

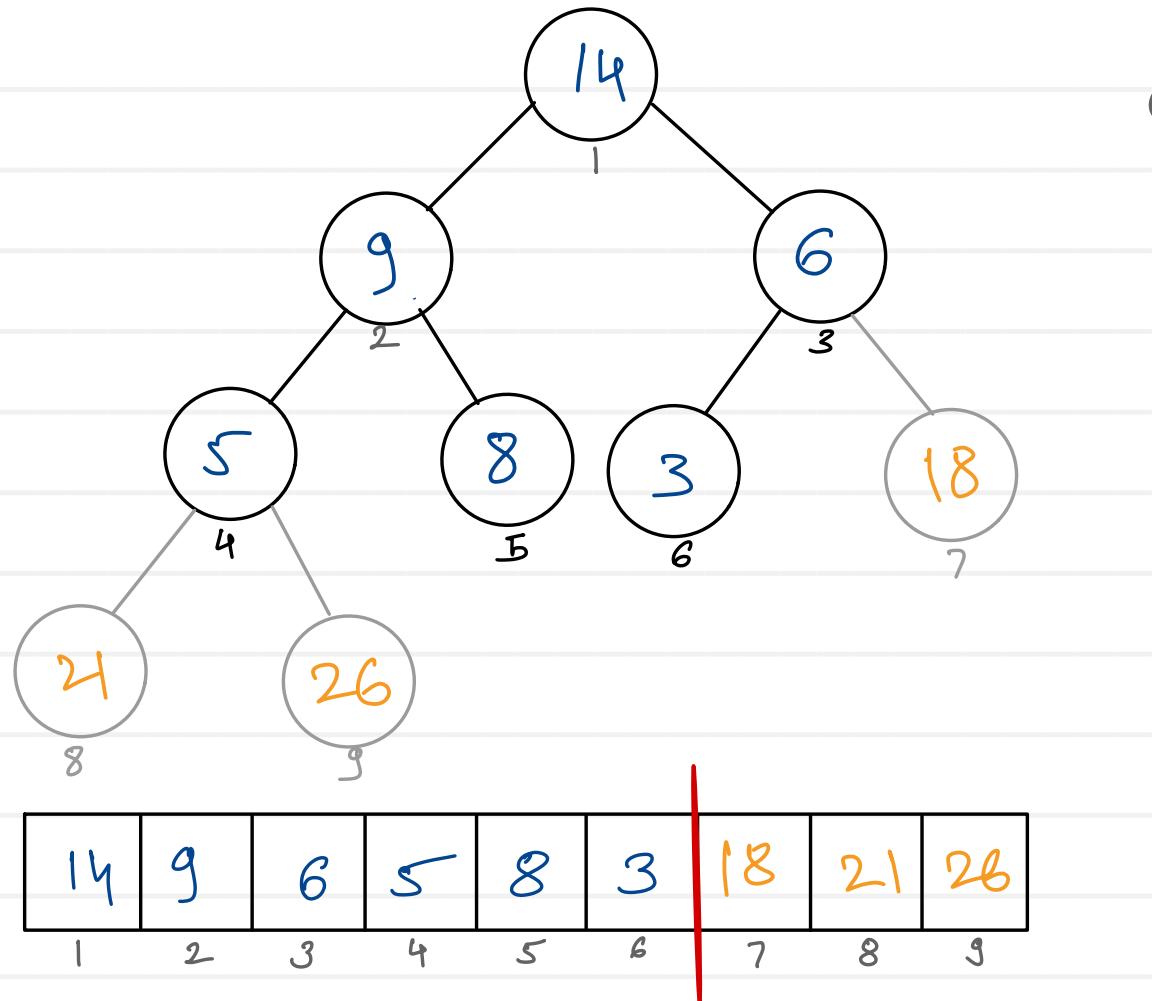


**Sunbeam Institute of Information Technology
Pune and Karad**

Algorithms and Data structures

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Heap sort



arr

| | | | | | | | | |
|---|----|---|----|---|----|----|---|---|
| 6 | 14 | 3 | 26 | 8 | 18 | 21 | 9 | 5 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

