

## Schrodinger's Time independent wave equation:

- ① As per DBH, particle of mass  $m$  moving with velocity  $v$  is associated with matter wave having wavelength ( $\lambda$ )

$$\lambda = \frac{h}{p} = \frac{h}{mv} \quad \text{--- (1)}$$

- ② Differential eq<sup>n</sup> of matter waves with wave vel ( $u$ )

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \left[ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right]$$

$$\frac{\partial^2 \psi}{\partial t^2} = u^2 \nabla^2 \psi \quad \text{--- (2)}$$

as Laplacian operator  $= \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

- ③ Solution of the above eq<sup>n</sup> will give  $\psi$  as function of space and time

$$\psi(x, y, z, t) = \psi_0(x, y, z) e^{-i\omega t} \quad \text{--- (3)}$$

- ④ Here,  $\psi_0$  is amplitude of wave at  $(x, y, z)$

- ⑤ Above eq<sup>n</sup> can be written as

$$\psi = \psi_0 e^{-i\omega t}$$

- ⑥ Differentiating above eq<sup>n</sup> twice we get

$$\frac{\partial \psi}{\partial t} = -i\omega \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi \quad \text{--- (4)}$$

⑦ Equating ② and ④

$$u^2 \nabla^2 \psi = -\omega^2 \psi$$

$$\nabla^2 \psi = -\frac{\omega^2}{u^2} \psi$$

$$\nabla^2 \psi + \frac{\omega^2}{u^2} \psi = 0 \quad \text{--- (5)}$$

⑧ But we know  $\omega = 2\pi\nu$  and  $u = \lambda\nu$

$$\frac{\omega}{u} = \frac{2\pi\nu}{\lambda\nu} = \frac{2\pi}{\lambda} \quad \text{--- (6)}$$

⑨ Substituting  $\frac{\omega}{u} = \frac{2\pi}{\lambda}$  in eq<sup>n</sup> ⑤

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2} \psi = 0$$

$$\nabla^2 \psi + \frac{4\pi^2 p^2}{h^2} \psi = 0 \quad \text{--- (7)} \quad \left[ \because \lambda = \frac{h}{p} \right]$$

⑩ Total Energy of particle =  $E = \text{P.E.} + \text{K.E.}$

$$E = V + \frac{1}{2} mv^2$$

$$E = V + \frac{1}{2} m^2 v^2$$

$$E = V + \frac{1}{2} p^2$$

$$p^2 = 2m(E - V) \quad \text{--- (8)}$$

⑪ Putting eq<sup>n</sup> ⑧ in eq<sup>n</sup> ⑦ we get

$$\nabla^2 \psi + \frac{4\pi^2 [2m(E - V)]}{h^2} \psi = 0$$

$$\nabla^2 \psi + \frac{8\pi^2 m(E - V)}{h^2} \psi = 0$$

This is Schrodinger's Time Independent Wave Equation.

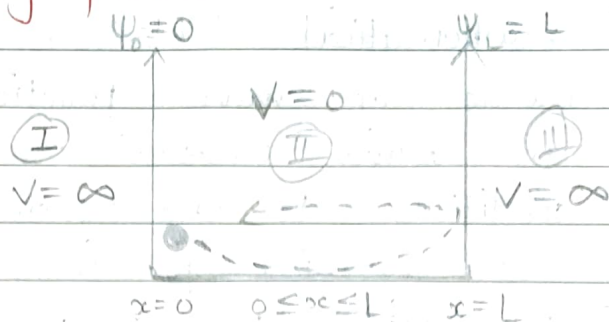
$\eta$  = efficiency

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## ✓ Significance of Schrodinger Wave Equation (SWE)

- ① SWE are the mathematical tools to cal energy of  $e^-$  and nature of wave functions, probability.
- ② This is done in active systems by applying proper boundary conditions to the system.
- ③ All these parameters regarding functioning of  $e^-$  helps engineers to modify the systems  $\uparrow$  its  $\eta$

