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A Critical Analysis of Design Flaws in the Death Star

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Report submitted in partial fulfilment of the requirements of the module Project (E) 448 for the degree Baccalaureus in Engineering in the Department of Electrical and Electronic Engineering at Stellenbosch University.

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I would like to thank my dog, Muffin. I also would like to thank the inventor of the incubator; without him/her, I would not be here. Finally, I would like to thank Dr Herman Kamper for this amazing report template.

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Abstract

English

The English abstract.

Afrikaans

Die Afrikaanse uittreksel.

Contents

List of Figures

List of Tables

Nomenclature

Variables and functions

$p(x)$	Probability density function with respect to variable x.
$P(A)$	Probability of event A occurring.
ε	The Bayes error.
ε_u	The Bhattacharyya bound.
B	The Bhattacharyya distance.
s	An HMM state. A subscript is used to refer to a particular state, e.g. s_i refers to the i^{th} state of an HMM.
$\mathbf{S}$	A set of HMM states.
$\mathbf{F}$	A set of frames.
$\mathbf{o}_f$	Observation (feature) vector associated with frame f.
$\gamma_s(\mathbf{o}_f)$	A posteriori probability of the observation vector $\mathbf{o}_f$ being generated by HMM state s.
μ	Statistical mean vector.
Σ	Statistical covariance matrix.
$L(\mathbf{S})$	Log likelihood of the set of HMM states $\mathbf{S}$ generating the training set observation vectors assigned to the states in that set.
$\mathcal{N}(\mathbf{x}\|\mu, \Sigma)$	Multivariate Gaussian PDF with mean μ and covariance matrix Σ.
a_{ij}	The probability of a transition from HMM state s_i to state s_j.
N	Total number of frames or number of tokens, depending on the context.
D	Number of deletion errors.
I	Number of insertion errors.
S	Number of substitution errors.

Acronyms and abbreviations

AE	Afrikaans English
AID	accent identification
ASR	automatic speech recognition
AST	African Speech Technology
CE	Cape Flats English
DCD	dialect-context-dependent
DNN	deep neural network
G2P	grapheme-to-phoneme
GMM	Gaussian mixture model
HMM	hidden Markov model
HTK	Hidden Markov Model Toolkit
IE	Indian South African English
IPA	International Phonetic Alphabet
LM	language model
LMS	language model scaling factor
MFCC	Mel-frequency cepstral coefficient
MLLR	maximum likelihood linear regression
OOV	out-of-vocabulary
PD	pronunciation dictionary
PDF	probability density function
SAE	South African English
SAMPA	Speech Assessment Methods Phonetic Alphabet

Chapter 1

Introduction

Chapter 2

Literature Review

2.1. Theory and applications of superconductivity

In order to understand and apply the methods described in [2] one must first understand the basic theory behind superconductivity as well as some examples of how this theory is used in practice.

2.1.1. Superconductivity

REVIEW NEEDED

Since the first discovery of superconductivity a couple of successfully theories have been put forth to explain the phenomenon. The London theory is a framework that describes the qualitative behaviour of superconductors and correctly describes perfect diamagnetic and zero resistance but fails to explain the effect on a microscopic level [3]. The London equations (equation 2.1 and equation 2.2) [4] is an addition to Maxwell's equations.

$$E = \frac{\partial}{\partial t}(\Lambda J_s) \tag{2.1}$$

$$h = -c\nabla \times (\Lambda J_s) \tag{2.2}$$

Here Λ is a phenomenological parameter determined through experimentation. The London equations allow us to calculate the current distribution in a superconductor which is very important to the objective of this project. BCS theory put forth a microscopic model of superconductors and explains the phenomenon as a quantum mechanical effect. The details are out of scope for this project but on a crude qualitative level BCS theory can be explained by the pairing of electrons in the crystal lattice of the superconductor allowing them to be considered one particle. These particles are known as cooper-pairs. At extremely low temperatures the formation of these cooper pairs are energetically favourable [5]. Electron pairs in this state can flow through the superconductor unimpeded. BCS theory is the most successful model of superconductivity discovered to date.