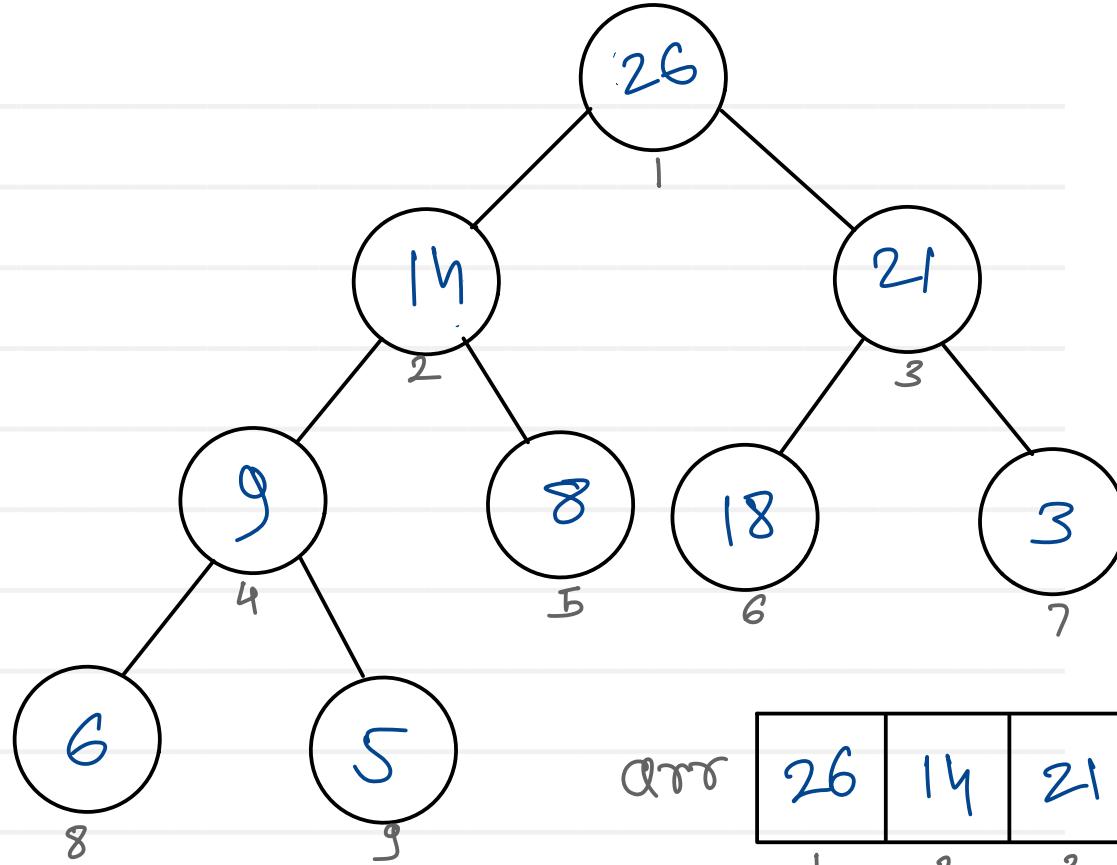
Algorithm:

1. create heap from given array
 ascending \leftarrow max $\min \rightarrow$ descending sort

2. delete all elements from heap one by one and place them on last index of array.

$$\begin{aligned} \text{Create heap} &= n * \log n \\ \text{delete heap} &= \frac{n * \log n}{2n \log n} \end{aligned}$$

$$T(n) = O(n \log n)$$



$$\text{size} = 9$$
$$\text{parent} = 9/2 = 4$$

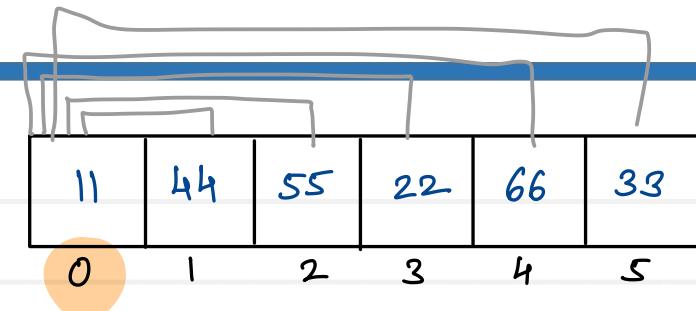
arr

| | | | | | | | | |
|----|----|----|---|---|----|---|---|---|
| 26 | 14 | 21 | 9 | 8 | 18 | 3 | 6 | 5 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



