Second Order Sliding Mode Control for Quadrotor

Sudhir
Electrical Engineering Department
National Institute of Technology Kurukshetra
Haryana, India
sudhir.nadda87@gmail.com

A. Swarup, Senior Member IEEE
Electrical Engineering Department
National Institute of Technology Kurukshetra
Haryana, India
a.swarup@ieee.org

Abstract- **The model of a quadrotor is nonlinear and dynamically unstable due to the high level of system nonlinearity and external disturbances. In this paper a robust second order sliding mode controller has been proposed for altitude tracking of a quadrotor. The controller is derived using Lyapunov stability approach and proved theoretically asymptotic stability. The performance of proposed control method is evaluated by comparing the performance with conventional sliding mode control from the literature. It is demonstrate that the proposed second order sliding mode control improves the tracking control performance with better transient performances.**

Keywords- Quadrotor, Sliding mode control

I. INTRODUCTION

The applications of an Unmanned Aerial Vehicles (UAVs) gained increased attention in the area of dangerous environment, surveillance, military and scientific research area. Quadrotor has advantages over fixed wing aircraft for their vertical take-off landing (VTOL) and payload capacity. The model of a quadrotor is nonlinear and dynamically unstable due to the high level of system nonlinearity and external disturbance. The control of quadrotor has received much interest due to its nonlinear and unstable dynamics.

There have been several linear control techniques such as PID and LQR, based on the linearized version of quadrotor [1-2]. These linear controls were restricted in hover flight condition. To overcome the hover flight condition, nonlinear control algorithms have been presented in [3-12]. Feedback linearization has been used for quadrotor presented [3-5]. In these techniques high-order derivative terms arising from the differentiation of derivative equations. Therefore, sliding mode control and backstepping control received more attention in the literature due to robustness and disturbance rejection [6-14]. Sliding mode control techniques based on the Lyapunov stability theory have been developed in [6-12], to stabilize the whole system and follow the desired trajectory. However, implementation of the sliding mode control technique has been presented some drawbacks such as chattering and constant gain. The chattering phenomena occur due to the non-ideal behaviour of system, actuators and due to the inclusion of the sign function in the switching control. It has been improved by smoothing the switching term [10] and use of higher-order sliding mode has been a good alternative solution [17]. But in the case of second order sliding order sliding mode control, it is difficult to directly choose the nonlinear sliding coefficients to get the desired sliding motion. To overcome this problem a second order sliding mode control is proposed in this paper with linear sliding manifold using Lyapunov stability theory, which guarantees the system stability. The effectiveness of proposed controller is compared with conventional sliding mode control.

The dynamical modeling of micro quadrotor is described in section II. In section III, the development of second order sliding mode control is discussed. The results and discussions are summed up in the section IV. The performance of proposed controller is concluded in section V.

II. DYNAMIC MODEL

Dynamics of quadrotor using Euler-Lagrange Formalism are presented in [7-12]. Defining $E = [e_x, e_y, e_z]$, the Earth fixed frame and $B = [x, y, z]$, the Body fixed frame. Let $\emptyset, \theta$ and φ represent the Euler angles; the outputs of the system are x, y and z, which are the positions of the center of gravity of the quadrotor; the p, q and r denote its angular velocities in the body-frame and F_i $(i = 1,2,3,4)$ is the thrust force produced by each propeller [24]. The orientation of UAV quadrotor is shown in fig.1.