2.1.2. The Josephson junction

In superconducting electronics the active component is the Josephson junction [6]. The Josephson junction refers to a situation where two superconductors are connected through a thin non-conductive barrier. If this barrier is thin enough one can observe what is known as the Josephson effect. This phenomena is can be explained by considering the effect of quantum tunnelling of cooper pairs through the non-conductive boundary. For a sufficiently large barrier one can express the ensemble average wave function in each superconductor independently [6]:

$$\Psi = |\Psi(\mathbf{r})| \exp\{i\theta(\mathbf{r})\} \tag{2.3}$$

This is due to the fact that it is energetically favourable for cooper pairs in proximity to one another to lock phases [6] allowing one to express a large collection of these cooper pairs as one ensemble wave function. The idea that it is energetically favourable for cooper-pairs in proximity to one another extends to the situation where the superconductors are separated by an insulating boundary. When the barrier is sufficiently small the energy of the system can be reduced by the coupling of wave functions in their respective superconductors [6]. This results in cooper-pairs being able to move across the boundary without energy loss. Following the derivation in [5] one can describe the system behaviour using Schrödinger's equation when a voltage is applied to the junction [5]:

$$i\hbar\frac{\partial\Psi_1}{\partial t} = U_1\Psi_1 + K\Psi_2 \tag{2.4}$$

$$i\hbar\frac{\partial\Psi_2}{\partial t} = U_2\Psi_2 + K\Psi_1 \tag{2.5}$$

Here $U_1 = qV/2$ and $U_2 = -qV/2$ [5] refer to the energy of the two wave functions and K refers to the coupling energy between each wave function. By taking 2.3 and setting $|\Psi(\mathbf{r})|$ to $\sqrt{\rho}$ where ρ refers to the cooper pair density in the super conductor one can substitute the result into equations 2.4 and 2.5. From this substitution one can find that the rate of change of electron density on either side of the junction to be described by the following equation [5]:

$$\dot{\rho}_1 = \frac{2}{\hbar}K\sqrt{\rho_1\rho_2}sin(\delta) \tag{2.6}$$

Where ρ_1 is the electron density on one side of the junction. The rate of change of charge on the other side of the junction is simply $\dot{\rho}_2 = -\dot{\rho}_1$ [5]. The rate of change of charge is the current and therefore the current through the junction can be expressed as follows [7]:

$$I = I_0 sin(\theta_2 - \theta_1) \tag{2.7}$$

Equation 2.7 is known as the current-phase relation of the DC Josephson effect. The phases of the currents on each side of the boundary is described by equation 2.8 [5].

$$\dot{\theta}_2 - \dot{\theta}_1 = \frac{qV}{\hbar} = \dot{\delta} \tag{2.8}$$

Integrating on both sides yields equation 2.9:

$$\delta(t) = \delta_0 + \frac{q}{\hbar}\int V(t)dt \tag{2.9}$$

It is important to note that 2.7 is only valid for a limited number of cases and the actual current-phase relation is often more complicated [7]. The applications of equation 2.7 is limited to analysing analogue and digital devices based on Josephson junctions [7].

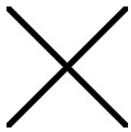

Figure 2.1: The circuit symbol for a Josephson junction

The RCSJ model

The math in this section needs review The phase-current and phase-voltage relation makes the assumption of a perfect Junction. This assumption is inaccurate and as such the RCSJ model is used to more accurately model the behaviour of a junction [4]. The resistively and capacitively shunted junction is a simple model used to describe the dynamics of a junction. As the name implies it consists of a resistor (R), capacitance (C) and a Josephson current (I_s). In a non-ideal junction a displacement current flows between the two superconductors. This displacement current is modelled by the capacitor. The resistive element models the effect of quasi-particle tunnelling across the boundary [6]. Figure Insert fig depicts the RCSJ model. To derive the current-voltage relation of an RCSJ model one can express the voltage across the parallel components as a function of the current flowing into the junction:

$$I = I_c sin(\delta) + C\frac{dV}{dt} + \frac{V}{R} \tag{2.10}$$

Equation 2.10 can be re-written using equation 2.9. The resulting equation is a second-order non-linear, inhomogeneous differential equation for which analytical solutions are not available. For the simple case when $C = 0$ a differential equation of the form of equation 2.11 can be obtained.