Selection sort

1. Select one position of the array
2. Find smallest element out of remaining elements
3. Swap selected position element and smallest element
4. Repeat above steps until array is sorted ($N-1$)



| | | | | | |
|----|----|----|----|----|----|
| 44 | 11 | 55 | 22 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

Pass 1

| | | | | | |
|----|----|----|----|----|----|
| 44 | 11 | 55 | 22 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

Pass 2

| | | | | | |
|----|----|----|----|----|----|
| 11 | 44 | 55 | 22 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

Pass 3

| | | | | | |
|----|----|----|----|----|----|
| 11 | 22 | 55 | 44 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

Pass 4

| | | | | | |
|----|----|----|----|----|----|
| 11 | 22 | 33 | 44 | 66 | 55 |
| 0 | 1 | 2 | 3 | 4 | 5 |

Pass 5

| | | | | | |
|----|----|----|----|----|----|
| 11 | 22 | 33 | 44 | 66 | 55 |
| 0 | 1 | 2 | 3 | 4 | 5 |

To select position of array : $i = 0 \rightarrow N-2$ ($i < N-1$)

To find smallest element : $j = i+1 \rightarrow N-1$ ($j < N$)

```
public static void selectionSort(int arr[], int N) {  
    // 1. select positions one by one  
    for (int i = 0; i < N - 1; i++) {  
        // 2. find min element from remaining elements  
        int minIndex = i;  
        for (int j = i + 1; j < N; j++) {  
            if (arr[j] < arr[minIndex])  
                minIndex = j;  
        }  
        // 3. swap selected position element & smallest element  
        int temp = arr[minIndex];  
        arr[minIndex] = arr[i];  
        arr[i] = temp;  
    }  
}
```



Selection sort

| | | | | | |
|----|----|----|----|----|----|
| 44 | 11 | 55 | 22 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | |
|---|----------|---|
| i | minIndex | j |
| 0 | 0 | 1 |
| | 1 | 2 |
| | 2 | 3 |
| | 3 | 4 |
| | 4 | 5 |
| | 5 | 6 |

| | | | | | |
|----|----|----|----|----|----|
| 11 | 44 | 55 | 22 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | |
|---|----------|---|
| i | minIndex | j |
| 1 | 1 | 2 |
| | 2 | 3 |
| | 3 | 4 |
| | 4 | 5 |
| | 5 | 6 |

```
minIndex = i
for(j=i+1; j<N; j++)
    if(arr[j] < arr[minIndex])
        minIndex = j;
```

Degree of polynomial;
- maximum power in polynomial.

- while writing complexities

consider only degree term because it is highest growing term in polynomial

| | |
|------|---------|
| n | n^2 |
| 1 | 1 |
| 10 | 100 |
| 100 | 10000 |
| 1000 | 1000000 |

N : no. of elements

N-1 : no. of passes

| pass | No. of comps |
|------|--------------|
| 1 | N-1 |
| 2 | N-2 |
| 3 | N-3 |
| : | : |
| N-1 | 1 |

Total comps $\approx 1 + 2 + 3 + \dots + (n-2) + (n-1)$
 $\approx \frac{n(n+1)}{2}$