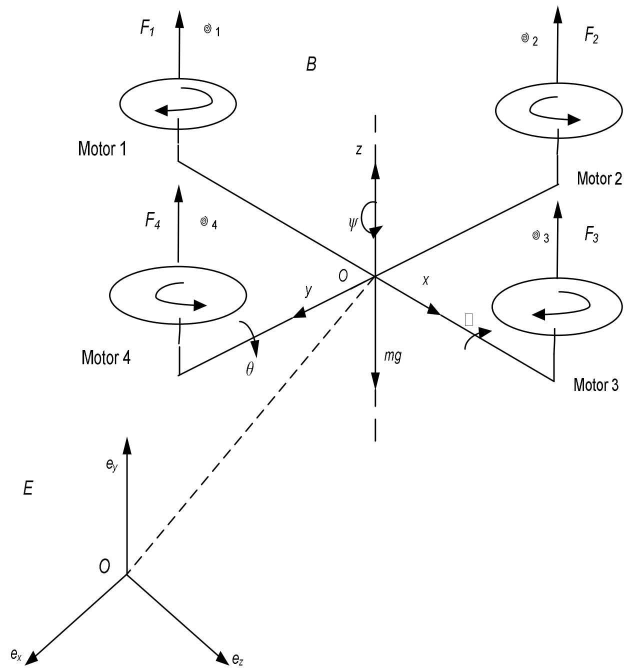

Figure 1: UAV Quadrotor

The dynamical model of quadrotor [17] with x, y and z motions as a consequence of roll, pitch and yaw rotation, can be expressed as:

$$\ddot{\emptyset} = \dot{\theta}\dot{\varphi}\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_r}{I_x}\dot{\theta}\Omega + \frac{l}{I_x}U_2 - \frac{K_1 l}{I_x}\dot{\emptyset}$$
$$\ddot{\theta} = \dot{\emptyset}\dot{\varphi}\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_r}{I_y}\dot{\emptyset}\Omega + \frac{l}{I_y}U_3 - \frac{K_2 l}{I_y}\dot{\theta}$$
$$\ddot{\varphi} = \dot{\emptyset}\dot{\theta}\left(\frac{I_x - I_y}{I_z}\right) + \frac{l}{I_z}U_4 - \frac{K_3 l}{I_z}\dot{\varphi}$$
$$\ddot{z} = -g + (cos\emptyset\ cos\theta)\frac{1}{m}U_1 - \frac{K_4 l}{m}\dot{z}$$
$$\ddot{x} = (cos\emptyset\ sin\theta\ cos\varphi + sin\emptyset\ sin\varphi)\frac{1}{m}U_1 - \frac{K_5 l}{m}\dot{x}$$
$$\ddot{y} = (cos\emptyset\ sin\theta\ sin\varphi - sin\emptyset\ sin\varphi)\frac{1}{m}U_1 - \frac{K_6 l}{m}\dot{y} \qquad (1)$$

where, I_x, I_y, I_z, g and m are physical parameters, K_i denote the positive drag coefficients, defined in table 1.1.
Where, U_2 and U_3 are the roll and pitch inputs; the U_1 is the total thrust on the body in the z-axis; U_4 is the yawing moment and Ω a disturbance, obtaining:

$$U_1 = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$$
$$U_2 = b(\Omega_2^2 + \Omega_4^2)$$
$$U_3 = b(\Omega_3^2 + \Omega_1^2)$$
$$U_4 = d(\Omega_2^2 + \Omega_4^2 - \Omega_1^2 - \Omega_3^2)$$

For the simulation exercise, physical parameters have been taken from [17].

Table 1.1

Symbol	Definition	Value
m	Mass	.23kg
l	Arm length	0.23 m
I_z	Moment of Inertia along z axis	1.3e-2 kgm^2
I_y	Moment of Inertia along y axis	7.5e-3 kgm^2
I_x	Moment of Inertia along x axis	7.5e-3 kgm^2
b	Drag coefficient	7.5e-7 Nms^2
d	Lift coefficient	3.13e-5 Ns^2
J_r	Rotor inertia	6e-5 kgm^2
g	Acceleration of gravity	9.8 m/s^2
$K_1 = K_2 = K_3$	Positive constants	0.01Ns/m
$K_4 = K_5 = K_6$		0.012Ns/m

The dynamic model developed in equation set (1) can be expressed in state space form

$$\dot{X} = f(X, U)$$

by introducing state vector as

$$X^T = [\emptyset, \dot{\emptyset}, \theta, \dot{\theta}, \varphi, \dot{\varphi}, z, \dot{z}, x, \dot{x}, y, \dot{y}].$$
$$x_1 = \emptyset, \quad x_2 = \dot{x}_1 = \dot{\emptyset}, \quad x_3 = \theta, \quad x_4 = \dot{x}_3 = \dot{\theta},$$
$$x_5 = \varphi, x_6 = \dot{x}_5 = \dot{\varphi}, x_7 = z, \ x_8 = \dot{x}_7 = \dot{z}, x_9 = x,$$
$$x_{10} = \dot{x}_9 = \dot{x}, \quad x_{11} = y, x_{12} = \dot{x}_{11} = \dot{y} \qquad (2)$$

