet.rs), Apr 28 2003

a(0) = 1; a(2m+2) = 2a(2m+1); a(2m+1) = 2a(2m) - c(m), where c(m)=A000108(m) are the Catalan numbers. - Christopher Hanusa (chanusa(AT)washington.edu), Nov 25 2003

a(n)=sum{k=0..n, (-1)^k*2^(n-k)*binomial(n, k)*A000108(k)} - Paul Barry, Jan 27 2005

a(n)=sum{k=0..floor(n/2), binomial(n, k)*binomial(1, n-2k)}. - Paul Barry (pbarry(AT)wit.ie), May 13 2005

a(n)=sum{k=0..floor((n+1)/2), binomial(n+1, k)(cos((n-2*k+1)*pi/2)+sin((n-2*k+1)*pi/2))}; a(n)=sum{k=0..n+1, binomial(n+1, (n-k+1)/2)*(1-(-1)^(n-k))*(cos(k*pi/2)+sin(k*pi))/2}. - Paul Barry, Nov 02 2004

a(n)=sum{k=floor(n/2)..n, C(n,n-k)-C(n,n-k-1)}. - Paul Barry, Sep 06 2007

Inverse binomial transform of A005773 starting (1, 2, 5, 13, 35, 96,...) and double inverse binomial transform of A001700. Row sums of triangle A132815. - Gary W. Adamson, Aug 31 2007

a(n)=Sum_{0<=k<=n} A120730(n,k). [From Philippe DELEHAM, Oct 16 2008]

a(n)=sum{k=0..floor(n/2), C(n,n-k)-C(n,n-k-1)}. - Nishant Doshi (doshinikki2004(AT)gmail.com), Apr 06 2009

Conjectured: a(n) = 2^n*2F1(1/2,-n;2;2), useful for number of paths in 1-d for which the coordinate is never negative. - Benjamin Phillabaum, Feb 20 2011

a(2*m+1) = (2*m+1)*a(2*m)/(m+1), e.g. a(7)=(7/4)*a(6) = (7/4)*20 = 35. - Jon Perry, Jan 20 2011

Contribution from Peter Bala, 28 Feb 2011: (Start)

Let F(x) be the logarithmic derivative of the o.g.f. A(x). Then 1+x*F(x) is the o.g.f. for A027306.

Let G(x) be the logarithmic derivative of 1+x*A(x). Then x*G(x) is the o.g.f. for A058622. (End)

Let M = an infinite tridiagonal matrix with 1's in the super and subdiagonals and [1,0,0,0,...] in the main diagonal; and V = the vector [1,0,0,0,...]. a(n) = M^n*V, leftmost term. [From Gary W. Adamson, Jun 013 2011]

A001462 Golomb's sequence: a(n) is the number of times n occurs, starting with a(1) = 1.

COMMENTS It is understood that a(n) is taken to be the smallest number >= a(n-1) which is compatible with the description.

Also called Silverman's sequence.

Vardi gives several identities satisfied by A001463 and this sequence.

We can interpret A001462 as a triangle: start with 1; 2,2; 3,3; and proceed by letting the row sum of row m-1 be the number of elements of row m. The partial sums of the row sums give 1, 5, 11, 38, 272, ... Conjecture: this proceeds as Lionel Levile's sequence A014644. See also A113676. - Floor van Lamoen, fvlamoen(AT)hotmail.com, Nov 06 2005.

The g.f. -z*(-1+z**4+z**7-z**8+z**9-z**3-z-z**11+z**12)/(1+z)/(z**2+1)/(z-1)**2 conjectured by S. Plouffe in his 1992 dissertationd is wrong. - N. J. A. Sloane (njas(AT)research.att.com), May 13 2008

FORMULA a(n) = phi^(2-phi)*n^(phi-1) + E(n), where phi is the golden number (1+sqrt(5))/2 (Marcus and Fine) and E(n) is an error term which Vardi shows is O(n^(phi-1) / log n).

a(1) = 1; a(n+1) = 1 + a(n+1-a(a(n))). - C. L. Mallows.

a(1)=1, a(2)=2 and for a(1)+a(2)+..+a(n-1) < k <= a(1)+a(2)+...+a(n) we have a(k)=n - Benoit Cloitre (benoit7848c(AT)orange.fr), Oct 07 2003

G.f.: Sum_{k>0} x^a(k). - Michael Somos Oct 21 2006

A001477 The nonnegative integers.

COMMENTS Although this is a list, and lists normally have offset 1, it seems better to make an exception in this case. - N. J. A. Sloane, Mar 13 2010

The subsequence 0,1,2,3,4 gives the known values of n such that 2^(2^n)+1 is a prime (see A019434, the Fermat primes). - N. J. A. Sloane, Jun 16 2010.

Contribution from Eric Desbiaux (moongerms(AT)wanadoo.fr), Nov 15 2009: It appears that, with the Bachet-Bezout theorem, A001477 = (2*A080425) + (3*A008611) and A000040 = (2*A039701)+(3*A157966).

a(n) = A007966(n)*A007967(n). [Reinhard Zumkeller, Jun 18 2011]

FORMULA a(n)=n; a(0) = 0, a(n) = a(n-1)+1; G.f.: x/(1-x)^2.

Multiplicative with a(p^e) = p^e. - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.

When seen as array: T(k, n) = n + (k+n)*(k+n+1)/2. Main diagonal is 2n(n+1) (A046092), antidiagonal sums are n(n+1)(n+2)/2 (A027480). - Ralf Stephan, Oct 17 2004

Dirichlet generating function: zeta(s-1). - Franklin T. Adams-Watters, Sep 11 2005.

E.g.f. x*e^x. - Franklin T. Adams-Watters, Sep 11 2005.

a(0)=0, a(1)=1, a(n)=2*a(n-1)-a(n-2). - Jaume Oliver i Lafont (joliverlafont(AT)gmail.com), May 07 2008

Contribution from Eric Desbiaux (moongerms(AT)wanadoo.fr), Oct 28 2008: Alternating partial sums give A001057 = A000217 - 2*(A008794)

A001477=(2*A028242)+(3*A059841) and A000040=(2*A067076)+3 [From Eric Desbiaux (moongerms(AT)wanadoo.fr), Dec 10 2009]

A001478 The negative integers.

FORMULA a(n)=-n; G.f.: -x/(1-x)^2.

G.f. A(x) satisfies A(x)+A(-x)=4A(x^2), A(x)A(-x)=A(x^2), A(x)^2+A(x^2)=4A(x)A(x^2). - Michael Somos Mar 23 2004

A001481 Numbers that are the sum of 2 nonnegative squares.

COMMENTS Numbers n such that n = x^2 + y^2 has a solution in nonnegative integers x, y.

Also, numbers whose cubes are the sum of 2 squares. - Artur Jasinski (grafix(AT)csl.pl), Nov

21 2006 (Cf. A125110.)

Terms are the squares of smallest radii of circles covering (on a square grid) a number of points equal to the terms of A057961. - Philippe Lallouet (philip.lallouet(AT)wanadoo.fr), Apr 16 2007. [Comment corrected by T. D. Noe (noe(AT)sspectra.com), Mar 28 2008]

Contribution from Ant King (mathstutoring(AT)ntlworld.com), Oct 05 2010: (Start)

Numbers with more 4k+1 divisors than 4k+3 divisors

If a(n) is a member of this sequence, then so too is any power of a(n) (End)

A000161(a(n)) > 0. [Reinhard Zumkeller, Aug 16 2011]

FORMULA n = square * 2^{0 or 1} * {product of distinct primes == 1 (mod 4)}.

The number of integers <N that are sums of two squares is asymptotic to constant*N/sqrt(log(N)).

Closed under multiplication. - David W. Wilson, Dec 20 2004

lim n->inf a(n)/n = inf.

Nonzero terms in expansion of Dirichlet series Product_p (1-(Kronecker(m, p)+1)*p^(-s)+Kronecker(m, p)*p^(-2s))^(-1) for m = -1.

A001489 The nonpositive integers.

FORMULA a(n)=-n; G.f.: -1/(1-x)^2.

A001511 The ruler function: 2^a(n) divides 2n. Or, a(n) = 2-adic valuation of 2n.

COMMENTS a(n) is the number of digits that must be counted from right to left to reach the first 1 in the binary representation of n. For example, a(12)=3 digits must be counted from right to left to reach the first 1 in 1100, the binary representation of 12. - anon, May 17 2002

If you are counting in binary and the least significant bit is numbered 1, the next bit is 2, etc., a(n) is the bit that is incremented when increasing from n-1 to n. - Jud McCranie, Apr 26, 2004

Number of steps to reach an integer starting with (n+1)/2 and using the map x -> x*ceiling(x) (cf. A073524).

a(n) = number of disk to be moved at n-th step of optimal solution to Tower of Hanoi problem (comment from Andreas M. Hinz (hinz(AT)appl-math.tu-muenchen.de)).

Shows which bit to flip when creating the binary reflected Gray code (bits are numbered from the right, offset is 1). This is essentially equivalent to Hinz's comment. - Adam Kertesz (adamkertesz(AT)worldnet.att.net), Jul 28 2001

a(n) is the Hamming distance between n and n-1 (in binary). This is equivalent to Kertesz's comments above. - Tak-Shing Chan (chan12(AT)alumni.usc.edu), Feb 25 2003

Let S(0) = {1}, S(n) = {S(n-1), S(n-1)-{x}, x+1} where x = last term of S(n-1); sequence gives S(infinity). - Benoit Cloitre (benoit7848c(AT)orange.fr), Jun 14 2003

The sum of all terms up to and including the first occurrence of m is 2^m-1. - Donald Sampson (marsquo(AT)hotmail.com), Dec 01 2003

m appears every 2^m terms starting with the 2^(m-1)th term. - Donald Sampson (marsquo(AT)hotmail.com), Dec 08 2003

Sequence read mod 4 gives A092412. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Mar 28 2004

If q = 2n/2^A001511(n) and if b(m) is defined by b(0)=q-1 and b(m)=2*b(m-1)+1, then 2n = b(A001511(n)) + 1. - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Dec 18 2004

Repeating pattern ABACABADABACABAE ... - Jeremy Gardiner (jeremy.gardiner(AT)btinternet.com), Jan 16 2005

Relation to C(n) = Collatz function iteration using only odd steps: a(n) is the number of right bits set in binary representation of A004767(n) (numbers of the form 4*m+3). So for m=A004767(n) it follows that there are exactly a(n) recursive steps where m<C(m). - Lambert Klasen (lambert.klasen(AT)gmx.de), Jan 23 2005

The ordinal transform of a sequence b_0, b_1, b_2, ... is the sequence a_0, a_1, a_2, ... where a_n is the number of times b_n has occurred in {b_0 ... b_n}.

Between every two instances of any positive integer m there are exactly m distinct values (1 through m-1 and one value greater than m). - Franklin T. Adams-Watters (FrankTAW(AT)Netscape.net), Sep 18 2006

Number of divisors of n of the form 2^k. - Giovanni Teofilatto (g.teofilatto(AT)tiscalinet.it), Jul 25 2007

Every subsequence through n = (2k - 1) is a palindrome. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 24 2008]

A001511(n) = A007814(n) + 1.

2*n = A001511 * A000265 [From Eric Desbiaux (moongerms(AT)wanadoo.fr), May 14 2009]

Multiplicative with a(2^k) = k + 1, a(p^k) = 1 for any odd prime p. [From Franklin T. Adams-Watters (FrankTAW(AT)Netscape.net), Jun 09 2009]

1 interleaved with (2 interleaved with (3 interleaved with (...))) [From Eric D. Burgess (ericdb(AT)gmail.com), Oct 17 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 26 2009: (Start)

A054525 (Mobius transform) * A001511 = A036987, the Fredholm-Rueppel

sequence, (1, 1, 0, 1, 0, 0, 0, 1,...) = A047999^(-1) * A001511 = the

inverse of Sierpinski's gaskit * the ruler sequence. (End)

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 26 2009: (Start)

Equals A051731 * A036987, (inverse Mobius transform of the Fredholm-Rueppel

sequence; = A047999 * A036987, (Sierpinski's gasket * A036987). (End)

Cf. A173238, showing links between generalized ruler functions and A000041 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 14 2010]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Feb 11 2010: (Start)

Given A000041, P(x) = A(x)/A(x^2) with P(x) = (1+x+2x^2+3x^3+5x^4+7x^5 + ...)

A(x) = (1+x+3x^2+4x^3+10x^4+13x^5 + ...),

A(x^2) = (1+x^2+3x^4+4x^6+10x^8+ ...), where A092119 = (1, 1, 3, 4, 10,...) =

Euler transform of the ruler sequence, A001511. (End)

FORMULA a(2n+1) = 1; a(2n) = 1 + a(n). - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Dec 08 2003

a(n) = 2-A000120(n)+A000120(n-1), n >= 1 - from Daniele Parisse (daniele.parisse(AT)m.dasa.de).

a(n) = 1 + lg(abs(A003188(n)-A003188(n-1))), where lg is the base-2 logarithm.

Multiplicative with a(p^e) = e+1 if p = 2; 1 if p > 2. - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01 2001.

For any real x > 1/2: lim N ->inf (1/N)*sum(n=1, N, x^(-a(n)))= 1/(2x-1); also lim N ->inf (1/N)*Sum(n=1, N, 1/a(n))=ln(2). - Benoit Cloitre (benoit7848c(AT)orange.fr), Nov 16 2001

s(n) = sum(k=1, n, a(k)) is asymptotic to 2*n since s(n)=2n-A000120(n). - Benoit Cloitre (benoit7848c(AT)orange.fr), Aug 31 2002

For any n>=0, for any m>=1, a(2^m*n+2^(m-1)) = m. - Benoit Cloitre, Nov 24 2002

a(n) = sum_{d divides n and d is odd} mu(d)*tau(n/d). - Vladeta Jovovic (vladeta(AT)eunet.rs), Dec 04 2002

G.f.: A(x) = sum_{k=0..infinity} x^(2^k)/(1-x^(2^k)). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Dec 24 2002

a(1) = 1; for n > 1, a(n) = a(n-1)+(-1)^n*a(floor(n/2)). - Vladeta Jovovic (vladeta(AT)eunet.rs), Apr 25 2003

Sum(k = 1 through n) a(k) = 2n - number of 1's in binary representation of n. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jun 15 2003

A fixed point of the mapping 1->12; 2->13; 3->14; 4->15; 5->16; .... - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Dec 13 2003

Product_{k>0}(1+x^k)^a(k) is g.f. for A000041(). - Vladeta Jovovic (vladeta(AT)eunet.rs), Mar 26 2004

G.f. A(x) satisfies A(x) = A(x^2) + x/(1-x). - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Feb 09 2006

a(A118413(n,k))=A002260(n,k); = a(A118416(n,k))=A002024(n,k); a(A014480(n))=A003602(A014480(n)). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Apr 27 2006

Ordinal transform of A003602. - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Aug 28 2006

Could be extended to n <= 0 using a(-n)=a(n), a(0)=0, a(2n)=a(n)+1 unless n=0. - Michael Somos Sep 30 2006

Sequence = A129360 * A000005 = M*V, where M = an infinite lower triangular matrix and V = d(n) as a vector: [1, 2, 2, 3, 2, 4,...]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Apr 15 2007

Row sums of triangle A130093. - Gary W. Adamson (qntmpkt(AT)yahoo.com), May 13 2007

Dirichlet g.f.: zeta(s)*2^s/(2^s-1). - Ralf Stephan, Jun 17 2007

a(n)= -sum_{d divides n} mu(2d)*tau(n/d). - Benoit Cloitre (benoit7848c(AT)orange.fr), Jun 21 2007

G.f.: x/(1-x) = Sum_{n>=1} a(n)*x^n*(1 - x^n) . - Paul D. Hanna (pauldhanna(AT)juno.com), Jun 22 2007

With S(n): 2^n - 1 first elements of the sequence then S(0) = {} (empty list) and if n > 0, S(n) = S(n-1), n, S(n-1) [From Yann David (yann_david(AT)hotmail.com), Mar 21 2010]

A001615 Dedekind psi function: n * Product_{p|n, p prime} (1 + 1/p).

COMMENTS Number of primitive sublattices of index n in generic 2-dimensional lattice; also index of GAMMA_0(n) in SL_2(Z).

A generic 2-dimensional lattice L = <V,W> consists of all vectors of the form mV + nW, (m,n integers). A sublattice S = <aV+bW, cV+dW> has index |ad-bc| and is primitive if gcd(a,b,c,d) = 1. L has precisely a(2) = 3 sublattices of index 2, namely <2V,W>, <V,2W> and <V+W,2V> (which = <V+W,2W>) and so on for other indices.

The sublattices of index n are in one-one correspondence with matrices [a b; 0 d] with a>0, ad=n, b in [0..d-1]. The number of these is Sum_{d|n} = sigma(n), which is A000203. A sublattice is primitive if gcd(a,b,d) = 1; the number of these is n * product_{p|n} (1+1/p), which is A001615.

SL_2(Z) = Gamma is the group of all 2 X 2 matrices [a b; c d] where a,b,c,d are integers with ad-bc = 1 and Gamma_0(N) is usually defined as the subgroup of this for which N|c. But conceptually Gamma is best thought of as the group of (positive) automorphisms of a lattice <V,W>, its typical element taking V -> aV + bW, W -> cV + dW and then Gamma_0(N) can be defined as the subgroup consisting of the automorphisms that fix the sublattice <NV,W> of index N. - J. H. Conway, May 05, 2001

Dedekind proved that if n = k_i*j_i for i in I represent all ways to write n as a product and e_i=gcd(k_i,j_i), then a(n)= sum(k_i / (e_i * phi(e_i)), i in I) [cf. Dickson, History of the Theory of Numbers, Vol. 1, p. 123].

Also a(n)= number of cyclic subgroups of order n in an Abelian group of order n^2 and type (1,1) (Fricke) - Len Smiley (smiley(AT)math.uaa.alaska.edu), Dec 04 2001

The polynomial degree of the classical modular equation of degree n relating j(z) and j(nz) is denoted by psi(n) by Fricke. - Michael Somos Nov 10 2006

Mobius transform of A001615 = A063659. - Gary W. Adamson (qntmpkt(AT)yahoo.com), May 23 2008

Riemann Hypothesis is true iff a(n)/n-e^gamma*log(log(n))<0 for any n>30 - Enrique Pérez Herrero, Jul 12 2011

FORMULA Dirichlet g.f.: zeta(s)*zeta(s-1)/zeta(2*s) - Michael Somos, May 19, 2000

Multiplicative with a(p^e) = (p+1)*p^(e-1). - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.

a(n) = A003557(n)*A048250(n) = A000203(A007947(n))/A007947(n). - Labos E. (labos(AT)ana.sote.hu), Dec 04 2001

a(n) = n*sum(d|n, mu(d)^2/d), Dirichlet convolution of A008966 and A000027. - Benoit Cloitre (benoit7848c(AT)orange.fr), Apr 07 2002

a(n) = sum(d|n, mu(n/d)^2 * d). [Joerg Arndt, Jul 06 2011]

Contribution from Enrique Perez Herrero (psychgeometry(AT)gmail.com), Aug 22 2010: (Start)

a(n) = J_2(n)/J_1(n) = J_2(n)/phi(n) = A007434(n)/A000010(n), where J_k is the k-th Jordan Totient Function

a(n) = sum(d|n,mu(n/d)*d^(b-1)/phi(n)), for b=3 (End)

A001699 Number of binary trees of height n; or products (ways to insert parentheses) of height n when multiplication is non-commutative and non-associative.

COMMENTS Approaches 1.5028368...^(2^n). Row sums of A065329 as square array. - Henry Bottomley (se16(AT)btinternet.com), Oct 29 2001. Also row sum of square array A073345 (AK).

FORMULA a(n+1) = 2*a(n)*(a(0)+...+a(n-1))+a(n)^2.

a(n+1) = a(n)^2+a(n)+a(n)*sqrt(4*a(n)-3), if n>0.

a(n+1) = A003095(n+1)-A003095(n) = A003095(n)^2- A003095(n)+1. - Henry Bottomley (se16(AT)btinternet.com), Apr 26 2001

a(n)=A059826(A003095(n-1))

A001700 C(2n+1, n+1): number of ways to put n+1 indistinguishable balls into n+1 distinguishable boxes = number of (n+1)-st degree monomials in n+1 variables = number of monotone maps from 1..n+1 to 1..n+1.

COMMENTS To show for example that C(2n+1, n+1) is the number of monotone maps from

1..n+1 to 1..n+1, notice that we can describe such a map by a nondecreasing sequence of length n+1 with entries from 1 to n+1. The number k of increases in this sequence is anywhere from 0 to n. We can specify these increases by throwing k balls into n+1 boxes, so the total is Sum_{k=0..n} C((n+1)+k-1, k) = C(2n+1, n+1).

Also number of ordered partitions (or compositions) of n+1 into n+1 parts. E.g. a(2)=10: 003 030 300 012 021 102 120 210 201 111 - Mambetov Bektur (bektur1987(AT)mail.ru), Apr 17 2003

Also number of walks of length n on square lattice, starting at origin, staying in first and second quadrants - David W. Wilson, May 05, 2001. E.g. for n = 2 there are 10 walks, all starting at 0,0: 0,1->0,0; 0,1->1,1; 0,1->0,2; 1,0->0,0; 1,0->1,1; 1,0->2,0; 1,0->1,-1; -1,0->0,0; -1,0->-1,1; -1,0->-2,0.

Also total number of leaves in all ordered trees with n+1 edges.

Also number of digitally balanced numbers [A031443] from 2^(2n+1) to 2^(2n+2). - Naohiro Nomoto, Apr 07 2001

Also number of ordered trees with 2n+2 edges having root of even degree and nonroot nodes of outdegree 0 or 2. - Emeric Deutsch, Aug 02 2002

Also number of paths of length 2*d(G) connecting two neighboring nodes in optimal chordal graph of degree 4, G(2*d(G)^2+2*d(G)+1, 2d(G)+1), where d(G) = diameter of graph G. - S. Bujnowski (slawb(AT)atr.bydgoszcz.pl), Feb 11 2002

Define an array by m(1,j)=1, m(i,1)=i, m(i,j)=m(i,j-1)+m(i-1,j); then a(n)=m(n,n) - Benoit Cloitre, May 07 2002

Also the numerator of the constant term in the expansion of Cos^2n(x) or Sin^2n(x) when the denominator is 2^(2n-1). - rgwv

Consider the expansion of cos^n(x) as a linear combination of cosines of multiple angles. If n is odd, then the expansion is a combination of a*cos((2k-1)*x)/2^(n-1) for all 2k-1<=n. If n is even, then the expansion is a combination of a*cos(2k*x)/2^(n-1) terms plus a constant. "The constant term, [a(n)/2^(2n-1)], is due to the fact that [cos^2n(x)] is never negative, i.e. electrical engineers would say the average or 'dc value' of [cos^2n(x)] is [a(n)/2^(2n-1)]. The dc value of [cos^(2n-1)(x)] on the other hand, is zero because it is symmetrical about the horizontal axis, i.e. it is negative and positive equally." Nahin[62] - rgwv Aug 01 2002

Also number of times a fixed Dyck word of length 2k occurs in all Dyck words of length 2n+2k. Example: if the fixed Dyck word is xyxy (k=2), then it occurs a(1)=3 times in the 5 Dyck words of length 6 (n=1): (xy[xy)xy], xyxxyy, xxyyxy, x(xyxy)y, xxxyyy (placed between parentheses). - Emeric Deutsch, Jan 02 2003

G.f. is C(x)/sqrt(1-4x) where C(x) is g.f. for Catalan numbers A000108.

a(n+1) is the determinant of the n X n matrix m(i,j)=binomial(2n-i,j) - Benoit Cloitre, Aug 26 2003

a(n-1) = (2n)!/(2*n!*n!), formula in [Davenport] used by Gauss for the special case prime p = 4*n+1: x = a(n-1) mod p and y = x*(2n)! mod p are solutions of p = x^2 + y^2. - Frank Ellermann. Example: For prime 29 = 4*7+1 use a(7-1) = 1716 = (2*7)!/(2*7!*7!), 5 = 1716 mod 29 and 2 = 5*(2*7)! mod 29, then 29 = 5*5 + 2*2.

a(n)=sum{k=0..n+1, binomial(2n+2,k)*cos((n-k+1)*pi)} - Paul Barry, Nov 02 2004

The number of compositions of 2n, say c_1+c_2+...c_k=2n, satisfy that Sum_(i=1..j)c_i <2j for all j=1..k, or equivalently, the number of subsets, say S, of [2n-1]={1,2,...2n-1} with at least n elements such that if 2k is in S, then there must be at least k elements in S smaller than 2k. E.g.

a(2)=3 because we can write 4=1+1+1+1=1+1+2=1+2+1 - Ricky X. F. Chen (ricky_chen(AT)mail.nankai.edu.cn), Jul 30 2006

a(n) = A122366(n,n). - Reinhard Zumkeller, Aug 30 2006

The number of walks of length 2n+1 on an infinite linear lattice that begin at the origin and end at node (1). Also the number of paths on a square lattice from the origin to (n+1,n) that use steps (1,0) and (0,1). Also number of binary numbers of length 2n+1 with n+1 1's and n 0's. - Stefan Hollos (stefan(AT)exstrom.com), Dec 10 2007

If Y is a 3-subset of an 2n-set X then, for n>=3, a(n-1) is the number of n-subsets of X having at least two elements in common with Y. - Milan R. Janjic (agnus(AT)blic.net), Dec 16 2007

Also the number of rankings (preferential arrangements) of n unlabeled elements onto n levels when empty levels are allowed. - Thomas Wieder, May 24 2008

Also the Catalan transform of A000225 shifted one index, ie, dropping A000225(0). [From R. J. Mathar, Nov 11 2008]

With offset 1. The number of solutions in nonnegative integers to X1+X2+...+Xn=n. The number of terms in the expansion of (X1+X2+...+Xn)^n. The coefficient of x^n in the expansion of (1+x+x^2+...)^n. The number of distinct image sets of all functions taking [n]into[n]. [From Geoffrey Critzer, Feb 22 2009]

The Hankel transform of the aerated sequence 1,0,3,0,10,0,... is 1,3,3,5,5,7,7,.... (A109613(n+1)). [From Paul Barry , Apr 21 2009]

Comment from Anthony Bachler, May 05 2010: This is also the number of unique network topologies for a network of n items with 1 to n-1 unidirectional connections to other objects in the network.

Contribution from Gary W. Adamson, May 15 2009: (Start)

Equals INVERT transform of the Catalan numbers starting with offset 1.