Time \propto comps

Time $\propto \frac{n(n+1)}{2}$

Time $\propto \frac{1}{2}(n^2+n)$

$T(n) = O(n^2)$

— Best
— Avg
— Worst

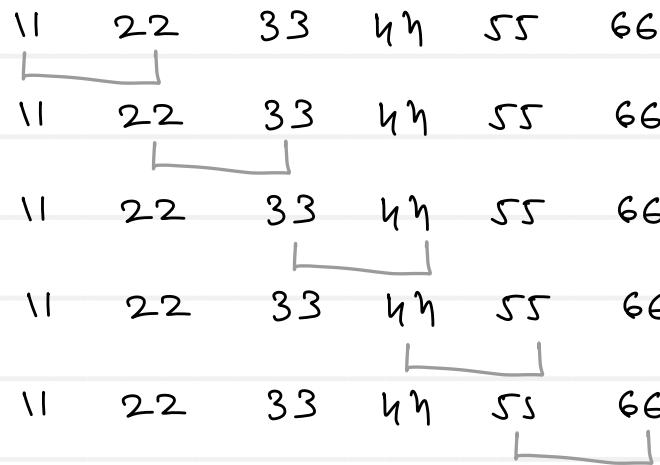
$S(n) = O(1)$





Bubble sort

1. Compare all pairs of consecutive elements of the array one by one
2. If left element is greater than right element , then swap both
3. Repeat above steps until array is sorted ($N-1$)



No. of comps = $n-1$

Time $\propto n-1$

$T(n) = O(n)$ - Best

$S(n) = O(1)$

| Pass | Comps |
|-------|-------|
| 1 | $n-1$ |
| 2 | $n-2$ |
| : | : |
| $n-2$ | 2 |
| $n-1$ | 1 |

$$\begin{aligned}
 \text{Total comps} &= 1 + 2 + 3 + \dots + (n-2) + (n-1) \\
 &= \frac{n(n+1)}{2} \\
 &= \frac{1}{2}(n^2+n)
 \end{aligned}$$

Time \propto comps

Time $\propto \frac{1}{2}(n^2+n)$

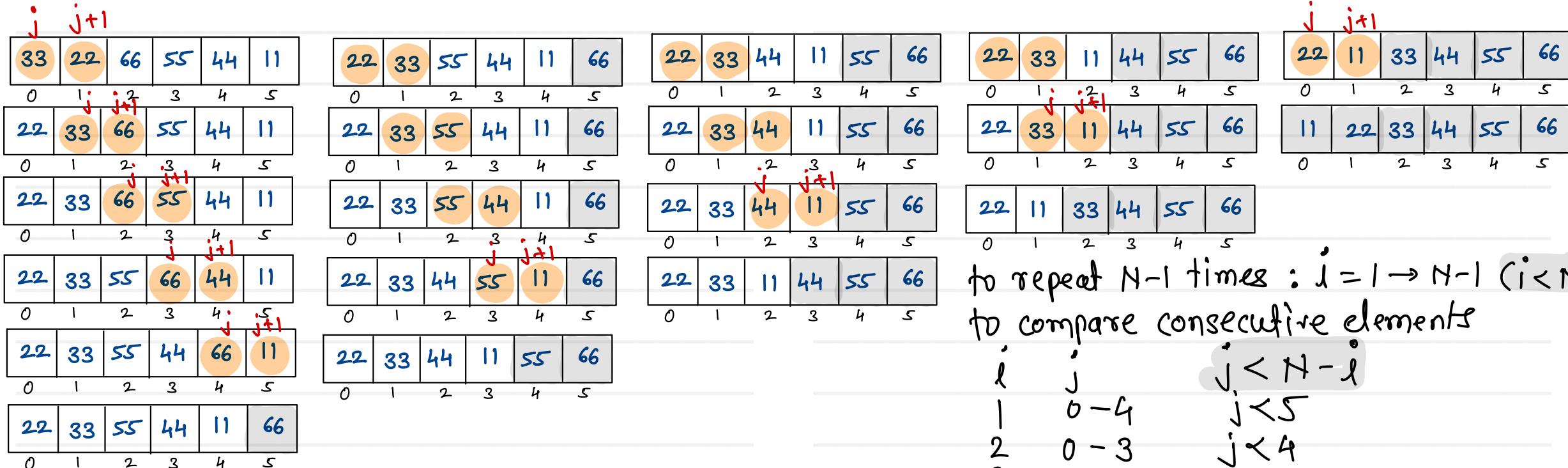
$T(n) = O(n^2)$ Avg worst





Bubble sort

| | | | | | |
|----|----|----|----|----|----|
| 33 | 22 | 66 | 55 | 44 | 11 |
| 0 | 1 | 2 | 3 | 4 | 5 |



to repeat N-1 times : $i = 1 \rightarrow N-1$ ($i < N$)
to compare consecutive elements

| i | j | $j < N-i$ |
|-----|-------|-----------|
| 1 | 0 - 4 | $j < 5$ |
| 2 | 0 - 3 | $j < 4$ |
| 3 | 0 - 2 | $j < 3$ |
| 4 | 0 - 1 | $j < 2$ |
| 5 | 0 - 0 | $j < 1$ |





