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Quadrotor Attitude Control Based on Fuzzy Sliding Mode Control Theory

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Abstract: The quadrotor is a strong coupled and nonlinear system. The attitude control of the quadrotor is the core of the stability of the quadrotor. In this study, an attitude controller based on sliding mode strategy is designed to stabilize the quadrotor. Firstly, the mathematical model of the quadrotor is demonstrated and the model is simplified at the equilibrium point of hover. Secondly, based on the simplified model, a fuzzy sliding mode controller is designed to track the reference value of attitude. The non-fuzzy hyperplane of the sliding mode is extended the fuzzy band and the fuzzy control is utilized to approach the supervisory control of sliding mode control and estimate the switching function. Finally, the simulation results show that the performance of the proposed control approach with superior response speed and stronger robustness.

Key Words: quadrotor, attitude control, sliding mode control, fuzzy control

1 Introduction

In recent years, as a kind of Micro Air Vehicles (MAVs), the quadrotor has attracted much attention from many researchers all over the world, due to the fact that it is unexpensive and very good maneuverability [1]. Because of the great mobility, precise hovering, vertical take-off and landing (VTAL) maneuver abilities [2], quadrotors have been widely used in many fields, such as military surveillance [3], search and rescue missions [4], cooperative manipulation [5] and so on. As a typical under-actuated, strongly coupled and nonlinear complex system [6], the control strategies of quadrotors have gradually become the academic research focus [7]. The attitude controller is the core part of the whole control system, it determines the trajectory of the flight. Therefore, the stability of the attitude system is the key for the whole quadrotor control system [8].

Over the past decades, different control techniques have been developed for the attitude control of the quadrotor. Since the dynamic model of the quadrotor can be linearized, some traditional linear control methods can be utilized to stabilize the quadrotor in a small range, such as proportional-integral-derivative (PID) control [9], linear quadratic regulator (LQR) control [10] and so on. Because of the loss of dynamic performance of linear control methods, more and more nonlinear control methods have been paied much attention in recent years. In [11], the backstepping control technique is adopted to control the position and attitude of the quadrotor. In [12], the nonlinear disturbance observer, the backstepping, and the sliding control strategies are combined to deliver attitude control with external disturbances. In [13], a sliding mode control (SMC) is designed to stabilize the attitude of the quadrotor. SMC is efficient for systems with large uncertainties, time varying properties and bounded external disturbances [14], but the switching function might create a chattering phenomenon which is a high frequency oscillation that may cause control system unstable.

In this paper, an attitude tracking controller of a quadrotor based on fuzzy sliding mode control (FSMC) is proposed. Considering the parametric uncertainties and external disturbances, using the SMC to stabilize the attitude of the quadrotor. The fuzzy band is designed based on fuzzy set and the fuzzy controller is presented to approach the supervisory control and estimate the switching function of SMC. Simulation results demonstrate that the FSMC achieves good tracking and disturbance rejection performance.

The remainder of this paper is organized as follows. In section 2, the dynamic model for the quadrotor is set up. Subsequently, controller development is detailed in section 3. The simulation results and the analysis have been shown in section 4. Finally, short conclusions are presented in section 5.

2 Quadrotor Dynamic Model

The traditional structures of the quadrotor can be divided into "$\times$" mode and "$+$" mode. We choose the "$+$" mode to establish the model of quadrotor. In order to establish the quadrotor system dynamic model, the Earth-fixed inertial frame N ($O_nX_nY_nZ_n$) and the Body-fixed frame B ($o_bx_by_bz_b$) are defined as Fig. 1 does. The Earth-fixed inertial frame is fixed on the ground with the quadrotor take-off point as the origin O_n, the axis X_n points to the geographical north, the axis Y_n points to the geographical east and the axis Z_n points to the ground. The Body-fixed frame is fixed on the quadrotor body.

Without losing the generality of the dynamic model of the quadrotors, we make the following assumption.

- The quadrotor is supposed to be rigid with a completely symmetrical structure. The elastic deformation won't occur during the flight.
- The quadrotor arms are fixed orthogonally, and the geometric center, the center of gravity and the origin of the Body-fixed frame B are assumed to coincide.
- Thrust is only related to the propeller's speed.