From equations (1) and (2) the dynamics is formulated as

$$\dot{x} = f(X, U)$$

$$= \begin{pmatrix} x_2 \\ x_4 x_6 a_1 + x_4 a_2 \Omega + b_1 U_2 - d_1 \\ x_4 \\ x_2 x_6 a_3 + x_2 a_4 \Omega + b_2 U_3 - d_2 \\ x_6 \\ x_4 x_6 a_5 + b_3 U_4 - d_3 \\ x_8 \\ -g + (cosx_1\ cosx_3)\frac{1}{m}U_1 - d_4 \\ x_{10} \\ u_x \frac{1}{m}U_1 - d_5 \\ x_{12} \\ u_y \frac{1}{m}U_1 - d_6 \end{pmatrix} \qquad (3)$$

where

$$a_1 = (I_y - I_z)/I_x \qquad a_2 = -J_R/I_x$$
$$a_3 = (I_z - I_x)/I_y, a_4 = -J_R/I_y \quad a_5 = (I_x - I_y)/I_z$$

and

$$b_1 = l/I_x, b_2 = l/I_y, b_3 = l/I_z, d_1 = \frac{K_1 l}{I_x}x_2, d_2 = \frac{K_2 l}{I_y}x_4,$$
$$d_3 = \frac{K_3 l}{I_z}x_6, d_4 = \frac{K_4 l}{m}x_8, d_5 = \frac{K_5 l}{m}x_{10}, d_1 = \frac{K_6 l}{m}x_{12}$$

and

$$u_x = cosx_1\ sinx_3\ cosx_5 + sinx_1\ sinx_5$$
$$u_y = cosx_1\ sinx_3\ cosx_5 - sinx_1\ cosx_5$$

The vertical input force U_1 is used to stabilize the altitude of quadrotor. By the position subsystem desired roll and pitch angle are formed and control inputs U_2, U_3 *and* U_4 are used to stabilize the quadrotor.

III SECOND ORDER SLIDING MODE CONTROLLER

A second order sliding mode control method for 6DOF of quadrotor UAV is presented in this section. The main objective of the control algorithm developed in this paper is to design an altitude tracking controller with better transient performance.

A. Altitude Control: From the dynamical model equation (1), the altitude subsystem containing vertical input force U_1 is given as:

$$\ddot{z} = -g + (cos\emptyset\ cos\theta)\frac{1}{m}U_1 - \frac{K_4 l}{m}\dot{z}$$

The sliding manifold are defined as

$$s_1 = c_z(z_d - z) + (z_d - \dot{z})$$

where the coefficient $c_z > 0$. Let, the switching function is introduced directly as $\dot{s}_1 = -k_1 sign(s_1) - \eta_1 s_1$. Then, the corresponding control laws are designed as

$$U_1 = \frac{m}{cosx_1 cosx_3}\{c_z(\dot{z}_d - x_8) + \ddot{z}_d + g + d_4 + k_1 sign(s_1) + \eta_1 s_1\} \qquad (4)$$

where the coefficients $k_1, \eta_1 > 0$.

B. Position Control: Let, desired speed in x and y direction are $\dot{x}_d$ and $\dot{y}_d$ respectively. Then the tracking error is defined as

$$e_x = \dot{x}_d - \dot{x}$$
$$e_y = \dot{y}_d - \dot{y}$$

In term of error between actual and desired speeds, the desired roll and pitch angles are separately given by

$$\phi_d = \arcsin\,(u_{e_x} sin\psi - u_{e_y} cos\psi)$$
$$\theta_d = arcsin\left(\frac{u_{e_x}}{cos\phi cos\psi} - \frac{sin\phi sin\psi}{cos\phi cos\psi}\right)$$

Where u_{e_x} and u_{e_y} are

$$u_{e_x} = \frac{K_x e_x m}{U_1},\ u_{e_y} = \frac{K_y e_y m}{U_1}$$

where K_x and K_y are the positive constants and U_1 is the desired vertical force input by the altitude control.