Insertion sort

1. Pick one element of the array (start from 2nd index)
2. Compare picked element with all its left neighbours one by one
3. If left neighbour is greater, move it one position ahead
4. Insert picked element at its appropriate position
5. Repeat above steps until array is sorted ($N-1$)



$$\text{Total itrs} = n-1$$

$$\text{Time} \propto n-1$$

$$T(n) = O(n)$$

- Best

$$S(n) = O(1)$$

| | passes | comps |
|-----------------------|--------|-----------------|
| No. of elements = n | 1 | 1 |
| No. of passes = $n-1$ | 2 | 2 |
| | 3 | 3 |
| | : | : |
| | $n-1$ | $\frac{n-1}{n}$ |

$$\text{Total comps} = 1+2+3+\dots+n$$

$$= \frac{n(n+1)}{2}$$

$$\text{Time} \propto \frac{1}{2}(n^2+n)$$

$$T(n) = O(n^2)$$

worst
Avg





Insertion sort

| | | | | | |
|----|----|----|----|----|----|
| 55 | 44 | 22 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

44
temp

22
temp

66
temp

11
temp

33
temp

| | | | | | |
|----|---|----|----|----|----|
| 55 | | 22 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|---|----|----|----|
| 44 | 55 | | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|---|----|----|
| 22 | 44 | 55 | | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|----|---|----|
| 22 | 44 | 55 | 66 | | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|----|----|---|
| 11 | 22 | 44 | 55 | 66 | |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|---|----|----|----|----|----|
| | 55 | 22 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|---|----|----|----|
| 44 | 55 | | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|----|----|----|
| 22 | 44 | 55 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|---|----|----|
| 22 | 44 | 55 | | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|----|----|---|
| 11 | 22 | 44 | 55 | 66 | |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|----|----|----|----|
| 44 | 55 | 22 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|---|----|----|----|----|
| 44 | | 55 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|---|----|----|----|----|----|
| | 44 | 55 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|---|----|----|----|
| 22 | 44 | | 55 | 66 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | |
|----|----|---|----|----|----|
| 11 | 22 | | 44 | 55 | 66 |
| 0 | 1 | 2 | 3 | 4 | 5 |

| | | | | | | |
|---|---|----|----|----|----|----|
| | | 44 | 55 | 66 | 11 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 | |

| | | | | | | |
|----|----|----|----|----|----|--|
| 22 | 44 | 55 | 66 | 11 | 33 | |
| 0 | 1 | 2 | 3 | 4 | 5 | |

to pick elements : $i = 1 \rightarrow N-1$ ($i < N$)

to compare with left neighbors : $j = i-1 \rightarrow 0$ ($j \geq 0$)





Insertion sort

```
for( i=1 ; i<N ; i++ ) {  
    temp = arr[i]  
    j;  
    for( j=i-1 ; j>=0 ; j-- ) {  
        if( arr[j] > temp)  
            arr[j+1] = arr[j];  
        else  
            break;  
    }  
    arr[j+1] = temp;  
}
```

| | | | | | |
|----|----|----|----|----|----|
| 11 | 22 | 33 | 44 | 55 | 66 |
| 0 | 1 | 2 | 3 | 4 | 5 |