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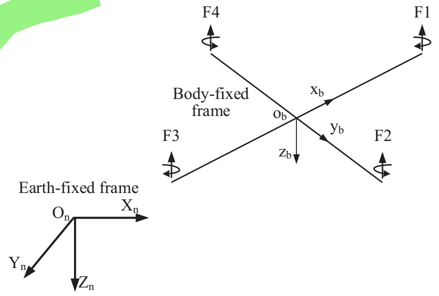


Fig. 1: The coordinate reference frames of quadrotors

The attitude of the quadrotor can be described by the Euler angles $\boldsymbol{\Theta} = \begin{bmatrix}\phi & \theta & \psi\end{bmatrix}^\top$, which are called the roll angle (ϕ, rotation around z-axis), pitch angle (θ, rotation around y-axis) and yaw angle (ψ, rotation around x-axis). The quadrotor dynamic model describing the roll, pitch and yaw rotations contains three terms which are the gyroscopic effect resulting from the rigid body rotation, the gyroscopic effect resulting from the propeller rotation coupled with the body rotation and finally the actuators action, neglecting the coupling, the dynamic model can be approximated by [15]:

$$\begin{cases} I_x\ddot{\phi} &= \dot{\theta}\dot{\psi}(I_y - I_z) + U_2 l + N_1 \\ I_y\ddot{\theta} &= \dot{\phi}\dot{\psi}(I_z - I_x) - U_3 l + N_2 \\ I_z\ddot{\psi} &= \dot{\phi}\dot{\theta}(I_x - I_y) - U_4 + N_3 \end{cases} \tag{1}$$

where I_x, I_y and I_z are the moment of inertia of the quadrotor when it rotates around the x, y and z-axes, respectively. l is the distance between the motor center and the quadrotor center. U_i $(i = 1, 2, 3)$ is the intermediate control inputs of the system, standing for rotary moment of attitude. N_j $(j = 1, 2, 3)$ is the uncertain external disturbance. U_i is related to the propeller's speed and can be expressed as:

$$\begin{bmatrix} U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} 0 & -b & 0 & b \\ -b & 0 & b & 0 \\ -d & d & -d & d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{2}$$

where Ω_z $(z = 1, \cdots, 4)$ is the speed of propellers. b is the thrust factor and d is the drag factor.

The system state formula (1) can be described as following:

$$\dot{\boldsymbol{X}} = \hat{f}(\boldsymbol{X}) + g(\boldsymbol{X})\boldsymbol{U} + \boldsymbol{N} \tag{3}$$

where $\boldsymbol{X} = \begin{bmatrix}\dot{\phi} & \dot{\theta} & \dot{\psi}\end{bmatrix}^\top$, $\boldsymbol{U} = \begin{bmatrix}U_2 & U_3 & U_4\end{bmatrix}^\top$ and $\boldsymbol{N} = \begin{bmatrix}N_1 & N_2 & N_3\end{bmatrix}^\top$. $\hat{f}(\boldsymbol{X}) = f(\boldsymbol{X}) + \Delta f(\boldsymbol{X})$ is a bounded unknown function due to the uncertainty of system. The uncertainty can be defined by $\Delta f(\mathbf{X})$ and bounded by $|\Delta f(\boldsymbol{X})| \le F(\boldsymbol{X})$.

3 Design of Adaptive Fuzzy Sliding Mode Controller

The control system structure of quadrotors is shown in Fig. 2. The control objective for the quadrotor is to track the given bounded reference signal $\boldsymbol{\Theta_d} = \begin{bmatrix}\phi_d & \theta_d & \psi_d\end{bmatrix}^\top$. And the controller consists of the sliding mode controller and the fuzzy controller.

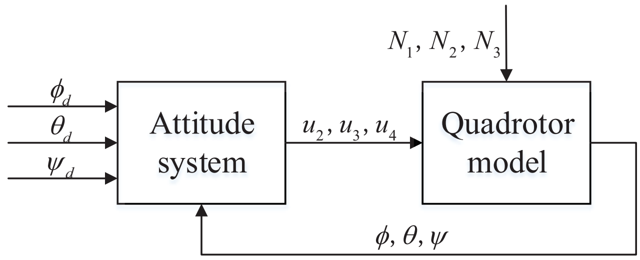


Fig. 2: The control system structure of quadrotors

3.1 Design of Sliding Mode Controller

Sliding-mode control has fast response and insensitive to some class of uncertainty. In this section, controller design for the quadrotor is proposed by using the sliding mode control technique. For a given attitude command $\boldsymbol{\Theta_d} = \begin{bmatrix}\phi_d & \theta_d & \psi_d\end{bmatrix}^\top$, the attitude tracking error is defined as following:

