Ternary Logic Machines

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Outline

Ternary Logic in Theory
Ternary Logic in Practice
Summary

Principles Of Ternary Logic
Ternary Logical Operations
Ternary Arithmetic
Why is Ternary Logic interesting?

What is Ternary Logic?

- Binary logic operates with two values: 0 and 1(or "true" and "false")
- Ternary logic adds one additional value to the mix
- This value is called -1, $\underline{1}$, "maybe" or "indeterminate"

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Ternary Logic: Electrical Engineering View

Say that we have a circuit wired to a 5V power source and to the ground.

- Binary Logic:
    - Voltage interval [GND, 5V] is divided in two parts
    - Signals > 5 mV are HIGH
    - Signals < 5 mV are LOW
    - LOW denotes 0(or "false"), HIGH denotes 1(or "true")
- Ternary Logic:
    - The interval [GND, 5V] is divided in three parts
- Other multi-valued logics are possible

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Logical Operations

Binary AND

	True	False
False	False	False
True	True	False

Ternary AND

	True	False	Maybe
False	False	False	False
True	True	False	Maybe
Maybe	Maybe	False	Maybe

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Ternary Arithmetic

Balanced Ternary is the most popular number representation:

$$\sum_{k=0}^{n} d_k * 3^k = d_n * 3^n + \ldots + d_1 * 3^1 + d_0 * 3^0$$

Example

Decimal	-4	-3	-2	-1	0	1	2	3	4
Balanced ternary	1̲1̲	1̲0	1̲1	1̲	0	1	11̲	10	11

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Advantages of Ternary Logic

- Arguably more "natural" for humans
- Balanced Ternary is in some sense "better" than Two's Complement
  - Symmetrical
  - Simpler
  - No "sign bit"
- More efficient(at least in theory)
  - Logical elements are more expensive, but less logical elements is needed

First Ternary Computer

Figure: Ternary Computer built by Thomas Fowler in 19th Century.

Research on Ternary Computing in USA

- Ternary logic was of interest to American researchers through 1950s-70s
  - Whirlpool – A military computer built at MIT
  - TERNAC – Research project by the NY University
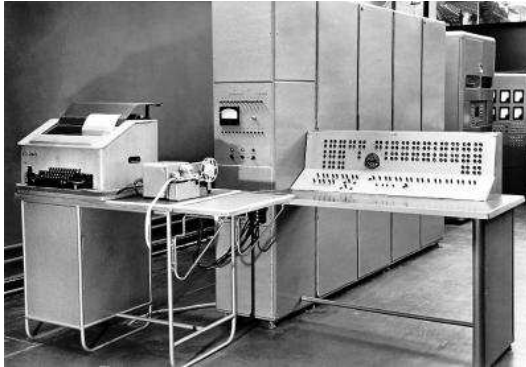- Technology never reached a practical stage.

Setun

Figure: Setun – the only general-purpose ternary architecture in history.

Setun, cont.

- Exceptionally simple architecture
- Only 24 instructions
- Three registers(two index registers and one accumulator)
- About 2 Kb of RAM and 6 Kb of permanent storage
- Used balanced ternary and had support for floating-point arithmetic

Setun, cont.

- Exceptionally reliable(according to its creator)
- Use of ternary logic was somewhat accidental
- About 50 such machines were built
- Setun 70 – improved version
- Project was not deemed perspective by the authorities
- Its creator still is a ternary logic advocate

Modern Applications of Ternary Logic

- Ternary logic is not used (obviously) in general-purpose architectures today
- But it has some specialized applications – fuzzy logic, error-correcting codes

Summary

- Ternary Logic was an interesting alternative at the dawn of digital computing.
- It still has some uses in specialized applications.
- It still is of some interest to the researchers.

Further References

- Third Base
  Brian Hayes, American Scientist vol. 89, Nov. 2001
- Boost.Tribool
  `http://www.boost.org`

Questions?

...perhaps the symmetric properties and simple arithmetic of this number system will prove quite important someday – when the "flip-flop" is replaced by "flip-flap-flop".