✓ Energy of particle enclosed in 1D rigid box / ∞ Potential well



① Consider, particle of mass  $m$  moving with velocity  $v$

② STIWE is  $\nabla^2 \psi + \frac{8\pi^2m(E - V)}{h^2} \psi = 0$  — ①

③ As we know particle is inside box where  $V=0$   
eq<sup>n</sup> ① becomes  $\nabla^2 \psi + \frac{8\pi^2mE}{h^2} \psi = 0$

$$\nabla^2 \psi + k^2 \psi = 0 \quad \text{--- ②}$$

④ Solution of eq<sup>n</sup> ② can be sine function/cosine function  
i.e.  $\psi(x) = A \cos kx + B \sin kx$

⑤ Since,  $\psi(x) = A e^{ikx} + B e^{-ikx}$

⑥ For boundary conditions

when  $x=0$  :  $\psi_{(0)} = A e^0 + B e^0 = A + B$

$$A + B = 0 \Rightarrow B = -A$$

when  $x=L$  :  $\psi_{(L)} = A e^{ikL} + B e^{-ikL}$

$$\psi_{(L)} = A e^{ikL} + (-A) e^{-ikL} = 0$$

$$\therefore B = -A$$

$$\frac{2iA[e^{ikL} - e^{-ikL}]}{2i} = 0$$

$$2A i \sin kL = 0$$

$$\sin kL = 0$$

$$kL \neq 0$$

$$kL = n\pi$$

$$\frac{e^{+ikL} - e^{-ikL}}{2i} = \sin kL$$

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$$k = \frac{n\pi}{L}$$

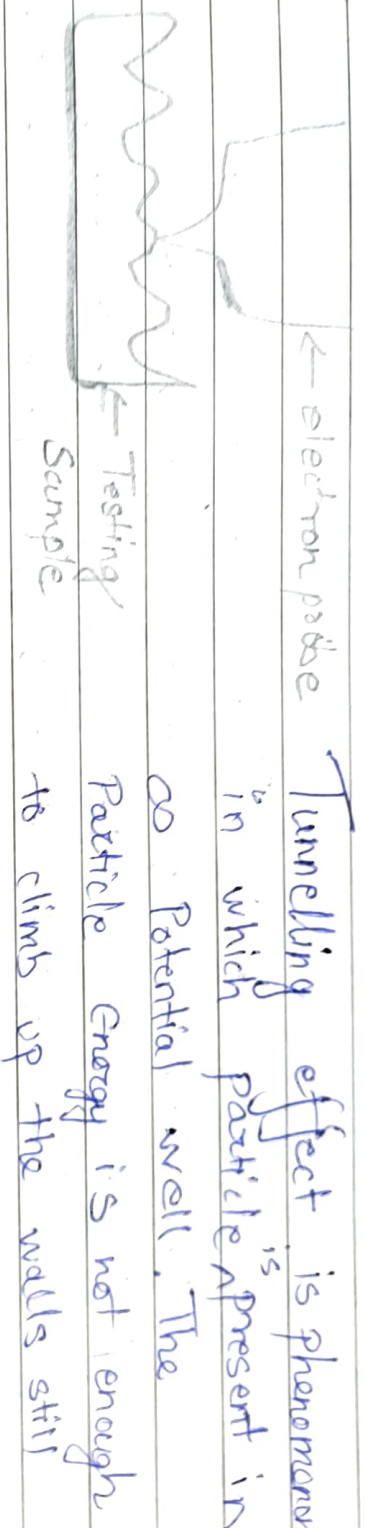
$$k^2 = \frac{n^2\pi^2}{L^2}$$

$$\frac{8\pi^2 m E}{h^2} = \frac{n^2 \pi^2}{L^2}$$

$$E = \frac{n^2 h^2}{8m L^2}$$



# Tunnelling effect + STM + Applications



it can penetrate the walls and leak out.