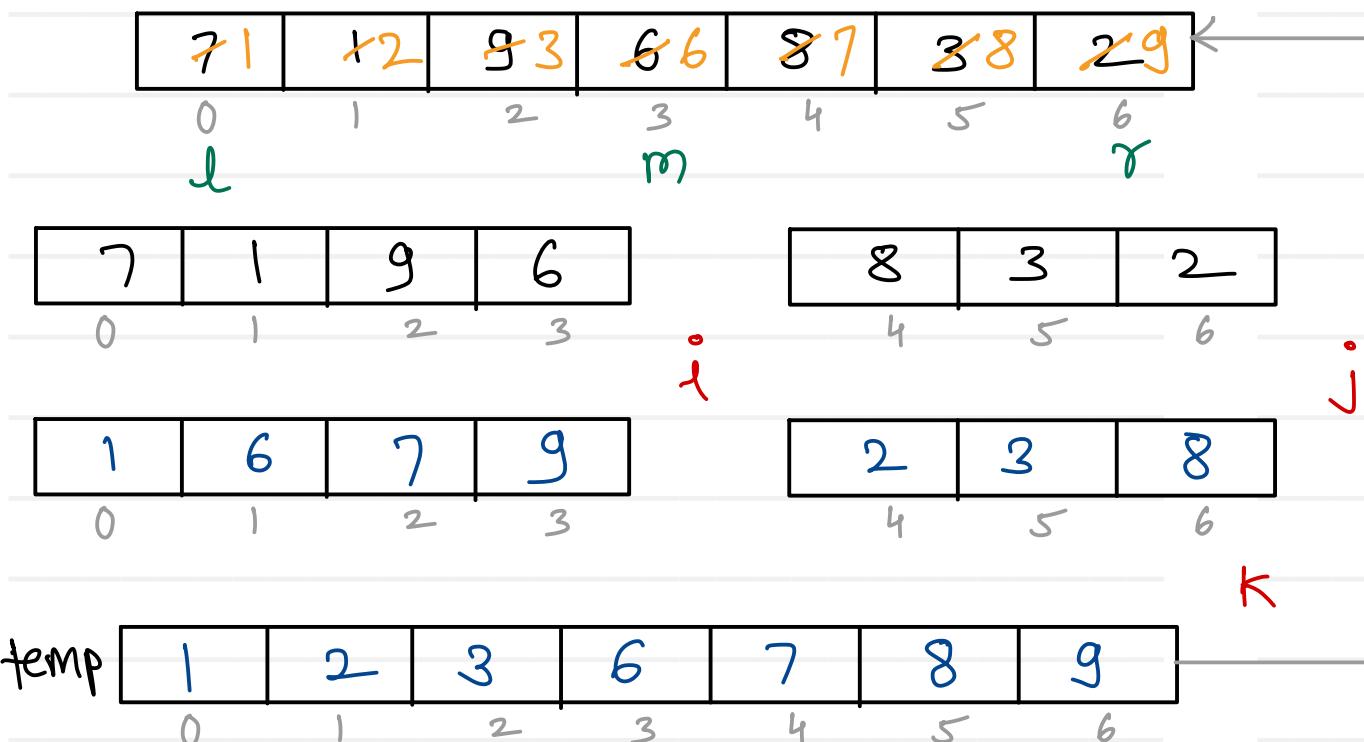
| i | i<6 | temp | j |
|---|-----|------|------------|
| 1 | T | 44 | 0,-1 |
| 2 | T | 22 | 1,0,-1 |
| 3 | T | 66 | 2 |
| 4 | T | 11 | 3,2,1,0,-1 |
| 5 | T | 33 | 4,3,2,1 |
| 6 | F | | |





Merge sort

1. Divide array in two parts
2. Sort both partitions individually (by merge sort only)
3. Merge sorted partitions into temporary array
4. Overwrite temporary array into original array