$$K = \frac{d\varphi}{d\tau} + sin(\varphi) \tag{2.11}$$

This equation can be integrated directly using separation of variables and a Weierstrass substitution [6]. A detailed derivation is listed in appendix List appendix. The resulting equation describes the dynamics of the phase shift for a constant input current. The frequency of the current through the junction is determined by the dynamics of the phase. The period of the phase-dynamics is the Josephson frequency. Using the Josephson frequency relation [6],

$$\omega = \frac{q}{\hbar}V \tag{2.12}$$

as well as the period of the equation for the phase, we find the time average voltage of the junction to be [6]:

$$V = \begin{cases} 0 & \text{for } I \leq I_c \\ I_c R[(\frac{I}{I_c})^2 - 1]^{\frac{1}{2}} & \text{for } I > I_c \end{cases} \tag{2.13}$$

Graphing equation 2.13 yields the figure below: Insert fig

2.1.3. SQUID's

An application of the Josephson effect is the superconducting quantum interference device (SQUID). In essence a SQUID refers to a superconducting ring that contains one or more Josephson junction. This interference effect is analogous to interference one might encounter in the field of optics [8]. The SQUID is a highly sensitive device that can in some cases measure fields as weak as $5 \times 10^{-14} T$ [9].

Flux quantisation

To understand the basic operation of a SQUID, one must first understand the concept of flux quantisation. To do so we consider a superconducting ring in the presence of a uniform magnetic field. The ring is superconducting, so it exhibits the Meissner effect and thus the current density inside the ring is zero. Recall that the flux through a ring is:

$$\Phi = \oint \mathbf{A} \cdot d\mathbf{s} \tag{2.14}$$

Now consider the equation for the current density in a superconductor [5]:

$$\mathbf{J} = \frac{\rho\hbar}{m}(\nabla\theta - \frac{q\mathbf{A}}{\hbar}) \tag{2.15}$$

The current density inside the ring in the superconducting state is zero so equation 2.15 becomes:

$$\nabla\theta = \frac{q}{\hbar}\mathbf{A} \tag{2.16}$$

Integrating on both sides around a curve deep inside the superconductor such that the assumption that the current density is zero holds we can express equation 2.16 as:

$$\oint \nabla\theta \cdot d\mathbf{s} = \frac{q\Phi}{\hbar} \tag{2.17}$$

Recognizing $\nabla\theta$ as vector field with potential function θ, we can simply write the left-hand side of equation 2.17 as $\theta(\mathbf{r_1}) - \theta(\mathbf{r_1})$. One might assume that the left-hand side of the equation 2.17 must be equal to zero. This is incorrect because the absolute phase, meaning the potential function θ cannot be determined and can only be determined up to some constant because the gradient of the scalar potential function will take any constant to zero. Therefore, the phase can only be determined relative to some point. According to [5] the only limitation we can place on the phase of the wave function is that the wave function must be singularly valued. This is intuitively understood as the fact that a particle cannot have two different amplitudes to be in a certain quantum state as that would imply that it has two probabilities associated with the exact same quantum state [5]. From equation 2.3 one can conclude that the phase can change by any integer multiple of 2π as this results in the exact same wave function. Equation 2.17 then becomes:

$$2\pi n = \frac{q\Phi}{\hbar} \tag{2.18}$$

Clearly equation 2.18 implies that the flux through the superconducting loop must be quantized as n can only take on integer values.

The DC SQUID

Spurred on by developments in reliable manufacturing processes of Josephson junctions the DC SQUID has become the dominant device in the field of niobium based sensors [9]. The initial discovery of the DC SQUID by [8] in 1964 came shortly after the discovery of the Josephson junction in 1962 [10]. The simplest realisation of the DC SQUID consists of two Josephson junctions connected in parallel as depicted in figure 2.2.