$$\boldsymbol{e} = \boldsymbol{\Theta} - \boldsymbol{\Theta_d} \tag{4}$$

The sliding mode variable $\boldsymbol{s}$ is constructed as

$$\boldsymbol{s} = \dot{\boldsymbol{e}} + \boldsymbol{\lambda}\boldsymbol{e} \tag{5}$$

where $\boldsymbol{\lambda}$ is a third-order diagonal matrix with positive entries.

For the attitude system, the three control variables U_i $(i = 1, 2, 3)$ are mutually independent. Choosing the roll angle ϕ as the example to design the sliding mode controller. And the pitch angle θ and the yaw angle ψ are similar.

The corresponding error state equation is

$$e_1 = \phi - \phi_d \tag{6}$$

Define $u_c = U_2 l$ as the new control input, with formula (1), we have

$$\ddot{e}_1 = \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + \frac{u_c}{I_x} + \frac{N_1}{I_x} - \ddot{\phi}_d \tag{7}$$

The sliding mode variable s_1 is constructed as

$$s_1 = \dot{e}_1 + \lambda_1 e_1, \qquad \lambda_1 > 0 \tag{8}$$

Taking the derivative of s_1 one time, then

$$\begin{aligned} \dot{s}_1 &= \ddot{e}_1 + \lambda_1\dot{e}_1 \\ &= \dot{\theta}\dot{\psi}\frac{I_y - I_z}{I_x} + \frac{u_c}{I_x} + N_1 - \ddot{\phi}_d + \lambda_1\dot{e}_1 \end{aligned} \tag{9}$$

The control input u_c can be derived:

$$u_c = I_x(\ddot{\phi}_d - N_1 - \lambda_1\dot{e}_1) - \dot{\theta}\dot{\psi}(I_y - I_z) \tag{10}$$

In the traditional sliding mode controller design, control input $\hat{u}_c$ is designed as $\hat{u}_c = u_c + u_s$. u_s is the switching control law:

$$u_s = -K sgn(s_1) \tag{11}$$

where $F(X) \le K \le K_m$ and

$$sgn(s_1) = \begin{cases} 1 & s_1 > 0 \\ 0 & s_1 = 0 \\ -1 & s_1 < 0 \end{cases} \tag{12}$$

It can be seen that u_s is discontinuous and this discontinuous sliding term is used to handle with the uncertainties in formula (10). However, the disadvantage of this method is chattering effect.

3.2 Design of Fuzzy Controller

Fuzzy logic control has been regarded as an effective method to solve the complex nonlinear system model and control problem. The fuzzy control is utilized to approach the supervisory control of SMC and estimate the switching function to decrease the chattering effect.

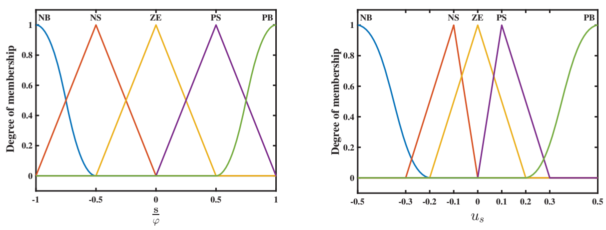
In formula (9), s_1 is a non-fuzzy hyperplane and it can be extended to the fuzzy hyperplane. $\widetilde{s} =$ ZERO is a fuzzy band near $s = 0$ and "ZERO" is the corresponding fuzzy set. Define φ as the thickness of sliding mode and $s \neq 0$ represents the extent which the system is dynamically out of stable mode. In fuzzy controller, $\frac{s}{\varphi}$ is regarded as the input variable and u_s is regarded as the output variable.

Define $\frac{s}{\varphi}$ as the fuzzy set and formula (11) can be written as

$$u_s = -K_m sgn(\frac{s}{\varphi}) \tag{13}$$

When $\left|\frac{s}{\varphi}\right| \geq 1$, it represents the thickness of the system that exceeds the sliding mode, the control variable should be increased to make system reach $\widetilde{s} =$ ZE as soon as possible. When $\left|\frac{s}{\varphi}\right| < 1$, the system has not exceeded the thickness of sliding mode and u_s is needed. In order to avoid overshoot, the control variable should not be too great. Considering this, fuzzy logic inference is utilized to obtain corresponding control variable in the range of $\left|\frac{s}{\varphi}\right| < 1$.