C. Rotational Control: Second order sliding mode control technique is designed for rotational subsystem, to control the quadrotor during hovering. Three separate control inputs are designed to stabilize the yaw roll and pitch and angles.
Considering sliding manifold for roll angle as

$$s_2 = c_\phi(\phi_d - \phi) + (\dot{\phi}_d - \dot{\phi}) \tag{5}$$

where the coefficient $c_\phi > 0$. Taking the first derivative of eq. (5), it becomes

$$\dot{s}_2 = c_\phi\big(\dot{\phi}_d - \dot{\phi}\big) + (\ddot{\phi}_d - \ddot{\phi})$$

or

$$\dot{s}_2 = c_\phi(\dot{\phi}_d - x_2) + (\ddot{\phi}_d - (x_4x_6a_1 + x_4a_2\Omega + b_1U_2 - d_1))$$
$$\dot{s}_2 = c_\phi(\dot{\phi}_d - x_2) + (\ddot{\phi}_d - x_4x_6a_1 - x_4a_2\Omega - b_1U_2 + d_1)$$
$$\dot{s}_2 - c_\phi\big(\dot{\phi}_d - x_2\big) - \ddot{\phi}_d + x_4x_6a_1 + x_4a_2\Omega - d_1 = -b_1U_2$$

The formulated control input U_2 is,

$$U_2 = \frac{1}{b_1}\left\{c_\phi\big(\dot{\phi}_d - x_2\big) + \ddot{\phi}_d - x_4x_6a_1 - x_4a_2\Omega + d_1\right\} - \dot{s}_2$$

By making $\dot{s}_i = k_i sign(s_i) - \eta_i s_i\ \ i = 2, 3, 4$, where the coefficients of exponential approach law $k_i, \eta_i > 0$. Let switching function is taken for roll angle as $\dot{s}_2 = -k_2 sign(s_2) - \eta_2 s_2$. Then control input U_2, is becoming

$$U_2 = \frac{1}{b_1}\{c_\phi\big(\dot{\phi}_d - x_2\big) + \ddot{\phi}_d - x_4x_6a_1 - x_4a_2\Omega + d_1 + k_2 sign(s_2) + \eta_2 s_2 \tag{6}$$

The second order sliding control for pitch and yaw subsystem has been designed to obtain U_3 and U_4, following the steps above similar to roll subsystem.
The control inputs U_3 and U_4 are calculated as:

$$U_3 = \frac{1}{b_2}\{c_\theta\big(\dot{\theta}_d - x_4\big) + \ddot{\theta}_d - x_2x_6a_3 - x_2a_4\Omega + d_2 + k_3 sign(s_3) + \eta_3 s_3 \tag{7}$$

$$U_4 = \frac{1}{b_3}\{c_\psi\big(\dot{\psi}_d - \psi_4\big) + \ddot{\psi}_d - x_4x_6a_5 + d_3 + k_4 sign(s_4) + \eta_4 s_4\} \tag{8}$$

where

$$s_3 = c_\theta(\theta_d - \theta) + (\dot{\theta}_d - \dot{\theta})$$
$$\dot{s}_3 = -k_3 sign(s_3) - \eta_3 s_3$$
$$s_4 = c_\psi(\psi_d - \psi) + (\dot{\psi}_d - \dot{\psi})$$
$$\dot{s}_4 = -k_4 sign(s_4) - \eta_4 s_4$$

Chattering occurs due to the sign function present in the above equations, to avoid this drawback, which affects the overall performance, this discontinuous function is replaced by a saturation function [10], defined as:

$$sat(s_i) = \begin{Bmatrix} s_i & if\ |s_i| \le 1 \\ sign\ (s_i) & if\ |s_i| > 1 \end{Bmatrix} \quad i = 1, 2 \ldots 4.$$