E.g.: a(3) = 35 = (1, 2, 5) dot (10, 3, 1) + 14 = 21 + 14 = 35. (End)

a(n)=2*a000984-a000108, that is, a(n)=2*c(2n,n)-catalan [From Joseph Abate, Jun 11 2010]

The integral of 1/(1+x^2)^(n+1) is given by a(n)/2^(2n-1) * (x/(1+x^2)^n*P(x) + arctan(x)), where P(x) is a monic polynomial of degree 2n-2 with rational coefficients. [From Christiaan van de Woestijne, Jan 25 2011]

a(n) is the number of Schroder paths of semilength n in which the (2,0)-steps at level 0 come in 2 colors and there are no (2,0)-steps at a higher level. Example: a(2)=10 because, denoting U=(1,1), H=(1,0), and D=(1,-1), we have 2^2 = 4 paths of shape HH, 2 paths of shape HUD, 2 paths of shape UDH, and 1 path of eah of the shapes UDUD and UUDD. - Emeric Deutsch, May 02 2011

a(n) is the number of Motzkin paths of length n in which the (1,0)-steps at level 0 come in 3 colors and those at a higher level come in 2 colors. Example: a(3)=35 because, denoting U=(1,1), H=(1,0), and D=(1,-1), we have 3^3 = 27 paths of shape HHH, 3 paths of shape HUD, 3 paths of shape UDH, and 2 paths of shape UHD. - Emeric Deutsch, May 02 2011

Also number of digitally balanced numbers having length 2*(n+1) in binary representation: a(n) = #{m: A070939(A031443(m))=2*(n+1)}. -- Reinhard Zumkeller, Jun 08 2011

a(n) equals 2^(2n+3) times the coefficient of pi in 2F1(1/2,n+2,3/2,-1). [From John M. Campbell, Jul 17 2011]

For positive n, a(n) equals 4^(n+2) times the coefficient of pi^2 in int 0...pi/2 x sin^(2n+2)x. [From John M. Campbell, Jul 19 2011]

FORMULA a(n-1) = binomial(2*n, n)/2 = (2*n)!/(2*n!*n!).

a(0) = 1, a(n) = 2*(2*n+1)*a(n-1)/(n+1) for n > 0.

G.f.: (1/sqrt(1-4*x)-1)/(2*x).

L.g.f. log((1-sqrt(1-4*x))/(2*x)) = sum(n>0, a(n)/n*x^n) [From Kruchinin Vladimir, Aug 10 2010]

G.f.: 2F1(1,3/2;2;4*x). [From Paul Barry, Jan 23 2009]

G.f.: 1/(1-2x-x/(1-x/(1-x/(1-x/(1-... (continued fraction). [From Paul Barry, May 06 2009]

G.f.: c(x)^2/(1-x*c(x)^2), c(x) the g.f. of A000108. [From Paul Barry, Sep 07 2009]

Convolution of A000108 (Catalan) and A000984 (central binomial): Sum(C(k)*binomial(2*(n-k), n-k), k=0..n), C(k) Catalan. [Wolfdieter Lang]

a(n) = sum(k=0..n, C(n+k, k)) - Benoit Cloitre, Aug 20 2002

a(n) = sum(k=0..n, C(n, k)*C(n+1, k+1)) - Benoit Cloitre , Oct 19 2002

a(n) = 4^n*binomial(n+1/2, n)/(n+1); - Paul Barry, May 10 2005

E.g.f. sum(n>=0, a(n)*x^(2*n+1)/(2*n+1)!) = BesselI(1, 2*x) . - Michael Somos Jun 22 2005

E.g.f. in Maple notation: exp(2*x)*(BesselI(0, 2*x)+BesselI(1, 2*x)). Integral representation as n-th moment of a positive function on [0, 4]: a(n)=int(x^n*((x/(4-x))^(1/2)), x=0..4)/(2*Pi), n=0, 1... This representation is unique. - Karol A. Penson, Oct 11 2001

Narayana transform of [1, 2, 3,...]. Let M = the Narayana triangle of A001263 as an infinite lower triangular matrix and V = the Vector [1, 2, 3,...]. Then A001700 = M * V. - Gary W. Adamson, Apr 25 2006

a(n) = C(2*n, n) + C(2*n, n-1) = A000984(n) + A001791(n); - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Jan 23 2007

a(n) = n*(n+1)*(n+2)*...*(2*n-1)/n! (product of n consecutive integers, divided by n!). - Jonathan Vos Post, Apr 09 2007

a(n) = (2*n-1)!/(n!*(n-1)!) - William A. Tedeschi (fynmun(AT)hotmail.com), Feb 27 2008

The Tedeschi formula above appears to be incorrect. It produces the correct sequence but with a shifted offset...ie a(2)=10 but 3!/(2!*1!)=3

a(n) = (2*n+1)*C(n); - Paul Barry, Aug 21 2007

Binomial transform of A005773 starting (1, 2, 5, 13, 35, 96,...) and double binomial transform of A001405. - Gary W. Adamson, Sep 01 2007

Row sums of triangle A132813. - Gary W. Adamson, Sep 01 2007

Row sums of triangle A134285 - Gary W. Adamson, Nov 19 2007

Conjectured: 4^n GaussHypergeometric(1/2,-n;2;1) --Solution for the path which stays in the first and second quadrant. [Benjamin Phillabaum, Feb 20 2011]

a(n)= sum(0<=k<=n, A038231(n,k)*(-1)^k*A000108(k)). [From Philippe DELEHAM, Nov 27 2009]

Let A be the Toeplitz matrix of order n defined by: A[i,i-1]=-1, A[i,j]=Catalan(j-i), (i<=j), and A[i,j]=0, otherwise. Then, for n>=1, a(n)=(-1)^n*charpoly(A,-2). [From Milan R. Janjic (agnus(AT)blic.net), Jul 08 2010]

a(n) is the upper left term of M^(n+1), where M is the infinite matrix in which a column of (1,2,3,...) is prepended to an infinite lower triangular matrix of all 1's and the rest zeros, as follows:

1, 1, 0, 0, 0,...

2, 1, 1, 0, 0,...

3, 1, 1, 1, 0,...

4, 1, 1, 1, 1,...

...

Alternatively, a(n) is the upper left term of M^n where M is the infinite matrix:

3, 1, 0, 0, 0,...

1, 1, 1, 0, 0,...

1, 1, 1, 1, 0,...

1, 1, 1, 1, 1,...

...

- Gary W. Adamson, Jul 14 2011

A001764 Binomial(3n,n)/(2n+1) (enumerates ternary trees and also non-crossing trees).

COMMENTS Smallest number of straight line crossing-free spanning trees on n points in the plane.

Number of dissections of some convex polygon by nonintersecting diagonals into polygons with an odd number of sides and having a total number of 2n+1 edges (sides and diagonals). - Emeric Deutsch (deutsch(AT)duke.poly.edu), Mar 06 2002

Number of lattice paths of n East steps and 2n North steps from (0,0) to (n,2n) and lying weakly below the line y=2x. - David Callan (callan(AT)stat.wisc.edu), Mar 14 2004

With interpolated zeros, this has g.f. 2sqrt(3)sin(arcsin(3sqrt(3)x/2)/3)/(3x) and a(n)=C(n+floor(n/2),floor(n/2))C(floor(n/2),n-floor(n/2))/(n+1). This is the first column of the inverse of the Riordan array (1-x^2,x(1-x^2)) (Essentially reversion of y-y^3). - Paul Barry (pbarry(AT)wit.ie), Feb 02 2005

Number of 12312-avoiding matchings on [2n].

Number of complete ternary trees with n internal nodes, or 3n edges.

Number of rooted plane trees with 2n edges, where every vertex has even outdegree ("even trees").

a(n) = number of noncrossing partitions of [2n] with all blocks of even size. E.g.: a(2)=3 counts 12-34, 14-23, 1234. - David Callan (callan(AT)stat.wisc.edu), Mar 30 2007

Pfaff-Fuss-Catalan sequence C^{m}_n for m=3. See the Graham et al. reference, p. 347. eq. 7.66.

Also 3-Raney sequence. See the Graham et al. reference, p. 346-7.

The number of lattice paths from (0,0) to (2n,0), using an Up-step=(1,1) and Down-steps=(1,k); where k<= -1 and stays above the x-axis. For example, a(2)= 3 ;[(1,1)(1,1)(1,-1)(1,-1)],[(1,1)(1,-1)(1,1)(1,-1)] and [(1,1)(1,-3)] Also the number of lattice paths from (0,0) to (2n,0) using an Up-step=(1,1) and a Down-step=(0,-2) and staying above the x-axis. E.g. a(2)=3; UUUUDD, UUUDUD, UUDUUD. - Charles Moore (chamoore(AT)howard.edu), Jan 09 2008

a(n) is (conjecturally) the number of permutations of [n+1] that avoid the patterns 4-2-3-1 and 4-2-5-1-3 and end with an ascent. For example, a(4)=55 counts all 60 permutations of [5] that end with an ascent except 42315, 52314, 52413, 53412, all of which contain a 4-2-3-1 pattern and 42513. - David Callan (callan(AT)stat.wisc.edu), Jul 22 2008

Central terms of pendular triangle A167763. [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 12 2009]

FORMULA G.f.: 2/sqrt(3x)sin(1/3 arcsin(sqrt(27x/4))). E.g.f.: hypergeom([1/3, 2/3], [1, 3/2], 27/4*x). Integral representation as n-th moment of a positive function on [0, 27/4]: a(n)=int(x^n*(1/12*3^(1/2)*2^(1/3)*(2^(1/3)*(27+3*sqrt(81-12*x))^(2/3)-6*x^(1/3))/Pi/x^(2/3)/(27+3*sqrt(81-12*x))^(1/3)), x=0..6.75), n=0, 1... This representation is unique. - Karol A. Penson (penson(AT)lptl.jussieu.fr), Nov 08 2001

G.f. A(x) satisfies A(x) = 1+xA(x)^3 = 1/(1-xA(x)^2) - Ralf Stephan (ralf(AT)ark.in-berlin.de), Jun 30 2003

a(n) = n-th coefficient in expansion of power series P(n), where P(0)=1, P(k+1) = 1/(1-x*P(k)^2).

Contribution from Paul Barry (pbarry(AT)wit.ie), Mar 26 2010: (Start)

G.f.: 2*sin(asin(3*sqrt(3x)/2)/3)/sqrt(3x);

a(n)=[x^(n+1)]Rev(x/c(x)), c(x) the g.f. of A000108 (Rev=reversion of). (End)

E.g.f: F(1/3,2/3;1,3/2;27*x/4), where F(a1,a2;b1,b2;z) is a hypergeometric series. [Emanuele Munarini, Apr 12 2011]

Let M = the production matrix:

1, 1

2, 2, 1

3, 3, 2, 1

4, 4, 3, 2, 1

5, 5, 4, 3, 2, 1

...

a(n) = upper left term in M^n. Top row terms of M^n = (n+1)-th row of triangle A143603, with top row sums generating A006013: (1, 2, 7, 30, 143, 728,...). - Gary W. Adamson, Jul 07 2011

A001969 Evil numbers: numbers with an even number of 1's in their binary expansion.

COMMENTS This sequence and A000069 give the unique solution to the problem of splitting the nonnegative integers into two classes in such a way that sums of pairs of distinct elements from either class occur with the same multiplicities [Lambek and Moser]. Cf. A000028, A000379.

En francais: les nombres paiens.

a(n)-A001285(n) = 2n-1 has been verified for n=0,1,2,...,400 - John W. Layman (layman(AT)math.vt.edu)

First differences give A036585. Observed by Franklin T. Adams-Watters, proved by Max Alekseyev, Aug 30 2006. This is equivalent to a(n) = 2*n + A010060(n). Proof: If the number of bits in n is odd then the last bit of a(n) is 1 and if the number of bits in n is even then the last bit of a(n) is 0. Hence the sequence of last bits is A010060. Therefore a(n) = 2*n + A010060(n).

Indices of zeros in the Thue-Morse sequence A010060. [From Tanya Khovanova (tanyakh(AT)yahoo.com), Feb 13 2009]

FORMULA Note that 2n+1 is in the sequence iff 2n is not and so this sequence has asymptotic density 1/2. - Franklin T. Adams-Watters, Aug 23 2006

a(n) = (1/2) * (4n + 1 - (-1)^A000120(n)). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Sep 14 2003

G.f.: sum[k>=0, t(3+2t+3t^2)/(1-t^2)^2 * prod(l=0, k-1, 1-x^(2^l)), t=x^2^k]. - Ralf Stephan (ralf(AT)ark.in-berlin.de), Mar 25 2004

n such that A010060(n)=0 - Benoit Cloitre (benoit7848c(AT)orange.fr), Nov 15 2003 a(2*n+1) + a(2*n) = A017101(n) = 8*n+3. a(2*n+1) - a(2*n) gives the Thue-Morse sequence (3, 1 version): 3, 1, 1, 3, 1, 3, 3, 1, 1, 3, ... A001969(n) + A000069(n) = A016813(n) = 4*n+1. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 04 2004

a(0) = 0, a(2n) = a(n) + 2n, a(2n+1) = -a(n) + 6n + 3.

Let b(n) = 1 if sum of digits of n is even, -1 if it is odd; then Shallit (1985) showed that Prod_{n>=0} ((2n+1)/(2n+2))^b(n) = 1/sqrt(2).

a(n) = 2n + A010060(n). - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Aug 28 2006

A002033 Number of perfect partitions of n.

COMMENTS A perfect partition of n is one which contains just one partition of every number less than n when repeated parts are regarded as indistinguishable. Thus 1^n is a perfect partition for every n; and for n = 7, 4 1^3, 4 2 1, 2^3 1 and 1^7 are all perfect partitions. [Riordan]

Also number of ordered factorizations of n+1, see A074206.

Also number of gozinta chains from 1 to n (see A034776) [David W. Wilson]

a(n) is the permanent of the n X n matrix with (i,j) entry = 1 if j|i+1 and = 0 otherwise. For n=3 the matrix is {{1, 1, 0}, {1, 0, 1}, {1, 1, 0}} with permanent = 2. - David Callan (callan(AT)stat.wisc.edu), Oct 19 2005

Appears to be the number of permutations that contribute to the determinant that gives the moebius function. Verified up to a(9). [From Mats O. Granvik (mgranvik(AT)abo.fi), Sep 13 2008]

Dirichlet inverse of A153881. [From Mats Granvik (mats.granvik(AT)abo.fi), Jan 03 2009]

Equals row sums of triangle A176917 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Apr 28 2010]

FORMULA a(n) = sum of all a(i) such that i divides n and i < n (Clark Kimberling).

a(p^k)=2^(k-1).

a(n) = A067824(n)/2 for n>1; a(A122408(n)) = A122408(n)/2. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Sep 03 2006

a(n-1) = sum of all a(i-1) such that i divides n and i < n. a(p^k-1)=2^(k-1). a(n-1) = A067824(n)/2 for n > 1; a(A122408(n)-1) = A122408(n)/2. - David Wasserman (dwasserm(AT)earthlink.net), Nov 14 2006

A002083 Narayana-Zidek-Capell numbers: a(2n)=2a(2n-1), a(2n+1)=2a(2n)-a(n).

COMMENTS Number of words beginning with 1, with sum of integers = n, in the sequence 1, 11, 111, 112, 1111, 1112, 1113, 1121, 1122, 1123, 1124, 11111, 11112, 11113, 11114, 11121, 11122, 11123, 11124, 11125, 11131, 11132, 11133, 11134, 11135, 11136, where any positive integer, in any word, is <= the sum of the preceding integers - Claude Lenormand (claude.lenormand(AT)free.fr), Jan 29 2001

a(n) = number of sequences (b(1),b(2),...) of unspecified length satisfying b(1) = 1, 1 <= b(i) <= 1 + Sum[b(j),{j,i-1}] for i>=2, Sum[b(i)] = n-1. For example, a(5) = 3 counts (1, 1, 1, 1), (1, 2, 1), (1, 1, 2). These sequences are generated by the Mathematica code below. - David Callan (callan(AT)stat.wisc.edu), Nov 02 2005

a(n+1) is the number of padded compositions (d_1,d_2,...,d_n) of n satisfying d_i <= i for all i. A padded composition of n is obtained from an ordinary composition (c_1,c_2,...,c_r) of n (r >= 1, each c_i >= 1, sum(c_i,i=1..r) = n) by inserting c_i - 1 zeros immediately after each c_i. Thus (3,1,2) -> (3,0,0,1,2,0) is a padded composition of 6 and a padded composition of n has length n. For example, with n=4, a(5) counts (1,1,1,1), (1,1,2,0), (1,2,0,1). - David Callan

(callan(AT)stat.wisc.edu), Feb 04 2006

Further comments from David Callan (callan(AT)stat.wisc.edu), Sep 25 2006: a(n) = # ordered trees on n edges in which (i) every node (= non-root non-leaf vertex) has at least 2 children and (ii) each leaf is either the leftmost or rightmost child of its parent. For example, a(4)=2 counts

./\.../\

/\...../\,

and a(5)=3 counts

.|.......|....../|\

/ \...../ \...../ \

../\.../\.

First column of A155092. [From Mats Granvik (mats.granvik(AT)abo.fi), Jan 20 2009]

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 15 2009: (Start)

Starting with offset 2 = eigensequence of triangle A101688 and row sums of

triangle A167948 (End)

FORMULA a(1)=1, else a(n) is sum of [n/2] previous terms.

Conjecture: lim n->inf a(n)/2^(n-3) = a constant ~ .633368 - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Jul 18 2004

a(n) is the permanent of the (n-1) X (n-1) matrix with (i, j) entry = 1 if i-1 <= j <= 2*i-1 and = 0 otherwise. - David Callan (callan(AT)stat.wisc.edu), Nov 02 2005

a(n)=sum(K(n-k+1, k)*a(n - k),k=1..n), where K(n,k) = 1 if 0 <= k AND k <= n and K(n,k)=0 else. (Several arguments to the K-coefficient K(n,k) can lead to the same sequence. For example, we get A002083 also from a(n)=sum(K((n - k)!,k!)*a(n - k),k=1..n), where K(n,k) = 1 if 0 <= k <= n and 0 else. See also the comment to a similar formula for A002487.) - Thomas Wieder (thomas.wieder(AT)t-online.de), Jan 13 2008

G.f. satisfies: A(x) = (1-x - x^2*A(x^2))/(1-2x). [From Paul D. Hanna (pauldhanna(AT)juno.com), Mar 17 2010]

A002106 Number of transitive permutation groups of degree n.

A002110 Primorial numbers (first definition): product of first n primes. Sometimes written p#.

COMMENTS See A034386 for the second definition of primorial numbers: product of primes in the range 2 to n.

p(n)# is the least number N with n distinct prime factors (i.e. omega(N)=n, cf. A001221). - Lekraj Beedassy (blekraj(AT)yahoo.com), Feb 15 2002

Phi(n)/n is a new minimum for each primorial. - Robert G. Wilson v Jan 10 2004.

Smallest number stroked off n times after the n-th sifting process in an Eratosthenes sieve. - Lekraj Beedassy (blekraj(AT)yahoo.com), Mar 31 2005

Apparently each term is a new minimum for phi(x)*sigma(x)/x^2. 6/pi^2 < sigma(x)*phi(x)/x^2 < 1 for n > 1. - Jud McCranie (JudMcCranie(AT)ugaalum.uga.edu), Jun 11 2005

Comment from David W. Wilson (davidwwilson(AT)comcast.net), Oct 23 2006: Let f be a multiplicative function with f(p) > f(p^k) > 1 (p prime, k > 1), f(p) > f(q) > 1 (p, q prime, p < q). Then the record maxima of f occur at n# for n >= 1. Similarly, if 0 < f(p) < f(p^k) < 1 (p prime, k > 1), 0 < f(p) < f(q) < 1 (p, q prime, p < q), then the record minima of f occur at n# for n >= 1.

Wolfe and Hirshberg give ?, ?, ?, ?, ?, 30030, ?, ... as a puzzle.

Records in number of distinct prime divisors - Artur Jasinski (grafix(AT)csl.pl), Apr 06 2008

Successive minimal records in value of EulerPhi[k]/k. [From Artur Jasinski (grafix(AT)csl.pl), Nov 05 2008]

The digital roots of primorial numbers are multiples of 3. [From Parthasarathy Nambi (PachaNambi(AT)yahoo.com), Aug 19 2009]

Denominators of the sum of the ratios of consecutive primes. Cf. A094661 [From Vladimir Orlovsky (4vladimir(AT)gmail.com), Oct 24 2009]

The xth root of the xth primorial has a magnitude on the order of its number of factors (ignoring the first trivial primorial 1 ie with 2 counted as the 1st). [From Bill R McEachen (bmceache(AT)centralsan.org), Feb 08 2010]

Where record values occur in A001221 [From Melinda Trang (mewithlinda(AT)yahoo.com), Apr 15 2010]

It can be proved that there are at least T prime numbers less than N, where the recursive function T is: T = N- N*sum(A005867(i)/A002110(i), i=0..T(sqrt(N))) This can show for example that at least .16*N numbers are prime less than N for 29^2>N>23^2 [From Ben Thurston (benpaulthurston(AT)gmail.com), Aug 23 2010]

Partial products of non-composite numbers. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 15 2010]

FORMULA Asymptotic expression for a(n): exp((1 + o(1)) * n * log(n)) where o(1) is the "little o" notation - Dan Fux (dan.fux(AT)OpenGaia.com or danfux(AT)OpenGaia.com), Apr 08 2001

a(n) = A054842(A002275(n))

Binomial transform = A136104: (1, 3, 11, 55, 375, 3731,...). Equals binomial transform of A121572: (1, 1, 3, 17, 119, 1509,...). - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 14 2007

a(1)=1, a(n+1)=prime(n)*a(n). [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 15 2010]

A002113 Palindromes in base 10.

COMMENTS n is a palindrome (i.e. a(k)=n for some k) iff n = A004086(n). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 10 2002

The g.f. -z*(1+10*z**9+10*z**10+81*z**11+9*z)/(8*z**10-9*z**9-9*z-1)/(z-1)**2 conjectured by S. Plouffe in his 1992 dissertation is wrong. - N. J. A. Sloane (njas(AT)research.att.com), May 12 2008

A178788(a(n)) = 0 for n > 9. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jun 30 2010]

A002275 Repunits: (10^n - 1)/9. Often denoted by R_n.

COMMENTS R_n is a string of n 1's.

Base 4 representation of Jacobsthal bisection sequence A002450. E.g. a(4)= 1111 because A002450(4)= 85 (in base 10) = 64 + 16 + 4 + 1 = 1*(4^3)+1*(4^2)+1*(4^1)+1. - Paul Barry (pbarry(AT)wit.ie), Mar 12 2004

Except for the first two terms, these numbers cannot be perfect squares, because x^2 =/= 11 (mod 100) - Zak Seidov (zakseidov(AT)yahoo.com), Dec 05 2008.

For n >= 2: a(n) = Sequence A000225(n) written in base 2. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Jul 27 2009]

Let A be the Hessenberg matrix of order n, defined by: A[1,j]=1, A[i,i]:=10, (i>1), A[i,i-1]=-1, and

A[i,j]=0 otherwise. Then, for n>=1, a(n)=det(A). [From Milan R. Janjic (agnus(AT)blic.net), Feb 21 2010]
a(n) = A075412(n)/A002283(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), May 31 2010]

FORMULA G.f.: x/((1-10*x)*(1-x)). Regarded as base b numbers, g.f. x/((1-b*x)*(1-x)). - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Jun 15 2006
a(n)=11*a(n-1)-10*a(n-2), a(0)=0, a(1)=1. - Lekraj Beedassy (blekraj(AT)yahoo.com), Jun 07 2006
a(n)=10*a(n-1)+1, a(0)=0.
a(n)=a(n-1)+10^(n-1) with a(0)=0. [From Vincenzo Librandi (vincenzo.librandi(AT)tin.it), Jul 22 2010]
Second binomial transform of Jacobsthal trisection A001045(3n)/3 (A015565). - Paul Barry (pbarry(AT)wit.ie), Mar 24 2004
a(n) = A125118(n,9) for n>8. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Nov 21 2006

A002322 Reduced totient function psi(n): least k such that x^k == 1 (mod n) for all x prime to n; also Carmichael lambda function (exponent of unit group mod n).

COMMENTS Largest period of repeating digits of 1/n written in different bases (i.e. largest value in each row of square array A066799 and least common multiple of each row). - Henry Bottomley (se16(AT)btinternet.com), Dec 20 2001

FORMULA If M=2^e*P1^e1*P2^e2*...*Pk^ek, lambda(2^e) =2^(e-1) if e=1 or 2, =2^(e-2) if e>2; lambda(M)=LCM{ lambda(2^e), (P1-1)*P1^(e1-1), (P2-1)*P2^(e2-1), ..., (Pk-1)*Pk^(ek-1)}.
a(p) = p-1 for prime p. - Paolo P. Lava (ppl(AT)spl.at), Oct 02 2006

A002378 Oblong (or promic, pronic, or heteromecic) numbers: n(n+1).