A relationship between the current and the flux through the loop of the SQUID would like to be derived. The current through each Josephson junction is denoted as I_1 and I_2 for junction 1 and 2 respectively. From figure 2.2 one can easily see that there are 2 possible paths through the SQUID. Following the derivation of [5] one can write the change in phase through each branch of the squid using equations 2.19 and 2.20 [5].

$$\Delta\theta_1 = \theta_1 + \frac{2q_e}{\hbar}\int_{\text{left branch}} \mathbf{A} \cdot d\mathbf{s} \tag{2.19}$$

$$\Delta\theta_2 = \theta_2 + \frac{2q_e}{\hbar}\int_{\text{right branch}} \mathbf{A} \cdot d\mathbf{s} \tag{2.20}$$

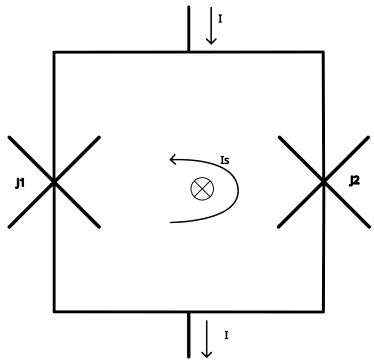

Figure 2.2: A circuit schematic depicting the configuration of a simple DC SQUID. The current I is the bias current and the current I_s is the screening current developed by an external magnetic field

Where θ_1 and θ_2 is the change in phase across their respective junctions. Similarly, $\Delta\theta_1$ and $\Delta\theta_2$ refer to the change in phase through the left and right branch respectively. As previously discussed, the integral of the phase gradient around a closed loop must be an integer multiple of 2π. As a consequence, subtracting equation 2.20 from 2.19 must yield $2\pi n$. Using equation 2.17 we derive equation 2.21.

$$\theta_2 = \theta_1 - 2\pi n + \frac{2q_e}{\hbar}\Phi \tag{2.21}$$

Using equation 2.7, 2.21 and a KCL at the top or bottom node, the relation between the current going into the loop and the flux through the loop is derived and equation is found. Here the $2\pi n$ term can be neglected as a $2\pi n$ shift in phase does not change the value of the function.

$$I = I_2 sin(\theta_2) + I_1 sin(\theta_2 - \frac{2q_e}{\hbar}\Phi) \tag{2.22}$$

Equation 2.22 describes the general behaviour of the DC SQUID. The current voltage characteristics are shown in the figure below Get figure

It is worth pointing out the mechanism through which flux is quantized in the SQUID loop. When an external field is applied to the washer a screening current begins to flow around the loop to oppose the applied field. The magnitude of the current is determined by the loop inductance. Assuming that the applied flux density is constant throughout the hole in the loop, the magnetic flux generated by the screening current is $\Phi_s = LI_s$. The total flux in the loop is $\Phi_{\text{ext}} + \Phi_s = n\Phi_0$. If applied flux is less than $\frac{\Phi}{2}$ the screening current flows to oppose the build up of flux in the loop by ensuring that $\Phi_s = -\Phi_{\text{ext}}$ and thus the total flux in the loop is 0. When the applied flux increases to $\frac{\Phi}{2}$ the flux state of the loop changes and the screening current changes direction [9] to ensure that the total flux in the loop is Φ_0. If the external flux is further increased the flux induced by the screening current decreases from $\frac{\Phi_0}{2}$ to 0 such that the total flux in the loop remains Φ_0. The trend continues as the external field strength is increased. Because of this mechanism the critical current and voltage-flux characteristics of the SQUID always has a period of Φ_0 [11]. As pointed out by [9], this allows the SQUID to be automatically calibrated as 1 period corresponds to 1 flux quantum in the loop.

2.1.4. Practical DC SQUIDS and design considerations

The resistively shunted junction

Practical realisation of a DC SQUID requires that the designer connect shunt resistors to each junction [9]. This is done such that the system is strongly over damped [11]. In the context of the discussion on the I-V characteristics of the Josephson junction the addition of the shunt resistors correspond to the effect of the junction capacitance becoming negligible. In the absence of a shunt resistor the I-V characteristics of a Josephson junction has hysteresis [9].

Effective Area and Inductance

Two important quantities for the design and analysis of SQUID sensors are the effective area and the SQUID inductance. The effective area of a SQUID washer is defined as [11]:

$$A_{\text{eff}} = \frac{\Phi_s}{B_a} \tag{2.23}$$

In equation 2.23 Φ_s refers to the flux coupled into the SQUID loop as a result of an applied magnetic field B_a. The effective area is typically higher than the geometric area of the washer hole due to the Meissner effect [9]. As will be shown in the next section there is a limit on the inductance of a SQUID loop and by extension a limit on how large one can make a SQUID. Critically, the field sensitivity is inversely proportional to the effective area. The magnetic field noise is inversely proportional to the geometric area of the hole in the SQUID loop. The design then comes down to creating a SQUID loop that maximises the effective area but minimizes the actual area of the SQUID loop to keep the SQUID loop inductance and thus the field noise to a minimum [11].