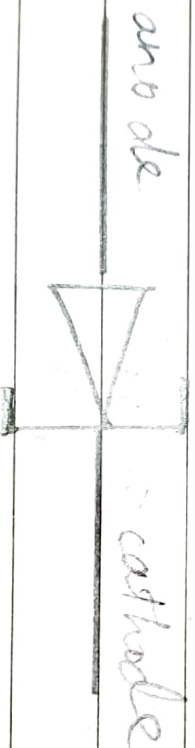
- ① STM is electron microscope
- ② STM shows 3D image of sample
- ③ STM helps to study structure of surface by scanning surface.
- ④ It slowly scans across surface @ dist of atom's diameter.
- ⑤ Stylus is raised & lowered in order to keep signal constant
- ⑥ This enables it to follow even smallest details of surface
- ⑦ Profile pic of surface is created. & from that computer generated contour map of surface is produced.

## Applications

- ① STM is used in the study of DNA molecules
- ② STM is used in chemistry as surface reactions play imp role.
- ③ STM is used in phy as study of surfaces is an important part of phy.

## ✓ Tunnel diode

- ① Tunnel diode is type of semiconductor diode which is capable of very fast operation
- ② Symbolically it is represented



- ③ Diode has heavily doped p-n junction.
- ④ It is fast switching device used in computers
- ⑤ Used in frequency converter & detectors, oscillators, amplifier, high frequency switch ( $10^9$  Hz)
- ⑥ Tunnel diode usually made from Germanium.

# State + Explain De Broglie's Hypothesis (DBH)

Any three properties of matter waves

① DBH states that moving particle always has wave associated with it and the motion of particle is guided by that wave.

② It says material particle has dual nature, i.e. particle nature & wave nature.

③ Light wave ~~of~~ frequency  $\nu$  is associated with energy  $E = h\nu$  [①  $h = \text{plank's constant}$ ]

④ Particle of mass  $m$  equivalent to amt of energy  $E = mc^2$  [② Einstein's Theory of relativity]

⑤ Equate eq<sup>n</sup> ① and eq<sup>n</sup> ②

$$h\nu = mc^2$$

$$\frac{h\nu}{c} = mc$$

$$mc = \frac{h\nu}{\lambda}$$

$$p = \frac{h}{\lambda}$$

$$\lambda$$

$$\lambda = \frac{h}{p}$$

③



⑥ Equation ⑥ shows relation bet<sup>n</sup>  $\lambda$  and  $P$ .

✓ DBH in terms of K.E

$$\bullet \quad E = \frac{1}{2} mv^2 = \frac{1}{2m} m^2 v^2 = \frac{p^2}{2m}$$

$$p^2 = 2mE \Rightarrow p = \sqrt{2mE}$$

$$\text{But } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \quad \text{--- ①}$$

• For electron with charge  $e$  accelerated with Potential  $V$  the K.E is given by

$$E = eV \quad \text{--- ②}$$

Substitute value of  $E$  in ①

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.25 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} V}}$$

$$\lambda = \frac{12.27 \times 10^{-10}}{\sqrt{V}}$$

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ \AA}$$

Properties of Matter waves

- $\lambda = \frac{h}{mv}$
- ①  $\lambda \propto \frac{1}{m}$  hence lighter the particle greater is the wavelength
  - ②  $\lambda \propto \frac{1}{v}$  greater the vel smaller is the wavelength.
  - ③ The wave<sup>(v)</sup> vel inversely depends on  $\lambda$ . This is basic diff bet<sup>n</sup> light waves & matter waves
  - ④ vel of matter waves not constant like EMW it depends of particle generating them.

# Heisenberg's Uncertainty Principle HUP

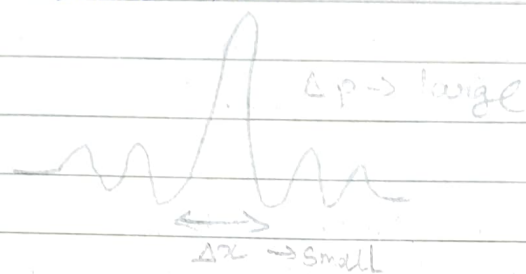
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- ① HUP states that in any simultaneous determination of Position and Momentum of particle of atomic size the product of uncertainties is equal to/greater than Planck's constant