COMMENTS 4*a(n)+1 are the odd squares A016754(n).
The word "pronic" (used by Dickson) is incorrect. - Michael Somos. According to the 2nd edition of Webster, the correct word is "promic" - R. K. Guy
a(n) is the number of minimal vectors in the root lattice A_n (see Conway and Sloane, p. 109).
Let M_n denotes the n X n matrix M_n(i,j)=(i+j); then the characteristic polynomial of M_n is x^(n-2) * (x^2-a(n)*x - A002415(n)). - Benoit Cloitre, Nov 09 2002
The greatest LCM of all pairs (j,k) for j<k=<n for n>1. - Robert G. Wilson v Jun 19 2004.
First differences are 2 4 6 8 10 12 14... (whilst first differences of the squares are 1 3 5 7 9 11 13...) - Alexandre Wajnberg (alexandre.wajnberg(AT)skynet.be), Dec 29 2005
25 appended to these numbers corresponds to squares of numbers ending in 5 (i.e. to squares of A017329). - Lekraj Beedassy, Mar 24 2006
Number of circular binary words of length n+1 having exactly one occurrence of 01. Example: a(2)=6 because we have 001, 010, 011, 100, 101 and 110. Column 1 of A119462. - Emeric Deutsch, May 21 2006
The sequence of iterated square roots sqrt(N+sqrt(N+...)) has for N=1,2,... the limit (1+sqrt(1+4*N))/2. For N=a(n) this limit is n+1, n=1,2,.... For all other numbers N, N>=1, this limit is not a natural number. Examples: n=1, a(1)=2: sqrt(2+sqrt(2+ ...)) = 1+1 =2; n=2, a(2)=6: sqrt(6+sqrt(6+ ...)) = 1+2 =3. W. Lang, May 05 2006.
Nonsquare integers m divisible by ceil(sqrt(m)), except m=0. - Max Alekseyev, Nov 27 2006

a(n) = 2*sum(1..n-1). - Artur Jasinski (grafix(AT)csl.pl), Jan 09 2007

The number of off-diagonal elements of an n+1 X n+1 matrix. - Artur Jasinski, Jan 11 2007

a(n) is equal to the number of functions f:{1,2}->{1,2,...,n+1} such that for a fixed x in {1,2} and a fixed y in {1,2,...,n+1} we have f(x)<>y. - Aleksandar M. Janjic and Milan R. Janjic (agnus(AT)blic.net), Mar 13 2007

Numbers m>=0 such that round(sqrt(m+1))-round(sqrt(m))=1. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Aug 06 2007

Numbers m>=0 such that ceiling(2*sqrt(m+1))-1=1+floor(2*sqrt(m)). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Aug 06 2007

Numbers m>=0 such that fract(sqrt(m+1))>1/2 and fract(sqrt(m))<1/2 where fract(x) is the fractional part (i.e. fract(x)=x-floor(x), x>=0). - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Aug 06 2007

Sequence allows us to find X values of the equation: 4*X^3 + X^2 = Y^2. To find Y values: b(n)=n(n+1)(2n+1). - Mohamed Bouhamida (bhmd95(AT)yahoo.fr), Nov 06 2007

Nonvanishing diagonal of A132792, the infinitesimal Lah matrix, so "generalized factorials" comprised of a(n) are given by the elements of the Lah matrix, unsigned A111596, e.g., a(1)*a(2)*a(3)/ 3! = -A111596(4,1) = 24 . - Tom Copeland, Nov 20 2007

If Y is a 2-subset of an n-set X then, for n>=2, a(n-2) is the number of 2-subsets and 3-subsets of X having exactly one element in common with Y. - Milan R. Janjic (agnus(AT)blic.net), Dec 28 2007

Infinite Sum[1/a[n+1],{n,1,Infinity}] = 1 but series is very slowly convergent. [From Artur Jasinski, Sep 28 2008]

a(n)=A061037(4*n)=(n+1/2)^2-1/4=((2n+1)^2-1)/4. [From Paul Curtz, Oct 03 2008]

a(n) coincides with the vertex of a parabola of even width in the Redheffer matrix, directed toward zero. An integer p is prime iff for all integer k, the parabola y = kx - x^2 has no integer solution with 1 < x < k when y = p; a(n) corresponds to odd k. [From Reikku Kulon , Nov 30 2008]

The third differences of certain values of the hypergeometric function 3F2 lead to the squares of the oblong numbers i.e. 3F2([1,n+1,n+1], [n+2,n+2], z=1) - 3*3F2([1,n+2,n+2], [n+3,n+3], z=1) + 3*3F2([1,n+3,n+3], [n+4,n+4], z=1) - 3F2([1,n+4,n+4], [n+5,n+5], z=1) = (1/((n+2)*(n+3)))^2 for n = -1, 0, 1,2, .. . See also A162990. [Johannes W. Meijer, Jul 21 2009]

a(A007018(n)) = A007018(n+1), see sequence A007018 (1,2,6,42,1806,...), i.e. A007018(n+1) = A007018(n) th oblong numbers. [From Jaroslav Krizek, Sep 13 2009]

a(j) = number of non-zero values of floor (j^2/n) taken over all n >= 1 for each j, with 1 <= j <= n-1.

a(n) = A035608(n) + A004526(n+1). [From Reinhard Zumkeller, Jan 27 2010]

a(n) = 2*(2*A006578(n) - A035608(n)). [From Reinhard Zumkeller, Feb 07 2010]

For n>1: a(n) = A173333(n+1,n-1). [From Reinhard Zumkeller, Feb 19 2010]

a(n) = A004202(A000217(n)). [Reinhard Zumkeller, Feb 12 2011]

a(n) = A188652(2*n+1) + 1. [Reinhard Zumkeller, Apr 13 2011]

FORMULA G.f.: (2*x)/(1-x)^3; a(n)=a(n-1)+2*n, a(0)=0.

Sum_{n >= 1} n*(n+1) = n(n+1)(n+2)/3 (cf. A007290).

Sum_{n >= 1} 1/(n*(n+1)) = 1. (Cf. Tijdeman)

1 = 1/2 + Sum(n = 1 through infinity) 1/[2*a(n)] = 1/2 + 1/4 + 1/12 + 1/24 + 1/40 + 1/60...with partial sums: 1/2, 3/4, 5/6, 7/8, 9/10, 11/12, 13/14... - Gary W. Adamson, Jun 16 2003

a(n)*a(n+1)=a(n*(n+2)); e.g. a(3)*a(4)=12*20=240=a(3*5) - Charlie Marion, Dec 29 2003
Sum_{k=1..n} 1/a(k) = n/(n+1). - Robert G. Wilson v Feb 04 2005.
a(n)=A046092/2. - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Jan 08 2006
Log 2 = Sum(n=0, inf.) 1/a(2n+1)= 1/2 + 1/12 + 1/30 + 1/56 + 1/90...; = (1 - 1/2) + (1/3 - 1/4) + (1/5 - 1/6) + (1/7 - 1/8) ...= Sum(n=0, inf.): (-1)^n/(Nn+1), with N=1. Log 2 = Integral_{0..1} 1/(1+x) dx = .69314718...; sum: 1/2 + 1/12 + 1/30 + 1/56 + 1/90 = 1627/2520 = .64563... - Gary W. Adamson, Jun 22 2003
a(n)=A049598-A124080; a(n)=A124080-A033996: a(n)=A033996-A028896: a(n)=A028896-A046092. - Zerinvary Lajos (zerinvarylajos(AT)yahoo.com), Mar 06 2007
a(n-1)=n^2-n = A000290(n)-A000027(n) for n>=1. a(n) = inverse (frequency distribution) sequence of A000194(n).- Mohammad K. Azarian, Jul 26 2007
(2, 6, 12, 20, 30,...) = binomial transform of (2, 4, 2). - Gary W. Adamson, Nov 28 2007
a(n)=A000217(n)*2. - Omar E. Pol, May 14 2008
a(n) = A006503(n) - A000292(n). [From Reinhard Zumkeller, Sep 24 2008]
a(0):=0, a(n)=a(n-1)+1+floor(x), where x is the minimal positive solution to fract(sqrt(a(n-1)+1+x))=1/2. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Dec 31 2008
E.g.f.:(x+2)*x*exp(x) [From Geoffrey Critzer, Feb 06 2009]
Product_{i=2..infinity} (1-1/a(i)) = -2*sin(Pi*A001622)/Pi = -2*sin(A094886)/A000796. [From R. J. Mathar, Mar 12 2009]
egf: ((-x+1)*ln(-x+1)+x)/x^2 also int(((-x+1)*ln(-x+1)+x)/x^2,x=0..1)=Zeta(2)-1 [From Stephen Crowley, Jul 11 2009]
a(n) = floor((n + 1/2)^2). [From Reinhard Zumkeller, Jan 27 2010]
a(n) = floor(n^5/(n^3+n^2+1)) with offset 1...a(1)=0 [From Gary Detles, Feb 11 2010]
a(n)=2*n+a(n-1) (with a(0)=0) [From Vincenzo Librandi]
For n>0 a(n)=1/(Integral_{x=0..Pi/2} 2*(sin(x))^(2*n-1)*(cos(x))^3). [From Francesco Daddi, Aug 02 2011]
A002426 Central trinomial coefficients: largest coefficient of (1+x+x^2)^n.
COMMENTS Number of ordered trees with n+1 edges, having root of odd degree and nonroot nodes of outdegree at most 2. - Emeric Deutsch, Aug 02 2002
Number of paths of length n with steps U=(1, 1), D=(1, -1) and H=(1, 0), running from (0, 0) to (n, 0) (i.e. grand Motzkin paths of length n). For example, a(3)=7 because we have HHH, HUD, HDU, UDH, DUH, UHD and DHU. - Emeric Deutsch, May 31 2003
Number of lattice paths from (0,0) to (n,n) using steps (2,0), (0,2), (1,1). It appears that 1/sqrt((1-x)^2-4*x^s) is the g.f. for lattice paths from (0,0) to (n,n) using steps (s,0), (0,s), (1,1). [Joerg Arndt, Jul 01 2011]
Number of lattice paths from (0,0) to (n,n) using steps (1,0), (1,1), (1,2). [Joerg Arndt, Jul 05 2011]
Binomial transform of A000984, with interpolated zeros. - Paul Barry, Jul 01 2003
Number of leaves in all 0-1-2 trees with n edges, n>0. (A 0-1-2 tree is an ordered tree in which every vertex has at most two children.) - Emeric Deutsch, Nov 30 2003
a(n)=number of UDU-free paths of n+1 upsteps (U) and n downsteps (D) that start U. For example, a(2)=3 counts UUUDD, UUDDU, UDDUU. - David Callan, Aug 18 2004
Diagonal sums of triangle A063007. - Paul Barry, Aug 31 2004

Number of ordered ballots from n voters that result in an equal number of votes for candidates A and B in a three candidate election. Ties are counted even when candidates A and B lose the election. For example, a(3)=7 because ballots of the form (voter-1 choice, voter-2 choice, voter-3 choice) that result in equal votes for candidates A and B are the following:(A,B,C), (A,C,B), (B,A,C), (B,C,A), (C,A,B), (C,B,A) and (C,C,C). - Dennis Walsh (dwalsh(AT)mtsu.edu), Oct 08 2004

a(n) = number of weakly increasing sequences (a_1,a_2,...,a_n) with each a_i in [n]={1,2,...,n} and no element of [n] occurring more than twice. For n=3, the sequences are 112, 113, 122, 123, 133, 223, 233. - David Callan, Oct 24 2004

Note that n divides a(n+1)-a(n). In fact, (a(n+1)-a(n))/n = A007971(n+1). - T. D. Noe, Mar 16 2005

Row sums of triangle A105868. - Paul Barry, Apr 23 2005

a(n) = A111808(n,n). - Reinhard Zumkeller, Aug 17 2005

Number of paths of length n with steps U=(1,1), D=(1,-1) and H=(1,0), starting at (0,0), staying weakly above the x-axis (i.e. left factors of Motzkin paths) and having no H steps on the x-axis. Example: a(3)=7 because we have UDU, UHD, UHH, UHU, UUD, UUH and UUU. - Emeric Deutsch, Oct 07 2007

Equals right border of triangle A152227; starting with offset 1, the row sums of triangle A152227. [Gary W. Adamson, Nov 29 2008]

Contribution from Gary W. Adamson, Jan 07 2009: (Start)

Starting with offset 1 = iterates of M * [1,1,1,...] where M = a tridiagonal

matrix with [0,1,1,1,...] in the main diagonal and [1,1,1,...] in the super and subdiagonals. (End)

Hankel transform is 2^n. [From Paul Barry, Aug 05 2009]

a(n) is prime for n=2, 3, and 4, with no others for n<=10^5 (E. W. Weisstein, Mar. 14, 2005). It has apparently not been proved that no [other] prime central trinomials exist. [From Jonathan Vos Post, Mar 19 2010]

a(n) is not divisible by 3 for n whose base 3 representation contains no 2, A005836.

FORMULA G.f.: 1/sqrt(1-2*x-3*x^2).

E.g.f.: exp(x) I_0(2x), where I_0 is Bessel function. - Michael Somos, Sep 09 2002.

a(n) = 2*A027914(n) - 3^n - Benoit Cloitre (benoit7848c(AT)orange.fr), Sep 28 2002

a(n) is asymptotic to d*3^n/sqrt(n) with d around 0.5.. - Benoit Cloitre, Nov 02, 2002

a(n)=((2*n-1)*a(n-1)+3*(n-1)*a(n-2))/n; a(0)=a(1)=1; see paper by Barcucci, Pinzani and Sprugnoli.

Inverse binomial transform of A000984. - Vladeta Jovovic (vladeta(AT)eunet.rs), Apr 28 2003

a(n)=sum{k=0..n, C(n, k)C(k, k/2)(1+(-1)^k)/2}; a(n)=sum{k=0..n, (-1)^(n-k)C(n, k)C(2k, k) }. - Paul Barry (pbarry(AT)wit.ie), Jul 01 2003

a(n) = Sum{k>=0, C(n, 2*k)*C(2*k, k)}. - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Dec 31 2003

a(n)=sum(i+j=n, 0<=j<=i<=n, binomial(n, i)*binomial(i, j)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Jun 06 2004

a(n) = 3* a(n-1) - 2*A005043(n) - Joost Vermeij (joost_vermeij(AT)hotmail.com), Feb 10 2005

a(n) is asymptotic to d*3^n/sqrt(n) with d=sqrt(3/Pi)/2=.488602512... - Alec Mihailovs (alec(AT)mihailovs.com), Feb 24 2005 a(n)=sum{k=0..n, C(n, k)C(k, n-k)}; - Paul Barry, Apr 23 2005

a(n) = (-1/4)^n*Sum_{k, 0<=k<=n} = binomial(2k, k)*binomial(2n-2k, n-k)*(-3)^k . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Aug 17 2005

a(n)=sum{k=0..n, ((1+(-1)^k)/2)*sum{i=0..floor((n-k)/2), C(n, i)C(n-i, i+k)((k+1)/(i+k+1))}}; - Paul Barry, Sep 23 2005

a(n)=3^n*sum{j=0..n,(-1/3)^j*C(n,j)*C(2j,j)}; follows from (a) in A027907. - Loic Turban (turban(AT)lpm.u-nancy.fr), Aug 31 2006

a(n)=(1/2)^n*sum{j=0..n,3^j*C(n,j)*C(2n-2j,n)}=(3/2)^n*sum{j=0..n,(1/3)^j*C(n,j)*C(2j,n)}; follows from (c) in A027907. - Loic Turban (turban(AT)lpm.u-nancy.fr), Aug 31 2006

a(n)=(1/pi)*int(x^n/sqrt((3-x)(1+x)),x,-1,3) is moment representation; - Paul Barry, Sep 10 2007

G.f.: 1/(1-x-2x^2/(1-x-x^2/(1-x-x^2/(1-... (continued fraction). [From Paul Barry, Aug 05 2009]

a(n) = sqrt(-1/3)*(-1)^n*hypergeom([1/2, n+1],[1],4/3) [From Mark van Hoeij (hoeij(AT)math.fsu.edu), Nov 12 2009]

a(n) = (1/pi)*int((1+2*x)^n/sqrt(1-x^2),x,-1,1) = (1/pi)*int((1+2*cos(t))^n,t,0,pi). - Eli Wolfhagen, Feb 01 2011

A002487 Stern's diatomic series: a(0) = 0, a(1) = 1; for n > 0: a(2*n) = a(n), a(2*n+1) = a(n) + a(n+1).

COMMENTS Also called fusc(n).

a(n)/a(n+1) runs through all the reduced nonnegative rationals exactly once [Stern; Calkin and Wilf]

If the terms are written as an array:

1

1,2

1,3,2,3

1,4,3,5,2,5,3,4

1,5,4,7,3,8,5,7,2,7,5,8,3,7,4,5

1,6,5,9,4,11,7,10,3,11,8,13,5,12,7,9,2,9,7,12,5,13,8,11,3,10,7,11,4,9,5,6

then the sum of the k-th row is 3^(k-1), each columns is an arithmetic progression and the steps are nothing but the original sequence. - Takashi Tokita (butaneko(AT)fa2.so-net.ne.jp), Mar 08 2003

Number of odd Stirling numbers S_2(n+1,2r+1) [Carlitz]

Moshe Newman proved that the fraction a(n+1)/a(n+2) can be generated from the previous fraction a(n)/a(n+1) = x by 1/(2*floor(x) + 1 - x). The successor function f(x) = 1/(floor(x) + 1 - frac(x)) can also be used.

a(n+1) = number of alternating bit sets in n (see Finch).

If f(x):=1/(1+floor(x)-frac(x)) then f(a(n-1)/a(n))=a(n)/a(n+1) except for n=0 or n=-1. If T(x):=-1/x and f(x)=y, then f(T(y))=T(x) except for x=0. - Michael Somos, Sep 03 2006

a(n+1) = number of ways of writing n as a sum of powers of 2, each power being used at most twice (the number of hyperbinary representations of n) [Carlitz; Lind]

a(n+1) = partitions of the n-th integer expressible as the sum of distinct even-subscripted Fibonacci numbers (=A054204(n)), into sums of distinct Fibonacci numbers numbers. [Bicknell-Johnson]

a(n+1) = number of odd binomial(n-k,k),0<=2k<n. [Carlitz].

a(2^k) = 1. a(3*2^k) = a(2^(k+1) + 2^k) = 2. Sequences of terms between a(2^k) = 1 and

a(2^(k+1)) = 1 are palindromes of length 2^k-1 with a(2^k + 2^(k-1)) = 2 in the middle. a(2^(k-1) + 1) = a(2^k - 1) = k+1 for k>1. - Alexander Adamchuk, Oct 10 2006
The coefficients of the inverse of the GF of this sequence form A073469 and are related to binary partitions A000123. - Philippe Flajolet, Sep 06 2008
It appears that the terms of this sequence are the number of odd entries in the diagonals of Pascal's triangle at 45 degrees slope. [From Javier Torres (adaycalledzero(AT)hotmail.com), Aug 06 2009]
Contribution from Gary W. Adamson, Dec 11 2009: (Start)
Let M = an infinite lower triangular matrix with (1, 1, 1, 0, 0, 0,..) in
every column shifted down twice:
1;
1, 0;
1, 1, 0;
0, 1, 0, 0;
0, 1, 1, 0, 0;
0, 0, 1, 0, 0, 0;
0, 0, 1, 1, 0, 0, 0;
...
A002487 = Lim_{n->inf.} M^n, a left-shifted vector considered as a sequence.
(Cf. A026741) (End)
Member of the infinite family of sequences of the form a(n) = a(2n); a(2n+1) = r*a(n) + a(n+1), r = 1 for A002487 = row 1 in the array of A178239. [Gary W. Adamson, May 23 2010]
Equals row 1 in an infinite array shown in A178568, sequences of the form
a(2n) = r*a(n), a(2n+1) = a(n) + a(n+1); r = 1. [Gary W. Adamson, May 29 2010]
Row sums of A125184, the Stern polynomials. Equivalently, B(n,1), the n-th Stern polynomial evaluated at x=1. - T. D. Noe, Feb 28 2011
The Kn1y and Kn2y triangle sums, see A180662 for their definitions, of A047999 lead to the sequence given above, e.g. Kn11(n) = A002487(n+1) - A000004(n), Kn12(n) = A002487(n+3) - A000012(n), Kn13(n) = A002487(n+5) - A000034(n+1) and Kn14(n) = A002487(n+7) - A157810(n+1). For the general case of the knight triangle sums see the Stern-Sierpinski triangle A191372. This triangle not only leads to Stern’s diatomic series but also to snippets of this sequence and, quite surprisingly, their reverse. [From Johannes W. Meijer, Jun 05 2011]
FORMULA a(0) = 0; a(1) = 1; a(2*k) = a(k); a(2*k+1) = a(k) + a(k+1) generates the same sequence with an initial 0 - David W. Wilson
a(n+1) = (2k+1)*a(n) - a(n-1) where k = [a(n-1) / a(n)] is the largest integer smaller than or equal to a(n-1)/a(n) - David Newman, Mar 04, 2001
Let e(n) = A007814(n) = exponent of highest power of 2 dividing n. Then a(n+1)=(2k+1)a(n)-a(n-1), n>0, where k=e(n). Moreover, floor(a(n-1)/a(n))=e(n), in agreement with D. Newman's formula. - Dragutin Svrtan (dsvrtan(AT)math.hr) and Igor Urbiha (urbiha(AT)math.hr), Jan 10, 2002.
Calkin and Wilf showed 0.9588 < limsup a(n)/n^(log(phi)/log(2))<1.1709 where phi is the golden mean. Does this supremum limit = 1? - Benoit Cloitre, Jan 18 2004
a(n)=sum{k=0..floor((n-1)/2), mod(binomial(n-k-1, k), 2)} - Paul Barry, Sep 13 2004
a(n)=sum{k=0..n-1, binomial(k, n-k-1) mod 2}; - Paul Barry, Mar 26 2005
G.f. A(x) satisfies 0=f(A(x), A(x^2), A(x^4)) where f(u, v, w)=v^3+2*u*v*w-u^2*w. - Michael

Somos, May 02 2005
G.f. A(x) satisfies 0=f(A(x), A(x^2), A(x^3), A(x^6)) where f(u1, u2, u3, u6)=u1^3*u6-3*u1^2*u2*u6+3*u2^3*u6-u2^3*u3. - Michael Somos, May 02 2005
For all integer n, a(-n)=-(-1)^n*a(n), a(n)=a(2n)sign(n)^n, a(n-1)+a(n)*sign(n)=a(2*n-1)*sign(n)^n. - Michael Somos, Sep 03 2006
G.f.: x * prod(k>=0, 1+x^(2^k)+x^(2^(k+1))). [Carlitz]
a(n)=a(n-2)+a(n-1)-2*(a(n-2) mod a(n-1)) where x mod y gives the remainder after dividing x by y - Mike Stay, Nov 06 2006
A079978(n)=(1+e^(i*pi*A002487(n)))/2, i=sqrt(-1); - Paul Barry, Jan 14 2005
a(n)=sum(k=1..n, K(k, n-k)*a(n - k)), where K(n,k) = 1 if 0 <= k AND k <= n AND n-k <= 2 and K(n,k) = 0 else. (When using such a K-coefficient several different arguments to K or several different definitions of K may lead to the same integer sequence. For example, if we drop the condition k <= n in the above definition, then we arrive at A002083 = Narayana-Zidek-Capell numbers.) - Thomas Wieder, Jan 13 2008
a(k+1)*a(2^n - k) - a(k)*a(2^n - (k+1)) = 1; a(2^n - k) + a(k) = a(2^(n+1) + k). Both formulas hold for 0 <= k <= 2^n - 1. G.f.: G(z) = a(1) + a(2)*z + a(3)*z^2 + ... + a(k+1)*z^k + ... Define f(z) = (1 + z + z^2), then G(z) = lim f(z)*f(z^2)*f(z^4)* ... *f(z^(2^n))*... =(1 + z + z^2)*G(z^2) - Arie Werksma (werksma(AT)tiscali.nl), Apr 11 2008
a(k+1).a(2^n - k) - a(k).a(2^n - (k+1)) = 1 (0 <= k <= 2^n - 1). - Arie Werksma (werksma(AT)tiscali.nl), Apr 18 2008
a(2^n + k) = a(2^n - k) + a(k) (0 <= k <= 2^n). - Arie Werksma (werksma(AT)tiscali.nl), Apr 18 2008
Let g(z) = a(1) + a(2)*z + a(3)*z^2 + ... + a(k+1)*z^k + ..., f(z) = 1 + z + z^2. Then g(z) = lim(n->00) f(z)*f(z^2)*f(z^4)*...*f(z^(2^n)), g(z) = f(z)*g(z^2). - Arie Werksma (werksma(AT)tiscali.nl), Apr 18 2008
For 0 <= k <= 2^n - 1, write k = b(0) + 2*b(1) + 4*b(2) + ... + 2^(n-1)*b(n-1) where b(0), b(1), etc. are 0 or 1. Define a 2 by 2 matrix X(m) with entries x(1,1) = x(2,2) = 1, x(1,2) = 1 - b(m), x(2,1) = b(m). Let P(n)= X(0)*X(1)* ... *X(n-1). The entries of the matrix P are members of the sequence: p(1,1) = a(k+1), p(1,2) = a(2^n - (k+1)), p(2,1) = a(k), p(2,2) = a(2^n - k). - Arie Werksma (werksma(AT)tiscali.nl), Apr 20 2008
Let f(x) = A030101(x); if 2^n + 1 <= x <= 2^(n + 1) and y = 2^(n + 1) - f(x - 1) then a(x) = a(y). - Arie Werksma (Werksma(AT)Tiscali.nl), Jul 11 2008
a(n) = A126606(n + 1) / 2. [From Reikku Kulon, Oct 05 2008]
Equals infinite convolution product of [1,1,1,0,0,0,0,0,0] aerated A000079 - 1 times, i.e. [1,1,1,0,0,0,0,0,0] * [1,0,1,0,1,0,0,0,0] * [1,0,0,0,1,0,0,0,1]. [From Mats Granvik, Gary W. Adamson, Oct 02 2009]
A002530 Denominators of continued fraction convergents to sqrt(3).
COMMENTS Also denominators of continued fraction convergents to sqrt(3) - 1. See A048788 for numerators. - N. J. A. Sloane (njas(AT)research.att.com), Dec 17 2007. Convergents are 1, 2/3, 3/4, 8/11, 11/15, 30/41, 41/56, 112/153, ...
Consider the mapping f(a/b) = (a + 3b)/(a + b). Taking a = b = 1 to start with and carrying out this mapping repeatedly on each new (reduced) rational number gives the following sequence 1/1,2/1,5/3,7/4,19/11,... converging to 3^(1/2). Sequence contains the denominators. The same mapping for N i.e. f(a/b) = (a + Nb)/(a+b) gives fractions converging to N^(1/2). - Amarnath

Murthy (amarnath_murthy(AT)yahoo.com), Mar 22 2003
Sqrt(3) = 2/2 + 2/3 + 2/(3*11) + 2/(11*41) + 2/(41*153) + 2/(153*571),...; where the sum of the first 6 terms of this series = 1.7320490367... and sqrt(3) = 1.7320508075... - Gary W. Adamson (qntmpkt(AT)yahoo.com), Dec 15 2007
Comment from Clark Kimberling (ck6(AT)evansville.edu), Aug 27 2008 (Start): Related convergents (numerator/denominator):
lower principal convergents: A001834/A001835
upper principal convergents: A001075/A001353
intermediate convergents: A005320/A001075
principal and intermediate convergents: A143642/A140827
lower principal and intermediate convergents: A143643/A005246 (End)
Row sums of triangle A152063 = (1, 3, 4, 11,...). [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 26 2008]
Also number of domino tilings of the 3 X (n-1) rectangle with upper left corner removed iff n is even. For n=4 the 4 domino tilings of the 3 X 3 rectangle with upper left corner removed are:

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. .___. . .___. . .___. . .___.
._|___| ._|___| ._| | | ._|___|
| |___| | | | | | |_|_| |___| |
|_|___| |_|_|_| |_|___| |___|_|    - Alois P. Heinz, Apr 13 2011
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FORMULA a(2n) = ((2+sqrt(3))^n-(2-sqrt(3))^n)/(2*sqrt(3)); a(2n) = A001353(n); a(2n-1) = ceil((1+1/sqrt(3))/2*(2+sqrt(3))^n) = ((3+sqrt(3))^(2n-1)+(3-sqrt(3))^(2n-1))/6^n; a(2n-1) = A001835(n).- Benoit Cloitre, Dec 15 2002
G.f.: x(1+x-x^2)/(1-4x^2+x^4). a(n) = 4a(n-2)+a(n-4) = -(-1)^n a(-n).
a(2n) = a(2n-1) + a(2n-2), a(2n+1) = 2a(2n) + a(2n-1).
a(n+1) = sum{k=0..floor(n/2), binomial(n-k, k)2^floor((n-2k)/2) } - Paul Barry (pbarry(AT)wit.ie), Jul 13 2004
a(n) = sum_{k=0..floor(n/2)} binomial(floor(n/2)+k,floor((n-1)/2-k))*2^k). - Paul Barry (pbarry(AT)wit.ie), Jun 22 2005

A002531 Numerators of continued fraction convergents to sqrt(3).
COMMENTS Consider the mapping f(a/b) = (a + 3b)/(a + b). Taking a = b = 1 to start with and carrying out this mapping repeatedly on each new (reduced) rational number gives the following sequence 1/1,2/1,5/3,7/4,19/11,... converging to 3^(1/2). Sequence contains the numerators. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Mar 22 2003
In the Murthy comment if we take a=0, b=1 then the denominator of the reduced fraction is a(n+1).
A083336(n)/a(n+1) converges to sqrt(3). - Mario Catalani (mario.catalani(AT)unito.it), Apr 26 2003
If signs are disregarded, all terms of A002316 appear to be elements of this sequence. - Creighton Dement (creighton.k.dement(AT)uni-oldenburg.de), Jun 11 2007
FORMULA G.f.: (1+x-2*x^2+x^3)/(1-4*x^2+x^4).
a(2*n) = a(2*n-1) + a(2*n-2), a(2*n+1) = 2*a(2*n) + a(2*n-1), n>0.
a(2*n)=(1/2)*((2+sqrt(3))^n+(2-sqrt(3))^n); a(2*n)=A003500(n)/2; a(2*n+1) = round(1/(1+sqrt(3))*(2+sqrt(3))^n) - Benoit Cloitre (benoit7848c(AT)orange.fr), Dec 15 2002