The universe of $\frac{s}{\varphi}$ is $[-1, 1]$ and the universe of u_s is $[-K_m, K_m]$. Define $K_m = 0.5$, the membership functions of two variables $\frac{s}{\varphi}$ and u_s are illustrated in Fig. 3.



(a) (b)

Fig. 3: Membership functions of $\frac{s}{\varphi}$ and u_s. (**a**) Membership function of $\frac{s}{\varphi}$. (**b**) Membership function of u_s.

The rules of the fuzzy controller are proposed as follows:

$$R^1: \text{ if } \frac{s}{\varphi} \text{ is NB then } u_s \text{ is PB},$$
$$R^2: \text{ if } \frac{s}{\varphi} \text{ is NS then } u_s \text{ is PS},$$
$$R^3: \text{ if } \frac{s}{\varphi} \text{ is ZE then } u_s \text{ is ZE},$$
$$R^4: \text{ if } \frac{s}{\varphi} \text{ is PS then } u_s \text{ is NS},$$
$$R^5: \text{ if } \frac{s}{\varphi} \text{ is PB then } u_s \text{ is NB}.$$

The control variable can be derived with the Mamdani algorithm under the rules, and u_s is calculated through centroid fuzzy decision principle as follows:

$$u_s = \frac{\int_{-K_m}^{K_m} u_s \cdot \mu(u_s)\, du_s}{\int_{-K_m}^{K_m} \mu(u_s)\, du_s} \tag{14}$$

u_s calculated by formula (14) is continuous when $\left|\frac{s}{\varphi}\right| < 1$ and the system chattering effect can be weakened.

4 Simulation Results

In this section, the performance of the proposed approach is evaluated. The algorithm is executed in MATLAB/SIMULINK simulation environment which version is MATLAB R2018B. The parameters of the quadrotor model in the simulation are listed in Table 1.

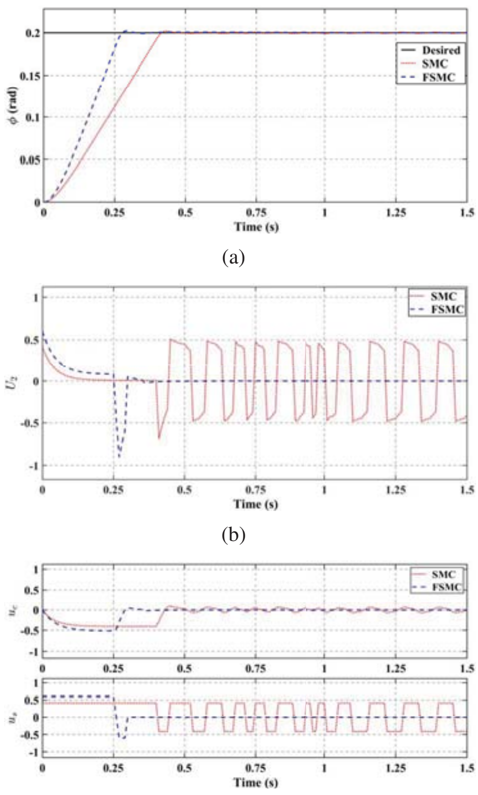
Table 1: Parameters of The Quadrotor

Parameter	Definition	Value
I_x	Roll inertia	7.5×10^{-3} kg $\cdot$ m^2
I_y	Pitch inertia	7.5×10^{-3} kg $\cdot$ m^2
I_z	Yaw inertia	1.3×10^{-2} kg $\cdot$ m^2
l	Arm length	0.23 m
b	Thrust factor	3.13×10^{-5}
d	Drag factor	7.5×10^{-7}

The sampling period Δt is set as $0.01s$. The attitude tracking experiment is to make the quadrotor in the initial attitude angles $\mathbf{\Theta_0} = \begin{bmatrix}0, & \frac{\pi}{4}, & \frac{\pi}{4}\end{bmatrix}^\top$, then following the desired attitudes $\mathbf{\Theta_d} = \begin{bmatrix}0.2, & 0.3, & 0.1\end{bmatrix}^\top$. The simulation results are shown in Fig. 4 and Fig. 5. Fig. 4 shows the response curves without any external disturbances, including attitude tracking of roll angle, control variable U_2, basic control input u_c and supervisory control input u_s.