Then second order sliding mode switching controller using the saturation function instead of sign function can be given as

$$\dot{s}_i = k_i sat(s_i) - \eta_i s_i\ \ i = 1,2,.,4.$$

Lyapunov stability approach is used to prove and evaluate the state convergence property of nonlinear flight controller equations (4, 6-8). Considering the Lyapunov function as [10],

$$V_i = \frac{1}{2}s_i^2 \qquad i = 1, 2, ..4$$

with $V(0) = 0$ and $V(t) > 0$ for $s(t) \neq 0$. A sufficient condition for the stability is guaranteed if the derivative of the Lyapunov function is negative definite:

$$\dot{V}_i = s_i\dot{s}_i$$
$$= s_i(-k_i sign(s_i) - k_i s_i)\ \ \forall\, k_i, i = 1, 2, .., 4$$
$$= -k_i sign|s_i| - k_i {s_i}^2$$
$$\le 0$$

Hence, $\dot{V}_i$ is negative definite and all the system state trajectories can reach and stay on the corresponding sliding surfaces, under the control laws.

IV SIMULATION EXERCISE

To verify the proposed control method, tracking of altitude has been obtained through simulation in MATLAB. The effectiveness of proposed controller is verified by comparative performance with conventional sliding mode control. The initial conditions are $\emptyset_0 = \theta_0 = \varphi_0 = .524$ rad and $z = 0\ m$ and references are $z_d = 1\ m, \dot{x}_d = \dot{y}_d = 0\frac{m}{s}$, and $\psi_d = 0\ rad$.

Fig. 3 show that the comparative performance of second order sliding mode control with conventional sliding mode control [10].

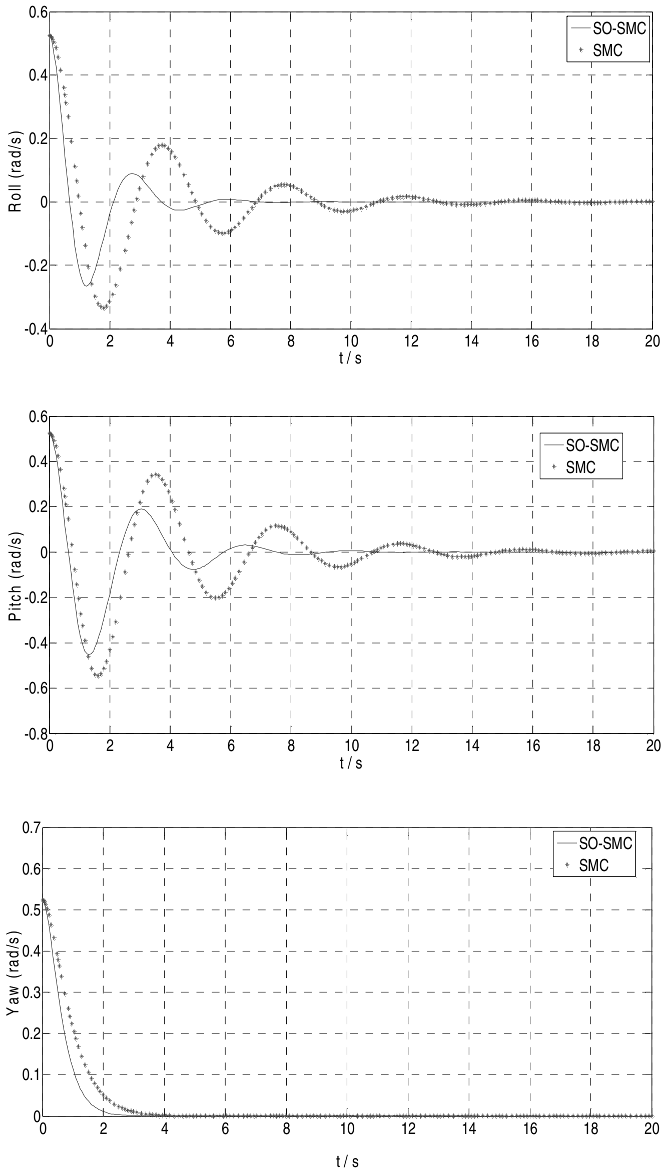

Figure 3: Comparison of proposed second order sliding mode control with conventional sliding mode control for rotational subsystem

From fig. 3, it is clear that the proposed controller presents better transient performance than the traditional sliding mode control. The positions rate and altitude rate responses of quadrotor are shown in fig. 4.