A002572
COMMENTS
Top row of Table 1 of Elsholtz.
FORMULA
Math. Rev. 22 #11020, Minc, H. A problem in partitions ... 1959: v(c, d) is the number of partitions of d into positive integers of the form d = c + c_1 + c_2 + ... + c_n, where c_1 <= 2*c, c_{i+1} <= 2*c_i.
A002620 Quarter-squares: floor(n/2)*ceiling(n/2). Equivalently, floor(n^2/4). (Formerly M0998 N0374)

COMMENTS b(n) = A002620(n+2) = number of multigraphs with loops on 2 nodes with n edges [so g.f. for b(n) is 1/((1-x)^2*(1-x^2))]. Also number of 2-covers of an n-set; also number of 2 X n binary matrices with no zero columns up to row and column permutation - Vladeta Jovovic (vladeta(AT)eunet.rs), Jun 08, 2000.

a(n) is also the maximal number of edges that a triangle-free graph of n vertices can have. For n = 2m the maximum is achieved by the bipartite graph K(m,m), For n = 2m+1 the maximum is achieved by the bipartite graph K(m,m+1). - Avi Peretz (njk(AT)netvision.net.il), Mar 18 2001

a(n) is the number of arithmetic progressions of 3 terms and any mean which can be extracted from the set of the first n natural numbers (starting from 1). - Santi Spadaro (spados(AT)katamail.com), Jul 13 2001

This is also the order dimension of the (strong) Bruhat order on the Coxeter group A_{n-1} (the symmetric group S_n). - Nathan Reading (reading(AT)math.umn.edu), Mar 07 2002

Let M_n denotes the n X n matrix m(i,j) = 2 if i =j; m(i,j) = 1 if (i+j) is even; m(i,j) = 0 if i+j is odd, then a(n+2) = det M_n - Benoit Cloitre (benoit7848c(AT)orange.fr), Jun 19 2002

Sums of pairs of neighboring terms are triangular numbers in increasing order. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Aug 19 2002

Also, from the starting position in standard chess, minimum number of captures by pawns of the same color to place n of them on the same file (column). Beyond a(6), the board and number of pieces available for capture are assumed to be extended enough to accomplish this task. - Rick L. Shepherd, Sep 17 2002

For example, a(2)=1 and one capture can produce "doubled pawns", a(3)=2 and two captures is sufficient to produce tripled pawns, etc. (Of course other, uncounted, non-capturing pawn moves are also necessary from the starting position in order to put three or more pawns on a given file.) - Rick L. Shepherd, Sep 17 2002

Terms are the geometric mean and arithmetic mean of their neighbors alternately. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Oct 17 2002

Maximum product of two integers whose sum is n. - Matthew Vandermast, Mar 04 2003

a(n+1) gives number of non-symmetric partitions of n into at most 3 parts, with zeros used as padding. E.g. a(6) = 12 because we can write 5 = 5+0+0 = 0+5+0 = 4+1+0 = 1+4+0 = 1+0+4 = 3+2+0 = 2+3+0 = 2+0+3 = 2+2+1 = 2+1+2 = 3+1+1 = 1+3+1. - Jon Perry, Jul 08 2003

a(n-1) gives number of distinct elements greater than 1 of non-symmetric partitions of n into at most 3 parts, with zeros used as padding, appear in the middle. E.g. 5 = 5+0+0 = 0+5+0 = 4+1+0 = 1+4+0 = 1+0+4 = 3+2+0 = 2+3+0 = 2+0+3 = 2+2+1 = 2+1+2 = 3+1+1 = 1+3+1. Of these 050,140,320,230,221,131 qualify and a(4)=6. - Jon Perry, Jul 08 2003

Union of square numbers (A000290) and oblong numbers (A002378). - Lekraj Beedassy, Oct

02 2003

Conjectured size of the smallest critical set in a Latin square of order n (true for n <= 8). - Richard Bean (rwb(AT)eskimo.com), Jun 12 2003 and Nov 18 2003

a(n) gives number of maximal strokes on complete graph K_n, when edges on K_n can be asigned directions in any way. A "stroke" is a locally maximal directed path on a directed graph. Examples: n=3, two strokes can exist, "x -> y -> z" and " x -> z", so a(3)=2 . n=4, four maximal strokes exist, "u -> x -> z" and "u -> y" and "u -> z" and "x -> y -> z", so a(4)=4. - Yasutoshi Kohmoto (zbi74583(AT)boat.zero.ad.jp), Dec 20, 2003

Number of symmetric Dyck paths of semilength n+1 and having three peaks. E.g. a(4)=4 because we have U*DUUU*DDDU*D, UU*DUU*DDU*DD, UU*DDU*DUU*DD and UUU*DU*DU*DDD, where U=(1,1), D=(1,-1) and * indicates a peak. - Emeric Deutsch, Jan 12 2004

Number of valid inequalities of the form j + k < n + 1, where j and k are positive integers, j <= k, n >= 0. Partial sums of A004526 (nonnegative integers repeated: partitions into two parts). - Rick Shepherd, Feb 27 2004

See A092186 for another application.

Also, the number of nonisomorphic transversal combinatorial geometries of rank 2. - Alexandr S. Radionov (rasmailru(AT)mail.ru), Jun 02 2004

a(n+1) is the transform of n under the Riordan array (1/(1-x^2),x). - Paul Barry, Apr 16 2005

a(n) = A108561(n+1,n-2) for n>2. - Reinhard Zumkeller, Jun 10 2005

1, 2, 4, 6, 9, 12, 16, 20, 25, 30, ... specifies the largest number of copies of any of the gifts you receive on the n-th day in the "Twelve Days of Christams" song. - Alonso Del Arte, Jun 17 2005

a(n) = Sum(Min{k,n-k}: 0<=k<=n), sums of rows of the triangle in A004197. - Reinhard Zumkeller, Jul 27 2005

a(n+1) is the number of noncongruent integer-sided triangles with largest side n - David W. Wilson. [Comment corrected Sep 26 2006]

A quarter-square table can be used to multiply integers since n*m = a(n+m)-a(n-m) for all integer n,m. - Michael Somos Oct 29 2006

The sequence is the size of the smallest strong critical set in a Latin square of order n. - G.H.J. van Rees (vanrees(AT)cs.umanitoba.ca), Feb 16 2007

Maximal number of squares (maximal area) in a polyomino with perimeter 2n. - Tanya Khovanova, Jul 04 2007

For n >= 3 a(n-1) is the number of bracelets with n+3 beads, 2 of which are red, 1 of which is blue. - Washington Bomfim (webonfim(AT)bol.com.br), Jul 26 2008

Equals row sums of triangle A122196 [From Gary W. Adamson, Nov 29 2008]

a(n+1) = a(n) + A110654(n). [From Reinhard Zumkeller, Aug 06 2009]

a(n)=(n*n - 2*n + nmod2)/4 [From Ctibor O. Zizka, Nov 23 2009]

Equals triangle A171608 * (1, 2, 3,...) [From Gary W. Adamson, Dec 12 2009]

a(n) gives the number of nonisomorphic faithful representations of the Symmetric group S_3 of dimension n. Any faithful representation of S_3 must contain at least one copy of the 2-dimensional irrep, along with any combination of the two 1-dimensional irreps. - Andrew Rupinski, Jan 20 2011

a(n+2) counts the number of ways to make change for "c" cents, letting n=floor(c/5) to account for the 5-repetitive nature of the task, using only pennies, nickels and dimes (see A187243). - Adam Sasson, Mar 07 2011

a(n) belongs to the sequence iff a(n) = floor(sqrt(a(n))) * ceiling(sqrt(a(n))), that is, a(n) = k^2 or a(n) = k*(k+1), k >= 0. - Daniel Forgues, Apr 17 2011

a(n) is the sum of the positive integers < n that have the opposite parity as n.

Deleting the first 0 from the sequence results in a sequence b = 0,1,2,4,.. such that b(n) is sum of the positive integers <= n that have the same parity as n. The sequence b(n) is the additive counterpart of the doublefactorial. - Peter Luschny, Jul 06 2011

FORMULA a(n)=(2*n^2-1+(-1)^(n))/8. - Paul Barry, May 27 2003

G.f.: x^2/((1-x)^2*(1-x^2)).

E.g.f.: exp(x)*(2*x^2+2*x-1)/8+exp(-x)/8.

a(n) = 2*a(n-1)-2*a(n-3)+a(n-4) [From Jaume Oliver Lafont, Dec 05 2008]

a(-n)=a(n).

a(n)=a(n-1)+int(n/2), n>0 - Adam Kertesz (adamkertesz(AT)worldnet.att.net), Sep 20 2000

a(n)=a(n-1)+a(n-2)-a(n-3)+1 [with a(-1)=a(0)=a(1)=0], a(2k)=k^2, a(2k-1)=k(k-1) - Henry Bottomley, Mar 08 2000

0*0, 0*1, 1*1, 1*2, 2*2, 2*3, 3*3, 3*4, ... with an obvious pattern.

a(n) = sum(floor(k/2), k=1..n) - Yong Kong (ykong(AT)curagen.com), Mar 10 2001

a(n) = n*floor((n - 1)/2) - (floor((n - 1)/2)*(floor((n - 1)/2)+ 1)); a(n)=a(n-2)+n-2 with a(1)=0, a(2)=0. - Santi Spadaro (spados(AT)katamail.com), Jul 13 2001

Also: a(n)=C(n, 2)-a(n-1)=A000217(n-1)-a(n-1) with a(0)=0. - Labos E. (labos(AT)ana.hu), Apr 26 2003

a(n)=sum{k=0..n, (-1)^(n-k)*C(k, 2) } - Paul Barry, Jul 01 2003

(-1)^n * partial sum of alternating triangular numbers. - Jon Perry, Dec 30 2003

a(n) = A024206(n+1) -n . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Feb 27 2004

Partial sums of A004526. - Lekraj Beedassy, Jun 30 2004

a(n)=a(n-2)+n-1, a(0)=0, a(1)=0. - Paul Barry, Jul 14 2004

a(n+1)=sum min(i, n-i), i=0..n. - Marc LeBrun, Feb 15 2005

a(n+1)=sum{k=0..floor((n-1)/2), n-2k}; a(n+1)=sum{k=0..n, k*(1-(-1)^(n+k-1))/2}; - Paul Barry, Apr 16 2005

1 + 1/(1 + 2/(1 + 4/(1 + 6/(1 + 9/(1 + 12/(1 + 16/(1 + . .))))))) = 6/(Pi^2 - 6) = 1.550546096730... - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Jun 20 2005

a(0) = 0; a(1) = 0; a(2) = 1; for n>2 a(n) = a(n-1) + ceiling(sqrt(a(n-1))). - Jonathan Vos Post, Jan 19 2006

Sequence starting (2, 2, 4, 6, 9,...) = A128174 (as an finite lower triangular matrix) * vector [1, 2, 3,...]; where A128174 = (1; 0,1; 1,0,1; 0,1,0,1;...). - Gary W. Adamson, Jul 27 2007

a(n) = sum(i=k..n, P(i,k)) where P(i,k) is the number of partitions of i into k parts. - Thomas Wieder, Sep 01 2007

a(n) = sum of row (n-2) of triangle A115514. - Gary W. Adamson, Oct 25 2007

For n>1: GreatestCommonDivisor(a(n+1), a(n)) = a(n+1) - a(n). - Reinhard Zumkeller, Apr 06 2008

a(n+3) = a(n) + A000027(n) + A008619(n+1) = a(n) + A001651 (n+1) with a(1)=0, a(2), a(3)=1 - Yosu Yurramendi (yosu.yurramendi(AT)ehu.es), Aug 10 2008

a(n) = SUM((k mod 2)*(n-k): 0<=k<=n), cf. A000035, A001477. [From Reinhard Zumkeller, Nov 05 2009]

a(n) = round((2*n^2-1)/8) = round(n^2/4) = ceil((n^2-1)/4). [From Mircea Merca, Nov 29 2010]

n*a(n+2) = 2*a(n+1)+(n+2)*a(n). Holonomic Ansatz with smallest order of recurrence. [From Thotsaporn Thanatipanonda, Dec 12 2010]

A002654 Number of ways of writing n as a sum of at most two nonzero squares, where order matters; also (number of divisors of n of form 4m+1) - (number of divisors of form 4m+3). (Formerly M0012 N0001)

COMMENTS Number of sublattices of Z X Z of index n that are similar to Z X Z; number of (principal) ideals of Z[i] of norm n.

a(A022544(n)) = 0; a(A001481(n)) > 0. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Sep 27 2008]

FORMULA Dirichlet series: (1-2^(-s))^(-1)*Product (1-p^(-s))^(-2) (p=1 mod 4) * Product (1-p^(-2s))^(-1) (p=3 mod 4) = Dedekind zeta-function of Z[i].

Coefficients in expansion of Dirichlet series Product_p (1-(Kronecker(m, p)+1)*p^(-s)+Kronecker(m, p)*p^(-2s))^(-1) for m = -16.

If n=2^k*u*v, where u is product of primes 4m+1, v is product of primes 4m+3, then a(n)=0 unless v is a square, in which case a(n) = number of divisors of u (Jacobi).

Multiplicative with a(p^e) = 1 if p = 2; e+1 if p == 1 (mod 4); (e+1) mod 2 if p == 3 (mod 4). - David W. Wilson, Sep 01, 2001

G.f. A(x) satisfies 0 = f(A(x), A(x^2), A(x^4)) where f(u, v, w) = (u - v)^2 - (v - w) * (4*w + 1). - Michael Somos, Jul 19 2004

G.f.: Sum((-1)^floor(n/2)*x^((n^2+n)/2)/(1+(-x)^n), n=1..infinity). - Vladeta Jovovic (vladeta(AT)eunet.rs), Sep 15 2004

Expansion of (eta(q^2)^10 / (eta(q) * eta(q^4))^4 - 1)/4 in powers of q.

G.f.: Sum_{k>0} x^k / (1 + x^(2*k)) = Sum_{k>0} -(-1)^k * x^(2*k-1) / (1 - x^(2*k-1)). - Michael Somos Aug 17 2005

a(4*n + 3) = a(9*n + 3) = a(9*n + 6) = 0. a(9*n) = a(2*n) = a(n). - Michael Somos Nov 01 2006

a(4*n + 1) = A008441(n). a(3*n + 1) = A122865(n). a(3*n + 2) = A122856(n). a(12*n + 1) = A002175(n). a(12*n + 5) = 2 * A121444(n). 4 * a(n) = A004018(n) unless n=0.

a(n) = SUM(A010052(k)*A010052(n-k): 1<=k<=n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Sep 27 2008]

a(n) = A001826(n) - A001842(n). - R. J. Mathar, Mar 23 2011

A002658 a(0) = a(1) = 1; for n > 0, a(n+1) = a(n)*(a(0)+...+a(n-1)) + a(n)*(a(n)+1)/2. (Formerly M1814 N0718)

COMMENTS Number of planted trees in which every node has degree <=3 and of height n; or products of height n when multiplication is commutative but non-associative.

Also called planted 3-trees or planted unary-binary trees.

The next term (which was incorrectly given) is in fact too large to include. See the b-file.

Comment from Marc LeBrun (mlb(AT)well.com): Maximum possible number of distinct new values after applying a commuting operation N times to a single initial value.

Divide the natural numbers in sets of consecutive numbers, starting with {1}, each set with number of elements equal to the sum of elements of the preceding set. The number of elements in the n-th (n>0) set gives a(n). The sets begin {1}, {2}, {3,4}, {5,6,7,8,9,10,11}, ... - Floor van

Lamoen (fvlamoen(AT)hotmail.com), Jan 16 2002

A002808 The composite numbers: numbers n of the form x*y for x > 1 and y > 1. (Formerly M3272 N1322)

COMMENTS The natural numbers 1,2,... are divided into three sets: 1 (the unit), the primes (A000040) and the composite numbers (A002808).

The number of composite numbers <= n (A065855) = n - pi(n) (A000720) - 1.

m is composite iff sigma(m)+phi(m)>2m. - Farideh Firoozbakht (mymontain(AT)yahoo.com), Jan 27 2005

The composite numbers have the semiprimes A001358 as primitive elements.

FORMULA a(n) = pi(a(n)) + 1 + n, where pi is the prime counting function.

A000005(a(n)) > 2. - [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 17 2009]

A001222(a(n)) > 1. - [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 30 2009]

A000203(a(n)) < A007955(a(n)). [From Juri-Stepan Gerasimov, Mar 17 2011]

A003136 Loeschian numbers: numbers of the form x^2 + xy + y^2; norms of vectors in A2 lattice. (Formerly M2336)

COMMENTS Equally, numbers of form x^2 - xy + y^2. [From Ray Chandler (rayjchandler(AT)sbcglobal.net), Jan 27 2009]

Also, numbers of form X^2+3Y^2 (X=y+x/2, Y=x/2). Cf. A092572 Numbers of the form x^2+3y^2 where x and y are positive integers. [From Zak Seidov (zakseidov(AT)yahoo.com), Jan 20 2009].

Equivalently, numbers n such that the coefficient of x^n in Theta3(x)*Theta3(x^3) is nonzero - Joerg Arndt, Jan 16 2011.

Equivalently, numbers n such that the coefficient of x^n in a(x) (resp. b(x)) is nonzero where a(), b() are cubic AGM functions. - Michael Somos, Jan 16 2011

Relative areas of equilateral triangles whose vertices are on a triangular lattice - Anton Sherwood (bronto(AT)pobox.com), Apr 05 2001

2 appended to a(n) (for positive n) corresponds to capsomere count in viral architectural structures (cf. A071336). - Lekraj Beedassy (blekraj(AT)yahoo.com), Apr 14 2006

The triangle in A132111 gives the enumeration: n^2 + k*n + k^2, 0<=k<=n.

The number of coat proteins at each corner of a triangular face of a virus shell. - Parthasarathy Nambi (PachaNambi(AT)yahoo.com), Sep 04 2007

FORMULA Either n=0 or else in the prime factorization of n all primes of the form 3a+2 must occur to even powers only (there is no restriction of primes congruent to 0 or 1 mod 3).

If n is in the sequence, then n^k is in the sequence (but the converse is not true). n is in the sequence iff n^(2k+1) is in the sequence. [From Ray Chandler (rayjchandler(AT)sbcglobal.net), Feb 03 2009]

A003418 a(0) = 1; for n >= 1, a(n) = least common multiple (or lcm) of {1, 2, ..., n}. (Formerly M1590)

COMMENTS Product over all primes of highest power of prime less than or equal to n. a(0) = 1 by convention.

Also smallest number such that its set of divisors contains an n-term arithmetic progression. - Reinhard Zumkeller, Dec 09 2002

An assertion equivalent to the Riemann hypothesis is: | log(a(n)) - n | < sqrt(n)*(ln(n))^2. - Lekraj Beedassy (blekraj(AT)yahoo.com), Aug 27 2006

Also the minimal exponent of the symmetric group S_n (i.e. the least positive integer a(n) for which x^a(n)=1 for all x in S_n). [From Franz Vrabec (franz.vrabec(AT)aon.at), Dec 28 2008]

Periods of the sequences b(n)=Sum{i=0..k-1}{(n+i} mod (k-i)} for k=0,1,2,3,... [From Paolo P. Lava (ppl(AT)spl.at), Feb 18 2009]

Corollary 3 of Farhi gives a simple proof that A003418(n) => 2^(n-1). The main theorem proved in Farhi is the identity lcm{binom{k,0}, binom(k,1), ..., binom(k,k) = lcm(1, 2, ..., k, k + 1)/(k + 1) for all k in N. [From Jonathan Vos Post, Jun 15 2009]

a[x]=exp(psi(x)) where psi(x)=log(lcm(1,2,...,floor(x))) is the Chebyshev function of the second kind. [From Stephen Crowley (crow(AT)crowlogic.net), Jul 04 2009]

The product of the gamma-function sampled over the set of all rational numbers in the open interval (0, 1) whose denominator in lowest terms is at most n equals ((2*pi)^(1/2)) * a(n)^(-1/2). [From Jonathan Vos Post, Jul 28 2009]

a(n) = LCM {A188666(n), A188666(n)+1, ... n}. [Reinhard Zumkeller, Apr 25 2011]

FORMULA The prime number theorem implies that LCM(1,2,...,n) = exp(n(1+o(1))) as n -> infinity. In other words, log(LCM(1,2,...,n))/n -> 1 as n -> infinity. - Jonathan Sondow (jsondow(AT)alumni.princeton.edu), Jan 17 2005

a(n)=product_{p^(floor(log n/log p))}, where p runs through primes not exceeding n (i.e. primes 2 through A007917(n)). - Lekraj Beedassy (blekraj(AT)yahoo.com), Jul 27 2004

Comment from Peter Luschny, Aug 08 2009: Greg Martin showed that a(n) = lcm{1,2,3,..,n} = Prod_{i=Farey(n),0<i<1} 2Pi/Gamma(i)^2. This can be rewritten (for n>1) as a(n) = (1/2)[Prod_{i=Farey(n),0<i<=1/2} 2sin(iPI)]^2.

Recursive formula useful for computations: a(0)=1; a(1)=1; a(n)=lcm(n,a(n-1))- Enrique Pérez Herrero, Jan 08 2011

Contribution from Enrique Pérez Herrero, Jun 01 2011: (Start)

a(n)/a(n-1)=A014963(n)

if n is a prime power p^k then a(n)=a(p^k)=p*a(n-1), otherwise a(n)=a(n-1).

a(n)=prod(k=2,n, 1+(A007947(k)-1)*floor(1/A001221(k))), for n>1. (End)

A003484 Radon function, also called Hurwitz-Radon numbers. (Formerly M0161)

COMMENTS Simon Plouffe (simon.plouffe(AT)gmail.com) observes that this sequence and A006519 (greatest power of 2 dividing n) are very similar, the difference being all zeros except for every 16-th term (see A101119 for nonzero differences). Dec 02, 2004.

FORMULA If n=2^(4*b+c)*d, 0<=c<=3, d odd, then a(n) = 8*b + 2^c.

If n=2^m*d, d odd, then a(n) = 2*m+1 if m=0 mod 4, a(n) = 2*m if m=1 or 2 mod 4, a(n) = 2*m+2 (otherwise, i.e., if m=3 mod 4).

Multiplicative with a(p^e) = 2e + a_(e mod 4) if p = 2; 1 if p > 2; where a = (1, 0, 0, 2). - David W. Wilson (davidwwilson(AT)comcast.net), Aug 01, 2001.

Dirichlet g.f. zeta(s) *(1-1/2^s)* {7*2^(-4*s) +1 +2^(3-3*s) +3*2^(1-5*s) +2^(1-s) +2^(2-6*s) +2^(2-2*s) }/ (1-2^(-4*s))^2 . - R. J. Mathar, Mar 04 2011

a((2*k+1)*2^t) = A003485(t). [Johannes W. Meijer, Jun 7 2011]

A004011 Theta series of D_4 lattice; Fourier coefficients of Eisenstein series E_{gamma,2}. (Formerly M5140)

COMMENTS D_4 is also the Barnes-Wall lattice in 4 dimensions.

E_{gamma,2} is the unique normalized modular form for Gamma_0(2) of weight 2.

FORMULA a(0) = 1; if n>0 then a(n) = 24 (sum_{d|n, d odd, d>0} d) = 24 * A000593(n).

G.f.: 1 + 24 Sum_{n>0} n x^n /(1 + x^n). a(n) = A000118(2*n) = A096727(2*n).

G.f.: Sum_{ a, b, c, d} x^(a^2 + b^2 + c^2 + d^2 + a*d + b*d + c*d). - Michael Somos Jan 11 2011

G.f. A(x) satisfies 0 = f(A(x), A(x^2), A(x^4)) where f(u, v, w) = u^2 - 2*u*v - 7*v^2 - 8*v*w + 16*w^2. - Michael Somos May 29 2005

Expansion of (1 + k^2) K(k^2)^2 / (pi/2)^2 in powers of nome q. - Michael Somos Jun 10 2006

Expansion of b(x) * b(x^2) + 3 * c(x) * c(x^2) in powers of x where b(), c() are cubic AGM functions. - Michael Somos Jan 11 2011

G.f.: (1/2) * (theta_3(z)^4 + theta_4(z)^4) = theta_3(2z)^4 + theta_2(2z)^4 = Sum_{k>=0} a(k) * x^(2*k).

G.f. is a period 1 Fourier series which satisfies f(-1 / (2 t)) = 2 (t/i)^2 f(t) where q = exp(2 pi i t). - Michael Somos Sep 11 2007

G.f. A(x) satisfies 0 = f(A(x), A(x^2), A(x^3), A(x^6)) where f(u1, u2, u3, u6) = u1^2 + 4*u2^2 + 9*u3^2 + 36*u6^2 - 2*u1*u2 - 10*u1*u3 + 10*u1*u6 + 10*u2*u3 - 40*u2*u6 - 18*u3*u6. - Michael Somos Sep 11 2007

A004018 Theta series of square lattice (or number of ways of writing n as a sum of 2 squares). (Formerly M3218)

COMMENTS Number of points in square lattice on the circle of radius sqrt(n).

Often denoted by r(n) or r_2(n).

Let b(n)=A004403(n), then sum(k=1..n, a(k)*b(n-k)) = 1 - John W. Layman

Theta series of D_2 lattice.

Let s = 16*q*(E1*E4^2/E2^3)^8 where Ek = prod(n>=1, 1-q^(k*n)) (s=k^2 where k is elliptic k), then the g.f. is hypergeom([+1/2, +1/2], [+1], s) (expansion of 2/Pi*elliptic_K(k) in powers of q). [Joerg Arndt, Aug 15 2011]

Ramanujan theta functions: f(q) := Prod_{k>=1} (1-(-q)^k) (see A121373), phi(q) := theta_3(q) := Sum_{k=-oo..oo} q^(k^2) (A000122), psi(q) := Sum_{k=0..oo} q^(k*(k+1)/2) (A10054), chi(q) := Prod_{k>=0} (1+q^(2k+1)) (A000700).

FORMULA Expansion of theta_3(q)^2 = sum(n=-inf..+inf, q^(n^2))^2 = prod(m>=1, (1-q^(2*m))^2 * (1+q^(2*m-1))^4).

Factor n as n = p1^a1 * p2^a2 * ... * q1^b1 * q2^b2 * ... * 2^c, where the p's are primes == 1 mod 4 and the q's are primes == 3 mod 4. Then a(n) = 0 if any b is odd, otherwise a(n) = 4*(1 + a1)*(1 + a2)*...