②  $\Delta x \cdot \Delta p \geq h$

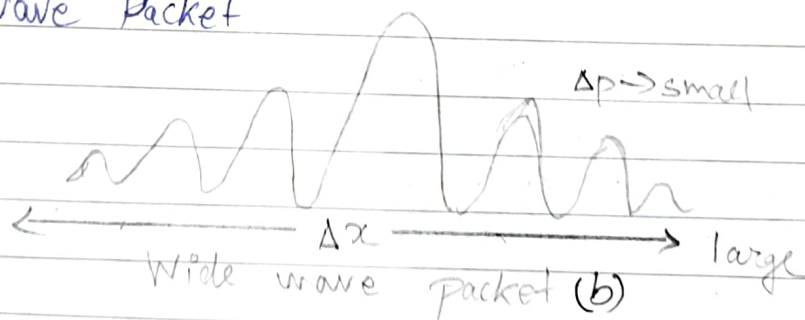
- ③ Narrow Wave Packet



Narrow Wave Packet (a)

- ① In case of abt small wave packet, the Amplitude is large over small region of space & negligible elsewhere (a)  
② The small region of space can be associated with Pos<sup>n</sup> of Part<sup>l</sup>  
③ Pos<sup>n</sup> of Part<sup>l</sup> can be fixed with min error.  
④ At the same time  $\lambda$  and hence <sup>momentum</sup> P cannot be measured accurately.

- ④ Wide Wave Packet



Wide wave packet (b)

- ① Wave Packets is wide so  $\lambda$  & P can be determined with more accuracy.  
② But at the same time Pos<sup>n</sup> of particle becomes very uncertain.  
⑤ Thus, it is impossible to det the Pos<sup>n</sup> & P simultaneously.

# What is wave function $\psi$

## Explain Physical Significance of $\psi^2$

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- ① Wave variable associated with matter wave is  $\psi$  + varies periodically
- ②  $\psi$  may be real / complex.
- ③  $\psi^2$  denotes (probability of finding the particle at a point in space / medium at time  $t$ )
- ④ If  $\psi^2$  is large,  $\star$  is max
- ⑤ If  $\psi^2$  is small,  $\star$  is min
- ⑥ If  $\psi = 0$  then there is no particle. Hence  $\star$  is zero.
- ⑦  $\psi$  may be complex fun<sup>n</sup> having real + img<sup>y</sup> part
- ⑧  $\psi = A + iB$   $\psi^* = A - iB$   
 $\psi \psi^* = A^2 + B^2$  which is +ve quantity
- ⑨ Particle under consideration will always be found somewhere hence total probability is 1  
 $\Rightarrow \iiint_{-\infty}^{+\infty} \psi \psi^* dv = 1$
- ⑩ Above con<sup>n</sup> of  $\psi$  is normalisation condition.



## Phase Velocity ( $V_p$ )

## Group Velocity ( $V_g$ )

①  $V_p$  or wave velocity of monochromatic ~~light~~ <sup>wave</sup> is vel with which a definite phase (crest/trough) of wave propagate in medium.

$V_g$  is velocity with which hump/wave group travels is called

② Expression for  $V_p$  is  $\frac{\omega}{k}$

Exp<sup>n</sup> for  $V_g$  is  $\frac{d\omega}{dk}$

③  $V_p$  always  $> c$

$V_g$  is always  $< c$

④  $V_p$  is not same for all the components, some of them propagate faster some slower

④  $V_g$  represents the vel of wave packet

⑤ This leads 2 dispersion of wave packet

⑤  $V_g$  is more significant



# Quantum Computing

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- ① Quantum computing came into picture when it was identified that computers use so much energy & get so hot even though they appear to do very less work,
- ② Landauer's showed how quantum computer can circumvent this pb by working in reversible way.
- ③ Quantum computer could carry massively complex computations without using massive amt of energy.

## Applications :

- ① Q.C could empower Machine Learning by enabling AI programs to search through gigantic datasets
- ② A Q.C we could expect it to be able to handle almost innumerable Permutation & Combinations.