The uncoupled SQUID

Below is an example of an uncoupled DC SQUID structure. The uncoupled SQUID is the simplest of designs as it only consists of a single superconducting loop with 2 resistively shunted Junctions. The SQUID characteristics are determined by the loop inductance, critical current, shunt resistance and parasitic junction capacitance [9]. Design involves choosing the optimal set of these parameters to satisfy the design requirements. This is often not a trivial task due to the non-linear nature of the equations describing the behaviour of the Josephson junction and by extension the DC SQUID. Through extensive

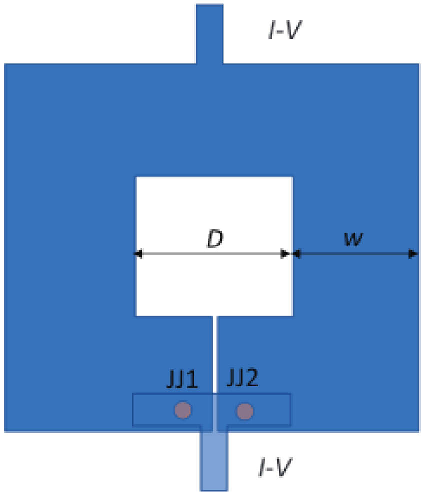

Figure 2.3: A diagram of a practical uncoupled DC SQUID structure [1]

simulation a number of key design parameters have been identified [9]. The first of which is described by equation 2.24 and is referred to as the noise parameter [11].

$$\Gamma = \frac{2\pi k_b T}{\Phi_0 I_c} \tag{2.24}$$

The noise parameter is the quotient of the thermal energy and the Josephson coupling energy [11]. It has the effect of an apparent reduction in the critical current of a junction at low voltages [9].
The second design consideration is the quality or "Q" factor. For currents the phase shift induced by the Josephson junction can be interpreted as an inductor [11]. Under this approximation it is clear to see how the combination of the shunt capacitance and junction create an RC resonator. As pointed out by [9] it is often desirable to keep the quality factor of resonating circuits in non-linear systems close to unity. This leads to the next design rule [11]:

$$\beta_c = Q_j^2 = 2\pi I_c R^2 \frac{C}{\Phi_0} \approx 1 \tag{2.25}$$

This parameter is largely controlled by designer by tuning the shunt resistance.
The next design rule sets a constraint on the choice of L (the inductance of the SQUID washer). The magnitude of the screening current in the loop is related to the applied flux through the inductance of the SQUID washer. Increasing the SQUID loop inductance results in a smaller screening current induced in the loop to keep the flux in the loop at an integer multiple of the flux quantum. This leads to a smaller modulation depth of the critical current and by extension the modulation depth of the voltage flux characteristic of the SQUID [9]. For this reason it is generally better to keep the inductance of the SQUID washer low. Simulations have shown that for very low inductances the SQUID

noise increases, so a compromise is in order [9] leading to the next design rule:

$$\beta_L = \frac{2LI_c}{\Phi_0} \approx 1 \tag{2.26}$$

Coupled SQUIDS

See pg 177

SQUID readout

Left off here. See pg 139 of SQUID hb

2.2. Noise in SQUIDS

List connection between Noise and squid sensitivity here aka - Why it is useful to know the noise figure ie the low frequency performance also list the characteristics of 1/f noise also why is low frequency performance important

2.2.1. Origin of noise in SQUIDS

proposed model for noise specifically 1/f noise

2.2.2. Models for noise in SQUIDS

this is basically steven antons work as well as some other people he links to observations made through measurements dependance on temp dependance on geometry some work on improving low frequency performance

2.2.3. Techniques for predicting mean square flux noise

go through methods previously used and explain Steven Antons work and discuss results for his noise modelling

2.3. InductEx and tetrahenry

Should this even be here?

Chapter 3

Solution development

asd

Chapter 4

Results

Chapter 5

Summary and Conclusion

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Appendix A

Project Planning Schedule

This is an appendix.

Appendix B

Outcomes Compliance

This is another appendix.