$$\text{mid} = \frac{\text{left} + \text{right}}{2}$$

left partition : $\text{left} \rightarrow \text{mid}$

right partition : $\text{mid}+1 \rightarrow \text{right}$

$$\text{size of temp array (size)} = \text{right} - \text{left} + 1$$

$$i = \text{left} \rightarrow \text{mid}$$

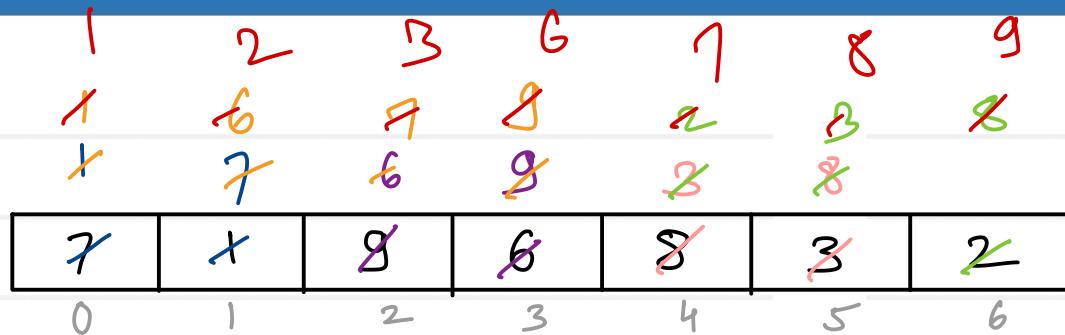
$$j = \text{mid}+1 \rightarrow \text{right}$$

$$k = 0 \rightarrow \text{size}$$





Merge sort



No. of levels = $\log n$
comps per level = n

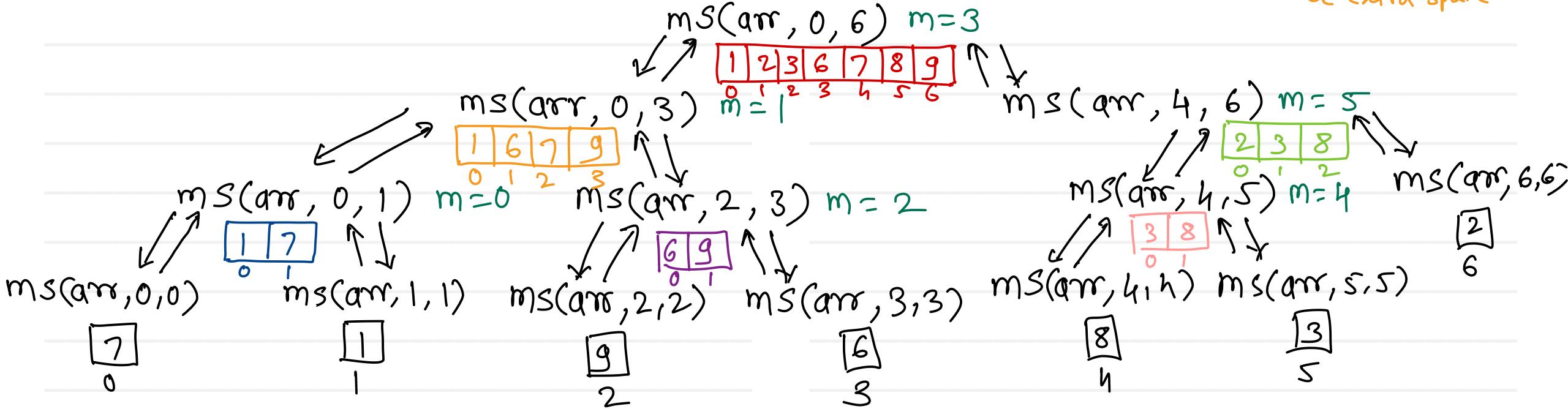
total comps = $n \log n$

Best
Avg
Worst

$$T(n) = O(n \log n)$$

$$S(n) = O(n)$$

↳ space of temp array will be extra space



| | Space |
|----------------|--|
| Selection sort | $O(n^2)$ |
| bubble sort | $O(n)$ |
| insertion sort | $O(1)$ in place sorting algorithm |
| Heap sort | $O(n)$ |
| Quick sort | $O(n)$ |
| Merge sort | $O(n)$ |

| Time | Best | Avg | Worst |
|---------------|---------------|---------------|---------------|
| $O(n^2)$ | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ |
| $O(n)$ | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ |
| $O(n)$ | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ |
| $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ |
| $O(n \log n)$ | $O(n \log n)$ | $O(n^2)$ | |
| $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ |



Two sum

Given an array of integers nums and an integer target, return indices of the two numbers such that they add up to target.

You may assume that each input would have exactly one solution, and you may not use the same element twice.