G.f. = s(2)^10/(s(1)^4*s(4)^4), where s(k) := subs(q=q^k, eta(q)) and eta(q) is Dedekind's function, cf. A010815. [Fine]

Expansion of eta(q^2)^10 / (eta(q) * eta(q^4))^4 in powers of q. - Michael Somos, Jul 19 2004

Expansion of (phi(q)^2 + phi(-q)^2) / 2 in powers of q^2 where phi() = theta3() is a Ramanujan theta function.

G.f. A(x) satisfies 0 = f(A(x), A(x^2), A(x^4)) where f(u, v, w) = (u - v)^2 - (v - w) * 4 * w. - Michael Somos, Jul 19 2004

Euler transform of period 4 sequence [4, -6, 4, -2, ...]. - Michael Somos, Jul 19 2004

Moebius transform is period 4 sequence [4, 0, -4, 0, ...]. - Michael Somos Sep 17 2007

G.f. is a period 1 Fourier series which satisfies f(-1/(4*t)) = 2*(t/i)*f(t) where q = exp(2*Pi*i*t).

The constant sqrt(Pi)/GAMMA(3/4)^2 produces the first 324 terms of the sequence when expanded in base exp(Pi), 450 digits of the constant are necessary. Simon Plouffe, March 3 2011.

A004526 Nonnegative integers repeated, floor(n/2).

COMMENTS Number of elements in the set {k: 1 <= 2k <= n}.

Apart from initial term(s), dimension of the space of weight 2n cusp forms for Gamma_0(2).

Number of ways 2^n is expressible as r^2-s^2 with s > 0. Proof: (r+s) and (r-s) both should be powers of 2, even and distinct hence a(2k)=a(2k-1)=(k-1) etc. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Sep 20 2002

Lengths of sides of Ulam square spiral; i.e. lengths of runs of equal terms in A063826. - Donald S. McDonald (don.mcdonald(AT)paradise.net.nz), Jan 09 2003

Number of partitions of n into two parts. A008619 gives partitions of n into at most two parts, so A008619(n) = A004526(n) + 1 for all n >= 0. Partial sums are A002620 (Quarter-squares). - Rick L. Shepherd (rshepherd2(AT)hotmail.com), Feb 27 2004

a(n+1) is the number of 1's in the binary expansion of the Jacobsthal number A001045(n). - Paul Barry (pbarry(AT)wit.ie), Jan 13 2005

Partitions of n+1 into two distinct parts. Example: a(8)=4 because we have [8,1],[7,2],[6,3] and [5,4]. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Apr 14 2006

Complement of A000035, since A000035(n)+2*a(n)=n. - Also equal to the partial sums of A000035. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jun 01 2007

Number of binary bracelets of n beads, two of them 0. For n>=2 a(n-2) is the number of binary bracelets of n beads, two of them 0, with 00 prohibited. [From Washington Bomfim (webonfim(AT)bol.com.br), Aug 27 2008]

Let A be the Hessenberg n by n matrix defined by: A[1,j]=j mod 2, A[i,i]:=1, A[i,i-1]=-1, and A[i,j]=0 otherwise. Then, for n>=1, a(n+1)=(-1)^n det(A). [From Milan R. Janjic (agnus(AT)blic.net), Jan 24 2010]

Let RT abbreviate rank transform (A187224). Then

RT(A004526)=A187484;

RT(A004526 without 1st term)=A026371;

RT(A004526 without 1st 2 terms)=A026367;

RT(A004526 without 1st 3 terms)=A026363. [From Clark Kimberling, Mar 10 2011]

The diameter (longest path) of the n-cycle. -Cade Herron (herrona (AT) goldmail.etsu.edu) April 14, 2011

For n>=3, a(n-1) is the number of two-color bracelets of n beads, three of them are black, having a diameter of symmetry. [Vladimir Shevelev (shevelev(AT)bgu.ac.il), May 3 2011]

FORMULA G.f.: x^2*(1+x)/(1-x^2)^2. a(n)=floor(n/2). a(n)=1+a(n-2). a(n)=a(n-1)+a(n-2)-a(n-3). a(2n)=a(2n+1)=n.

For n>0, a(n)=sum(i=1, n, (1/2)/cos(Pi*(2*i-(1-(-1)^n)/2)/(2*n+1))). - Benoit Cloitre (benoit7848c(AT)orange.fr), Oct 11 2002

a(n)=(2n-1)/4+(-1)^n/4; a(n+1)=sum{k=0..n, k*(-1)^(n+k)}; - Paul Barry (pbarry(AT)wit.ie), May 20 2003

E.g.f.: ((2x-1)exp(x)+exp(-x))/4; - Paul Barry (pbarry(AT)wit.ie), Sep 03 2003

G.f.: 1/(1-x) * sum(k>=0, t^2/(1-t^4), t=x^2^k). - Ralf Stephan, Feb 24 2004

a(n+1)=A000120(A001045(n)); - Paul Barry (pbarry(AT)wit.ie), Jan 13 2005

a(n+1)=n-a(n) - Jeremy Bem (jeremy1(AT)gmail.com), Feb 22 2007 a(n)=(n-(1-(-1)^n)/2)/2=1/2*(n-|sin(n*Pi/2)|). Likewise: a(n)=(n-A000035(n))/2. Also: a(n)=sum{0<=k<=n, A000035(k)}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), Jun 01 2007

The expression floor((x^2-1)/(2*x)) (x >= 1) produces this sequence. - Mohammad K. Azarian (azarian(AT)evansville.edu), Nov 08 2007; corrected by Maximilian Hasler, Nov 17 2008

a(n+1) = A002378(n) - A035608(n). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jan 27 2010]

a(n+1) = A002620(n+1) - A002620(n) = floor((n+1)/2)*ceiling((n+1)/2) - floor(n^2/4). [From Jonathan Vos Post (jvospost3(AT)gmail.com), May 20 2010]

For n>=2, a(n)=floor(log_2(2^a(n-1)+2^a(n-2))) [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jun 22 2010]

a(n)=A180969(2,n) - Adriano Caroli (adriano_caroli(AT)virgilio.it), Nov 24 2010

A005036 Number of ways of dissecting a polygon into n quadrilaterals. (Formerly M1491)

COMMENTS The subsequence of primes begins: 2, 5, 6257, no more through a(100). [Jonathan Vos Post, April 8, 2011]

A005100 Deficient numbers: numbers n such that sigma(n) < 2n. (Formerly M0514)

COMMENTS A number n is abundant if sigma(n) > 2n (cf. A005101), perfect if sigma(n) = 2n (cf. A000396), deficient if sigma(n) < 2n (this entry), where sigma(n) is the sum of the divisors of n (A000203).

Also, numbers n such that A033630(n) = 1. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Mar 02 2007

A005101

COMMENTS A number n is abundant if sigma(n) > 2n (this entry), perfect if sigma(n) = 2n (cf.A000396), deficient if sigma(n) < 2n (cf. A005100), where sigma(n) is the sum of the divisors of n (A000203).

While the first even abundant number is 12 = 2^2*3, the first odd abundant is 945 = 3^3*5*7, the 232th abundant number!

It appears that for n > 23, the result of (2*A001055)-A101113 is NOT 0 if n=A005101. [From Eric Desbiaux (moongerms(AT)wanadoo.fr), Jun 01 2009]

If n is a member so is every positive multiple of n. "Primitive" members are inA091191.

If n=6k (k>=2), then sigma(n)>=1+k+2*k+3*k+6*k>12*k=2*n. Thus all such n are in the sequence.

FORMULA a(n) is asymptotic to C*n with C=4.038.. (Deleglise 1998) - Benoit Cloitre (benoit7848c(AT)orange.fr), Sep 04 2002

A005117

COMMENTS 1 together with the numbers that are products of distinct primes.

Also smallest sequence with the property that a(m)*a(n) is never a square for n <> m. - Ulrich Schimke (ulrschimke(AT)aol.com), Dec 12 2001

Numbers n such that there is only one Abelian group with n elements, the cyclic group of order n (the numbers such that A000688(n) = 1). - Ahmed Fares (ahmedfares(AT)my-deja.com), Apr 25 2001

Numbers n such that A007913(n)>phi(n) - Benoit Cloitre, Apr 10 2002

a(n) = smallest m with exactly n squarefree numbers <= m. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), May 21 2002

n is squarefree <=> n divides n# where n# = product of first n prime numbers - Mohammed Bouayoun (bouyao(AT)wanadoo.fr), Mar 30 2004

Numbers n such that omega(n)=Omega(n)=A072047(n). - Lekraj Beedassy, Jul 11 2006

The lcm of any subsequence of a(n) is in a(n). - Lekraj Beedassy, Jul 11 2006

This sequence and the Beatty Pi^2/6 sequence (A059535) are "incestuous": the first 20000 terms are bounded within (-9, 14). [Ed Pegg Jr, Jul 22 2008]

Let us introduce a function D(n)=sigma_0(n)/(2^(alfa(1)+...+alfa(r)), sigma_0(n) number of divisors of n (A000005), prime factorization of n=p(1)^alfa(1) * ... * p(r)^alfa(r), alfa(1)+...+alfa(r) is sequence (A086436). Function D(n) splits the set of positive integers into subsets, according to the value of D(n). Squarefree numbers (A005117) has D(n)=1, other numbers are "deviated" from the squarefree ideal and have 0<D(n)<1. For D(n)=1/2 we have A048109, for D(n)=3/4 we haveA067295. [From Ctibor O. Zizka, Sep 21 2008]

A122840(a(n)) <= 1; A010888(a(n)) < 9. [From Reinhard Zumkeller, Mar 30 2010]

a(n)=A055229(A062838(n)) and a(n)>A055229(m) for m < A062838(n). [From Reinhard Zumkeller, Apr 09 2010]

Numbers n such that gcd(n,n')=1 where n' is the arithmetic derivative (A003415) of n. - Giorgio Balzarotti, Apr 23 2011

Numbers n such that A007913(n)=core(n)=n. [Franz Vrabec, Aug 27 2011]

FORMULA Lim n -> infinity a(n)/n=Pi^2/6 - Benoit Cloitre, May 23 2002

A039956 UNION A056911. - R. J. Mathar, May 16 2008

A005130

COMMENTS An alternating sign matrix is a matrix of 0's, 1's and -1's such that (a) the sum of each row and column is 1; (b) the nonzero entries in each row and column alternate in sign.

Contribution from Gary W. Adamson (qntmpkt(AT)yahoo.com), May 27 2009: (Start)

Starting with offset 1 = row sums of triangle A160708, and convolution square ofA160707.

a(n) is odd iff n is a Jacobsthal number [Frey and Sellers, 2000].

Starting with offset 1 = row sums of triangle A160708.

Starting (1, 2, 7,...) = convolution square of A160707: [1, 1, 3, 18, 192,...]. (End)

FORMULA a(n) = Product_{k=0..n-1} (3k+1)!/(n+k)!.

The Hankel transform of A025748 is a(n)3^binomial(n,2).

a(n) = sqrt(A049503).

A005230

COMMENTS The subsequence of primes in this partial sum begins: 2, 3, 11, a(41) = 262364233421, and no more through a(200). [From Jonathan Vos Post (jvospost3(AT)gmail.com), Feb 18 2010]

FORMULA Partial sums give Conway-Guy sequence A005318. Cf. A066777.

2*a(n*(n+1)/2 + 1) = a(n*(n+1)/2 + 2) for n>=1; limit_{n->infty} a(n+1)/a(n) = 2. - Paul D. Hanna (pauldhanna(AT)juno.com), Aug 28 2006

A005408

COMMENTS Leibniz's series: Pi/4 = Sum_{n=0..inf} (-1)^n/(2n+1) (cf. A072172).

Beginning of the ordering of the natural numbers used in Sharkovski's theorem - see the Cielsielski-Pogoda paper.

The Sharkovski ordering begins with the odd numbers >= 3, then twice these numbers, then 4 times them, then 8 times them, etc., ending with the powers of 2 in decreasing order, ending with 2^0 = 1.

Apart from initial term(s), dimension of the space of weight 2n cusp forms for Gamma_0(6).

Also continued fraction for coth(1) (A073747 is decimal expansion). - Rick L. Shepherd, Aug 07 2002

a(1) = 1; a(n) = smallest number such that a(n) + a(i) is composite for all i = 1 to n-1. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Jul 14 2003

Smallest number greater than n, not a multiple of n, but containing it in binary representation. - Reinhard Zumkeller, Oct 06 2003

Numbers n such that phi(2n)=phi(n), where phi is the Euler's totient(A000010). - Lekraj Beedassy, Aug 27 2004

Pi*sqrt(2)/4 = Sum_{n=0..inf} (-1)^floor(n/2)/(2n+1) = 1 + 1/3 - 1/5 - 1/7 + 1/9 + 1/11 ... [since periodic f(x)=x over -Pi<x<Pi = 2(sin(x)/1 - sin(2x)/2 + sin(3x)/3 - ...) using x = Pi/4 (Maor)] - Gerald McGarvey, Feb 04 2005

a(n) = L(n,-2)*(-1)^n, where L is defined as in A108299. - Reinhard Zumkeller, Jun 01 2005

For n>1, numbers having 2 as an anti-divisor. - Alexandre Wajnberg (alexandre.wajnberg(AT)skynet.be), Oct 02 2005

a(n) = shortest side a of all integer-sided triangles with sides a<=b<=c and inradius n >= 1.

First differences of squares (A000290). - Lekraj Beedassy, Jul 15 2006

The odd numbers are the solution to the simplest recursion arising when assuming that the algorithm "merge sort" could merge in constant unit time, i.e. T(1):=1, T(n):=T(floor(n/2)) + T(ceiling(n/2)) + 1. - Peter C. Heinig (algorithms(AT)gmx.de), Oct 14 2006

2n-5 counts the permutations in S_n which have zero occurrences of the pattern 312 and one occurrence of the pattern 123. - David Hoek (david.hok(AT)telia.com), Feb 28 2007

For n>0: number of divisors of (n-1)th power of any squarefree semiprime: a(n)=A000005(A001248(k)^(n-1)); a(n) = A000005(A000302(n-1)) = A000005(A001019(n-1)) = A000005(A009969(n-1)) = A000005(A087752(n-1)). - Reinhard Zumkeller, Mar 04 2007

For n > 2, a(n-1) is the least integer not the sum of < n n-gonal numbers (0 allowed). - Jonathan Sondow, Jul 01 2007

A134451(a(n)) = ABS(A134452(a(n))) = 1; union of A134453 and A134454. - Reinhard Zumkeller, Oct 27 2007

Numbers n such that sigma(2n)=3*sigma(n). - Farideh Firoozbakht, Feb 26 2008

a(n) = A139391(A016825(n)) = A006370(A016825(n)). - Reinhard Zumkeller, Apr 17 2008

Number of divisors of 4^(n-1) for n>0. - J. Lowell (jhbubby(AT)mindspring.com), Aug 30 2008

Equals INVERT transform of A078050 (signed - Cf. comments); and row sums of triangleA144106. [From Gary W. Adamson, Sep 11 2008]

odd numbers(n) = 2*n+1 = square pyramidal number(3*n+1) / triangular number(3*n+1) [From Pierre CAMI, Sep 27 2008]

A000035(a(n))=1, A059841(a(n))=0. [From Reinhard Zumkeller, Sep 29 2008]

Multiplicative closure of A065091. [From Reinhard Zumkeller, Oct 14 2008]

a(n) is also the maximum number of triangles that n+2 points in the same plane can determine. 3 points determine max 1 triangle; 4 points can give 3 triangles; 5 points can give 5; 6

points can give 7 etc. [From Carmine Suriano, Jun 08 2009]

Binomial transform of A130706, inverse binomial transform of A001787(without the initial 0). [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Sep 17 2009]

Also the 3-rough numbers: positive integers that have no prime factors less than 3. [From Michael Porter, Oct 08 2009]

0r n without 2 as prime factor. [From Juri-StepanGerasimov, Nov 19 2009]

Given an L(2,1) labeling l of a graph G, let k be the maximum label assigned by l. The minimum k possible over all L(2,1) labelings of G is denoted by lambda(G). For n > 0, this sequence gives lambda(K_{n+1}) where K_{n+1} is the complete graph on n+1 vertices. [From Kailasam Viswanathan Iyer (kvi(AT)nitt.edu), Dec 19 2009]

A176271 = odd numbers seen as a triangle read by rows: a(n)=A176271(A002024(n+1),A002260(n+1)). [From Reinhard Zumkeller, Apr 13 2010]

For n >= 1, a(n-1) = numbers k such that arithmetic mean of the first k positive integers is integer. A040001(a(n-1)) = 1. See A145051 and A040001. [From Jaroslav Krizek, May 28 2010]

Union of A179084 and A179085. [From Reinhard Zumkeller, Jun 28 2010]

For n>0, continued fraction [1,1,n] = (n+1)/a(n); e.g. [1,1,7] = 8/15 [From Gary W. Adamson, Jul 15 2010]

Numbers that are the sum of two sequential integers. [From Dominick Cancilla, Aug 09 2010]

Cf. property described by Gary Detlefs in A113801: more generally, these numbers are of the form (2*h*n+(h-4)*(-1)^n-h)/4 (h and n in A000027), then ((2*h*n+(h-4)*(-1)^n-h)/4)^2-1=0 (mod h); in our case, a(n)^2-1=0 (mod 4). Also a(n)^2-1=0 (mod 8). [From Bruno Berselli, Nov 17 2010]

A004767 = a(a(n)). [Reinhard Zumkeller, Jun 27 2011]

For n>=3 they are the numbers for which the product of their proper divisors divides the product of their anti-divisors [Paolo P. Lava, Jul 7 2011]

FORMULA a(n) = 2*n + 1. a(-1 - n) = -a(n). a(n+1) = a(n) + 2.

G.f.: (1 + x) / (1 - x)^2.

E.g.f.: (1 + 2*x) * exp(x).

Euler transform of length 2 sequence [3, -1]. - Michael Somos Mar 30 2007

G.f. A(x) satisfies 0 = f(A(x), A(x^2)) where f(u, v) = v * (1 + 2*u) * (1 - 2*u + 16*v) - (u - 4*v)^2 * (1 + 2*u + 2*u^2). - Michael Somos Mar 30 2007

a(n) = b(2*n + 1) where b(n) = n if n is odd is multiplicative.

a(n)=(n+1)^2-n^2. G.f. g(x)=sum{k>=0, x^floor(sqr(k))}=sum{k>=0, x^A000196(k)}. - Hieronymus Fischer (Hieronymus.Fischer(AT)gmx.de), May 25 2007

a(0)=1, a(1)=3, a(n)=2a(n-1)-a(n-2). - Jaume Oliver i Lafont (joliverlafont(AT)gmail.com), May 07 2008

A005408(n)=A000330(A016777(n))/A000217(A016777(n)) [From Pierre CAMI, Sep 27 2008]

a(n)=2*a(n-1)-a(n-2); a(0)=1, a(1)=3. [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 03 2008]

a(n) = A034856(n+1) - A000217(n) = A005843(n) + A000124(n) - A000217(n) = A005843(n) + 1. [From Jaroslav Krizek, Sep 05 2009]

a(n) = (n - 1) + n (sum of two sequential integers) [From Dominick Cancilla, Aug 09 2010] a(n) = 4*A000217(n)+1 - 2*sum[i=1..n-1] a(i) for n>1. [From Bruno Berselli, Nov 17 2010]

n*a(2n+1)^2+1 = (n+1)*a(2n)^2; e.g., 3*15^2+1 = 4*13^2 - Charlie Marion, Dec 31 2010

A005470

COMMENTS Euler transform of A003094 - Christian G. Bower (bowerc(AT)usa.net)

A005588

FORMULA Reference gives a complicated recurrence.

A005811

COMMENTS Starting with a(1)=0 mirror all initial 2^k segments and increase by one.

a(n) gives the net rotation (measured in right angles) after taking n steps along a dragon curve. - Christopher Hendrie (hendrie(AT)acm.org), Sep 11 2002

This sequence generates A082410: (0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 1...) and A014577; identical to the latter except starting 1, 1, 0...; by writing a "1" if a(n+1) > a(n); if not, write "0". E.g. A014577(2) = 0, since a(3) < a(2), or 1 < 2. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 20 2003

Starting with 1 = partial sums of A034947: (1, 1, -1, 1, 1, -1, -1, 1, 1, 1,...). - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jul 23 2008

The composer Per Norgard's name is also written in the OEIS as Per Noergaard.

FORMULA a(2^k + i) = a(2^k - i + 1) + 1 for k >= 0 and 0 < i <= 2^k. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Aug 14 2001

a(2n+1) = 2a(n)-a(2n)+1, a(4n) = a(2n), a(4n+2) = 1+a(2n+1).

a(j+1) = a(j) + (-1)^A014707[j] - Christopher Hendrie (hendrie(AT)acm.org), Sep 11 2002

G.f.: 1/(1-x) * sum(k>=0, x^2^k/(1+x^2^(k+1))). - Ralf Stephan, May 2 2003

Delete the 0, make subsets of 2^n terms; and reverse the terms in each subset to generate A088696. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 19 2003

a(0)=0, a(2n) = a(n) + [n odd], a(2n+1) = a(n) + [n even]. - Ralf Stephan (ralf(AT)ark.in-berlin.de), Oct 20 2003

a(n) = sum(k=1, n, (-1)^((k/2^A007814(k)-1)/2)) = sum(k=1, n, (-1)^A025480(k-1)). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Oct 29 2003

A005843

COMMENTS -2, -4, -6, -8, -10, -12, -14, ... are the trivial zeros of the Riemann zeta function. - Vivek Suri (vsuri(AT)jhu.edu), Jan 24 2008

If a 2-set Y and an (n-2)-set Z are disjoint subsets of an n-set X then a(n-2) is the number of 2-subsets of X intersecting both Y and Z. - Milan R. Janjic (agnus(AT)blic.net), Sep 19 2007

A134452(a(n)) = 0; A134451(a(n)) = 2 for n>0. - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Oct 27 2007

Omitting the initial zero gives the number of prime divisors with multiplicity of product of terms of n-th row of A077553. - Ray Chandler (rayjchandler(AT)sbcglobal.net), Aug 21 2003

A059841(a(n))=1, A000035(a(n))=0. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Sep 29 2008]

Contribution from Eric Desbiaux (moongerms(AT)wanadoo.fr), Oct 28 2008: (Start)

(APSO) Alternating partial sums of

(a-b+c-d+e-f+g...)=(a+b+c+d+e+f+g...)-2*(b+d+f...)

it appears that APSO A005843 =

A052928 = A002378 - 2*(A116471)

A116471=2*A008794

(End)

A056753(a(n)) = 1. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Aug 23 2009]

Twice the nonnegative numbers. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Dec 12 2009]

The number of hydrogen atoms in straight-chain (C(n)H(2n+2)), branched (C(n)H(2n+2), n > 3), and cyclic, n-carbon alkanes (C(n)H(2n), n > 2). [From Paul Muljadi (paulmuljadi(AT)yahoo.com), Feb 18 2010]

For n >= 1; a(n) = the smallest numbers m with the number of steps n of iterations of {r - (smallest prime divisor of r)} needed to reach 0 starting at r = m. SeeA175126 and A175127. A175126(a(n)) = A175126(A175127(n)) = n. Example (a(4)=8): 8-2=6, 6-2=4, 4-2=2, 2-2=0; iterations has 4 steps and number 8 is the smallest number with such result. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Feb 15 2010]

For n >= 1, a(n) = numbers k such that arithmetic mean of the first k positive integers is not integer. A040001(a(n)) > 1. See A145051 and A040001. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), May 28 2010]

Union of A179082 and A179083. [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jun 28 2010]

The Hosoya index H(n) of the n-star graph S_n is given by H(2n-1) = 0 and H(2n) = a(n). [From Eric Weisstein, Jul 11 2011]

FORMULA G.f.: 2*x/(1-x)^2.

Inverse binomial transform of A036289, n*2^n. - Joshua Zucker (joshua.zucker(AT)stanfordalumni.org), Jan 13 2006

a(0)=0, a(1)=2, a(n)=2a(n-1)-a(n-2). - Jaume Oliver i Lafont (joliverlafont(AT)gmail.com), May 07 2008

a(n)=Sum{k=1,n}floor(6n/4^k+1/2) [From Vladimir Shevelev (shevelev(AT)bgu.ac.il), Jun 04 2009]

a(n) = A034856(n+1) - A000124(n) = A000217(n) + A005408(n) - A000124(n) = A005408(n) - 1. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Sep 05 2009]

A006318

COMMENTS The number of perfect matchings in a triangular grid of n squares (n=1,4,9,16,25...). - Roberto E. Martinez II (martinez(AT)deas.harvard.edu), Nov 05 2001

a(n)=number of subdiagonal paths from (0,0) to (n,n) consisting of steps East (1,0), North (0,1) and Northeast (1,1) (sometimes called royal paths). - David Callan (callan(AT)stat.wisc.edu), Mar 14 2004

Twice A001003 (except for the first term).

a(n)=number of dissections of a regular (n+4)-gon by diagonals that do not touch the base. (A diagonal is a straight line joining two nonconsecutive vertices and dissection means the diagonals are noncrossing though they may share an endpoint. One side of the (n+4)-gon is designated the base.) Example. a(1)=2 because a pentagon has only 2 such dissections: the empty one and the one with a diagonal parallel to the base. - David Callan (callan(AT)stat.wisc.edu), Aug 02 2004

Comments from Jonathan Vos Post, Dec 23, 2004: "The only prime in this sequence is 2. The semiprimes (intersection with A001358) are a(2)=6, a(3)=22, a(4)=394, a(9)=206098 and a(215) correspond 1-to-1 with prime super-Catalan numbers also called prime little Schroeder numbers (intersection of A001003 and A000040) which are listed as A092840 and indexed as A092839.

"The 3-almost prime large Schroeder numbers a(7)=8558, a(11)=5293446, a(17)=111818026018, a(19)=3236724317174, a(21)=95149655201962 (intersection ofA006318 and A014612) correspond 1-to-1 with semiprime super-Catalan numbers also called semiprime little Schroeder numbers (intersection of A001003 and A001358) which are listed as A101619 and indexed as A101618. These relationships all derive from the fact that a(n) = 2*A001003(n).