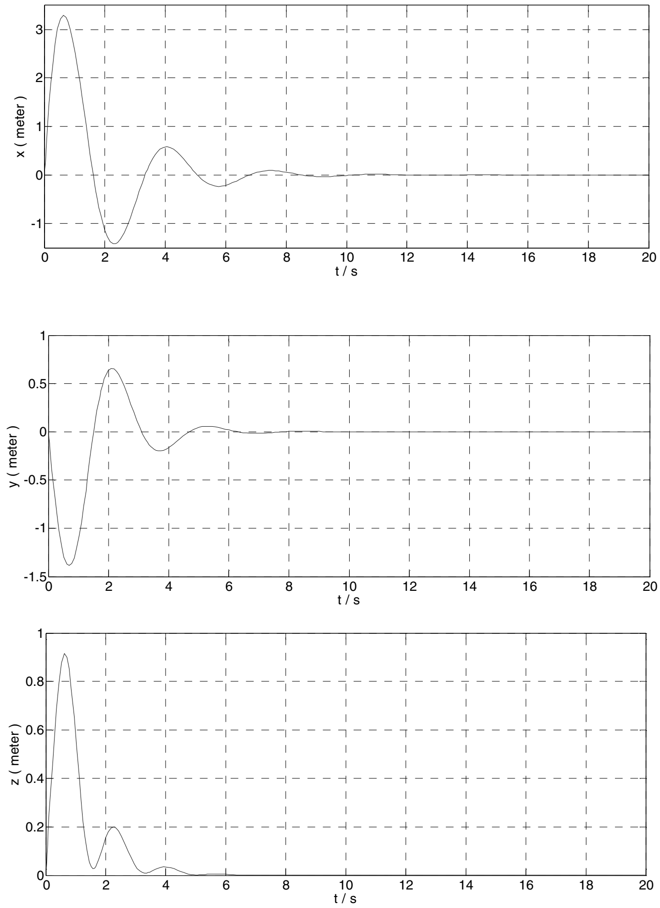

Figure 4: Position and altitude rate response of a quadrotor with second order sliding mode control

There rates quickly reaches to zero. The response of altitude subsystem is shown in fig. 5.

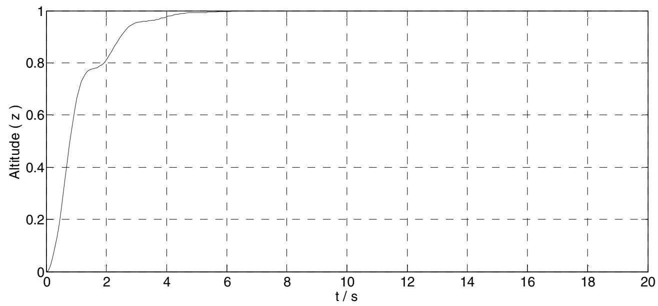

Figure 5: Altitude control of a quadrotor

The altitude reaches the desired value in 6 second. As are indicated in figs. 3-5, the proposed second order sliding mode controller is able to stabilize the attitude angles and positions of the quadrotor.

V. CONCLUSION

Second order sliding mode controller is developed in this paper for a quadrotor. Second order sliding mode controller design method is based on the Lyapunov stability theory and used to stabilize the quadrotor. The stabilization and robustness of the proposed control method has been demonstrated and compared with conventional sliding mode control [10]. From the simulation exercise it is clear that the proposed controller is able to stabilize the quadrotor. Also results show that, proposed control method has better transient performances.

References

[1]. Y.M. Younes, M.A.A. Jarrah and A.A. Jhemi, "Linear vs Nonlinear Control Technology for a Quadrotor Vehicle", Proc. of the 7th Int. Symposium on Mechatronics and its Application, Shajah, UAE, April 20-22, pp. 1-10, 2012.

[2]. S. Bouabdallah, A. Noth and R. Siegwart, "PID vs LQ Control Techniques applied to an Indoor Micro Quadrotor", Int. Conf. on Intelligent Robots and Syst., vol.3, pp. 2451 – 2456, 2004.