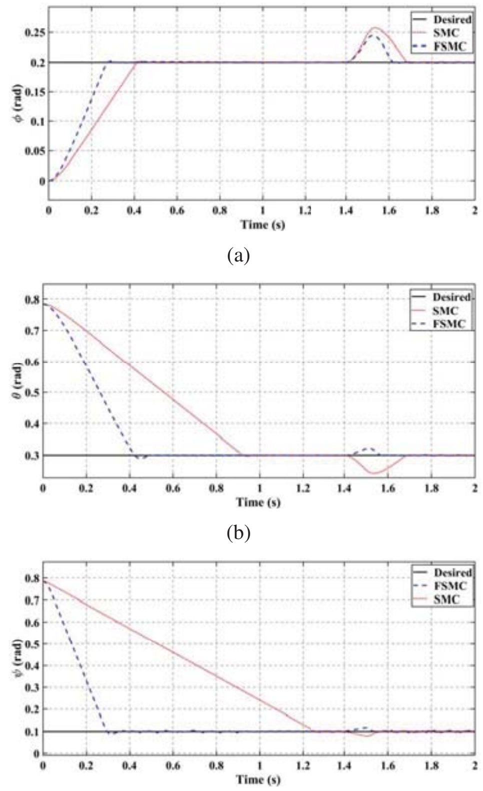
From Fig. 4(a), for the roll angle, the rise time of SMC is 302 ms and FSMC is 235 ms. The settling time of SMC is 394 ms and FSMC is 259 ms. The overshoot of SMC is 1% and FSMC is 1.5%. Compared with SMC, FSMC has superior response speed and the chattering effect would be weakened.

From Fig. 4(b), for SMC, the chattering phenomenon is clearly observed in the control variable U_2 with a large amplitude even if the angles reach in the desire attitudes, which means a large energy consumption. This phenomenon reduces the lifetime of batteries and motors in the system. And the sliding mode hyperplane is approached at 0.4 s. For FSMC, the fuzzy sliding mode hyperplane is approached at 0.25 s and the switching is occurred. This phenomenon is consistent with the settling time roughly in Fig. 4(a). We can also find that the control performance is stronger when using FSMC to stabilize the attitude. From Fig. 4(c), the effectiveness of switching function designed in FSMC is verified according to the response curve of supervisory control input u_s.



(a) (b) (c)

Fig. 4: Simulation results for attitude stabilization. (**a**) Comparision on FSMC and SMC for roll angle. (**b**) Comparision on FSMC and SMC for control variable U_2.(**c**) Comparision on FSMC and SMC for u_c and u_s of U_2.



(a) (b) (c)

Fig. 5: Response curves of anti-disturbance experiment. (**a**) Roll angle. (**b**) Pitch angle.(**c**) Yaw angle.

To demonstrate the tracking and disturbance rejection property of the proposed control system, the disturbance signal with the amplitude of 1 and the width of 0.1 s as the mutation signal is added on the the three control variables at 1.4 s, respectively. The simulation results are shown in Fig. 5.

It is clear that the attitudes of the quadrotor remain at the desired states despite constant disturbance. As can be seen from Fig. 5, the anti-disturbance performance of the SMC is significantly weaker than that of the FSMC. Compared with the SMC, the FSMC has less fluctuating and better inhibitory effect when disturbed.

5 Conclusions

In this paper, an attitude controller of the quadrotor based on fuzzy sliding mode control theory is proposed. The controller can track the desired roll, pitch and yaw angles, even if in the presence of external disturbance. First, the dynamic model of the quadrotor is built. Then, a sliding mode controller is designed. Considering the uncertain parameters in the model and various disturbances during flight, the fuzzy logic strategy is utilized to design the equivalent control of sliding mode control and estimate the switching function. Compared with the conventional sliding mode control, the fuzzy sliding mode control achieves good tracking and disturbance rejection performance. At the same time, the chattering effect is decreased. Extensive simulation results demonstrate that the proposed control strategy enhances the robustness of the quadrotor and improves the convergence speed.

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