You can return the answer in any order.

Example 1:

Input: nums = [2,7,11,15], target = 9
Output: [0,1]

Example 2:

Input: nums = [3,2,4], target = 6
Output: [1,2]

Example 3:

Input: nums = [3,3], target = 6
Output: [0,1]

$$T(n) = O(n^2)$$

$$S(n) = O(1)$$

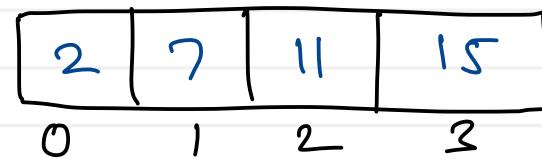
```
int[] twoSum(int[] nums, int target) {  
    for (int i = 0; i < nums.length - 1; i++) {  
        for (int j = i + 1; j < nums.length; j++) {  
            if (nums[i] + nums[j] == target)  
                return new int[]{i, j};  
        }  
    }  
    return new int[]{};  
}
```





Two pointers Technique

- The two-pointer technique is a widely used approach to solving problems efficiently, which involves arrays or linked lists.
- This method involves traversing arrays or lists with two pointers moving at different speeds or in different directions.
- This technique is used to solve problems more efficiently than using a single pointer or nested loops.



- Find a pair of elements that sum up to a target.
 - Array: [2, 7, 11, 15]
 - Target Sum: 9
- Use two index variables **left** and **right** to traverse from both corners.
 - Initialize: **left** = 0, **right** = n – 1
 - Run a loop while **left < right**, do the following inside the loop
 - Compute current sum, **sum** = arr[**left**] + arr[**right**]
 - If the **sum** equals the **target**, we've found the pair.
 - If the **sum** is less than the **target**, move the **left** pointer to the right to increase the **sum**.
 - If the **sum** is greater than the **target**, move the **right** pointer to the left to decrease the **sum**.





Two sum

sorted

Given an array of integers `nums` and an integer `target`, return indices of the two numbers such that they add up to `target`.

You may assume that each input would have exactly one solution, and you may not use the same element twice.

You can return the answer in any order.

Example 1:

Input: `nums` = [2,7,11,15], `target` = 9

Output: [0,1]

Example 2:

Input: `nums` = [3,2,4], `target` = 6

Output: [1,2]

Example 3:

Input: `nums` = [3,3], `target` = 6

Output: [0,1]

| | | | |
|---|---|----|----|
| 2 | 7 | 11 | 15 |
| 0 | 1 | 2 | 3 |
| l | r | | |

| l | r | sum | target |
|---|---|-----|--------|
| 0 | 3 | 17 | > |
| 0 | 2 | 13 | > |
| 0 | 1 | 9 | = |

```
int[] twoSum(int[] nums, int target) {
    int left = 0, right = nums.length - 1;
    while (left < right) {
        sum = nums[left] + nums[right];
        if (sum == target)
            return new int[]{left, right};
        else if (sum < target)
            left++;
        else
            right++;
    }
    return new int[] {};
}
```



Two sum

Given an array of integers nums and an integer target, return indices of the two numbers such that they add up to target.

You may assume that each input would have exactly one solution, and you may not use the same element twice.

You can return the answer in any order.

Example 1:

Input: nums = [2,7,11,15], target = 9
Output: [0,1]

| HashMap | |
|---------|-------|
| Key | value |
| 2 | 0 |

Example 2:

Input: nums = [3,2,4], target = 6
Output: [1,2]

| HashMap | |
|---------|-------|
| Key | value |
| 3 | 0 |
| 2 | 1 |

Example 3:

Input: nums = [3,3], target = 6
Output: [0,1]

$$T(n) = O(n)$$
$$S(n) = O(n)$$

```
int [] twoSum(int[] nums, int target){  
    Map<Integer, Integer> tbl = new HashMap<>();  
    for( int i = 0 ; i < nums.length; i++ ) {  
        if( tbl.containsKey(target - nums[i]) )  
            return new int[]{tbl.get(target - nums[i]), i};  
        tbl.put(nums[i], i);  
    }  
    return new int[]{};  
}
```



Thank you!!!

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