"Eric Weisstein comments that the Schroeder numbers bear the relationship to the Delannoy numbers [A001850] as the Catalan numbers [A000108] do to the binomial coefficients."

a(n)=number of lattice paths from (0,0) to (n+1,n+1) consisting of unit steps north N=(0,1) and variable-length steps east E=(k,0) with k a positive integer, that stay strictly below the line y=x except at the endpoints. For example, a(2)=6 counts 111NNN, 21NNN, 3NNN, 12NNN,11N1NN, 2N1NN (east steps indicated by their length). If the word "strictly" is replaced by "weakly", the counting sequence becomes the little Schroeder numbers A001003 (offset). - David Callan (callan(AT)stat.wisc.edu), Jun 07 2006

a(n)=number of dissections of a regular (n+3)-gon with base AB that do not contain a triangle of the form ABP with BP a diagonal. Example. a(1)=2 because the square D-C | | A-B has only 2 such dissections: the empty one and the one with the single diagonal AC (although this dissection contains the triangle ABC, BC is not a diagonal). - David Callan (callan(AT)stat.wisc.edu), Jul 14 2006

a(n) = number of (colored) Motzkin n-paths with each upstep and each flatstep at ground level getting one of 2 colors and each flatstep not at ground level getting one of 3 colors. Example. With their colors immediately following upsteps/flatsteps, a(2) = 6 counts U1D, U2D, F1F1, F1F2, F2F1, F2F2. - David Callan (callan(AT)stat.wisc.edu), Aug 16 2006

a(n)=number of separable permutations, i.e. permutations avoiding 2413 and 3142, see Shapiro and Stephens. - Vince Vatter (vince(AT)mcs.st-and.ac.uk), Aug 16 2006

The Hankel transform of this sequence is A006125(n+1)=[1, 2, 8, 64, 1024, 32768, ...] ; example : Det([1,2,6,22 ; 2,6,22,90 ; 6,22,90,394 ; 22,90,394,1806])= 64 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Sep 03 2006

Triangle A144156 has row sums = A006318 with left border A001003. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Sep 12 2008]

a(n) is also the number of order-preserving and order-decreasing partial transformations (of an n-chain). Equivalently, it is the order of the Schroeder monoid, PC sub n. [From A. Umar (aumarh(AT)squ.edu.om), Oct 02 2008]

Sum_{n=0...infinity} a(n)/10^n-1 = [9-sqrt(41)]/2. 1/Sqrt(41)= sum_{n=0...infinity} Delannoy number(n)/10^n. [From M. Dols (markdols99(AT)yahoo.com), Jun 22 2010]

a(n) is also the dimension of the space Hoch(n) related to Hochschild two cocyles. [From Ph. Leroux (ph_ler_math(AT)yahoo.com), Aug 24 2010]

Let W=(w(n,k)) denote the augmentation triangle (as at A193091) of A154325; then w(n,n)=A006318(n). [From Clark Kimberling, Jul 30 2011]

FORMULA G.f.: (1-x-(1-6*x+x^2)^(1/2))/(2*x).

a(n) = 2*hypergeom([-n+1, n+2], [2], -1). - Vladeta Jovovic (vladeta(AT)eunet.rs), Apr 24 2003

For n>0, a(n)=(1/n)*sum(k=0, n, 2^k*C(n, k)*C(n, k-1)) - Benoit Cloitre (benoit7848c(AT)orange.fr), May 10 2003

The g.f. satisfies (1-x)A(x)-xA(x)^2 = 1. - Ralf Stephan (ralf(AT)ark.in-berlin.de), Jun 30 2003

For the asymptotic behavior see A001003 (remembering that A006318 = 2*A001003). - N. J. A. Sloane, Apr 10 2011.

Row sums of A088617 and A060693. a(n) = sum (k=0..n, C(n+k, n)*C(n, k)/k+1). - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Nov 28 2003

With offset 1 : a(1)=1, a(n)=a(n-1)+sum(i=1, n-1, a(i)*a(n-i)). - Benoit Cloitre (benoit7848c(AT)orange.fr), Mar 16 2004

a(n)=sum(k=0, n, A000108(k)*binomial(n+k, n-k)) - Benoit Cloitre (benoit7848c(AT)orange.fr), May 09 2004

a(n) = Sum_{k = 0..n} A011117(n, k). - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jul 10 2004

a(n) = (CentralDelannoy[n+1] - 3 CentralDelannoy[n])/(2n) = (-CentralDelannoy[n+1] + 6 CentralDelannoy[n] - CentralDelannoy[n-1])/2 for n>=1 where CentralDelannoy isA001850. - David Callan (callan(AT)stat.wisc.edu), Aug 16 2006

The Hankel transform of this sequence is A006125(n+1)=[1, 2, 8, 64, 1024, 32768, ...] ; example : Det([1,2,6,22 ; 2,6,22,90 ; 6,22,90,394 ; 22,90,394,1806])= 64 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Sep 03 2006

Comment from Peter John (peter.john(AT)tu-ilmenau.de), Oct 19 2006: Define the general Delannoy numbers d(i,j) as in A001850. Then a(k) = d(2*k,k) - d(2*k,k-1) and a(0).=1, sum[{(-1)^j}*{d(n,j)+d(n-1,j-1)}*a(n-j)] = 0, j=0,1,...,n.

Comment from Gary W. Adamson (qntmpkt(AT)yahoo.com), Oct 27 2008: Given an integer t >= 1 and initial values u = [a_0, a_1, ..., a_{t-1}], we may define an infinite sequence Phi(u) by setting a_n = a_{n-1} + a_0*a_{n-1} + a_1*a_{n-2} + ... + a_{n-2}*a_1 for n >= t. For example Phi([1]) is the Catalan numbers A000108. The present sequence is (essentially) Phi([2]).

G.f.: 1/(1-2x/(1-x/(1-2x/(1-x/(1-2x/(1-x/(1-2x/(1-x/(1-2x/(1-x.... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Dec 08 2008]

G.f.: 1/(1-x-x/(1-x-x/(1-x-x/(1-x-x/(1-x-x/(1-... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Jan 29 2009]

a(n) ~ ((3+2*sqrt(2))^n)/(n*sqrt(2*pi*n)*sqrt(3*sqrt(2)-4))*(1-(9*sqrt(2)+24)/(32*n)+...) [From G. Nemes (nemesgery(AT)gmail.com), Jan 25 2009]

Logarithmic derivative yields A002003. [From Paul D. Hanna (pauldhanna(AT)juno.com), Oct 25 2010]

a(n) = the upper left term in M^(n+1), M = the production matrix:

1, 1, 0, 0, 0, 0,...
1, 1, 1, 0, 0, 0,...
2, 2, 1, 1, 0, 0,...
4, 4, 2, 1, 1, 0,...
8, 8, 8, 2, 1, 1,...
... - Gary W. Adamson, Jul 08 2011

a(n) is the sum of top row terms in Q^n, Q = an infinite square production matrix as follows:

1, 1, 0, 0, 0, 0,...

1, 1, 2, 0, 0, 0,...
1, 1, 1, 2, 0, 0,...
1, 1, 1, 1, 2, 0,...
1, 1, 1, 1, 1, 2,...
... - Gary W. Adamson, Aug 23 2011

A006530

COMMENTS The unit 1 is not a prime number (although it has been considered so in the past). 1 is the empty product of prime numbers, thus 1 has no largest prime factor. - Daniel Forgues, Jul 05 2011

a(n) = A027748(n, A001221(n)) = A027746(n, A001222(n)); a(n)^A071178(n) = A053585(n). [Reinhard Zumkeller, Aug 27 2011]

A006882

COMMENTS Product of pairs of successive terms gives factorials in increasing order. - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Oct 17 2002

a(n) = number of down-up permutations on [n+1] for which the entries in the even positions are increasing. For example, a(3)=3 counts 2143, 3142, 4132. Also, a(n) = number of down-up permutations on [n+2] for which the entries in the odd positions are decreasing. For example, a(3)=3 counts 51423, 52413, 53412. - David Callan (callan(AT)stat.wisc.edu), Nov 29 2007

The double factorial of a positive integer n is the product of the positive integers <= n that have the same parity as n. - Peter Luschny, Jun 23 2011

FORMULA a(n) = prod(i=0..floor((n-1)/2), n-2*i)

E.g.f.: 1+e^(x^2/2) x (1+Sqrt[Pi/2] Erf[x/Sqrt[2]]) - wouter.meeussen(AT)pandora.be Thu Mar 08 07:17:05 2001

Satisfies a(n+3)*a(n) - a(n+1)*a(n+2) = n! [Putnam Contest]

n!! = 2^[(n + 1)/2]/sqrt(Pi)*Gamma(n/2 + 1)*{[sqrt(Pi)/2^(1/2) + 1]/2 + (-1)^n*[sqrt(Pi)/2^(1/2)-1]/2} - Paolo P. Lava (ppl(AT)spl.at), Jul 24 2007

a(n)=2^{[1+2*n-cos(n*Pi)]/4}*Pi^{[cos(n*Pi)-1]/4}*Gamma(1+1/2*n) - Paolo P. Lava (ppl(AT)spl.at), Jul 24 2007

A006894

COMMENTS Representation requires n triangular numbers with greedy algorithm.

Comment from Marc LeBrun (mlb(AT)well.com): Maximum possible number of distinct values after applying a commuting operation from 0 to N times to a single initial value.

Divide the natural numbers in sets of consecutive numbers, starting with {1}, each set with number of elements equal to the sum of elements of the preceding set. The greatest element of the n-th set gives a(n). The sets begin {1}, {2}, {3,4}, {5,6,7,8,9,10,11}, ... - Floor van Lamoen (fvlamoen(AT)hotmail.com), Jan 16 2002

a(n+1) = (a(n)) th triangular numbers + 1 = A000217(a(n)) + 1. a(n) = A072638(n-1) + 1. [From Jaroslav Krizek (jaroslav.krizek(AT)atlas.cz), Sep 11 2009]

FORMULA Partial sums of A002658; a(n+1) = a(n)(a(n)+1)/2 + 1 (from Marc LeBrun).

Sequence arises from a self-recursive process: a[1]=1, a[n]=a[n-1]*(a[n-1]+1)/2+1. E.g. a(1)=1, a(2)=1*2/2+1=2, a(3)=2*3/2+1=4, a(4)=4*5/2+1=11, a(5)=11*12/2+1=67... - Miklos Kristof (kristmikl(AT)freemail.hu), Dec 11 2007

A006966 Number of lattices on n unlabeled nodes.(Formerly M1486)

COMMENTS Also commutative idempotent monoids. Also commutative idempotent semigroups of order n-1.

A007318 Pascal's triangle read by rows: C(n,k) = binomial(n,k) = n!/(k!*(n-k)!), 0<=k<=n.(Formerly M0082

COMMENTS C(n,k) = number of k-element subsets of an n-element set.

Row n gives coefficients in expansion of (1+x)^n.

C(n+k-1,n-1) is the number of ways of placing k indistinguishable balls into n boxes (the "bars and stars" argument - see Feller).

C(n-1,m-1) is the number of compositions of n with m summands.

C(n,k) is the number of lattice paths from (0,0) to (n,k) using steps (1,0) and (1,1). [Joerg Arndt, Jul 01 2011]

If thought of as an infinite lower triangular matrix, inverse begins:

+1

-1 +1

+1 -2 +1

-1 +3 -3 +1

+1 -4 +6 -4 +1

The string of 2^n palindromic binomial coefficients starting after the A006516(n)-th entry are all odd. - Lekraj Beedassy (blekraj(AT)yahoo.com), May 20 2003

C(n+k-1,n-1) is the number of standard tableaux of shape (n,1^k). - Emeric Deutsch (deutsch(AT)duke.poly.edu), May 13 2004

Can be viewed as an array, read by antidiagonals, where the entries in the first row and column are all 1's and A(i,j) = A(i-1,j) + A(i,j-1) for all other entries. The determinants of all its n X n subarrays starting at (0,0) are all 1. - Gerald McGarvey (Gerald.McGarvey(AT)comcast.net), Aug 17 2004

Also the lower triangular readout of the exponential of a matrix whose entry {j+1,j} equals j+1 (and all other entries are zero). - Joseph Biberstine (jrbibers(AT)indiana.edu), May 26 2006

C(n-3,k-1) counts the permutations in S_n which have zero occurrences of the pattern 231 and one occurrence of the pattern 132 and k descents. C(n-3,k-1) also counts the permutations in S_n which have zero occurrences of the pattern 231 and one occurrence of the pattern 213 and k descents. - David Hoek (david.hok(AT)telia.com), Feb 28 2007

Inverse of A130595 (as an infinite lower triangular matrix). - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Aug 21 2007

Consider integer lists LL of lists L of the form LL=[m#L]=[m#[k#2]] (where '#' means 'times') like LL(m=3,k=3) = [[2,2,2],[2,2,2],[2,2,2]]. The number of the integer list partitions of LL(m,k) is equal to C(m+k,k) if multiple partitions like [[1,1],[2],[2]] and [[2],[2],[1,1]] and [[2],[1,1],[2]] are count only once. For the example we find 4*5*6/3! = 20 = C(6,3). - Thomas Wieder (thomas.wieder(AT)t-online.de), Oct 03 2007

The infinitesimal generator for the Pascal triangle and its inverse is A132440. - Tom Copeland (tcjpn(AT)msn.com), Nov 15 2007

Row n>=2 gives the number of k-digit (k>0) base n numbers with strictly decreasing digits; e.g. row 10 for A009995. Similarly, row n-1>=2 gives the number of k-digit (k>1) base n numbers with strictly increasing digits; see A009993 and compare A118629. - Rick L. Shepherd (rshepherd2(AT)hotmail.com), Nov 25 2007

Comments from Lee Naish (lee(AT)cs.mu.oz.au), Mar 07 2008: (Start) C(n+k-1, k) is the number of ways a sequence of length k can be partitioned into n subsequences (see the Naish link).

C(n+k-1, k) is also the number of n- (or fewer) digit numbers written in radix at least k whose digits sum to k. For example, in decimal, there are C(3+3-1,3)=10 3-digit numbers whose digits sum to 3 (see A052217) and also C(4+2-1,2)=10 4-digit numbers whose digits sum to 2 (see A052216). This relationship can be used to generate the numbers of sequences A052216 to A052224 (and further sequences using radix greater than 10). (End)

Denote by sigma_k(x_1,x_2,...,x_n) the elementary symmetric polynomials. Then: C(2n+1,2k+1)=sigma_{n-k}(x_1,x_2,...,x_n), where x_i=tan^2(i*Pi/(2n+1)),(i=1,2,...,n). C(2n,2k+1)=2n*sigma_{n-1-k}(x_1,x_2,...,x_{n-1}), where x_i=tan^2(i*Pi/(2n)), (i=1,2,...,n-1). C(2n,2k)=sigma_{n-k}(x_1,x_2,...,x_n), where x_i=tan^2((2i-1)Pi/(4n)), (i=1,2,...,n). C(2n+1,2k)=(2n+1)sigma_{n-k}(x_1,x_2,...,x_n), where x_i=tan^2((2i-1)Pi/(4n+2)), (i=1,2,...,n). - Milan R. Janjic (agnus(AT)blic.net), May 07 2008

Given matrices R and S with R(n,k) = C(n,k)*r(n-k) and S(n,k) = C(n,k)*s(n-k), then R*S = T where T(n,k) = C(n,k)*[r(.)+s(.)]^(n-k), umbrally. And, the e.g.f.s for the row polynomials of R, S and T are, respectively, exp(x*t)*exp[r(.)*x], exp(x*t)*exp[s(.)*x] and exp(x*t)*exp[r(.)*x]*exp[s(.)*x] = exp{[t+r(.)+s(.)]*x}. The row polynomials are essentially Appell polynomials. See A132382 for an example. [From Tom Copeland (tcjpn(AT)msn.com), Aug 21 2008]

Contribution from Clark Kimberling (ck6(AT)evansville.edu), Sep 15 2008: (Start)

As the rectangle R(m,n)=C(m+n-2,m-1), the weight array W (defined

generally at A114112) of R is essentially R itself, in the sense that

if row 1 and column 1 of W=A144225 are deleted, the remaining array is R. (End)

If A007318 = M as infinite lower triangular matrix, M^n gives A130595, A023531, A007318, A038207, A027465, A038231, A038243, A038255, A027466, A038279, A038291, A038303, A038315, A038327, A133371, A147716, A027467 for n=-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 respectively. [From Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 11 2008]

The coefficients of the polynomials with e.g.f. exp(x*t)*(cosh(t)+sinh(t)). [From Peter Luschny , Jul 09 2009]

Contribution from Johannes W. Meijer, Sep 22 2010: (Start)

The triangle or chess sums, see A180662 for their definitions, link Pascal's triangle with twenty different sequences, see the crossrefs. All sums come in pairs due to the symmetrical nature of this triangle. The knight sums Kn14 - Kn110 have been added. It is remarkable that all knight sums are related to the Fibonacci numbers, i.e. A000045, but none of the others.

(End)

C(n,k) is also the number of ways to distribute n+1 balls into k+1 urns so that each urn gets at least one ball. See example in the example section below.

C(n,k) is the number of increasing functions from {1,...,k} to {1,...,n} since there are C(n,k) ways to choose the k distinct, ordered elements of the range from the codomain {1,...,n}. See example in the example section below. [From Dennis Walsh, April 7 2011]

FORMULA a(n, k) = C(n,k) = binomial(n, k).

C(n, k) = C(n-1, k) + C(n-1, k-1).

a(n+1, m) = a(n, m)+a(n, m-1), a(n, -1) := 0, a(n, m) := 0, n<m; a(0, 0)=1.

C(n, k)=n!/(k!(n-k)!) if 0<=k<=n, otherwise 0.

G.f.: 1/(1-y-x*y)=Sum(C(n, k)*x^k*y^n, n, k>=0)

G.f.: 1/(1-x-y)=Sum(C(n+k, k)*x^k*y^n, n, k>=0).

G.f. for row n: (1+x)^n = sum(k=0..n, C(n, k)x^k).

G.f. for column n: x^n/(1-x)^n.

E.g.f.: A(x, y)=exp(x+x*y).

E.g.f. for column n: x^n*exp(x)/n!.

In general the m-th power of A007318 is given by: T(0, 0) = 1, T(n, k) = T(n-1, k-1) + m*T(n-1, k), where n is the row-index and k is the column; also T(n, k) = m^(n-k) C(n, k).

Triangle T(n, k) read by rows; given by A000007 DELTA A000007, where DELTA is Deleham's operator defined in A084938.

With P(n+1) = the number of integer partitions of (n+1), p(i) = the number of parts of the i-th partition of (n+1), d(i) = the number of different parts of the i-th partition of (n+1), m(i, j) = multiplicity of the j-th part of the i-th partition of (n+1), sum_[p(i)=k]_{i=1}^{P(n+1)} = sum running from i=1 to i=P(n+1) but taking only partitions with p(i)=(k+1) parts into account, prod_{j=1}^{d(i)} = product running from j=1 to j=d(i) one has B(n, k) = sum_[p(i)=(k+1)]_{i=1}^{P(n+1)} 1/prod_{j=1}^{d(i)} m(i, j)! E.g. B(5, 3) = 10 because n=6 has the following partitions with m=3 parts: (114), (123), (222). For their multiplicities one has: (114): 3!/(2!*1!) = 3, (123): 3!/(1!*1!*1!) = 6, (222): 3!/3! = 1. The sum is 3+6+1=10=B(5, 3). - Thomas Wieder (wieder.thomas(AT)t-online.de), Jun 03 2005

C(n, k) = Sum_{j, 0<=j<=k} = (-1)^j*C(n+1+j, k-j)*A000108(j) . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 10 2005

G.f.: 1 + x(1 + x) + x^3(1 + x)^2 + x^6(1 + x)^3 + ... - Michael Somos Sep 16 2006

Sum_{k, 0<=k<=[n/2]} x^(n-k)*T(n-k,k)= A000007(n), A000045(n+1), A002605(n), A030195(n+1), A057087(n), A057088(n), A057089(n), A057090(n), A057091(n), A057092(n), A057093(n) for x= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 respectively . Sum_{k, 0<=k<=[n/2]} (-1)^k*x^(n-k)*T(n-k,k)= A000007(n), A010892(n), A009545(n+1), A057083(n), A001787(n+1), A030191(n), A030192(n), A030240(n), A057084(n), A057085(n+1), A057086(n), A084329(n+1) for x= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20 respectively . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Sep 16 2006

C(n,k) <= A062758(n) for n > 1. - Reinhard Zumkeller, Mar 04 2008

C(t+p-1, t) = Sum(i=0..t, C(i+p-2, i)) = Sum(i=1..p, C(i+t-2, t-1)) A binomial number is the sum of its left parent and all its right ancestors, which equals the sum of its right parent and all its left ancestors. - Lee Naish (lee(AT)cs.mu.oz.au), Mar 07 2008

Contribution from Paul D. Hanna, Mar 24 2011: (Start)

Let A(x) = Sum_{n>=0} x^(n(n+1)/2)*(1+x)^n be the g.f. of the flattened triangle:

A(x) = 1 + (x + x^2) + (x^3 + 2*x^4 + x^5) + (x^6 + 3*x^7 + 3*x^8 + x^9) +...

then A(x) equals the series Sum_{n>=0} (1+x)^n*x^n*Product_{k=1..n} (1-(1+x)*x^(2k-1))/(1-(1+x)*x^(2k));

also, A(x) equals the continued fraction 1/(1- x*(1+x)/(1+ x*(1-x)*(1+x)/(1- x^3*(1+x)/(1+ x^2*(1-x^2)*(1+x)/(1- x^5*(1+x)/(1+ x^3*(1-x^3)*(1+x)/(1- x^7*(1+x)/(1+ x^4*(1-x^4)*(1+x)/(1- ...))))))))).

These formulae are due to (1) a q-series identity and (2) a partial elliptic theta function expression. (End)

A008275 Triangle read by rows of Stirling numbers of first kind, s(n,k), n >= 1, 1<=k<=n.

COMMENTS The unsigned numbers are also called Stirling cycle numbers: |s(n,k)| = number of permutations of n objects with exactly k cycles.

With P(n) = the number of integer partitions of n, T(i,n) = the number of parts of the i-th partition of n, D(i,n) = the number of different parts of the i-th partition of n, p(j,i,n) = the j-th part of the i-th partition of n, m(j,i,n) = multiplicity of the j-th part of the i-th partition of n, sum_[T(i,n)=k]_{i=1}^{P(n)} = sum running from i=1 to i=p(n) but taking only partitions with T(i,n)=k parts into account, prod_{j=1}^{T(i,n)} = product running from j=1 to j=T(i,n), prod_{j=1}^{D(i,n)} = product running from j=1 to j=D(i,n) one has S1(n,k) = sum_[T(i,n)=k]_{i=1}^{P(n)} (n!/prod_{j=1}^{T(i,n)} p(j,i,n))* (1/prod_{j=1}^{D(i,n)} m(j,i,n)!). For example, S1(6,3) = 225 because n=6 has the following partitions with k=3 parts: (114), (123), (222). Their complexions are: (114): (6!/1*1*4)*(1/2!*1!) = 90, (123): (6!/1*2*3)*(1/1!*1!*1!) = 120, (222): (6!/2*2*2)*(1/3!) = 15. The sum of the complexions is 90+120+15=225=S1(6,3). - Thomas Wieder (wieder.thomas(AT)t-online.de), Aug 04 2005

Row sums equal 0 - Jon Perry (perry(AT)globalnet.co.uk), Nov 14 2005

|s(n,k)| enumerates unordered n-vertex forests composed of k increasing non-plane (unordered) trees. Proof from the e.g.f. of the first column and the F. Bergeron et al. reference, especially Table 1, last row (non plane ``recursive"), given in A049029. W. Lang Oct 12 2007.

|s(n,k)| enumerates unordered increasing n-vertex k-forests composed of k unary trees (out-degree r from {0,1}) whose vertices of depth (distance from the root) j>=0 come in j+1 colors (j=0 for the k roots). W. Lang, Oct 12 2007, Feb 22 2008

FORMULA s(n, k)=s(n-1, k-1)-(n-1)*s(n-1, k), n, k >= 1; s(n, 0)=s(0, k)=0; s(0, 0)=1.

The unsigned numbers a(n, k)=|s(n, k)| satisfy a(n, k)=a(n-1, k-1)+(n-1)*a(n-1, k), n, k >= 1; a(n, 0)=a(0, k)=0; a(0, 0)=1.

E.g.f. for m-th column (unsigned): ((-ln(1-x))^m)/m!.

s(n, k) = T(n-1, k-1), n>1 and k>1, where T(n, k) is the triangle [-1, -1, -2, -2, -3, -3, -4, -4, -5, -5, -6, -6, ...] DELTA [1, 0, 1, 0, 1, 0, 1, 0, 1, ...]and DELTA is Deleham's operator defined in A084938. The unsigned numbers are also |s(n, k)| = T(n-1, k-1), for n>0 and k>0, where T(n, k) = [1, 1, 2, 2, 3, 3, 4, 4, 5, 5, ...] DELTA [1, 0, 1, 0, 1, 0, 1, 0, ...].

Sum[(-1)^(n-i) StirlingS1[n, i] binomial[i, k], {i,0,n}] == (-1)^(n-k) StirlingS1[n+1, k+1]. - Carlo Wood (carlo(AT)alinoe.com), Feb 13 2007

G.f.: S(n) = product[j=1, n, (x-j)] (i.e. (x-1)(x-2)(x-3) = x^3 - 6x^2 + 11x - 6) - Jon Perry (perry(AT)globalnet.co.uk), Nov 14 2005

a(n,k) = s(k,n) = (-1)^(k-n) * S1(k,n) = ((-1)^(k-n)) * (k!/{(n-1)!*2^(k-n)}) * [{ 1/(k-n)! }*k^(k-n-1) - { (1/6)*(1/(k-n-2)!) }*k^(k-n-2) + { (1/72)*(1/(k-n-4)!) }*k^(k-n-3) - { (1/6480)*(5/(k-n-6)! -36/(k-n-4)!) }*k^(k-n-4) + { (1/155520)*(5/(k-n-8)!-144/(k-n-6)!) }*k^(k-n-5) - { (1/6531840)*(7/(k-n-10)! -504/(k-n-8)!+2304/(k-n-6)!) }*k^(k-n-6) + { (1/1175731200)*(35/(k-n-12)!-5040/(k-n-10)!+87264/(k-n-8)!) }*k^(k-n-7) - { (1/7054387200)*(5/(k-n-14)!-1260/(k-n-12)!+52704/(k-n-10)!-186624/(k-n-8)!) }*k^(k-n-8) + { (1/338610585600)*(5/(k-n-16)!-2016/(k-n-14)!+164736/(k-n-12)!-2156544/(k-n-10)!) }*k^(k-n-9) - .....]. - Andre F. Labossiere (boronali(AT)laposte.net), Mar 27 2006

A008277 Triangle of Stirling numbers of 2nd kind, S2(n,k), n >= 1, 1<=k<=n.

COMMENTS Also known as Stirling set numbers and written {n, k}. S2(n,k) enumerates partitions of an n-set into k non-empty subsets.

Triangle S2(n,k), 1<=k<=n, read by rows, given by [0, 1, 0, 2, 0, 3, 0, 4, 0, 5, 0, 6, ...] DELTA [1, 0, 1, 0, 1, 0, 1, 0, 1, 0, 1, ...] where DELTA is Deleham's operator defined in A084938.

Number of partitions of {1, ...,n+1} into k+1 subsets of nonconsecutive integers, including the partition 1|2|...|n+1 if n=k. E.g. S2(3,2)=3 since the number of partitions of {1,2,3,4} into three subsets of nonconsecutive integers is 3, i.e., 13|2|4, 14|2|3, 1|24|3. - A. O. Munagi (amunagi(AT)yahoo.com), Mar 20 2005

Draw n cards (with replacement) from a deck of k cards. Let prob(n,k) be the probability that each card was drawn at least once. Then prob(n,k) = S2(n,k)*k!/k^n (see A090582). - Rainer Rosenthal (r.rosenthal(AT)web.de), Oct 22 2005

Define f_1(x),f_2(x),... such that f_1(x)=e^x and for n=2,3,... f_{n+1}(x)=diff(x*f_n(x),x). Then f_n(x)=e^x*sum(stirling2(n,k)*x^(k-1),k=1..n). - Milan R. Janjic (agnus(AT)blic.net), May 30 2008

Contribution from Peter Bala (pbala(AT)toucansurf.com), Oct 03 2008: (Start)

For tables of restricted Stirling numbers of the second kind see A143494 - A143496.