[3]. M. Belkheiri, A. Rabhi, A.E. Hajjaji and C. Pegard, "Different Linearization Control Techniques for a Quadrotor System", 2nd Int. Conf. on Comm., Computing and Control Applications, Marseilles, Dec 6-8, pp. 1-6, 2012.

[4]. D. Lee, H.J. Kim and S. Sastry, "Feedback Linearization vs Adaptive Sliding Mode Control for a Quadrotor Helicopter", Int. Journal of Control Autom. and Syst. vol. 7, no. 3, pp. 419-428, 2009.

[5]. H. Voos, "Nonlinear Control of a Quadrotor Micro-UAV using Feedback-Linearization", IEEE Int. Conf. on Mechatronics. Spain April 14-16, pp. 1-6, 2009.

[6]. A.R. Patel, M.A. Patel and D.R. Vyas, "Modeling and Analysis of Quadrotor Using Sliding Mode Control", 44th IEEE Southeastern Symposium on Syst. Theory, Florida, March 11-13, pp. 111-114, 2013.

[7]. M.F. Khelfi and A. Kacimi, "Robust control with sliding mode for a quadrotor unmanned aerial vehicle", IEEE Int. Symposium on Ind. Electronics (ISIE), Hangzhou, May 28-31, pp. 886-892, 2012.

[8]. I.C. Dikmen, A. Arisoy and H. Temeltas, "Attitude Control of a Quadrotor", Proc. of Recent Advances in Space Technologies, Istanbul, June 11-13, pp. 722-727, 2009.

[9]. R. Xu and U. Ozguner, "Sliding Mode Control of a Quadrotor Helicopter", Proc. of the 45th IEEE Conf. on Decision and Control, USA, Dec. 13-15, pp. 4957-4960, 2006.

[10]. K. Runcharoon and V. Srichatrapimuk, "Sliding Mode Control of Quadrotor" Int. Conf. of Technological Advances in Electrical, Electronics and Computer Eng., Konya, May 9-11, pp. 552-556, 2013.

[11]. Y. Yali, S. Feng and W. Yuanxi, "Controller Design of Quadrotor Aerial Robot", Int. Conf. on Medical Physics and Biomedical Engineering, Singapore, vol. 33, pp. 1254-1260, 2012.

[12]. X. Rong and U. Ozguner, "Sliding Mode Control of Under actuated Systems", Science Direct Automatica, vol. 44, no. 1, pp. 233-241, 2008.

[13]. A.A. Main, W. Daobo, "Modeling and Backstepping-based Nonlinear Control Strategy for a 6 DOF Quadrotor Helicopter", Chinese Journal of Aeronautics, vol. 21, no. 3, pp. 261-268, June 2008.

[14]. S. Bouabdallah and R. Siegwart, "Backstepping and Sliding-mode Techniques Applied to an Indoor Micro Quadrotor", Proceeding of IEEE Int. Conf. on Robotics and Autom. Barcelona, Spain, pp. 2259-2264, April 2005.

[15]. L. Luque-Vega, B. Castillo-Toledo and A. G. Loukianov, "Robust Block Second Order Sliding Mode Control for a Quadrotor", Journal of the Franklin Institute, vol. 349, no. 2, pp. 719–739, 2012.

[16]. H. Bouadi, S.S. Cunha, A. Drouin and F. Mora-Camino, "Adaptive Sliding Mode Control for Quadrotor Attitude Stabilization and Altitude Tracking", 12th IEEE Int. Symposium on Computational Intelligence and Informatics, Budapest, Hungary, Nov 21-22, pp. 449-455, 2011.

[17]. E.H. Zheng, J.J. Xiong and J.L. Luo, "Second Order Sliding Mode Control for a Quadrotor", ISA Transactions, vol. 53, no. 4, pp. 1350-1356, 2014.

[18]. Sudhir and A. Swarup, "Development of Backstepping Based Sliding Mode Control for a Quadrotor", Proc. IEEE 10th Int. Colloquium on Signal Processing & its Applications, Kula Lumpur, Malaysia, March 07-09, ISBN: 978-1-4799-3090-6, pp. 10-13, 2014.