Stirling2(n,k) gives the number of 'patterns' of words of length n using k distinct symbols - see [Cooper & Kennedy] for an exact definition of the term 'pattern'. As an example, the words AADCBB and XXEGTT, both of length 6, have the same pattern of letters. The five patterns of words of length 3 are AAA, AAB, ABA, BAA and ABC giving row 3 of this table as (1,3,1).

Equivalently, Stirling2(n,k) gives the number of sequences of positive integers (N_1,...,N_n) of length n, with k distinct entries, such that N_1 = 1 and N_(i+1) <= 1 + max{j = 1..i} N_j for i >= 1. For example, Stirling(4,2) = 7 since the sequences of length 4 having 2 distinct entries that satisfy the conditions are (1,1,1,2), (1,1,2,1), (1,2,1,1), (1,1,2,2), (1,2,2,2), (1,2,2,1) and (1,2,1,2). (End)

Number of combinations of subsets in the plane. [From Mats Granvik (mats.granvik(AT)abo.fi), Jan 13 2009]

Contribution from Geoffrey Critzer (critzer.geoffrey(AT)usd443.org), Apr 06 2009: (Start)

S2(n+1,k+1) is the number of size k collections of pairwise disjoint, nonempty subsets of [n]. For example: S2(4,3)=6 because there are six such collections of subsets of [3] that have cardinality two: {(1)(23)},{(12)(3)},{(13)(2)},

{(1)(2)},{(1)(3)},{(2)(3)} (End)

Consider a set of A000217(n) balls of n colors in which, for each integer k = 1 to n, exactly one color appears in the set a total of k times. (Each ball has exactly one color and is indistinguishable from other balls of the same color.) a(n+1, k+1) equals the number of ways to choose 0 or more balls of each color in such a way that exactly (n-k) colors are chosen at least once, and no two colors are chosen the same positive number of times. [From Matthew Vandermast(ghodges14(AT)comcast.net), Nov 22 2010]

FORMULA S2(n, k) = k*S2(n-1, k)+S2(n-1, k-1), n>1. S2(1, k) = 0, k>1. S2(1, 1)=1.

E.g.f.: A(x, y)=exp(y*exp(x)-y). E.g.f. for m-th column: ((exp(x)-1)^m)/m!.

S2(n, k) = (1/k!) * Sum_{i=0..k} (-1)^(k-i)*C(k, i)*i^n.

Row sums: Bell number A000110(n) = sum(S2(n, k)) k=1..n, n>0.

The k-th row (k>=1) contains a(n, k) for n=1 to k where a(n, k) = (1/(n-1)!) * Sum_{q=1..[2*n+1+(-1)^(n-1)]/4} [C(n-1, 2*q-2)*(n-2*q+2)^(k-1) -C(n-1, 2*q-1)*(n-2*q+1)^(k-1)]. E.g. Row 7 contains S2(7, 3)=301 { A001298, S2(n+4, n) } and will be computed as the following: S2(7, 3) = a(3, 7) = 1/(3-1)! * Sum_{q=1..2} [C(3-1, 2*q-2)*(3-2*q+2)^(7-1) -C(3-1, 2*q-1)*(3-2*q+1)^(7-1)] = Sum_{q=1..2} [C(2, 2*q-2)*(5-2*q)^6 -C(2, 2*q-1)*(4-2*q)^6]/2! = [C(2, 0)*3^6 -C(2, 1)*2^6 +C(2, 2)*1^6 -C(2, 3)*0^6]/2! = [729 -128 +1 -0]/2 = 301. - Andre F. Labossiere (boronali(AT)laposte.net), Jun 07 2004

For k>0 and for all x sufficiently small, Sum[n>=0, T(n, k)*x^n] = x^k/[(1-x)(1-2x)(1-3x)...(1-kx)].

With P(n) = the number of integer partitions of n, p(i) = the number of parts of the i-th partition of n, d(i) = the number of different parts of the i-th partition of n, p(j, i) = the j-th part of the i-th partition of n, m(i, j) = multiplicity of the j-th part of the i-th partition of n, sum_[p(i)=m]_{i=1}^{P(n)} = sum running from i=1 to i=p(n) but taking only partitions with p(i)=m parts into account, prod_{j=1}^{p(i)} = product running from j=1 to j=p(i), prod_{j=1}^{d(i)} = product running from j=1 to j=d(i) one has S2(n, m) = sum_[p(i)=m]_{i=1}^{P(n)} (n!/prod_{j=1}^{p(i)} p(i, j)!) (1/prod_{j=1}^{d(i)} m(i, j)!). For example, S2(6, 3) = 90 because n=6 has the following partitions with m=3 parts: (114), (123), (222). Their complexions are: (114): (6!/1!*1!*4!)*(1/2!*1!) = 15, (123): (6!/1!*2!*3!)*(1/1!*1!*1!) = 60, (222): (6!/2!*2!*2!)*(1/3!) = 15. The sum of the complexions is 15+60+15=90=S2(6, 3). - Thomas Wieder (wieder.thomas(AT)t-online.de), Jun 02 2005

Sum(k*S2(n,k), k=1..n)=B(n+1)-B(n), where B(q) are the Bell numbers (A000110). - Emeric Deutsch (deutsch(AT)duke.poly.edu), Nov 01 2006

Recurrence: S2(n+1,k) = sum_{i=0}^n C(n,i) S2(i, k-1). With the starting conditions S2(n,k) = 1 for n = 0 or k = 1 and S2(n,k) = 0 for k = 0 we have the closely related recurrence S2(n,k) = sum_{i=k}^n C(n-1,i-1) S2(i-1, k-1) where C(n,m) is the binomial coefficient. - Thomas Wieder (thomas.wieder(AT)t-online.de), Jan 27 2007

Representation of Stirling numbers of the second kind S2(n,k), n=1,2..., k=1,2...n, as special values of hypergeometric function of type (n)F(n-1): stirling2(n,k)= (-1)^(k-1)*hypergeom([-k+1,2,2...2],[1,1...1],1)/(k-1)!, i.e. having n parameters in the numerator: one equal to -k+1 and n-1 parameters all equal to 2 ; and having n-1 parameters in the denominator all equal to 1 and the value of the argument equal to 1. Example: stirling2(6,k)= seq(evalf((-1)^(k-1)*hypergeom([-k+1,2,2,2,2,2],[1,1,1,1,1],1)/(k-1)!),k=1..6)=1,31,90,65,15,1. - Karol A. Penson (penson(AT)lptl.jussieu.fr), Mar 28 2007

Formulae and comments from Tom Copeland, Oct 10 2007 (Start): Bell(n,x) = sum(j=0,...,n) S2(n,j) * x^j = sum(j=0,...,n) E(n,j) * Lag(n,-x, j-n) = sum(j=0,...,n) [E(n,j)/n!] * [n!*Lag(n,-x, j-n)] = sum(j=0,...,n) E(n,j) * C(Bell(.,x)+j,n) umbrally where Bell(n,x) are the Bell / Touchard / exponential polynomials; S2(n,j), the Stirling numbers of the second kind; E(n,j), the Eulerian numbers; Lag(n,x,m), the associated Laguerre polynomials of order m; and C(x,y) = x!/[y!*(x-y)!].

By substituting the umbral compositional inverse of the Bell polynomials, the lower factorial n!*C(x,n), for x in the equation, the equation becomes x^n = sum(j=0,...,n) S2(n,j) * j! * C(x,j)

Note that E(n,j)/n! = E(n,j)/ {sum(k=0,..,n)E(n,k)}. Also [n!*Lag(n,-1, j-n)] is A086885 with a simple combinatorial interpretation in terms of seating arrangements, giving a combinatorial interpretation to the equation for x = 1; n!*Bell(n,1) = n!*sum(j=0,...,n) S2(n,j) = sum(j=0,...,n) {E(n,j) * [n!*Lag(n,-1, j-n)]} . (End)

n-th row = leftmost column of nonzero terms of A127701^(n-1). Also, (n+1)-th row of the triangle = A127701 * n-th row; deleting the zeros. Example: A127701 * [1, 3, 1, 0, 0, 0,...] = [1, 7, 6, 1, 0, 0, 0,...]. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 21 2007

Contribution from Roger L. Bagula (rlbagulatftn(AT)yahoo.com), Jan 11 2009: (Start)

p(x,n)=Sum[m^n*x^m/m!, {m, 0, Infinity}]/(x*Exp[x]);

s(n,m)=coefficients(p(x,n)) (End)

A008279 Triangle T(n,k) = n!/(n-k)! (0 <= k <= n) read by rows, giving number of permutations of n things k at a time.

COMMENTS Also called permutation coefficients.

Also falling factorials triangle A068424 with column a(n,0)=1 and row a(0,1)=1 else a(0,k)=0, added. - Wolfdieter Lang, Nov 07 2003

Contribution from Johannes W. Meijer (meijgia(AT)hotmail.com), Oct 16 2009: (Start)

The higher order exponential integrals E(x,m,n) are defined in A163931 and for information about the asymptotic expansion of E(x,m=1,n) see A130534. The asymptotic expansions for n = 1, 2, 3, 4, ..., lead to the right hand columns of the triangle given above.

(End)

The number of injective functions from a set of size k to a set of size n. [From Dennis Walsh, Feb 10 2011]

The number of functions f from {1,2,...,k} to {1,2,...,n} that satisfy f(x)>=x for all x in {1,2,...,k}. [From Dennis Walsh, April 20 2011]

FORMULA E.g.f.: sum T(n,k) x^n/n! y^k = exp(x)/(1-x*y). - Vladeta Jovovic (vladeta(AT)eunet.rs), Aug 19 2002

Equals A007318 * A136572 - Gary W. Adamson (qntmpkt(AT)yahoo.com), Jan 07 2008

T(n, k) = n*T(n-1, k-1) = k*T(n-1, k-1)+T(n-1, k) = n*T(n-1, k)/(n-k) = (n-k+1)*T(n, k-1) - Henry Bottomley (se16(AT)btinternet.com), Mar 29 2001

T(n, k) = n!/(n-k)! if n >= k >= 0 else 0. G.f. for k-th column k!*x^k/(1-x)^(k+1), k >= 0. E.g.f. for n-th row (1+x)^n, n >= 0.

Sum T(n, k)x^k = permanent of n X n matrix a_ij = (x+1 if i=j, x otherwise). - Michael Somos Mar 05 2004

Ramanujan psi_1(k, x) polynomials evaluated at n+1. - Ralf Stephan, Apr 16 2004

E.g.f. sum T(n,k) x^n/n! y^k/k! = e^{x+xy}. - Frank Adams-Watters (FrankTAW(AT)Netscape.net), Feb 07 2006

The triangle is the binomial transform of an infinite matrix with (1, 1, 2, 6, 24...) in the main diagonal and the rest zeros. - Gary W. Adamson (qntmpkt(AT)yahoo.com), Nov 20 2006

G.f.: 1/(1-x-xy/(1-xy/(1-x-2xy/(1-2xy/(1-x-3xy/(1-3xy/(1-x-4xy/(1-4xy/(1-... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Feb 11 2009]

T(n,k)=sum(binomial(k,j)*T(x,j)*T(y,k-j),j=0..k) for x+y=n. [From Dennis Walsh, Feb 10 2011]

E.g.f (with k fixed): x^k*exp(x) [From Dennis Walsh, April 20 2011]

G.f. (with k fixed): k!x^k/(1-x)^(k+1) [From Dennis Walsh, April 20 2011]

A008292 Triangle of Eulerian numbers T(n,k) (n>=1, 1 <= k <= n) read by rows.

COMMENTS The indexing used here follows that given in the classic books by Riordan and Comtet. For two other versions see A173018 and A123125. - N. J. A. Sloane, Nov 21 2010

Coefficients of Eulerian polynomials. Number of permutations of n objects with k-1 rises.

Number of increasing rooted trees with n+1 nodes and k leaves.

T(n,k)=number of permutations of [n] with k runs. T(n,k)=number of permutations of [n] requiring k readings (see the Knuth reference). T(n,k)=number of permutations of [n] having k distinct entries in its inversion table. - Emeric Deutsch (deutsch(AT)duke.poly.edu), Jun 09 2004

T(n,k) = number of ways to write the Coxeter element s_{e1}s_{e1-e2}s_{e2-e3}s_{e3-e4}...s_{e_{n-1}-e_n} of the reflection group of type B_n, using s_{e_k} and as few reflections of the form s_{e_i+e_j}, where i = 1, 2, ..., n and j is not equal to i, as possible. - Pramook Khungurn (pramook(AT)mit.edu), Jul 07 2004

Subtriangle for k>=1 and n>=1 of triangle A123125 . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Oct 22 2006

Comment from Stefano Zunino, Oct 25 2006: T(n,k)/n! also represents the n-dimensional volume of the portion of the n-dimensional hyper-cube cut by the (n-1)-dimensional hyperplanes x_1 + x_2 + ... x_n = k, x_1 + x_2 + ... x_n = k-1; or, equivalently, it represents the probability that the sum of n independent random variables with uniform distribution between 0 and 1 is between k-1 and k.

[E(.,t)/(1-t)]^n = n!*Lag[n,-P(.,t)/(1-t)] and

[-P(.,t)/(1-t)]^n = n!*Lag[n, E(.,t)/(1-t)] umbrally comprise a combinatorial Laguerre transform pair, where E(n,t) are the Eulerian polynomials and P(n,t) are the polynomials in A131758. - Tom Copeland (tcjpn(AT)msn.com), Sep 30 2007

Comment from Tom Copeland (tcjpn(AT)msn.com), Oct 07 2008: (Start)

G(x,t) = 1/(1 + (1-exp(x*t))/t) = 1 + 1*x + (2+t)*x^2/2! + (6+6*t+t^2)*x^3/3! + ...

gives row polynomials for A090582-- reverse f-polynomials for the permutohedra (see A019538).

G(x,t-1) = 1 + 1*x + (1+t)*x^2/2! + (1+4*t+t^2)*x^3/3! + ...

gives row polynomials for A008292, the h-polynomials for permutohedra.

G((t+1)*x,-1/(t+1)) = 1 + (1+t)*x + (1+3*t+2*t^2)*x^2/2! + ...

gives row polynomials for A028246.

(End)

A subexceedant function f on [n] is a map f:[n] -> [n] such that 1 <= f(i) <= i for all i, 1 <= i <= n. T(n,k) equals the number of subexceedant functions f of [n] such that the image of f has cardinality k [Mantaci & Rakotondrajao]. Example T(3,2) = 4: if we identify a subexceedant function f with the word f(1)f(2)...f(n) then the subexceedant functions on [3] are 111, 112, 113, 121, 122 and 123 and four of these functions have an image set of cardinality 2. [From Peter Bala (pbala(AT)toucansurf.com), Oct 21 2008]

Further to the comments of T. Copeland above, the n-th row of this triangle is the h-vector of the simplicial complex dual to a permutohedron of type A_(n-1). The corresponding f-vectors are the rows of A019538. For example, 1 + 4*x + x^2 = y^2 + 6*y + 6 and 1 + 11*x + 11*x^2 + x^3 = y^3 + 14*y^2 + 36*y + 24, where x = y + 1, give [1,6,6] and [1,14,36,24] as the third and fourth rows of A019538. The Hilbert transform of this triangle (see A145905 for the definition) is A047969. See A060187 for the triangle of Eulerian numbers of type B (the h-vectors of the simplicial complexes dual to permutohedra of type B). See A066094 for the array of h-vectors of type D. For tables of restricted Eulerian numbers see A144696 - A144699. [From Peter Bala (pbala(AT)toucansurf.com), Oct 26 2008]

For a natural refinement of A008292 with connections to compositional inversion and iterated derivatives, see A145271. [From Tom Copeland (tcjpn(AT)msn.com), Nov 06 2008]

Contribution from Johannes W. Meijer (meijgia(AT)hotmail.com), May 24 2009: (Start)

The polynomials E(z,n) = numer(sum((-1)^(n+1)*k^n*z^(k-1), k=1..infinity)) for n >=1 lead directly to the triangle of Eulerian numbers. (End)

Contribution from Walther Janous (walther.janous(AT)tirol.com), Nov 01 2009: (Start)

The (Eulerian) polynomials e(n,x) = SUM(T(n,k+1)*x^k, k = 0 to n-1) turn out to be also as the numerators of the closed-form-expressions of the infinite sums

S(p,x):= SUM((j+1)^p*x^j, j = 0 to infinity), that is

S(p,x) = e(p,x)/(1-x)^(p+1), whenever |x| < 1 and p is a positive integer.

(Note the inconsistent use of T(n,k) in the section listing the formula section. I adhere tacitly to the first one.) (End)

If n is an odd prime, then all numbers of the (n-2)-th and (n-1)-th rows are in the progression k*n+1.-Vladimir Shevelev, Jul 01 2011.

FORMULA T(n, k) = k * T(n-1, k) + (n-k+1) * T(n-1, k-1), T(1, 1) = 1.

T(n, k) = Sum (-1)^j * (k-j)^n * C(n+1, j), j=0..k.

Row sums = n! = A000142(n) unless n=0. - Michael Somos Mar 17 2011

E.g.f. A(x, q) = Sum_{n>0} (Sum_{k=1..n} T(n, k) * q^k) * x^n / n! = q * (e^(q*x) - e^x) / (q*e^x - e^(q*x)) satisfies dA / dx = (A + 1) * (A + q). - Michael Somos Mar 17 2011

For a column listing, n-th term: T(c, n)=c^(n+c-1)+sum(i=1, c-1, (-1)^i/i!*(c-i)^(n+c-1)*prod(j=1, i, n+c+1-j)) - Randall L. Rathbun (randallr(AT)abac.com), Jan 23 2002

Four characterizations of Eulerian numbers T(i, n) from John Robertson (jpr2718(AT)aol.com), Sep 02, 2002: (Start)

1. T(0, n)=1 for n>=1, T(i, 1)=0 for i>=1, T(i, n) = (n-i)T(i-1, n-1) + (i+1)T(i, n-1).

2. T(i, n) = sum_{j=0}^{i} (-1)^j (n+1 combin j) (i-j+1)^n for n>=1, i>=0.

3. Let Cn be the unit cube in R^n with vertices (e_1, e_2, ..., e_n) where each e_i is 0 or 1 and all 2^n combinations are used. Then T(i, n)/n! is the volume of Cn between the hyperplanes x_1 + x_2 + ... + x_n = i and x_1 + x_2 + ... + x_n = i+1. Hence T(i, n)/n! is the probability that i <= X_1 + X_2 + ... + X_n < i+1 where the X_j are independent uniform [0, 1] distributions. - See Ehrenborg & Readdy reference.

4. Let f(i, n) = T(i, n)/n!. The f(i, n) are the unique coefficients so that (1/(r-1)^(n+1)) sum_{i=0}^{n-1} f(i, n) r^{i+1} = sum_{j=0}^{infinity} (j^n)/(r^j) whenever n>=1 and abs(r)>1. (End)

O.g.f. for n-th row: (1-x)^(n+1)*polylog(-n, x)/x. - Vladeta Jovovic (vladeta(AT)eunet.rs), Sep 02 2002

Triangle T(n, k), n>0 and k>0, read by rows; given by [0, 1, 0, 2, 0, 3, 0, 4, 0, 5, 0, 6, ...] DELTA [1, 0, 2, 0, 3, 0, 4, 0, 5, 0, 6, ...] (positive integers interspersed with 0's) where DELTA is Deleham's operator defined in A084938.

Sum_{k = 1..n} T(n, k)*2^k = A000629(n) . - DELEHAM Philippe (kolotoko(AT)wanadoo.fr), Jun 05 2004

a(n,k) = Sum_{i=1..n} (-1)^(n-i) * (i^k) * C(k+1,n-i). - Andre F. Labossiere (boronali(AT)laposte.net), Aug 16 2006

From Tom Copeland, Oct 10 2007: (Start)

Bell(n,x) = sum(j=0,...,n) S2(n,j) * x^j = sum(j=0,...,n) E(n,j) * Lag(n,-x, j-n) = sum(j=0,...,n)

[E(n,j)/n!] * [n!*Lag(n,-x, j-n)] = sum(j=0,...,n) E(n,j) * C(Bell(.,x)+j,n) umbrally where Bell(n,x) are the Bell / Touchard / exponential polynomials; S2(n,j), the Stirling numbers of the second kind; E(n,j), the Eulerian numbers; Lag(n,x,m), the associated Laguerre polynomials of order m; and C(x,y) = x!/[y!*(x-y)!].

For x = 0, the equation gives sum(j=0,...,n) E(n,j) * C(j,n) = 1 for n = 0 and 0 for all other n. By substituting the umbral compositional inverse of the Bell polynomials, the lower factorial n!*C(y,n), for x in the equation, the Worpitzky identity is obtained; y^n = sum(j=0,...,n) E(n,j) * C(y+j,n) .

Note that E(n,j)/n! = E(n,j)/ {sum(k=0,..,n)E(n,k)}. Also [n!*Lag(n,-1, j-n)] is A086885 with a simple combinatorial interpretation in terms of seating arrangements, giving a combinatorial interpretation to the equation for x=1; n!*Bell(n,1) = n!*sum(j=0,...,n) S2(n,j) = sum(j=0,...,n) {E(n,j) * [n!*Lag(n,-1, j-n)]} . (End)

From the relations between the h- and f-polynomials of permutohedra and reciprocals of e.g.f.s described in A049019: (t-1)[(t-1)d/dx]^n 1/(t-exp(x)) evaluated at x=0 gives the n-th Eulerian row polynomial in t and the n-th row polynomial in (t-1) of A019538 and A090582. From the Comtet and Copeland references in A139605: [(t+exp(x)-1)d/dx]^(n+1) x gives pairs of the Eulerian polynomials in t as the coefficients of x^0 and x^1 in its Taylor series expansion in x. [From Tom Copeland (tcjpn(AT)msn.com), Oct 05 2008]

G.f: 1/(1-x/(1-x*y/1-2x/(1-2x*y/(1-3x/(1-3x*y/(1-... (continued fraction). [From Paul Barry (pbarry(AT)wit.ie), Mar 24 2010]

If n is odd prime, then the following consecutive 2*n+1 terms are 1 modulo n: a((n-1)*(n-2)/2+i), i=0,...,2*n.

Such chain of terms is maximal in the sense that neither the previous term nor the following one are 1 modulo n. [Vladimir Shevelev, Jul 01 2011]

A008683 Moebius (or Mobius) function mu(n).

COMMENTS Moebius inversion: f(n) = Sum_{ d divides n } g(d) for all n <=> g(n) = Sum_{ d divides n } mu(d)*f(n/d) for all n.

a(n) depends only on prime signature of n (cf. A025487). So a(24) = a(375) since 24=2^3*3 and 375=3*5^3 both have prime signature (3,1).

A008683 = A140579^(-1) * A140664 - Gary W. Adamson, May 20 2008

See last sentence of abstract of Coons and Borwein: We give a new proof of Fatou's theorem: if an algebraic function has a power series expansion with bounded integer coefficients, then it must be a rational function.} This result is applied to show that for any non--trivial completely multiplicative function from N to {-1,1), the series sum_{n=1 to infinity) f(n)z^n is transcendental over {Z}[z]; in particular, sum_{n=1 to infinity) lambda(n)z^n is transcendental, where lambda is Liouville's function. The transcendence of sum_{n=1 to infinity) mu(n)z^n is also proved. - Jonathan Vos Post, Jun 11 2008

Equals row sums of triangle A144735 (the square of triangle A054533). [From Gary W. Adamson, Sep 20 2008]

Conjecture: a(n) = determinant of Redheffer matrix A143104 where T(n,n)=0. Verified for 50 first terms. - Mats O. Granvik, Jul 25 2008

Contribution from Mats Granvik, Dec 06 2008: (Start)

The Editorial Office of the Journal of Number Theory kindly provided

(via B. Conrey) the following proof of the conjecture:

Let A be A143104 and B be A143104 where T(n,n)=0.

"Suppose you expand det(B_n) along the bottom row. There is only a 1 in the first position and so the answer is (-1)^n times det(C_{n-1}) say, where C_{n-1} is the (n-1) by (n-1) matrix obtained from B_n by deleting the first column and the last row.

Now the determinant of the Redheffer matrix is det(A_n)=M(n) where M(n) is the sum of mu(m) for 1<=m<=n. Expanding det(A_n) along the bottom row, we see that det(A_n)=(-1)^n*det(C_{n-1})+M(n-1). So we have det(B_n)=(-1)^n*det(C_{n-1})=det(A_n)-M(n-1)=M(n)-M(n-1)=mu(n)."

(End)

Conjecture: Consider the table A051731 and treat 1 as a divisor. Move the value in the lower right corner vertically to a divisor position in the transpose of the table and you will find that the determinant is the Moebius function. The number of permutation matrices that contribute to the Moebius function appears to be A074206. - Mats Granvik, Dec 08 2008

Convolved with A152902 = A000027, the natural numbers. [From Gary W. Adamson, Dec 14 2008]

Contribution from Gary W. Adamson, Aug 13 2009: (Start)

[Pickover, p. 226]: "The probability that a number falls in the -1 mailbox turns out to be 3/Pi^2 - the same probability as for falling in the +1 mailbox". (End)

Let A=A176890*A176890, B=A*A, C=B*B, D=C*C and so on, then the leftmost column in the last matrix converges to the Moebius function. [From Mats Granvik, Gary W. Adamson, Apr 28 2010]

Equals row sums of triangle A176918 [From Gary W. Adamson, Apr 29 2010]

Calculate matrix powers: A175992^0-A175992^1+A175992^2-A175992^3+A175992^4... Then the mobius function is found in the first column. Compare this to the binomial series for (1+x)^-1=1-x+x^2-x^3+x^4...[From Mats Granvik, Gary W. Adamson, Dec 6 2010]

FORMULA mu(1)=1; mu(n)=(-1)^k if n is the product of k different primes; otherwise mu(n)=0.

sum(d divides n, mu(d)) = 1 if n=1 else 0.

Dirichlet generating function: Sum_{n >= 1} mu(n)/n^s = 1/zeta(s). Also Sum_{n >= 1} mu(n)*x^n/(1-x^n) = x.

phi(n) = sum(d divides n, mu(d)*n/d).

a(n) = A091219(A091202(n)).

Multiplicative with a(p^e) = -1 if e = 1; 0 if e > 1. - David W. Wilson, Aug 01, 2001.

a(n)=sum(d divides n, 2^A001221(d)*a(n/d)) - Benoit Cloitre, Apr 05 2002

sum(d divides n, (-1)^(n/d)*mobius(d)) = 0. - Emeric Deutsch, Jan 28 2005

a(n) = (-1)^omega(n) * 0^(bigomega(n)-omega(n)) for n>0, where bigomega(n) and omega(n) are the numbers of prime factors of n with and without repetition (A001222, A001221, A046660). - Reinhard Zumkeller, Apr 05 2003

Dirichlet generating function for the absolute value: zeta(s)/zeta(2s). - Franklin T. Adams-Watters, Sep 11 2005.

mu(n) = A129360 * (1, -1, 0, 0, 0,...). - Gary W. Adamson, Apr 17 2007

mu(n) = -sum(d<n,d divides n, mu(d)) if n>1 and mu(1) = 1. [From Alois P. Heinz, Aug 13 2008] a(n)=A174725-A174726. [From Mats Granvik, Mar 28 2010]

a(n) = first column in the matrix inverse of a triangular table with the definition: T(1,1)=1, n>1: T(n,1) = any number or sequence, k=2: T(n,2)=T(n,k-1)-T(n-1,k), k>2 and n>=k: T(n,k) = (sum from i = 1 to k-1 of T(n-i,k-1)) - (sum from i = 1 to k-1 of T(n-i,k)). [From Mats Granvik, Jun 12 2010]

prod(n>=1, (1-x^n)^(-a(n)/n)) = exp(x) (product form of the exponential function). [Joerg Arndt, May 13 2011]

a(n)= sum(exp(2*pi*k/n), k=1..n and gcd(k,n)=1), the sum over the primitive n-th roots of unity. See the Apostol reference, p. 48, Exercise 14 (b). [Wolfdieter Lang, Jun 13 2011]

A010060 Thue-Morse sequence: let A_k denote the first 2^k terms; then A_0 = 0 and for k >= 0, A_{k+1} = A_k B_k, where B_k is obtained from A_k by interchanging 0's and 1's.

COMMENTS Also called the Thue-Morse infinite word.

The sequence is cube-free (does not contain three consecutive identical blocks) and is overlap-free (does not contain XYXYX where X is 0 or 1 and Y is any string of 0's and 1's).

a(n) = "parity sequence" = parity of number of 1's in binary representation of n.

To construct the sequence: alternate blocks of 0's and 1's of successive lengths A003159(k)-A003159(k-1), k=1,2,3,... (A003159(0)=0). Example: since the first seven differences of A003159 are 1,2,1,1,2,2,2, the sequence starts with 0,1,1,0,1,0,0,1,1,0,0. - Emeric Deutsch, Jan 10 2003

Characteristic function of A000069 (odious numbers). a(n) = 1-A010059(n) = A001285(n)-1. - Ralf Stephan, Jun 20 2003

a(n)=S2(n)mod 2, where S2(n) = sum of digits of n, n in base-2 notation. There is a class of generalized Thue-Morse sequences : Let Sk(n) = sum of digits of n; n in base-k notation. Let F(t) be some arithmetic function. Then a(n)= F(Sk(n)) mod m is a generalised Thue-Morse sequence. The classical Thue-Morse sequence is the case k=2, m=2, F(t)= 1. - Ctibor O. Zizka, Feb 12 2008

More generally, the partial sums of the generalized Thue-Morse sequences a(n)=F(Sk(n)) mod m are fractal, where Sk(n) is sum of digits of n, n in base k; F(t) is an arithmetic function; m integer. - Ctibor O. ZIZKA, Feb 25 2008

Starting with offset 1, = running sums mod 2 of the kneading sequence (A035263, 1, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1,...); also parity of A005187: (1, 3, 4, 7, 8, 10, 11, 15, 16, 18, 19,...). - Gary W. Adamson, Jun 15 2008

Generalized Thue-Morse sequences mod n (n>1) = the array shown in A141803. As n -> Inf. the sequences -> (1, 2, 3,...). - Gary W. Adamson, Jul 10 2008

The Thue-Morse sequence for N = 3 = A053838, (sum of digits of n in base 3, mod 3): (0, 1, 2, 1, 2, 0, 2, 0, 1, 1, 2,...) = A004128 mod 3. [From Gary W. Adamson, Aug 24 2008]

For all positive integers k, the subsequence a(0) to a(2^k-1) is identical to the subsequence a(2^k+2^(k-1)) to a(2^(k+1)+2^(k-1)-1). That is to say, the first half of A_k is identical to the second half of B_k, and the second half of A_k is identical to the first quarter of B_{k+1}, which consists of the k/2 terms immediately following B_k.

Proof: The subsequence a(2^k+2^(k-1)) to a(2^(k+1)-1), the second half of B_k, is by definition formed from the subsequence a(2^(k-1)) to a(2^k-1), the second half of A_k, by interchanging its 0s and 1s. In turn, the subsequence a(2^(k-1)) to a(2^k-1), the second half of A_k, which is by definition also B_{k-1}, is by definition formed from the subsequence a(0) to a(2^(k-1)-1), the first half of A_k, which is by definition also A_{k-1}, by interchanging its 0s and

1s. Interchanging the 0s and 1s of a subsequence twice leaves it unchanged, so the subsequence a(2^k+2^(k-1)) to a(2^(k+1)-1), the second half of B_k, must be identical to the subsequence a(0) to a(2^(k-1)-1), the first half of A_k.

Also, the subsequence a(2^(k+1)) to a(2^(k+1)+2^(k-1)-1), the first quarter of B_{k+1}, is by definition formed from the subsequence a(0) to a(2^(k-1)-1), the first quarter of A_{k+1}, by interchanging its 0s and 1s. As noted above, the subsequence a(2^(k-1)) to a(2^k-1), the second half of A_k, which is by definition also B_{k-1}, is by definition formed from the subsequence a(0) to a(2^(k-1)-1), which is by definition A_{k-1}, by interchanging its 0s and 1s, as well. If two subsequences are formed from the same subsequence by interchanging its 0s and 1s then they must be identical, so the subsequence a(2^(k+1)) to a(2^(k+1)+2^(k-1)-1), the first quarter of B_{k+1}, must be identical to the subsequence a(2^(k-1)) to a(2^k-1), the second half of A_k.

Therefore the subsequence a(0),..., a(2^(k-1)-1), a(2^(k-1)),..., a(2^k-1) is identical to the subsequence a(2^k+2^(k-1)),..., a(2^(k+1)-1), a(2^(k+1)),..., a(2^(k+1)+2^(k-1)-1), QED.

According to the German chess rules of 1929 a game of chess was drawn if the same sequence of moves was repeated three times consecutively. Euwe, see the references, proved that this rule could lead to infinite games. For his proof he reinvented the Thue-Morse sequence. [From Johannes W. Meijer, Feb 04 2010.]

"Thue-Morse 0->01 & 1->10, at each stage append the previous with its complement. Start with 0,1,2,3 and write them in binary. Next calculate the sum of the digits (mod 2) - that is divide the sum by 2 and use the remainder." Pickover, The Math Book.

FORMULA

a(2n)=a(n), a(2n+1)=1-a(n), a(0)=0. Also, a(k+2^m)=1-a(k) if 0<=k<2^m.

Let S(0) = 0 and for k >=1, construct S(k) from S(k-1) by mapping 0 -> 01 and 1 -> 10; sequence is S(infinity).

G.f.: (1/(1-x) - product_{k>=0} 1-x^(2^k))/2. - Benoit Cloitre, Apr 23 2003

a(0)=0, a(n)=(n+a(floor(n/2))) mod 2; also a(0)=0, a(n)=(n-a(floor(n/2))) mod 2 - Benoit Cloitre, Dec 10 2003

a(n)=-1+sum(k=0, n, binomial(n, k){mod 2}) {mod 3}=-1+A001316(n) {mod 3} - Benoit Cloitre, May 09 2004

Let b(1)=1 and b(n)=b(ceil(n/2))-b(floor(n/2)) then a(n-1)=(1/2)*(1-b(2n-1)) - Benoit Cloitre, Apr 26 2005

a(n) = A001969(n) - 2n. - Frank Adams-Watters, Aug 28 2006

a(n) = A115384(n) - A115384(n-1) for n>0. - Reinhard Zumkeller, Aug 26 2007

For n>=0, a(A004760(n+1))=1-a(n). [From Vladimir Shevelev, Apr 25 2009]

a(A160217(n))=1-a(n). [From Vladimir Shevelev, May 05 2009]

A018252 The nonprime numbers (1 together with the composite numbers, A002808).

COMMENTS

d(n)<>2 (cf. A000005). [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 17 2009]

Number of prime divisors of n (counted with multiplicity)<>1. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 30 2009]

Largest nonprime<nth composite. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Oct 29 2009]

The nonnegative nonprimes A141468 without zero; the natural nonprimes; the whole nonprimes; the counting nonprimes. If the nonprime numbers A141468 which are also the nonnegative integers A001477, then the nonprimes A141468 also called the nonnegative nonprimes. If the nonprime numbers A018252 which are also the natural (or whole or counting) numbers A000027, then the nonprimes A018252 also called the natural nonprimes, the whole nonprimes and the counting nonprimes. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Nov 22 2009]

Smallest nonprime>nth nonnegative nonprime. [From Juri-Stepan Gerasimov (2stepan(AT)rambler.ru), Dec 04 2009]

a(n)=A175944(A014284(n))=A175944(A175965(n)). [Reinhard Zumkeller, Mar 18 2011]

FORMULA Let b(0)=n+pi(n) and b(n+1)=n+pi(b(n)), with pi(n)=A000720(n); then a(n) is the limit value of b(n). - Floor van Lamoen (fvlamoen(AT)hotmail.com), Oct 08 2001

a(n) = A137621(A137624(n)). - Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Jan 30 2008

A020639 Lpf(n): least prime dividing n (with a(1)=1).

COMMENTS a(1) = 1, a(2) = (2*1)/1 = 2; a(n+1) = a(n)*(the smallest prime divisor of (n+1) divided by the largest prime divisor of a(n)). - Amarnath Murthy (amarnath_murthy(AT)yahoo.com), Nov 28 2004

a(n) = the maximum number of integers such that all pairwise differences are coprime to n. - Max Alekseyev, Mar 17 2006

The unit 1 is not a prime number (although it has been considered so in the past.) 1 is the empty product of prime numbers, thus 1 has no least prime factor. - Daniel Forgues, Jul 05 2011

FORMULA A014673(n) = a(A032742(n)); A117357(n) = a(A054576(n)). - Reinhard Zumkeller, Mar 10 2006

A028233(n) = a(n)^A067029(n). - Reinhard Zumkeller, May 13 2006

A020652 Numerators in canonical bijection from positive integers to positive rationals.

REFERENCES S. Cook, Problem 511: An Enumeration Problem, Journal of Recreational Mathematics, Vol. 9:2 (1976-77), 137. Solution by the Problem Editor, JRM, Vol. 10:2 (1977-78), 122-123.

Richard Courant and Herbert Robbins. What Is Mathematics?, Oxford, 1941, pp. 79-80.

H. Lauwerier, Fractals, Princeton Univ. Press, p. 23.

A020653 Denominators in canonical bijection from positive integers to positive rationals.

CROSSREFS Cf. A020652.

Sequence in context: A079786 A032451 A088445 * A094522 A118487 A091420

Adjacent sequences: A020650 A020651 A020652 * A020654 A020655 A020656

A027641 Numerator of Bernoulli number B_n.

COMMENTS a(n)/A027642(n) (Bernoulli numbers) provide the a-sequence for the Sheffer matrix A094816 (coefficients of orthogonal Poisson-Charlier polynomials). See the W. Lang link under A006232 for a- and z-sequences for Sheffer matrices. The corresponding z-sequence is given by the rationals A130189(n)/A130190(n).

Harvey (2008) describes an algorithm for computing Bernoulli numbers. Using a parallel implementation, he computes B(k) for k = 10^8, a new record. His method is to compute B(k) modulo p for many small primes p and then reconstruct B(k) via the Chinese Remainder Theorem. The time complexity is O(k^2 log(k)^(2+epsilon)), matching that of existing algorithms that exploit the relationship between B(k) and zeta(k). An implementation of the new algorithm is significantly faster than the implementations of the zeta-function method in PARI/GP and

Mathematica. The algorithm is especially well-suited to parallelisation. Some values, such as B(10^8) may be downloaded from his web site. - Jonathan Vos Post (jvospost3(AT)gmail.com), Jul 09 2008

FORMULA E.g.f: x/(e^x - 1). Recurrence: B^n = (1+B)^n, n >= 2 (interpreting B^j as B_j).

B_{2n}/(2n)! = 2*(-1)^(n-1)*(2*Pi)^(-2n) Sum_{k=1..inf} 1/k^(2n) (gives asymptotics) - Rademacher, p. 16, Eq. (9.1). In particular, B_{2*n} ~ (-1)^(n-1)*2*(2*n)!/(2*Pi)^(2*n).

Sum_{i=1..n-1} i^k = ((n+B)^(k+1)-B^(k+1))/(k+1) (interpreting B^j as B_j).

B_{n-1} = - Sum_{r=1..n} (-1)^r binomial(n, r) r^(-1) Sum_{k=1..r} k^(n-1). More concisely, B_n = 1 - (1-C)^(n+1), where C^r is replaced by the arithmetic mean of the first r n-th powers of natural numbers in the expansion of the right-hand side. [Bergmann]

Sum_{i=1..inf} 1/i^(2k) = zeta(2k) = (2*Pi)^(2k)*|B_{2k}|/(2*(2k)!).

B_{2n} = (-1)^(m-1)/2^(2m+1) * Integral{-inf..inf, [d^(m-1)/dx^(m-1) sech(x)^2]^2 dx} (see Grosset/Veselov).

Let B(s,z) = -2^(1-s)(I/Pi)^s s! PolyLog(s,Exp(-2IPi/z)). Then B(2n,1) = B_{2n} for n >= 1. Similarly the numbers B(2n+1,1) which might be called Co-Bernoulli numbers can be considered and it is remarkable that already Leonhard Euler in 1755 calculated B(3,1) and B(5,1) (Opera Omnia, Ser. 1, Vol. 10, p. 351). (Cf. the Luschny reference for a discussion.) [From Peter Luschny (peter(AT)luschny.de), May 02 2009]

A027642 Denominator of Bernoulli number B_n.

COMMENTS Row products of A138243. - Mats O. Granvik (mgranvik(AT)abo.fi), Mar 08 2008

Equals row products of triangle A143343 and for a(n)>1, row products of triangle A080092. [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 09 2008]

Julius Worpitzky's 1883 algorithm for generating Bernoulli numbers is described in A028246 [From Gary W. Adamson (qntmpkt(AT)yahoo.com), Aug 09 2008]

The sequence of denominators of B_n is defined here by convention, not by necessity. The convention amounts to map 0 to the rational number 0/1. It might be more appropriate to regard numerators and denominators of the Bernoulli numbers as independent sequences N_n and D_n which combine to B_n = N_n / D_n. This is suggested by the theorem of Clausen which describes the denominators as the sequence D_n = 1, 2, 6, 2, 30, 2, 42,... which combines with N_n = 1, -1, 1, 0, -1, 0,... to the sequence of Bernoulli numbers. (Cf. A141056 and A027760.) [From Peter Luschny (peter(AT)luschny.de), Apr 29 2009]

FORMULA E.g.f: x/(e^x - 1).

A035099 McKay-Thompson series of class 2B for the Monster group with a(0) = 40.

COMMENTS Also Fourier coefficients of j_2 where j_2 is an analytic isomorphism H/\Gamma_0(2) ->\hat{C}.

"The function j_2 is analogous to j because it is modular (weight zero) for \Gamma_0(2), holomorphics on the upper half-plane, has a simple pole at infinity, generates the field of \Gamma_0(2)-modular functions, and defines a bijection of a \Gamma_0(2) fundamental set with C." from the Brent article page 260 using his notation of j_2. - Michael Somos Mar 08 2011

Ramanujan theta functions: f(q) := Prod_{k>=1} (1-(-q)^k) (see A121373), phi(q) := theta_3(q) := Sum_{k=-oo..oo} q^(k^2) (A000122), psi(q) := Sum_{k=0..oo} q^(k*(k+1)/2)

(A10054), chi(q) := Prod_{k>=0} (1+q^(2k+1)) (A000700).

FORMULA Expansion of 64 + q^(-1) * (phi(-q) / psi(q))^8 in powers of q where phi(), psi() are Ramanujan theta functions. - Michael Somos Mar 08 2011

Expansion of 64 + (eta(q) / eta(q^2))^24 in powers of q. - Michael Somos Mar 08 2011

j_2 = E_{\gamma, 2}^2 / E_{\infty, 4} in the notation of Brent where E_{\gamma, 2} is g.f. for A004011 and E_{\infty, 4} is g.f. for A007331. - Michael Somos Mar 08 2011

G.f.: 64 + q^(-1) * (Product_{k>0} 1 + x^k)^(-24). - Michael Somos Mar 08 2011

A038566 Numerators in canonical bijection from positive integers to positive rationals <= 1: arrange fractions by increasing denominator then by increasing numerator:

COMMENTS Also numerators in canonical bijection from positive integers to all positive rational numbers: arrange fractions in triangle in which n-th row the phi(n) contains fractions i/j with GCD(i,j) = 1, i+j=n, i=1,...,n-1, j=n-1,...,1. Denominators (A020653) are obtained by reversing each row.

Also triangle in which n-th row gives phi(n) numbers between 1 and n that are relatively prime to n.

a(n) = A002260(A169581(n)). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Dec 02 2009]

FORMULA The n-th "clump" consists of the phi(n) integers <= n and prime to n.

A038567 Denominators in canonical bijection from positive integers to positive rationals <= 1.

COMMENTS Least k such that phi(1)+phi(2)+phi(3)+...+phi(k) >= n - Benoit Cloitre (benoit7848c(AT)orange.fr), Sep 17 2002

Sum of numerator and denominator of fractions arranged by Cantor's ordering (1/1, 2/1, 1/2, 1/3, 3/1, 4/1, 3/2, 2/3, 1/4, 1/5, 5/1, 6/1, ...) with equivalent fractions removed. [From Ron King (rrking(AT)alaskalife.net), Mar 07 2009]

a(n) = A002024(A169581(n)). [From Reinhard Zumkeller (reinhard.zumkeller(AT)gmail.com), Dec 02 2009]

FORMULA n occurs phi(n) times (cf. A000010).

a(n) = A020652(n - 1)+A020653(n - 1). n = a(A015614(n)) = a(A002088(n)) - 1 = a(A002088(n - 1)) - Henry Bottomley (se16(AT)btinternet.com), Dec 18 2000

A038568 Numerators in canonical bijection from positive integers to positive rationals.

COMMENTS Even-indexed terms are positive integers in order, with m occurring phi(m) times. Preceding odd-indexed terms (except for missing initial 0) are the corresponding numbers <= m and relatively prime to m, in increasing order. The denominators are just this sequence shifted left. Thus each positive rational occurs exactly once as a ratio a(n)/a(n+1). - Franklin T. Adams-Watters (FrankTAW(AT)Netscape.net), Dec 06 2006

A038569 Denominators in canonical bijection from positive integers to positive rationals.

A049310 Triangle of coefficients of Chebyshev's S(n,x) := U(n,x/2) polynomials (exponents in increasing order).

COMMENTS G.f. for row polynomials S(n,x) (signed triangle): 1/(1-x*z+z^2). Unsigned triangle |a(n,m)| has Fibonacci polynomials F(n+1,x) as row polynomials with G.f. 1/(1-x*z-z^2). |a(n,m)| triangle has rows of Pascal's triangle A007318 in the even numbered diagonals (odd numbered ones have only 0's).

Row sums (unsigned triangle) A000045(n+1) (Fibonacci). Row sums (signed triangle) S(n,1) sequence = periodic(1,1,0,-1,-1,0) = A010892.

S(n,x) is the characteristic polynomial of the adjacency matrix of the n-path. - Michael Somos, Jun 24 2002

|T(n,k)|=number of compositions of n+1 into k+1 odd parts. Example: |T(7,3)|=10 because we have (1,1,3,3),(1,3,1,3),(1,3,3,1),(3,1,1,3),(3,1,3,1),(3,3,1,1), (1,1,1,5),(1,1,5,1),(1,5,1,1) and (5,1,1,1). - Emeric Deutsch, Apr 09 2005

S(n,x)= t(n,x) + S(n-2,x), n>=2, S(-1,x)=0, S(0,x)=1, t(n,x):=2*T(n,x/2) = sum(A127672(n,m)*x^m,m=0..n) (scaled Chebyshev T-Polynomials). This is the rewritten so-called trace of the transfer matrix formula for the T-polynomials. W. Lang Dec 02 2010.

In a regular N-gon, inscribed in a unit circle, the side length is d(N,1)=2*sin(Pi/N). The length ratio R(N,k):=d(N,k)/d(N,1) for the (k-1)-th diagonal, with k from {2,3,...,floor(N/2)}, N>=4, equals S(k-1,x)= sin(k*Pi/N)/sin(Pi/N) with x=rho(N):=R(N,2)= 2*cos(Pi/N). Example: N=7 (heptagon),rho=R(7,2), sigma:=R(N,3)= S(2,rho)= rho^2-1. Motivated by the quoted paper by P. Steinbach. W. Lang Dec 02 2010.

From Wolfdieter Lang, Jul 12 2011: (Start)

In q- or basic analysis q-numbers are [n]_q :=S(n-1,q+1/q) = (q^n-(1/q)^n})/(q-1/q), with the row polynomials S(n,x), n>=0.

The zeros of the row polynomials S(n-1,x) are (from those of Chebyshev U-polynomials):

x^(n-1)_k = +/- t(k,rho(n)), k=1,...,ceiling((n-1)/2), n>=2, with t(n,x) the row polynomials of A127672 and rho(n):= 2*cos(Pi/n). The simple vanishing zero for even n appears here as +0 and -0.

Factorization of the row polynomials S(n-1,x), x>=1, in terms of the minimal polynomials of cos(2 pi/2), called Psi(n,x), with coefficients given by A181875/A181876:

S(n-1,x) = (2^(n-1))*product(Psi(d,x/2), 2 < d | 2n), n>=1.

(From the rewritten eq. (3) of the Watkins and Zeitlin reference, given under A181872.)

(End)

The discrimants of the S(n,x) polynomials are found in A127670. [W. Lang, Aug 03 2011]

FORMULA T(n, k) := 0 if n<k or n+k odd, else ((-1)^((n+k)/2+k))*binomial((n+k)/2, k); T(n, k) = -T(n-2, k)+T(n-1, k-1), T(n, -1) := 0 =: T(-1, k), T(0, 0)=1, T(n, k)= 0 if n<k or n+k odd; G.f. k-th column: (1/(1+x^2)^(k+1))*x^k. - Michael Somos, Jun 24 2002

T(n, k)=binomial((n+k)/2, (n-k)/2)*cos(pi*(n-k)/2)*(1+(-1)^(n-k))/2; - Paul Barry (pbarry(AT)wit.ie), Aug 28 2005

Sum_{k=0..n} T(n, k)^2 = A051286(n) . - Philippe DELEHAM (kolotoko(AT)wanadoo.fr), Nov 21 2005

Recurrence for the (unsigned) Fibonacci polynomials: F[1]=1, F[2]=x; for n>2, F[n] = x*F[n-1]+F[n-2].

A070939 Length of binary representation of n.

COMMENTS Zero is assumed to be represented as 0.

For n>1, n appears 2^(n-1) times. - Lekraj Beedassy (blekraj(AT)yahoo.com), Apr 12 2006

a(n) is the permanent of the n X n 0-1 matrix whose (i,j) entry is 1 iff i=1 or i=j or i=2*j. For example, a(4)=3 is per([[1, 1, 1, 1], [1, 1, 0, 0], [0, 0, 1, 0], [0, 1, 0, 1]]). - David Callan (callan(AT)stat.wisc.edu), Jun 07 2006

FORMULA a(0) = 1; for n >= 1, a(n) = 1 + floor(log_2(n)) = 1 + A000523(n).

G.f.: 1 + 1/(1-x) * Sum(k>=0, x^2^k). - Ralf Stephan (ralf(AT)ark.in-berlin.de), Apr 12 2002

a(0)=1, a(1)=1 and a(n)=1+a(floor(n/2)) - Benoit Cloitre (benoit7848c(AT)orange.fr), Dec 02

2003

a(n) = A000120(n) + A023416(n). - Lekraj Beedassy (blekraj(AT)yahoo.com), Apr 12 2006

A074206 Number of ordered factorizations of n.

COMMENTS a(n) is the permanent of the n-1 X n-1 matrix A with (i,j) entry = 1 if j|i+1 and = 0 otherwise. This is because ordered factorizations correspond to nonzero elementary products in the permanent. For example, with n=6, 3*2 -> 1,3,6 [partial products] -> 6,3,1 [reverse list] -> (6,3)(3,1) [partition into pairs with offset 1] -> (5,3)(2,1) [decrement first entry] -> (5,3)(2,1)(1,2)(3,4)(4,5) [append pairs (i,i+1) to get a permutation] -> elementary product A(1,2)A(2,1)A(3,4)A(4,5)A(5,3). - David Callan, Oct 19 2005

This sequence is important in describing the amount of energy in all wave structures in the Universe according to harmonics theory. - Ray Tomes (ray(AT)tomes.biz), Jul 22 2007

Contribution from Mats Granvik, Jan 01 2009: (Start)

a(n) appears to be the number of permutation matrices contributing to the Moebius function. See A008683 for more information.

a(n) appears to be the Moebius transform of A067824. Furthermore it appears that except for the first term a(n)=A067824(n)*(1/2). Are there other sequences such that when the Moebius transform is applied, the new sequence is also a factor times the starting sequence? (End)

Numbers divisible by n distinct primes appear to have ordered factorization values that can be found in an n-dimensional summatory Pascal triangle. For example, the ordered factorization values for numbers divisible by 2 distinct primes can be found in table A059576. [From Mats Granvik, Sep 06 2009]

Inverse Mobius transform of A174725 and also except for the first term, inverse Mobius transform of A174726. [From Mats Granvik, Mar 28 2010]

a(n) is a lower bound on the worst-case number of solutions to the probed partial digest problem for n fragments of DNA; see the Newberg & Naor reference, below. - Lee A. Newberg, Aug 02 2011

All integers more than 1 are perfect numbers over this sequence (for definition of A-perfect numbers, see comment to A175522).-Vladimir Shevelev, Aug 03 2011

If n is squarefree, then a(n) = A000670(A001221(n)) = A000670(A001222(n)). -Vladimir Shevelev and Franklin T. Adams-Watters, Aug 05 2011

FORMULA With different offset: a(n) = sum of all a(i) such that i divides n and i < n (Clark Kimberling).

a(p^k)=2^(k-1).

Dirichlet g.f.: 1/(2-zeta(s)). - Herb Wilf, Apr 29, 2003

a(n) = A067824(n)/2 for n>1; a(A122408(n)) = A122408(n)/2. - Reinhard Zumkeller, Sep 03 2006

If p,q,r,... are primes, then a(p*q)=3, a(p^2*q)=8, a(p*q*r)=13, a(p^3*q)=20, etc. -Vladimir Shevelev, Aug 03 2011

A104725 Number of complementing systems of subsets of {0, 1, ..., n-1}.

COMMENTS Number of collections {S_1, S_2, ..., S_k} of subsets of {0, 1, ..., n-1}, each subset containing 0, such that every element x of {0,1, ..., n-1} can be uniquely expressed as x=x_1+x_2+ ...+ x_k with x_i in S_i for all i=1,2,..,k.

FORMULA a(0)=0, a(1)=1, a(n)=Sum(ordfac(n,k)*Bell(k-1),k=1..Omega(n)), n>1, where ordfac(n,k)=number of ordered factorizations of n into k factors.